Lecture-3

* Linear transformation

Transformation is another word for function ($f(x)$).

In context of LA, it means something which takes input as a vector and gives output as another vector.

$$\underset{\text{Ip}}{\begin{bmatrix} 5 \\ 7 \end{bmatrix}} \quad L(\vec{v}) \quad \underset{\text{op}}{\begin{bmatrix} 2 \\ -3 \end{bmatrix}}$$

Transformation can be thought of as movement of vectors.

Properties of Linear transformation :

① All lines must remain lines without getting curved.

② Origin must remain fixed in place.

The grid lines must be parallel and evenly spaced.

Example: $\vec{v} = -1\hat{i} + 2\hat{j}$

We just need to know where $\hat{i}$ and $\hat{j}$ will land after the transformation. $\vec{V}$ will remain the same linear combination of $\hat{i}$, $\hat{j}$ even after transformation.

$\therefore \vec{V}_{New} = -1\,\hat{i}_{New} + 2\,\hat{j}_{New}$

$\hat{i} \rightarrow \begin{bmatrix}1\\-2\end{bmatrix}$ $\quad j \rightarrow \begin{bmatrix}3\\0\end{bmatrix}$

$$\begin{bmatrix}x\\y\end{bmatrix} \rightarrow x\begin{bmatrix}1\\-2\end{bmatrix} + y\begin{bmatrix}3\\0\end{bmatrix} = \begin{bmatrix}1x+3y\\-2x+0y\end{bmatrix}$$

where any vector lands

Applying ~~tran~~ 2-D linear transformation:

Where 1st basis vector / $\hat{i}$ lands

Where 2nd basis vector / $\hat{j}$ lands

~~Ass~~ vector to be transformed

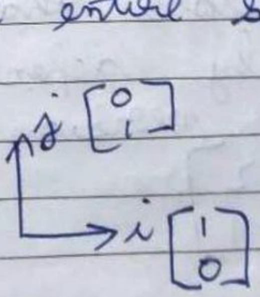
$$\begin{bmatrix}a & c\\b & d\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}$$

$$= x\begin{bmatrix}a\\b\end{bmatrix} + y\begin{bmatrix}c\\d\end{bmatrix}$$

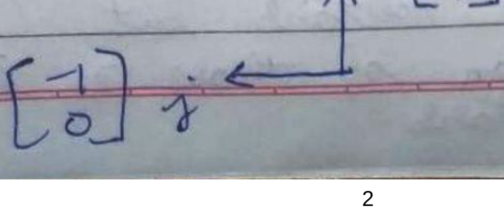
$$= \begin{bmatrix}ax+cy\\bx+dy\end{bmatrix}$$

Ex:

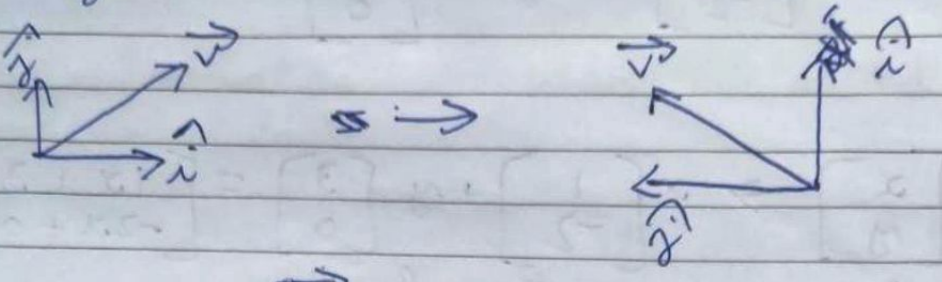
If we rotate entire space 90° anti-clockwise.



becomes



∴ Any vector $\vec{v}$ will be transformed as



$$\vec{v} \downarrow$$

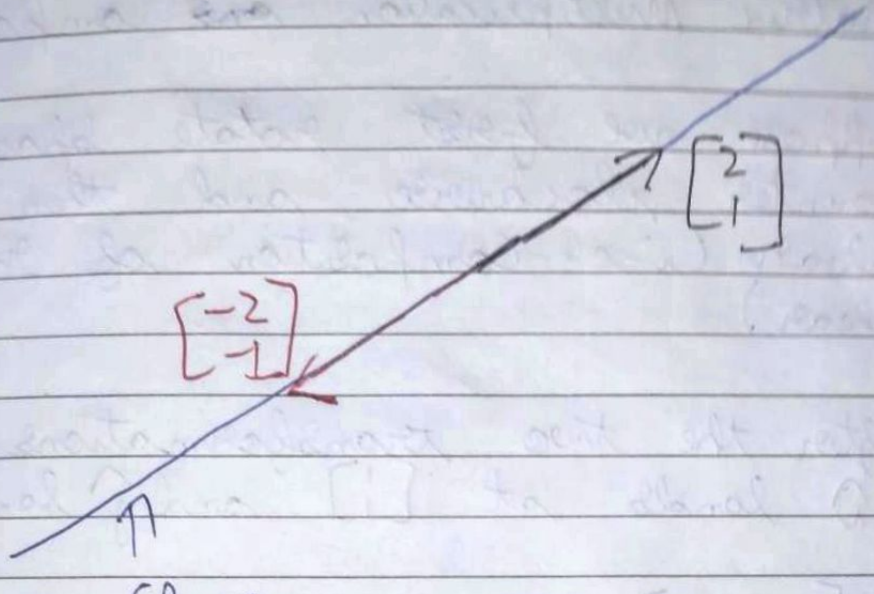
$$\begin{bmatrix}0 & -1\\1 & 0\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}$$

* Shear transformation
$\hat{i}$ remains fixed at $\begin{bmatrix}1\\0\end{bmatrix}$ and $\hat{j}$ changes from $\begin{bmatrix}0\\1\end{bmatrix}$ to $\begin{bmatrix}1\\1\end{bmatrix}$

$$\begin{bmatrix}1 & 1\\0 & 1\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}$$

* If the vectors that $\hat{i}$ and $\hat{j}$ land on are linearly dependent (i.e. one is a scaled version of the other),
Ex: $\begin{bmatrix}2 & -2\\1 & -1\end{bmatrix}$

It means the linear transformation squishes the space onto the line where those vectors sit. (Also known as 1-D span of those 2 linearly dependent vectors.



Span

Linear Transformations are a way to move space.

A matrix can be interpreted as a certain transformation of space.