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Lecture -15

* A quick trick for computing eigenvalues.

① $\frac{1}{2} \overset{\text{trace}}{tr}\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = \frac{a+d}{2} = \frac{\lambda_1 + \lambda_2}{2} = m$ (mean)

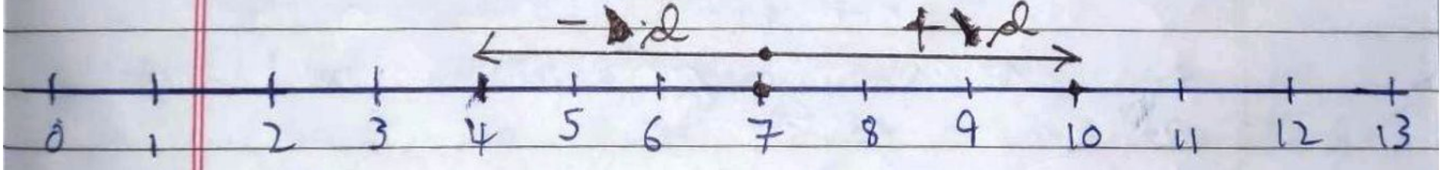
② $\det\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = ad - bc = \lambda_1 \lambda_2 = p$
(product)

③ Ex: Find eigen values of $\begin{bmatrix} 8 & 4 \\ 2 & 6 \end{bmatrix}$.

$m = \frac{8+6}{2} = 7$

$p = 48 - 8 = 40$

Now having a mean of 7 means both the eigen values are evenly spaced around 7.



Also product $= 40$

$\therefore 40 = (7+d)(7-d)$

$\therefore 40 = 7^2 - d^2$

$\therefore d^2 = 9$

$\therefore d = 3$

$\therefore \cancel{\lambda_1 = 7+3} \; \cancel{\lambda_1 = -3}, \; \cancel{\lambda_2 = 8}$

$\therefore \lambda_1 = 7 + d = 10, \; \lambda_2 = 7 - d = 4$

General formula

$$d^2 = m^2 - p$$

$\therefore \boxed{d = \pm\sqrt{m^2 - p}}$ $\quad \therefore \boxed{\lambda_1, \lambda_2 = m \pm \sqrt{m^2 - p}}$

Where m = mean, p = product, d = distance and λ_1, λ_2 = eigen values

$$\begin{bmatrix} 2 & 7 \\ 1 & 8 \end{bmatrix}$$

$m = (2+8)/2 = 5, \quad p = 16 - 7 = 9$

$\therefore \lambda_1 = 5 + \sqrt{25 - 9}, \quad \lambda_2 = 5 - \sqrt{25 - 9}$

$\therefore \lambda_1 = 5 + 4, \quad \lambda_2 = 5 - 4$

$\therefore \lambda_1 = 9, 1$