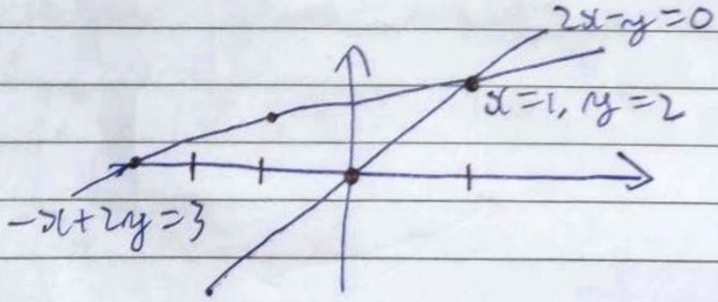
MIT 18.06
Prof Gilbert Strang
Lect 1

$2x - y = 0$
$-x + 2y = 3$

→ Matrix form: $AX = B$

$$\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \end{bmatrix}$$

→ Row Picture:
The row pictures mean each row is seen individually.
The row picture of 2x2 matrix is a line in a 2D plane.

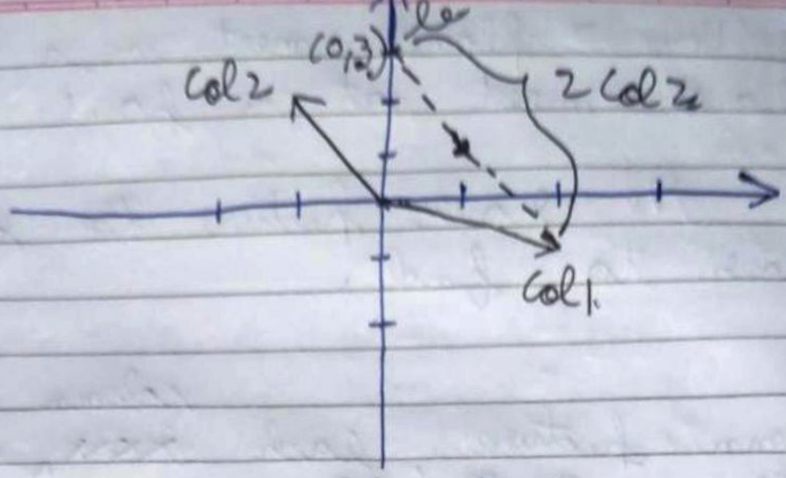


→ Column picture:
It means each column is taken, one at a time.
Each column acts as a vector in 2D space

$$x\begin{bmatrix} 2 \\ -1 \end{bmatrix} + y\begin{bmatrix} -1 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \end{bmatrix}$$

Linear combination of columns

$x = 1, y = 2$



We we take any x and y, then the diff values of x and y will fill the whole plane.

→ For 3D,

$2x - y = 0$

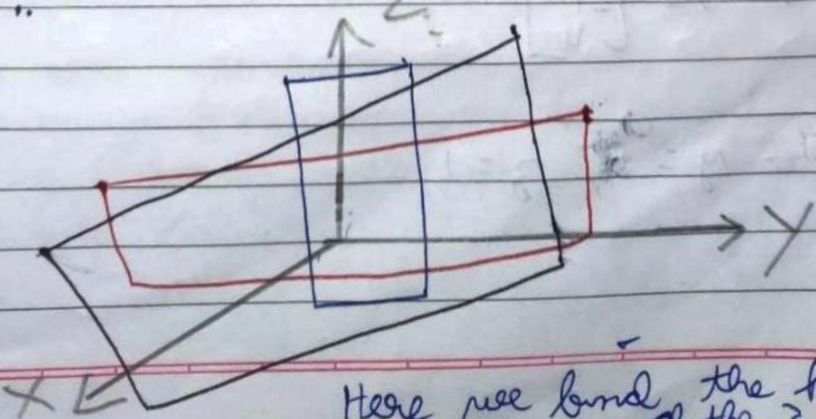
$-x + 2y - z = -1$

$-3y + 4z = 4$

Matrix form: $AX = B$

$$\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -3 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \\ 4 \end{bmatrix}$$

→ In row picture, here each row is a plane..



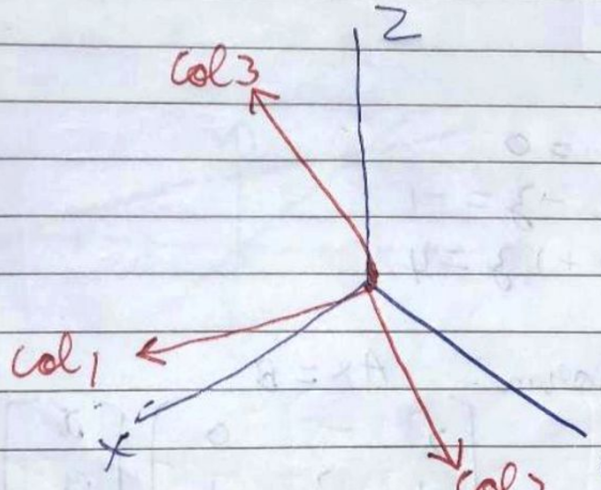
Here we find the pt of intersection of the 3 planes..

Note: Here 2 planes meet in a line. But 3 planes meet at a point.

Here the row picture is getting harder to find.

→ Column picture, each column is a vector in 3D space.

$$x\begin{bmatrix}2\\-1\\0\end{bmatrix}+y\begin{bmatrix}-1\\2\\-3\end{bmatrix}+z\begin{bmatrix}0\\-1\\4\end{bmatrix}=\begin{bmatrix}0\\-1\\4\end{bmatrix}$$



Here we find what linear combination of the 3 columns are required to produce $\begin{bmatrix}0\\-1\\4\end{bmatrix}$.

$x = y = 0, \; z = 1$

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→ Can we solve $Ax = b$ for ~~every~~ every B ?

OR

Do the linear combs of the columns fill 3-D space ?

For this A given above, yes.
∵ A is a non-singular i.e. invertible matrix.

So, when will the answer be no ?
→ When all the 3 ~~vector~~ column vectors lie in the same plane. i.e. Two matrices add up to give the 3rd one.

In such a case, the only B we will be getting will be those lying in the plane.
Here the matrix A will be singular i.e. non-invertible matrix.

→ Now imagine 9 eqs and 9 unknowns.
That means we have 9 column vectors.
If each of these column vectors are independent of each other, then we can fill the 9D space.

But if, for ex, the last column is the same as the 2nd last column, then we cannot span the whole 9D space. Instead we would get a 8D plane in 9D space.

$Ax = b$

Here 'A' is a matrix and 'x' is a vector.

$$\begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

How to multiply a matrix and a vector?

Method I : ~~Multif~~ ~~A~~ Multiply a column at a time.

$$\begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \underbrace{1 \begin{bmatrix} 2 \\ 1 \end{bmatrix} + 2 \begin{bmatrix} 5 \\ 3 \end{bmatrix}}^{\text{LC of columns}} = \begin{bmatrix} 12 \\ 7 \end{bmatrix}$$

Method II : Take one row at a time. This method is the idea of the dot product. It's the std method we use

$$\begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 2 \times 1 + 5 \times 2 \\ 1 \times 1 + 3 \times 2 \end{bmatrix}$$

$$= \begin{bmatrix} 12 \\ 7 \end{bmatrix}$$

Ax is a combination of columns of 'A'.