Lecture - 4

* Matrix Multiplication ~~and~~ as Composition

Suppose we first rotate space by 90° counter clockwise and then apply shear (i.e. Composition of rotation and shear)

After the two transformations,
$\hat{i}$ lands at $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ and $\hat{j}$ lands at $\begin{bmatrix} -1 \\ 0 \end{bmatrix}$

$\therefore \begin{bmatrix} 1 & -1 \\ 1 & 0 \end{bmatrix}$.

This matrix captures both rotation and shear but as a single action rather than 2 separate actions.

$$\underbrace{\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}}_{\text{Shear}} \left(\underbrace{\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}}_{\text{Rotation}} \begin{bmatrix} x \\ y \end{bmatrix} \right) = \underbrace{\begin{bmatrix} 1 & -1 \\ 1 & 0 \end{bmatrix}}_{\text{Composition}} \begin{bmatrix} x \\ y \end{bmatrix}$$

Read right to left

$$\therefore \underbrace{\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}}_{\text{Product}} = \begin{bmatrix} 1 & -1 \\ 1 & 0 \end{bmatrix}$$

The product of two matrices means applying two transformations. First transformation is due to the matrix on the right and second transformation is due to the matrix on the left.

This habit of right to left stems from functions $f(g(x))$ which are read from right to left.

$$\underbrace{\begin{bmatrix} a & b \\ c & d \end{bmatrix}}_{M_2} \underbrace{\begin{bmatrix} e & f \\ g & h \end{bmatrix}}_{M_1} = \begin{bmatrix} ? & ? \\ ? & ? \end{bmatrix}$$

$\begin{bmatrix} e \\ g \end{bmatrix}$ denotes the new position of $\hat{i}$ after the 1st transformation.

$$\therefore \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} e \\ g \end{bmatrix} = e \begin{bmatrix} a \\ c \end{bmatrix} + g \begin{bmatrix} b \\ d \end{bmatrix}$$

$$= \begin{bmatrix} ae + bg \\ ce + dg \end{bmatrix}$$

This denotes the position of $\hat{i}$ after 2nd transformation.

Same logic would be used for $\hat{j}$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} f \\ h \end{bmatrix} = f \begin{bmatrix} a \\ c \end{bmatrix} + h \begin{bmatrix} b \\ d \end{bmatrix} = \begin{bmatrix} af + bh \\ cf + dh \end{bmatrix}$$

$$\therefore \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} ae+bg & af+bh \\ ce+dg & cf+dh \end{bmatrix}$$

∴ In matrix multiplication, generally,
$AB \neq BA$

∵ If we first apply rotation then shear and if we apply shear, then rotation, the final position of $\hat{i}$ and $\hat{j}$ are different for both cases.

Also, Matrix multiplication is associative
i.e. $(AB)C = A(BC)$

Because it doesn't matter which two we multiply first as long as we maintain order since ultimately we are just applying 3 transformations.