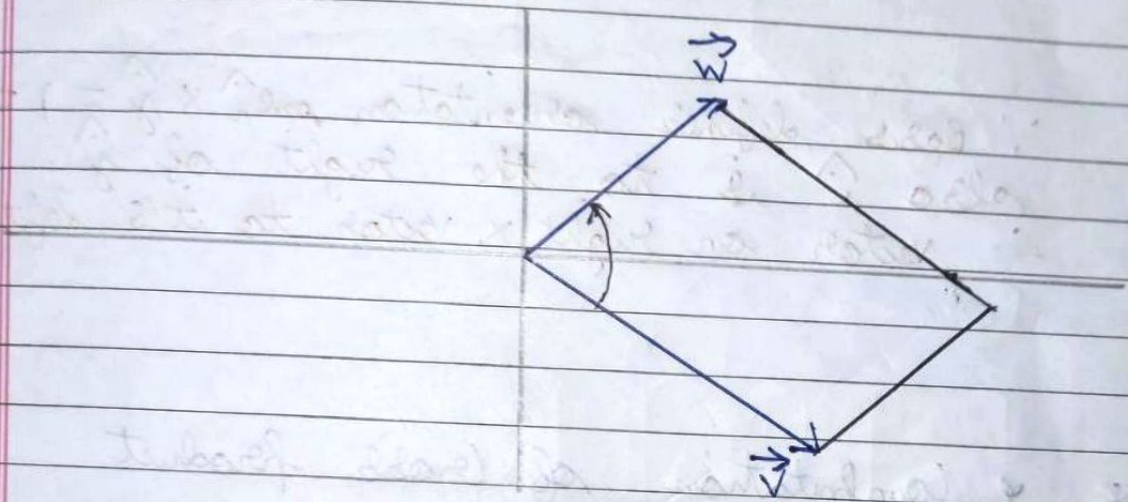
Lecture - 10

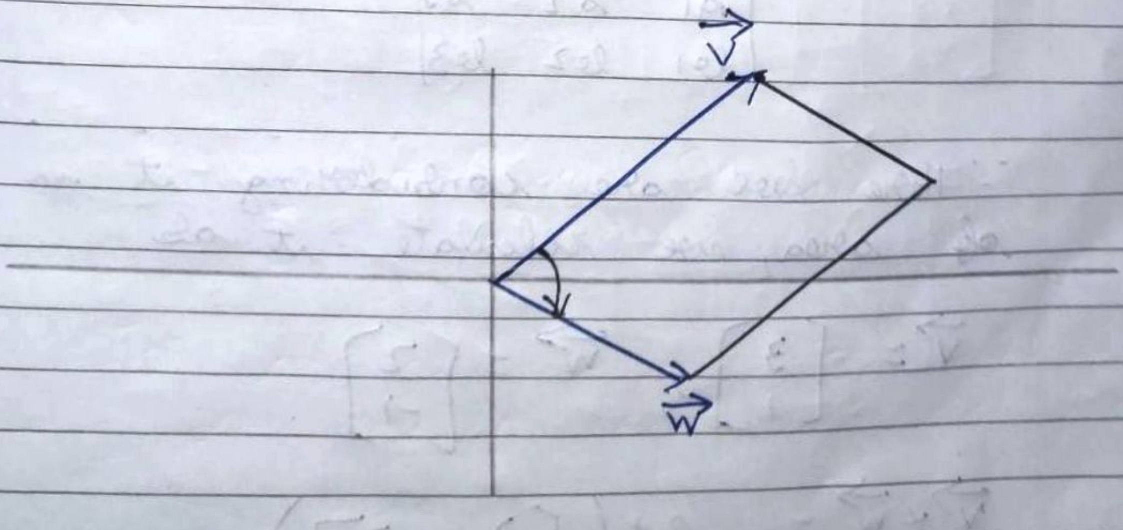
* Cross Products

→ Case 1: If $\vec{V}$ is to the right of $\vec{W}$, area of parallelogram is +ve



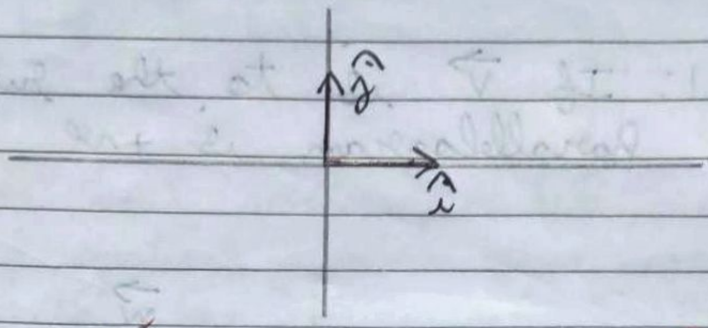
$\vec{V} \times \vec{W}$ = Area of parallelogram = +ve

→ Case 2: If $\vec{V}$ is to the left of $\vec{W}$, area of parallelogram is -ve



$\vec{V} \times \vec{W}$ = -Area of parallelogram = -ve

Note: How to remember it?



∵ Basis defines orientation and $\hat{i} \times \hat{j} = 1 = +ve$, also $\hat{i}$ is to the right of $\hat{j}$.
∴ Vector on right × vector to it's left = +ve

* Computation of Cross product

Generally,

If $\vec{V} = a_1 \hat{i} + a_2 \hat{j} + a_3 \hat{k}$

$\vec{W} = b_1 \hat{i} + b_2 \hat{j} + b_3 \hat{k}$

$$\vec{V} \times \vec{W} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

∵ Here we are considering it in terms of area, we calculate it as

$$\vec{V} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}, \quad \vec{W} = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$$

$$\vec{V} \times \vec{W} = \det\left(\begin{bmatrix} 3 & 2 \\ 1 & -1 \end{bmatrix}\right)$$

Now $\begin{bmatrix} 3 & 2 \\ 1 & -1 \end{bmatrix}$ is a transformation and determinant ~~so~~ is related to areas.

∴ The unit square made by $\hat{i}$ and $\hat{j}$, after transformation becomes a parallelogram.
∴ The initial area of 1, the determinant which gives us factor by which area has changed, gives us are of parallelogram in this case.

→ More perpendicular $\vec{v}$ and $\vec{w}$, larger is the value of $\vec{v} \times \vec{w}$.

→ $(3\vec{v}) \times \vec{w} = 3(\vec{v} \times \vec{w})$

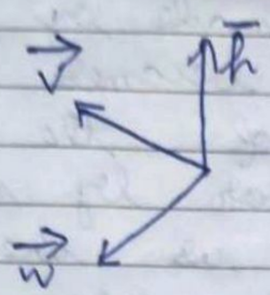
* Actually the cross product of two vectors (3-D vectors) gives us a 3-D vector whose length is equal ~~of~~ to the area ~~formed of~~ the parallelogram formed by the cross product of those two vectors.

Also, the resultant vector will be formed perpendicular to the plane parallelogram ~~found~~.

But ⊥ on which side of the parallelogram?
Ans: Use Right Hand Rule

Point the fore finger ~~so~~ of right hand in direction of $\vec{v}$ and point middle finger in direction of $\vec{w}$, then the thumb goes direction of cross product.

$$\vec{v} \times \vec{w} = \bar{h}$$



Formula:

$$\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} \times \begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix} = \begin{bmatrix} v_2 \cdot w_3 - w_2 \cdot v_3 \\ v_3 \cdot w_1 - w_3 \cdot v_1 \\ v_1 \cdot w_2 - w_1 \cdot v_2 \end{bmatrix}$$

OR

$$\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} \times \begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix} = \det\left(\begin{bmatrix} \hat{i} & v_1 & w_1 \\ \hat{j} & v_2 & w_2 \\ \hat{k} & v_3 & w_3 \end{bmatrix} \right)$$

$$= \hat{i}\,(v_2 w_3 - v_3 w_2) + \hat{j}\,(v_3 w_1 - v_1 w_3) + \hat{k}\,(v_1 w_2 - v_2 w_1)$$

Some Number