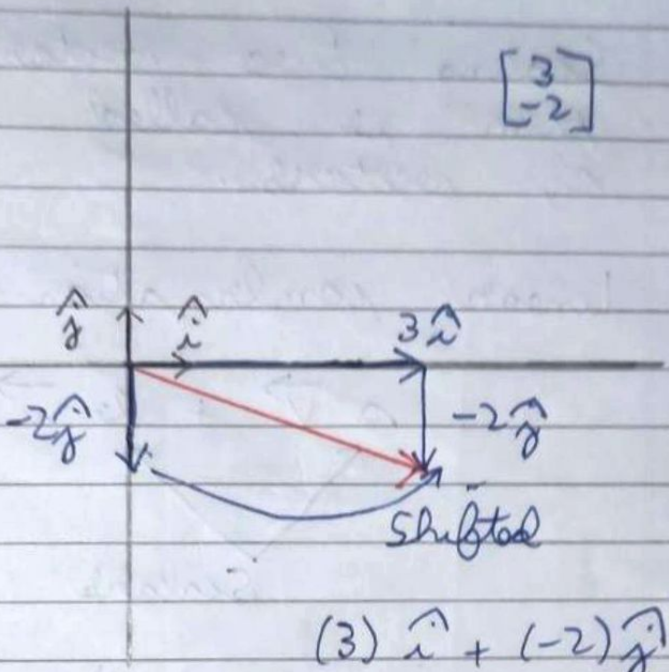
Lecture - 2

* Basis :

$\hat{i}$ - Unit vector in X direction
$\hat{j}$ - Unit vector in Y direction



$(3)\hat{i} + (-2)\hat{j}$

$\hat{i}$ and $\hat{j}$ are the "basis vectors" of the xy coordinate system.

The coordinates of vectors ~~scale~~ scale the basis vectors.

Note: If we choose the basis vectors different from $\hat{i}$ and $\hat{j}$, then too we can still reach every pair of vectors in the 2-D space using scalars.

* Linear Combination of Vectors

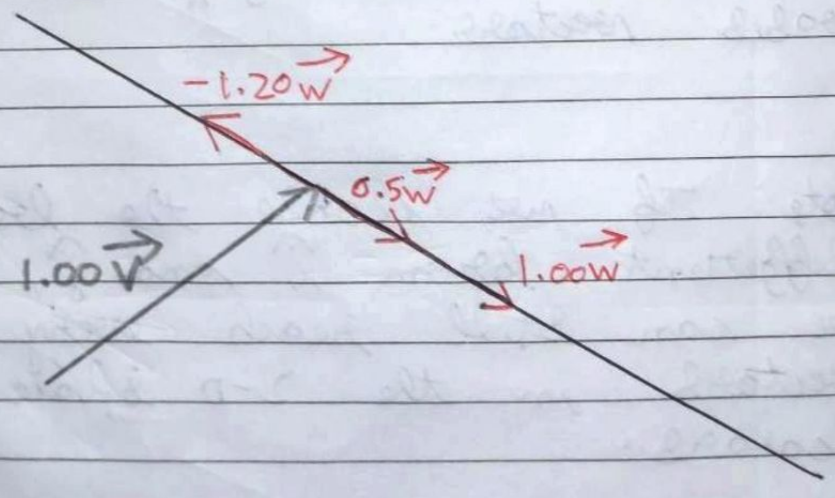
Scaling two vectors and adding them is called linear combination of vectors.

Linear combination of $\vec{v}$ and $\vec{w}$

$$\underbrace{a}_{\text{Scalars}}\vec{v} + \underbrace{b}_{\text{Scalars}}\vec{w}$$

Why is it a "linear" combination?

Ans: Because if you fix the scalar of one vector and let the other vector scale freely, you get a straight line.



* Span:

If we let both scalars range freely, then for most cases, we can reach every point in the plane. i.e. Every 2-D vector is within our reach.

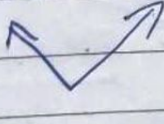
But if our two vectors overlap each other, the tip of the resulting vector is limited to a single line passing through the origin.

If both our initial vectors are zero, then we'll be stuck at the origin.

The set of all possible vectors that one can reach with a linear combination of given pair of vectors is called the span of those two vectors.

The "span" of $\vec{v}$ and $\vec{w}$ is the set of all their linear combinations:

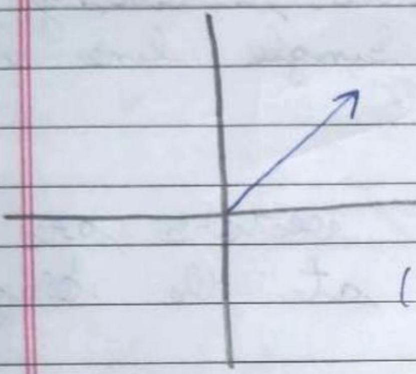
$$a\vec{v} + b\vec{w}$$



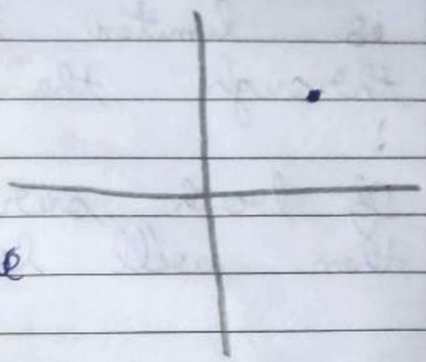
let a and b vary over all real numbers

* Vectors vs Points

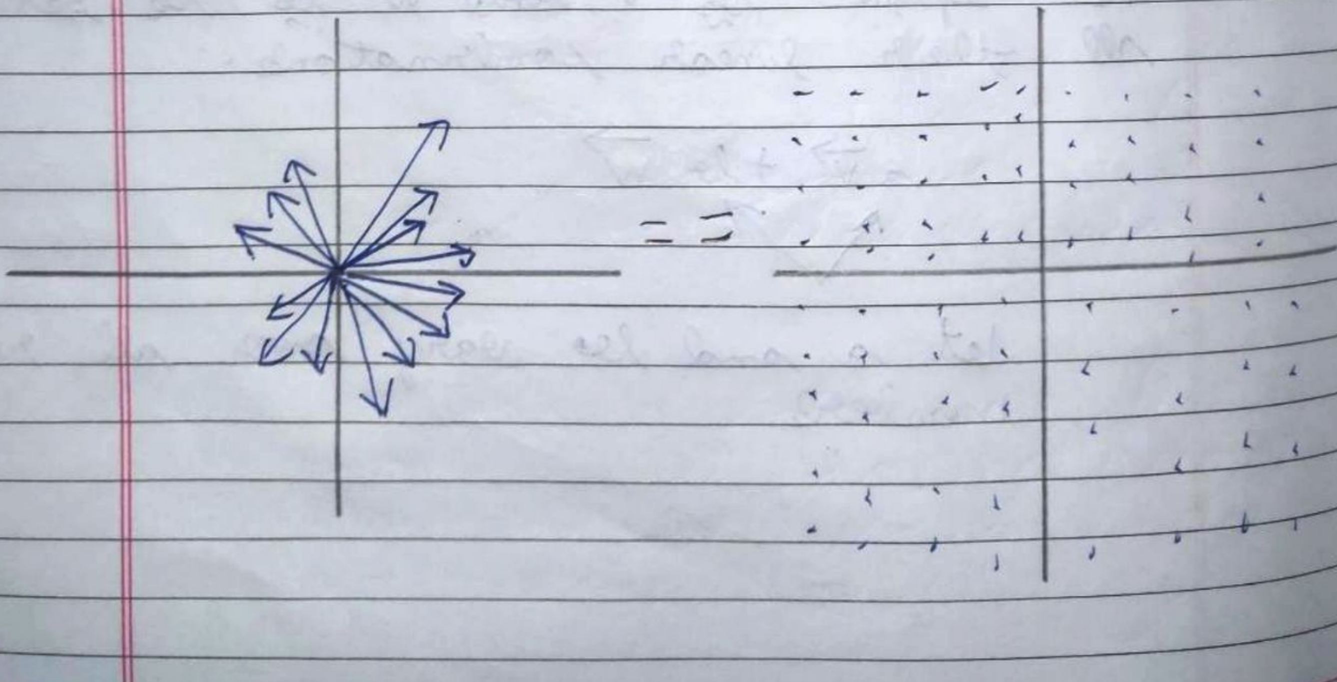
When we deal with collection of vectors, it's important to treat them as points



is same as (Tip is the point)



For most pair of vectors, their span is the entire 2-D sheet of space



But if the two initial vectors line up, their span is just a line

* For 2 vectors in 3-D space,

The linear combination of two vectors in 3-D space is called forming a plane

We scale the two vectors and add them. The tip of the resultant vector traces a flat sheet passing through origin in 3-D space.

* For 3 vectors in 3-D space,

Linear combination of $\vec{v}$, $\vec{w}$ and $\vec{u}$:

$$a\vec{v} + b\vec{w} + c\vec{u}$$

The span of these vectors is the set of all possible linear combinations of these vectors

$$a\vec{v} + b\vec{w} + c\vec{u}$$

For span we let the constants a, b, c vary.

Case 1 : If the 3rd vector happens to be sitting on the span of first two, then the span doesn't change. It is still that same flat sheet. Adding a scaled version of the 3rd vector to 1st two doesn't really give access to any new vector.

Case 2 : But in most cases, the 3rd vector won't sit on the span of the first two. This gives access to every possible 3-D vector. As we move the 3rd scaled vector, it moves that sheet through space.

When we have a vector such that if we remove it, it does not change the span then it's called linearly dependent vectors.

i.e. one of the vectors can be expressed as the ~~span~~ linear combination of the others since it's just sitting in the span of the other two.

$\vec{u} = a\vec{v} + b\vec{w}$ For all values of a and b

If each vector adds an another dimension to the span, they are said to be linearly independent.

$\vec{u} \neq a\vec{w} + b\vec{w}$ For all values of a and b

* Technical definition of basis:

The basis of a vector space is a set of linearly independent vectors that span the full space.