EECE5554 Robotics Sensing and Navigation

Lab 3: IMU Noise Characterization with Allan Variance

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Link to 5 hour dataset:
$https://drive.google.com/drive/folders/1nwLgW-H63xkK9kq_prT2830t6kOU83u?usp=drive_link$

1. Intoduction

The lab had two main data collection time intervals for analysis: 15 minutes and 5 hours. I have analyzed the data obtained from the VN-100 IMU below that was published by my "imu_driver.py". Also, the link to the 5 hour dataset is given here: $https://drive.google.com/drive/folders/1nwLgW-H63xkK9kq_prT2830t6kOU83u?usp=drive_link$. There are a few important things to note. I have published the angular X, Y, and Z velocities in rad/s as given by the VN-100 and required by the "sensor_msgs/Imu.msg". But for the ease of visualization, I have converted them to deg/s for the analysis part. Similarly, the data for the X, Y, and Z magnetic fields is published in Tesla (T) as required by the "sensor_msgs/MagneticField". But for convenience, I have converted them to Micro Tesla (μT) for analysis at certain places. Also, the Quaternions published by the driver have been converted into Euler angles for the analysis utilizing the formulae:

$$yaw = atan2(2(q_w * q_z + q_x * q_y), 1 - 2(q_y^2 + q_z^2)) \quad (1)$$

$$pitch = asin(2(q_w * q_y - q_z * q_x)) \quad (2)$$

$$roll = atan2(2(q_w * q_x + q_y * q_z), 1 - 2(q_x^2 + q_y^2)) \quad (3)$$

The above equations give the Euler angles in radians. I have further converted them into degrees for ease of analysis.

2. Data collected for short time interval

I collected the IMU data for about 15 minutes (917.400 seconds to be precise).

2.1. Analysis of Time Series Plots

As shown in Figures 1, 2, 3, and 4, I have plotted a time series graph of the orientations, the angular velocity, linear acceleration,and magnetometer data, respectively. The Euler angles and the accelerometer data remain stable throughout the entire time duration. The accelerometer data about the Z-axis is close to -9.81 m/s^2, which indicates the effect of the gravitational force on it. As for the gyroscope data, it appears to be affected by a very small amount of noise, causing deviations of -0.2 to 0.2 deg/s for the Y-axis and of -0.1 to 0.1 deg/s for the X and Z axes. The distribution appears to be centered around zero with very low variance. As far as the magnetometer is concerned, there is a certain magnetic field affecting all three axes. It causes small fluctuations in the magnetometer's X and Y axes while a bit more fluctuation in it's Z-axis.

2.2. Analysis based on Mean and Standard Deviation

The mean values (degrees) obtained for yaw, pitch, and roll are approximately 0.775399, 0.000309, and -0.002696, respectively, whereas the standard deviation values (degrees) obtained for yaw, pitch, and roll are 0.001060, 0.000136, and 0.000118, respectively. The specs for VN-100 state a 0.5 degree pitch/roll accuracy. As far as pitch and roll are concerned, their mean values are close to zero, indicating minimal bias or drift. Their standard deviation values also fit perfectly within the specified error margin of 0.5 degrees, indicating that the sensor is not affected much by noise. The mean value for yaw indicates there might be some bias or drift due to noise on that axis, but it's value for standard deviation indicates that it's relatively stable and the Extended Kalman Filter in VN-100 is working effectively to reduce that noise.

The mean values (deg/s) obtained for the gyroscope's X, Y, and Z axes are -0.00124606, -0.00139934, and -0.00206343, respectively, whereas the standard deviation values (deg/s) obtained for the gyroscope's X, Y, and Z axes are 0.03942763, 0.04903864, and 0.03702651, respectively. The value of the "Gyro In-Run Bias" specified in the specs of VN-100 is 5 deg/hr which approximates to 0.00139 deg/s. In-run bias is the systematic error or offset that accumulates over time due to imperfections in the gyroscope. The mean value of the gyroscope data indicates the central tendency or average of the readings. If there's a bias, it will show up in the mean because the sensor consistently reads an offset from zero. The mean values for the axes of the gyroscope are

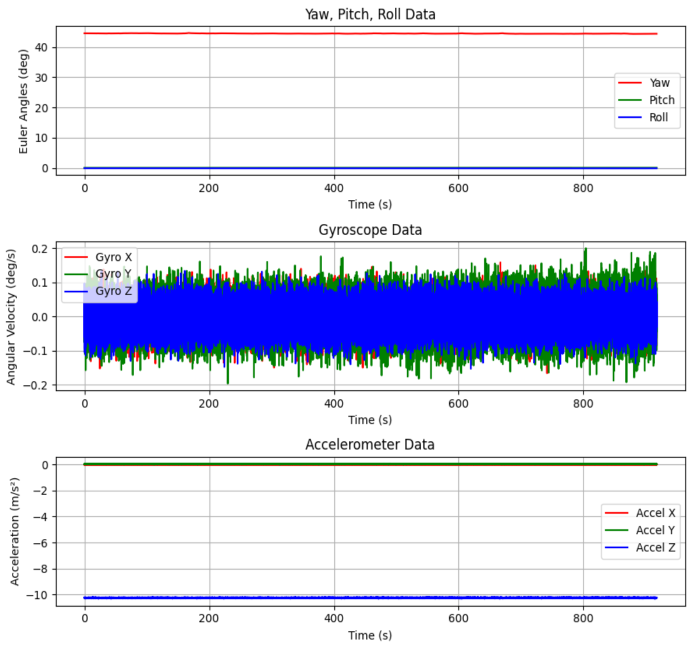

Figure 1: Time series plot for the orientation, the three gyroscope axes, and the three accelerometer axes.

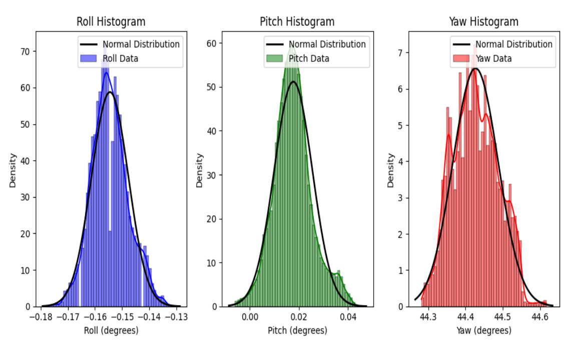

Figure 2: Histogram for the roll, pitch, and yaw data.

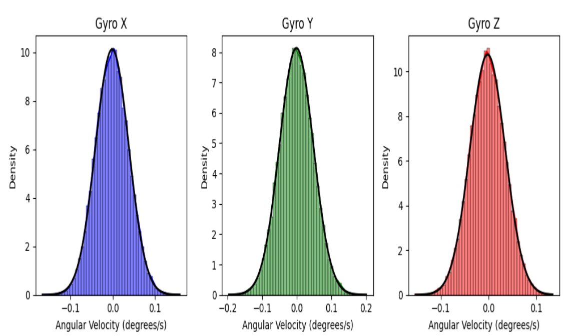

Figure 3: Histogram for the Gyroscope's X, Y, and Z axes.

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quite close to the specified In-Run Bias, indicating that the gyroscope is performing well. The noise density for the gyroscope as per the specs of the VN-100 is 0.0035 $deg/s/\sqrt{Hz}$. The standard deviations of the axes are a little above this, but are well within the acceptable limit.

The mean values (m/s^2) obtained for the accelerometer's X, Y, and Z axes are 0.00350132, 0.02728912, and -10.24579546, respectively, whereas the standard deviation values (m/s^2) obtained for the gyroscope's X, Y, and Z axes are 0.01231397, 0.0116576, and 0.01721513, respectively. The In-Run Bias Stability of the accelerometer is specified as $<$ 0.04 mg (equivalent to $<$ 0.000392 m/s^2) in the VN-100 specs. The mean values for the X and Y axes are relatively small, but higher than the expected in-run bias stability. However, these values are still reasonable and could be due to environmental factors. The mean value for the Z-axis is -10.24579546 m/s^2 which is close to -9.81 m/s^2, indicating the effect of earth's gravity on it. The VN-100's accelerometer has a noise density of 0.14 $mg/\sqrt{Hz}$ (0.001372 $m/s/\sqrt{Hz}$), which refers to the random noise in the signal per root Hertz of bandwidth. The standard deviation values of the three axes are a bit higher, indicating the presence of a small amount of noise.

The mean values (T) obtained for the magnetometer's X, Y, and Z axes are $1.59734665 * 10^{-5}$, $-1.57253187 * 10^{-5}$, and $1.86796861 * 10^{-5}$, respectively, whereas the standard deviation values (T) obtained for the magnetometer's X, Y, and Z axes are $1.43781846 * 10^{-7}$, $2.80602093 * 10^{-7}$, and $4.02154124 * 10^{-7}$, respectively. There are no specs provided for the In-Run Bias of the VN-100's magnetometer. However, there are no significant drifts in the data that would indicate poor in-run bias stability. The noise density of the VN-100 according to it's specs is 140 μGauss/$\sqrt{Hz}$, or 0.014 μT/$\sqrt{Hz}$. The measured noise as indicated by the standard deviations appears to be small.

2.3. Analysis of the distributions based on the Histogram Plots

As shown in Figure 5, the distributions for roll, pitch, and yaw somewhat resemble a Gaussian distribution, but have a certain amount of deviation from it. This might me due the presence of the Extended Kalman Filter in the VN-100. From Figure 6, the Gyro X, Y, and Z histograms also appear Gaussian, which is expected when the IMU is stationary. The tightness of the distribution implies that the gyroscope noise is low and stable. The histograms for Accel X and Z are very tightly clustered around zero and -10.25 m/s^2, respectively, as seen in Figure 7, indicating low drift and high stability, whereas Accel Y has more spread, with a slight shift in the mean, suggesting slightly higher noise or bias in that axis. For the magnetometer, in Figure 8, the distribution of the X-axis magnetic field has several sharp peaks instead of a smooth Gaussian shape. The Y-axis magnetic field distribution also shows some peaks. The Z-axis distribution has the most Gaussian-like shape among the three axes, though it still shows some sharp peaks.

3. Data collected for long time interval

I collected the IMU data for approximately 5.28 hours. The Allan Deviation plots can be used to identify various noise sources present in stationary gyroscope and accelerometer's measurements.

Consider L samples of data from a gyroscope with a sample time of τ_0. The Allan variance is computed using the formula,

$$\sigma^2(\tau) = \frac{1}{2\tau^2(L-2m)} \sum_{k=1}^{L-2m} (\theta_{k+2m} - 2\theta_{k+m} + \theta_k)^2 \tag{4}$$

where τ = mτ_0, $<>$ is the ensemble average, and θ is the output angle for each discrete sample obtained by calculating the product of the cumulative sum of the angular velocity about an axis (Ω) and the sample time (τ_0). For accelerometer, there's no need to calculate the cumulative sum; we need to calculate Allan variance directly on the acceleration data. The Allan Deviation $\sigma(t)$ is given by:

$$\sigma(t) = \sqrt{\sigma^2(t)} \tag{5}$$

3.1. Sources of noise in the data

I found several sources of noise in the stationary IMU data, such as “Angle Random Walk“ for gyroscope and “Velocity Random Walk“ for accelerometer, “Rate Random Walk“, and “Bias Instability“. Figures 9 and 10 show the Allan deviation plots along with the noise parameters.

3.2. Modeling of the sources of noise

To determine the noise parameters for the gyroscope, the equation involving the Allan variance and the two-sided power spectral density (PSD) of the noise components in the original data set Ω given below

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can be used.

$$\sigma^2(\tau) = 4\int_0^\infty S_\Omega(f)\frac{sin^4(\pi f\tau)}{(\pi f\tau)^2}df \tag{6}$$

Angle Random Walk (ARW)/ Velocity Random Walk (VRW) is modeled as white noise, and it is characterized by the random fluctuations in the sensor output over short time intervals. The ARW/VRW coefficient N is used in the model. The units of N are $(deg/\sqrt{s})$ for the gyroscope and $(m/s)/\sqrt{s}$ for the accelerometer. The PSD given below

$$S_\Omega(f) = N^2 \tag{7}$$

is substituted in eq 6 and integrated to obtain the following equation:

$$\sigma^2(\tau) = \frac{N^2}{\tau} \tag{8}$$

Rate Random Walk (RRW) models the red noise (Brownian noise) spectrum of the gyroscope output. The RRW coefficient K is used in the model. The units of K are $(deg/s)\sqrt{Hz}$ for the gyroscope and $(m/s^2)\sqrt{Hz}$ for the accelerometer. The PSD below

$$S_\Omega(f) = (\frac{K}{2\pi})^2\frac{1}{f^2} \tag{9}$$

is substituted in eq 6 and integrated to obtain the following equation:

$$\sigma^2(\tau) = \frac{K^2\tau}{3} \tag{10}$$

The bias instability is characterized by the pink noise spectrum of the gyroscope output. The bias instability coefficient B is used in the model. The units of B are (deg/s) for the gyroscope and (m/s^2) for the accelerometer. The PSD given below

$$S_\Omega(f) = \begin{cases} (\frac{B^2}{2\pi})\frac{1}{f} : f \leq f_0 \\ 0 : f > f_0 \end{cases} \tag{11}$$

where f_0 = cut-off frequency, is substituted in eq 6 and integrated to obtain the following equation:

$$\sigma^2(\tau) = \frac{2B^2}{\pi}[\ln 2 + \frac{sin^3 x}{2x^2}(sinx + 4xcosx) + Ci(2x) - Ci(4x)] \tag{12}$$

where $x = \pi f_0 \tau$ and Ci = cosine-integral function

3.3. Measuring these errors

From the Allan deviation plots, we can extract these parameters at different time intervals.

Angle/Velocity Random Walk is measured from the slope at shorter time intervals (τ) on the Allan deviation plot. In the log-log plot, N is the point where the slope is -0.5. The value of ‘N‘ can be read directly off of this line at $\tau = 1$. Rate Random Walk is extracted from the slope at intermediate time intervals, where the slope of the Allan deviation is +0.5. The value of ‘K‘ can be read directly off of this line at $\tau = 3$. The flat portion of the Allan deviation curve at long time intervals corresponds to the Bias Instability. The minimum value in this region represents the bias instability. The value of ‘B‘ can be read directly off of this line with a scaling of $\sqrt{\frac{2\ln 2}{\pi}} \approx 0.664$.

3.4. Relating these errors with the VN-100 data sheet

The values of N $(deg/\sqrt{s})$, K$(deg/s/\sqrt{Hz})$, and B (deg/s) obtained from the IMU's gyroscope data for the X-axis are 0.0062984, 0.000045306, and 0.0010662, for the Y-axis are 0.019884, 0.000019182, and 0.00081526, and for the Z-axis are 0.0061063, 0.000070454, and 0.0010692. The In-Run Bias Stability (Allan variance) given for the gyroscope in VN-100's data sheet is $<$ 10 deg/hr(5-7 deg/hr typ.) or $<$ 0.00278 deg/s (0.00139-0.00194 deg/s typ.). The values of B for the gyroscope all fit well within the specified limit. The noise density given for the gyroscope in VN-100's data sheet is 0.0035 $deg/s/\sqrt{Hz}$. Gyro X and Z have slightly higher values of N than this indicating the presence of some random noise. The value of N of Gyro Y possibly indicates more noise or higher instability in this axis.Since the K values are in $deg/s/\sqrt{Hz}$, they provide a sense of how the drift evolves over time. Without a direct Rate Random Walk value in the VN-100's data sheet, we can only qualitatively say that the Gyro Y axis has the highest K value, indicating it experiences the largest long-term drift compared to the other axes.

The values of N $(m/s/\sqrt{s})$, K$(m/s^2/\sqrt{Hz})$, and B (m/s^2) obtained from the IMU's accelerometer data for the X-axis are 0.060762, 0.41411, and 0.014471, for the Y-axis are 0.023226, 0.39042, and 0.15175, and for the Z-axis are 0.021859, 0.014771, and 0.087153. According to the VN-100 data sheet, the in-run bias stability for the accelerometer is $<$ 0.04 mg (0.0003924 m/s^2). The values of B for Accel Y and Z are higher than the typical spec, indicating potential sources of drift or noise. The noise density according to the data sheet is 0.14 $mg/\sqrt{Hz}$ (0.001372 $m/s^2/\sqrt{Hz}$). The values of N for the three axes are a little higher than expected.