Lab 12 - Dijkstra's Algorithm

Single Source Shortest path

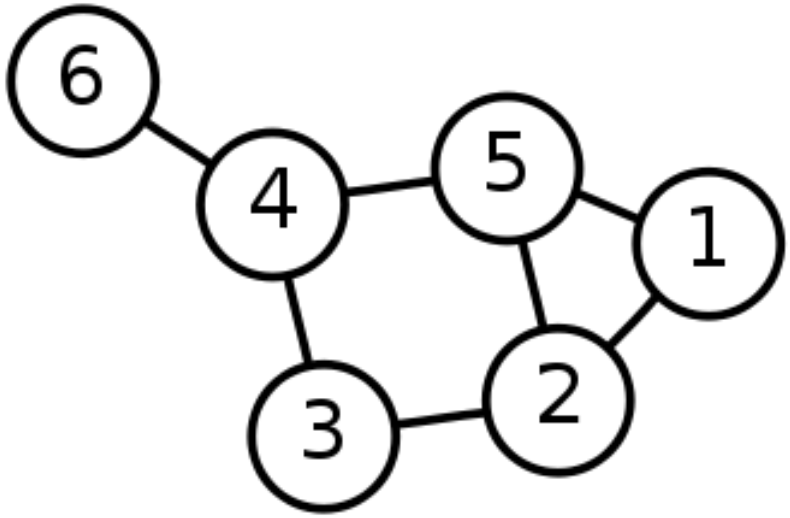
For clarifications watch this video
https://www.youtube.com/watch?v=XB4MIexjvY0

Animated Example : Source –IIITD resources from Google

Compiled by Dr.Saleena /Dr.Nisha

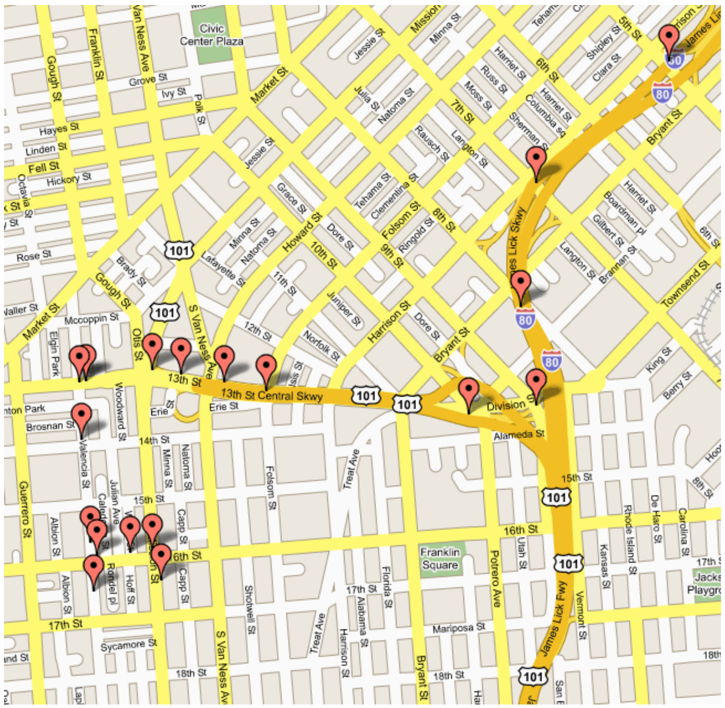
Single-Source Shortest Path Problem

Single-Source Shortest Path Problem - The problem of finding shortest paths from a source vertex *v* to all other vertices in the graph.



Applications

- Maps (Map Quest, Google Maps)
- Routing Systems



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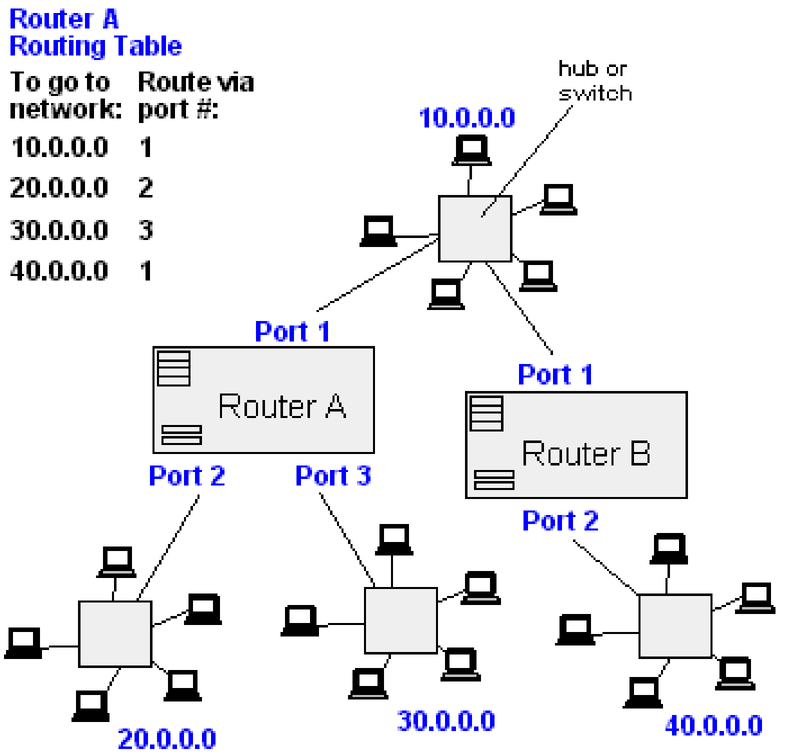


Dijkstra's algorithm

Dijkstra's algorithm - is a solution to the single-source shortest path problem in graph theory.

Works on both directed and undirected graphs. However, all edges must have nonnegative weights.

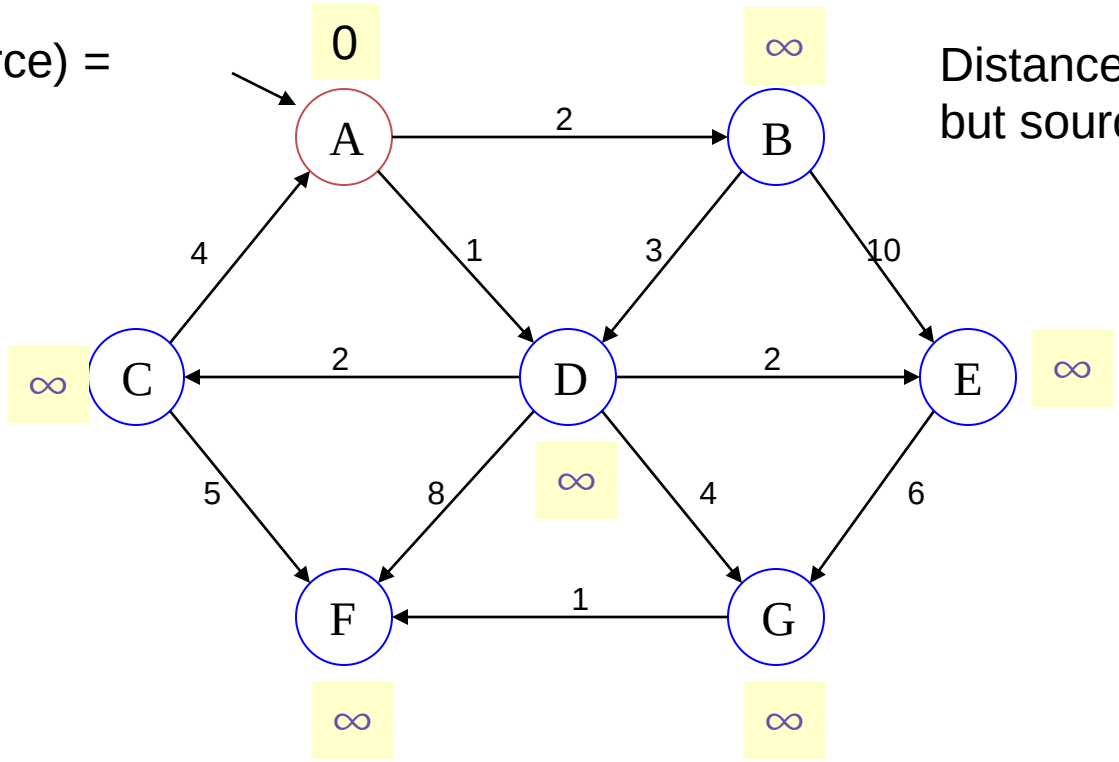
Input: Weighted graph G={E,V} and source vertex $v \in V$, such that all edge weights are nonnegative

Output: Lengths of shortest paths (or the shortest paths themselves) from a given source vertex $v \in V$ to all other vertices

Example: Initialization

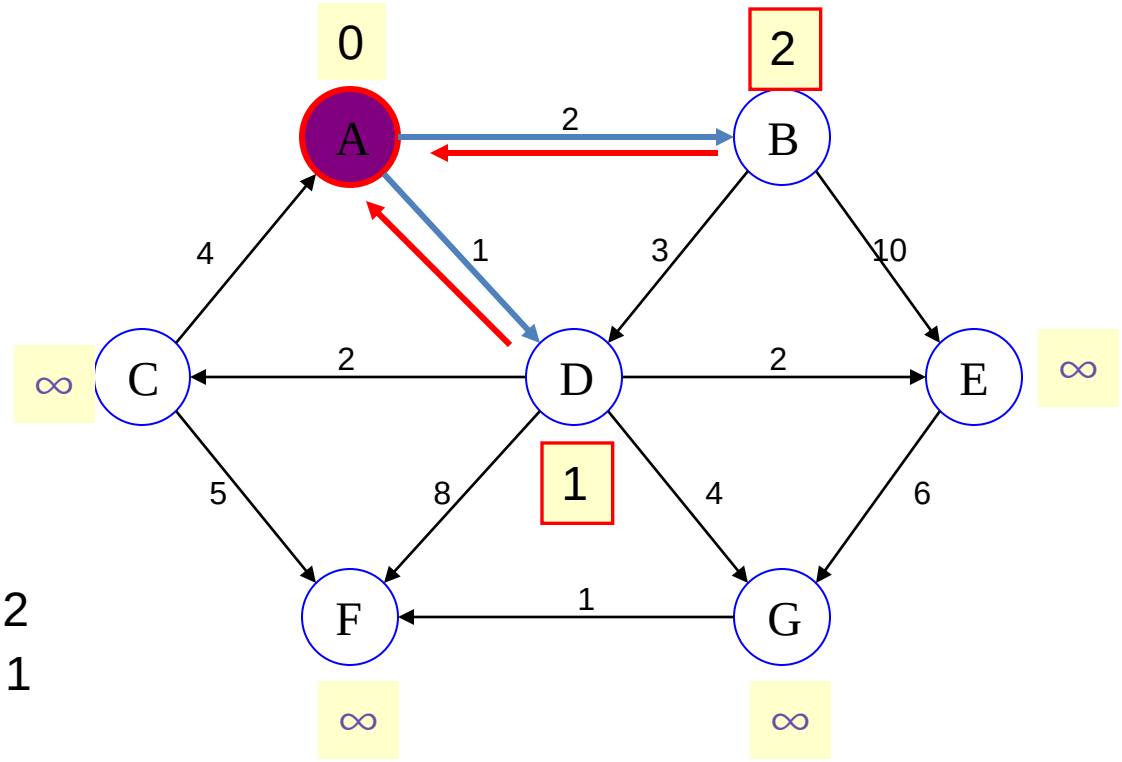
Distance(source) = 0

Distance (all vertices but source) = ∞



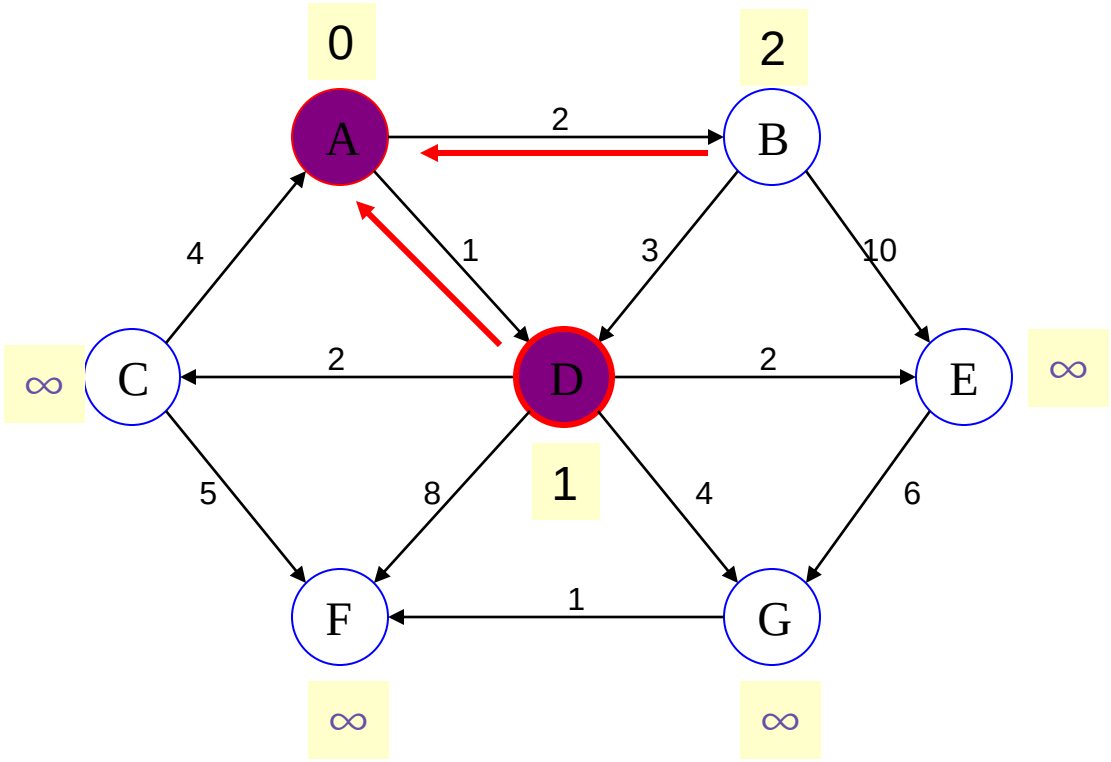
Pick vertex in List with minimum distance.

Example: Update neighbors' distance



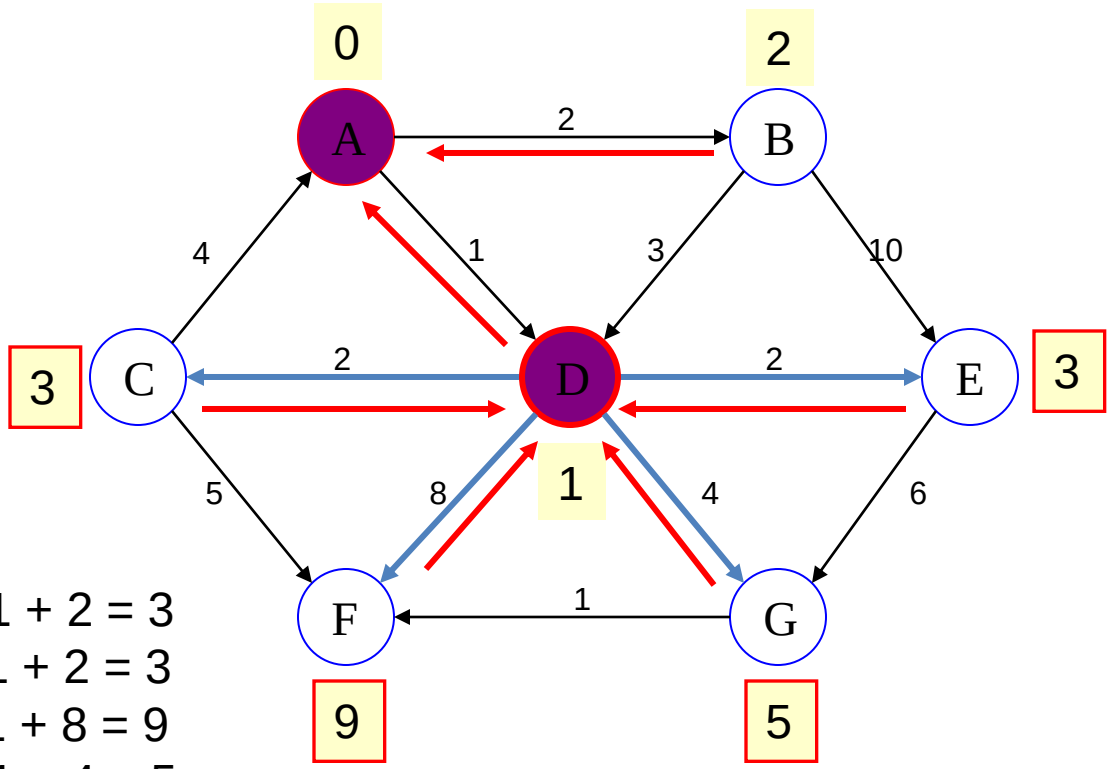
Distance(B) = 2
Distance(D) = 1

Example: Remove vertex with minimum distance



Pick vertex in List with minimum distance, i.e., D

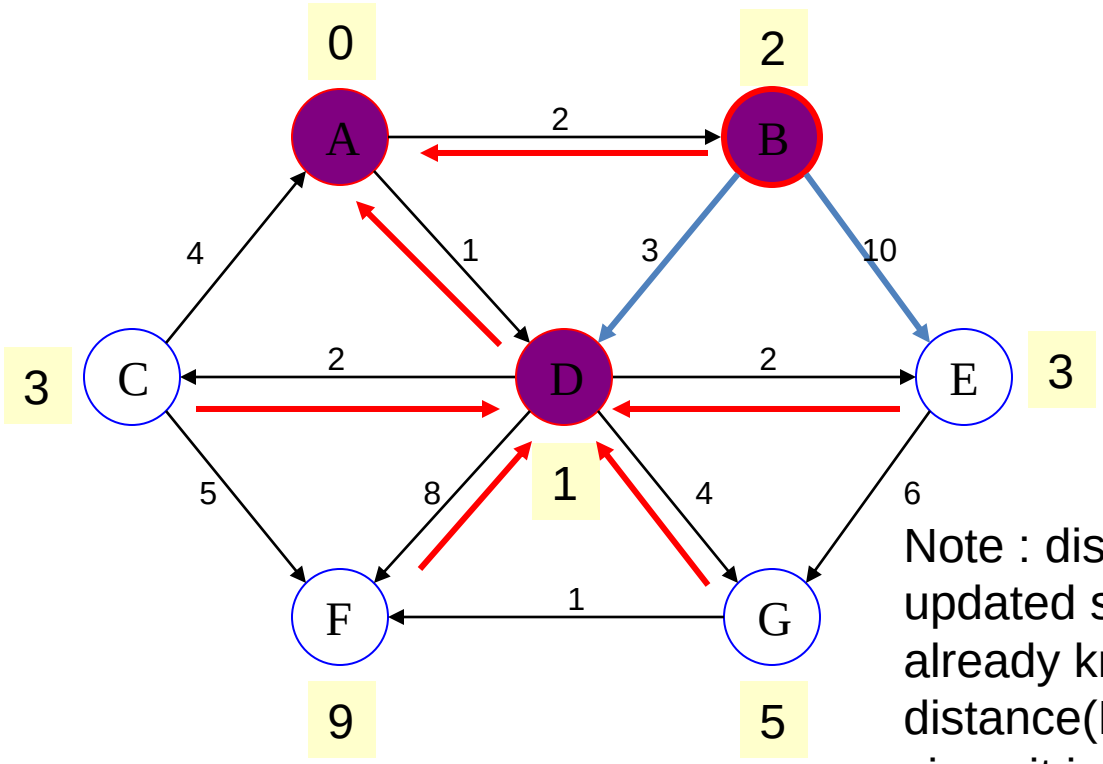
Example: Update neighbors



Distance(C) = 1 + 2 = 3
Distance(E) = 1 + 2 = 3
Distance(F) = 1 + 8 = 9
Distance(G) = 1 + 4 = 5

Example: Continued...

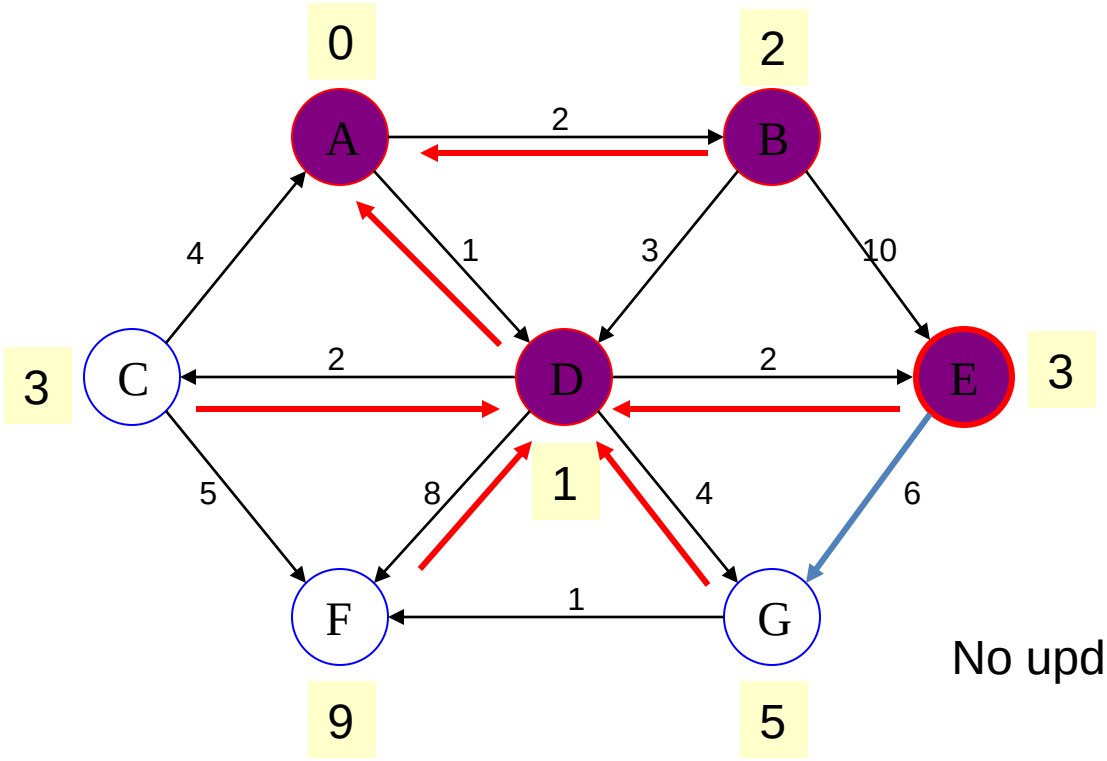
Pick vertex in List with minimum distance (B) and update neighbors



Note : distance(D) not updated since D is already known and distance(E) not updated since it is larger than previously computed

Example: Continued...

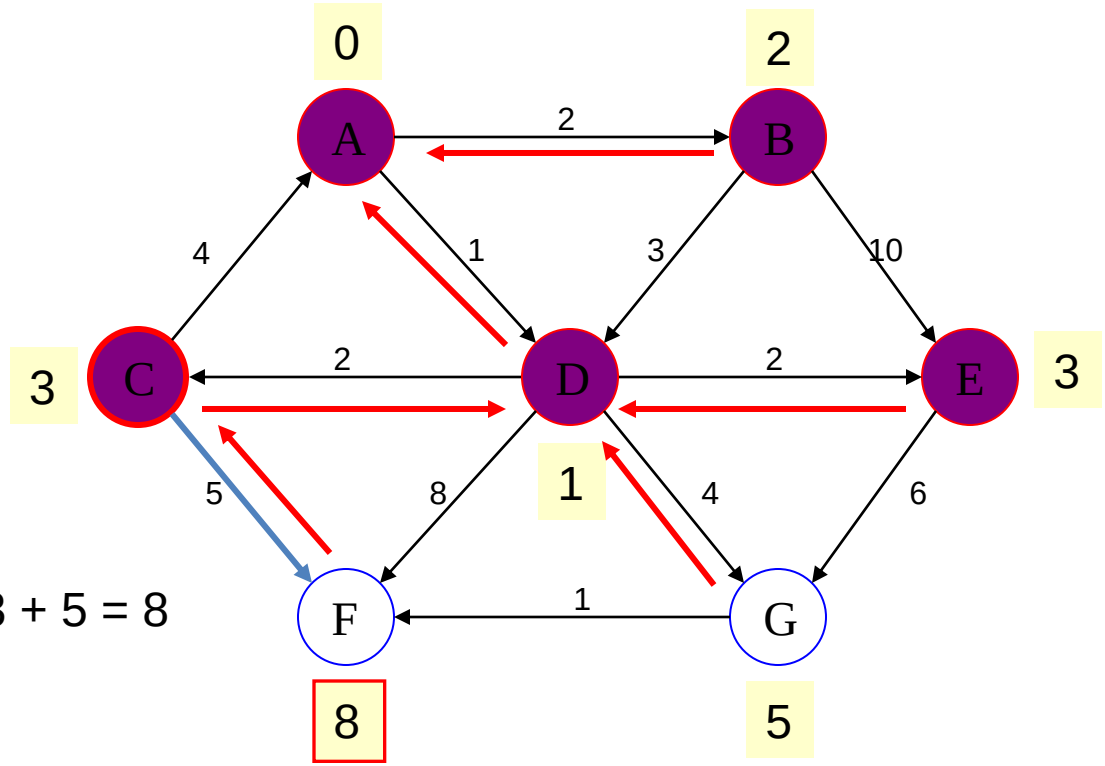
Pick vertex List with minimum distance (E) and update neighbors



No updating

Example: Continued...

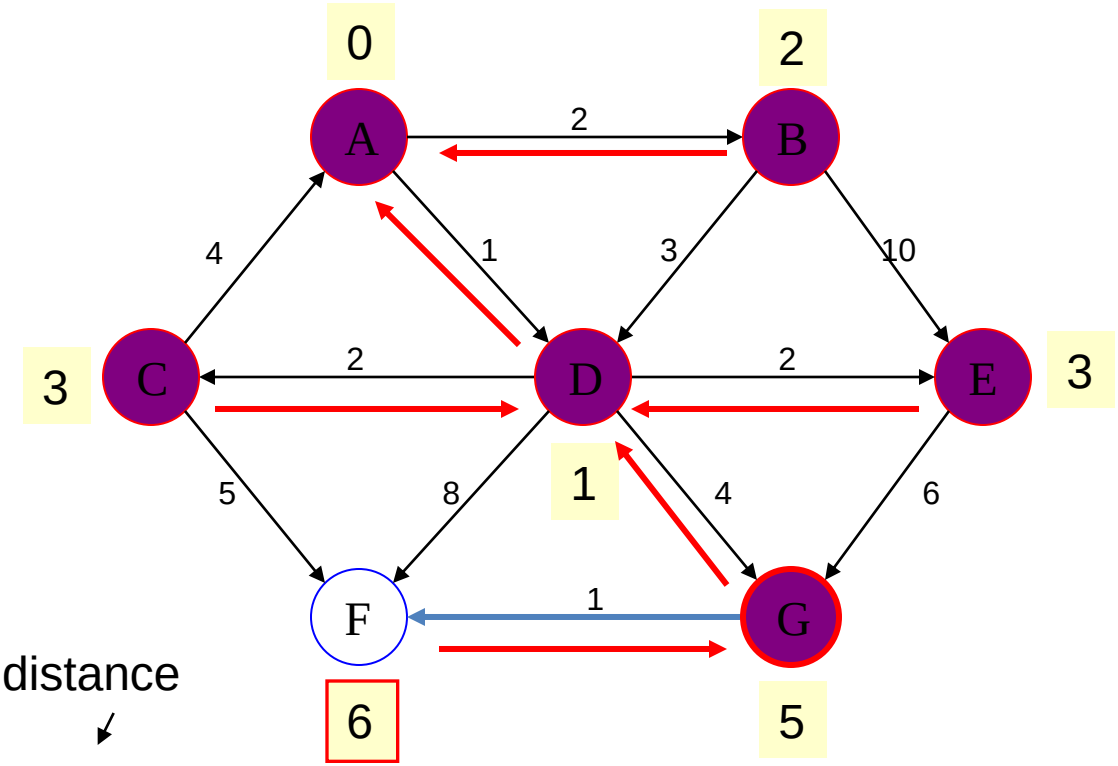
Pick vertex List with minimum distance (C) and update neighbors



Distance(F) = 3 + 5 = 8

Example: Continued...

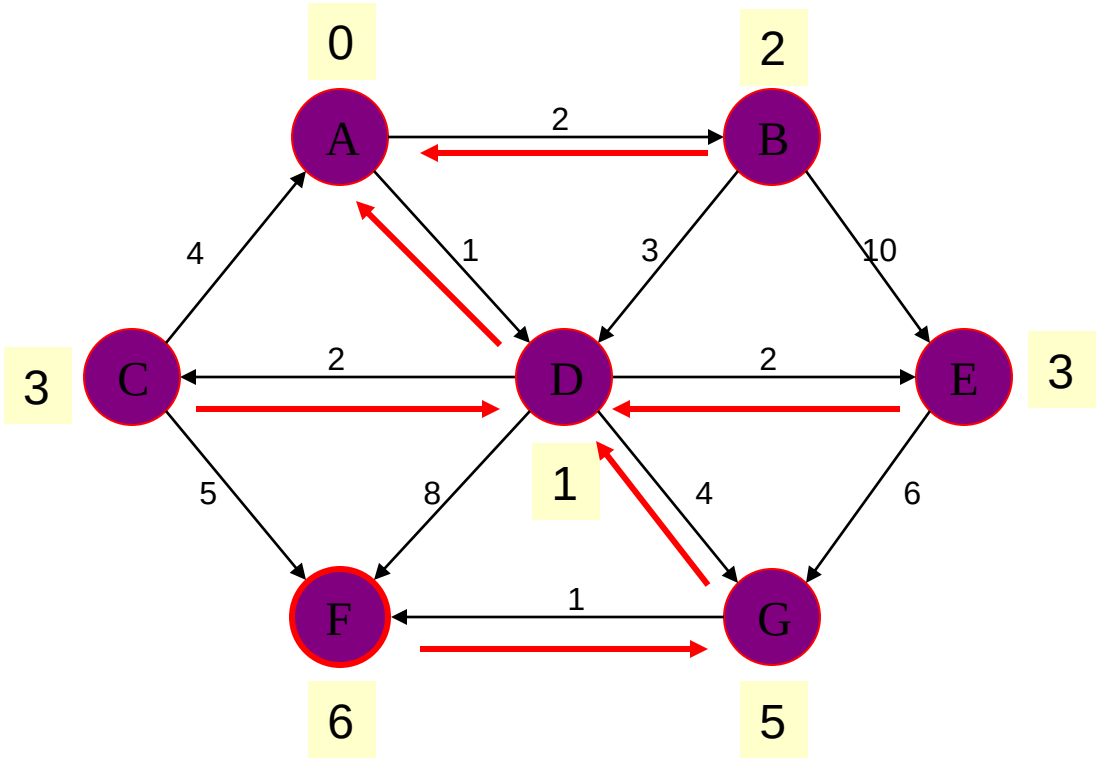
Pick vertex List with minimum distance (G) and update neighbors



Previous distance

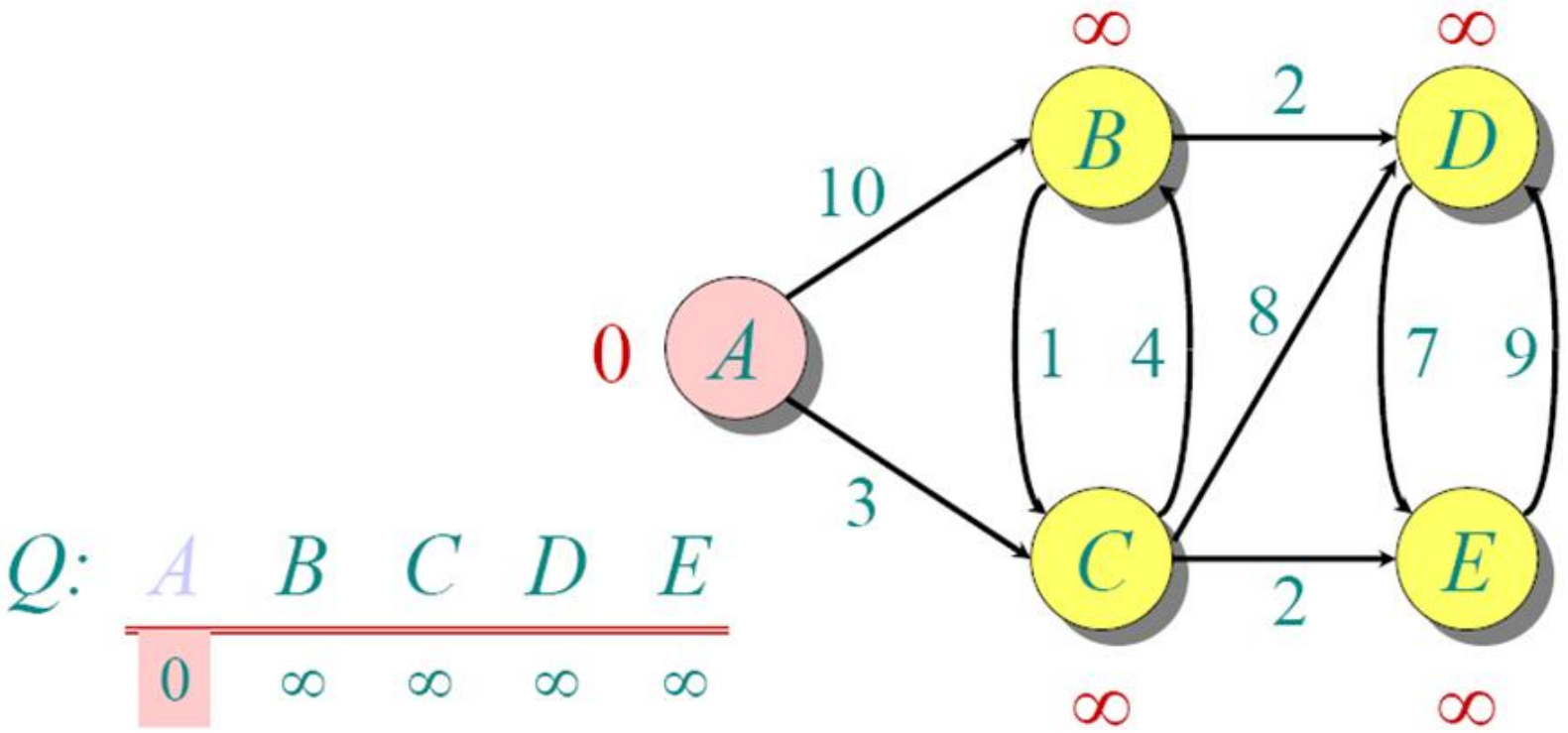
Distance(F) = min (8, 5+1) = 6

Example (end)



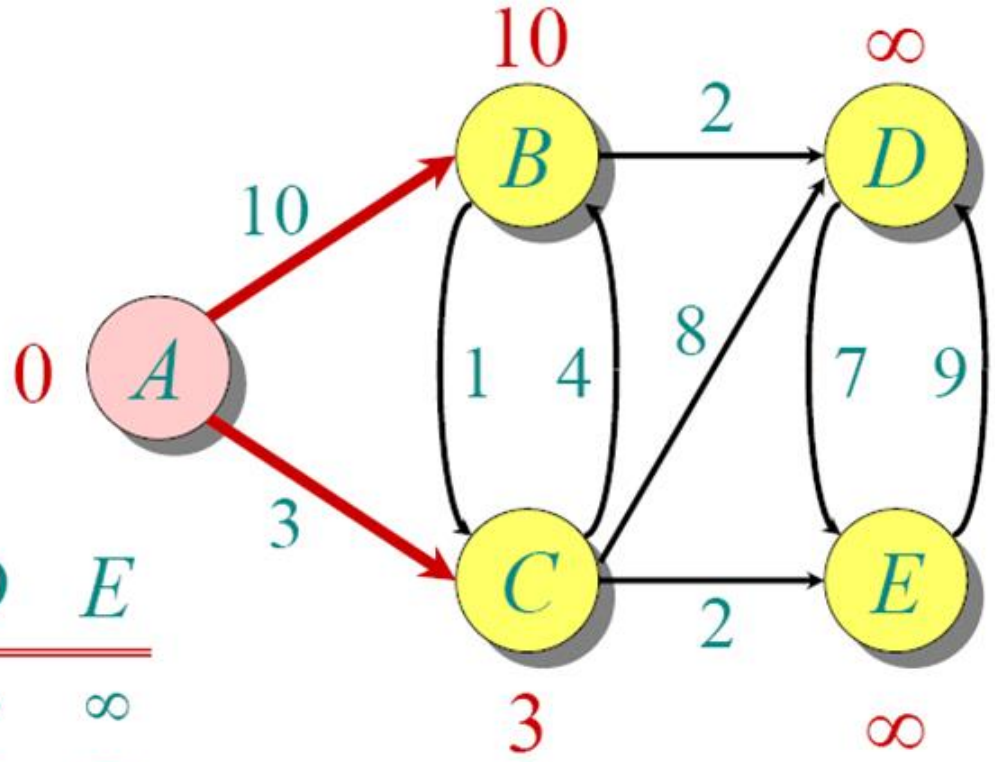
Pick vertex not in S with lowest cost (F) and update neighbors

Another Example



Q:	A	B	C	D	E
	0	∞	∞	∞	∞

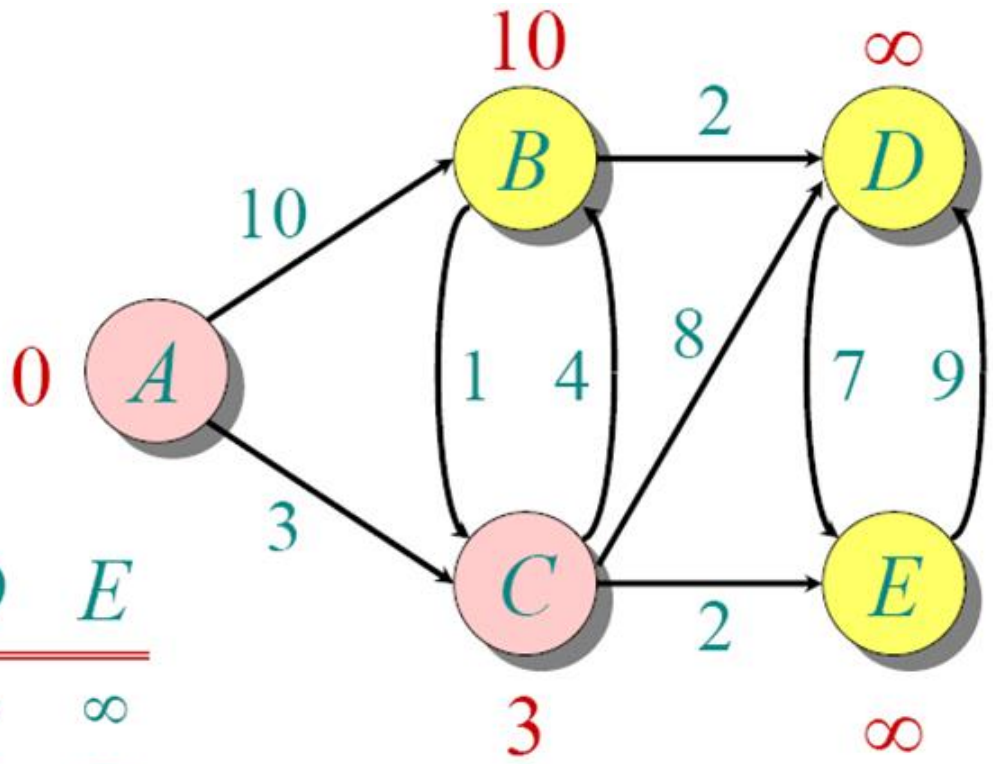
Another Example



Q:	A	B	C	D	E
	0	∞	∞	∞	∞
		10	3	∞	∞

$S: \{ A \}$

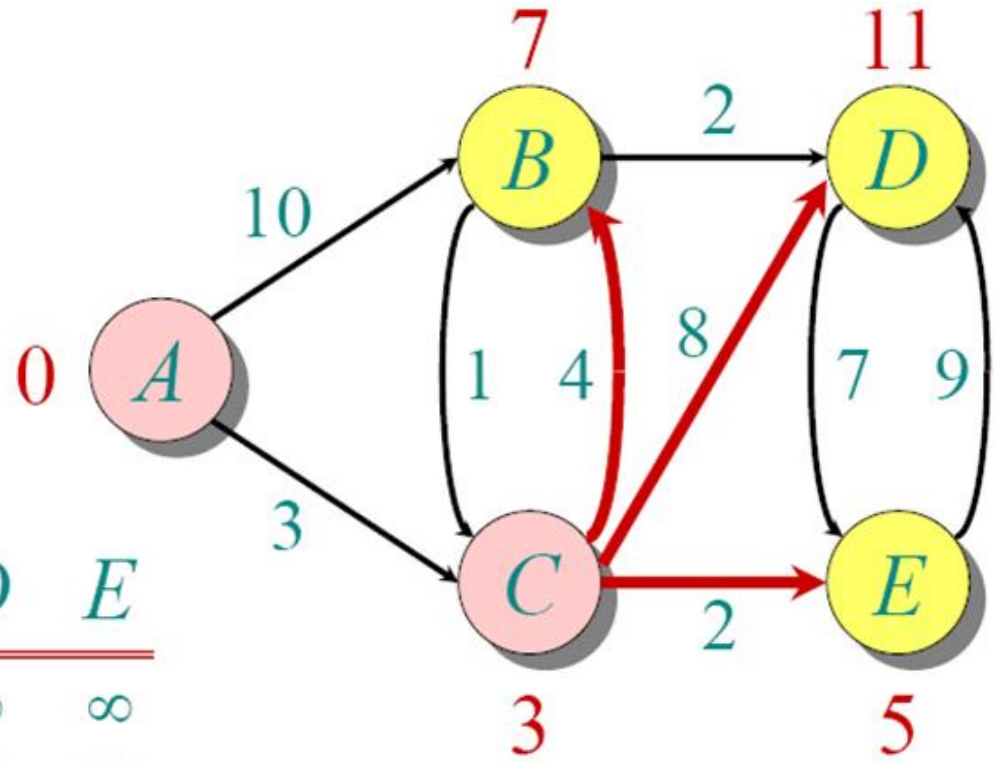
Another Example



Q:	A	B	C	D	E
	0	∞	∞	∞	∞
		10	3	∞	∞

$S: \{ A, C \}$

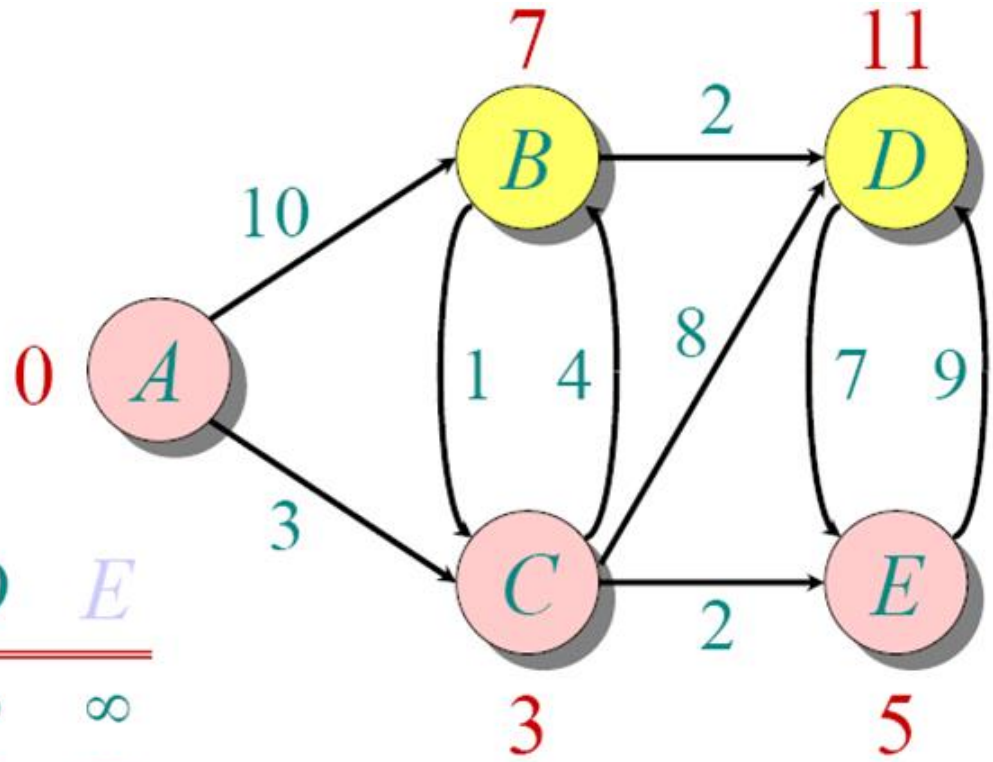
Another Example



Q:	A	B	C	D	E
	0	∞	∞	∞	∞
		10	3	∞	∞
		7		11	5

$S: \{ A, C \}$

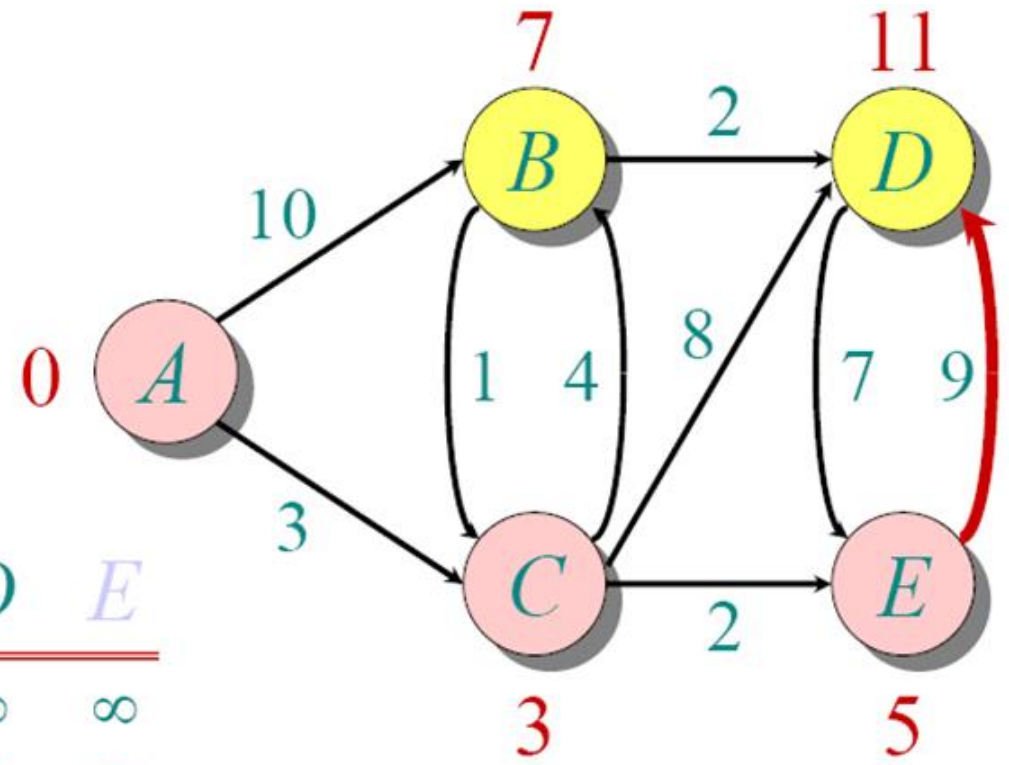
Another Example



Q:	A	B	C	D	E
	0	∞	∞	∞	∞
		10	3	∞	∞
		7		11	5

$S: \{ A, C, E \}$

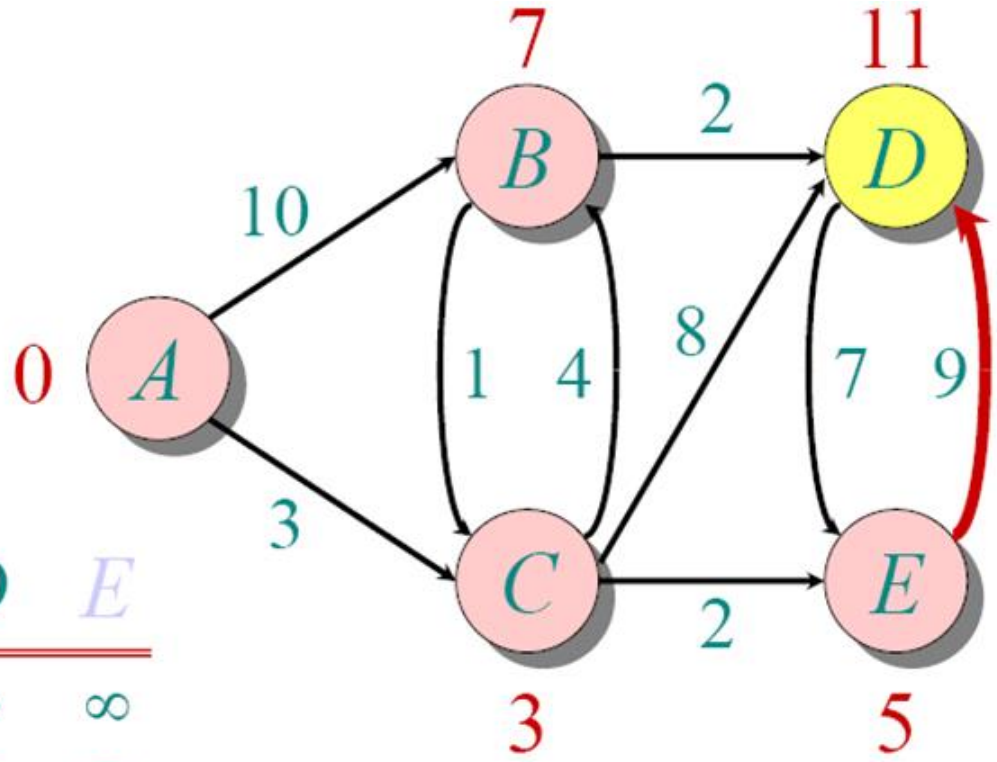
Another Example



Q:	A	B	C	D	E
	0	∞	∞	∞	∞
		10	3	∞	∞
		7		11	5
		7		11	

$S: \{ A, C, E \}$

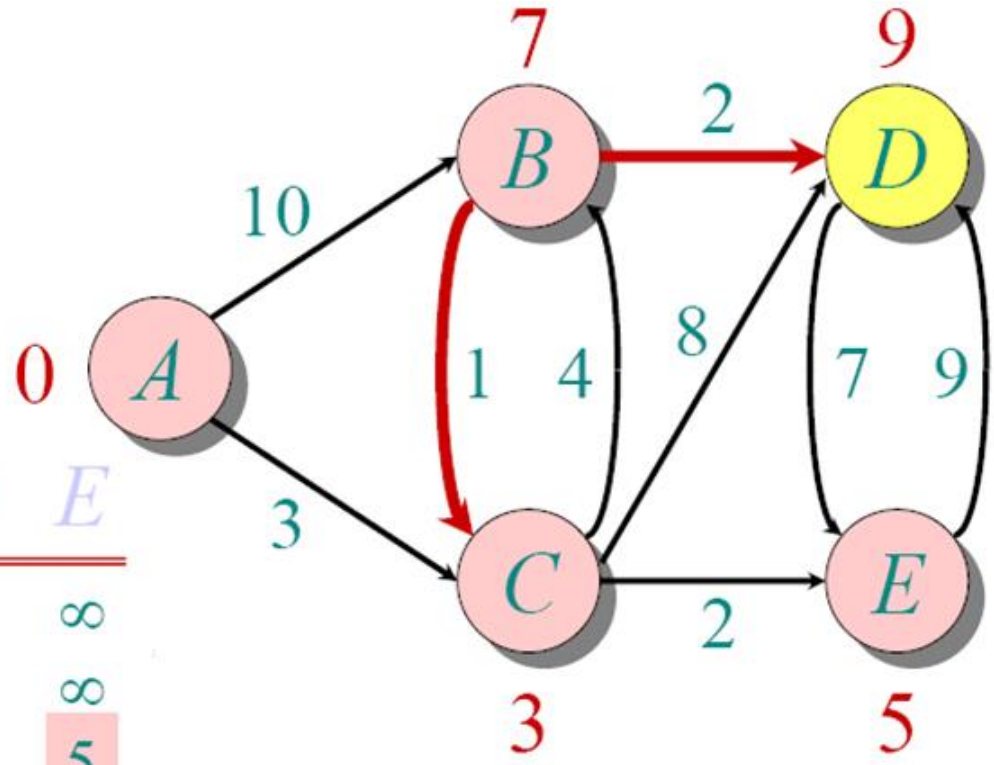
Another Example



Q:	A	B	C	D	E
	0	∞	∞	∞	∞
		10	3	∞	∞
		7		11	5
		7		11	

S: { A, C, E, B }

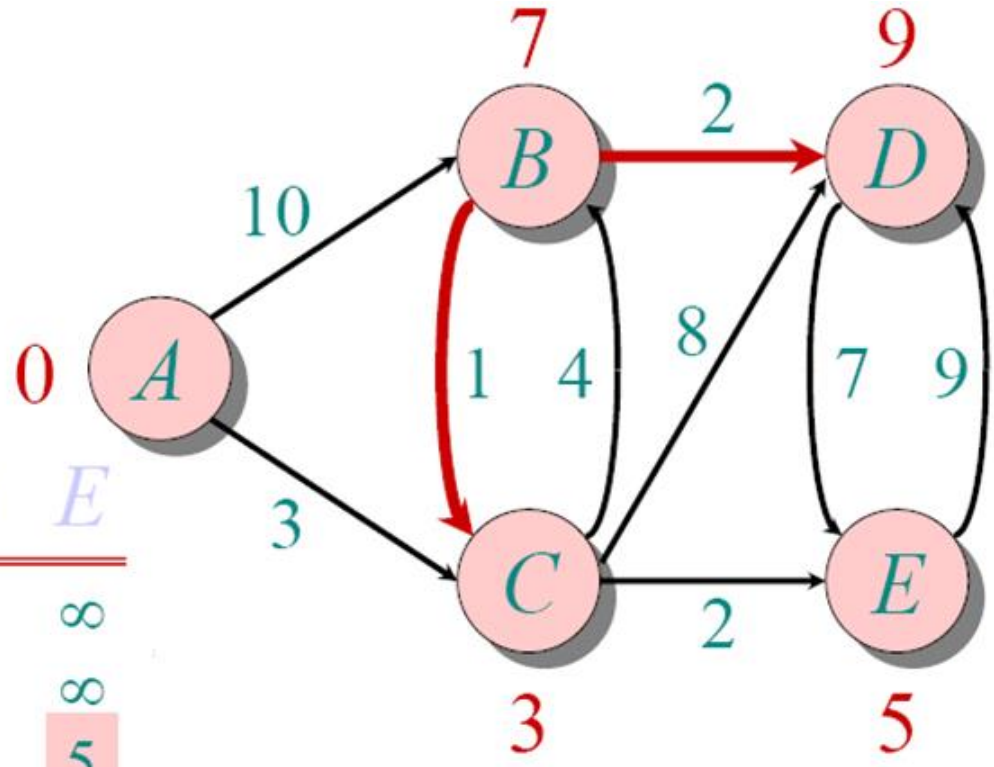
Another Example



Q:	A	B	C	D	E
	0	∞	∞	∞	∞
		10	3	∞	∞
		7		11	5
		7		11	
				9	

S: { A, C, E, B }

Another Example



Q:	A	B	C	D	E
	0	∞	∞	∞	∞
		10	3	∞	∞
		7		11	5
		7		11	
				9	

S: { A, C, E, B, D }

Single Source All Destinations

```
void shortestpath(int v, int
  cost[][MAX_ERXTICES], int distance[], int n,
  short int found[])
{
  int i, u, w;
  for (i=0; i<n; i++) {
    found[i] = FALSE;
    distance[i] = cost[v][i];
  }
  found[v] = TRUE;
  distance[v] = 0;
```

O(n)

```
  for (i=0; i<n-2; i++) {determine n-1 paths from v
    u = choose(distance, n, found);
    found[u] = TRUE;
    for (w=0; w<n; w++)
      if (!found[w])
  (distance[u]+cost[u][w]<distance[w])
          distance[w] = distance[u]+cost[u][w];
  }
}
```

Pseudocode

```
dist[s] ←o                          (distance to source vertex is zero)
for all v ∈ V–{s}
      do dist[v] ←∞                 (set all other distances to infinity)
S←∅                                 (S, the set of visited vertices is initially empty)
Q←V                                 (Q, the queue initially contains all vertices)
while Q ≠∅                          (while the queue is not empty)
do  u ← mindistance(Q,dist)         (select the element of Q with the min. distance)
    S←S∪{u}                         (add u to list of visited vertices)
     for all v ∈ neighbors[u]
          do if  dist[v] > dist[u] + w(u, v)        (if new shortest path found)
                then    d[v] ←d[u] + w(u, v)       (set new value of shortest path)
                                                   (if desired, add traceback code)
return dist
```