

# Electric Vehicles Charging Scheduling Optimization for Total Elapsed Time Minimization

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**Abstract**—With the rapid advancement of electric vehicle (EV) technology, EV has been emerging as a promising transportation due to the low carbon emission. However, the frequent and long time charging is indispensable to continue travelling. During peak hours, EVs further spend long time on the path routing because of the traffic congestion and queuing in the charging stations. Therefore, we study the EV charging scheduling problem that minimizes the total elapsed time which includes charging time for EVs through jointly optimizing the charging path routing and charging station selection in this paper. Considering the NP-hardness of this optimization problem, we propose an efficient EV charging scheduling method to obtain the optimal solution based on crowd sensing through considering the remaining energy in the battery, traffic condition, and the queue length of charging stations. Simulation results demonstrate that the proposed backtracking method based on crowd sensing can effectively reduce the total elapsed time, in comparison with the greedy algorithm.

**Index Terms**—electric vehicles, crowd sensing, total elapsed time, matrix decomposition method, backtracking method

## I. INTRODUCTION

Nowadays, the rapid development of the automobile industry has brought more convenience to people's daily life and logistics transportation at the cost of energy crisis and environmental pollution [1]. With the rapid advancement of electric vehicle (EV) technology, EVs have been emerging as a promising solution to replace the traditional fuel vehicles with plenty of advantages, such as high energy efficiency, zero pollution, and low noise [2]. However, different from traditional fuel vehicles, EVs powered by the equipped charging batteries require the frequent and long time charging to continue travelling. Furthermore, EVs have to spend long time on the path routing because of the traffic condition and queuing in the charging stations during peak hours. To guarantee the travelling quality of EVs, it is important to study the EV charging scheduling problem through jointly considering the remaining energy in the battery, traffic information as well as the location and queue length of charging stations.

Research efforts have been spent on designing effective charging scheduling algorithms for EVs to satisfy different requirements, such as queueing time minimization [3], travel energy consumption minimization [8] [9], total elapsed time

minimization [4] [5] [8], charging cost minimization [5] [7], etc. With the goal of minimizing the queueing time, a decentralized policy is proposed to assign EVs to a network [3]. Pourazarm et al. [4] solved a path-finding problem within a graph of charging station nodes using a dynamic programming solution. They applied a grouping technique based on traffic flows with multi-vehicle routing to minimize the total elapsed time towards their destinations for EVs. In [5], an optimal EV routing model based on a learnable partheno-genetic algorithm was proposed to minimize the total elapsed time. A big-data analytics-based framework which collects different standard, historical, and real-time data from different resources was introduced in [6] to estimate the remaining driving range of EV. Tan et al. [7] proposed an integrated EV charging navigation framework, which takes into consideration the impacts from both the power system and transportation system, and used a hierarchical game approach to effectively navigate EVs to charging stations. Based on the traffic signal control technologies in urban areas and an EV energy consumption model, a green wave band-based multi-objective ant colony optimization algorithm was proposed in [8] to optimize the driving route. In [9], an energy optimal real time navigation system was proposed to focus mainly on energy optimal route calculation, it is fully independent and capable of retrieving real time traffic network information and real time route to destination updates. The above papers all obtain the charging path of the EV only consider one aspect of the charging station information and traffic condition. Different from previous work, this paper jointly considers the remaining energy in the battery, traffic condition as well as the location and queue length of charging stations to obtain the the best charging path for EVs.

The aim of this paper is to minimize the total elapsed time which includes charging time for EVs through jointly optimizing the charging path routing and charging station selection. The main contributions of this paper are summarized as follows. We first use the coulomb counting method to calculate the remaining energy of the battery. Then, we construct an information matrix containing the traffic condition and charging station service information based on crowd sensing, and present the corresponding optimization problem. Considering the incompleteness of information due to crowd sensing, we adopt the matrix decomposition method to obtain an entire information matrix. Third, we propose a backtracking method to obtain the optimal solution. Finally, simulation

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results demonstrate that the proposed EV charging scheduling method based on crowd sensing can effectively reduce the EV total elapsed time.

The rest of this paper is organized as follows. In Section II, we introduce the system framework and problem formulation. In Section III, we propose an optimization method to solve the proposed optimization problem. The numerical results are presented in Section IV. Finally, we summarize this paper and present the future work direction in Section V.

## II. SYSTEM MODEL AND PROBLEM FORMULATION

### A. System Model

In this paper, we consider an EV charging system consisting of EVs, charging stations, and edge computing units, as shown in Fig. 1. Sensors in the EVs sense the EVs' positions and

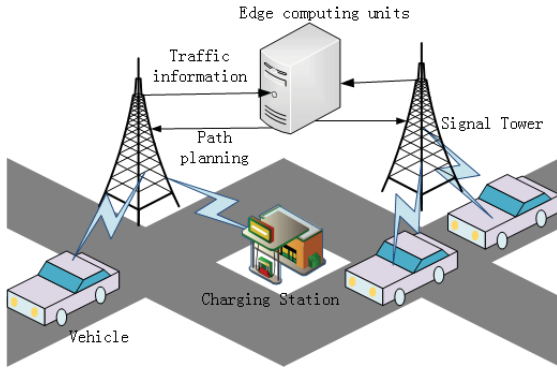


Fig. 1. System model.

EVs' speed and upload these information to edge computing units. Charging stations upload the number of EVs arriving at the charging stations and the queue length of charging stations to the edge computing units. Edge computing units provide an EV charging scheduling optimization method to aim at minimizing through running an EV charging scheduling optimization algorithm the EVs' total elapsed time, and then feed back the optimal scheduling scheme to the EV users via a wireless communication network. In particular, edge computing units obtain traffic and charging stations information based on crowd sensing [10] [11]. It implies that not all EVs participate in crowd sensing and we can still get the optimal solution even if some information is not completely acquired at some time.

### B. Problem Formulation

In this paper, we aim at minimizing the total elapsed time by jointly optimizing the charging path route and charging station selection based on the aforementioned EV charging system. Before presenting the optimization problem, we have to consider the following constraints.

#### 1) Remaining Battery Energy

The length of charging path route is highly dependent on the remaining battery energy of EV according to the current traffic conditions. Therefore, it is important to estimate the remaining battery energy of EV for optimizing the charging path route. Coulomb counting method is a common and effective method to estimate the remaining battery capacity accurately. In this

paper, the remaining energy of the EV is estimated based on the inflow and outflow coulomb amount and using the coulomb counting method [12] [13]. During the measurement process, the battery capacity is measured in ampere-hour. We use  $E_{u,max}$  and  $E_{u,c}$  to denote the battery capacity and consumed battery energy of EV  $u$ , respectively. Thus, the remaining battery energy, denoted by  $E_{u,l}$ , can be expressed as

$$E_{u,l} = E_{u,max} - E_{u,c}. \quad (1)$$

#### 2) Driving Time

In general, it is difficult to accurately describe the traffic condition. Thus, we use the driving time as a characterization indicator of traffic condition. We can estimate the driving time needed by EV  $u$  from node  $i$  to node  $j$  in the road segment  $x_{ij}$  as follows. First, edge computing units use the positioning information uploaded by the EV at the time  $t-1$  and the time  $t$  to calculate the driving speed  $v_{tu,ij}$  of EV  $u$  at the time  $t$  in the road segment  $x_{ij}$

$$v_{tu,ij} = \frac{f(d_{u,t}, d_{u,t-1})}{\Delta t}, \quad (2)$$

where  $d_{u,t}$  and  $d_{u,t-1}$  are the positions of the EV  $u$  at times  $t$  and  $t-1$ , respectively,  $f(\cdot)$  is the distance function for calculating the road network distance between two positions, and  $\Delta t$  is the sampling period. Then, according to a large amount of location information which is uploaded by the EVs participating in crowd sensing, edge computing units calculate the average driving speed in the road segment  $x_{ij}$  at time  $t$ , i.e.,

$$\bar{v}_{t,ij} = \frac{1}{U} \sum_{u=1}^U v_{tu,ij}, \quad (3)$$

where  $U$  is the total number of EVs participating in crowd sensing in the road segment  $x_{ij}$ . Thus, the driving time  $T_{t,ij}$  of the EV arriving at node  $j$  from the node  $i$  at time  $t$  can be expressed as

$$T_{t,ij} = \frac{s_{ij}}{\bar{v}_{t,ij}}, \quad (4)$$

where  $s_{ij}$  is the distance between node  $i$  and node  $j$  in the road network.

#### 3) Waiting Time

In order to accurately obtain the charging station service information, the number of EVs arriving at the charging station during the duration between time  $t$  and time  $t-1$ , denoted by  $n_t^a$ , is obtained and uploaded to edge computing units. The charging station service information will be updated and uploaded during once every sampling period  $\Delta t$ . Thus, we can calculate the average arrival rate  $\varphi_{s,t}$  (vehicles/h) of the charging station  $s$  at time  $t$  as

$$\varphi_{s,t} = \frac{n_t^a + n_{t-1}^a}{2\Delta t}. \quad (5)$$

The average waiting time  $T_s^w$  of the charging station  $s$  at time  $t$  can be calculated as

$$T_s^w = \frac{L}{\varphi_{s,t}}, \quad (6)$$

where  $L$  is the number of EVs waiting to be charged at time  $t$ .

#### 4) Charging Time

The charging station adopts fast charging mode for charging. Thus, the fast charging energy needed by EV  $u$  at the charging station  $s$ , denoted by  $E_{s,u}$ , can be expressed as

$$E_{s,u} = E_{u,max} - (E_{u,l} - \hat{E}_u), \quad (7)$$

where  $\hat{E}_u$  is the total energy consumption of the EV before it reaches the charging station, the calculation formula for  $\hat{E}_u$  is given below. The fast charging time of EV  $u$  at the charging station  $s$ , denoted by  $T_{s,u}$ , can be calculated as

$$T_{s,u} = \frac{E_{s,u}}{P_{s,u}\eta_u}, \quad (8)$$

where  $P_{s,u}$  is the fast charging power that the charging station  $s$  provides to EV  $u$ , and  $\eta_u$  is the fast charging efficiency of EV  $u$ .

With the time as the row, and the road segments or charging stations as the column in the matrix, we constructed the time information matrix, as illustrated in Fig. 2, including  $X$  road segments or charging stations and  $T$  moments. The time information matrix represents information about a certain road segment or charging station at the moment of  $T$ .

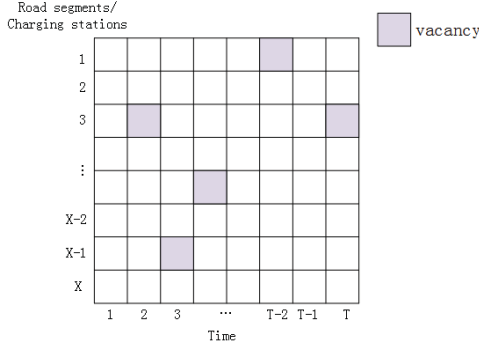


Fig. 2. Time information matrix.

Our goal is to minimize the total elapsed time which is the sum of travelling time and charging time through jointly optimizing the charging path routing and charging station selection. We denote all nodes as a set  $W$ , the EV starts from the initial node  $S$  and reaches the target node  $D$  through the candidate path. In the driving route selection of the EV, in addition to the initial and target nodes, each node must satisfy the flow balance, that is, the EV must leave a certain node after reaching it. The problem of EV charging scheduling and driving path optimization can be modeled as

$$\min T_e = \sum_{i,j \in W} T_{ij}p_{ij} + T_s^w + T_{s,u} \quad (9)$$

$$s.t. \quad p_{ij} = \begin{cases} 1, & i, j \in W, \text{ the vehicle travels from } i \text{ to } j \\ 0, & \text{otherwise} \end{cases} \quad (10)$$

$$\sum_{i,j \in W} p_{ij} - \sum_{i,j \in W} p_{ji} = \begin{cases} 1, & i = S \\ 0, & i \neq S, D \\ -1, & i = D \end{cases} \quad (11)$$

$$\hat{E}_u = E_l - \sum_{i,j \in W} E_{ij}p_{ij} \geq 0 \quad (12)$$

$$\text{vars. } p_{ij}, T_{ij}, T_s^w, T_{s,u}, E_l, E_{ij} \quad (13)$$

where  $T_e$  is the total elapsed time we have obtained,  $p_{ij}$  is the decision variable and  $E_{ij}$  represents the energy consumed by the EV from the road network node  $i$  to  $j$ .

### III. ALGORITHM DESIGN

In this section, we want to obtain the optimal the charging path routing by optimizing multiple variables. For specific, we first use the matrix decomposition to obtain an entire information matrix. Then, we develop a backtracking method to obtain the optimal solution to (9)-(13).

#### A. Matrix Decomposition Method

In this paper, not all EVs in the road segments participating in crowd sensing, and thus the information of certain road segments or charging stations at a certain time moment cannot be obtained. For this reason, there might be incomplete and unknown vacancies in the information matrix  $M(x, t)$ . The information matrix is a non-negative matrix of data generated by a plurality of EVs and charging stations at a set of continuous time points. Considering the information vacancy in some road segments or charging stations at some time, we use matrix decomposition method to fill the gaps in the information matrix. The matrix is decomposed into the product of two low-rank matrices without gaps. An error function is defined to describe the recovery accuracy before and after the decomposition, and then the optimal solution is solved under the non-negative constraint, so as to obtain the values of some vacancy terms in the original matrix.

Based on the matrix elementary transformation, the matrix  $M(x, t)$  can be decomposed into the product of two low rank matrices

$$M(x, t) = \hat{M}(x, t) \approx P(x, k)Q(k, t), \quad (14)$$

where  $k$  is an intermediate variable and  $k < \min(x, t)$ . Define the error  $e_{xt}$  as the difference between  $M(x, t)$  and  $\hat{M}(x, t)$  corresponding to the sum of the elements in row  $x - th$  and column  $t - th$ . To ensure the accuracy of matrix decomposition, the overall error  $e_{xt}$  between the original matrix  $M(x, t)$  and the decomposed matrix  $\hat{M}(X, T)$  should be reduced as much as possible, that is,

$$\hat{M}(x, t) = \sum_{k=1}^K P(x, k)Q(k, t) = P_x^T q_t, \quad (15)$$

$$e_{xt} = M(x, t) - \hat{M}(x, t), \quad (16)$$

where  $p_x^T$  and  $q_t$  are row  $x$  and column  $t$  of matrix  $P$  and  $Q$  respectively.

To avoid the use of negative error, the square value of the difference between the corresponding elements in the matrix before and after decomposition is defined as the optimization objective. The information matrix acquisition problem with partial vacancies is described as an optimization model that minimizes the sum of squared errors corresponding to all elements in the matrix,

$$\min \sum e_{xt}^2. \quad (17)$$

We increase the regularization term to prevent over-fitting, and the error function of the above model is

$$e_{xt}^2 = \| M(x, t) - \hat{M}(x, t) \|^2 + \alpha (\| P(x, k) \|^2 + \| Q(k, t) \|^2), \quad (18)$$

where  $\| \cdot \|$  is a Euclid norm,  $0 \leq \alpha \leq 1$ . We use the gradient descent method adjust  $P(x, k)$  and  $Q(k, t)$  components continuously until  $e_{xt}$  is less than a preset threshold  $e$ .

#### B. Backtracking Method

After fulfilling the information matrix, we want to solve the formulated problem described in (9)-(13) with the backtracking method. Backtracking method is an orderly searching method, and can avoid unnecessary search that may execute in exhaustivity search. Backtracking method's advantage is that it can reduce search space and improve the computational efficiency with large information [14] [15]. It searches for the solution space tree according to depth-first strategy, judges whether to continue searching through some comparison purpose, uses constraint conditions or boundary conditions, and returns directly if the constraint conditions are not satisfied. The main ingredients of backtracking method are as follows.

- Build and initialization an information matrix and input all the lists of nodes can be selected.
- Find out if results meets the requirements, If the requirements are met, return path plan to user. If not, start with this node and traverse next node in its list successively. And when traversing, judge whether the current node can satisfy constraints, if not, skip the node. If satisfying, continue traversing until find the satisfied path plan.
- If there are no satisfied path plans after finishing traversing  $i$  nodes, trace back to  $i - 1$  node and then traverse other nodes successively. Repeat the above steps until get the satisfactory solution. At last, return the optimal plan to user.

#### IV. SIMULATION RESULTS

In this section, we evaluate the performance and efficiency of the proposed algorithms through several simulation examples. We consider a road network of  $20\text{km} \times 20\text{km}$ , consisting of 28 nodes, 50 roads and 5 charging stations (node 4, node 13, node 19, node 23, node 27), where the initial node  $S$  is 1 and the target node  $D$  is 11, as shown in Fig. 3. We set the charging station's fast charging power  $P_f$  for electric vehicles as 100kw, and fast charging efficiency as 90%, the proportion of EVs participating in crowd sensing is 50% and the sampling period  $\Delta t$  is 5 min, that is to say, the information based on crowd sensing is updated every 5 minutes. Each decision includes the acquisition of traffic information matrix and the solution of the optimization problem. In order to verify the effectiveness of the method, we have done the following simulations.

**Example 1:** In this simulation example, we want to verify the efficiency of matrix decomposition method. For the Lagrange interpolation [16] as the baseline. Generally, the condition of the traffic and the charging station change little within two hours, so we adopt the time information matrix (54 rows 24 columns) obtained within two hours to verify the accuracy of the results. In Fig. 4, we plot the matrix

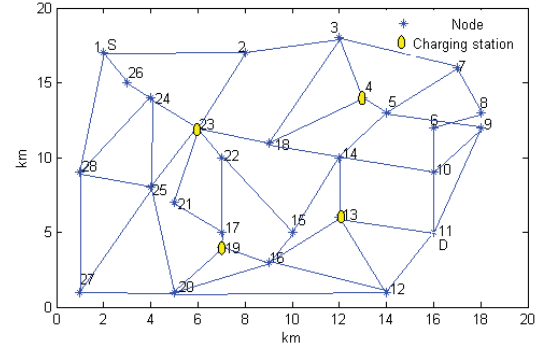


Fig. 3. Road network model diagram.

error value before and after the recovery. We can see that the maximum error of matrix decomposition method is 4.87%, and the maximum error of Lagrange interpolation is 13.54%. This observation implies that the matrix decomposition method can recover the information matrix better.

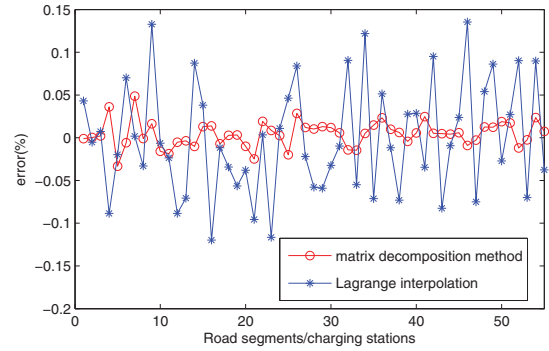


Fig. 4. Matrix error value before and after recovery.

**Example 2:** In this simulation example, we want to verify the efficiency of backtracking method, and how the number of EVs participating in crowd sensing affect the optimal elapsed time. For the comparison purpose, we consider three simulation settings as follows. In scheme 1 and scheme 2, we use the backtracking method. The difference is that the information is obtained without crowd sensing in scheme 1, the information is acquired with crowd sensing in scheme 2. In scheme 3, we get the information with crowd sensing which is the same as the scheme 2 but use the greedy algorithm to solve the optimization problem. Three schemes' charging schedule optimization results are shown in Tab. I and optimal charging routings are shown in Fig. 5. The total elapsed time  $T_e$  obtained in scheme 1 is 1.76h. The total elapsed time  $T_e$  obtained in scheme 2 is 1.15h and the time is 34.86 percent less than scheme 1. The total elapsed time  $T_e$  obtained in scheme 3 is 1.38h, the time is 20 percent more than scheme 2. Experimental results show that the proposed EV charging scheduling method based on crowd sensing can effectively reduce the EV total elapsed time.

Furthermore, we illustrate the relation between total elapsed time  $T_e$  and the proportion of EVs participating in crowd sensing in Fig. 6. We can see that for the backtracking method, the stable elapsed time can be obtained when the proportion of EVs participating in crowd sensing is more than 75%. On the contrary, the greedy algorithm obtains the stable



TABLE I  
THE CHARGING SCHEDULE OPTIMIZATION RESULTS

Schemes	Driving time(h)	Waiting time(h)	Charging time(h)	Total elapsed time $T_e$ (h)
Scheme 1	1.06	0.42	0.28	1.76
Scheme 2	0.65	0.27	0.23	1.15
Scheme 3	0.89	0.32	0.17	1.38

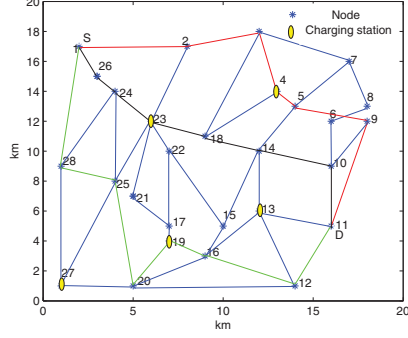


Fig. 5. The optimal charging path routing diagram. The green line denotes the charging routing in scheme 1. The red line denotes the charging routing in scheme 2. The black line denotes the charging routing in scheme 3.

elapsed time only when the proportion of EVs participating in crowd sensing exceeds 85%. Also, in comparison with the greedy algorithm, the backtracking method can reduce the elapsed time by 15.29%. These observations imply that the backtracking method is better than the greedy algorithm for the practical implementation.

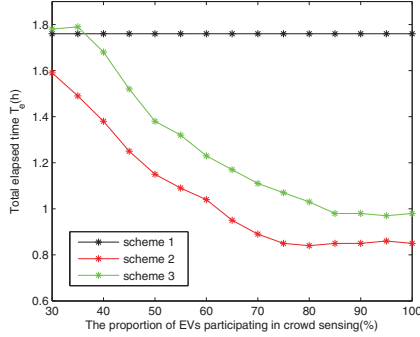


Fig. 6. The relation between total elapsed time  $T_e$  and the proportion of EVs participating in crowd sensing.

## V. CONCLUSION

In this paper, we focused on designing a backtracking method based on crowd sensing for the EV charging scheduling optimization. With the aid of crowd sensing, we obtained information such as traffic condition and charging station services. We first adopted the matrix decomposition method to obtain an entire information matrix. Then, we proposed the backtracking method to solve the EV charging scheduling problem that minimizes the total elapsed time. The simulation results shew that the information acquisition based on crowd sensing can effectively reduce EVs total elapsed time. In the future work, we want to consider the charging price and different objectives, such as minimum charging cost, minimum

travel time.

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