Course: Engineering Physics
PHY 1701

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Dr. R. Navamathavan

Physics Division

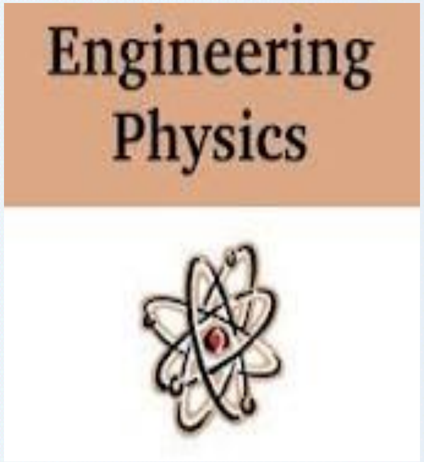
School of Advanced Sciences (SAS)

Vellore Institute of Technology
(Deemed to be University under section 3 of UGC Act, 1956)

navamathavan.r@vit.ac.in

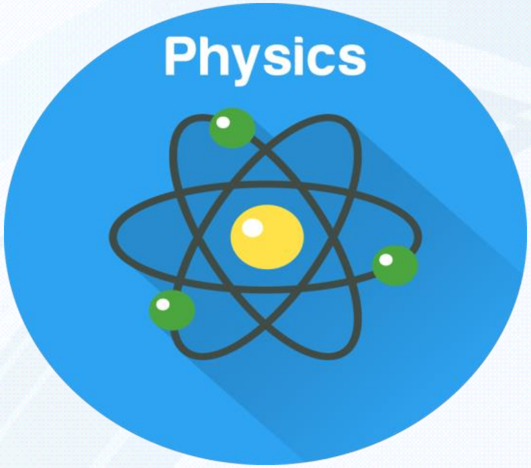
Outline

- ➢ **Davisson – Germer Experiment**
- ➢ **Heissenberg Uncertainty Principle**



Resources:

Concepts of Modern Physics (Arthur Beiser)
Pages: 115 – 117 & 119-128



de Broglie Wavelength

Relativity

$$E = mc^2 = \sqrt{p^2c^2 + m_0^2c^4}$$

Kinetic energy term

Rest mass energy term

rest mass = 0

Momentum of a photon

$$p = \frac{E}{c}$$

Photoelectric effect $E = hf = \dfrac{hc}{\lambda}$

Wavelength-energy relation

$$\frac{h}{\lambda} = \frac{E}{c}$$

$$\lambda = \frac{h}{p}$$

for photon

The de Broglie Hypothesis

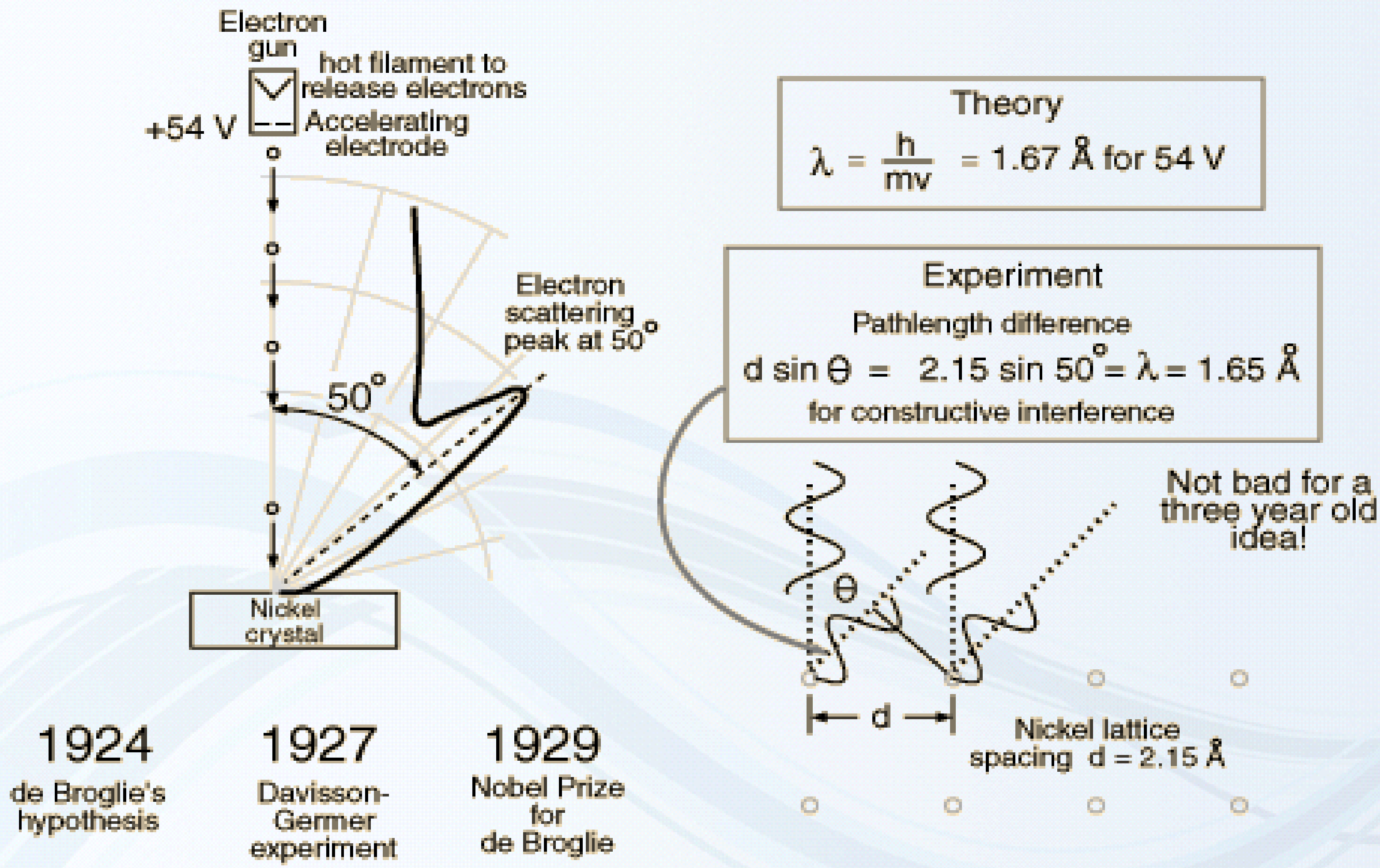
$$\lambda \overset{?}{=} \frac{h}{p} = \frac{h}{mv}$$

for electron?

DeBroglie Wavelength

$$\boxed{\lambda = \frac{h}{p}}$$

Davison-Germer Experiment



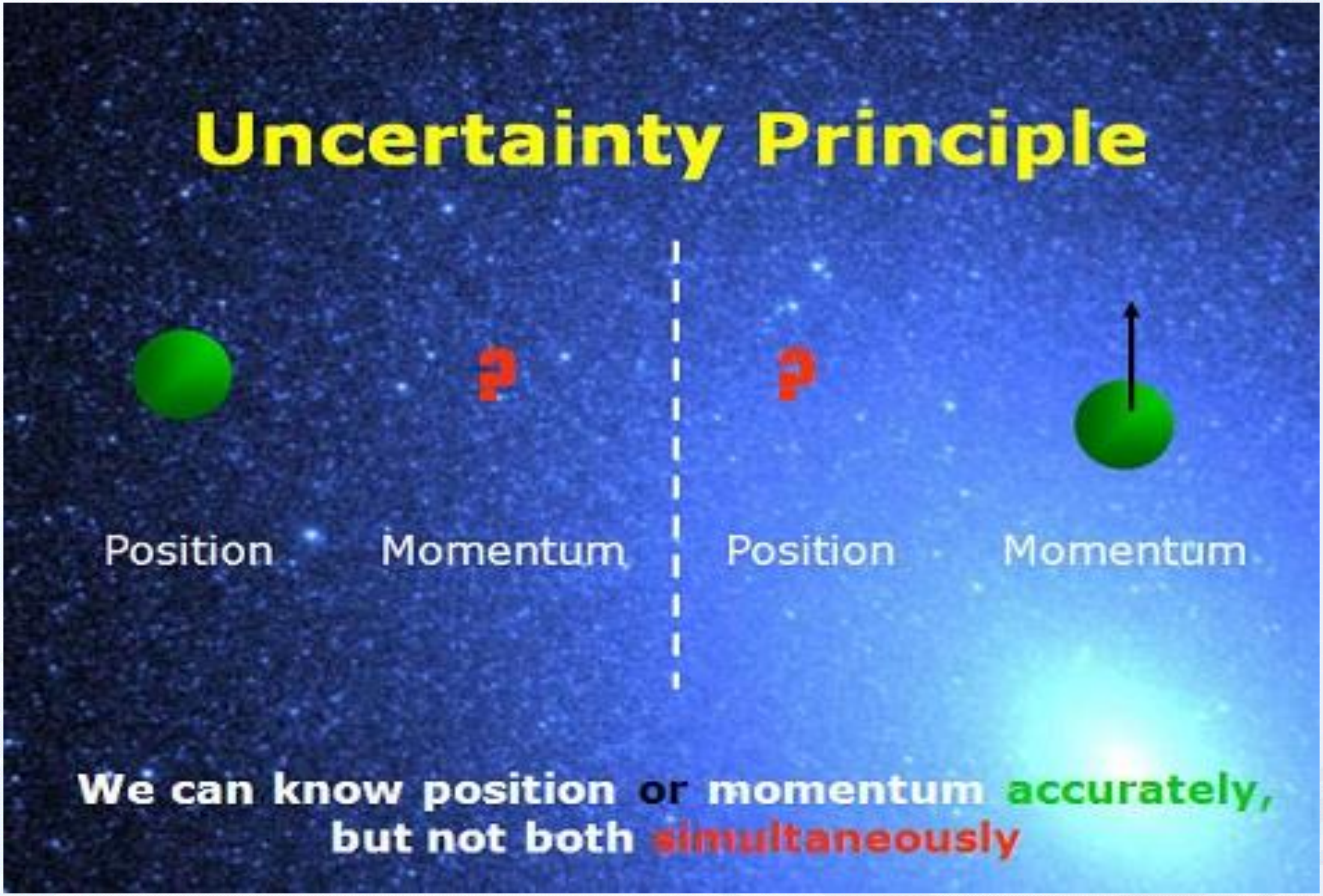
Introduction

The position and momentum of a particle cannot be simultaneously measured with arbitrarily high precision.

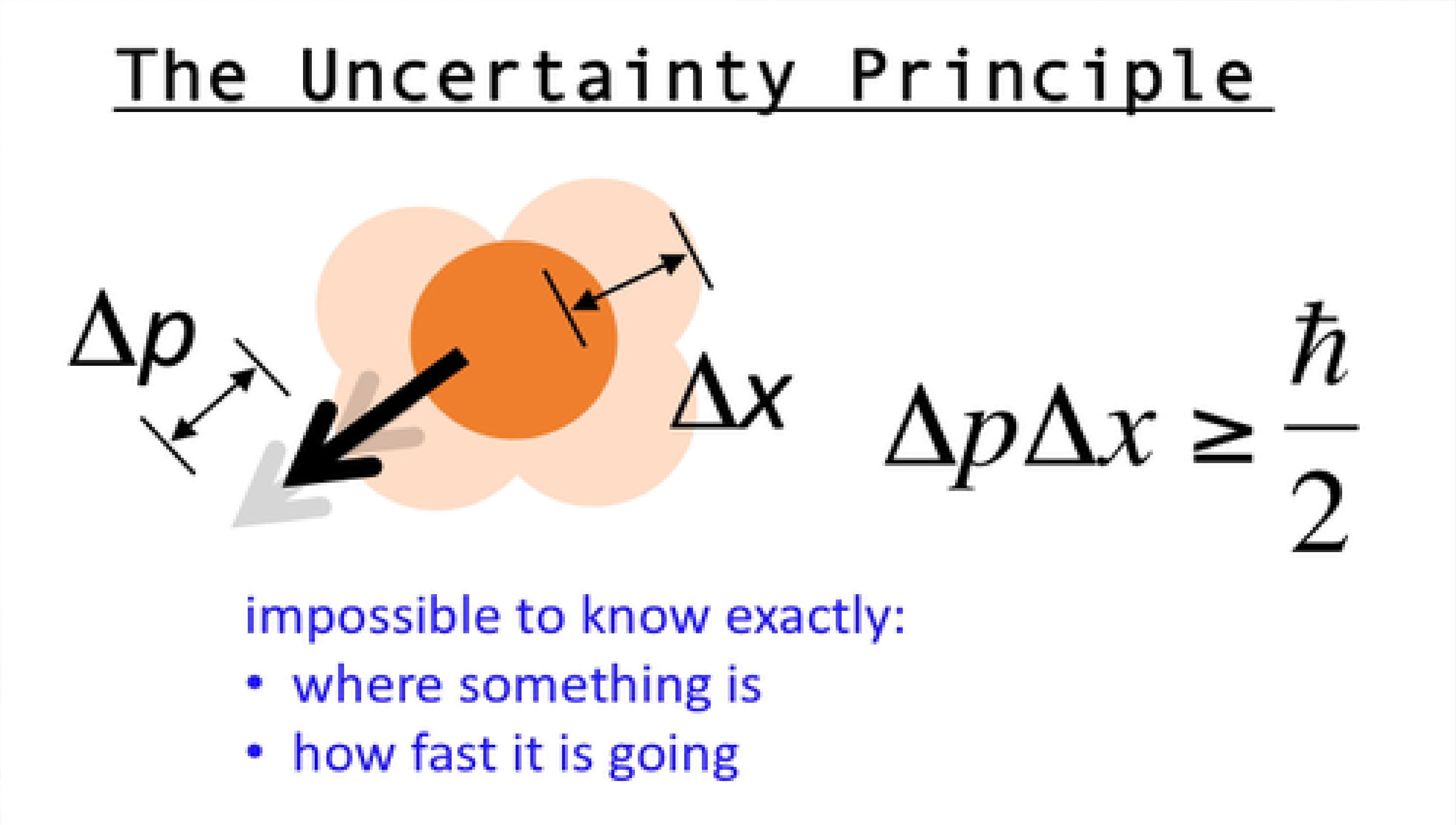
There is a minimum for the product of the uncertainties of these two measurements.

There is likewise a minimum for the product of the uncertainties of the energy and time.

Introduction



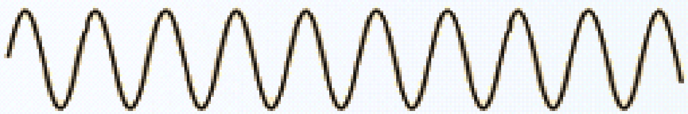
Introduction



It states that the more precisely the position of some particle is determined, the less precisely its momentum can be known, and vice versa

Uncertainty Principle

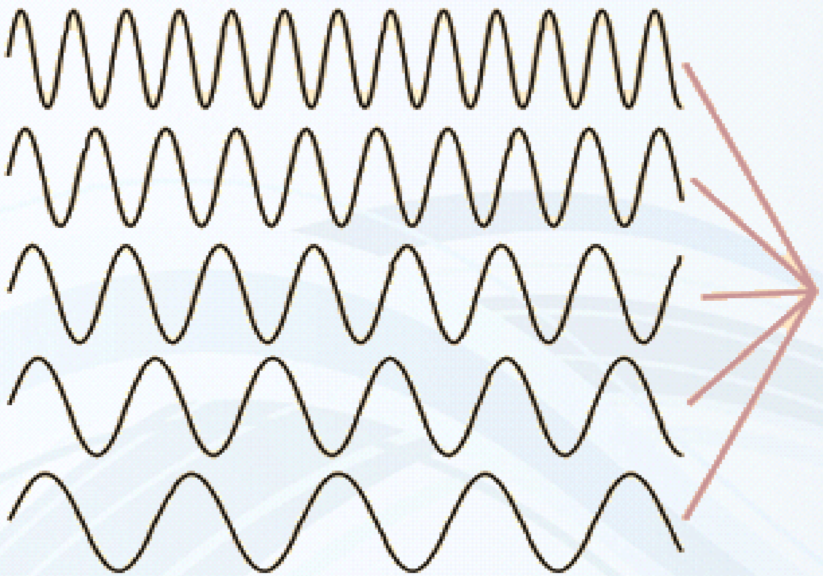
Precisely determined momentum



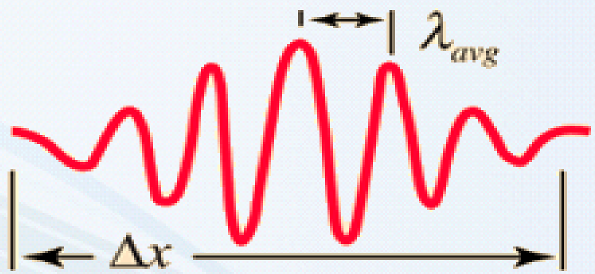
A sine wave of wavelength λ implies that the momentum is precisely known. But the wavefunction and the probability of finding the particle $\Psi^*\Psi$ is spread over all of space!

$$\boxed{p = \frac{h}{\lambda}}$$

p precise
x unknown



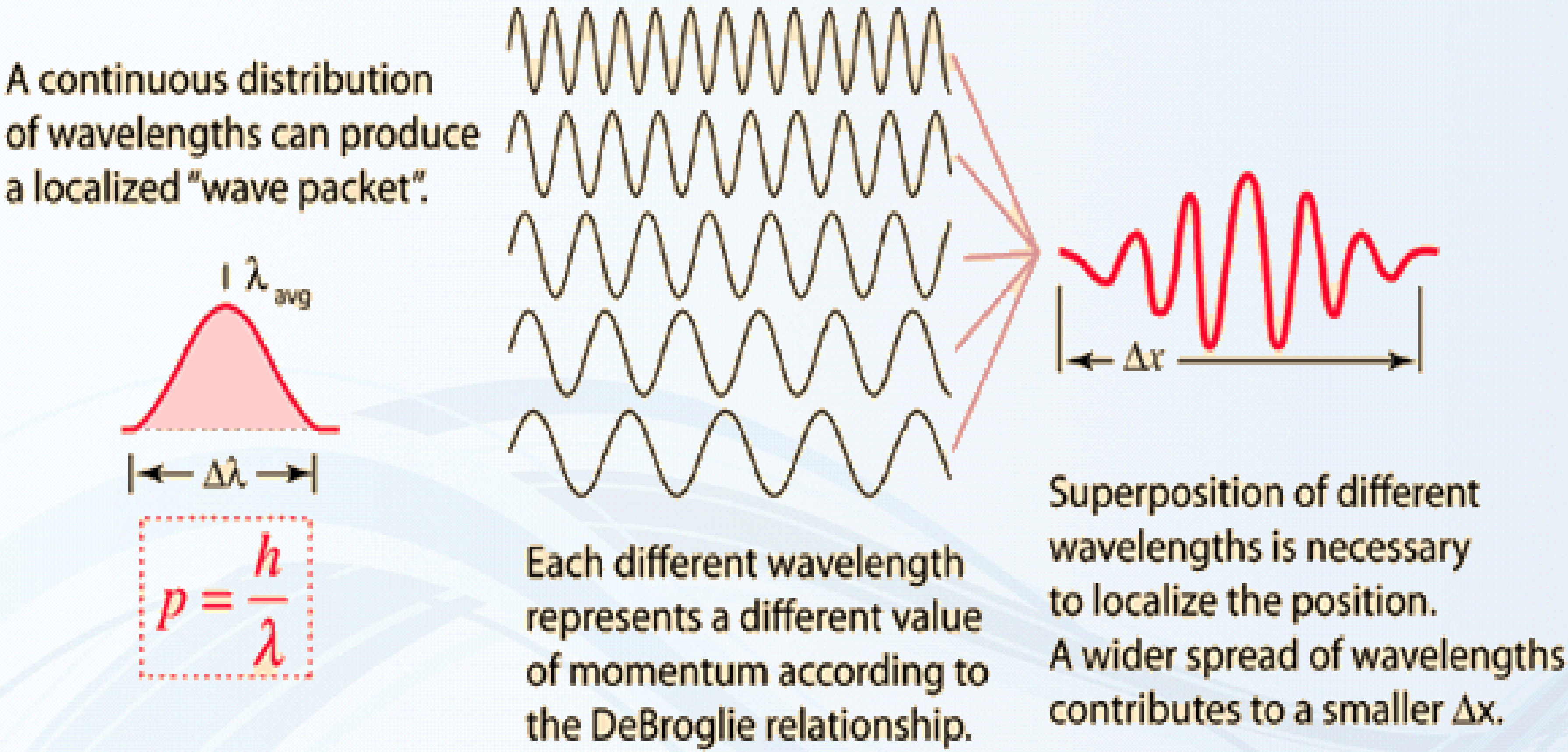
Adding several waves of different wavelength together will produce an interference pattern which begins to localize the wave.



But that process spreads the momentum values and makes it more uncertain. This is an inherent and inescapable increase in the uncertainty Δp when Δx is decreased.

$$\boxed{\Delta x \Delta p > \frac{\hbar}{2}}$$

Uncertainty Principle



Uncertainty Principle

The narrower the group, the broader the range of wave numbers needed to describe it, and vice versa

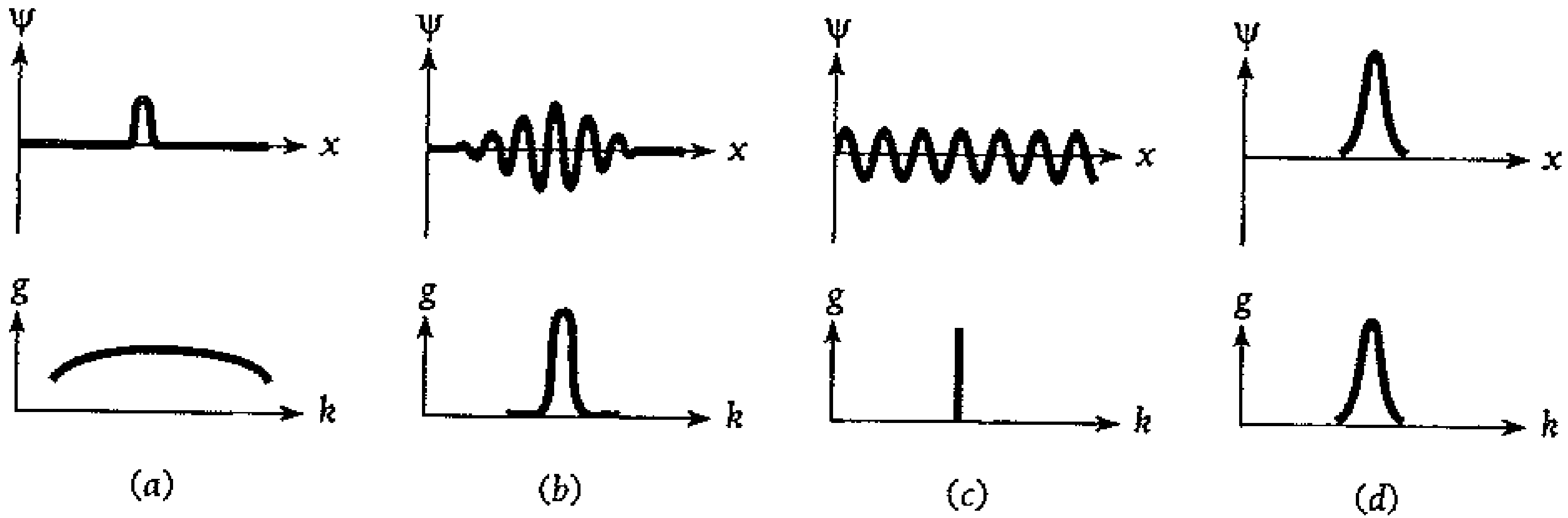
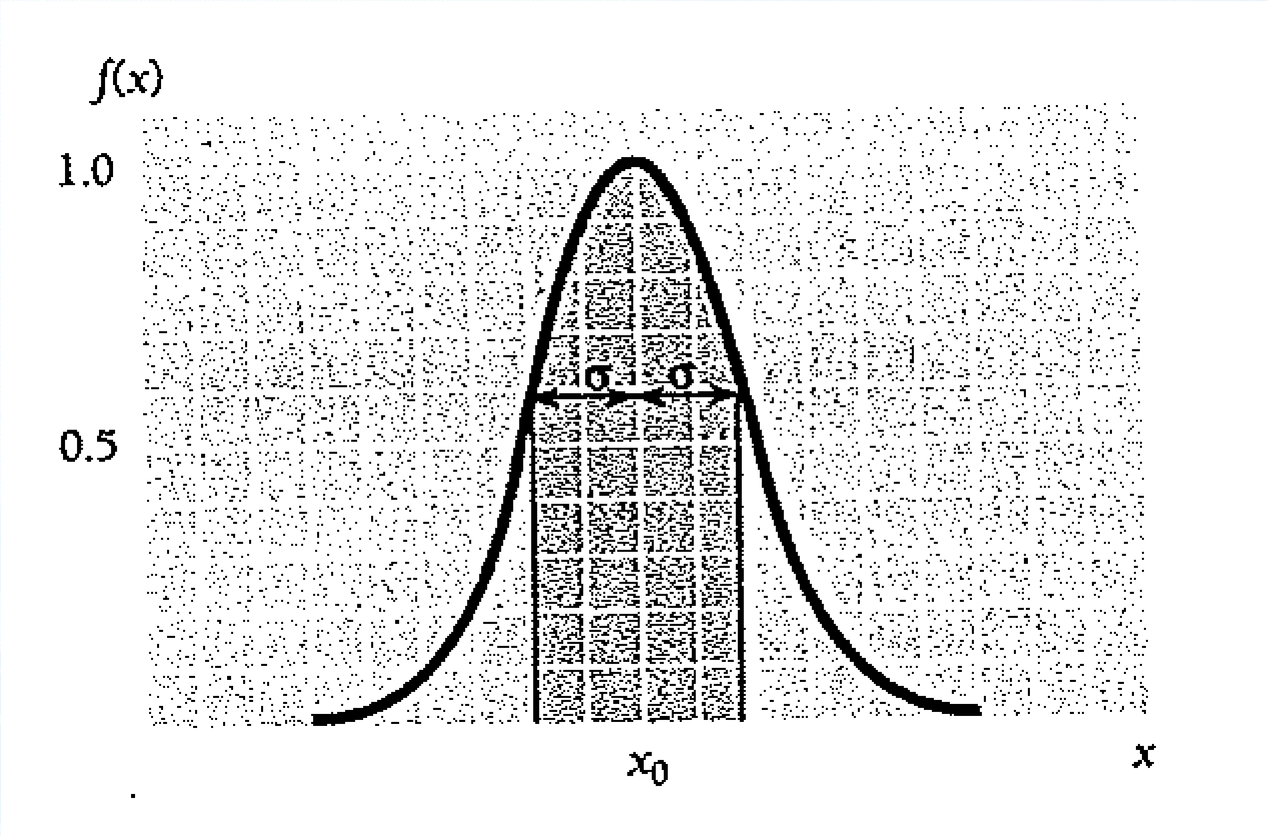


Figure 3.14 The wave functions and Fourier transforms for (*a*) a pulse, (*b*) a wave group, (*c*) a wave train, and (*d*) a Gaussian distribution. A brief disturbance needs a broader range of frequencies to describe it than a disturbance of greater duration. The Fourier transform of a Gaussian function is also a Gaussian function.

Uncertainty Principle



Uncertainty Principle

Standard deviation $$\sigma = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(x_1 - x_0)^2}$$

The width of a Gaussian curve at half its maximum value is 2.35σ.

The *Gaussian function* $f(x)$ that describes the above curve is given by

Gaussian function $$f(x) = \frac{1}{\sigma\sqrt{2\pi}}e^{-(x-x_0)^2/2\sigma^2}$$

where $f(x)$ is the probability that the value x be found in a particular measurement. Gaussian functions occur elsewhere in physics and mathematics as well. (Gabriel Lippmann had this to say about the Gaussian function: "Experimentalists think that it is a mathematical theorem while mathematicians believe it to be an experimental fact.")

The probability that a measurement lie inside a certain range of x values, say between x_1 and x_2, is given by the area of the $f(x)$ curve between these limits. This area is the integral

$$P_{x_1x_2} = \int_{x_1}^{x_2} f(x)\,dx$$

An interesting questions is what fraction of a series of measurements has values within a standard deviation of the mean value x_0. In this case $x_1 = x_0 - \sigma$ and $x_2 = x_0 + \sigma$, and

$$P_{x_0\pm\sigma} = \int_{x_0-\sigma}^{x_0+\sigma} f(x)\,dx = 0.683$$

Hence 68.3 percent of the measurements fall in this interval, which is shaded in Fig. 3.15. A similar calculation shows that 95.4 percent of the measurements fall within two standard deviations of the mean value.

Uncertainty Principle

The de Broglie wavelength of a particle of momentum p is $\lambda = h/p$ and the corresponding wave number is

$$k = \frac{2\pi}{\lambda} = \frac{2\pi p}{h}$$

In terms of wave number the particle's momentum is therefore

$$p = \frac{hk}{2\pi}$$

Hence an uncertainty Δk in the wave number of the de Broglie waves associated with the particle results in an uncertainty Δp in the particle's momentum according to the formula

$$\Delta p = \frac{h\,\Delta k}{2\pi}$$

Since $\Delta x\,\Delta k \geq \frac{1}{2}$, $\Delta k \geq 1/(2\Delta x)$ and

Uncertainty principle

$$\Delta x\,\Delta p \geq \frac{h}{4\pi} \tag{3.21}$$

This equation states that the product of the uncertainty Δx in the position of an object at some instant and the uncertainty Δp in its momentum component in the x direction at the same instant is equal to or greater than $h/4\pi$.

If we arrange matters so that Δx is small, corresponding to a narrow wave group, then Δp will be large. If we reduce Δp in some way, a broad wave group is inevitable and Δx will be large.