Course: Engineering Physics
PHY 1701

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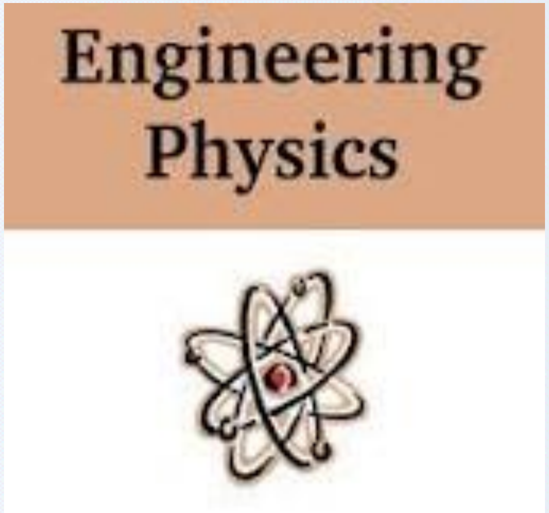
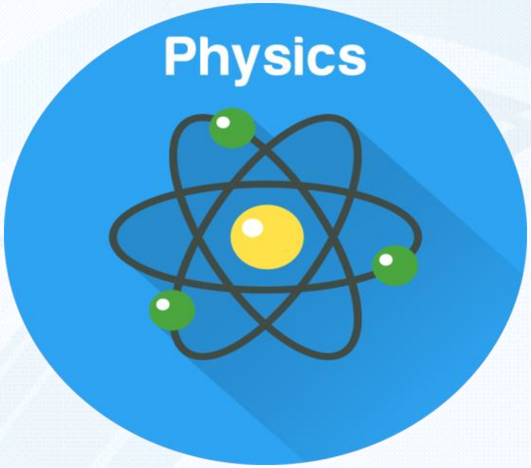
Outline

- Schrodinger Wave Equation
- Time independent
- Time dependent



Resources:

Concepts of Modern Physics (Arthur Beiser)
Pages: 187 – 190 & 195 – 197



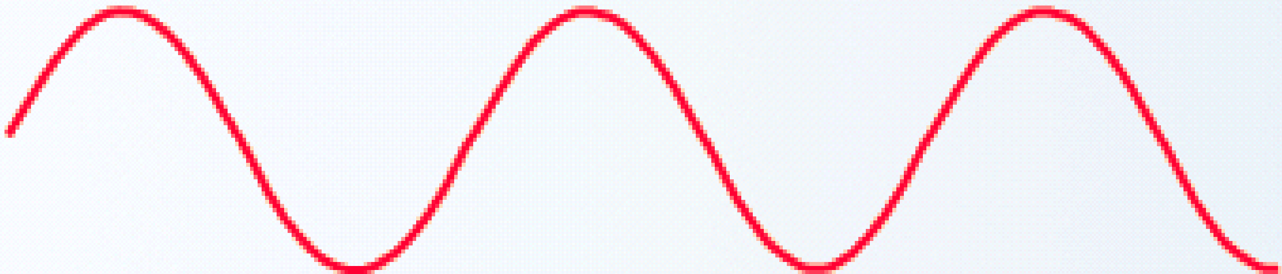
Schrodinger Wave Equation

Schrodinger's equation shows all of the wave like properties of matter and was one of greatest achievements of 20th century science.

It is used in physics and most of chemistry to deal with problems about the atomic structure of matter.

It is an extremely powerful mathematical tool and the whole basis of wave mechanics.

Schrodinger Wave Equation



Ψ = wavefunction for electron

$$\Psi = A\cos\left(\frac{2\pi}{\lambda}x - \omega t\right)$$

Using the deBroglie relationship

$$\frac{2\pi}{\lambda} = \frac{2\pi p}{h} = k$$

p = electron momentum

Using the Planck relationship

$$\omega = \frac{\hbar\omega}{\hbar} = \frac{E}{\hbar}$$

E = electron energy

Schrodinger Wave Equation

Time-Independent Schrodinger Wave Equation

The motion of the particles in waves

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial z^2} - \frac{1}{u^2}\frac{\partial^2 \psi}{\partial t^2} = 0$$

u → velocity

$$\psi(x, y, z, t) = \psi_o(x, y, z)e^{-i\omega t}$$

$$\frac{\partial^2 \psi}{\partial t^2} = -\omega^2 \psi_o e^{-i\omega t}$$

$$\frac{\partial^2 \psi}{\partial t^2} = -\omega^2 \psi$$

The above eqn. becomes

$$\frac{\partial^2 \Psi}{\partial x^2} + \frac{\partial^2 \Psi}{\partial y^2} + \frac{\partial^2 \Psi}{\partial z^2} + \frac{\omega^2}{u^2}\Psi = 0$$

$$\omega = 2\pi\gamma$$

$$\omega = 2\pi\frac{u}{\lambda}$$

$$\frac{\omega}{u} = \frac{2\pi}{\lambda}$$

Schrodinger Wave Equation

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial z^2} + \frac{4\pi^2}{\lambda^2}\psi = 0$$

Substituting

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

$$\nabla^2 \psi + \frac{4\pi^2}{\lambda^2}\psi = 0$$

Schrodinger Wave Equation

We know that de Broglie relation is

$$\lambda = \frac{h}{mv}$$

$$\nabla^2 \psi + \frac{4\pi^2}{\lambda^2}\psi = 0$$

Therefore, we get

$$\nabla^2 \psi + \frac{4\pi^2 m^2 v^2}{h^2}\psi = 0$$

Schrodinger Wave Equation

$$\nabla^2 \psi + \frac{p^2}{\hbar^2} \psi = 0$$

P=mv

$$\hbar = \frac{h}{2\pi}$$

According to the principle of conservation of energy

TE = KE + PE

$$E = \frac{p^2}{2m} + V(x, y, z)$$

$$\nabla^2 \psi + \frac{2m}{\hbar^2}(E - V)\psi = 0$$

$$\boxed{\nabla^2 \psi + \frac{2m}{\hbar^2} E \psi = 0}$$

For free particle
V=0

Schrodinger Wave Equation

Second derivative with respect to X

Shrodinger Wave Function

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{8\pi^2 m}{h^2}(E - V)\psi = 0$$

Position

Energy

Potential Energy

Schrodinger Wave Equation

Time-Dependent Schrodinger Wave Equation

The periodic displacement of a particle is represented by

$$\psi = \psi_o e^{-i\omega t}$$

$$\frac{\partial \psi}{\partial t} = -i\omega\psi_o e^{-i\omega t}$$

$$\frac{\partial \psi}{\partial t} = -i\frac{E}{\hbar}\psi$$

$$E\psi = i\hbar\frac{\partial \psi}{\partial t}$$

$$E = \hbar\omega$$

$$\omega = \frac{E}{\hbar}$$

Schrodinger Wave Equation

We have, time-independent Schrodinger equation as..

$$\nabla^2 \psi + \frac{2m}{\hbar^2}(E - V)\psi = 0$$

$$E\psi = i\hbar \frac{\partial \psi}{\partial t}$$

$$\nabla^2 \psi + \frac{2m}{\hbar^2}\left(i\hbar \frac{\partial \psi}{\partial t} - V\psi\right) = 0$$

$$-\frac{\hbar^2}{2\mathrm{m}}\nabla^2 \psi + V\psi = i\hbar \frac{\partial \psi}{\partial t}$$

H $\rightarrow -\frac{\hbar^2}{2\mathrm{m}}\nabla^2 + V$

& E $\rightarrow i\hbar \frac{\partial}{\partial t}$

$$\boxed{H\psi = E\psi}$$

Schrodinger Wave Equation

The **Schrodinger equation** is used to find the allowed energy levels of quantum mechanical systems (such as atoms, or transistors). The associated **wave function** gives the probability of finding the particle at a certain position. The solution to this **equation** is a **wave** that describes the quantum aspects of a system.