Error Analysis in Physics Laboratory

1.1 Introduction to Uncertainty

In introductory lab work, such as in Physics labs, you usually know in advance what the result is supposed to be. You can compare your actual result with the anticipated result, and calculate an actual error value. In real-world laboratory work, on the other hand, you usually *don't* know in advance what the result is supposed to be. If you *did*, you probably wouldn't be doing the experiment in the first place! When you state your final result, it's important to state also, how much you think you can trust that result, in the form of a numerical **uncertainty (or error in measurement)**. For example, you might state the volume of an object as

$$V = 43.25 \pm 0.13\,\text{cm}^3 \tag{1.1}$$

When we state the uncertainty in this form, without further elaboration, it generally means that we think that the true value has about a 68% chance of being within that range. A more precise statement would include the confidence level of the uncertainty range, which might be 68% or 95% or even 99%. Usually, in an experiment we measure some number of quantities directly, and combine them mathematically to get a final result. Therefore, estimating the final uncertainty usually involves two steps. First, we must estimate the uncertainties in the individual quantities that we measure directly. Second, we must combine those uncertainties to get the overall uncertainty, in a way that corresponds to the way that we combine individual measurements to get the final result.

1.2 Estimating the Uncertainty in a Single Measurement

1.2.1 Normal analog scale

(e.g. meter stick) Estimate the final digit by interpolating between the smallest scale divisions, and make the uncertainty ±1 or ±2 in that last digit (use your judgment in deciding).

1.2.2 Analog scale with vernier

(e.g. vernier caliper or micrometer) Use the vernier scale to get the last digit, and make the uncertainty ±0.5 of that last digit.

1.2.3 Digital scale

(e.g. digital multimeter) If the reading is steady, make the uncertainty ±0.5 of the last digit; otherwise take several instantaneous readings, average them, and find the standard deviation of the mean as described below.

1.3 Estimating the Uncertainty in an Averaged Measurement

If you can make several measurements $x_1, x_2, x_3 \ldots\ldots\ldots x_N$ calculate the mean, $\bar{x}$, and use that as "the" measurement. Then calculate the standard deviation of the mean:

$$\sigma_m = \frac{\sqrt{(x_1 - \bar{x})^2 + (x_2 - \bar{x})^2 + \ldots\ldots\ldots\ldots + (x_N - \bar{x})^2}}{N} \tag{1.2}$$

and use this as the uncertainty, Δx. If your calculator has a standard deviation function, divide its result by $\sqrt{N}$ to get the standard deviation of the mean.

1.4 Combining Uncertainties in Calculated Results

In the following equations, Δx means the absolute uncertainty in x, which is the number you get from one of the methods above; it has units just like the measurement itself has. $\Delta x\%$ means the percent (or fractional) uncertainty in *x*, which is the uncertainty expressed as a percentage or fraction of the measurement; it has no units.

1.4.1 Addition and Subtraction

If $z = x + y$ or $z = x - y$

$$\Delta z = \sqrt{\Delta x^2 + \Delta y^2} \tag{1.3}$$

If you're adding and subtracting more variables, simply add more terms inside the square root.

1.4.2 Multiplication and Division

If $z = xy$ or $z = x/y$, then

$$\frac{\Delta z}{z} = \sqrt{\left(\frac{\Delta x}{x}\right)^2 + \left(\frac{\Delta y}{y}\right)^2} \tag{1.4}$$

or (same thing in different notation).

$$\Delta z\% = \sqrt{(\Delta x\%)^2 + (\Delta y\%)^2} \tag{1.5}$$

If you're multiplying and dividing more variables, simply add more terms inside the square root.

1.4.3 Powers, Including Roots

If $z = x^n$, then

$$\frac{\Delta z}{z} = n\left(\frac{\Delta x}{x}\right) \tag{1.6}$$

1.4.4 More Complicated Calculations

Sometimes you can combine the three rules given above, doing the calculation one step at a time, combining uncertainties as you go along, and switching back and forth between absolute and percent uncertainties as necessary. However, you cannot do this if the same variable appears more than once in the equation or calculation, or if you have situations not covered by the rules given above, such as trig functions. In such cases you must use the general procedure given below.

The following table gives an idea about the relation between error and actual equation:

Table 1.1

Sr. No.	Relation between Z and (A, B)	Relation between Δz and (ΔA and ΔB)
1.	$Z = A + B$	$(\Delta z)^2 = (\Delta A)^2 + (\Delta B)^2$
2.	$Z = A - B$	$(\Delta z)^2 = (\Delta A)^2 + (\Delta B)^2$
3.	$Z = AB$	$\left(\frac{\Delta z}{z}\right)^2 = \left(\frac{\Delta A}{A}\right)^2 + \left(\frac{\Delta B}{B}\right)^2$
4.	$Z = A / B$	$\left(\frac{\Delta z}{z}\right)^2 = \left(\frac{\Delta A}{A}\right)^2 + \left(\frac{\Delta B}{B}\right)^2$
5.	$Z = An$	$\frac{\Delta z}{z} = n\left(\frac{\Delta A}{A}\right)$
6.	$Z = \ln A$	$\Delta Z = \frac{\Delta A}{A}$
7.	$Z = eA$	$\frac{\Delta Z}{Z} = \Delta A$

1.5 General Procedure

If $z = f(x, y)$ first calculate the differences caused by the uncertainty in each variable separately:

$$(\Delta z)_x = f((x + \Delta x), y) - f(x, y)$$
$$(\Delta z)_y = f(x, (y + \Delta y)) - f(x, y) \tag{1.7}$$

Then combine the differences to get the total uncertainty:

$$(\Delta z) = \sqrt{(\Delta z)_x^2 + (\Delta z)_y^2} \tag{1.8}$$

If there are more variables, extend these equations appropriately by adding more terms. If a variable occurs more than once in the formula for *f*(*x*,*y*), change all occurrences simultaneously when calculating the difference for that variable.

To illustrate the procedure for calculation the error in an experiment, we will work out the average (mean) value *x* and the standard deviation of the mean, $\overline{\sigma}$ and the standard deviation of an individual data point, σ, using the position measurements in the accompanying Table 1.2

Table 1.2

x_i (m)	$(x_i - \bar{x}_i)$ (m)	$(x_i - \bar{x}_i)^2$ (m²)
15.68	0.15	0.0225
15.42	0.11	0.0121
15.03	0.50	0.2500
15.66	0.13	0.0169
15.17	0.36	0.1296
15.89	0.36	0.1296
15.35	0.18	0.0324
15.81	0.28	0.0784
15.62	0.09	0.0081
15.39	0.14	0.0196
15.21	0.32	0.1024
15.78	0.25	0.0625

15.46	0.07	0.0049
15.12	0.41	0.1681
15.93	0.40	0.1600
15.23	0.30	0.0900
15.62	0.09	0.0081
15.88	0.35	0.1225
15.95	0.42	0.1764
15.37	0.16	0.0256
15.51	0.02	0.0004

From the above table we can make the following calculations:

$$N = 21\sum_{i=1}^{N} x_i = 326:08m, \sum_{i}^{N}(x_i - \bar{x}_i)^2 = 1:61998m^2$$

and then evaluate the following quantities:

$$\bar{x} = \frac{\sum_{i=1}^{N} x_i}{N} = \frac{326.08}{21} = 15.53m \tag{1.9}$$

$$\bar{\sigma} = \sqrt{\frac{\sum_{i=1}^{N}(x_i - \bar{x})^2}{N(N-1)}} = \sqrt{\frac{1.6201}{20.21}} = 0.062m \tag{1.10}$$

$$\sigma = \sqrt{\frac{\sum_{i=1}^{N}(x_i - \bar{x})^2}{(N-1)}} = \sqrt{\frac{1.6201}{20.0}} = 0.063m \tag{1.11}$$

The error or spread in individual measurements is σg 0:28 m. But for the mean $\bar{x} + \bar{\sigma} = 15.53 \pm 0.06$ m. This says the average is 15.53 m which has an error of 0.06m. Or putting it another way, there is about a 68% probability that the true value of *x* falls in the range 15.47m to 15.59m. In some cases the fractional error $(\sigma/\bar{x})$, or relative error, is of more interest than the absolute value of σ. It is possible that the size of σ is large while the fractional error is small. Note that increasing the number of individual measurements on the uncertainty of the average reduces the statistical uncertainty (random errors); this improves the "precision". On the other hand, more measurements do not diminish systematic error in the mean because these are always in the same direction; the "accuracy" of the experiment is limited by systematic errors.

1.6 Suggested Experiments:

1) Measure the diameter of a wire using a screw gauge at 10 different places on the wire. Calculate the standard deviation in your measurements.
2) Measure the thickness of a tabletop at using a scale in cm. Calculate the error in your measurements.
3) Measure the period of oscillations of a pendulum using your wrist watch and record your data ten times. Estimate the standard deviation and error in your measurements.
4) Ask your partner to drop a solid object at a same height for 10 times. Measure the time of flight with your wrist watch. The same can be repeated by your other partners also. Compare the standard deviation of each of your measurements.

Assessment of purity of a given liquid by determining its refractive index

Aim

To determine the purity of a liquid in terms of refractive index using travelling microscope

Apparatus required

Travelling microscope, reading lens, 50 ml beaker, water, saw dust, etc.,

Formula

Refractive Index of the liquid $\mu = \frac{(C-A)}{(C-B)}$ (no unit)

where

A- Reading of the microscope when the ink dot is focused directly (cm)

B- Reading of the microscope when the ink dot is focused through water (cm)

C- Reading of the microscope when the saw dust is focused (cm)

Procedure

1. Focus ink dot marked at the bottom of the beaker directly and take the readings (MSR and VSC) in the vertical scale (A).
2. Pour ~ 20 ml of water into beaker without disturbing it.
3. Now, focus the ink dot through water by adjusting the vertical adjustment screw (Do not use the focusing knob) and take the reading (MSR and VSC) in the vertical scale. (B).
4. Sprinkle some saw dust on the surface of water gently without disturbing the set-up.
5. Focus the saw dust by adjusting the vertical adjustment screw (Do not use the focusing knob) and take the readings (MSR and VSC) in the vertical scale (C).
6. Pour out the water.
7. Repeat the experiment (step 1 and step 5) for different quantities/levels (~ 40 ml, 60 ml, 80 ml) of water and tabulate the readings.
8. Calculate the refractive index μ using the formula.

Tabulation

S. No	Reading when the ink dot is directly focused (A)			Reading when the ink dot is directly focused through water (B)			Reading when the saw dust is focused (C)			(C- A)	(C - B)	μ
	MSR	VSC	TR	MSR	VSC	TR	MSR	VSC	TR			
	cm	div	cm	cm	div	cm	cm	div	cm			
										Mean μ =		

TR= MSR + (VSC x LC)

Calculations

$$\mu = \frac{(C - A)}{(C - B)} \text{ (no unit)}$$

μ = ____________

Result

The purity of the given liquid evaluated in terms of Refractive Index is determined to be μ = ____________

Viva voce questions:

1. Why should we focus the ink dot in the beaker in this experiment?
2. Why should we use sawdust in this experiment?
2. Differentiate between actual depth and apparent depth.
3. How do you calculate the percentage of error?
4. What is the significance of this experiment is day to day life?

Determination of the number lines of a given grating using a laser source for display applications

Aim

To determine the number of lines in a given grating using a laser source of light.

Apparatus required

He-Ne laser or semiconducting laser, grating, scale, grating stand

Formula

$$N = \frac{\sin(\theta)}{n\lambda} \text{ lines per meter}$$

where

λ- Wavelength of the laser light used in the experiment (nm),

θ - Angle of diffraction (degree)

n - Order of diffraction,

N-The density of lines in the grating = ———— lines/meter.

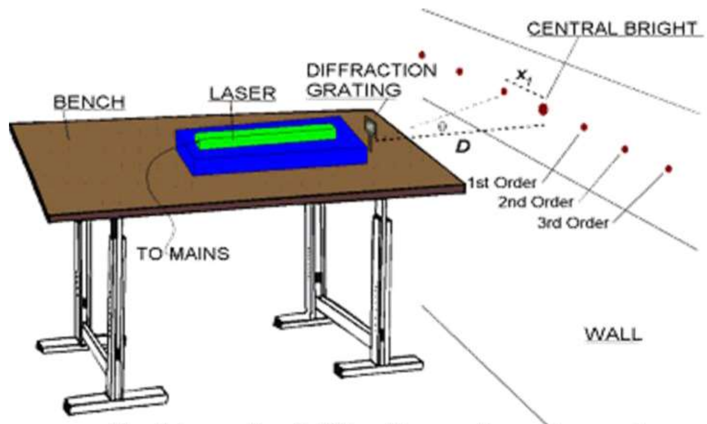

Fig. Schematic of diffraction grating setup and its diffraction pattern

Procedure

1. The grating is held normal to the laser beam at a distance D (~ 30 cm) from the screen.
2. The laser light is switched on and it is diffracted by the grating.
3. Symmetric weaker spots corresponding to different orders (n= 1,2,3,4,5,..) of diffraction will be observed around a central bright spot
4. The distances (2L) between the spots on either side of the central spot corresponding to various orders is measured and tabulated.
5. Step 4 is repeated for different values of D (~ 35 cm, 40 cm,45 cm, 50 cm).
6. The wavelength λ is calculated using the formula.

Tabulation

Diffraction Order n	D	2L	L	tan θ (L/D)	θ $\tan^{-1}$ (L/D)	Sin θ	Mean sin θ	N
	centimeters				*degrees*			*Lines/meter*
1	30							
	35							
	40							
	45							
	50							
2	30							
	35							
	40							
	45							
	50							
3	30							
	35							
	40							
	45							
	50							
4	30							
	35							
	40							
	45							
	50							
5	30							
	35							
	40							
	45							
	50							
							Mean N =	

Observations

For n = 1; Mean sin θ = ___________________________

For n = 2; Mean sin θ = ___________________________

Result

The density of the lines in the given grating was determined to be N = ____________________

Viva voce question

1. What is the type of the laser you used in the laboratory?
2. What is the reason for serial of light spots appearing on the measuring scale?
3. Distinguish between laser source and conventional light source.
4. Define grating element.
5. What are the requisites of good grating?
6. What is meant by diffraction of light?
7. Comment on: Grating with larger number of rulings per cm is always preferable.
8. what is the significance of the experiment

~~Date:~~ ~~Reg. No.~~

INTEGRATED OPTICS

Apparatus Available:

- Spectrometer
- Spirit level
- Magnifying glass
- Glass prism
- Mercury vapour lamp

SLO:

✓ To determine the apex angle of the given prism using a spectrometer

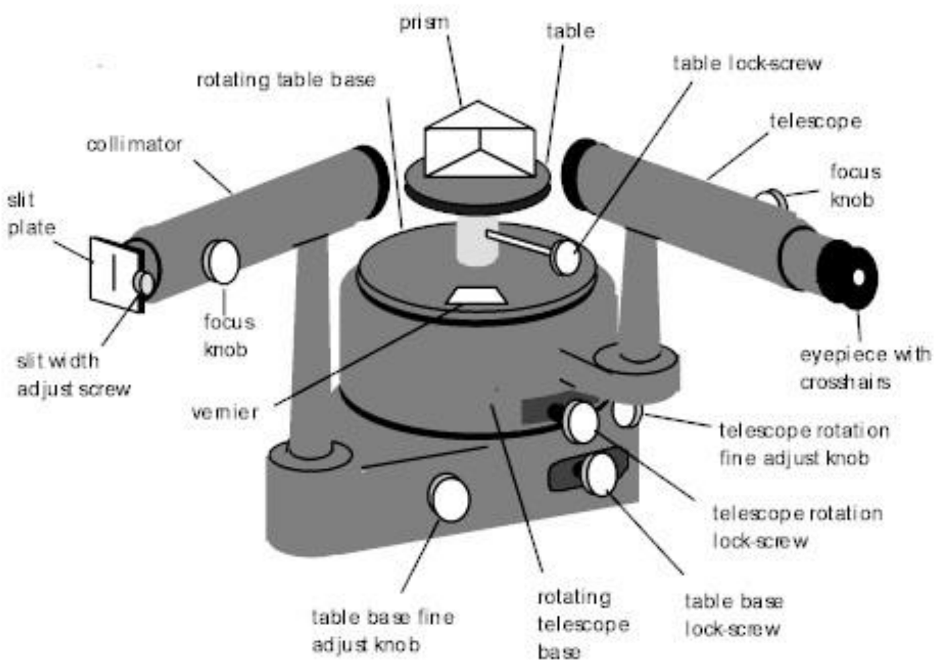

Figure 1. *Schematic diagram of a spectrometer*

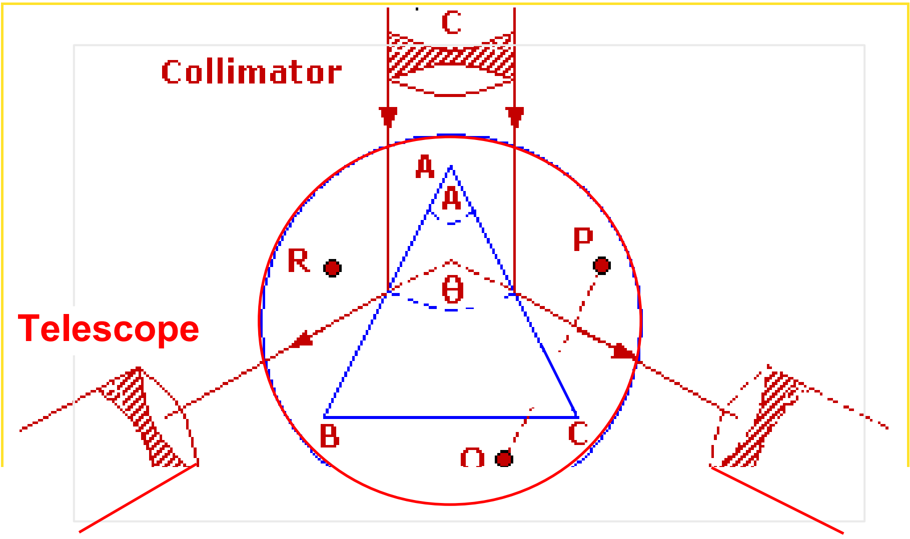

Figure 2 *Ray diagram for Angle of Prism*

Tabulation:

Least count = -----------

Reading of reflected ray	Vernier A			Vernier B		
	MSR	VSR	Total	MSR	VSR	Total
Reflection from side (a)AB						
Reflection from side (b) AC						
Difference between a & b						

Mean θ = …………

Result

The Apex angle of the given equilateral prism = ___________________