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Tutorial Sheet I(PHY1701)

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Useful formula

1. BBR: $u(\nu, T)d\nu = \frac{8\pi\nu^3 h\ d\nu}{c^3(e^{\frac{h\nu}{kT}}-1)}$;$u(\lambda, T)d\lambda = \frac{8\pi hc\ d\lambda}{\lambda^5(e^{\frac{hc}{\lambda kT}}-1)}$
2. Wien's energy dist: $u(\nu, T)d\nu = A\nu^3 e^{-\frac{\beta\nu}{T}} d\nu$
3. Heisenberg's Uncertainty: $\Delta p_x \Delta x \geq \frac{\hbar}{2}$, $\Delta p_y \Delta y \geq \frac{\hbar}{2}$, $\Delta p_z \Delta z \geq \frac{\hbar}{2}$, $\Delta E \Delta t \geq \frac{\hbar}{2}$
4. Rayleigh's energy dist: $u(\nu, T)d\nu = \frac{8\pi\nu^2(kT)}{c^3} d\nu$
5. Wien's displacement law: $\lambda_{\max} T = 2.898 \times 10^{-3}$ m K
6. Compton's Effect: $\Delta\lambda = \lambda_c(1 - \cos\theta)$ here $\lambda_c = 2.426 \times 10^{-12}$m
7. Bragg's condition: $2d \sin\theta = n\lambda$
8. de Broglie: $\lambda = \frac{h}{p} = \frac{h}{\gamma m_0 u}$
9. STDE: $-i\hbar\frac{\partial\Psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\Psi + V\Psi$; STIE: $\nabla^2\psi + \frac{2m}{\hbar^2}(E - V)\psi = 0$
10. Particle in 1D box (centered at$L/2$): Normalized wave function $\psi_n(x) = \sqrt{\frac{2}{L}} \sin(\frac{n\pi x}{L})$;
    Energy eigenvalue: $E_n = \frac{n^2\pi^2\hbar^2}{2mL^2}$
11. Particle in 3D cubic box:
    Normalized wave function: $\psi(x, y, z) = (\sqrt{\frac{2}{L}})^3 \sin(\frac{n_x\pi x}{L}) \sin(\frac{n_y\pi y}{L}) \sin(\frac{n_z\pi z}{L})$
12. Tunneling effect: Probability $T = e^{-2k_2 L}$ where $k_2 = \frac{\sqrt{2m(V-E)}}{\hbar}$

Problems

0.1 *Compton Effect*

3. What is the frequency of an x-ray photon whose momentum is 1.1×10^{-23} kg m/s?

4. How much energy must a photon have if it is to have the momentum of a 10-MeV proton?

5. A monochromatic x-ray beam whose wavelength is 55.8 pm is scattered through 46 deg. Find the wavelength of the scattered beam.

6. A beam of x-rays is scattered by a target. At 45 deg from the beam direction the scattered x-rays have a wavelength of 2.2 pm. What is the wavelength of the x-rays in the direct beam?

7. An x-ray photon whose initial frequency was 1.5×10^{19} Hz emerges from a collision with an electron with a frequency of 1.2×10^{19} Hz. How much kinetic energy was imparted to the electron?

8. An x-ray photon of initial frequency 3.0×10^{19} Hz collides with an electron and is scattered through 90 deg. Find its new frequency.

9 Find the energy of an x-ray photon which can impart a maximum energy of 50 keV to an electron.

10. At what scattering angle will incident 100-keV x-rays leave a target with an energy of 90 keV?

11. (a) Find the change in wavelength of 80-pm x-rays that are scattered 120 deg by a target. (b) Find the angle between the directions of the recoil electron and the incident photon. (c) Find the energy of the recoil electron.

12. A photon of frequency ν is scattered by an electron initially at rest. Verify that the maximum kinetic energy of the recoil electron is $\text{KE(max)} = \frac{2h^2\nu^2/mc^2}{(1+2h\nu/mc^2)}$

13. In a Compton-effect experiment in which the incident x-rays have a wavelength of 10.0 pm, the scattered x-rays at a certain angle have a wavelength of 10.5 pm. Find the momentum (magnitude and direction) of the corresponding recoil electrons.

14. A photon whose energy equals the rest energy of the electron undergoes a Compton collision with an electron. If the electron moves off at an angle of 40 deg with the original photon direction, what is the energy of the scattered photon?

0.2 *de Broglie Waves*

15. A photon and a particle have the same wavelength. Can anything be said about how their linear momenta compare? About how the photon's energy compares

with the particle's total energy? About how the photon's energy compares with the particle's kinetic energy?

16. Find the de Broglie wavelength of (a) an electron whose speed is 1.0×10^8 m/s, and (b) an electron whose speed is 2.0×10^8 m/s.

17. Find the de Broglie wavelength of a 1.0 mg grain of sand blown by the wind at a speed of 20 m/s.

18. Find the de Broglie wavelength of the 40-keV electrons used in a certain electron microscope.

19. By what percentage will a nonrelativistic calculation of the de Broglie wavelength of a 100 keV electron be in error?

20. Find the de Broglie wavelength of a 1.00 MeV proton. Is a relativistic calculation needed?

21. The atomic spacing in rock salt, NaCl, is 0.282 nm. Find the kinetic energy (in eV) of a neutron with a de Broglie wavelength of 0.282 nm. Is a relativistic calculation needed? Such neutrons can be used to study crystal structure.

22. Find the kinetic energy of an electron whose de Broglie wavelength is the same as that of a 100 keV x-ray.

23. Green light has a wavelength of about 550 nm. Through what potential difference must an electron be accelerated to have this wavelength?

0.3 *Particle Diffraction*

24. A beam of neutrons that emerges from a nuclear reactor contains neutrons with a variety of energies. To obtain neutrons with an energy of 0.050 eV, the beam is passed through a crystal whose atomic planes are 0.20 nm apart. At what angles relative to the original beam will the desired neutrons be diffracted?

25. Consider a beam of 54-eV electrons directed at a nickel target. The potential energy of an electron that enters the target changes by 26 eV. (a) Compare the electron speeds outside and inside the target. (b) Compare the respective de Broglie wavelengths.

26. A beam of 50 keV electrons is directed at a crystal and diffracted electrons are found at an angle of 50 deg relative to the original beam. What is the spacing of the atomic planes of the crystal?

0.4 *Particle in a Box*

27. Obtain an expression for the energy levels (in MeV) of a neutron confined to a one-dimensional box 1.00×10^{14} m wide. What is the neutron's minimum energy? (The diameter of an atomic nucleus is of this order of magnitude.)

28. The lowest energy possible for a certain particle trapped in a certain box is 1.00 eV. (a) What are the next two higher energies the particle can have? (b) If the particle is an electron, how wide is the box?

29. A proton in a one-dimensional box has an energy of 400 keV in its first excited state. How wide is the box?

0.5 *Heisenberg's Principle*

30. Discuss the prohibition of E_0 for a particle trapped in a box L wide in terms of the uncertainty principle. How does the minimum momentum of such a particle compare with the momentum uncertainty required by the uncertainty principle if we take $\Delta x = L$?

31. The atoms in a solid possess a certain minimum zero-point energy even at 0 K, while no such restriction holds for the molecules in an ideal gas. Use the uncertainty principle to explain these statements.

32. Compare the uncertainties in the velocities of an electron and a proton confined in a 1.00 nm box.

33. The position and momentum of a 1.00 keV electron are simultaneously determined. If its position is located to within 0.100 nm, what is the percentage of uncertainty in its momentum?

34. (a) How much time is needed to measure the kinetic energy of an electron whose speed is 10.0 m/s with an uncertainty of no more than 0.100 percent? How far will the electron have traveled in this period of time? (b) Make the same calculations for a 1.00 g insect whose speed is the same. What do these sets of figures indicate?

35. How accurately can the position of a proton with $u << c$ be determined without giving it more than 1.00 keV of kinetic energy?

36. A marine radar operating at a frequency of 9400 MHz emits groups of electromagnetic waves 0.0800 s in duration. The time needed for the reflections of these groups to return indicates the distance to a target. (a) Find the length of each group and the number of waves it contains. (b) What is the approximate minimum bandwidth (that is, spread of frequencies) the radar receiver must be able to process?

0.6 *Wave Function and Schroedinger's Equation*

37. A particle limited to the x axis has the wave function $\Psi(x) = a\ x$ between $x = 0$ and $x = 1$; $\Psi(x) = 0$ elsewhere. (a) Find the probability that the particle can be found between $x = 0.45$ and $x = 0.55$. (b) Find the expectation value $< x >$ of the particle's position.

38. An eigenfunction of the operator $\frac{d^2}{dx^2}$ is $f(x) = e^{4x}$. Find the corresponding eigenvalue.

39. Find the probability that a particle trapped in a box L wide can be found between $0.45L$ and $0.55L$ for the ground and first excited states.

40. Find the expectation value $< x >$ of the position of a particle trapped in a box L wide.

41. Find the normalization constant B for the combination $\psi = B(\sin\frac{\pi x}{L} + \sin\frac{2\pi x}{L})$

42. A particle of mass m in a one-dimensional box has the following wave function in the region $x = 0$ to $x = L$: $\Psi(x,t) = \frac{1}{\sqrt{2}}\psi_1(x)e^{-\frac{iE_1 t}{\hbar}} + \frac{1}{\sqrt{2}}\psi_3(x)e^{-\frac{iE_3 t}{\hbar}}$ Here $\psi_1(x)$and $\psi_3(x)$ are the normalized stationary-state wave functions for the $n = 1$ and $n = 3$ levels, and E_1 and E_3 are the energies of these levels. The wave function is zero for $x < 0$ and for $x > L$. (a) Find the value of the probability distribution function at $x = L/2$ as a function of time. (b) Find the angular frequency at which the probability distribution function oscillates.

0.7 *Tunnel Effect*

43. Electrons with energies of 1.0 eV and 2.0 eV are incident on a barrier 10.0 eV high and 0.50 nm wide. (a) Find their respective transmission probabilities. (b) How are these affected if the barrier is doubled in width?

44. (a) An electron with initial kinetic energy 32 eV encounters a square barrier with height 41 eV and width 0.25 nm. What is the probability that the electron will tunnel through the barrier? (b) A proton with the same kinetic energy encounters the same barrier. What is the probability that the proton will tunnel through the barrier?