

## ☞ RATIO AND PROPORTION

A ratio is a relation between two or more quantities normally of same kind. The ratio a "to b" is written as a:b and is also expressed as a fraction  $\frac{a}{b}$  ( $b \neq 0$ ). While considering homogeneous quantities, the ratio is also expressed as percentage.

If two quantities are in the ratio a:b, it means that if the first quantity is ax, then the second quantity is bx. A and B are in the ratio a:b  $\Rightarrow$  as a proportion of the total, A is  $\frac{a}{a+b}$  and B is  $\frac{b}{a+b}$ .

Ratio is a mere number and to find the ratio of two quantities, they must be expressed in the same units.

If two ratios are equal, then the four quantities are in proportion. Thus if a:b is equal to c:d, then a, b, c, d are in proportion and the proportion is written as a:b::c:d, where a and d are called the extremes and b and c the means. Thus if a:b::c:d, then  $ad = bc$ .

**Proportionality:** The values of two quantities may not be independent. For example, the volume of a cube is proportional to its side and vice versa. Two mutually dependent quantities are proportional if the ratio of their values remains constant.

If  $\frac{a}{b} = \frac{c}{d}$ , then each term of the ratio  $\frac{a}{b}$  and  $\frac{c}{d}$  is called proportional. a, b, c and d are respectively the first, second, third and fourth proportionals. Here, a and d are known as extremes and b, c are known as means.

**Direct Proportion:** If two quantities are so related that an increase or decrease in one quantity produces a proportional increase or decrease in the other, then the two quantities are said to be directly proportional to each other, e.g. the weight of a solid material is proportional to its volume.

**Inverse Proportion:** If two quantities are so related that an increase in one of them produces a proportional decrease in the other or vice versa, then the two quantities are said to be inversely proportional to each other, e.g. the time a train takes to travel a certain distance is inversely proportional to its speed.

### Application 1

If all the members of a team are either juniors or seniors and if the ratio of the juniors to seniors in the team is 3:5, what percent of the team members are seniors?

- (a) 50%                      (b) 60%                      (c) 62.5%                      (d) 65%

**Ans: (c)**

#### Solution

Ratio of juniors to seniors

$$= 3:5 \text{ (i.e. 5 out of 8 members are seniors)}$$

Seniors as a percent of the team

$$= \frac{5}{8} \times 100 = \frac{500}{8} = 62.5$$

Percentage of seniors = 62.5%

### Application 2

A club had 3 boys and 5 girls. During a membership drive, same number of boys and girls joined the club. How many members does the club have now if the ratio of boys to girls is 3:4?

- (a) 12                      (b) 14                      (c) 16                      (d) 15

**Ans: (b)**

#### Solution

Number of boys = 3

Number of girls = 5

Let the number of boys and girls who joined during the membership drive be x each.

The increased number of boys =  $3 + x$

The increased number of girls =  $4 + x$

$$\frac{3+x}{5+x} = \frac{3}{4}$$

$$\Rightarrow 4(3+x) = 3(5+x)$$

$$\Rightarrow 12 + 4x = 15 + 3x$$

$$\Rightarrow x = 3$$

The number of boys now =  $3 + 3 = 6$  and the number of girls  
=  $5 + 3 = 8$

The club now has  $6 + 8 = 14$  members

### Application 3

If two numbers are in the ratio 6:13 and their least common multiple is 312, what is the sum of the numbers?

- (a) 60                      (b) 76                      (c) 65                      (d) 75

**Ans: (b)**

#### Solution

Let the two numbers be  $6x$  and  $13x$ .

LCM of the numbers =  $78x = 312$

$$x = \frac{312}{78} = 4$$

The two numbers are  $6 \times 4 = 24$  and  $13 \times 4 = 52$ . The sum of the 2 numbers =  $24 + 52 = 76$ .

### Application 4

The sum of the salaries of John and Mary is Rs.12000 per month. John spends 80% of his salary and Mary spends 70% of her salary. If their savings are in the ratio 4:3, what is the salary of John?

- (a) Rs.8000                      (b) Rs.8500                      (c) Rs.6500                      (d) Rs.7850

**Ans: (a)**

#### Solution

Let John's salary be Rs.X and Mary's salary be Rs.(12000 - X).

John's spending is 80% and his savings is 20%

$$\Rightarrow X \times \frac{20}{100} = 0.2X.$$

Mary's spending is 70% and her savings is 30%

$$\Rightarrow \frac{30}{100} (12000 - X).$$

The ratio of John's savings to Mary's savings = 4:3

$$\rightarrow 0.2X : 0.3(12000 - X) = 4:3$$

$$\rightarrow 3(2X) = 4 \times 3(12000 - X)$$

$$\rightarrow 6X = 144000 - 12X$$

$$\rightarrow 18X = 144000 \Rightarrow X = 8000$$

Salary of John = Rs.8000.

## Application 5

Anand, Krish and Selva have invested a total of Rs.1,20,000 in a business in the ratio 4:5:6. At the end of one year, the profit earned was Rs.12,000. What is the profit share of Krish?

- (a) Rs.3000 (b) Rs.4000 (c) Rs.5000 (d) Rs.6000

**Ans: (b)**

## Solution

$$\text{Anand : Krish : Selva} \\ 4 : 5 : 6$$

$$\text{Proportion of Krish} = \frac{5}{15}$$

$$\text{Total Profit} = \text{Rs.12,000}$$

$$\therefore \text{Profit share of Krish} = \frac{5}{15} \times \text{Rs.12000} = \text{Rs.4000}$$

## Application 6

Find the number that must be subtracted from the terms of the ratio 13:17 to make it equal to 5:9.

- (a) 7 (b) 9 (c) 8 (d) 5

**Ans: (c)**

## Solution

Let the number to be subtracted be x

$$\text{Then } \frac{13-x}{17-x} = \frac{5}{9}$$

$$\rightarrow 117 - 9x = 85 - 5x$$

$$\rightarrow x = 8$$

## Application 7

The incomes of A and B are in the ratio 7:2 and their expenditures are in the ratio 4:1. If each saves Rs.1000 find their expenditures.

- (a) Rs.20000, Rs.5000 (b) Rs.22000, Rs.7000  
(c) Rs.18000, Rs.5000 (d) Rs.19000, Rs.6000

**Ans: (a)**

## Solution

Let the incomes of A and B be 7x & 2x and their expenditures 4y & y respectively.

$$\therefore 7x - 4y = 1000 \quad \dots (1)$$

$$2x - y = 1000 \quad \dots (2)$$

$$\text{Multiplying (1) by 2, } 14x - 8y = 2000 \quad \dots (3)$$

$$\text{and multiplying (2) by 7, } 14x - 7y = 7000 \quad \dots (4)$$

$$\text{Solving (3) and (4), } y = 5000$$

$$\therefore \text{Expenditure of A} = 4y = \text{Rs.20,000} \text{ and that of B} = \text{Rs.5,000}$$

## Application 8

A mixture of 180 ml contains milk and water in the ratio 7:2. How much more water is to be added to get a new mixture contain milk and water in the ratio 7:3.

- (a) 25 ml (b) 30 ml (c) 18 ml (d) 20 ml

**Ans: (d)**

## Solution

Mixture	
Milk	Water
7	: 2
↓	↓

$$180 \text{ ml of mixture } \rightarrow 140 \text{ ml } 40 \text{ ml}$$

Let the water to be added now be x ml.

$$\therefore \text{Diluted } (180 + x) \text{ ml} \rightarrow 140 \text{ ml } 40 + x \text{ ml} \rightarrow 7:3$$

$$\rightarrow 140 \times 3 = (40 + x) 7$$

$$\text{Solving, } x = 20$$

$$\text{Water to be added} = 20 \text{ ml}$$

## Application 9

There are 3 sections in standard III of a school. The ratio of students in sections A and B is 5:7 and that in B and C is 8:9. If the total number of students in the class is 318, find the total number of students in section B.

- (a) 122 (b) 132 (c) 102 (d) 112

**Ans: (d)**

## Solution

	A	B	C
Ratio of students in the sections	5	: 7	
			8 : 9
$\therefore$ Combined ratio	40	: 56	: 63

$$\text{Total students in the three sections} = 318$$

$$\therefore \text{Students in section B} = 318 \times \frac{56}{159} = 112$$

## Application 10

The ratio of present ages of Suresh and Naresh is 7:5. If after 6 years, their ages will be in the ratio of 4:3. What is the present age difference of the two?

- (a) 10 years (b) 12 years (c) 14 years (d) 11 years

**Ans: (b)**

## Solution

	Suresh	Naresh
Present age	7	: 5 $\rightarrow 7x$ and $5x$
After 6 years, the ratio of ages	4	: 3 $\rightarrow (7x + 6)$ and $(5x + 6)$
$\therefore$	$4(5x + 6) = 3(7x + 6)$	

$$\text{Solving, } x = 6 \text{ and the present ages are 42 and 30 years.}$$

$$\therefore \text{Present age difference} = 12 \text{ years}$$

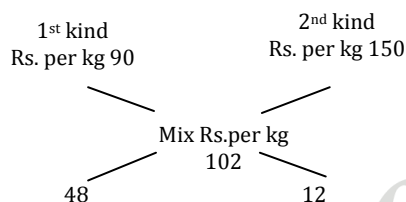
## Application 11

In what ratio the two kinds of tea, one at Rs.90 per Kg and another at Rs.150 per kg, must be mixed so that the mixture may cost Rs.102 per kg?

- (a) 4:1                      (b) 5:2                      (c) 4:3                      (d) 7:5

**Ans: (a)**

**Solution**



$$\frac{1^{\text{st}}}{2^{\text{nd}}} = \frac{48}{12} = \frac{4}{1} \rightarrow 4:1$$

The two kinds are to be mixed in the ratio 4:1.

## Application 12

Mary finishes the first half of her examination in two-third the time it takes her to finish the second half. If the whole exam takes her 75 minutes, in how much time did she finish the first half of the exam?

- (a) 40 min                      (b) 30 min                      (c) 45 min                      (d) 20ml

**Ans: (b)**

**Solution**

Let the time taken to finish the second half be x min.

$$\therefore \text{Time for finishing first half} = \frac{2}{3}x$$

$$\rightarrow x + \frac{2}{3}x = 75$$

$$\rightarrow x = 45 \text{ min}$$

$$\therefore \text{Time taken to finish the first half} = \frac{2}{3} \times 45 = 30 \text{ min}$$

### GUARD AGAINST POPULAR ERRORS

- If the elements of a ratio a:b are multiplied by a constant say m i.e. ma:mb, then the ratio remains the same i.e. a:b.
- If the elements of a ratio are increased by a constant say n i.e. a + n:b + n, then the ratio will not remain the same.
- Difference between the ratio of two quantities is always directly proportional to the difference when the ratio is multiplied by a constant.
- Difference between the ratio of two quantities is always the same when the ratio is added by a constant.
- In case of addition of ratio by a constant, the proportion of elements of the ratio will not remain the same.

## TWO QUESTIONS

1. The numbers are in the ratio 3:4. LCM of the numbers is 360. Find HCF of the numbers.
2. For every 3 litres of water, 2 litres of milk is mixed. After adding 10 litres of water, the ratio of milk and water is 2:4. Initially the mixture was 50 litres. What is the total quantity of milk (in litres) in the final mixture?
3. An automobile travels 176 miles on 8 gallons of gasoline. How far can it go on a tankful of gasoline if the tank holds 14 gallons?
4. Place 10 dishes along the 4 edges of a table so that each edge has the same number of dishes.
5. The ratio of Arun's marks to Babu's marks is 3:2 and their total marks are 165. What are Babu's marks?

## KEY POINTS

- Ratio is nothing but a comparison of two or more different quantities based on the same units.
- If the ratio is a:b, then proportion of 'a' is (a/a + b). Similarly, proportion of 'b' is (b/a + b).
- If the ratio of a large container containing milk and water is 3:2, then each and every drop of the container will have the same ratio.
- When Ramu eats 'A' cookies, Peter eats 'B' cookies; and when Peter eats 'C' cookies, Syed eats 'D' cookies. Here, the weighted ratio is A:(B+C):D
- If rice and sugar are in the ratio a:b and sugar and wheat are in the ratio c:d, then the combined ratio must be ac:bc:db. For instance, if the ratio of rice and sugar is 4:3 and the ratio of sugar and wheat is 5:2, then the combined ratio of rice, sugar and wheat is  $4 \times 5:3 \times 5:2 \times 3 = 20:15:6$ .

## PRACTICE EXERCISE

1. If two numbers are in the ratio 4:13 and their least common multiple is 312, then the larger number is  
(a) 72                      (b) 76                      (c) 78                      (d) 44
2. The difference between a two digit number and the number obtained by interchanging the digits is 36. What is the difference between the sum and the difference of the digits of the number, if the ratio between the digits of the number is 2:1?  
(a) 16                      (b) 4                      (c) 8                      (d) None of these
3. The wages of labourers in a factory have increased in the ratio 22:25 and their number has decreased in the ratio 15:11. What was the original wage bill of the factory if the present bill is Rs.5000?  
(a) Rs.4000                      (b) Rs.5000  
(c) Rs.6000                      (d) Rs.7000

4. Ramu spends Rs.3620 for buying pants at the rate of Rs.480 each and shirts at the rate of Rs.130 each. What will be the ratio of the number of pants to the number of shirts when maximum number of pants is bought?  
(a) 7:2      (b) 7:3      (c) 2:7      (d) None of these
5. A bag contains Rs.600 in the form of one-rupee, 50 paise and 25 paise coins in the ratio 3:4:12. The number of 25 paise coins is  
(a) 600      (b) 900      (c) 1200      (d) 1376



### Lateral Thinking

I need to pay an employee each day for one week. His wages are 1 piece of the gold bar per day. How can I cut the bar to allow for fair payment using minimum cuts. You cannot fold the bar because it is gold.

6. Rs.79.20 is divided among 7 men, 11 women and 5 boys, so that each woman may have thrice as much as a boy and a man as much as a woman and a boy together. Find the share of a man.  
(a) Rs.1.20      (b) Rs.2.40      (c) Rs.4.80      (d) Rs.5
7. The monthly income of A and B are in the ratio 4:5 and their expenses are in the ratio 5:6. If A saves Rs.25 per month and B saves Rs.50 per month, what are their respective incomes in rupees?  
(a) 400 and 500      (b) 240 and 300  
(c) 320 and 400      (d) 440 and 550
8. A, B and C play cricket. The ratio of A's runs to B's runs and that of B's runs to C's are both 3:2. They got altogether 342 runs. How many runs did A make?  
(a) 162      (b) 108      (c) 72      (d) None of these
9. The ratio of marks obtained by Vinod and Basu is 6:5. If the combined average of their percentage is 68.75 and sum of their marks is 275, find the total marks for which examination was conducted.  
(a) 150      (b) 200      (c) 400      (d) None of these
10. The ratio of present age of A to that of B is 6:4. Five years ago, the ratio was 5:3. Find their present age in years?  
(a) 42, 28      (b) 36, 24      (c) 30, 20      (d) 25, 15
11. A, B and C working alone can complete a job in 6, 8 and 12 days respectively. If all three of them work together to complete the job and earn Rs.2340, what will be C's share of the earnings?  
(a) Rs.520      (b) Rs.1080      (c) Rs.1170      (d) Rs.630
12. The marks scored by a student in three subjects are in the ratio of 4:5:6. If the candidate scored an overall aggregate of 60% of the sum of the maximum marks and the maximum marks is the same in all three subjects, in how many subjects did he score more than 60%?  
(a) 1      (b) 2  
(c) 3      (d) None of the subjects

### Directions for Q13 to Q15:

**Answer (a)** if statement I alone is sufficient to answer the question but statement 2 alone is not sufficient to answer the question;

**Answer (b)** if statement II alone is sufficient to answer the question but statement 1 alone is not sufficient to answer the question;

**Answer (c)** if statements I and II together are sufficient to answer the question but neither statement alone is sufficient to answer the question;

**Answer (d)** if each statement alone is sufficient to answer the question;

**Answer (e)** if both statements I and II together are not sufficient to answer the question and additional data specific to the problem are needed.

13. In what ratio sugar and sand must be mixed to sell at Rs.12/Kg?  
(a) Cost price of 1 kg sugar is Rs.15.  
(b) Cost price of 1 kg sand is Rs.5.
14. What is the profit in selling one kg of diluted HCl?  
(a) 80% HCl solution is mixed in 3 litres of water.  
(b) 100% HCl costs Rs.200/litre.
15. What is the resultant ratio of coffee and chicory, when two cups A and B are mixed?  
(a) Coffee and chicory are mixed in the ratio 3:2 in Cup A.  
(b) Coffee and chicory are mixed in the ratio 4:7 in Cup B.

### MIXTURES AND ALLIGATIONS

**Mixture** is a material system made up of two or more different substances which are mixed but are not combined chemically. A mixture refers to the physical combination of two or more substances in which the identities are retained.

**A Solution** is basically two substances that are evenly mixed together. One of them is called the solute and the other is the solvent. A solute is the substance to be dissolved (sugar). The solvent is the one doing the dissolving (water). Both the substances may be of liquid form.

### ALLIGATION

It is the rule that enables us to find the ratio in which two or more ingredients at the given prices must be mixed to produce a mixture of desired price.

#### MEAN PRICE

The cost of a unit quantity of the mixture is called the mean price.

#### RULE OF ALLIGATION

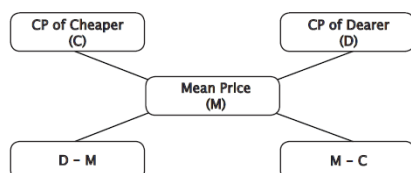
When two or more quantities of different products are mixed together to produce a mixture of a mean value, the ratios of their amounts are inversely proportional to the difference of their values from the mean value.

If two ingredients are mixed, then

Quantity of cheaper = C.P. of dearer – Mean Price

Quantity of dearer = Mean price – C.P. of cheaper

Alligation is represented as:



The ratio in which the quantity of the cheaper is mixed with the quantity of the dearer to obtain the mean value is given as:

$$(\text{Cheaper quantity}) : (\text{Dearer quantity}) = (D - M) : (M - C)$$

To be known:

1. Choose any one constituent of the mixture or solution. (There is no rule in selection)
2. Check out the proportion or percentage or amount of the chosen constituent in the two mixtures or solutions available and also in the resultant mixture.

**PROBLEMS ON MIXTURES AND SOLUTIONS FALL UNDER FIVE DIFFERENT CATEGORIES:**

## Type 1

**Given:** Two similar mixtures or solutions containing the same two constituents in two different ratios.

**To Find:** Ratio in which these two mixtures are to be mixed to obtain a desired ratio of the two constituents in the resultant mixture.

### Example

In what ratio should a coffee blend containing coffee and chicory in the ratio 3:7 and another blend with coffee to chicory ratio as 2:3 be mixed so that the resultant blend will have a coffee to chicory ratio of 7:13?

### Solution

Let x and y be the quantities of the first and the second blends respectively to be mixed. Then in the resultant blend, the

$$\text{quantity of coffee} = \frac{3}{10}x + \frac{2}{5}y$$

$$\text{and the quantity of chicory} = \frac{7}{10}x + \frac{3}{5}y.$$

By the given condition,

$$\left\{ \frac{3}{10}x + \frac{2}{5}y \right\} : \left\{ \frac{7}{10}x + \frac{3}{5}y \right\} = 7 : 13 \text{ or}$$

$$\frac{\left\{ \frac{3}{10}x + \frac{2}{5}y \right\}}{\left\{ \frac{7}{10}x + \frac{3}{5}y \right\}} = \frac{7}{13} \text{ or}$$

$$13 \left\{ \frac{3}{10}x + \frac{2}{5}y \right\} = 7 \left\{ \frac{7}{10}x + \frac{3}{5}y \right\}$$

$$\Rightarrow \left\{ \frac{26}{5} - \frac{21}{5} \right\} y = \left\{ \frac{49}{10} - \frac{39}{10} \right\} x \text{ or}$$

$$\Rightarrow y = x \quad \therefore x : y = 1 : 1$$

**Aliter:**

$$\begin{array}{ccc} \frac{3}{10} & & \frac{2}{5} \\ & \searrow \quad \swarrow & \\ & \frac{7}{20} & \\ & \swarrow \quad \searrow & \\ \frac{1}{20} & & \frac{1}{20} \end{array}$$

Therefore, the required ratio is 1:1.

## Type 2:

**Given:** Quantity of one mixture or solution with the ratio of the constituents.

**To find:** The quantity of similar mixture or solution of same constituents to be added to obtain a desired ratio of constituents in the resultant mixture.

### Example

A container has 60 liters of 40% alcohol solution. How many liters of a 50% alcohol solution should be added to this solution so that the resultant solution will have 42% alcohol?

- (a) 2:3                      (b) 5:1                      (c) 4:1                      (d) 3:4

**Ans: (c)**

### Solution

Let the quantity of 50% alcohol to be added be x litres.

Quantity of alcohol in 60 litres of 40% solution =  $60 \times 0.4 = 24$  litres

Quantity of alcohol in x litres of 50% solution =  $x \times 0.5 = 0.5x$

Quantity of alcohol in  $(60 + x)$  litres of the resultant solution =  $24 + 0.5x$ .

Percentage of alcohol in  $(60 + x)$  litres of resultant solution =  $\left\{ \frac{24 + 0.5x}{(60 + x)} \right\} \times 100$ .

This is required to be 42%.

Equating,

$$\left\{ \frac{24 + 0.5x}{(60 + x)} \right\} \times 100 = 42 \text{ or}$$

$$(24 + 0.5x) = 0.42(60 + x) \text{ or}$$

$$24 + 0.5x = 25.2 + 0.42x \text{ or}$$

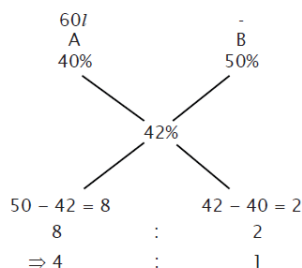
$$0.08x = 1.2 \text{ or}$$

$$x = 1.2 / 0.08 = 15.$$

So, 15 litres of 50% alcohol are to be added.



**Aliter:**



Therefore, 15 litres of 50% alcohol solution are to be added.

**Type 3:**

**Given:** Quantity of one mixture or solution of two constituents in a particular ratio.

**To find:** The quantity of either of the constituents to be added to obtain a desired ratio of the two constituents in the resultant mixture or solution.

**Example**

Sea water contains 5% salt by weight. How many kilograms of fresh water must be added to 40 kg of sea water for the salt content of the solution to be 2%?

- (a) 50      (b) 60      (c) 65      (d) 70

**Ans: (b)**

**Solution**

Let the quantity of fresh water to be added be  $x$  kg.

Quantity of salt in 40 kg of sea water =  $40 \times 0.05 = 2$  kg

Total quantity of the solution after adding water =  $(40 + x)$ .

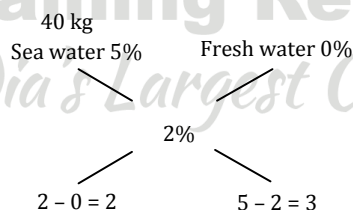
Percentage of salt in the resultant solution

$$= \left\{ \frac{2}{(40 + x)} \times 100 \right\} = 2 \text{ (given)}$$

$$40 + x = 100 \text{ or } x = 60.$$

So, 60 kg of fresh water should be added.

**Aliter:**



By the theory of alligation,  $\frac{\text{Sea water}}{\text{Fresh water}} = \frac{2}{3}$

$$\Rightarrow \frac{40}{\text{Fresh water}} = \frac{2}{3}$$

$$\text{Fresh water} = \frac{120}{2} = 60 \text{ kg}$$

60 kg of fresh water must be added.

**Type 4:**

**Given:** The quantity of one mixture or solution of two constituents in a particular ratio.

**To Find:** The ratio of the two constituents in the resultant mixture or solution when certain quantity is removed and replaced with one of the constituents.

**Example**

A vessel is filled with a liquid, 3 parts of which are water and 4 parts milk. How much of the mixture must be drawn off and replaced with water so that the mixture may be half water and half milk?

- (a)  $\frac{1}{7}$       (b)  $\frac{1}{4}$       (c)  $\frac{1}{8}$       (d)  $\frac{1}{6}$

**Ans: (c)**

**Solution**

Suppose the vessel initially contains 7 litres of liquid.

Let  $x$  litres of this liquid be drawn off and replaced with water.

$$\text{Quantity of water in the new mixture} = 3 - \frac{3x}{7} + x \text{ litres}$$

$$\text{Quantity of milk in the new mixture} = 4 - \frac{4x}{7} \text{ litres}$$

Ratio of water to milk in the resulting mixture = 1:1

$$\Rightarrow 3 - \frac{3x}{7} + x = 4 - \frac{4x}{7}$$

$$\Rightarrow \frac{1}{7}x + x = 1$$

$$\Rightarrow \frac{8}{7}x = 1.$$

$$x = \frac{7}{8}$$

So, the part of the mixture to be drawn off and replaced

$$= \frac{1}{7} \times \frac{7}{8} = \frac{1}{8}.$$

**Type 5:**

**A variation of type 4 where the process of removing and replacing is repeated**

Solutions to the problems of this type can be worked out quickly using the following formula.

$$\boxed{\text{F.C.} = \text{I.C.} \times (1 - P)^K}$$

where

**F.C.** - Final Concentration of constituent

**I.C.** - Initial Concentration of constituent

**P** - Proportion of the quantity taken out and replaced

**K** - Number of repetitions

## Example

A milkman has in his container 80 litres of pure milk. He sells 8 litres to his first customer. After adding 8 litres of water to his container, he sells 8 litres to the second customer. Again after adding 8 litres of water to the container, he sells 8 litres to the third customer. How many litres of pure milk does the third customer get?

### Solution

In the problem, the initial concentration is 100% since the container contains pure milk. The quantity taken out and replaced is 8 litres and the full quantity is maintained at 80 litres each time before sale. The process is repeated twice.

So, initial concentration = 100%

$$\text{Concentration after first sale} = 100 \left( 1 - \frac{8}{80} \right)$$

$$\begin{aligned} \text{Concentration after second sale} &= 100 \left\{ 1 - \left( \frac{8}{80} \right) \times 1 - \left( \frac{8}{80} \right) \right\} \\ &= 100 \left( 1 - \left( \frac{8}{80} \right)^2 \right) \\ &= 100(0.9)^2 = 81\% \end{aligned}$$

Hence, the third customer gets 81% of pure milk or  $8 \times 0.81 = 6.48$  liters of pure milk.

## Application 1

In what ratio must a grocer mix two varieties of pulses costing Rs.15 and Rs.20 per kg respectively so as to get a mixture worth Rs.16.50 kg?

- (a) 2:3 (b) 5:1 (c) 7:3 (d) 3:4

Ans: (c)

### Solution

Cost of 1 kg pulses of variety 1 = Rs.15

Let the quantity of variety 1 be x kg.

Cost of 1 kg pulses of variety 2 = Rs.20

Let the quantity of variety 2 be y kg

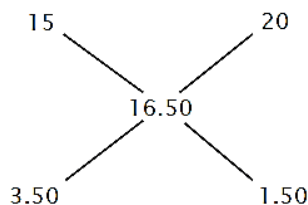
Cost of 1 kg pulses of the mixture = Rs.16.50

Quantity of the mixture = (x + y) kg

So, we get  $15x + 20y = 16.50(x + y)$

$1.50x = 3.50y \Rightarrow x:y = 3.5:1.5 = 7:3$

Aliter:



Therefore, required ratio = 3.50:1.50 = 7:3.

## Application 2

A merchant has 1000 kg of sugar part of which he sells at 8% profit and the rest at 18% profit. He gains 14% on the whole. The quantity sold at 18% profit is

- (a) 400 kg (b) 560 kg (c) 600 kg (d) 640 kg

Ans: (c)

### Solution

Let the quantity sold at 8% profit be x kg and that at 18% profit be y kg

$\therefore$  Profit on x kg = 8% x and profit on y kg = 18% y

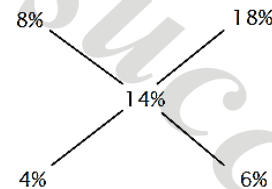
Profit on (x + y) kg = 14% (x + y)

So, we get  $8\%x + 18\%y = 14\%(x + y) \Rightarrow 6\%x = 4\%y$

Hence, the required ratio of x and y = 4:6

Therefore, the quantity sold at 18% profit is 600 kg.

Aliter:



$\therefore$  Quantity sold at 18% profit =  $1000 \times \frac{60}{100} = 600$  kg

## Application 3

In what ratio must a grocer mix two varieties of tea worth Rs.60 a kg and Rs.65 a kg so that by selling the mixture at Rs.68.20 a kg, he may gain 10%?

- (a) 3:2 (b) 3:4 (c) 3:5 (d) 3:7

Ans: (a)

### Solution

Selling price of the mixture per kg = Rs.68.20

Gain = 10%

$\therefore$  Cost price of the mixture =  $68.20 \times \frac{100}{110} = \text{Rs.}62$

Cost price of 1 kg of variety 1 = Rs.60

Cost price of 1 kg of variety 2 = Rs.65

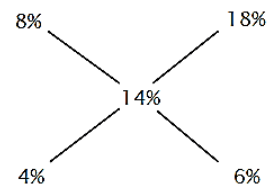
Let the quantity of variety 1 be x kg

Let the quantity of variety 2 be y kg

So, we get  $60x + 65y = 62(x + y) \Rightarrow 2x = 3y$

So, the required ratio = 3:2

Aliter:



Therefore, the required ratio = 3:2.

## Application 4

A vessel is filled with a mixture 3 parts of which are water and 5 parts syrup. How much part of the mixture must be drawn off and replaced with water so that the final mixture may be half water and half syrup?

- (a)  $\frac{1}{3}$                       (b)  $\frac{1}{4}$                       (c)  $\frac{1}{5}$                       (d)  $\frac{1}{6}$

**Ans: (c)**

### Solution

Let the vessel initially contain 8 litres of mixture.

Let x litres of this mixture be drawn off and replaced with water.

Quantity of water in the new mixture =  $3 - \frac{3x}{8} + x$  litres

Quantity of syrup in the new mixture =  $5 - \frac{5x}{8}$  litres

$$\therefore 3 - \frac{3x}{8} + x = 5 - \frac{5x}{8}$$

$$\Rightarrow 5x + 24 = 40 - 5x \Rightarrow 10x = 16 \Rightarrow x = \frac{8}{5}$$

So, the part of the mixture drawn off and replaced with water

$$= \frac{8}{5} \times \frac{1}{8} = \frac{1}{5}$$

## Application 5

A container contains 40 litres of milk. From this container, 4 litres of milk were taken out and replaced with water. This process was repeated further two times. How much milk is now contained by the container?

- (a) 26.34 litres                      (b) 27.36 litres  
(c) 28 litres                      (d) 29.16 litres

**Ans: (d)**

### Solution

Applying the formula, F.C. = I.C.  $\times (1 - \frac{4}{40})^3$

amount of milk left after 3 repetitions

$$= 40 \left(1 - \frac{4}{40}\right)^3 \text{ litres}$$

$$= 40 \times \frac{9}{10} \times \frac{9}{10} \times \frac{9}{10} = 29.16 \text{ litres.}$$

## Application 6

Four litres of 100% pure sulphuric acid is added to six litres of 20% sulphuric acid. What is the strength of the acid in the new mixture?

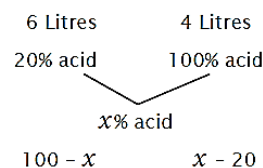
- (a) 48%                      (b) 42%                      (c) 52%                      (d) 51%

**Ans: (c)**

### Solution

Let the strength of the acid in the new mixture be x%.

Applying the principle of alligation,



$$\therefore \frac{6}{4} = \frac{100 - x}{x - 20} \rightarrow 6(x - 20) = 4(100 - x) \rightarrow x = 52$$

Strength of acid in the new mixture = 52%

## Application 7

Ratio of milk and water in one container is 5:1 and that in the other container is 7:2. In what ratio the contents of the two containers should be mixed so that the quantity of milk in the resulting mixture may become 80%?

- (a) 3:4                      (b) 2:3                      (c) 4:3                      (d) 2:5

**Ans: (b)**

### Solution

Let the contents of the two containers be added in the ratio x:y

$$\therefore \text{Quantity of milk in the new mixture} = \frac{5}{6}x + \frac{7}{9}y$$

$\therefore$  Percentage of milk in the new mixture

$$= \frac{\frac{5}{6}x + \frac{7}{9}y}{x + y} \times 100 = \frac{45x + 42y}{54(x + y)} \times 100 = 80\% \text{ given}$$

$$(45x + 42y) \times 5 = 4 \times 54(x + y)$$

$$\rightarrow x : y = 2 : 3$$

The contents of the two containers should be mixed in the ratio 2:3.

## Application 8

Two alloys contain zinc and copper in the ratio of 2:1 and 4:1 respectively. In what ratio the two alloys should be melted to get a new alloy having zinc and copper in the ratio 3:1?

- (a) 3:5                      (b) 3:7                      (c) 4:7                      (d) 7:5

**Ans: (a)**

### Solution

Let the two alloys be mixed in the ratio x:y.

$$\text{Quantity of zinc in the new alloy} = \frac{2}{3}x + \frac{4}{5}y$$

$\therefore$  Percentage of zinc

$$\frac{\frac{2}{3}x + \frac{4}{5}y}{x + y} \times 100 = \frac{10x + 12y}{15(x + y)} \times 100 \quad \dots (1)$$

Ratio of zinc and copper in the new alloy = 3:1

$$\therefore \text{Percentage of zinc in the new alloy} = \frac{3}{3+1} \times 100 \quad \dots (2)$$

From (1) and (2),

$$\frac{10x + 12y}{15(x + y)} = \frac{3}{4} \rightarrow x : y = 3 : 5$$



## Application 9

In what ratio must a person mix three kinds of solution costing Rs.12, Rs.14.40 and Rs.17.40 per kilogram so that the mixture may be worth Rs.14.10 per kilogram?

- (a) 11:77:7 (b) 11:17:7 (c) 11:7:77 (d) 13:11:77

Ans: (a)

### Solution

Let the ratio in which the three kinds of solution are to be mixed be  $x:y:z$ . Applying first the principle of alligation to the 1st and 2nd kinds and then to the 1st and 3rd kinds, we get

	$x$ parts 1 <sup>st</sup> kind	$y$ parts 2 <sup>nd</sup> kind	$z$ parts 3 <sup>rd</sup> kind
Price	Rs.12 per kg	Rs.14.40 per kg	Rs.17.40 per kg
		Rs.14.10 per kg	Rs.14.10 per kg

$$\frac{x}{y} = \frac{14.40 - 14.10}{14.40 - 12.00} = \frac{0.30}{2.10} \rightarrow \frac{1}{7}$$

$$\text{and } \frac{x}{z} = \frac{17.40 - 14.10}{14.10 - 12.00} = \frac{3.30}{2.10} \rightarrow \frac{11}{7}$$

$$\therefore 1^{\text{st}}:2^{\text{nd}} = 1:7$$

$$1^{\text{st}}:3^{\text{rd}} = 11:7$$

$$\therefore \text{Combined ratio } 1^{\text{st}}:2^{\text{nd}}:3^{\text{rd}} = 11:77:7$$

## Application 10

A vessel contains a mixture of two liquids A and B in the ratio 7:5. When 9 litres of mixture are drawn off and the flask is filled with liquid B, the ratio of A and B becomes 7:9.

How many litres of liquid A was present initially in the flask?

- (a) 18 (b) 20 (c) 24 (d) 21

Ans: (d)

### Solution

Let the volume of the vessel be  $a$  litres.

$$\text{Initial Concentration of liquid A} = \frac{7}{12}$$

$$\text{Final Concentration of liquid A} = \frac{7}{16}$$

Applying the formula,

$$\frac{\text{Final Concentration(FC)}}{\text{Initial Concentration(IC)}} =$$

$$= \frac{\text{Initial Volume} - \text{Volume of mixture taken out and replaced}}{\text{Initial Volume}}$$

$$\rightarrow \frac{\frac{7}{16}}{\frac{7}{12}} = \frac{a-9}{a} \Rightarrow \frac{12}{16} = \frac{a-9}{a} \Rightarrow a = 36 \text{ litres}$$

$$\therefore \text{liquid A in the vessel} = 36 \times \frac{7}{12} = 21 \text{ litres}$$

## GUARD AGAINST POPULAR ERRORS

- Mixing 80% milk solution and 90% milk solution does not always result in a 85% milk solution - this is possible only if equal quantities are mixed.
- In 20 litres of milk solution, milk and water are in the ratio 3:2. When 10 litres of water added to the solution, the ratio becomes 2:3, but not at 3:2.
- In 10 litres of mango juice, milk and mango are in the ratio 1:2. Then every drop of the 10 litres will bear the same ratio.
- In case of solid mixtures, the ingredients should be powdered and mixed so that they become homogeneous. Then only each and every part of the mixture will bear the same ratio.

## TWO QUESTIONS

- How many kgs of Basmati rice costing Rs.42/kg should a shopkeeper mix with 25 kgs of ordinary rice costing Rs.24 per kg so that he makes a profit of 25% on selling the mixture at Rs.40/kg?
- A zookeeper counted the heads of the animals in a zoo and found it to be 80. When he counted the legs of the animals he found it to be 260. If the zoo had only pigeons and horses, how many horses were there in the zoo?
- How many litres of a 12 litre mixture containing milk and water in the ratio of 2:3 are to be replaced with pure milk so that the resultant mixture contains milk and water in equal proportions?
- The milk and water in two vessels A and B are in the ratio 4:3 and 2:3 respectively. In what ratio the liquids in the two vessels be mixed to obtain a new mixture in vessel C consisting half milk and half water?
- A goldsmith has two qualities of gold, one of 10 carat and the other of 15 carat purity. In what proportion should he mix both to make an ornament of 12 carat purity?

## PRACTICE EXERCISE

- In what ratio two varieties of milk costing Rs.8 and Rs.9 per litre respectively are to be mixed, so that the mixture costs Rs.8.30 per litre?  
(a) 9:8 (b) 7:3 (c) 3:7 (d) 8:9
- How many litres of pure alcohol should be added to 10 litres of a 30% alcohol solution to make it a 44% alcohol solution?  
(a) 7.5 (b) 3.5 (c) 5.0 (d) 2.5
- A grocer bought two kinds of rice at Rs.38 and Rs.32 per kg respectively. In what ratio should these be mixed, so that by selling the mixture at Rs.42.5 per kg, 25% may be gained?  
(a) 2:1 (b) 3:2 (c) 3:4 (d) 1:2

4. In a mixture of 60 litres, the ratio of milk to water is 2:1. If the ratio of the milk to water is to be 1:2, then the amount of water to be added is  
(a) 20 litres (b) 30 litres  
(c) 40 litres (d) 60 litres
5. A trader mixes 'Ariel' detergent costing Rs.64 per kg with 'Surf Excel' which costs Rs.76 per kg in the ratio 1:3. If the cost of 'Surf excel' drops to Rs.74 per kg, then in what ratio should he mix the two to leave the cost of mixture unchanged?  
(a) 1:9 (b) 1:8 (c) 9:1 (d) None of these
6. A milkman dilutes 36 litres of milk with water. The percentage of milk in the solution is now 80%. How many litres of water did he add?  
(a) 9 (b) 10 (c) 4 (d) 36
7. From a vessel containing only milk, 5 litres are drawn and replaced with water. This action is repeated once more (i.e. 5 litres of the mixture are drawn and replaced with water). The ratio of milk to water now is 36:13. How many litres of solution does the vessel hold?  
(a) 35 (b) 30 (c) 25 (d) None of these
8. A vessel contains wine solution, wine and water being in the ratio 4:1. How many litres of water should be added to this solution so that the ratio of wine to water becomes 3:2 in the resultant solution of 100 litres?  
(a) 25 (b) 9 (c) 23 (d) None of these
9. The cost price of three kinds of sugar is Rs.5, Rs.6 and Rs.6.80 per bag respectively. In what ratio should they be mixed so that the price of the mixture is Rs.6.50 per bag?  
(a) 1:3:10 (b) 1:10:3 (c) 1:2:3 (d) None of these
10. Two vessels contain a litre each of kerosene and petrol. 100 ml of kerosene is added to the vessel containing petrol. Now, 100ml from the vessel originally containing only petrol is poured back into the vessel originally containing only kerosene. Then, the fraction of kerosene in petrol is same as the fraction of petrol in kerosene. State whether the above statement is true or false.  
(a) True (b) False  
(c) Cannot say (d) None of these
11. A grocer mixes 38 kg of rice that cost him Rs.8 per kg with 42 kg of rice that cost him Rs.12.50 per kg. He sells the mixture at Rs.11.25 per kg. Find his gain or loss percentage.  
(a) 8.05% (b) 7%  
(c) 7.05% (d) None of these
12. A can contains 200 litres of pure spirit. From this can, 20 litres of spirit is removed and replaced with water. This process is repeated two more times. Find the concentration of spirit in the resultant solution.  
(a) 72.9% (b) 70.9%  
(c) 52.9% (d) None of these

13. Two glasses are filled with a solution of water and spirit. In the first glass, the volume of water is thrice the volume of spirit, while in the second glass the volume of spirit is twice the volume of water. The contents of the two glasses are emptied into a third glass of capacity 15 litres, so that it contains an equal amount of spirit and water. What is the volume (in litres) of the solution taken from the second glass?  
(a) 9 (b) 12 (c) 10 (d) None of these

## Directions for Q14 and Q15:

**Answer (a)** if statement I alone is sufficient to answer the question but statement 2 alone is not sufficient to answer the question;

**Answer (b)** if statement II alone is sufficient to answer the question but statement 1 alone is not sufficient to answer the question;

**Answer (c)** if statements I and II together are sufficient to answer the question but neither statement alone is sufficient to answer the question;

**Answer (d)** if each statement alone is sufficient to answer the question;

**Answer (e)** if both statements I and II together are sufficient to answer the question and additional data specific to the problem are needed.

14. What is the ratio of the gold added to the gold present in the alloy originally?  
I. In an alloy, the ratio of gold and tin present is 5:3.  
II. By adding certain quantity of gold, the ratio becomes 2:1.
15. A cowboy dilutes 36 litres of milk with water. How many litres of water did he add?  
I. After adding water, percentage of milk in the solution is 80%.  
II. Cost of milk is Rs.15 per litre

## SIMPLE EQUATION BASED PROBLEMS

### WORD PROBLEMS

#### PRACTICE EXERCISE

1. There are 3 societies A, B, C having some tractors each. A gives B and C as many tractors as they already have. After a few days B gives A and C as many tractors as they have. After a few more days C gives A and B as many tractors as they have. Finally, each society has 24 tractors. What is the original No. of tractors each had in the beginning?  
(a) A-39, B-21, C-12 (b) A-49, B-21, C-12  
(c) A-39, B-26, C-12 (d) A-29, B-21, C-11

2. 24 apples were divided among 3 brothers. Each of them got the same No of apples as their age 3 years ago. The youngest of them kept half of his share and divided the rest of them equally between other two. The other two did the same thing. They all ended up with 8 apples each. Determine their ages.

(a) 8, 12, 16 (b) 9, 12, 15  
(c) 7, 10, 16 (d) 5, 10, 15

**Directions for Q3 and Q4:** The cost of fuel for running the engine of an army tank is proportional to the square of the speed and is Rs.64 per hour for a speed of 16 kmph. Other costs amount to Rs.400 per hour. The tank has to make a journey of 400 km at a constant speed.

3. The most economical speed for this January is  
(a) 20 kmph (b) 30 kmph  
(c) 35 kmph (d) 40 kmph
4. The total cost for the journey at this most economical speed is  
(a) 8000 (b) 10000  
(c) 12000 (d) 14000
5. Abhishek had a certain number of Rs.1 coins, Rs.2 coins and Rs.10 coins. If the number of Rs.1 coins he had is six times the number of Rs.2 coins Abhishek had, and the total worth of his coins is Rs 160, find the maximum number of Rs.10 coins Abhishek could have had.  
(a) 5 (b) 6 (c) 12 (d) 10
6. In a family of husband, wife and a daughter, the sum of the husband's age, twice the wife's age, and thrice the daughter's age is 85; while the sum of twice the husband's age, four times the wife's age, and six times the daughter's age is 170. It is also given that the sum of five times the husband's age, ten times the wife's age and fifteen times the daughter's age equals 450. The number of possible solutions, in terms of the ages of the husband, wife and the daughter, to this problem is  
(a) 1 (b) 0 (c) 2 (d) many
7. The currencies in countries X and Y are denoted by Xs. and Ys. respectively. The exchange rate in 1990 was 1 Xs. = 0.6 Ys. The price level in 2006 in X and Y are 150 and 400 respectively with 1990 as a base of 100. The exchange rate in 2006, based solely on this purchasing power parity consideration, is 1 Xs.=  
(a) 1.6 Ys (b) 0.8 Ys (c) 2 Ys (d) 1.2 Ys

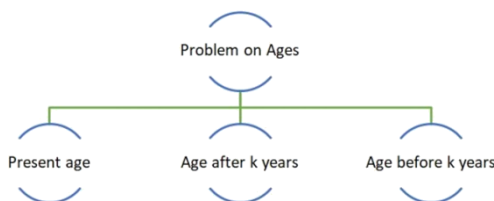
**Directions for Q8 to Q10:** A boy is roaming on the roof of his house observes that insects are there in bunches. He found 'p' insects on day 1. On day two, the number of insects doubled and he takes out 'q' insects and kills them. On the third day again, the insects left have doubled and he again takes out 'q' insects and kills them. On the fourth day also he found the same, insects left have doubled and he takes out 'q' insects again. He is surprised to find that there are no insects left on his roof.

8. If the number of insects on the roof, on day 1 is 63, then the number of insects that are killed on the fourth day is  
(a) 72 (b) 63 (c) 50 (d) 55
9. The minimum number of insects that he kills on any of the last three days is  
(a) 8 (b) 7 (c) 0 (d) 1
10. The minimum number of insects that he left on day 1 is  
(a) 8 (b) 7 (c) 0 (d) 1
11. Three persons Suresh, Devesh and Prashant were born on different days in the same year. If the date and month of birth of Suresh, Devesh and Prashant are numerically equal, then what could be the minimum difference in the ages of youngest and oldest in days?  
(a) 61 (b) 57 (c) 60 (d) 70
12. In a green view apartment, the houses of a row are numbered consecutively from 1 to 49. Assuming that there is a value of 'x' such that the sum of the numbers of the houses preceding the house numbered 'x' is equal to the sum of the numbers of the houses following it. Then, what will be the value of 'x'?  
(a) 35 (b) 49 (c) 17 (d) 20
13. In a cricket match, Team A scored 232 runs without losing a wicket. The score consisted of byes, wides and runs scored by two opening batsmen: Raj and Shyam. The runs scored by the two batsmen are 26 times wides. There are 8 more byes than wides. If the ratio of the runs scored by Raj and Shyam is 6:7, then the runs scored by Raj is  
(a) 40 (b) 70 (c) 50 (d) 10
14. A and B collect fruits from some trees such that A picks 4 from each tree while B picks 3 from each tree and the number of trees that B used is 4 more than that of A. If A and B have equal number of fruits then what will be the total number of fruits in their collection?  
(a) 40 (b) 70 (c) 50 (d) 96
15. The time taken by a hose to fill up a cistern is less than the time taken by another hose used for the same purpose by 6 minutes. If 1st one is  $5/2$  times faster than the second, What is the time taken (minutes) by the 1st hose to fill up?  
(a) 4 (b) 7 (c) 5 (d) 1
16. Village A has a population of 6,800, which is decreasing at a rate of 120 per year. Village B has a population of 4, 200, which is increasing at a rate of 80 per year. In how many years will the population of the two villages be equal?  
(a) 9 (b) 11 (c) 13 (d) 16

17. The number of blues was two more than the total number of reds and greens. Three times the number of reds added to twice the number of greens was twice the number of blues. If there were 22 total, how many were there of each colour (red, green and blue)?  
(a) 4, 6, 12 (b) 6, 4, 12  
(c) 8, 2, 12 (d) None of these
18. Ram and Mohan are friends. Each has some money. If Ram gives Rs.30 to Mohan, then Mohan will have twice the money left with Ram. But, if Mohan gives Rs.10 to Ram, then Ram will have thrice as much as is left with Mohan. How much money does each have?  
(a) Rs.62, Rs.34 (b) Rs.6, Rs.2  
(c) Rs.170, Rs.124 (d) Rs.43, Rs.26
19. A lady went to the market with a few one-rupee notes and a few 20-paise coins. When she returned, she had as many one-rupee notes as she originally had 20-paise coins and vice versa. She actually came back with about one-third of what she had gone with. How much does she spend in the market?  
(a) Rs.14.40 (b) Rs.14.70  
(c) Rs.15.50 (d) Rs.17.40
20. Ramesh has two examinations on Wednesday - Engineering Mathematics in the morning and Engineering Drawing in the afternoon. He has a fixed amount of time to read the textbooks of both these subjects on Tuesday. During this time he can read 80 pages of Engineering Mathematics and 100 pages of Engineering Drawing. Alternatively, he can also read 50 pages of Engineering Mathematics and 250 pages of Engineering Drawing. Assume that the amount of time it takes to read one page of the textbook of either subject is constant. Ramesh is confident about Engineering Drawing and wants to devote full time in reading Engineering Mathematics. The number of Engineering Mathematics text book pages he can read on Tuesday is  
(a) 100 (b) 150 (c) 200 (d) 175

## PROBLEMS ON AGES

**Problems on ages** can be categorized into three types, i.e. questions based on calculating the present age, questions to determine the age of person after k years and questions to calculate the age of a person before k years. These three types may cover cases of various types with different combinations of ratios, fractions etc.



- If the present age of 'A' is 'x' years, then  
'n' years ago  $\Rightarrow$  Age of 'A' was  $(x - n)$  years.  
'n' years after  $\Rightarrow$  Age of 'A' will be  $(x + n)$  years.
- In general, at the starting of the solution we have to assume the present age of one person. It is better to assume the present age as 'x' years.
- It is advisable to assume the age of the younger person as 'x' years.
- If the present age 'A' and 'B' are 'x' and 'y' years respectively, then 'n' years ago  $\Rightarrow$  Age of 'A' and 'B' were  $(x - n)$  and  $(y - n)$  years respectively.  
'n' years after  $\Rightarrow$  Age of 'A' and 'B' will be  $(x + n)$  and  $(y + n)$  years respectively.
- The age difference between you and your friend is let's suppose 6 years. And after 10 or 15 years, this will be same, because clock will run equally for all.

### Why should I learn:

- 'Problems on ages' is an important topic in quantitative exams which is very essential in bank exam. It is based on algebra equation solving
- Algebra is a very powerful branch of Mathematics. It provides solution to real-world problems. It helps in transforming word problems into mathematical expressions in the form of equations using variables to denote unknown quantities or parameters thus providing numerous techniques to solve these mathematical equations and hence, determining the answer to the problem.
- In CAT you will be getting very tricky problems and there would be multiple instances where you have to make use of linear equations to find the solution of the problem. So, to successfully apply linear equations, we first need to learn to translate aptly the word problem into them.

### Example 1

The present age of the mother is 5 times the age of her son. Five years ago, the age of the mother was ten times the age of her son at that time. How old is the mother at present?

- (a) 45 years (b) 40 years  
(c) 48 years (d) 49 years  
(e) None of these

Ans: [a]

### REGULAR METHOD TO SOLVE AGE PROBLEM

#### Step 1:

Let us assume mother's present age as M.

Let us assume the present age of the son to be as S.

#### Step 2:

As mentioned in the question, the present age of the mother is 5 times the age of son

$$M = 5 \times S \quad \dots (i)$$



## Step 3:

Now an equation now

$$(M - 5) = 10(S - 5) \quad \dots(ii)$$

## Step 4:

$$M - 5 = 10S - 50$$

$$5S - 5 = 10S - 50 \text{ (Applying equation (i))}$$

$$5S = 45$$

$$S = 9$$

$$F = 9 \times 5 = 45$$

## SMART METHOD TO SOLVE AGE PROBLEMS

### Step 1:

To solve this question using the smart method, we need to understand the question carefully.

As mentioned in the question that the mother's age is 5 times the age of son, we can clearly make out that the mother's age should be a multiple of 5.

By this, we can easily eliminate the options which are not the multiples of 5.

Therefore, option 3 and option 4 are eliminated.

### Step 2:

Now we take the second condition into consideration and check which option satisfies the condition.

Option 1: 45 years.

If the mother's present age is 45 then the son's age would be 9, since the mother is 5 times the son's age.

Now, five years ago mother's age would be 40 and son's age would be 4.

Hence, the second statement states that 5 years ago mother's age was 10 times the age of the son.

As,  $40 = 10 \times 4$ ; option 1 satisfies both the condition.

Therefore the mother's present age is 45.

### Example 2

The total age of P and Q is 25 years more than the total age of Q and R. R is how many years younger than P?

- (a) 11 (b) 12 (c) 13 (d) 14

Ans: [b]

$$\text{Given that } P + Q = 12 + Q + R$$

$$\Rightarrow P - R = 25 + Q - Q = 25$$

$$\Rightarrow R \text{ is younger than P by 12 years}$$

### Example 3

Eight years ago, Krishna's age was  $\frac{4}{3}$  times that of Vishal. Eight years hence, Krishna's age will be  $\frac{6}{5}$  times that of Vishal. What is the present age of Krishna?

- (a) 41 yrs (b) 40 yrs (c) 37 yrs (d) 33 yrs

Ans: [b]

Let the present ages of Krishna and Vishal be 'K' and 'V' years respectively.

$$\Rightarrow K - 8 = \frac{4}{3}(V - 8) \text{ and } K + 8 = \frac{6}{5}(V + 8)$$

$$\Rightarrow \frac{3}{4}(K - 8) = V - 8 \text{ and } \frac{5}{6}(K + 8) = V + 8$$

$$V = \frac{3}{4}(K - 8) + 8 = \frac{5}{6}(K + 8) - 8$$

$$\Rightarrow \frac{3}{4}K - 6 + 8 = \frac{5}{6}K + \frac{20}{3} - 8$$

$$\Rightarrow 10 - \frac{20}{3} = \frac{10}{12}K - \frac{9}{12}K$$

$$\Rightarrow \frac{10}{3} = \frac{K}{12} \Rightarrow K = 40.$$

### Example 4

When Vijay was born, his mother was 32 years older than his cousin brother whose mother was 25 years older than Vijay's sister. If Vijay's brother is 6 years older than Vijay and the brother's mother is 3 years younger than Vijay's mother, how old was Vijay's sister when Vijay was born?

- (a) 6 years (b) 8 years (c) 10 years (d) 12 years

Ans: [c]

Vijay age when he was born = 0 years

$\Rightarrow$  His cousin brother's age = 6 years

$\Rightarrow$  Vijay's mother's age = brother age + 32 years =  $6 + 32 = 38$

$\Rightarrow$  Brother's mother's age =  $38 - 3 = 35$

So Vijay's sister's age =  $35 - 25 = 10$  years.

### Example 5

On a woman's tombstone, it is said that one sixth of her life was spent in childhood and one twelfth as a teenager. One seventh of her life passed between the time she became adult and the time she married; five years later, her son was born. Siva, her son, died four years before she did. She lived to be twice as old as her son did. How old did the woman live to be?

Let the woman live to be x years and her son live to be y years.

$$\therefore \frac{x}{6} + \frac{x}{12} + \frac{x}{7} + 5 + y + 4 = x \quad \dots (1)$$

$$x = 2y \quad \dots (2)$$

Solving (1) and (2),

$$x = 84$$

The woman lived to be 84 years.

## PRACTICE EXERCISE

- The ratio of Abhishek's age 4 years ago and Salman's age after 4 years is 1 : 1. If at present, the ratio of their ages is 5 : 3, then find the ratio between Abhishek's age 4 years hence and Salman's age 4 years ago.  
(a) 1 : 3 (b) 3 : 1 (c) 4 : 3 (d) 3 : 4
- 4 years ago, sister's age was 6 times the age of her brother and the sum of the present ages of the sister and the brother is 29 years. What will be the age of her brother after 5 years?  
(a) 12 years (b) 13.5 years  
(c) 15 years (d) 20 years



3. Father is 3 times more aged than his son. If after 5 years, he would be 3 times of son's age, then further after 5 years, how many times he would be of his son's age?  
(a) 1 ( $\frac{1}{2}$ ) times (b) 2 times  
(c) 2.5 times (d) 3 times
4. Paviesh is 15 years older than Mohan. If 5 years ago, Paviesh was 3 times as old as Mohan, then find Paviesh's present age.  
(a) 32.5 years (b) 27.5 years  
(c) 25 years (d) 24.9 years
5. What is Akshay's present age, if after 20 years his age will be 10 times his age 10 years back?  
(a) 6.2 years (b) 7.7 years  
(c) 13.3 years (d) 10 years
6. The ratio of Sachin's age 4 years ago and Dravid's age after 4 years is 1: 1. Presently, the ratio of their ages is 5: 3. Find the ratio between Sachin's age 4 years hence and Dravid's age 4 years ago.  
(a) 1 : 3 (b) 3 : 1 (c) 4 : 3 (d) 3 : 4
7. Six years ago Aditi was P times as old as Bhema was. If Aditi is now 17 years old, how old is Bhema now in terms of P?  
(a)  $11/P + 6$  years (b)  $P/11 + 6$  years  
(c)  $17 - P/6$  years (d)  $17/P$  years
8. Sum of present ages of Pooja and Quincy is 41. Age of Pooja 2 year hence is equal to age of Riya 1 year ago. Age of Pooja, 4 year hence is equal to age of Quincy 1 year ago and ratio of present age of Pooja and Sruthi is 3 : 4. Find the difference of age of Riya and Sruthi.  
(a) 2 years (b) 3 years (c) 4 years (d) 5 years
9. The average age of Vijay and Paviu is 24 years. If a new person Uma is also included, the average age increases by 1.5 years. What is the age of Uma?  
(a) 28.5 years (b) 29.5 years  
(c) 30.5 years (d) 27 years
10. Arun's present age is 9 years more than that of what Pavi's age will be after five years. Pavi's present age is seven years more than that of what Vino's age was 4 years ago. Vino's present age is 19 years. What will be Arun's age after 5 years?  
(a) 39 yrs (b) 36 yrs (c) 46 yrs (d) 41 yrs
11. Mr. Anand got married 12 years ago to Priya and she became the 7<sup>th</sup> member of the family. Today his mother died and Mrs. Priya gave birth to a girl baby. The average age of Anand's family at the time of his marriage was the same as of today. What is the age of his mother when she died?  
(a) 60 (b) 72 (c) 84 (d) 54
12. Peter's current age is  $\frac{2}{5}$ th of the age of his father. After 8 years, he will be  $\frac{1}{2}$  of the age of his father. What is the age of the father at present?  
(a) 50 (b) 40 (c) 33 (d) 30
13. The age of a person is ten times the total age of his 2 daughters. 0.5 decade hence, his age will be five times the total age of his daughters. Then what is the father's current age? [0.5 Decades = 5 Years]  
(a) 55 years (b) 80 years  
(c) 90 years (d) 87 years
14. 0.1 decade ago, Vimala was quadrice as old as her daughter Sasi. 0.6 decades hence, Vimala's age will beat her daughter's age by 0.9 decades. The proportion of the current ages of Vimala and Sasi is [0.1 Decades = 1 Year; quadrice = 4 times]  
(a) 8:1 (b) 10:2 (c) 11:4 (d) 13:4
15. The ratio of Kishore's and Kaviya's age is 4: 5. If the difference between the present age of Kaviya and the age of Kishore 5 years hence is 3 years, then what is the total of the present age of Kishore and Kaviya?  
(a) 56 years (b) 62 years  
(c) 69 years (d) 72 years
16. The average of the ages of W, X and Y is 43 years and the average of the ages of W, Y and Z is 49 years. If Z is 54 years old, what is the age of X?  
(a) 48 yrs (b) 42 yrs (c) 40 yrs (d) 36 yrs
17. Dileep is twice more than the age of Rohini. After 10 years, he would be twice the age of Rohini. After 30 years, how many times would he be of Rohini's age?  
(a) 2.5 times (b) 1.5 times  
(c) 2 times (d) 3 times
18. Joseph is six years older than Sangeetha. 30 years ago, the difference between thrice Joseph's age and four times Sangeetha's age was 6. They got married when Sangeetha was 22 years old. How old would they be when they celebrate their 50<sup>th</sup> wedding anniversary?  
(a) 74, 80 (b) 68, 74  
(c) 70, 76 (d) 78, 72
19. A father has a daughter and a son. The son is four years older than the daughter. In two years, the father will be seven times as old as the daughter is now. In ten years the father will be fourteen years older than the combined ages of his children. Find the present age of the father.  
(a) 32 years (b) 40 years  
(c) 25 years (d) 50 years

20. Father's age is twice more than his son's age. After 6 years, father's age will be  $\frac{7}{3}$ rd of his son's age. How many times would his age be of his son's age after 12 years?  
(a) Twice (b) Two and half times  
(c) Thrice (d) One and half times
21. Three sisters Rina, Meena and Tina whose ages are in the ratio 7:4:3, had to divide the total of 770 pearls among them in the ratio of their ages. How many pearls did Tina get?  
(a) 330 (b) 75 (c) 144 (d) 165
22. Twice the age of the eldest child in a family is just 6 years short of thrice the age of the youngest child. Twice the age of the youngest child exceeds the age of the eldest child by 19 years. By how many years is the youngest child younger than the eldest child?  
(a) 13 (b) 15 (c) 14 (d) 16
23. The average age of 10 lecturers of a department is the same as it was 5 years ago, because a lecturer has been replaced by another lecturer. What is the age of the lecturer who left the department?  
(a) 50 yrs (b) 55 yrs  
(c) 60 yrs (d) Cannot say
24. Sampath is as much as younger to Vamsi as he is older to Rakesh. If the sum of ages of Vamsi and Rakesh is 56, how old is Sampath now?  
(a) 28 years (b) 26 years  
(c) 24 years (d) 25 years
25. Jerry went to get his driving license. When asked his age, he said "My age today is three times what it will be three years from now minus three times what my age was three years ago". How old is Jerry?  
(a) 22 years (b) 21 years  
(c) 18 years (d) 20 years
26. A man has a daughter and a son. The son is 3 years older than the daughter. In 1 year, the man will be 6 times as old as daughter is now. In ten years, man will be 14 years older than the combined age of children at that time. What is man's present age?  
(a) 42 years (b) 41 years  
(c) 44 years (d) 45 years
27. Three times the age of my father added to seven times the age of my sister is 183 years. Six times the difference of their ages added to 9 years is three times the sum of the ages. Determine the sister's present age.  
(a) 12 years (b) 10 years  
(c) 16 years (d) 18 years
28. Mr. Nagaraj got married 12 years ago to Parvathy and she became 7<sup>th</sup> member of the family. Today his mother died and Mrs. Parvathy gave birth to a girl baby. The average age of Nagaraj's family at the time of his marriage is the same as of today. What is the age of his mother when she died?  
(a) 60 years (b) 72 years  
(c) 84 years (d) 54 years
29. The sum of present ages of father and son is 55 years. If the father is to live another year's equal to present age of son, then the sum of their ages will be 93 years. Find the present ages of father and son respectively (in years)?  
(a) 38, 20 (b) 36, 19 (c) 37, 20 (d) 32, 18
30. In January of the year 2000, I was one more than eleven times as old as my son William. In January of 2009, I was seven more than three times as old as him. How old was my son in January of 2000 (in years)?  
(a) 5 years (b) 4 years  
(c) 3 years (d) 6 years
31. A says to B, "I am twice as old as you were, when I was as old as you are". The sum of their present ages is 63 years, what are the present ages (in years) of A and B respectively?  
(a) 36, 27 (b) 35, 25 (c) 35, 27 (d) 38, 24
32. Father is aged three times more than his son Sunil. After 8 years, he would be two and a half times of Sunil's age. After further 8 years, how many times would he be of Sunil's age?  
(a) 4 times (b) 3 times  
(c) 2 times (d) None of these
33. Aftab tells his daughter "Seven years ago, I was 7 times as old as you were then. Also, three years from now, I shall be three times as old as you will be". Find the present ages of Aftab and his daughter (in years)?  
(a) 42, 12 (b) 40, 10 (c) 45, 12 (d) 41, 10
34. Ravi is three years older than Vimala. When they got married 40 years ago, the difference of twice Ravi age from thrice Vimala's age was 16 years. How old will each of them be when they celebrate their golden anniversary (50<sup>th</sup> year) (in years)?  
(a) 75, 72 (b) 70, 68 (c) 74, 72 (d) 74, 69
35. Nitin's age was equal to square of some number last year and the following year it would be cube of a number. If again Nitin's age has to be the cube of some number, then for how long he will have to wait?  
(a) 16 years (b) 30 years  
(c) 38 years (d) 64 years

36. The present age of a father is 3 years more than three times the age of son. Three years later, father's age will be 10 years more than twice the age of son. Find the present age of father?
- (a) 33 years (b) 28 years  
(c) 34 years (d) 30 years
37. In three more years, Arun's grandfather will be six times as old as Arun was last year. When Arun's present age is added to his grandfather's present age, the total is 68. How old is Arun's grandfather?
- (a) 57 years (b) 60 years  
(c) 63 years (d) 68 years
38. A is two years older than B who is twice as old as C. The total of the ages of A, B and C is 27. How old is B?
- (a) 10 years (b) 9 years  
(c) 8 years (d) 7 years
39. I am three times as old as my son. Five years later, I shall be two and half times as old as my son. How old is my son?
- (a) 10 years (b) 15 years  
(c) 12 years (d) 8 years
40. Sobha's father was 38 years of age when she was born while her mother was 36 years old when her brother four years younger to her was born. What is the difference between the ages of her parents?
- (a) 6 years (b) 5 years  
(c) 4 years (d) 3 years

## PERCENTAGES

### DEFINITION:

The term 'per cent' means one out of a hundred. In Mathematics percentages are used to describe parts of a whole – the whole being made up of a hundred equal parts.

Percentage is a concept evolved so that there can be a uniform platform for comparison of various things. (Since each value is taken to a common platform of 100)

For example,

To compare three different students depending on the marks they scored, we cannot directly compare their marks until we know the maximum marks for which they took the test. But by calculating the percentages, they can directly be compared with one another.

There are lots of reasons why you may want to find the percentage of a number. Let's suppose that you want to buy a new laptop computer for £500 – you have checked with the suppliers and one company has offered to give you 20% off the price. How much will the laptop cost taking into account the discount offered?

Like this, percentages are used frequently in all walks of life. "30% increase in fuel prices this winter", "20% off all shoes", "79% of people in UK have an Internet connection", "50% extra free".

In one recent study...51% of adults could not calculate a 10% tip on a lunch bill.

### Why should I learn This Topic?

Percentages are used in two main ways in everyday life:

#### 1. As operators

In many real life situations you find a percentage of an amount. For example, if you buy something at 30% discount you pay 70% of the usual price.

70% operates on the usual price, e.g. 70% of Rs.60 is Rs.42.

#### 2. As proportions

Percentages are often used to compare two or more proportions. For example, to compare two shooters in a netball game, you might convert the statistics into percentages.

Selma gets 32 out of 40 shots and so her shooting percentage is 80%.

Niki gets 33 out of 44 shots and so her percentage is 75%.

### How to find percentage:

As we have already learnt, percentages are ways of dividing the whole into 100 equal parts. The whole can be anything, an amount of money, a period of time, your body weight – the whole is simply the whole amount of something or 100%.

1 % of £500 is, therefore,  $£500 \div 100$ . That is £5. 1% of £500 = £5.

Once you have worked out what 1% is equal to, you can multiply it by the percentage you are looking for, in this case 20%. So  $£5 \times 20 = £100$ . Therefore 20% of £500 = £100. The laptop computer will therefore cost £500 – 20% which is  $£500 - £100 = £400$ .

### The easy way to work out 1% of any number:

1% is the whole (whatever that may be) divided by 100. When we divide something by 100, we simply move the place values two columns to the right,

Once you have mastered dividing by 100 to work out 1% of the whole, the relevant questions can be answered as per following steps.

1. Find your whole value and the number that you want to turn into a percentage.
2. Turn the two numbers into a fraction.
3. Place the number you want to turn into a percentage over the whole value.
4. Convert the fraction into a decimal.
5. Multiply the decimal by 100 to get the percent.
6. Use the percent as your answer.

## PERCENTAGES – FRACTIONS CONVERSIONS

For faster calculations we can convert the percentages or decimal equivalents into their respective fraction notations. The following is a table showing the conversion of percentages and decimals into fractions:

FRACTIONS	PERCENTAGE (%)
1	100
1/2	50
1/3	33.33
1/4	25
1/5	20
1/6	16.66
1/7	14.28
1/8	12.5
1/9	11.11
1/10	10

Finding 100%, 50%, 10%, 5%, 1% and 0.5% of any value helps to find the given percentage easily.

### PROPERTIES OF PERCENTAGE:

a)  $X\% \text{ of } y = y\% \text{ of } x$

$$\left(\frac{x}{100}\right) * y = \left(\frac{y}{100}\right) * x$$

b)  $X \text{ as a \% of } y \neq y \text{ as a \% of } x$

$$\frac{a}{100} \times y \neq \frac{a}{100} \times x$$

### Example 1:

Find 100%, 50%, 10%, 5%, 1% and 0.5% of 80.

Solution:

$$100\% = 80$$

$$50\% = 40$$

$$10\% = 8$$

$$5\% = 4$$

$$1\% = 0.8$$

$$0.5\% = 0.4$$

### Example 2:

Find 52% of 60.

Solution:

$$50\% \text{ of } 60 = 30$$

$$1\% \text{ of } 60 = 0.6$$

$$\text{So } 2\% \text{ of } 60 = 1.2$$

$$\text{Hence, } 52\% \text{ can be written as } 50\% + 2\% \Rightarrow 30 + 1.2 = 31.2$$

### Example: 3

Find 49% of 70.

Solution:

$$50\% \text{ of } 70 = 35$$

$$1\% \text{ of } 70 = 0.7$$

$$\text{So } 49\% \text{ of } 70 = 50\% - 1\% \Rightarrow 35 - 0.7 = 34.3$$

### Example 4:

Find 33.33% of 60.

Solution:

We know that fraction value of 33.33% is  $\frac{1}{3}$ .

Applying, we get,  $\frac{1}{3}$  of 60 = 20

### Example 5:

Find 120 % of 50.

Solution:

$$120 \% \text{ of } 50 = 50\% \text{ of } 120$$

Finding 50% of 120, we get 60.

### PRACTICE EXERCISE

- Find 62.5% of 160.  
(a) 100 (b) 150 (c) 120 (d) 200
- Find 166.66% of 1200.  
(a) 1000 (b) 2000 (c) 1500 (d) 2100
- If 40% of a = b, then b% of 40 is the same as:  
(a) 33.33% of a (b) 16 % of a  
(c) 40% of a (d) a% of b
- Orange fruit contains 68% water and the dry fruit contains 15% water. How much dry fruit will be equivalent to 15 kg of oranges?  
(a) 34 kg (b) 15 kg (c) 68 kg (d) 100 kg
- Hari spends 45% of his income on food, 25% on children's education and 80% of the remaining on house rent. What percent of his income he is left with?  
(a) 5% (b) 6% (c) 8.25% (d) 10%
- $72\% \text{ of } 180 + 24\% \text{ of } 210 - x = 420$   
(a) - 240 (b) - 244.8 (c) - 278 (d) - 120
- A bag contains 400 coins of 25 p denomination and 600 coins of 50 p denomination. If 24% of 25 p coins and 12% of 50 p coins are removed, the percentage of coins removed from the bag is nearly:  
(a) 15.2 (b) 36% (c) 16.8% (d) 25%
- A bag contains 400 coins of 25 p denomination and 600 coins of 50 p denomination. If 24% of 25 p coins and 12% of 50 p coins are removed, the percentage of money removed from the bag is nearly:  
(a) 16% (b) 15% (c) 14% (d) 36%



9. In some quantity of milk, 60% is pure milk and 40% is water. If 10 litres of pure milk is added, then the concentration of water becomes 20%. The original quantity was:

(a) 5 litres (b) 15 litres  
(c) 10 litres (d) 20 litres

**Directions for Q10 and Q11:** George spends 10 % of his income on travel, 25% on food, 20% on house rent and 15% on medical and he is left with Rs.594/-.

10. What is his income?  
(a) 2010 (b) 1980 (c) 1680 (d) 1800
11. What is the amount spent on medical expenses?  
(a) 297 (b) 300 (c) 500 (d) 420
12.  $79\% \text{ of } 8400 + 66.66\% \text{ of } 4800 = 120\% \text{ of } 7200 + x\% \text{ of } 4784$ . What is the value of x?  
(a) 33.33 (b) 25 (c) 11.11 (d) 20
13. In a town of total population, 40% are educated and the remaining are uneducated. 35% educated are employed and 45% uneducated are unemployed. What is the total percentage of people who are employed?  
(a) 55% (b) 58% (c) 49% (d) 47%

**Directions for Q14 to Q16:** A school is conducting exams for Physics, Chemistry, Mathematics and English and the maximum marks are 120, 150, 125 and 75 respectively. A student scored 65% in Physics, 78% in Chemistry, 84% in Mathematics and totally he scored 354 marks.

14. What is the percentage of marks secured by him in English?  
(a) 66.67% (b) 54% (c) 72% (d) 68%
15. What is the overall percentage of the student?  
(a) 75.31% (b) 63.28% (c) 70.8% (d) 76.66%
16. If 35% of A = B, what is B% of 35?  
(a) 35%A (b) 70%A (c) 12.25%A (d) 25%A
17. A and B are 2 digit numbers. If 25% of A + B = A and 33.33% of B = A, what is the ratio between A and A + B?  
(a) 1:4 (b) 3:4 (c) 4:3 (d) 1:2
18. 20% of 1750 is what percentage over 20?  
(a) 20% (b) 1250% (c) 1500% (d) 1750%

**Directions for Q19 and Q20:** Out of total, party A secured 35% votes and party B got 52% of votes. Difference in the number of votes between party A and party B is 255. Remaining votes are not valid.

19. What is the total number of votes?  
(a) 1800 (b) 1500 (c) 1200 (d) 1000

20. How many votes are not valid?  
(a) 100 (b) 205 (c) 185 (d) 195

## SUCCESSIVE PERCENTAGES:

If a value is increased by some percentage and decreased by some percentage subsequently or vice-versa then it is known as successive percentages or percentage increase or decrease.

You can use the formula for effective percentage change in case of successive percentage changes of a% and b% is  $(a + b + ab/100)$

Note: Please keep in mind the +/- sign while using the formula. + sign is used to increase the price amount in case of tax, GST etc. and - sign is used to decrease the price amount in case of discount offers.

## PRODUCT CONSTANCY:

Petrol/Diesel price is regularly hiked. But we are not affected as we always fill for 100 Rupees.

## Formula:

If  $Z = XY$ , where X and Y are the price and quantity of an object and Z is the total expenditure and if there is p% increase in X (or Y) then Y (or X) should decrease by  $p/(100 + p) \times 100$  to keep Z constant.

If there is p% decrease in X(or Y) then Y (or X) should increase by  $p/(100 - p) \times 100$  to keep Z constant.

## Example 1:

The price of a bokeh flower increases from Rs.5 to Rs.7. What percentage increase is this?

(a) 40% (b) 20% (c) 35% (d) 50%

Ans: [a]

Percentage increase =  $7 - 5 \times 100 / 5 \rightarrow 2 \times 100 / 5 \rightarrow 40\%$

Therefore, it is 40% increase in the price of bokeh flower.

## Example 2:

A number 98 is misread as 89. Find the reading error per cent.

(a) 8.5 (b) 7.3 (c) 9.18 (d) 10.32

Ans: [c]

Reading error percentage =  $98 - 89 \times 100 / 98 \rightarrow 900/98 \rightarrow 9.18\%$

## Example 3:

What would be the final value of 120 if increased by 20% and then decreased by 25%?

(a) 144 (b) 108 (c) 180 (d) 132

Ans: [b]

20% of 120 =  $1/5$  of 120 = 24 yielding 144

25% of 144  $\rightarrow \frac{1}{4}$  of 144 = 36 yielding 108

The final value of 120 when it is increased by 20% and then decreased by 25% would be 108.



## Example 4:

The price of an article has been reduced by 20%. In order to restore the original price, the new price must be increased by how much percentage?

- (a) 25% (b) 20% (c) 33 1/3% (d) 50%

Ans: [a]

The price of an article is reduced to 20%.

The new price of an article is to be increased by  $20/(100-20) \times 100$

$$\Rightarrow 20/80 \times 100 \Rightarrow 25\%$$

So, the new price must be increased by 25% to restore the original price.

## Example 5:

If the price of oil be raised by 20%, then the percentage by which a person must reduce his consumption so as not to increase his expenditure on oil is

- (a) 20% (b) 25% (c) 50% (d) 75%

Ans: [b]

The price of oil is raised by 20%.

To maintain the expenditure, the person has to reduce the consumption by  $b = a/(100 - a) \times 100$

$$\Rightarrow 20/100-20 \times 100 \Rightarrow 20/80 \times 100 \Rightarrow 25\%$$

The person has to reduce the consumption by 25% to maintain the expenditure.

## PRACTICE EXERCISE

21. What is the overall change when 120 is increased by 20% and then decreased by 25%?  
(a) 12% (b) 11% (c) 10% (d) 8%
22. A new bike costs Rs.85000. The customer was asked to pay 10% for extra accessories and 5% was added as road tax. Finally, how much did the customer pay?  
(a) Rs.98175 (b) Rs.98000  
(c) Rs.97355 (d) Rs.98375
23. The population of a town was 280000 two years ago. If it increased by 7% & 5% respectively in the last two years, then the present population is:  
(a) 400170 (b) 296150 (c) 314580 (d) 382110
24. Initially, Ramesh has Rs.800 in his phonepe wallet. Then he increased it by 25%. Once again he increased his amount by 20%. The final value of money in his wallet will be how much percentage greater than the initial amount?  
(a) 40% (b) 50% (c) 80% (d) 60%
25. The revenue of a shop in the month of June was Rs 75,000. In the month of July, the shopkeeper announced a discount of 30% and hence, his sales went up by 30%. What will be the revenue in the month of July?  
(a) 65750 (b) 68250 (c) 69420 (d) 64550
26. Mr. Sam purchases some articles with market price of Rs.6000. If the tax of the article increased from 7% to 28% then what will be the difference in his purchases?  
(a) Rs.1275 (b) Rs.2303  
(c) Rs.1960 (d) Rs.1260
27. The price of a washing machine is increased by 20 % and then decreased by 20%. If the original price of washing machine was 8900, then what is the final price of washing machine and the percentage change in the price?  
(a) 7896 and 8% decrease (b) 9078 and 2% increase  
(c) 7990 and 10% decrease (d) 8989 and 10% increase
28. The population of a town increased by 10% in the first year, decreased by 20% in the 2nd year and increased by 10% in the next year. If the population of the town was 28500 three years before, then the population of the town at present is  
(a) 24010 (b) 27588 (c) 25004 (d) 24545
29. In a test, minimum passing percentage for boys and girls is 40% and 50% respectively. A girl scored 360 marks and failed by 140 marks. How many more marks did a boy require to pass in the test if he scored 220 marks?  
(a) 112 (b) 160 (c) 180 (d) 160
30. If K is 200 percentage of S, then S is what percentage of S +K?  
(a) 33.33% (b) 40% (c) 75% (d) 80%
31. Mahesh buys a television for Rs.7580. He gets discount of 8% on it. After getting the discount, he pays service tax at 18%. Find the amount he will have to pay for the television.  
(a) Rs.6876.10 (b) Rs.8228.848  
(c) Rs.8216.78 (d) Rs.8118.36
32. Three successive discounts of 10%, 16%, 20% are equal to a single discount of  
(a) 53.12% (b) 49.23% (c) 36.78% (d) 39.52%
33. What conditions must be suitable for the percentage increase and the subsequent percentage decrease of a good resulting in no change in the price?  
(a)  $\frac{A}{(100+A)} \times 100$  (b)  $\frac{A}{(100-A)} \times 100$   
(c)  $\frac{(100-A)}{A} \times 100$  (d)  $\frac{(100+A)}{A} \times 100$
34. The output of a company was increased by 15% to keep up with rising demand. To handle the holiday rush, this new output was increased by 30%. By approximately what percentage would the output now have to be decreased in order to restore the original output?  
(a) 20.36% (b) 24.96% (c) 30.45% (d) 33.11%

35. A person started from Bangalore at 60 kmph to reach Chennai at 600 kms in certain time. Due to some engine problem, he wasted 1 hr while starting. By how much percentage should the person increase the speed to reach on time?  
(a) 10% (b) 11.11% (c) 12.5% (d) 14.28%
36. The price of sugar increased by 25%. What should be the percentage decrease in the consumption of sugar by a family, such that their expenditure on sugar remains the same?  
(a) 25% (b) 22.5% (c) 20% (d) 17.5%
37. 30 books in the length of 50 cm are arranged in box. If the length of the book is reduced by 50%, then, how many books can be arranged in that box?  
(a) 40 (b) 35 (c) 45 (d) 60
38. 50 men can do a work in 10 days. If the number of men is decreased by 50%, then by how much percentage the number of days gets increased?  
(a) 75% (b) 80% (c) 90% (d) 100%
39. The price of petrol increases by 45%. A family reduces its consumption so that the expenditure of the petrol is up by 20% only. If the total consumption of petrol before the price rise was 20 litres per month, then the consumption of petrol per month at present ( in lts)  
(a) 17.65 litres (b) 16.55 litres  
(c) 18.92 litres (d) 19.24 litres
40. If the price of barley be raised by 37%, find by how much percentage a person must reduce his consumption so as not to increase his expenditure?  
(a) 24% (b) 25% (c) 26% (d) 27%

## ☞ INTEREST CALCULATION

- Principal:**  
The amount of money initially borrowed for a certain period is called the principal or the sum.
- Interest:**  
Extra money paid for use of the principal after the certain period is called the interest.
- Amount:**  
The sum of the principal and the interest after a certain period is called the amount.
- Rate of Interest:**  
The rate at which the interest is calculated is called the rate of interest, which is always specified as a percentage.
- Simple Interest (S.I.):**  
If the interest on a sum borrowed for a certain period is reckoned uniformly, it is called simple interest.

If P is the principal, R% is the rate of interest per annum and T the time in years,

$$S.I. = \frac{P \times R \times T}{100}$$

$$\text{Amount} = P + \frac{PRT}{100}$$

### 6. Compound Interest:

If after specific intervals, the interest is also added to the principal for calculation of further interest, then the interest is called compound interest.

The specific interval may be annual, half yearly and quarterly.

### 7. Formulae for compound interest:

Let P be the principal, rate of interest r% per annum and 'n' the time in number of years.

- (i) When the interest is compounded annually

Compound Interest = A - P where

$$A = P \left( 1 + \frac{r}{100} \right)^n$$

- (ii) When the interest is compounded half-yearly

$$\text{Compound Interest} = A - P \text{ where } A = P \left( 1 + \frac{r}{200} \right)^{2n}$$

- (iii) When the interest compound quarterly,

$$\text{Compound Interest} = A - P, \text{ where } A = P \left( 1 + \frac{r}{400} \right)^{4n}$$

- (iv) When the interest is compounded annually, but time is in fraction, say  $3\frac{1}{2}$  years

$$\text{Amount} = P \left( 1 + \frac{R}{100} \right)^3 \left( 1 + \frac{R}{100} \right)^{1/2}$$

- (v) Present worth of Rs.x due n years hence is,

$$\text{Present worth} = \frac{x}{\left( 1 + \frac{R}{100} \right)^n}$$

### Application 1

Find the simple interest on a principal of Rs.5000 at an interest rate of 6% p.a. for a period of 6 years.

Solution:

$$P = \text{Rs.}5000, \quad R = 6\% \text{ p.a.}, \quad T = 6 \text{ years}$$

$$\text{Simple interest} = \frac{PRT}{100} = \frac{5000 \times 6 \times 6}{100} = \text{Rs.}1800$$

## Application 2

If Rs.5000 becomes Rs.5700 in a year's time, what will Rs.7000 become at the end of 5 years at the same rate of interest?

**Solution**

Amount = Principal + Interest

⇒ Interest = Amount – Principal

$$= 5700 - 5000 = \text{Rs.}700$$

$$I = \frac{PRT}{100}$$

$$\Rightarrow 700 = \frac{5000 \times r \times 1}{100} \Rightarrow r = \frac{7000}{500} = \frac{70}{5} = 14\%$$

∴ The rate of interest = 14%

The interest on Rs.7000 at 14% interest for 5 years

$$= \frac{7000 \times 14 \times 5}{100} = \text{Rs.}4900$$

∴ Amount = 7000 + 4900 = Rs.11900

## Application 3

Find the compound interest on Rs.16,000 at 20% per annum for 9 months, compounded quarterly.

**Solution**

Principal = Rs.16,000, Rate = 20% per annum = 5% per quarter

Time = 9 months = 3 quarters

$$\begin{aligned} \therefore \text{Amount} &= 16000 \times \left(1 + \frac{5}{100}\right)^3 \\ &= 16000 \times \left(\frac{21}{20}\right)^3 = \text{Rs.}16000 \times \frac{21 \times 21 \times 21}{20 \times 20 \times 20} = \text{Rs.}18522 \end{aligned}$$

$$\begin{aligned} \therefore \text{Compound interest} &= \text{Amount} - \text{Principal} \\ &= 18522 - 16000 = \text{Rs.}2522 \end{aligned}$$

## Application 4

A sum of money doubles itself at compound interest in 15 years. In how many years will it become 8 times?

**Solution**

$$P \left(1 + \frac{r}{100}\right)^{15} = 2P \Rightarrow \left(1 + \frac{r}{100}\right)^{15} = 2 \quad \dots (i)$$

Let it become 8 times in n years.

$$\therefore P \left(1 + \frac{r}{100}\right)^n = 8P \Rightarrow \left(1 + \frac{r}{100}\right)^n = 8 \quad \dots (ii)$$

From (i) and (ii)

$$\left(1 + \frac{r}{100}\right)^n = 8 = \left[\left(1 + \frac{r}{100}\right)^{15}\right]^3$$

$$\Rightarrow \left(1 + \frac{r}{100}\right)^n = \left(1 + \frac{r}{100}\right)^{45}$$

$$\Rightarrow n = 45 \text{ years}$$

∴ The required time is 45 years.

## Application 5

If the difference between the compound interest and the simple interest on a certain sum of money is Rs.72 at 12% per annum for 2 years, find the amount.

**Solution**

Let the amount be P.

$$\text{Simple interest} = \frac{P \times 12 \times 2}{100} = \frac{24P}{100} = 0.24P$$

$$\begin{aligned} \text{Compound interest} &= P \left(1 + \frac{12}{100}\right)^2 - P \\ &= P \left[\left(\frac{28}{25}\right)^2 - 1\right] \\ &= P [1.12^2 - 1] = 0.2544P \end{aligned}$$

$$\text{Difference} = 0.2544P - 0.24P = \text{Rs.}72$$

$$\Rightarrow 0.144P = \text{Rs.}72$$

$$\Rightarrow P = \frac{72}{0.144} = \frac{72000}{144} = \text{Rs.}5000$$

∴ The amount is Rs.5000.

## GUARD AGAINST POPULAR ERRORS

- While calculating the interest, the time period is always years. If months are given, it has to be converted to years by dividing by 12.
- If the rate of interest is specified as half-yearly or quarterly, time to be taken in terms of 6 months or 3 months.
- The half yearly interest rate will be  $\frac{1}{2}$  of the annual rate of interest and the quarterly interest rate will be  $\frac{1}{4}$  of the annual rate of interest.

- While calculating compound interest, the principal should be subtracted from the amount calculated.

## TWO QUESTIONS

- Ranjan deposited Rs.2000 in a bank. The bank pays 5% interest per annum. What will be his annual income?
- Shyam lent Rs.2100 at compound interest of 5% p.a. for 2 years to his friend. Find the amount after 2 years.
- Find the rate of interest if the amount after 2 years of simple interest on a capital of Rs.1200 is Rs.1440.
- At what rate percent per annum simple interest will a sum of money double in 16 years?
- At what time will Rs.1000 become Rs.1331 at 10% per annum compounded annually?

## PRACTICE EXERCISE

- Find the difference between the simple interest and compound interest at 5% per annum for 2 years on a principal of Rs.2000.  
(a) Rs.5 (b) Rs.105 (c) Rs.50 (d) Rs.5.50
- The SBI lent Rs.1331 to the Tata group at compound interest and got Rs.1728 after 3 years. What is the rate of interest charged if the interest was compounded annually?  
(a) 11% (b) 9.09% (c) 12% (d) 8.33%
- Ranjan purchased a Maruthi van for Rs.1,96,000 and the rate of depreciation is  $14\frac{2}{7}\%$  per annum. Find the value of the van after 2 years.  
(a) Rs.1,40,000 (b) Rs.1,44,000  
(c) Rs.1,50,000 (d) Rs.1,60,000
- A sum was invested at simple interest at certain rate for 2 years. It would have fetched Rs.60 more had it been invested at 2% higher rate. Find the sum.  
(a) Rs.1500 (b) Rs.1300  
(c) Rs.2500 (d) Rs.1000
- Shashikanth derives an annual income of Rs.688.25 from Rs.10,000 invested partly at 8% per annum and partly at 5% per annum simple interest. Find the sum of money invested at 5%.  
(a) Rs.5000 (b) Rs.4225  
(c) Rs.4800 (d) Rs.3725
- A sum of money doubles itself in 5 years. In how many years will it become four-fold if interest is compounded annually?  
(a) 15 (b) 10 (c) 20 (d) 12
- If a person repaid Rs.22500 after 10 years of borrowing a loan at 10% per annum simple interest, find out the amount he took as loan.  
(a) Rs.11225 (b) Rs.11250  
(c) Rs.10,000 (d) Rs.7500
- The RBI lends a certain amount to the SBI on simple interest for 2 years at 20%. The SBI gives this entire amount to Bharathi Telecom on compound interest for 2 years at the same rate annually. Find the percentage earning of the SBI at the end of 2 years on the entire amount.  
(a) 4% (b)  $3\frac{1}{7}\%$  (c)  $3\frac{2}{7}\%$  (d)  $3\frac{6}{7}\%$
- Find the compound interest at the rate of 10% for 2 years on the principal which in 3 years at the rate of 10% p.a. gives Rs.300 as simple interest.  
(a) Rs.331 (b) Rs.310  
(c) Rs.330 (d) Rs.333
- The difference between compound interest and simple interest on a certain sum of money at 10% p.a. for 3 years is Rs.620. Find the principal if it is known that the interest is compounded annually.  
(a) Rs.200,000 (b) Rs.20,000  
(c) Rs.10,000 (d) Rs.100,000
- Find the least number of complete years in which a sum of money put out at 20% compound interest will be more than doubled.  
(a) 3 (b) 4 (c) 5 (d) 6
- Mr. Dhas invested money in 2 schemes A and B offering compound interest at 8% per annum and 9% per annum respectively. If the total amount of interest accrued through the two schemes together in 2 years was Rs.4818.30 and the total amount invested was Rs.27,000. Find the amount invested in scheme A.  
(a) Rs.12,000 (b) Rs.13,500  
(c) Rs.15,000 (d) None of these

## Directions for Q13 to Q15

Answer (a) if Statement I alone is sufficient to answer the question but statement II alone is not sufficient to answer the question.

Answer (b) if Statement II alone is sufficient to answer the question but Statement I alone is not sufficient to answer the question.

Answer (c) if Statements I and II together are sufficient to answer the question but neither Statement alone is sufficient to answer the question.

Answer (d) if each Statement alone is sufficient to answer the question.

Answer (e) if both Statements I and II together are not sufficient to answer the question and additional data is needed.

- What is the compound interest earned by Akash at the end of 2 years?  
I: Simple interest at the same rate for 1 year in Rs.1020 and the rate of interest is 12% per annum.  
II: The amount invested is Rs.8500.
- An amount of money was lent for 3 years. Find the difference between the simple and the compound interest earned on it at the same rate.  
I: The rate of interest was 8% per annum.  
II: The total amount of simple interest was Rs.1200.
- What is the principal deposited by Krishna in a bank?  
I: The interest earned on a fixed deposit for 2 years is Rs.4830.  
II: The bank offers 10% interest compounded annually on fixed deposits.



16. Kaviya borrowed a sum of Rs.4800 from Anitha as a loan. She promised Anitha that she would pay it back in two equal instalments. If the rate of Interest be 5% per annum compounded annually, find the amount of each instalment  
 (a) Rs.2581.46 (b) Rs.4232.075  
 (c) Rs.2845.16 (d) Rs.4243.075
17. Equal sums of money are deposited in two different banks by M/s ABC Company, one at compound interest (compounded annually) and the other at simple interest, both at 15% per annum. If after two years, the difference in the amounts comes to Rs.360, what is the amount deposited with each bank?  
 (a) Rs.17200 (b) Rs.16400  
 (c) Rs.16000 (d) Rs.18400
18. Mr. X puts equal amount of money in two schemes: one at 10% per annum compound interest payable half yearly and the second at a certain percent per annum compound interest payable yearly. If he gets equal amounts after 3 yrs, what is the value of the interest percent in the second scheme?  
 (a) 10.25% (b) 15% (c) 22% (d) 21%
19. A sum of Rs.5887 is divided between Prince and Fernando, such that Prince's share at the end of 9 yrs is equal to Fernando's share at the end of 11 yrs both compounded annually at the rate of 5%. The share of Prince is  
 (a) Data inadequate (b) Rs.3087  
 (c) Rs.2000 (d) Rs.2088
20. A sum of money is accumulating at compound interest at a certain rate. If simple interest instead of compound were reckoned, the interest for the first two years would be diminished by Rs.20 and that for the first three years by Rs.61. Find the sum.  
 (a) Rs.7000 (b) Rs.47405  
 (c) Rs.45305 (d) Rs.8000
21. An amount of Rs.7500 is invested in a compound interest scheme for 4 years. The rate of interest is 2% for the first year, 3% for the next 2 years and for the last year, it is 4%. The final amount approximately is  
 (a) Rs.8356.7 (b) Rs.8557.45  
 (c) Rs.8440 (d) Rs.8635
22. A took a sum of Rs.4500 from B. He promised B that he would give back her money at the end of the year but she gave an option to him that he could pay her in two equal annual instalments. A agreed on her suggestion. If the rate of interest on the sum was 10% per annum, compounded annually, find the amount of the instalment given by A.  
 (a) Rs.2390 (b) Rs.3429  
 (c) Rs.2560 (d) Rs.2592
23. The simple interest accrued on an amount of Rs.27,500 at the end of three years is Rs.9900. What would be the difference between compound interest and simple interest on the same amount at the same rate in the same period?  
 (a) Rs.11550 (b) Rs.1262.52  
 (c) Rs.1235.52 (d) Rs.11135.52
24. Shawn invested one half of his savings in a bond that paid simple interest for 2 years and received \$ 550 as interest. He invested the remaining in a bond that paid compound interest, interest being compounded annually, for the same 2 years at the same rate of interest and received \$605 as interest. What was the value of his total savings before investing in these two bonds?  
 (a) \$5500 (b) \$7750 (c) \$3550 (d) \$2750
25. Raj borrowed Rs.7000 from a bank at 5% simple annual rate of interest. The amount is to be paid after five years. Raj paid Rs.4000 after two years. How much amount should he pay at the end of 5 years to pay off his debt completely?  
 (a) Rs.4050 (b) Rs.4100 (c) Rs.4150 (d) Rs.4200
26. Effective annual rate of interest corresponding to nominal rate of 6% per annum compounded half yearly will be  
 (a) 6.09% (b) 6.10% (c) 6.12% (d) 6.14%
27. Adam borrowed some money at the rate of 6% p.a. for the first two years, at the rate of 9% p.a. for the next three years, and at the rate of 14% p.a. for the period beyond five years. If he pays a total interest of Rs.11,400 at the end of nine years, how much money did he borrow ?  
 (a) Rs.15000 (b) Rs.11000  
 (c) Rs.12000 (d) Rs.18000
28. The compound interest on a sum for 2 years is Rs.832 and the simple interest on the same sum for the same period is Rs.800. The difference between the compound and simple interest for 3 years will be  
 (a) Rs.98.56 (b) Rs.99.86 (c) Rs.96.62 (d) Rs.97.77
29. Sally borrowed a sum of Rs.14000 at 9% rate of interest per annum from Martha for a term of 2 years. After completion of one year, Sally repaid some amount and finally at the end of two years Sally completed the debt by discharging a sum of Rs.11990. What amount did Sally pay at the end of the first year?  
 (a) Rs.4560 (b) Rs.4260 (c) Rs.4460 (d) Rs.4360
30. What annual instalment will discharge a debt of Rs.6450 due in 4 years at 5% simple interest?  
 (a) Rs.1000 (b) Rs.1200  
 (c) Rs.1450 (d) Rs.1500



31. A sum of Rs.10 is given as a loan to be returned in 6 monthly instalments at Rs.3. What is the rate of interest?  
(a) 500% (b) 620% (c) 640% (d) 580%
32. Rs.7500 is to be paid in 3 years at 5% C.I in equal annual instalments. Find the value of the instalment?  
(a) Rs.2754 (b) Rs.2457 (c) Rs.2547 (d) Rs.2745
33. Interest on a sum of money at the end of 3 years is  $\frac{3}{8}$ th of the sum itself. Find the rate of interest.  
(a) 11.5% (b) 12.2% (c) 12.5% (d) 13.5%
34. Arun borrowed a certain sum from Manish at a certain rate of simple interest for 2 years. He lent this sum to Sunil at the same rate of interest compounded annually for the same period. At the end of two years, he received Rs.2400 as compound interest but paid Rs.2000 only as simple interest. Find the rate of interest.  
(a) 40% (b) 30% (c) 10% (d) 20%
35. Divide Rs.3364 between A and B, so that A's share at the end of 5 years may equal to B's share at the end of 7 years, the compound interest being at 5 percent.  
(a) Rs.1764, Rs.1600 (b) Rs.1756, Rs.1608  
(c) Rs.1722, Rs.1642 (d) None of these
8. The product of the two positive integers multiplied with the square of its HCF is 240. How many such pairs exist?  
(a) 9 (b) 7 (c) 5 (d) None of these
9. If a two digit number pq has 3 divisors, then how many divisors does the number pqpq have?  
(a) 3 (b) 6 (c) 9 (d) 12
10. Find the number of divisors of 1400 which are not perfect squares.  
(a) 24 (b) 20 (c) 18 (d) 16
11. Find the number of even divisors of 2016.  
(a) 20 (b) 22 (c) 30 (d) 24
12. Find the sum of divisors of 2475.  
(a) 2418 (b) 3627  
(c) 4836 (d) Cannot be determined
13. How many divisors of  $2^7 \times 3^3 \times 11^2$  are perfect squares?  
(a) 15 (b) 16 (c) 17 (d) 18
14. The numbers A, B, C and D have 10, 15, 27 and 49 divisors respectively. Which of these could be a perfect cube?  
(a) A and B (b) B and C (c) C and D (d) A and D
15. What is the unit's digit of  $127^{127}$ ?  
(a) 7 (b) 9 (c) 3 (d) 1
16. Find the last non zero digit of the number  $80^{1230}$ .  
(a) 8 (b) 4 (c) 2 (d) 6
17. What is the unit's digit of  $52^{52}$ ?  
(a) 2 (b) 4 (c) 8 (d) 6
18. Find the number of trailing zeros in  $229!$   
(a) 50 (b) 55 (c) 60 (d) 65
19. Find the least number n such that no factorial has n trailing zeros or n+1 trailing zeros.  
(a) 29 (b) 30 (c) 49 (d) 50
20. Find the greatest value of n such that  $180^n$  exactly divides  $128!$   
(a) 30 (b) 15  
(c) 31 (d) such n does not exist
21. A number N gives a remainder of 6 when divided by D and a remainder of 19 when divided by 3D. What will be the remainder when 2N is divided by D?  
(a) 6 (b) 12 (c) 19 (d) 24
22. What is the remainder when  $6^{90} - 5^{90}$  is divided by 91?  
(a) 0 (b) 1 (c) 89 (d) 90
23. Find the remainder when  $3^{555} + 5^{333}$  is divided by 23.  
(a) 18 (b) 7 (c) 0 (d) None of these
24. What is the remainder when  $23^{24^{25}}$  is divided by 7?  
(a) 1 (b) 2 (c) 4 (d) 6

## NUMBER PROPERTIES

1. Write 0.0512512... as a fraction in its lowest terms.  
(a)  $\frac{512}{9990}$  (b)  $\frac{256}{4995}$  (c)  $\frac{512}{9900}$  (d)  $\frac{128}{2475}$
2. The number  $43xy20xy$  is exactly divisible by 12. How many two digit numbers as xy are possible?  
(a) 4 (b) 6  
(c) 8 (d) Cannot be determined
3. Let  $N = 5ab42ab$ . If N is exactly divisible by 180, then the sum of the digits in N is  
(a) 18 (b) 24 (c) 27 (d) 36
4. The HCF and LCM of two numbers are 73 and 2555 respectively. If one number is 365, then the other number is  
(a) 73 (b) 146 (c) 438 (d) 511
5. Which of the following is the largest four digit number that can be added to 5793 so that the sum is divisible by each of 12, 14, 33 and 42?  
(a) 9854 (b) 9924 (c) 9963 (d) 9915
6. Find the greatest number of five digits which when divided by 10, 13, 15 and 26 gives remainders 4, 7, 9 and 20 respectively.  
(a) 99854 (b) 99834 (c) 99844 (d) 99914
7. How many pairs of positive integers a, b exist such that  $HCF(a, b) + LCM(a, b) = 77$ ?  
(a) 10 (b) 8 (c) 7 (d) 6

25. What is the remainder when  $35^{182}$  is divided by 37?  
(a) 15 (b) 11 (c) 8 (d) 4
26. Find the remainder when the number  $3451 + 3452 + \dots + 3794$  is divided by 345.  
(a) 340 (b) 175 (c) 53 (d) None of these
27. How many integers are there from 1 to 500 that leaves a remainder of 2 on division by 5 and a remainder of 5 on division by 7?  
(a) 12 (b) 13 (c) 14 (d) 15
28. A number when divided by 33 leaves a remainder 10. The same number when divided by 22 leaves a remainder n. How many values can n take?  
(a) 1 (b) 2 (c) 3 (d) 4
29. Find the remainder when  $2^{64}$  is divided by 160.  
(a) 36 (b) 66 (c) 96 (d) None of these
30. What is the last two digits in the expression  $91 \times 92 \times 93 \times \dots \times 98$ ?  
(a) 80 (b) 30 (c) 40 (d) 50
31. Find the last two digits in  $31^{77} \times 75^{84}$ .  
(a) 25 (b) 55 (c) 75 (d) 95
32. Find the last two digits in  $59^{27}$ .  
(a) 19 (b) 29 (c) 39 (d) 49
33. Find the last two digits in  $78^{35}$ .  
(a) 18 (b) 32 (c) 58 (d) 72
34. What is the last two digits of  $16^{26}$ ?  
(a) 16 (b) 26 (c) 06 (d) None of these
35. What is the last two digits in the expression  $31^{15} - 24^{15}$ ?  
(a) 25 (b) 75 (c) 27 (d) 57
36. How many divisors of  $140^5$  will have at least one zero at its end?  
(a) 225 (b) 300 (c) 350 (d) 425
37. Let K be the largest number with exactly three factors that divide  $40!$ . How many factors does  $K - 1$  have?  
(a) 16 (b) 24 (c) 32 (d) 40
38. How many factors does  $6N$  have, if  $N$  is a number such that  $2N$  has 28 factors and  $3N$  has 30 factors?  
(a) 35 (b) 32 (c) 28 (d) None of these
39. What is the unit digit in the expression  $38^{38}$ ?  
(a) 8 (b) 4 (c) 2 (d) 6
40. What is the last non zero digit in  $25!$ ?  
(a) 9 (b) 4 (c) 2 (d) 1
41. Write  $0.22727\dots$  as a fraction in its lowest terms.  
(a)  $\frac{227}{990}$  (b)  $\frac{5}{11}$  (c)  $\frac{5}{22}$  (d)  $\frac{10}{22}$
42. If  $74p58p4$  is divisible by 9 and if  $757qp$  is divisible by 8, then the minimum value of  $p + q$  is  
(a) 0 (b) 2 (c) 3 (d) 4
43. The least perfect square number which is divisible by 4, 11, 13 and 18 is  
(a) 736164 (b) 361524 (c) 1472328 (d) 723048
44. Two numbers are in the ratio 16:15. If their HCF is 13, what are the numbers?  
(a) 210, 197 (b) 188, 173  
(c) 228, 205 (d) 208, 195
45. How many pairs of non coprime positive integers x, y exist such that sum of the numbers x, y and twice the HCF(x, y) is 221?  
(a) 5 (b) 7  
(c) 9 (d) None of these
46. How many divisors of 98000 are odd numbers?  
(a) 60 (b) 48 (c) 12 (d) 8
47. Find the unit digit of  $2323^{479} \times 529^{479}$ .  
(a) 3 (b) 9 (c) 7 (d) 1
48. What is the unit digit of  $2321^{78}$ ?  
(a) 2 (b) 4  
(c) 8 (d) None of these
49. Find the difference of number of trailing zeros in  $112!$  and  $115!$ .  
(a) 0 (b) 1 (c) 2 (d) 3
50. Find the greatest value of n such that  $15^n$  exactly divides  $87!$ .  
(a) 20 (b) 22 (c) 24 (d) 26
51. Find the remainder when  $6^{70} + 8^{70}$  is divided by 100.  
(a) 86 (b) 68 (c) 42 (d) 0
52. What is the remainder when  $(10^3 + 9^3)^{3089}$  is divided by  $12^3$ ?  
(a) 1 (b) 729 (c) 1450 (d) 1727
53. What is the remainder when  $52^{53^{54}}$  is divided by 17?  
(a) 15 (b) 7 (c) 1 (d) 0
54. Find the remainder when  $72 \times 1$  is divided by 73.  
(a) 0 (b) 1 (c) 2 (d) 3
55. What is the remainder when  $3^{101}$  is divided by 101?  
(a) 99 (b) 66 (c) 33 (d) 3
56. What is the tens digit in the expression  $1! + 2! + \dots + 150!$ ?  
(a) 1 (b) 2 (c) 3 (d) 4
57. Find the last two digits in  $83^{26}$ .  
(a) 39 (b) 69 (c) 99 (d) 97
58. Find the last two digits in  $44^{63}$ .  
(a) 26 (b) 24 (c) 86 (d) None of these
59. What is the difference between the number of even and odd factors of 1080?  
(a) 16 (b) 14 (c) 12 (d) 8
60. If the sum of the  $(4n + 3)$  prime numbers is an even number, then one of the prime numbers must be  
(a) 3 (b) 5 (c) 11 (d) 2