Course: Engineering Physics
PHY 1701

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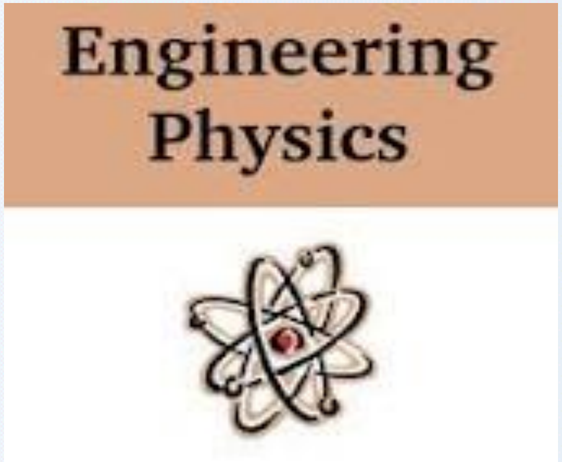
Outline

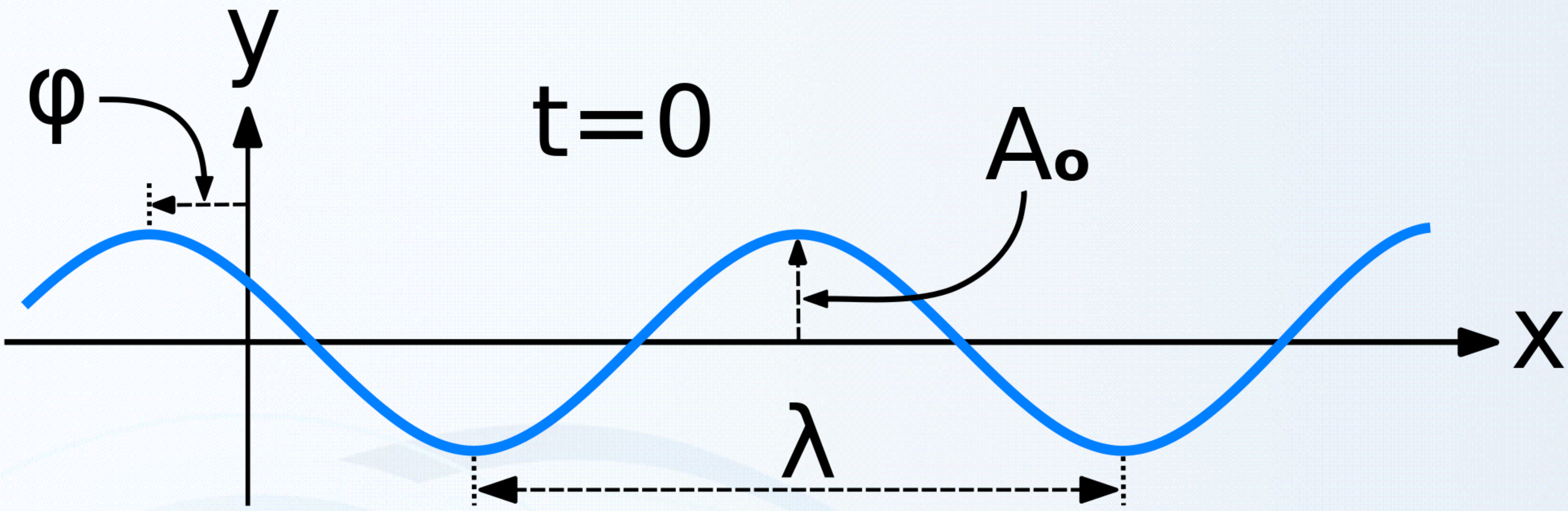
- Introduction
- Wave Function
- Schrodinger Equation
  - Time independent
  - Time dependent



Resources:

Concepts of Modern Physics (Arthur Beiser)
Pages: 181 – 190

Wave Equation



$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2},$$

$$y(x,t) = A\cos\omega\left(t - \frac{x}{v}\right)$$

Wave Function

Each "particle" is represented by a wavefunction Ψ (position, time) such that $\Psi^*\Psi$ = the probability of finding the particle at that position at that time.

The wavefunction is used in the Schrodinger equation. The Schrodinger equation plays the role of Newton's laws and conservation of energy in classical mechanics - i.e., it predicts the future behavior of a dynamic system. It predicts analytically and precisely the probability of events or outcome. The detailed outcome depends on chance, but given a large nummber of events, the Schrodinger equation will predict the distribution of results.

Wave Function

- ψ contains all the measurable information about the particle
- ψ $\Psi^*\Psi$ summed over all space = 1 (if the particle exists, the probability of finding it somewhere must be one)
- ψ is continuous
- ψ allows energy calculations via the Schrodinger equation
- ψ establishes the probability distribution in three dimensions
- ψ permits calculation of the effective average value (expectation value) of a given variable
- ψ for a free particle is a sine wave, implying a precisely determined momentum and a totally uncertain position (uncertainty principle)

Schrodinger Equation

It is a wave equation in terms of the wave function which predicts analytically and precisely the probability of events or outcome.

The detailed outcome is not strictly determined, but given a large number of events, the Schrodinger equation will predict the distribution of results.

Schrodinger Equation

The kinetic and potential energies are transformed into the Hamiltonian which acts upon the wave function to generate the evolution of the wave function in time and space.

The Schrodinger equation gives the quantized energies of the system and gives the form of the wave function so that other properties may be calculated.

Schrodinger Equation

$$\text{Kinetic Energy} + \text{Potential Energy} = E$$

Classical

Conservation of Energy

Newton's Laws

$$\frac{1}{2}mv^2 + \frac{1}{2}kx^2 = E$$

$$F = ma = -kx$$

Harmonic oscillator example.

Quantum

Conservation of Energy

Schrodinger Equation

$$\frac{p^2}{2m} + \frac{1}{2}kx^2$$

The energy becomes the Hamiltonian operator

$$H\Psi = E\Psi$$

Wavefunction

Energy "eigenvalue" for the system.

In making the transition to a wave equation, physical variables take the form of "operators".

$$p \rightarrow \frac{\hbar}{i}\frac{\partial}{\partial x} \qquad x \rightarrow x$$

$$H \rightarrow \frac{-\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + \frac{1}{2}kx^2$$

The form of the Hamiltonian operator for a quantum harmonic oscillator.

Time Independent

$$\frac{\partial^2\psi}{\partial x^2}+\frac{\partial^2\psi}{\partial y^2}+\frac{\partial^2\psi}{\partial z^2}+\frac{8\pi^2 m}{h^2}(E-V)\psi=0,$$

In some situations the potential energy does not depend on time t.

In this case we can often solve the problem by considering the simpler *time-independent* version of the Schrödinger equation for a function Ψ depending only on space Ψ(x,y,z)

Time Dependent

$$\frac{ih}{2\pi}\frac{\partial\Psi}{\partial t} = -\frac{h^2}{8\pi^2 m}\left(\frac{\partial^2\Psi}{\partial x^2}+\frac{\partial^2\Psi}{\partial y^2}+\frac{\partial^2\Psi}{\partial z^2}\right)+V\Psi.$$

Here V is the potential energy of the particle (a function of x, y, z and t),

i=$\sqrt{-1}$,

m is the mass of the particle and

h is Planck's constant.

The solution to this equation is the wave function

Ψ(x,y,z,t)