Theory of Computation and Compiler Design

Turing Machines

Prof. Nisha V M,
VIT Chennai

Introduction-Chomsky Hierarchy

- In formal language theory, Noam Chomsky classified the grammars and the correspoding language it generates.
- Chomsky categorised the language acceptor machines also along with the grammar and languages based on the computation power of each machines.

Chomsky hierarchy

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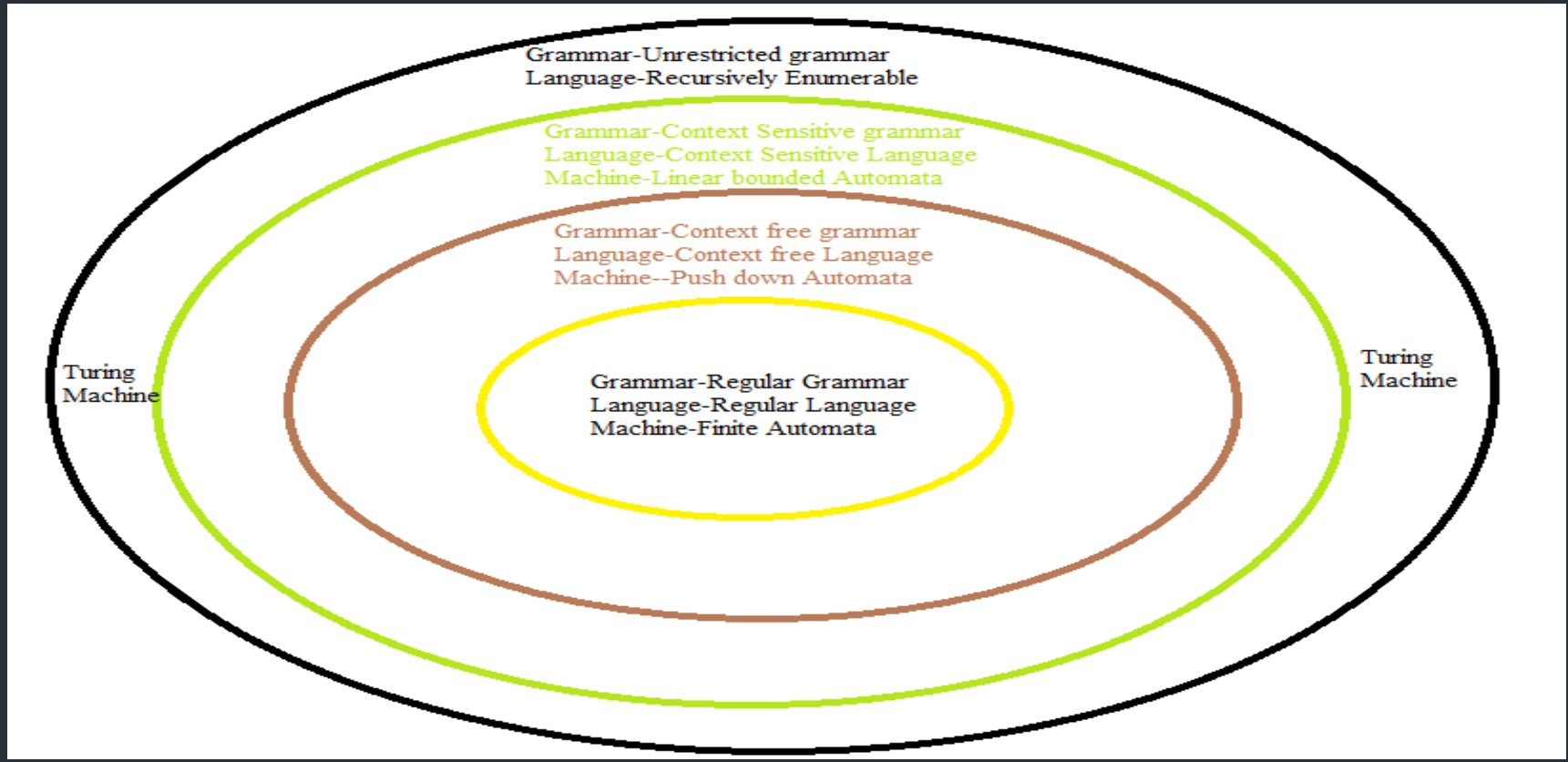
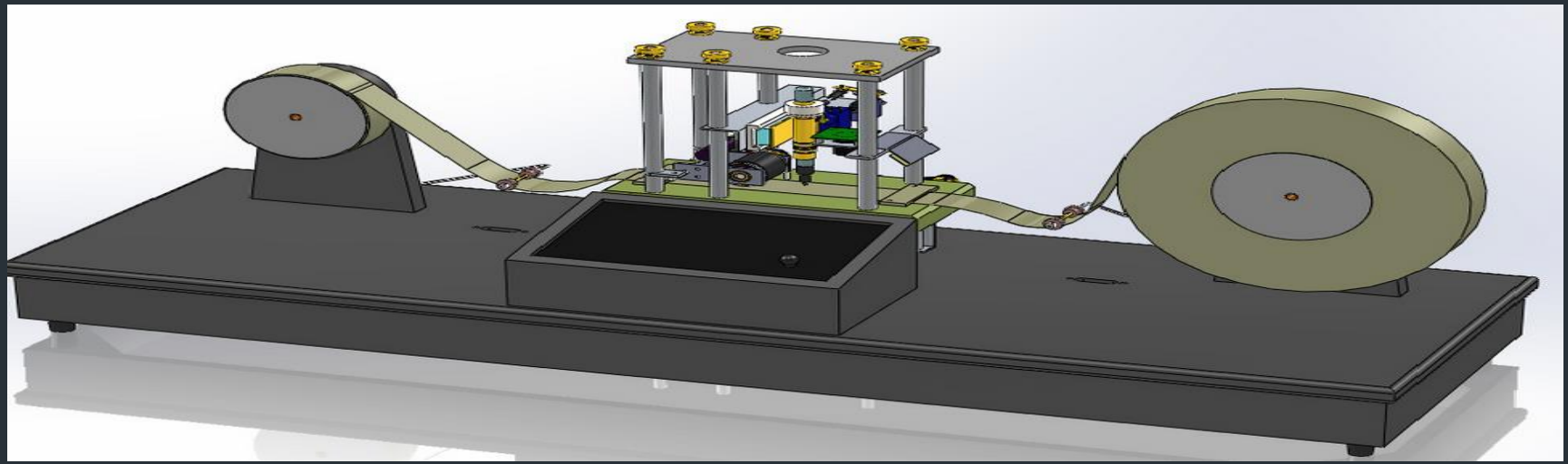


Turing Machine

- Alan Turing, proposed a machine for computation known as Turing machine.

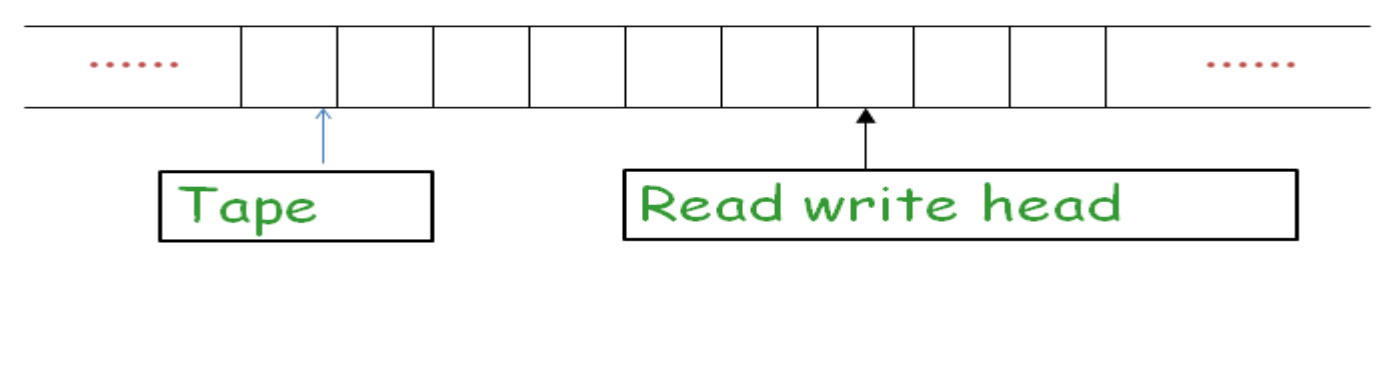


Turing Machine

- It is a general computation model.
- It has the potential to compute any algorithm
- It is a sequential machine
- It is used as language acceptor
- It is used for computing any function
- Different types of variants of Turing machines:
  - Multi tape Turing Machine
  - Multiple track Turing Machine
  - Multi tape –multi head Turing Machine.

Basics of Turing Machine

- Turing machine consist of an infinite sized tape as memory.
- Tape is divided into equal sized cells.
- A read –write head points to each cells in the tape



Basics of Turing Machine

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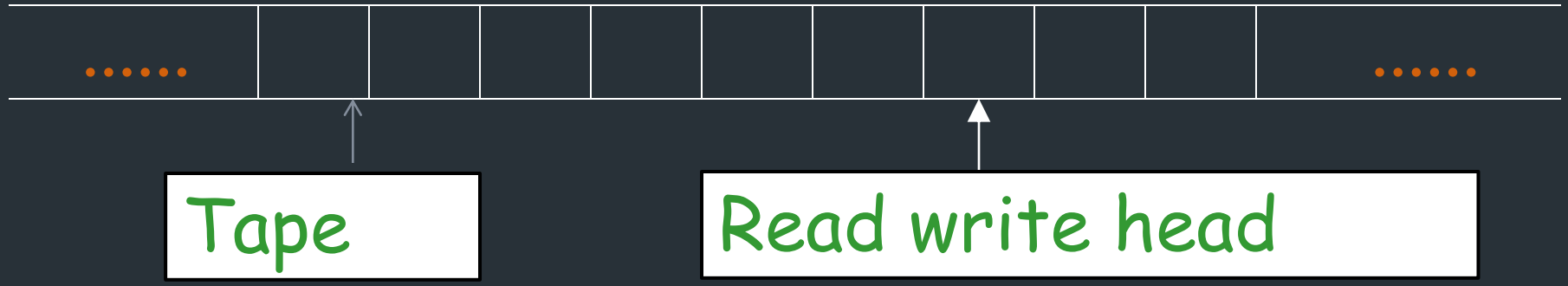

Basics of Turing Machine

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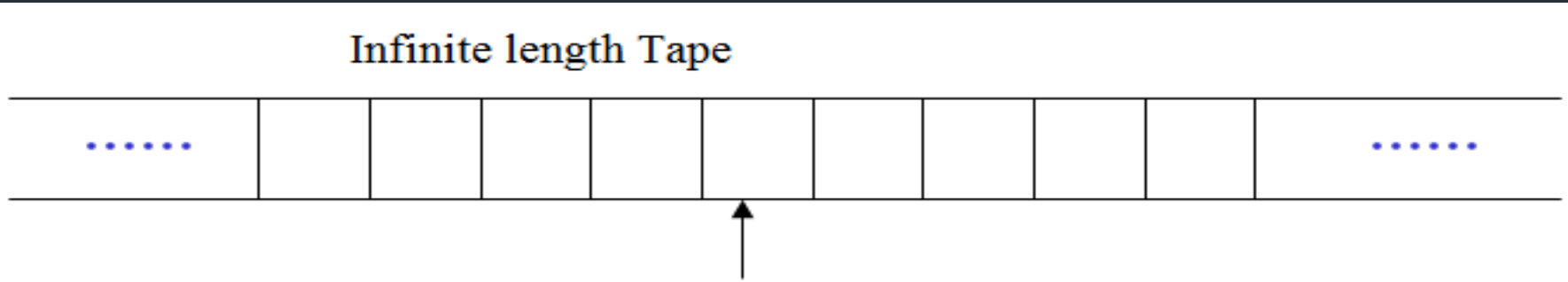


The head moves Left or Right

Goal of a Turing Machine

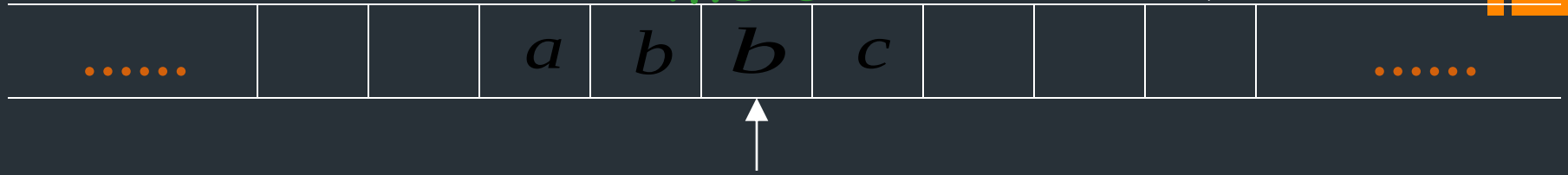
Goal of Turing's Machine: A model that can compute anything that a human can compute.

The process of Turing machine

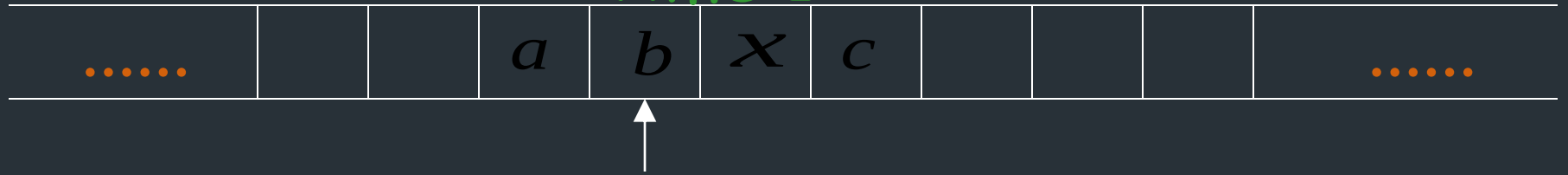
- At each time step, the Turing machine head points to a symbol in the tape
- The operations of the head at each time step:
  - Read a symbol from the tape
  - Write a symbol into the tape
  - Move left or write

Example:

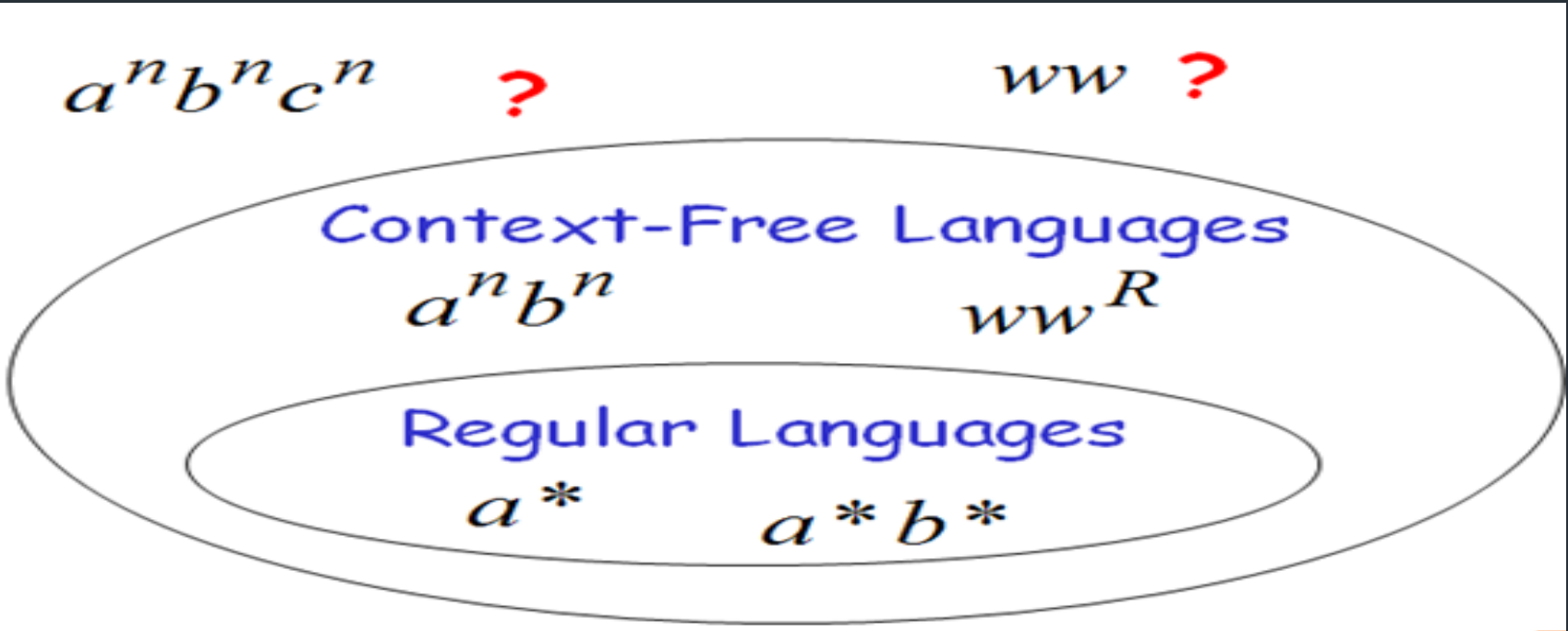
Time 0

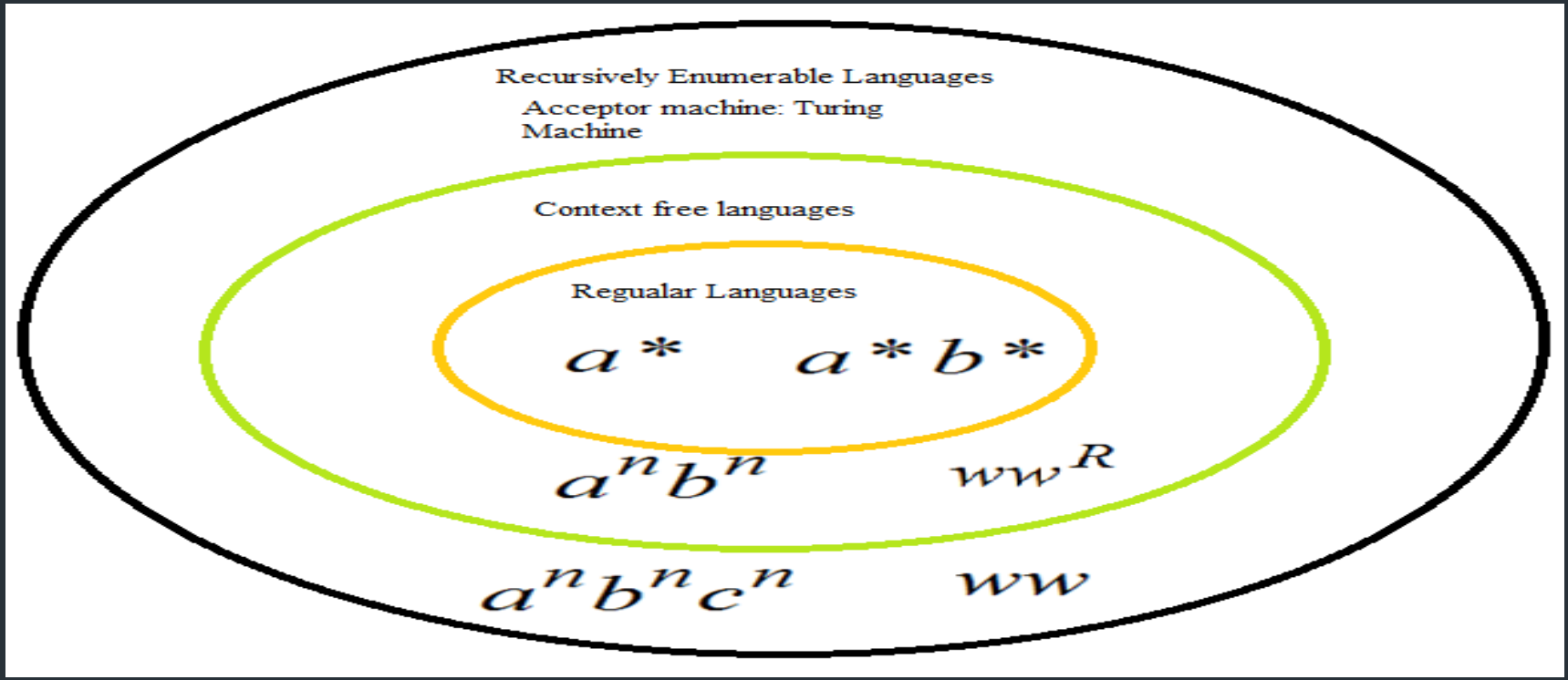


Time 1



1. Reads b
2. Writes x
3. Moves Left


$a^n b^n c^n$?
ww ?
Context-Free Languages
$a^n b^n$
ww^R
Regular Languages
a^*
$a^* b^*$


Recursively Enumerable Languages
Acceptor machine: Turing Machine
Context free languages
Regualar Languages
$a*$
$a*b*$
$a^n b^n$
ww^R
$a^n b^n c^n$
ww

Definition of Turing Machine

- A Turing machine is defined as a 5-tuple,
- TM=(S,∑,**τ**, δ, B,F, S_0)
- where S indicate- set of states
- **τ** – Tape symbol
- ∑- input alphabet ------ it is a subset of **τ**

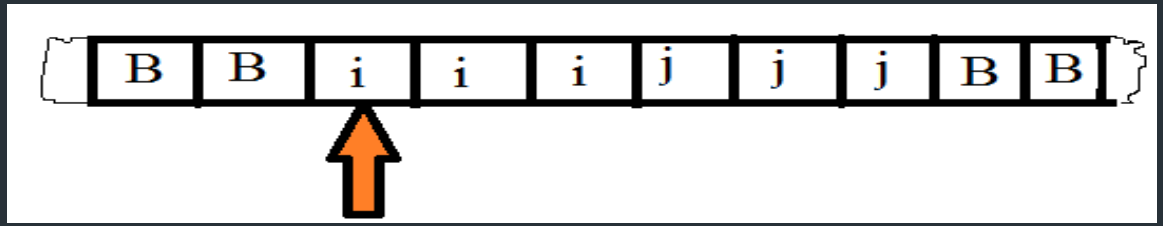
  δ is the transition function,
- δ: S x **τ** →S, **τ**, {L, R}
- B—blank symbol in the tape
- S_0 --- Initial state
- F – Final state

Transition function in TM

- $\delta: S \times \tau \rightarrow S, \tau, \{L, R\}$
- A transition is a function(δ), takes two parameters such as present state and the tape symbol pointed by the read write head, and it returns the next state, the new tape symbol in the location pointed by the read write head and the direction of the head movement.

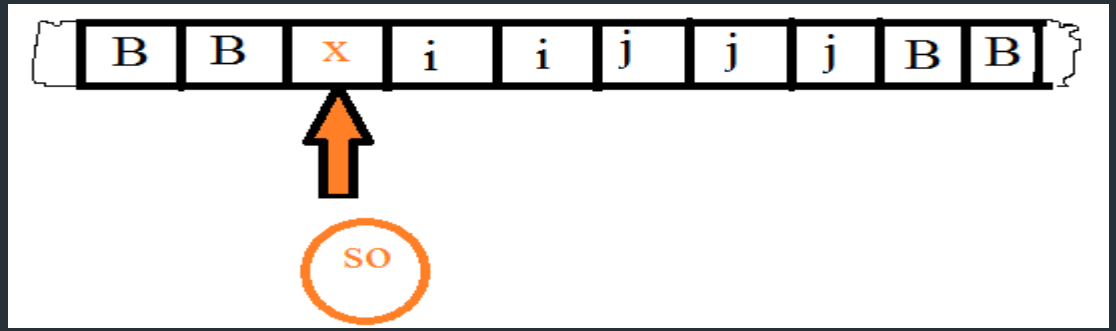
Turing machine as language acceptor

- Consider a language, $L = \{i^n j^n \mid n>=1\}$
- The basic idea:
- For each i in the tape, check for a corresponding j
- If any mismatch, TM halts(enter into the reject state)
- Otherwise, TM enter into the accepting state
- Consider an example :$L = \{i^3 j^3\}$

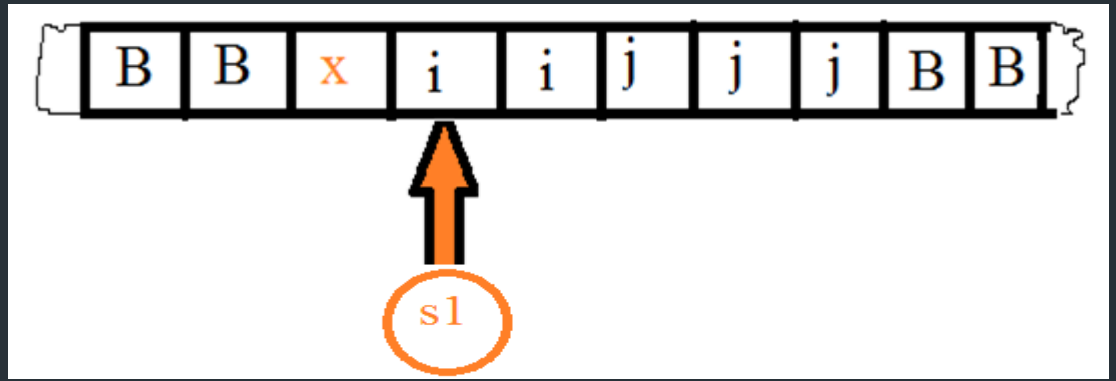


The working of TM for the example, $L = \{i^3 j^3\}$

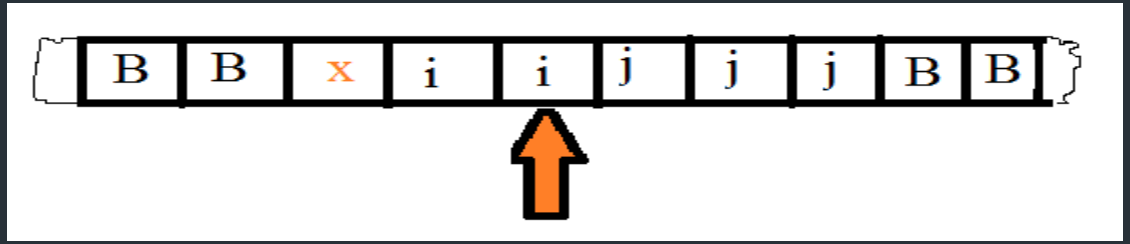
- The machine is initially at the start state S0
- Read leftmost i
- Replace i with x
- Move the read write head to right direction



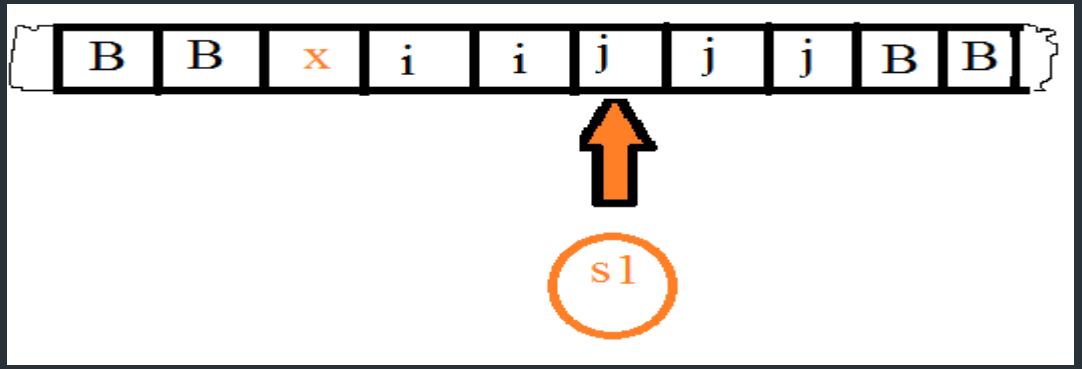
- Move the head to right
- Change the state as S1



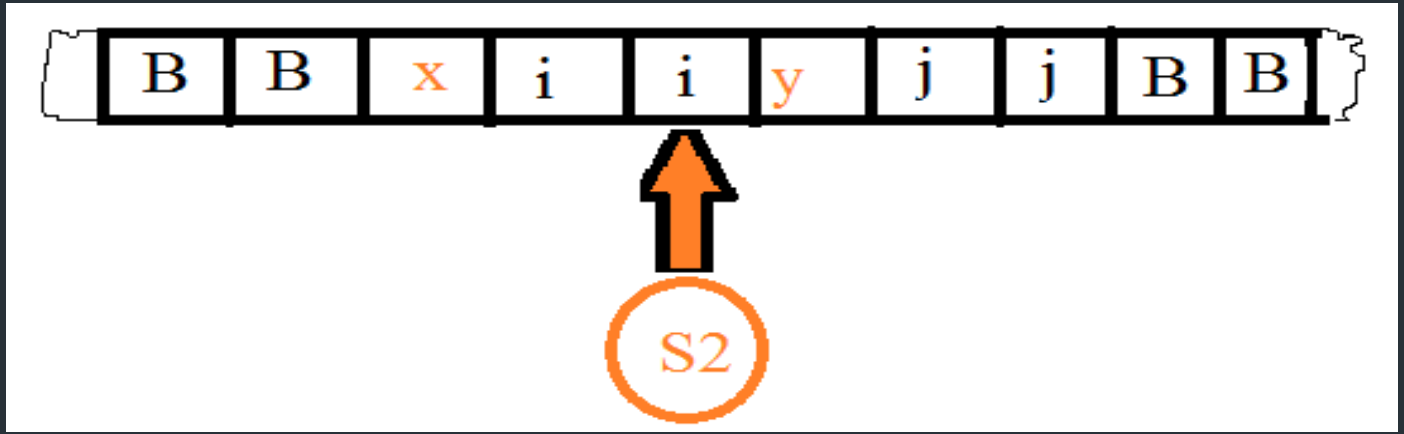
- Move the head to right till to find the leftmost j



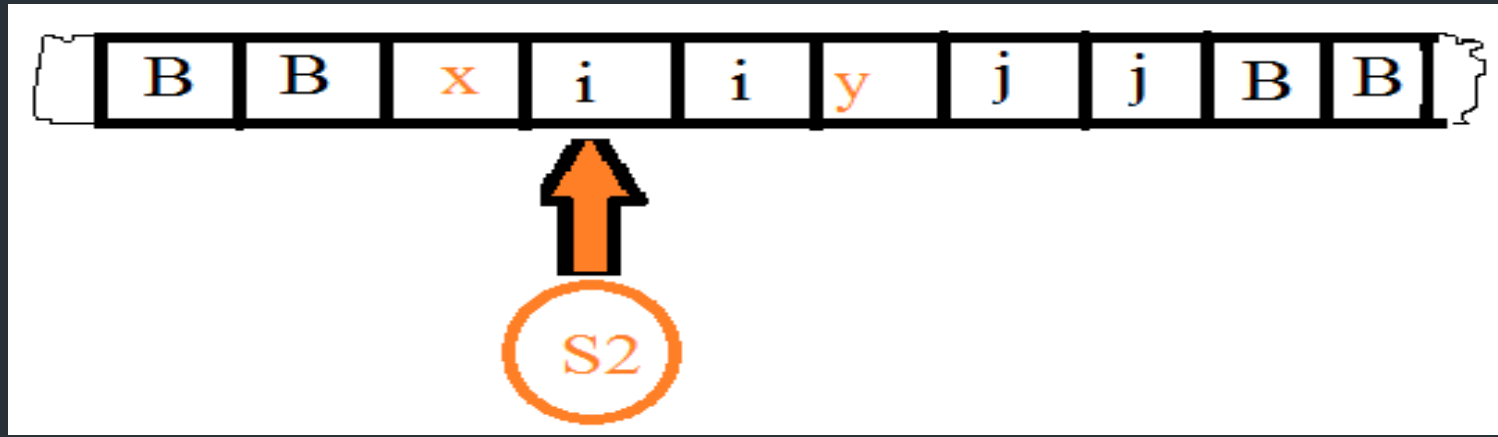
- Read j in the tape

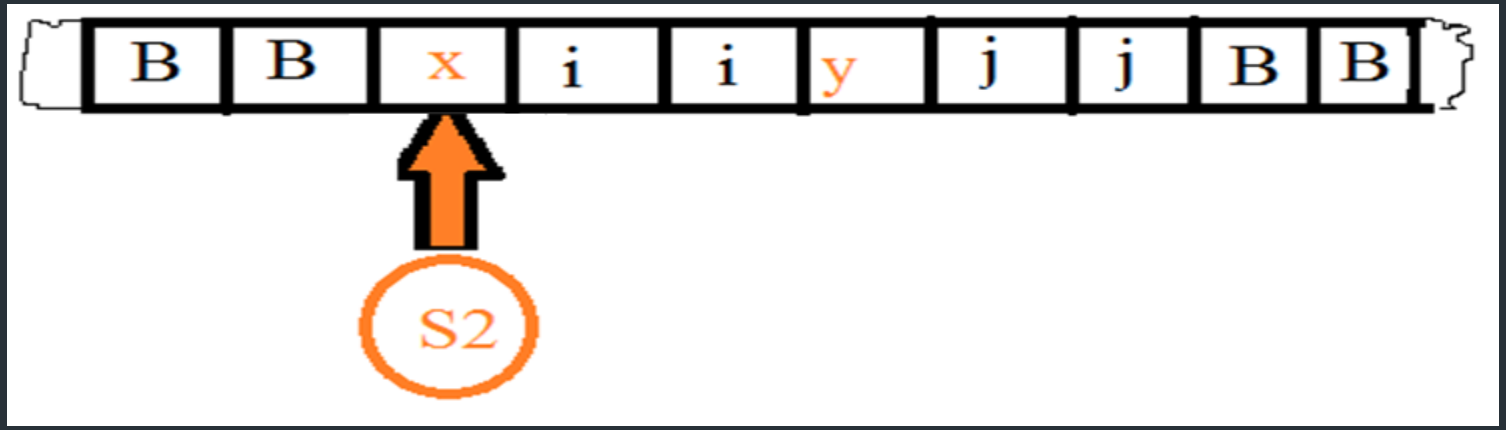


- Change the state as S2
- Mark j as y
- Move left

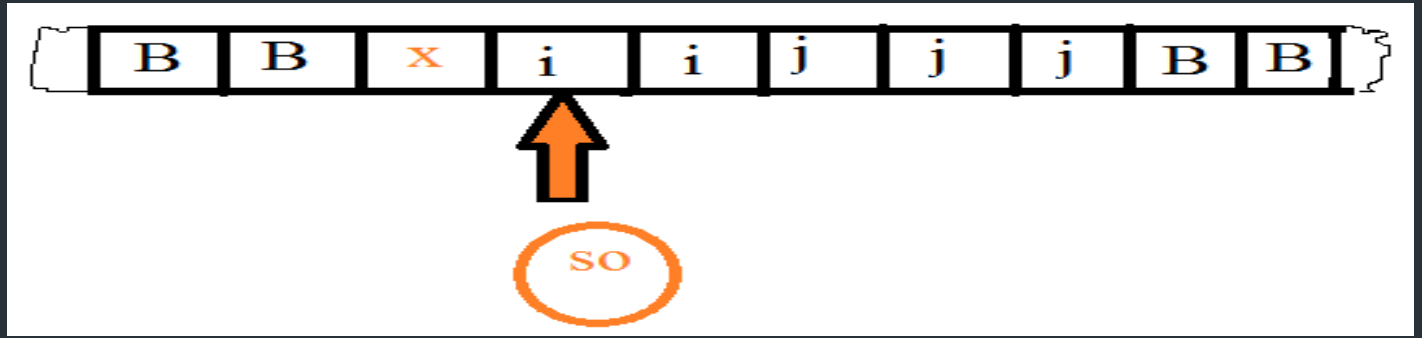


- Move left till reach the rightmost x

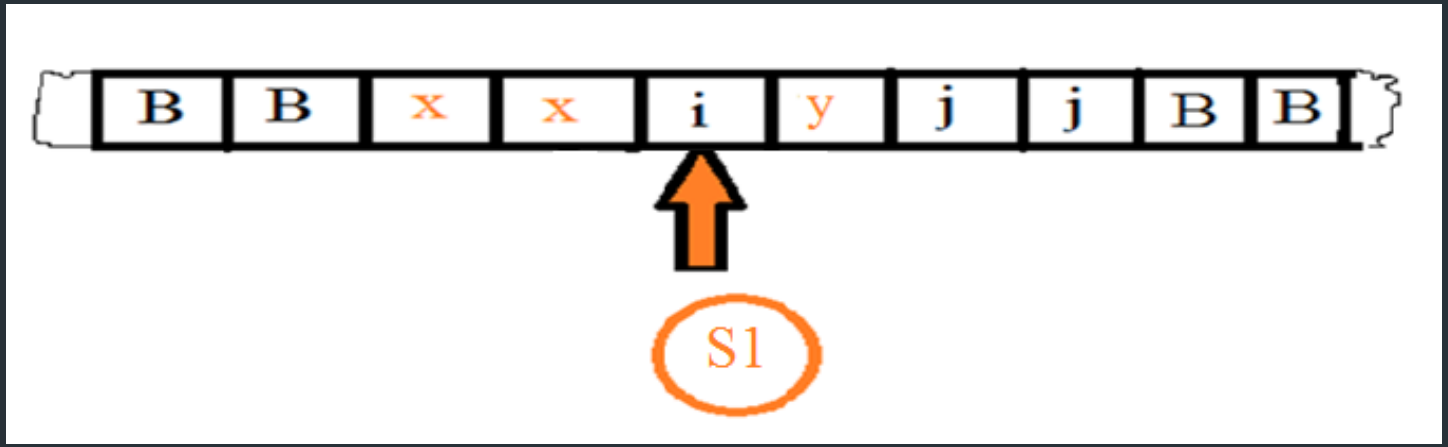



B
B
x
i
i
y
j
j
B
B
S2

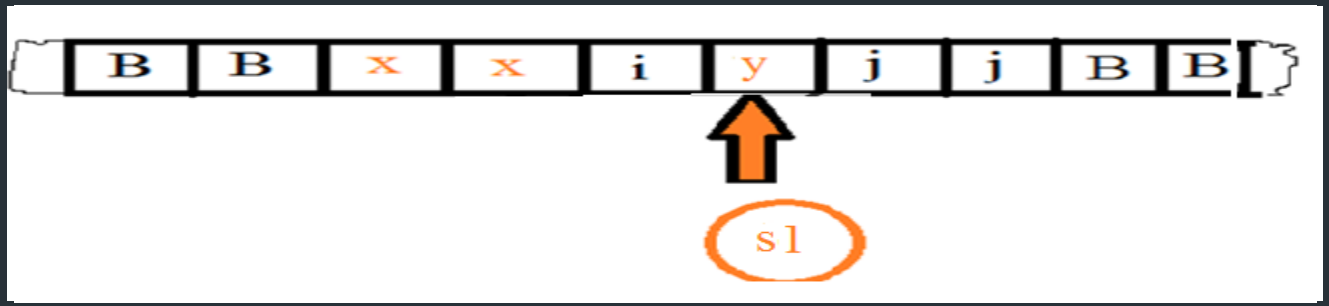
- When the head encounters rightmost x, change the state as q0 and move right

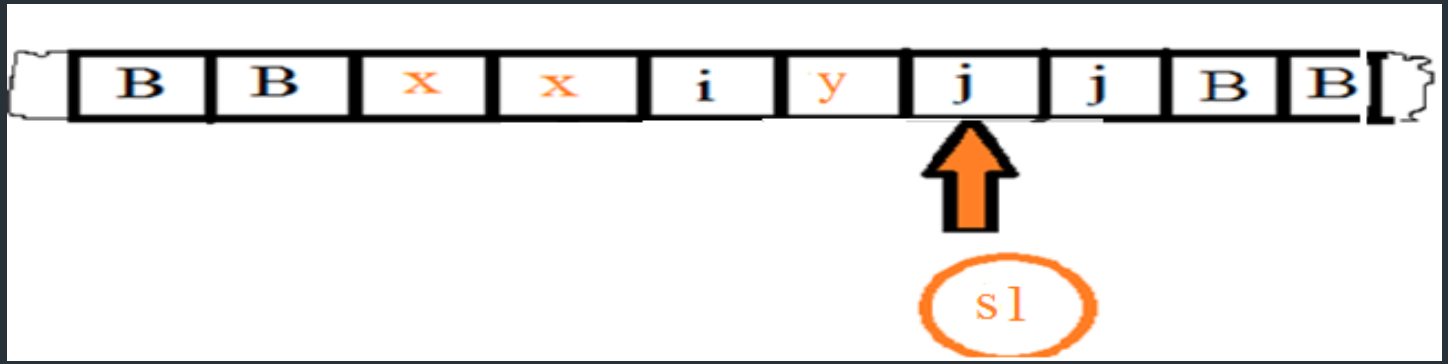


- The process continues….

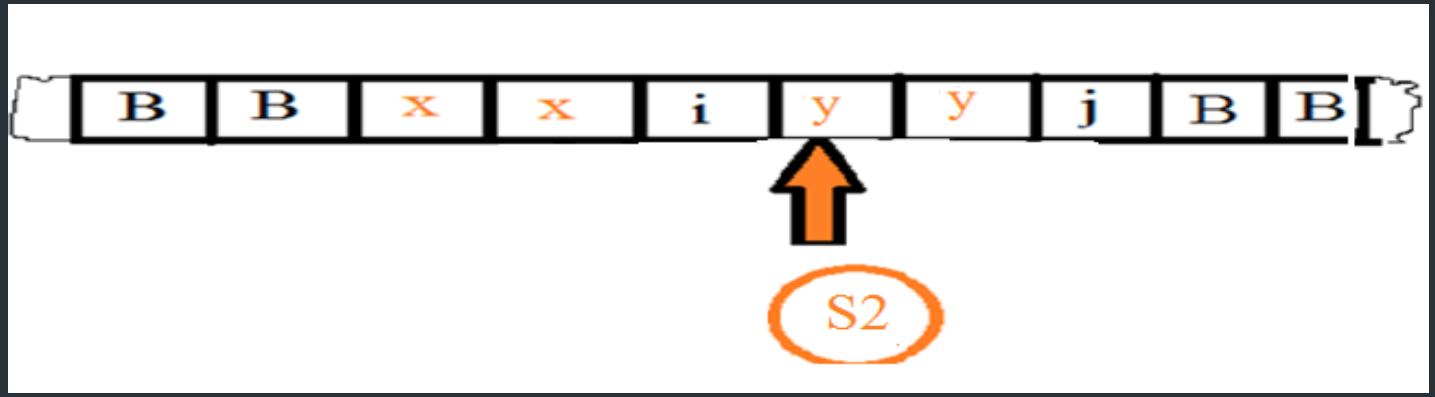


- The process continues….

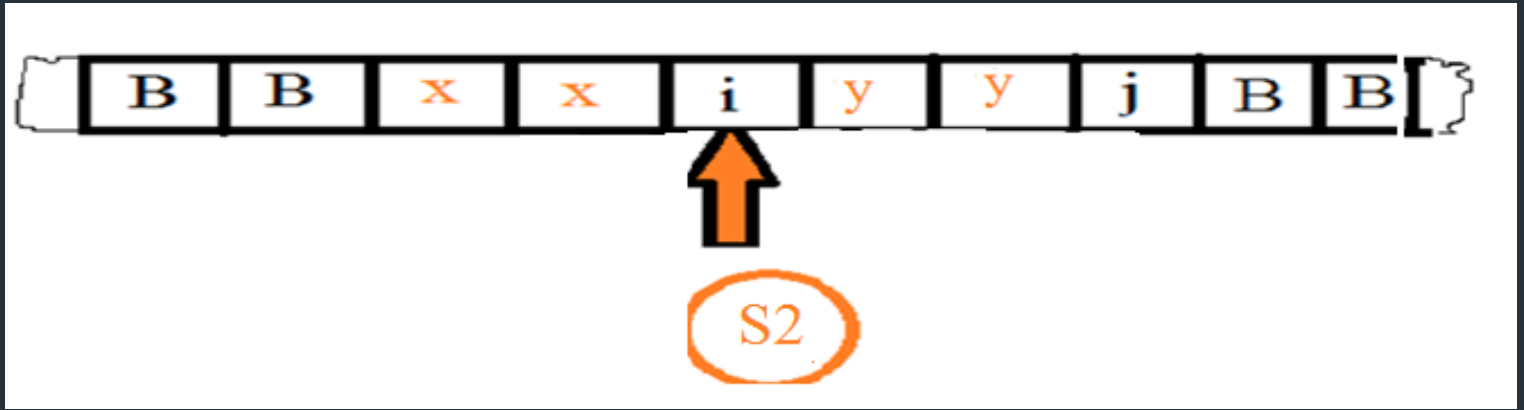


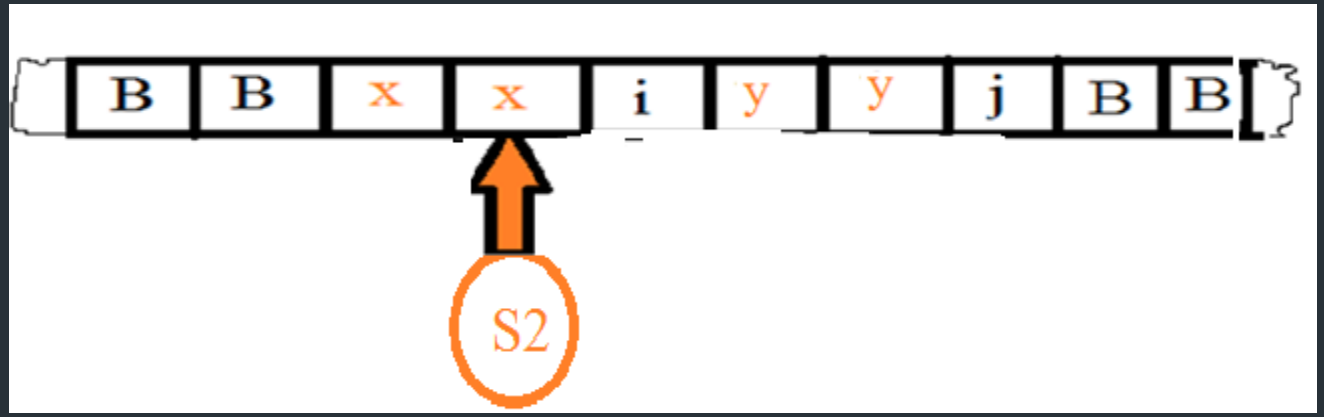

B
B
x
x
i
y
j
j
B
B
s1

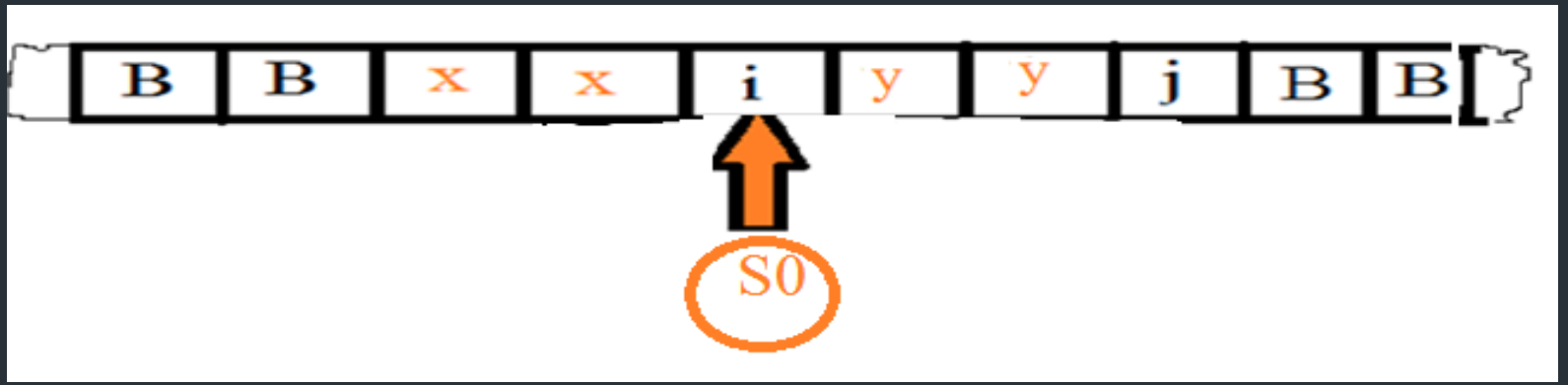
- In state S1, move left to read the next rightmost x

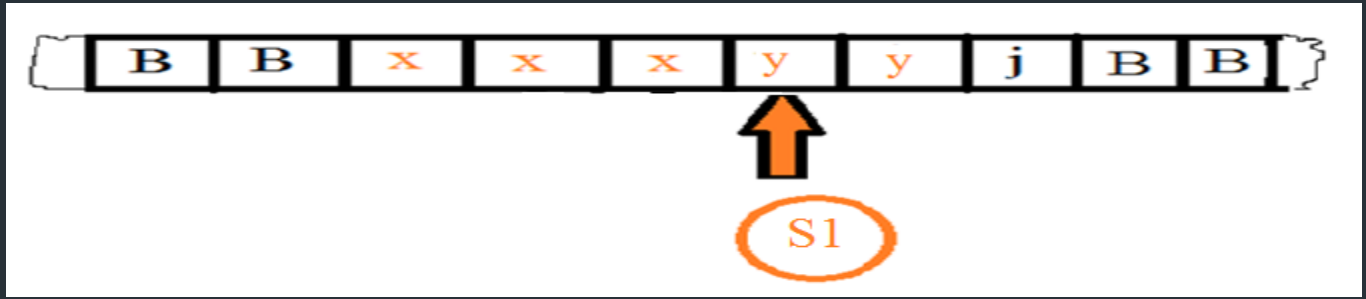


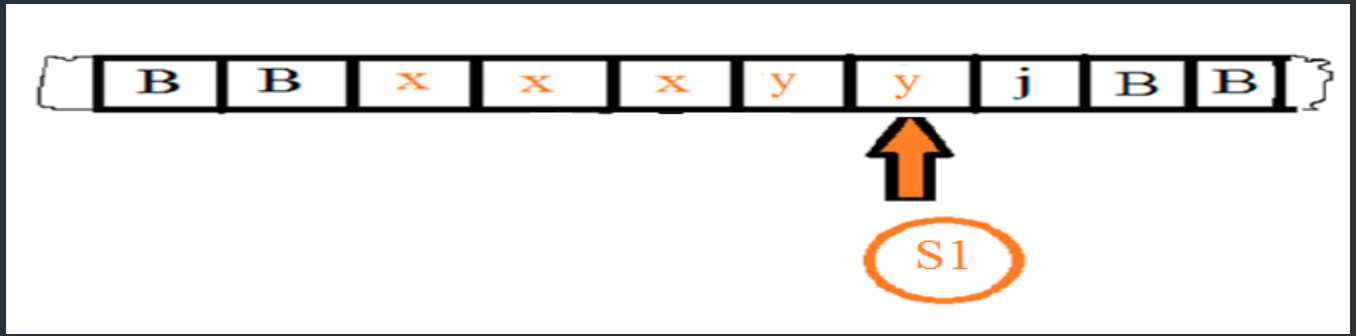
- Move left

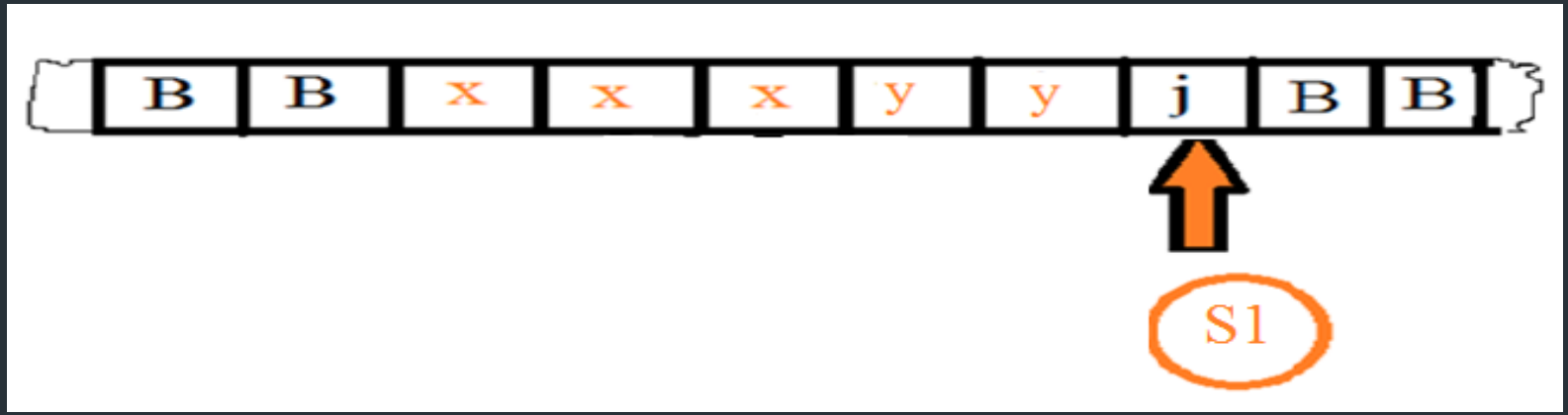


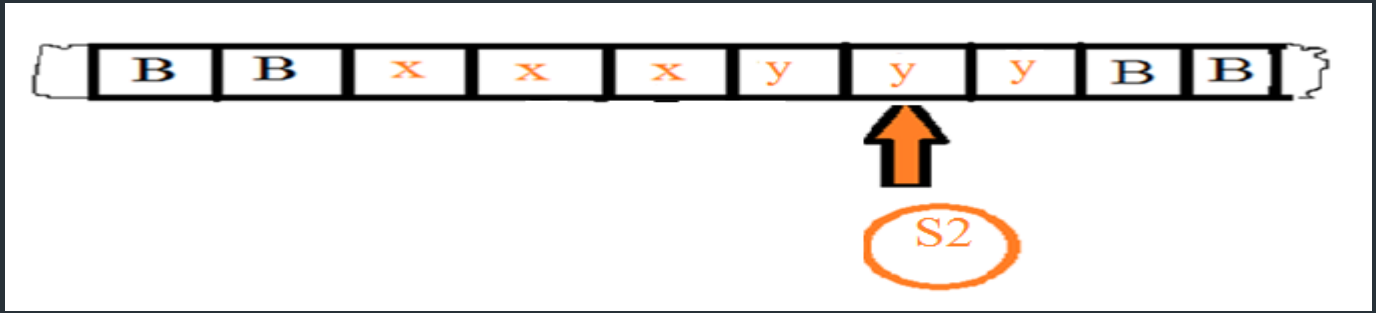

B
B
x
x
i
y
y
j
B
B
S2

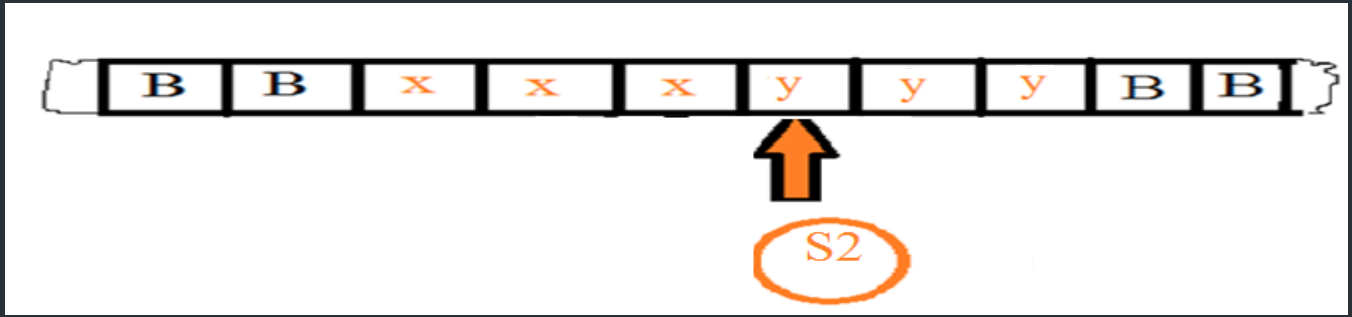

B
B
x
x
i
y
y
j
B
B
S0

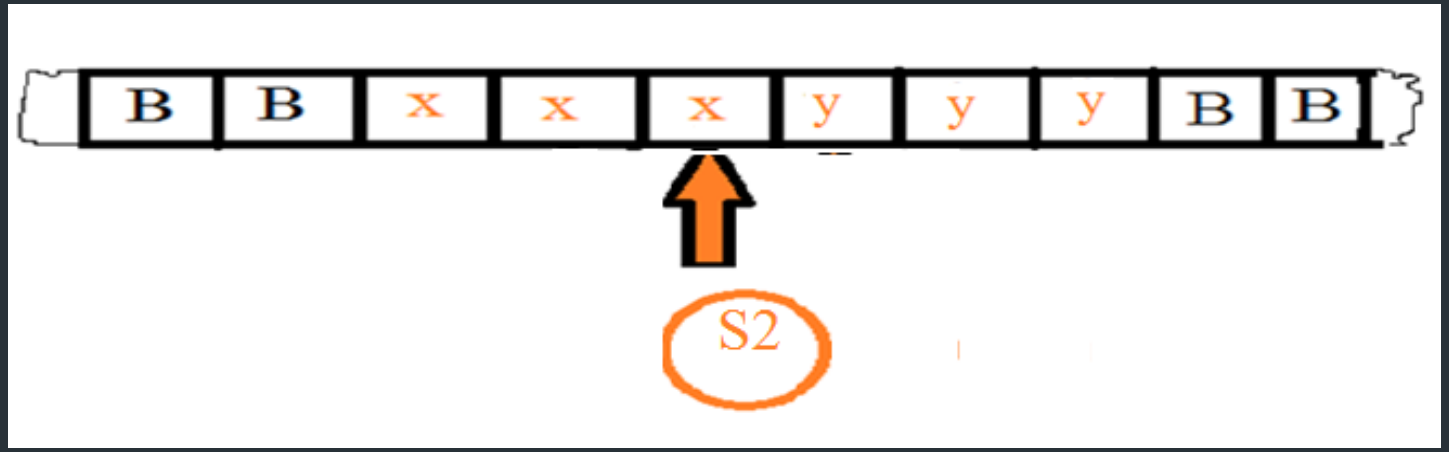

B
B
x
x
x
y
y
j
B
B
S1

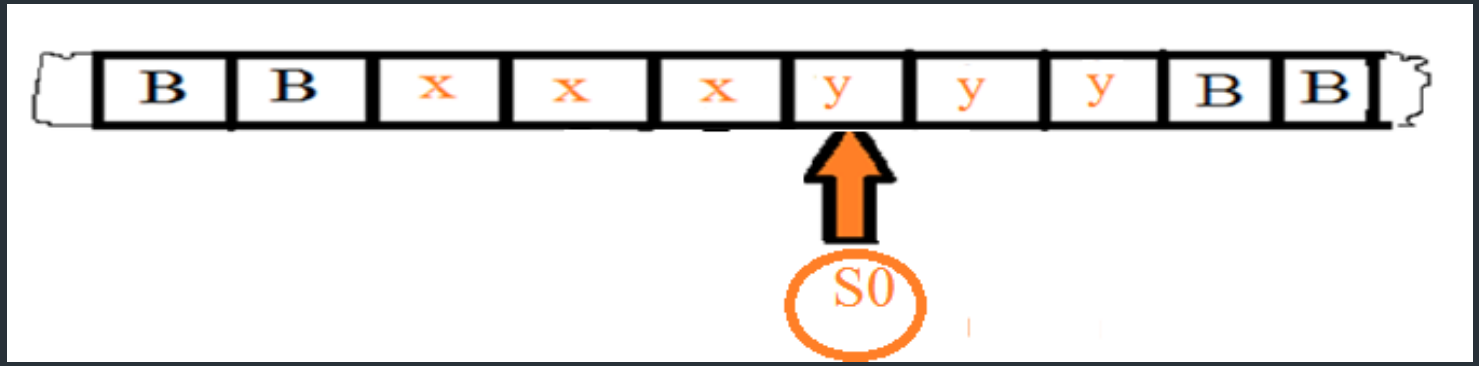

B
B
x
x
x
y
y
j
B
B
S1


B
B
x
x
x
y
y
j
B
B
S1

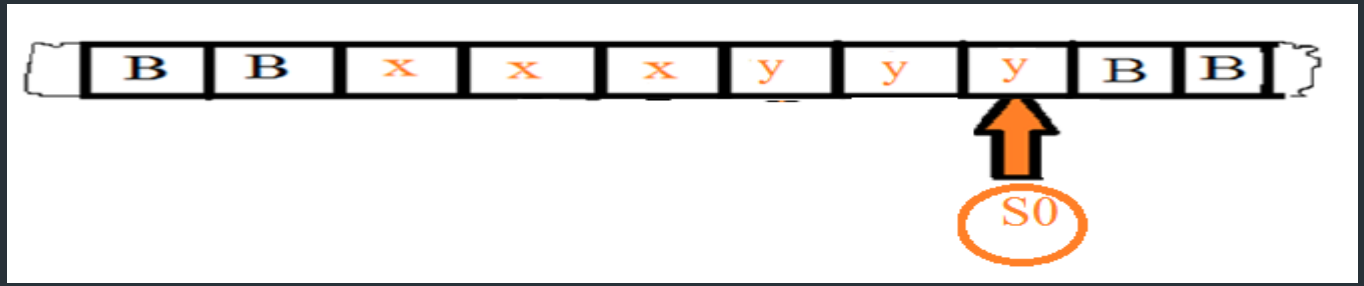

B
B
x
x
x
y
y
y
B
B
S2

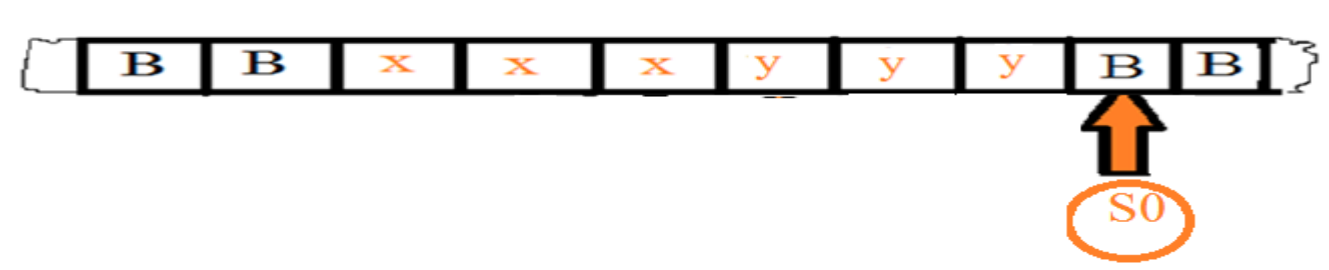

B
B
x
x
x
y
y
y
B
B
S2

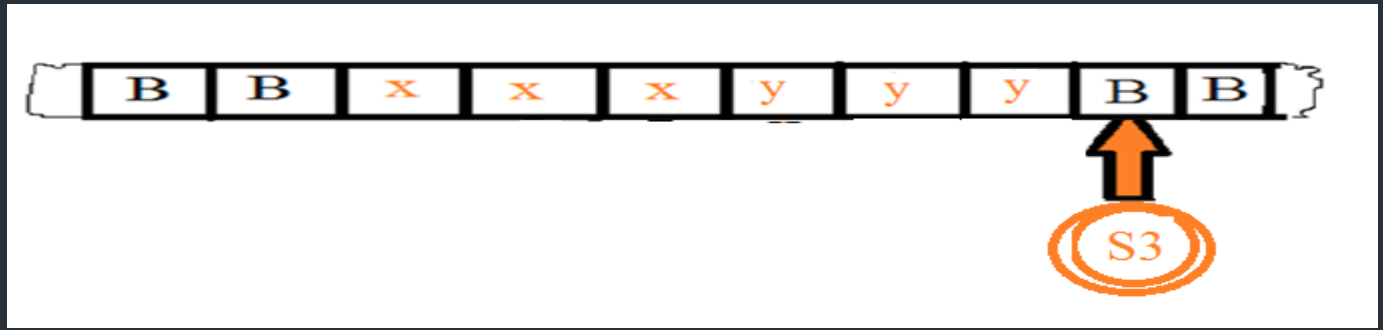

B
B
x
x
x
y
y
y
B
B
S2


B
B
x
x
x
y
y
y
B
B
S0


B
B
x
x
x
y
y
y
B
B
S0


B
B
x
x
x
y
y
y
B
B
S0


B
B
x
x
x
y
y
y
B
B
S0


B
B
x
x
x
y
y
y
B
B
S3

Turing machine for $L = \{i^n j^n \mid n>=1\}$