THE IMPACT OF TECHNOLOGY ON MATHEMATICS

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INTRODUCTION

In this dissertation, the impact of technology on mathematics will be considered and discussed, from the early calculators to the future impacts of artificial intelligence. This topic is being studied due to the rapid increase in reliance on technology in mathematics. Accordingly, this research paper will seek to show how technology has impacted mathematics in different areas. The following paper aims to cover how technology has impacted mathematics in the past and how it will further develop mathematics in the future. There are many theories on the extent of the impact of technology on mathematics, and how much the reliance on technology will increase in the future. Through the points mentioned above, an argument will be made concerning the impacts of technology on mathematics.

THE HISTORY OF THE IMPACTS OF TECHNOLOGY ON MATHEMATICS

There is a long history of the use of aide-mémoire for calculation. In ancient civilizations, calculations were performed with the help of physical objects such as pebbles, marks in sand or dust, or beads on counting boards and abacus frames (Hansson, 2018). Then in 1642, philosopher and mathematician Blaise Pascal invented the ‘Pascaline’. It could only do addition and subtraction, with numbers being entered by manipulating its dials. The ‘Pascaline’ was the first calculator to be produced in quantity and actually used (Freiberger, 2021). The ‘Pascaline’ allowed tax collectors to do calculations that used to take them days, in minutes. However, in other facets of life, these machines remained rarities without much practical usage. Commercial production and widespread use of mechanical calculators only began in the second half of the nineteenth century (Hansson, 2018).

In 1834, mathematician Charles Babbage invented two constructions that were the most advanced computing machines to be conceived in the pre electronic era (Hansson, 2018). These were the difference engine and the analytical engine. And though Babbage synthesised both, both would have been huge mechanical constructions, and neither were completed in his life-time. The difference engine was constructed to calculate a series of values for instance for logarithmic tables. The analytical machine was a general-purpose computational machine. It would be controlled with punched cards, a technology already in use for the control of automatic looms. The punched cards had instructions, based on the subdivision of complex mathematical tasks into a large number of small, simple tasks that had been developed for the organisation of large-scale calculations by human computists (Hansson, 2018). These two machines solved cumbersome mathematical calculations, that in the past would have taken months to fully solve, in days. These constructions both heavily impacted mathematics and made solving and proving formulas far easier. This led to an increase in the rate of progress in mathematics.

Subsequently came the Turing Machine. In 1937 Alan Turing provided a characterization of routine symbol manipulations. Every such operation that a human can perform can be reduced to a set of very simple, truly “mechanical” operations (Nicaise, 2019). These operations were in fact also mechanical in another sense: they can be performed by a machine. A digital computer can do everything that a human can do routinely (Hansson, 2018). The Turing Machine thus gave the first idea that machines could solve and prove mathematical equations that humans could not, as before these machines were only thought to increase efficiency.

Although this idea came in 1937, the first major proof solved using a computer was in 1976 (Rehmeyer, 2013). The proof of the Four Colour Theorem was a combined effort of mathematicians and computers working together (Budd, 2018). The Four Colour Theorem is a theorem that states that any map in the plane can be coloured using four-colours in such a way that regions that share a common boundary do not share the same colour (Carnielli, 2020). In 1976, mathematicians Appel and Haken created a proof based on an analysis of several cases by a computer. However, at the time, some mathematicians did not accept it because it could not be verified by a human, even though the proof seemed valid (Carnielli, 2020). A shorter proof was given then by mathematicians Robertson and Sanders in 1996. In 2008, George Gonthier and Benjamin Werner, proved this Four Colour Theorem using Coq (Carnielli, 2020). This proof was far more widely accepted partly due to the progress in technology and the increasing trust in technology in the early 21st century.

Another example of technology impacting mathematics in history is the Cannonball problem. The Cannonball problem was a problem conjectured by mathematician Kepler in 1611. Kepler’s speculation was that the maximum density of a sphere packing in a three dimensional space is achieved by the familiar or cannonball arrangement in a pile (Carnielli, 2020). In 1998, Thomas Hales solved this problem using a computer and linear programming. Hales solved the puzzle by treating spheres as the vertices of graphs and connecting neighbouring vertices with edges. He reduced the infinite possibilities to a list of the few thousand densest graphs, setting up a proof-by-exhaustion. “We then used a method called linear programming to show that none of the possibilities are a counterexample,” said Hales, now a mathematician at the University of Pittsburgh. The proof consisted of about 300 written pages and an estimated 50,000 lines of computer code (Wolchover, 2013), but the proof used codes in such a way that could not be verified by humans, and again was not widely accepted. The proof was only verified in 2014 (Carnielli, 2020). These examples show the influence of technology in proving sophisticated mathematical facts even before the modern era of computer-assisted proofs. However, the challenges Appel, Hakken and Hales faced also highlights the lack of trust in technology and computers.

THE IMPACTS OF TECHNOLOGY ON MATHEMATICS RESEARCH

Although computer-assisted proofs were not fully trusted and accepted in the 20th century, in the 21st century computer-assisted proofs and proofs with the help of technology have become the industry standard in mathematics research. The most notable case of a 21st century computer-assisted proof is Wiles' proof of Fermat's Last Theorem in 2003. Fermat's Last Theorem states that no three positive integers a, b, and c satisfy the equation $a^n + b^n = c^n$ for any integer value of n greater than 2 (Nicaise, 2019). Wiles found that when the representation of an elliptic curve using p=3 is reducible, it was easier to work with p=5 and use his new lifting theorem to prove that ρ(E, 5) will always be modular, than to try and prove directly that ρ(E,3) itself is modular (Boston, 2003). However, this proof could only be created by way of the modularity conjecture for semistable elliptic curves (Boston, 2003). The modelling of these elliptical curves was only possible due to the technological advancements that came in the late 20th century and in the early 21st century. Without these advancements Wiles would have never been able to prove Fermat's last theorem, further displaying the effect of technology on mathematics. This also further reinforces how technology has impacted mathematics due to the fact that Wiles' Proof has opened new doors for Number Theory which are still being explored and tested today.

An additional exhibition of how mathematics has been progressed by technology is the Coq logic system. Coq was designed to verify computer algorithms, for use in abstract mathematics. In the Coq logic system, the user first suggests a tactic, or logically airtight operation, that the computer should employ to check whether a step in the proof is valid. If the tactic confirms the step, then the user suggests another tactic for assessing the next step (Wolchover, 2013). "Verifying a paper is becoming just as hard as writing a paper," a mathematical researcher said. "For writing, you get some reward, a promotion, perhaps, but to verify someone else's paper, no one gets a reward. So the dream here is that the paper will come to a journal together with a file in this formal language, and referees simply verify the statement of the theorem and verify that it is interesting " (Wolchover, 2013).

Many researchers postulate that this is the only solution to maths' growing complexity problem (Wolchover, 2013). The fact that researchers are explaining this themselves further expresses how, without technology, mathematics research will progress far slower. Without progress in technology, the rate of progress in mathematics will also decline. The complexity of mathematics problems in research has continued to increase, and though mathematicians have better base knowledge now, the complexity has increased exponentially, leading to the progression of mathematics to slow down. However, technology has helped alleviate some of these problems as they have allowed for greater efficiency in calculations. This has helped the progression of mathematical research to not get stalled.

A further illustration of the impact of technology on mathematical research in the modern era is the Stokes-Navier equations in computational fluid dynamics (Barr, 2020). The Stokes-Navier equations are a set of coupled differential equations and could, in theory, be solved for a given flow problem by using methods from calculus. But, in practice, these equations are too difficult to solve analytically. In the past, engineers made further approximations and simplifications to the equation set until they had a group of equations that they could solve. Recently, high speed computers have been used to solve approximations to the equations using a variety of techniques like finite difference, finite volume, finite element, and spectral methods (NASA, 2021). This has helped in the research of mathematics as it has helped create formulas and equations in the very closely related field of Theoretical Physics. Advancements in Theoretical Physics can lead to progress in mathematics, and these advancements would not be possible without the use of high speed computers. This further conveys the influence of technology on mathematical research.

Another case of the effect of technology on mathematical research is Number Theory. Number Theory has been affected by technology in numerous areas. Some examples are low-dimensional topology and Knot Theory. However, one of the biggest examples of the effect of technology on Number Theory at present is computing large Mersene's primes (Barr, 2020). Without supercomputers mathematicians would be unable to compute large Mersene's primes. A Mersenne prime is a prime number that is one less than a power of two. That is, it is a prime number of the form $M = 2^n - 1$ for some integer n. The largest known prime number, 2^(82,589,933) − 1, is a Mersenne prime (Ford, Luca, Shparlinski, 2009). This is also the largest known mersenne prime. These large prime numbers would have been impossible to find without supercomputers.

All of these cases spotlight the impact that computer-assisted proofs have had on mathematical research. Without the strides humanity has made in technology, most of these proofs and advancements in mathematics would have been impossible to comprehend in a lifetime. Technology has allowed for mathematicians to make strides in their research and has allowed for greater efficiency. Improvements in technology have also meant that mathematicians have been able to share their knowledge and research with each other in ways far simpler compared to before, allowing for mathematical research to further prosper.

THE IMPACTS OF TECHNOLOGY ON MATHEMATICS IN FIRMS

Mathematics is used in almost every facet of life, and mathematics research is just one example of an area which has been impacted by technology. Additionally, mathematics has been impacted by technology is the workplace. Firms that used to have to employ specialised mathematicians can now rely more on technology to take care of their complex calculations. Large computations in firms, such as those underlying weather forecasts are performed on computers that do routinely what was practically impossible a generation or two ago (Hansson, 2018).

Genuine forecasts are created regularly not only over the entire globe, but over many local regions for periods up to 5 days or longer on as many as 80 vertical levels and for a host of variables today (Baer, 2000). This development is based on dramatic improvements in the models used for prediction, including the numerical methods applied to integrate the prediction equations and a better understanding of the dynamics and physics of the system. However, the most important factor is the progress in technology and the growth of computing power during this era which has allowed the models to expand by more than five orders of magnitude, significantly reducing errors and increasing the number and range of variables that can be used to create forecasts. At the same time, data processing used by models as initial conditions has also benefited from this explosion in computing resources through the development of highly sophisticated and complex methodology to extract the most information from accessible data. Furthermore, increased communication speeds has allowed for the use of more data to input into the models and for the rapid dissemination of forecast products (Baer, 2000).

The improvements in technology have led to the mathematical aspects being covered by machines. This has allowed the workload for mathematicians to decrease as now they only need to create algorithms and no longer need to compute the algorithms. This has allowed for weather forecasting firms to be able to predict the weather at a very fast rate. Without this improvement in technology, weather forecasting firms would be unable to predict the weather at the rate or accuracy with which they predict the weather now.

Cryptography is another area where mathematics is used in the workplace. Cryptography is the practice and study of techniques for secure communication in the presence of adversarial behaviour (Kim, 2016). Cryptography is used in lots of different ways, like banking transactions cards, computer passwords, and e-commerce transactions (Kim, 2016). The newest type of cryptography is quantum cryptography. The security of quantum cryptography is based on quantum mechanics laws, like how RSA (RSA is a public-key cryptosystem that is widely used for secure data transmission) relies on the difficulty of factorization (Zulkifli, 2007). The quantum mechanics laws that quantum cryptography is based on are very complicated. Without computers, the laws used in quantum cryptography would be too hard to compute to use commercially. But with computers, quantum cryptography is not actually very hard to use in encryption.

The speed of calculations that computers can do allows quantum cryptography to be used commercially in encryption. This means that secure communication can have better encryption and so transmissions are harder to hack. On the other hand, the encryptions would not have to be this complex if computers were not this powerful, as without powerful computers, people would be unable to crack even RSA encryption. This conveys how though technology has impacted cryptography by progressing it, technological advancements are also the reason for the need for the improved cryptography.

THE IMPACTS OF TECHNOLOGY ON MATHEMATICS EDUCATION

Technology has impacted every part of mathematics, and one of the biggest aspects of mathematics technology has influenced is mathematics education. Mathematical education has completely changed due to the impact of technology. In an era of computers and the internet, everyone no longer needs to memorise the so-called number facts and everyone also no longer needs to have fast calculations, as even simple calculators can compute numbers far faster than what is required in mathematics (Volkov, 2018). Being more reliant on technology has meant that mathematical education can focus more on other skills such as creativity and innovativity in solving problems.

Most teachers see this as a major positive. This is reflected in the number of mathematics teachers who support the usage of technology in their classrooms. Teachers use a wide variety of technology and use them in a range of ways. Almost 90% of the teachers used a computer or laptop, 62% made use of scientific calculators, 53% had used a tablet, and a smaller proportion of teachers employed cameras (37%), graphing calculators (27%), and data loggers (10%). Websites were popular tools for 70% of these mathematics teachers primarily for accessing online tutorials and exercises for students to practice their proficiency with mathematics problems (Attard, Holmes, 2019). Desmos and GeoGebra were popular tools, primarily with secondary teachers, for producing mathematics visualisations and for exploring mathematics in a dynamic way. Spreadsheets were also very popular tools used by almost 60% of mathematics teachers (Attard, Holmes, 2019).

FIGURE 1

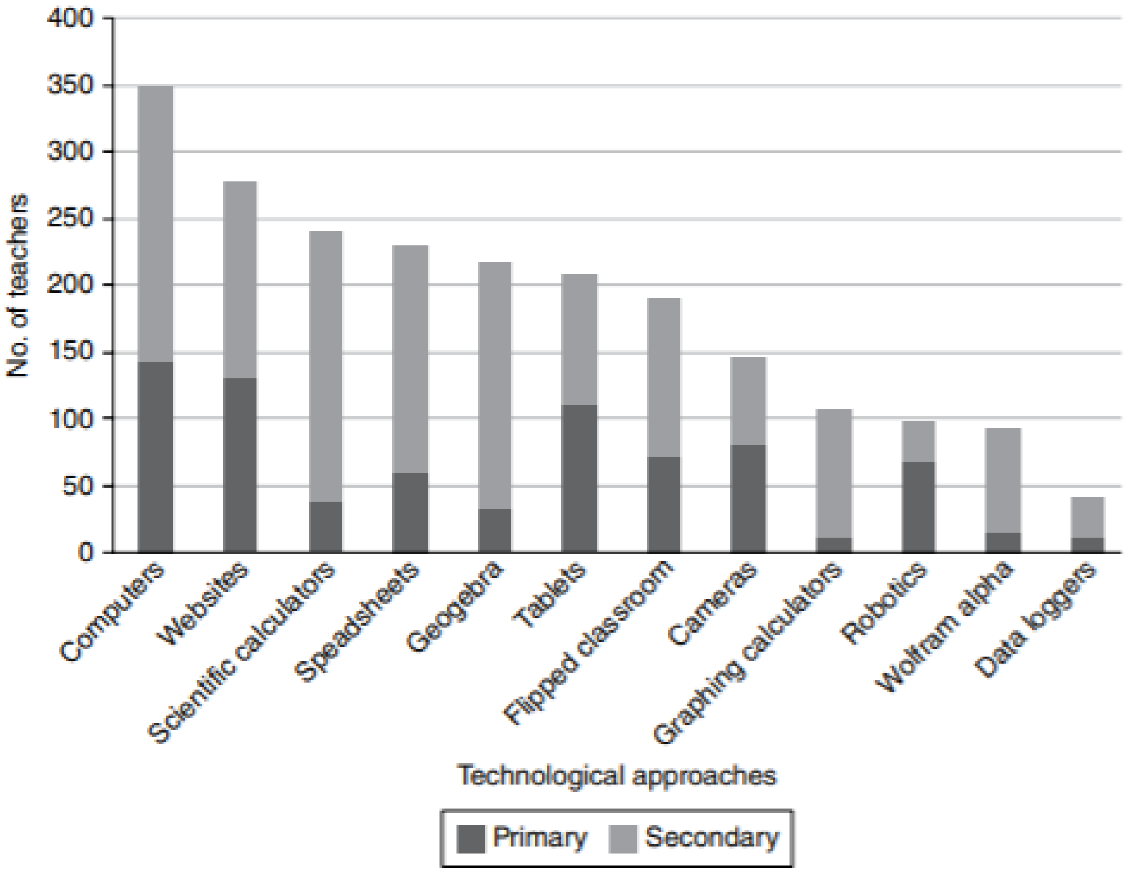

Figure 1 (Attard, Holmes, 2019).

The proportion of primary and secondary teachers using various technological approaches is displayed in Figure 1 (Attard, Holmes, 2019).

A survey conducted for this dissertation showed that different types of mathematics students had different usages of different technologies. PhD students used technology the most often and also used the widest range of technology in mathematics. PhD students and Masters Students were also the only students that used artificial intelligence at any level in their mathematics. The results also showed a trend of an increase in reliance on technology as the level of mathematics education increased. The results also showed that some examples of technologies used by students included Calculators, Photomath, MathsGenie, Desmos, Graphing Calculators, and a Python Program created by a participant. Every participant in the survey also said that they used technology consistently in mathematics.

This prevalence of technology in mathematical education that is exhibited may however be influenced by the area where the survey was conducted. Both the survey conducted for this dissertation and the research in the journal were conducted in developed countries. In developing countries these results might be different as the resources in schools in developing countries might not be as effective as they are in developed countries.

But in developed countries at least, technology is heavily relied on in mathematical education, especially at higher levels of education. One example of the use of technology in mathematical education is interpolation. This application of interpolation involves integrating the secant function. A practical application for this calculation resulted in the need to construct navigational charts. Deriving the integral of the secant function is often a difficult task for calculus students. It involves making a trigonometric substitution that seems contrived and logically convenient. However, understanding how this integral was approximated before there was even an understanding of calculus can be illuminating to students and give them a full appreciation for the creativity that can arise through approximations (Tassell, 2018). Without technology, the calculations needed for the approximations are very hard for students. Technology allows these computations to be done and understood by students, helping them further appreciate the creativity that arises from approximations.

THE IMPACTS OF ARTIFICIAL INTELLIGENCE ON MATHEMATICS

The previous sections in this dissertation have evaluated the impacts of technology on different parts of mathematics, but this section will instead focus on how one specific aspect of technology, artificial intelligence, has impacted mathematics. The area of research in artificial intelligence can be defined as the design of machines the behaviour of which in a given situation would be labelled intelligent if observed in human activity (Persson, 1964). Machine learning is a component of artificial intelligence that solves specific tasks by learning from data and making predictions. Machine learning is how a computer system develops its intelligence.

Artificial intelligence and machine learning have impacted mathematics in numerous ways. One of these ways is how artificial intelligence and machine learning have changed how big data is handled and manipulated by organisations like BIS: an organisation owned by 63 central banks, whose mission is to support central banks' pursuit of monetary and financial stability through international cooperation, and to act as a bank for central banks. One object in applying machine learning methods by BIS is to develop tools, heuristics, and techniques that can supplement domain expertise in modelling tasks: such as stock prediction (Jude, 2019). Before, mathematicians had to spend hours working out algorithms and had to constantly update their algorithms so that the algorithms did not become obsolete. But now with the use of machine learning methods, the algorithms created by mathematicians can update by themselves. Therefore, mathematicians can focus more on the creation of the original algorithm. This shows another positive impact of technology on mathematics.

A subsection of machine learning is deep learning. Deep learning is a type of machine learning and artificial intelligence that imitates the way humans gain certain types of knowledge. Deep learning is based on artificial neural networks with representation learning. Deep learning has impacted mathematics through its applications in solving cumbersome differential equations. This application of deep learning uses neural networks to approximate answers of differential equations.

FIGURE 2

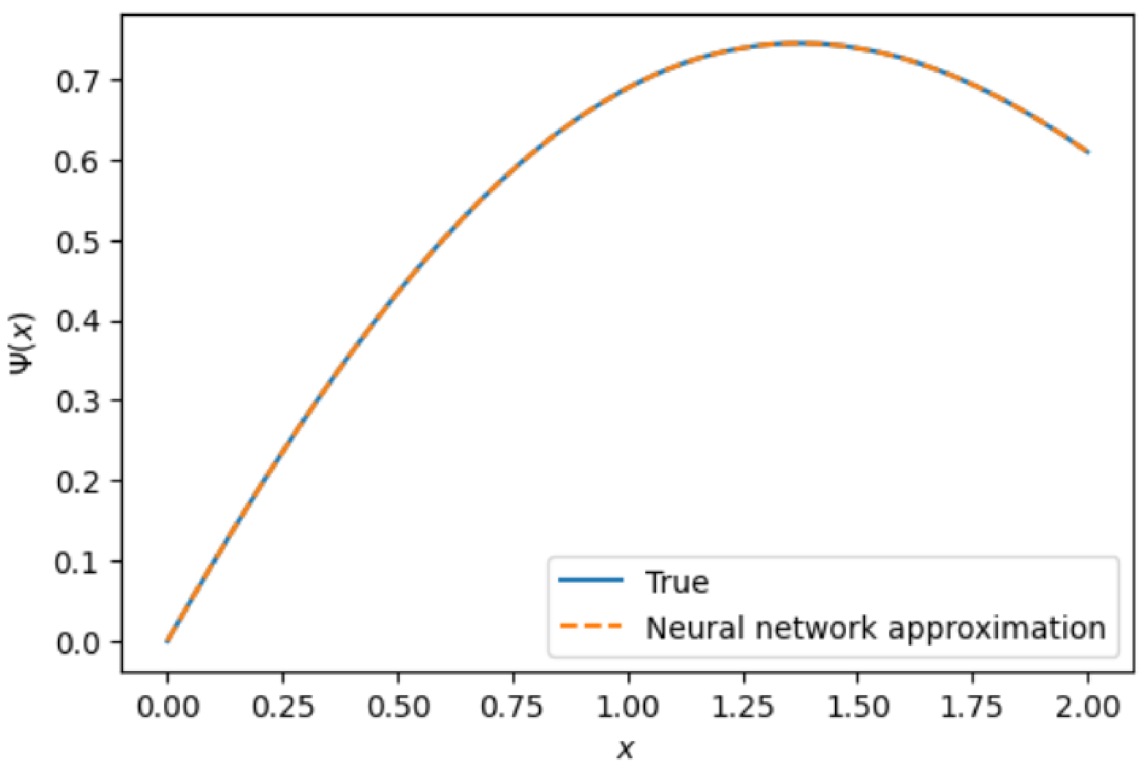

Figure 2 (Singh, 2021)

This graph displays a comparison of the answers generated by the neural network approximation compared to the true answers to the differential equations. The answers generated by the neural network approximations took less than a minute to be computed after the algorithm was created, while solving the differential equation in the traditional way would have taken far longer. This shows how artificial intelligence can impact mathematics through its ways of improving the efficiency of solving problems.

Artificial intelligence can also impact the future of mathematical research very significantly. Artificial intelligence has improved very quickly, and it seems as if it will continue to do so for some time, as the first experiments which would help us to answer this fundamental question:

“Could artificial intelligence be real?” could not even be performed until the twentieth century, when the first computers were constructed (Flasiński, 2016). The first artificially intelligent computer program was written in 1950 (Alexander, 2000). Since then artificial intelligence has improved to the point where it can generate music and art, though the algorithms for such music or art creation systems are still designed by humans (Kankanhalli, 2020). In the future, artificial intelligence could even grow to the point of replacing mathematical researchers, which would be an extremely significant impact of technology on mathematics. However, this is very unlikely as artificial intelligence would have to develop to a level comparable to a human before it would be able to replace researchers. This is as researchers are more interested in understanding how things work, rather than simply getting results. Artificial intelligence algorithms using machine learning typically lack transparency and explainability, thus hindering research progress (Kankanhalli, 2020). Also, the part that artificial intelligence would struggle the most to replace mathematical researchers in is problem formulation. In some disciplines like organic chemistry, there are programs to help organic chemists in identifying unknown organic molecules, this is not the case in mathematics where often the problem domains are not well-defined (Kankanhalli, 2020). It has been suggested that artificial intelligence can use data mining tools to generate hypotheses. But, even if artificial intelligence generates hypotheses from big data sets, there will be more than one relationship or proof for the artificial intelligence to solve, so there would still be a need for a mathematician to make sense of these suggestions and then decide which option has the most merit (Kankanhalli, 2020). So, artificial intelligence will have to develop to extremes to fully replace mathematical researchers. Nevertheless, artificial intelligence could still help speed up the process of finding possible new proofs and relationships among variables that mathematicians would not have thought of on their own (Kankanhalli, 2020).

These examples show how artificial intelligence has had significant impacts on mathematics. Artificial intelligence can increase the efficiency of mathematical operations and in the future can further progress and influence mathematics in unimaginable ways.

THE FUTURE IMPACTS OF TECHNOLOGY ON MATHEMATICS

Mathematics has been crucially affected by technology in the past and the present, but the future of mathematics will be impacted by technology to an even greater extent.

One aspect of mathematics that most mathematicians agree needs to change in the future is the level of complexity mathematics is reaching. Currently mathematics is becoming too complex and this has led to specialists in one area not understanding the results obtained in another area. This means that new theories have to appear. These theories must unify and simplify the current overly complex branches of mathematics. Generally, periods of complex science are followed by periods of simplification and vice-versa. Before Newton and Leibniz invented calculus, the mathematics of calculating areas and volumes was very complex. Calculus simplified it. Similarly, Copernicus' Theory replaced the complex Ptolemy's Theory. Currently, mathematics is overly complex like Ptolemy's system. It has to be replaced by much simpler mathematics (Karasik, 2021). This is the biggest problem in mathematics, but it is also a problem that has nothing to do with actual mathematics.

Technology, however, could be the variable that leads to mathematics evolving in the future. One way that technology could lead to solving this problem is through a project spearheaded by Voevodsky (Rehmeyer, 2013). The project aims to unite the fields of Homotopy Theory, mathematical logic, and the theory of programming languages- and in the process, it also aims to be able to create computer verifiable proofs usable for the working mathematician. Coq is the name of the program that the project is based on, and the dissertation considered the program earlier on for its usage in current mathematical research. In the future, Voevodsky's project could be used to unify several parts of mathematics. Furthermore, as the technology improves and the tactics become smarter, similar programs may someday perform higher-order reasoning on par with or beyond that of humans (Rehmeyer, 2013).

Another way technology can lead to mathematics changing in the future is through Type Theory. Type Theory is a formal logic system that was developed by computer scientists (Wolcher, 2013). Type Theory could be used to recreate the entire mathematical universe from scratch (Wolcher, 2013). A mathematical universe would in theory unify every area of mathematics. Type Theory is consistent with the mathematical axioms, but is entrenched in the language of computers (Wolcher, 2013). This could even be an alternative way of formalising mathematics, which has been renamed the univalent foundations of mathematics (Wolcher, 2013). This could further streamline the process of formal theorem proving (Wolcher, 2013). A formal theory proof is a proof where all of the intermediate logical steps of a proof are supplied (Hales, 2021). Formal theorem proving is still relatively rare in mathematics, but that could change as programs like Voevodsky's adaptation of Coq improve (Wolcher, 2013). In a formal proof no appeal is made to intuition, even if the translation from intuition to logic is routine. Thus, a formal proof is less intuitive and yet less susceptible to logical errors than a traditional proof (Hales, 2021). So making formal proofs easier to create will help in the progress of mathematics.

Some mathematicians, for instance Hale, envision a future in which computers are so adept at higher-order reasoning that they will be able to prove huge chunks of a theorem at a time with little, or no, human guidance (Wolcher, 2013). Other mathematicians, for instance Ellenberg, see a more significant role for humans in the future of his field (Wolcher, 2013). They claim that mathematicians are very good at figuring out things that computers can not. If there were a future in which all the known theorems could be proven on a computer, they claim that mathematicians would just figure out other things that a computer can not solve, and that would become mathematics. Though both sides of the argument are opposing on the level of impact, both sides agree that technology and computers will have significant impacts on mathematics.

No one knows what the future of mathematics holds, but most mathematicians agree that the impacts of technology on mathematics will not diminish, and are expected to escalate.

CONCLUSION

In conclusion, the impacts of technology on mathematics are significant and widespread throughout the past, present and future and throughout several areas of mathematics.

The impact of technology on mathematics in research is very significant, as without technology, mathematics research would not be able to develop at the rate it is developing at. Technology has allowed researchers to not have to worry about computing numbers. This has improved their efficiency, leading to more progress in mathematics research.

The impact of technology on mathematics in firms is also very significant as without technology firms would have to hire several numerical analysts whose only job would be to compute numbers and calculate arithmetic operations. Technology has meant that firms can compute these numbers automatically, and it also improves their accuracy and efficiency as generally computers will be more accurate and efficient than mathematicians in calculations.

The impact of technology on mathematics in the future is also very significant. Even though what happens in the future is all speculation, the impacts of technology in the future have a chance of growing exponentially.
Technology is advancing very rapidly, and so its applications in mathematics will also advance. However, out of all the impacts of technology on mathematics, the most significant is the impacts on mathematics education. Technology has led to mathematics being taught completely differently. Mathematics education used to focus on computing numbers and a good mathematics student used to be someone who could calculate numbers very quickly. However, a computer can be programmed to do mathematical calculations faster than any human. So now the skill of calculating numbers quickly is no longer the deciding factor in a student's mathematical excellence. Instead, students can now be evaluated on how well they can apply their mathematical knowledge and do deep mathematics. This focus on deep mathematics means that students can develop their problem solving and creative thinking skills, leading to them becoming better mathematicians. This is the most important impact of technology on mathematics as better mathematics education will lead to better mathematicians, which will in turn lead to further advancements in mathematics.

On the other hand, though technology has impacted mathematics, mathematics is one of the underlying foundations for the development of advanced technologies. Future research could be done to explore the inverse relationship between technology and mathematics, on how technology has been affected by mathematics.

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