Find the dercivative of the follwing formula

a) $f(z) = \log_e (1+z)$

where, $z = x^T x$, $x \in R^d$

Now Applying chain raule,

$$\frac{df}{dx} = \frac{df}{dz} \cdot \frac{dz}{dx}$$

now,

$$\frac{df}{dz} = \frac{1}{1+z}$$

$$\frac{dz}{dx} = 2x$$

$$\therefore \frac{df}{dx} = \frac{1}{1+x^T x} \cdot 2x$$

$$= \frac{2x}{1+x^T x}$$

01/20/2024 16:55

b) $f(z) = e^{-z/2}$

given

$$z = g(y) = y^T s^{-1} y$$

$$y = h(x) = x - \mu$$

Now Applying chain rule,

$$\frac{df}{dx} = \frac{df}{dz} \cdot \frac{dz}{dy} \cdot \frac{dy}{dx}$$

As $\frac{d}{dx} e^{mx} = m e^{mx}$

Hence,

$$\frac{dz}{dy} = 2 s^{-1} y$$

$$\frac{dy}{dx} = I \text{ [Identity matrix}$$

$$\therefore \frac{df}{dz} = -\frac{1}{2} e^{\frac{-z}{2}} \cdot 2 s^{-1} y$$

$$= \frac{1}{2} e^{-\frac{\{(x-\mu)^T s^{-1} (x-\mu)\}}{2}} \cdot 2 s^{-1} (x-\mu)$$

01/20/2024 16:56