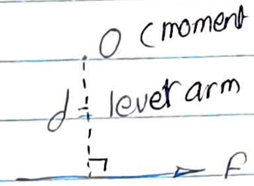
Moment is a tendency of the force to make a rigid body to rotate. ①

* Moment :-

Pana used for opening Nuts.
ex: opening/closing the door, jack used for lifting car,

It is the turning effect produced by a force. Moment of a force about any point is the product of magnitude of the force & perpendicular distance about that point.

The point about which moment is taken is called moment centre.



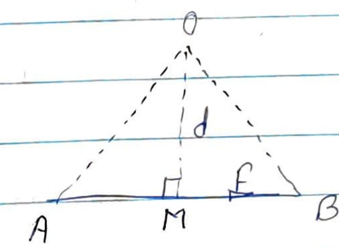
Taking moment about O, $M_o = F \times d$

∴ Moment about O $M_o = F \times d$

SI unit is Nm or kN.m

Sign Convention, Clockwise moment ↻ +ve
Anticlockwise moment ↺ +ve

Graphical Representation of Moment:-



A force F is represented by line AB
Let O be the moment centre.
d be the perpendicular distance (OM)

∴ By definition moment of force about O is,

$$M_o = F \times d$$

Now, join OA & OB

∴ Area of triangle OAB $= \frac{1}{2} \times AB \times OM$

$= \frac{1}{2} \times F \times d$

∴ 2 Area of $\triangle OAB = F \times d$

From eqⁿ ① & ②

$$M_o = 2 \text{ Area of } \triangle OAB$$

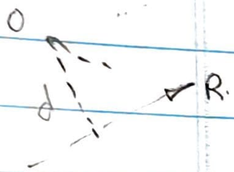
It is concluded that moment of a force about any point is numerically equal to area of triangle in which base of triangle must represent force & altitude (height) of triangle must represent perpendicular distance.

* Varignon's Theorem (Law of Moments):-

The moment of resultant of a force system about any point is equal to the algebric sum of moments of all other forces about the same point.

Mathematically, $R x = \Sigma M$

where x = perpendicular distance of Resultant R about a point where moment is taken.



$\therefore$ ⊥er distance $x = \frac{\Sigma M}{R}$

$\Sigma M = R_x \times \bar{x} + R_y \times \bar{y}$

By using varignon's theorem we can also find the horizontal & vertical distances of resultant.

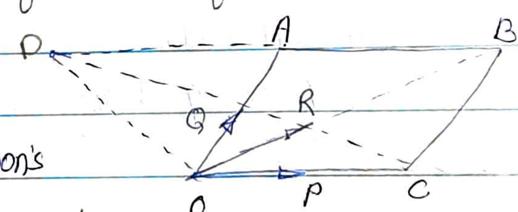
$\therefore \bar{x} = \frac{\Sigma M}{\Sigma f_y}$

Horizontal distance $a = \frac{\Sigma M}{R_y} = \frac{\Sigma M}{\Sigma f_y}$

$\bar{y} = \frac{\Sigma M}{\Sigma f_x}$

& Vertical distance $b = \frac{\Sigma M}{R_x} = \frac{\Sigma M}{\Sigma f_x}$

Use:- Varignon's theorem is useful to find out position or (Moment centre) location of resultant of non-concurrent force-system.



Proof:- We shall prove varignon's theorem by considering two concurrent forces P & Q having resultant R.

Let P & Q be the concurrent forces at O.

Let R be the resultant of P & Q

Now, select a convenient moment centre 'D'.

As per concept of Graphical representation of moment.

The moment of P about D is

$M_P = 2 \triangle ODC$ —— ①

The moment of Q about D is, $M_Q = 2 \triangle ODA$ —— ②

The moment of R about D is $M_R = 2 \triangle ODB$

But $\Delta ODB = 2\Delta ODA + 2\Delta OAB$

~~But from~~

$$M_R = 2\Delta ODA + 2\Delta OAB \quad \text{——— (3)}$$

$$\therefore \cancel{M_R = M_Q + M_P}$$

But from geometry areas of following three Δ's are same.

$$\Delta OAB = \Delta OBC = \Delta ODC$$

Now, substituting $\Delta OAB = \Delta ODC$ in eq^n (3)

$$M_R = 2\Delta ODA + 2\Delta ODC$$

$$M_R = M_Q + M_P$$

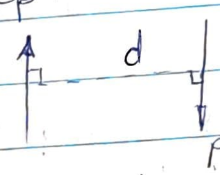
$\therefore$ Moment of resultant at D is equal to total moments of P & Q about D.

Moment of Couple is Torque.

eg: Water tap, steirring of car, Nut-bolt, Screw driver, pedaling cycle.

* Couple

Two unlike parallel, non-collinear forces having same magnitude form a couple.



The distance between two forces is known as arm or lever of the couple.

Properties of a couple:-

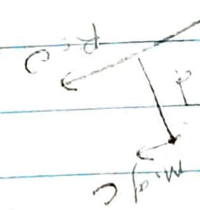
1. Two unlike parallel, non-collinear forces of same magnitude form a couple.
2. The resultant of a couple is always zero.
3. The moment of a couple is the product of one of the forces and lever-arm of the couple.

$$\therefore M = Pd$$

4. A couple can not be balanced by a single force.
5. It can be balanced only by another couple of opposite nature.
6. The moment of couple is independent of the moment centre.

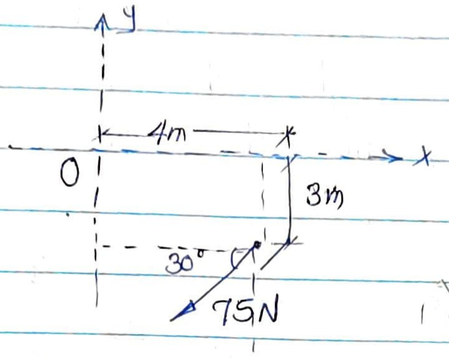
Important Notes:

1. For moment of a force any reference point (moment centre) is required but for couple such a point is not required as its moment is independent of the moment centre.
2. The effect of a couple is unchanged if
   1. The couple is shifted to any other position in its plane.
   2. The couple is shifted to a parallel plane.
   3. The couple is rotated through any ~~plane~~ angle in its plane.



* Moment Examples

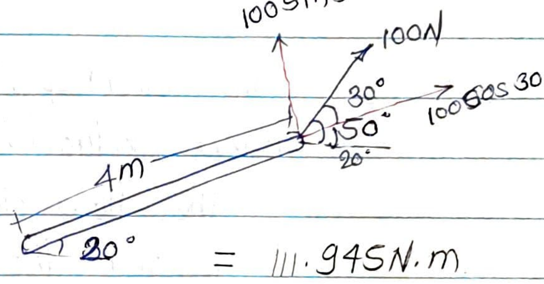
① Determine the moment of the 75N force shown in fig about O



$$M_O = -75\cos 30 \times 3 - 75\sin 30 \times 4$$

$$= -344.85\ N.m$$

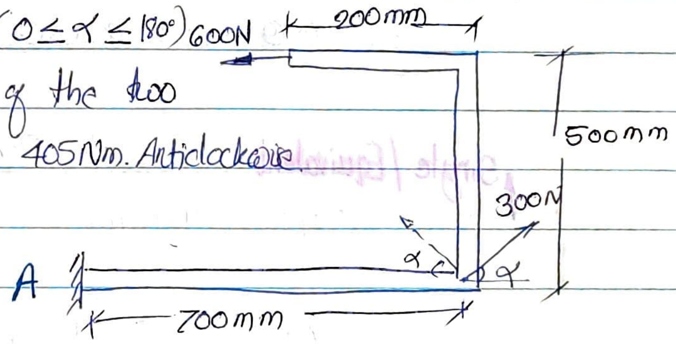
②



$$M_O = 100\sin 30 \times 4 \qquad = 111.945 N.m$$

$$M_o = -40\cos 20 \times 2 + 40\sin 20 \times 3 + 30\cos 40 \times 3 + 30\sin 40 \times 4$$

③ Determine the two values of α $(0 \le \alpha \le 180°)$ for which the combined moment of the two forces shown in fig. about A is 405Nm. Anticlockwise.



$$M_A = 405 N.m.$$

$$600 \times 0.5 + 300\sin\alpha \times 0.7 = 405$$

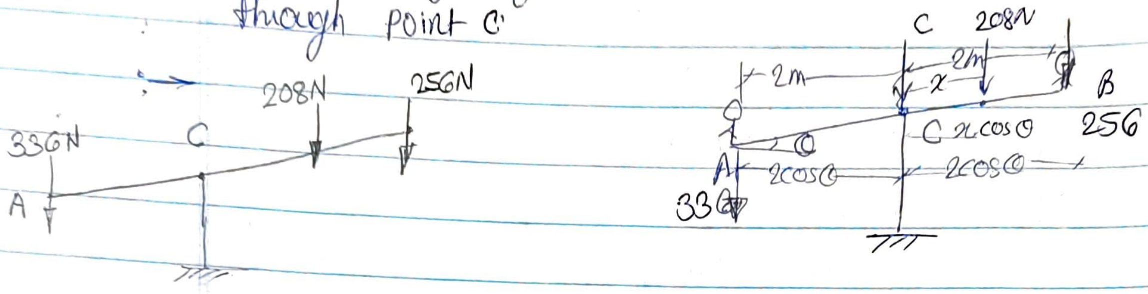
$$\alpha = 30°$$

In the second case the angle with +ve x axis will be

$$180 - 30 = 150°$$

$$\alpha = 30° \text{ or } 150°$$

④ The weights of two children sitting at ends A&B of a seesaw are 336N & 256N respectively. Where should a third child of weight 208N sit so that the resultant of the weights of the three children will pass through point C.



The moment produced by 336N about C is larger than moment produced by 256N @ C.

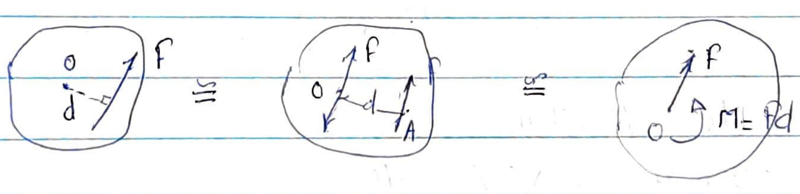
∴ The third child should sit to the right of C

Varignon's theorem

$$336 \times 2\cos\theta - 208 \times x\cos\theta - 256 \times 2 \times \cos\theta = 0$$

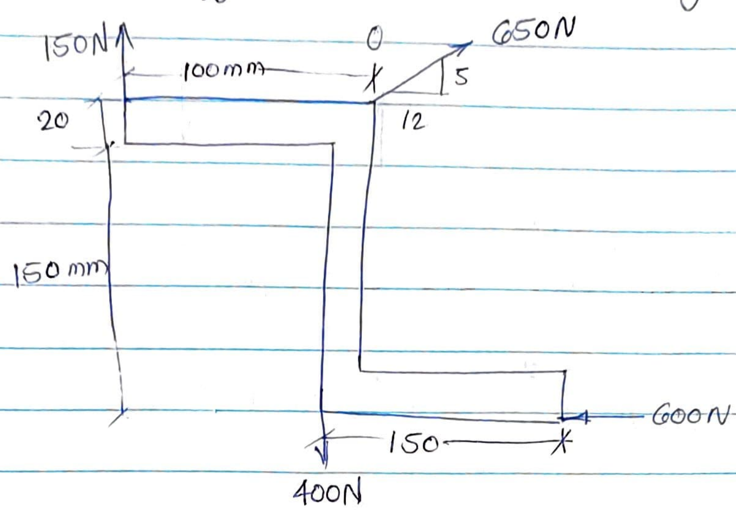
$$\therefore x = 0.769\,m$$

* Single / Equivalent force- Couple System:-



Force acting on an object produces translation & rotation of the object. The translational effects are represented by the magnitude & direction of the force, whereas the rotational effects are represented by the moment of force. Hence a force can be replaced by a force & couple as shown in fig.

(5) A z-shaped lamina of uniform width of 20mm is subjected to four forces as shown in fig. Find equilibriant in magnitude & direction.



$\tan\theta = \frac{5}{12}$

$= 22.619°$

To find equilibriant, we have to find resultant.

$R_x = \Sigma F_x = 650 \cos 22.619 - 600$

$= 0$

$R_y = \Sigma F_y = 150 - 400 + 650 \sin 22.619 = 0$

$R = 0$

When $R = 0$, the resultant can be a moment which will be same about any point in the plane. Taking moment about the point of application of 650N force.

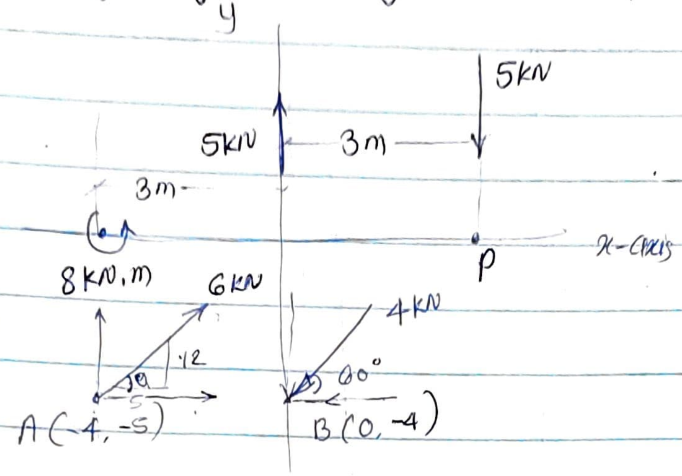
$M = -150 \times 100 + 400 \times 20 - 600 \times 170$

$= -109000$ N.mm

$M = 109$ N.m ↻

Equilibriant is a moment of 109 N.m ↺

⑥ Replace the force & couple system shown in fig by an equivalent single force & single moment at point P.



~~cosθ~~ $\tan\theta = \frac{12}{5}$

$\theta = 67.380°$

$$\Sigma F_x = 6\cos 67.380 - 4\cos 60$$

$$= 0.308 \text{ kN}$$

$$\Sigma F_y = 6\sin 67.380 - 4\sin 60$$

$$= 2.074 \text{ kN}$$

$$R = \sqrt{0.308^2 + 2.074^2}$$

$$= 2.097 \text{ kN}$$

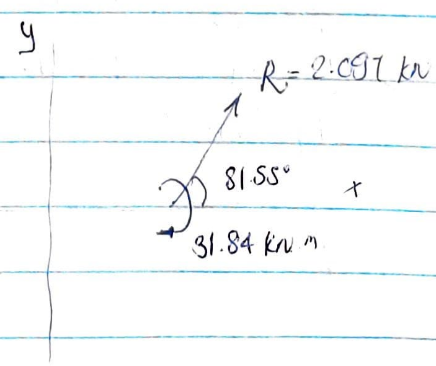
$$\theta = \tan^{-1}\frac{2.074}{0.308}$$

$$= 81.55$$

$$M_p = 8 - 5\times3 + 6\cos 67.380\times5 - 6\sin 67.380\times7$$
$$+ 4\sin 60\times3 - 4\cos 60\times4$$

$$= -31.84 \text{ kN.m}$$

$$= 31.84 \text{ kN.m}$$

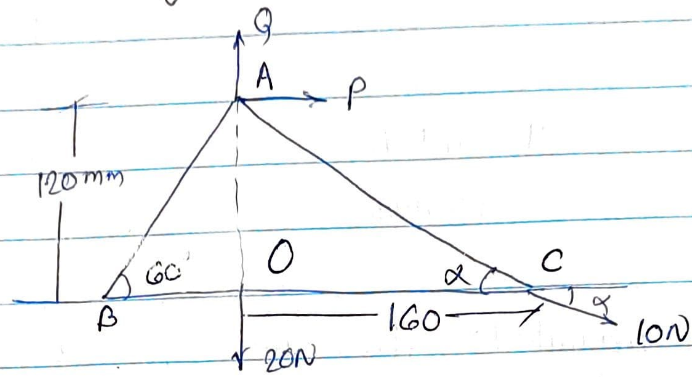


$R\times d = -31.84$

$2.097\times d = -31.84$

$d = -15.183$

⑦ Four forces acting on a triangle ABC are shown in fig. The sum of moments of these forces at point C is 2000 N.mm clockwise. If resultant of force system is in horizontal direction, find its magnitude & point of application.



As resultant is horizontal

$R_x = R$ & $R_y = 0$

$M_c = -2000$ N. mm

From fig. $\alpha = \tan^{-1}\frac{120}{160} = 36.87°$

$R_y = 0$

$Q - 20 - 10\sin\alpha = 0$

$Q = 20 + 10\sin 36.87 = 26N$

$\boxed{Q = 26N}$

$M_c = -2000$ N. mm

$-P\times 120 + 20\times 160 - Q\times 160 = -2000$

$-P\times 120 + 3200 - 26\times 160 = -2000$

$120P = 2000 + 3200 - 4160$

$120P = 1040$

$P = 8.667$ N

$R_x = R = P + 10\cos\alpha$

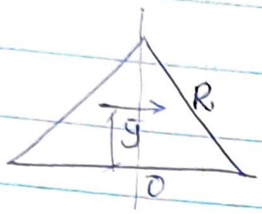
$= 8.667 + 10\cos 36.87$

$= 16.667$ N.

As resultant is horizontal, it will only act the y axis as shown in fig at dist $\bar{y}$ from O. Applying Varignon's theorem at O,

$$-P \times 120 - 10\sin 36.87 \times 160 = -R \times \bar{y}$$

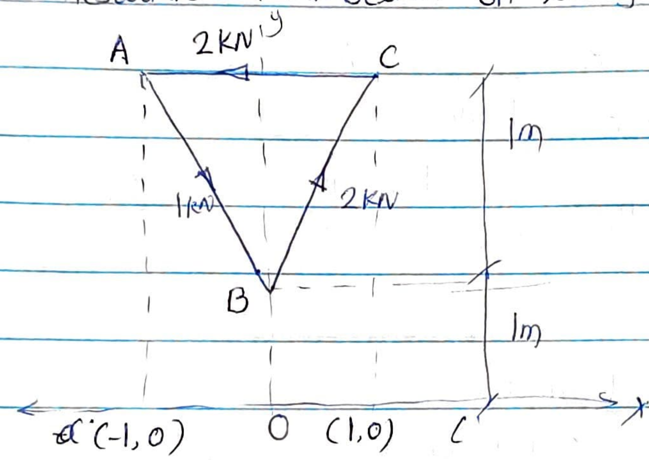
$$-8.667 \times 120 - 10\sin 36.87 \times 160 = -16.667 \times \bar{y}$$



$\therefore \bar{y} = 119.97$

$= 120mm$

(8) forces of magnitude 1kN, 2kN & 2kN act along the sides of a triangular frame ABC as shown in fig. Find resultant & intersection on x & y axis.



The angles for AB & BC are 45° with horizontal.

$R_x = \Sigma f_x = -2 + 1\cos 45 + 2\cos 45$

$= 0.1213$ kN.

$R_y = \Sigma f_y = -1\sin 45 + 2\sin 45$

$= 0.7071$ kN

$R = \sqrt{0.1213^2 + 0.7071^2}$

$R = 0.7174$ kN

$\theta = \tan^{-1} \dfrac{0.7071}{0.1213}$

$= 80.26°$

Let the resultant act at a distance x from O, as shown in
Using Varignon's theorem,

$$2\times2-(1\cos45)1 - 2\cos45\times1 = 0.7071x$$

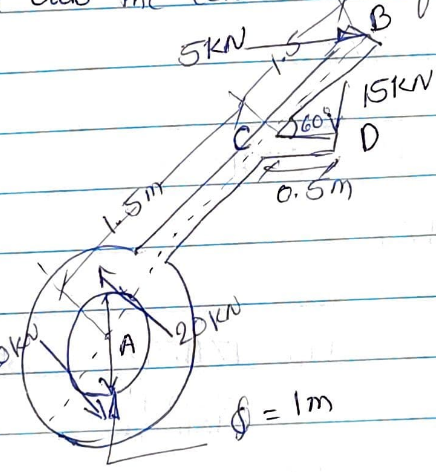
$$x = 2.657\,m$$

$$\tan\theta = \frac{y}{x}$$

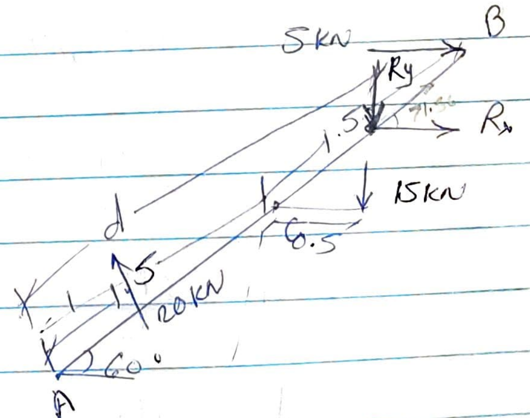
$$\tan 80.26 = \frac{y}{2.657}$$

$$\bar{y} = 15.48\,m$$

9) A machine part is subjected to forces as shown in fig. Find resultant of forces in magnitude & direction. Also locate the point where the resultant cuts the central line of bar AB.



$d = -0.74m$



$\Sigma f_x = 5\,kN$

$\Sigma f_y = -15\,kN$

$R = \sqrt{5^2+15^2} = 15.81\,kN$

$\theta = \tan^{-1}\frac{15}{5} = 71.565$

Let the resultant act at a distance d as shown in fig

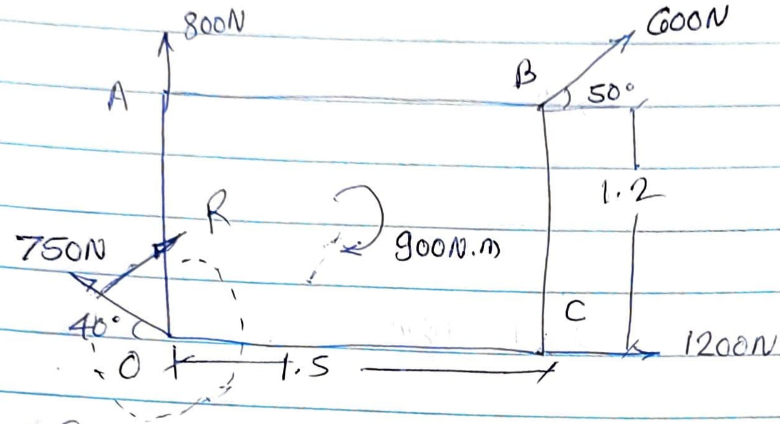
Using Varignon's theorem. ΣM_A

$$20\times1 - 5\times3\sin60 - 15(1.5\cos60+0.5) = -5d\sin60 - 15d\cos60$$

$$-11.740 = -d(11.830)$$

$$d = 0.9924\,m$$

(10) Find the resultant of the force system acting on a body OABC as shown in fig. Also find the points where the resultant will cut the x & y axis



$\Sigma F_x = 1200 + 600 \cos 50 - 750 \cos 40$

$= 1011.139$

$\Sigma F_y = 800 + 600 \sin 50 + 750 \sin 40$

$= 1741.717 \, N$

$R = \sqrt{(1011.139)^2 + (1741.717)^2}$

$= 2013.97 \, N$

$\theta = \tan^{-1} \frac{\Sigma F_y}{\Sigma F_x} = \tan^{-1} \frac{1741.717}{\cancel{2013.97}\ 1011.139} = 59.86°$

$\Sigma M_O = -600 \cos 50 \times 1.2 + 600 \sin 50 \times 1.5 - 900$

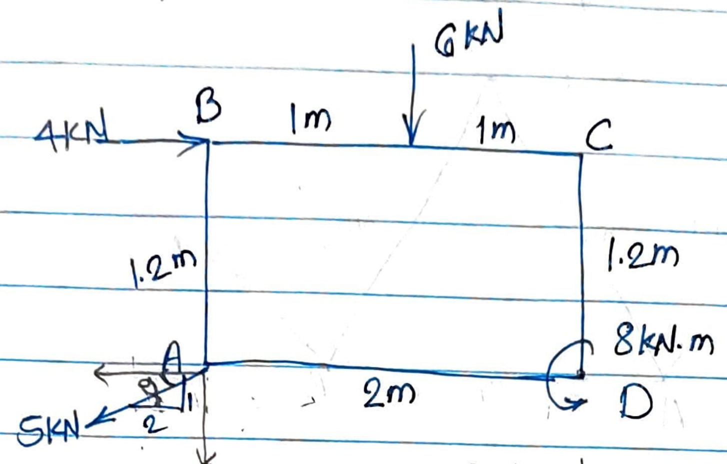
$= -673.367$

$\bar{x} = \frac{\Sigma M}{\Sigma F_y} = \frac{-673.367}{1741.717} = -0.387 \, m$

$\bar{y} = \frac{\Sigma M}{\Sigma F_x} = \frac{-673.367}{1011.139} = -0.665 \, m$

$d = \frac{\Sigma M_O}{R} = \frac{673.367}{2013.97} = 0.334 \, m.$

(11) Determine the magnitude & direction of the resultant force for the force system shown in fig. Locate the resultant force with respect to point D.



$\theta = \tan^{-1}\frac{1}{2} = 26.565$

$\Sigma F_x = 4 - 5\cos 26.565 = -0.472\ N$

$\Sigma F_y = -6 - 5\sin 26.565 = -8.212\ N.$

$\therefore R = \sqrt{(-0.472)^2 + (-8.212)^2}$

$= 8.225\ N.$

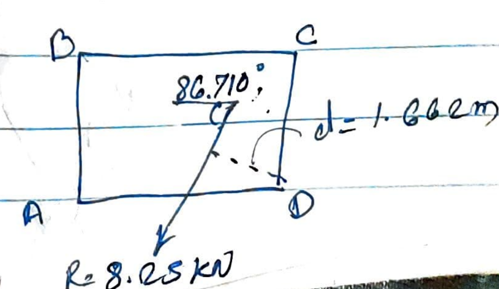
$\alpha = \tan^{-1}\left(\frac{-8.212}{-0.472}\right) = 86.710°$

Using Varignon's theorem
Taking moment at D.

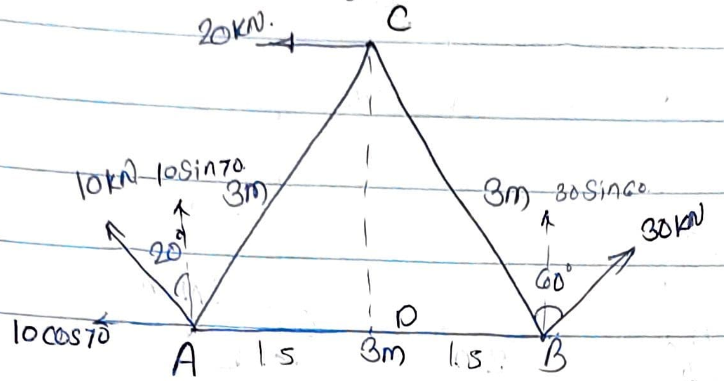
$8 + 6\times1 - 4\times1.2 + 5\sin 26.565\times2 = R\times d.$

$13.672 = 8.225\times d$

$\therefore d = \frac{13.672}{8.225} = 1.662\ m$



(12) An equilateral triangular plate of side 3m is acted on by three forces as shown in fig. Replace them by an equivalent force- couple system at A.



$\Sigma F_x = -10 \cos 70 - 20 + 30 \cos 60$

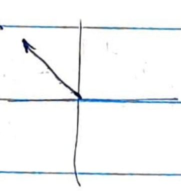
$= -8.42 \text{ kN}.$

$\Sigma F_y = 10 \sin 70 + 30 \sin 60$

$= 35.377 \text{ kN}$

$R = \sqrt{\Sigma F_x^2 + \Sigma F_y^2} = \sqrt{(-8.42)^2 + (35.377)^2}$

$R = 36.365 \text{ kN}$

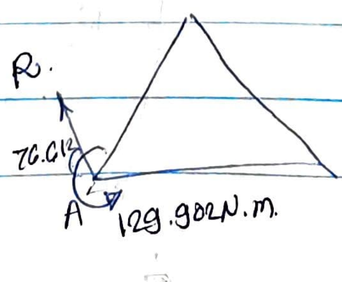


$\alpha = \tan^{-1} \frac{35.377}{8.42} = 76.612$

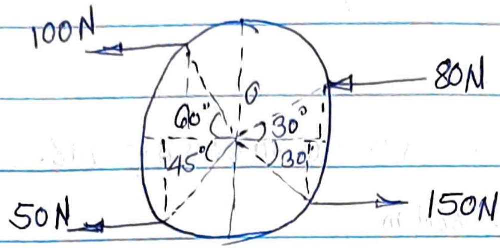
$l\ CD = \sqrt{3^2 - 1.5^2} = 2.598 \text{ m}$

$\Sigma M @ A = 20 \times 2.598 + 30 \sin 60 \times 3$

$= 129.902 \text{ N.m.}$



(13) Determine resultant of the following parallel forces & locate w.r.to 'O'. Radius is 1m.



$\Sigma F_x = R = -100 - 80 - 50 + 150 = -80N$

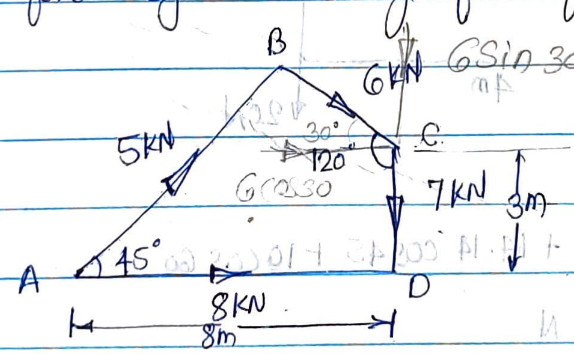
$R = 80N \leftarrow$

Using Varignon's theorem,

$80 \times 1 \sin 30 + 100 \times 1 \sin 60 + 150 \times 1 \sin 30 - 50 \times 1 \sin 45 = 80 \times d.$

$\therefore d = 2.078\,m$

(14) Find magnitude, direction & line of action of the resultant of the coplaner force system consisting of the four forces shown in fig.

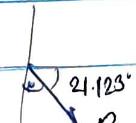


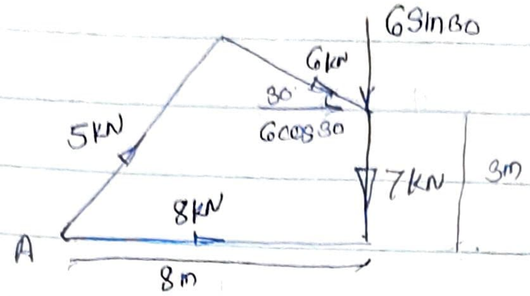
$\Sigma F_x = 6 \cos 30 + 5 \cos 45 + 8 = 16.731\,N$

$\Sigma F_y = -6 \sin 30 - 7 + 5 \sin 45 = -6.464N$

$\therefore R = \sqrt{\Sigma F_x^2 + \Sigma F_y^2} = 17.936N.$

$\alpha = \tan^{-1} \frac{\Sigma F_y}{\Sigma F_x} = 21.123°$





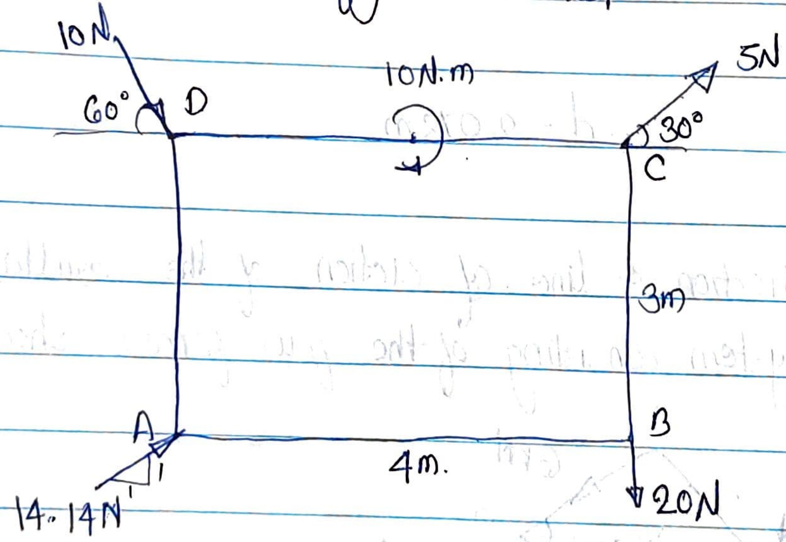
$\Sigma M@A = -6\cos30 \times 3 - 6\sin30 \times 8 - 7\times8$

$= -95.588 N.m$

$x = \dfrac{\Sigma M}{\Sigma fy} = \dfrac{95.588}{6.464} = 14.787m$

$y = \dfrac{\Sigma M}{\Sigma fx} = \dfrac{95.588}{16.731} = 5.713m.$

(15) Determine resultant of the force system acting on the plate as shown in fig. with respect to AB & AD.



$\Sigma Fx = 5\cos30 + 14.14\cos45 + 10\cos60$

$= 19.328 N$

$\Sigma Fy = -10\sin60 + 5\sin30 - 20 + 14.14\sin45$

$= -16.161$

$R = \sqrt{\Sigma Fx^2 + \Sigma Fy^2} = + 25.194 kN$

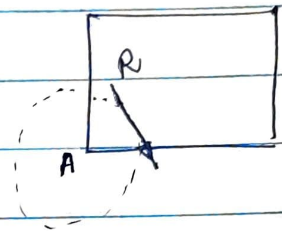
$\alpha = \tan^{-1} \frac{16.161}{19.328} = 39.900°$

$M_A = -20 \times 4 - 5\cos 30 \times 3 + 5\sin 30 \times 4 - 10 - 10\cos 60 \times 3$

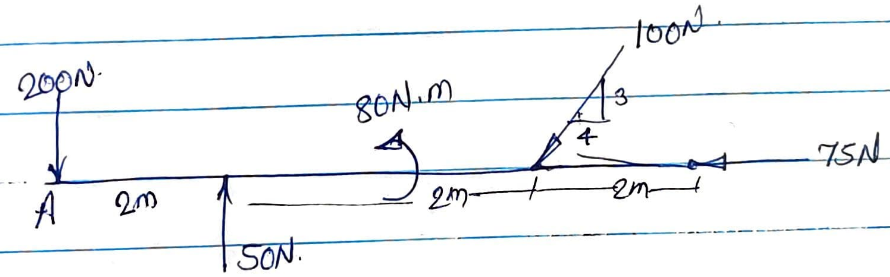
$= -107.990$ kN.m.

$x = \frac{\Sigma M_A}{\Sigma F_y} = \frac{-107.990}{-16.161} = 6.682$ m

$y = \frac{\Sigma M_A}{\Sigma F_x} = \frac{-107.990}{19.328} = 5.587$ m



(16) Replace the system of forces & couple by a single force couple system at A. Refer. Fig.



$\theta = \tan^{-1} \frac{3}{4}$

$= 36.869°$

$\Sigma F_x = -100\cos 36.869 - 75$

$= -155.$ N

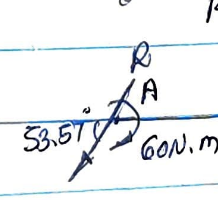
$\Sigma F_y = -200 + 50 - 100\sin 36.869$

$= -209.998$ N

$R = \sqrt{\Sigma F_x^2 + \Sigma F_y^2} = 261$ N.

$d = \frac{\Sigma M}{R} = \frac{60}{261} = 0.229$ m.

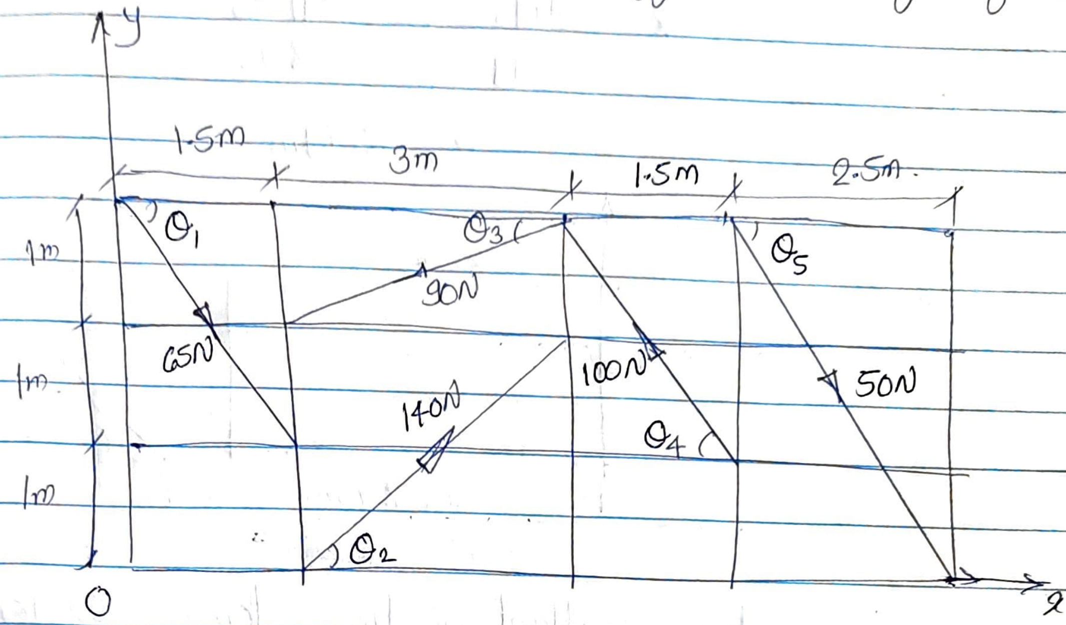
$\alpha = \tan^{-1} \frac{209.998}{155} = 53.569°$



$\Sigma M_{@A} = 50 \times 2 + 80 - 100\sin 36.869 \times 4$

$= -60$ N.m $= 60$ N.m

Replace the force system shown in fig. by a single force



$\tan\theta_1 = \frac{2}{1.5}$ $\therefore \theta_1 = 53.13°$

$\tan\theta_4 = \frac{2}{1.5}$ $\theta_4 = 53.13°$

$\tan\theta_2 = \frac{2}{3}$ $\theta_2 = 33.69°$

$\tan\theta_5 = \frac{3}{2.5}$ $\theta_5 = 50.19°$

$\tan\theta_3 = \frac{1}{3}$ $\theta_3 = 18.44°$

$\Sigma F_x = 65\cos 53.13 + 140\cos 33.69 - 90\cos 18.44 - 100\cos 53.13 + 50\cos 50.19$

$= 42.12\,N$

$\Sigma F_y = -65\sin 53.13 + 140\sin 33.69 - 90\sin 18.44 + 100\sin 53.13 - 50\sin 50.19$

$= 38.78\,N$

$R = \sqrt{42.12^2 + 38.78^2} = 57.25\,N$

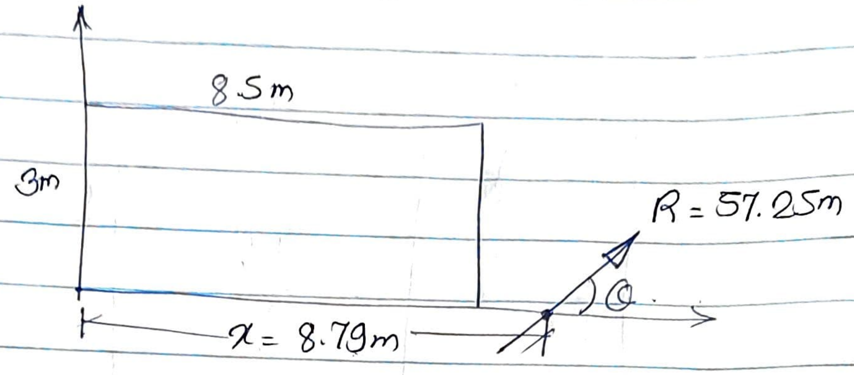
$\alpha = \tan^{-1}\frac{38.78}{42.12} = 42.64°$

$\Sigma M_O = -65\cos 53.13 \times 3 + 140\sin 33.69 \times 1.5 - 90\sin 18.44 \times 4.5 + 90\cos 18.44 \times 3 + 100\cos 53.13 \times 1 + 100\sin 53.13 \times 6 - 50\sin 50.19 \times 6 - 50\cos 50.19 \times 3 = 341.03\,N.m$ ↺

By Varignon's theorem,

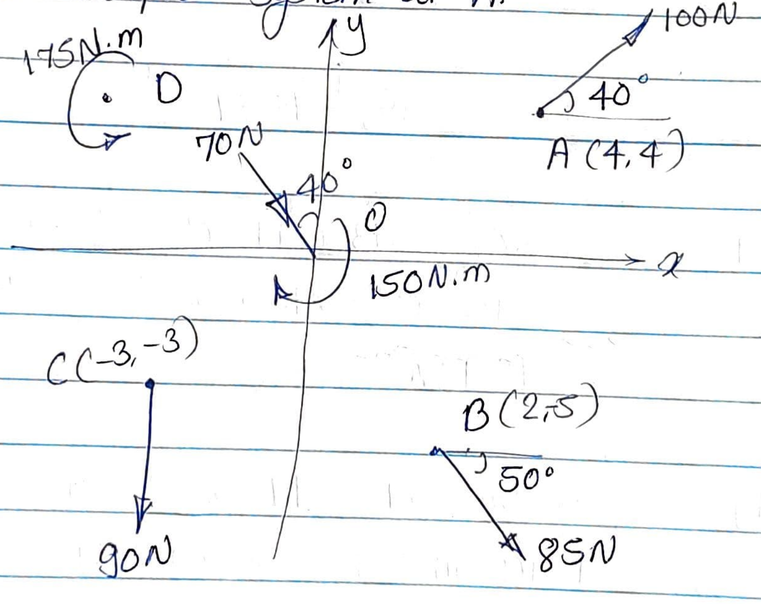
$$x = \frac{\Sigma M_o}{\Sigma F_y} = \frac{341.03}{38.78}$$

$$x = 8.79m$$



* (i) Find the resultant of the force system shown in fig.

(ii) Replace the given force and couple by a single force and couple system at A.



(i) $\Sigma F_x = 100 \cos 40 + 85 \cos 50 + 70 \cos 50$

$= 176.24N \rightarrow$

$\Sigma F_y = 100 \sin 40 - 85 \sin 50 - 90 - 70 \sin 50$

$= -144.46N$

$= 144.46N \downarrow$

$$R = \sqrt{(176.24)^2 + (144.46)^2}$$

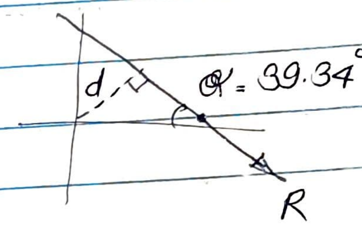
$= 227.88\,N.$

$$\alpha = \tan^{-1}\left(\frac{144.46}{176.24}\right) = 39.34^\circ$$

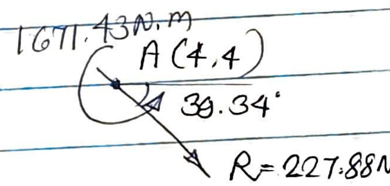
$\Sigma M@O = -100\cos 40 \times 4 + 100\sin 40 \times 4 + 85\cos 50 \times 5$
$- 85\sin 50 \times 2 + 90 \times 3 + 175 - 150$
$= -2369.10\ N.m$

$\Sigma M_O = R \times d$

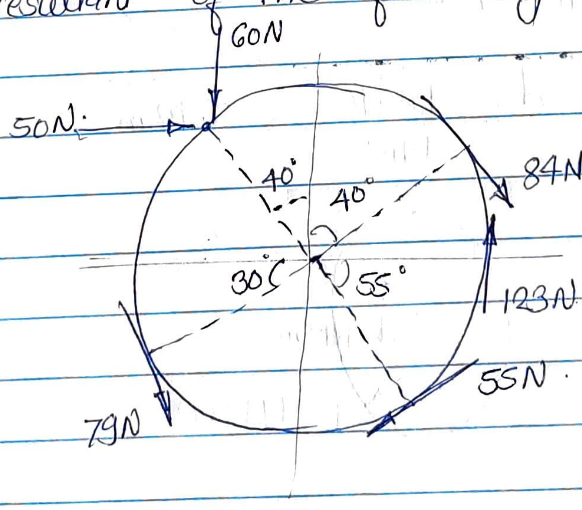
$$d = \frac{2369.10}{227.88} = 10.4\,m$$

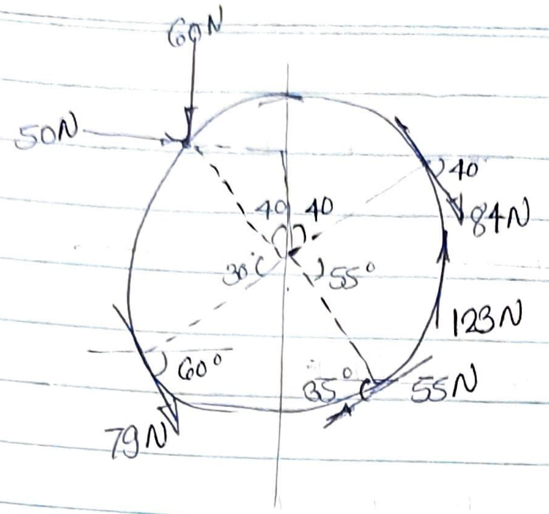


(ii) $M@A = 85\cos 50 \times 9 + 85\sin 50 \times 2 + 90 \times 7 + 70\cos 50 \times 4$
$70\sin 50 \times 4 + 175 - 150.$
$= 1671.43\ N.m.$



* Find the resultant of the force system as shown in fig.
$r = 2.5m.$



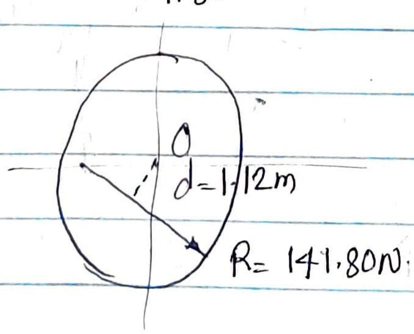


$$\Sigma F_x = 84\cos 40 - 55\cos 35 + 79\cos 60 + 50$$
$$= 108.79\,N \rightarrow$$
$$\Sigma F_y = -60 - 84\sin 40 - 55\sin 35 - 79\sin 60$$
$$= -90.96\,N \downarrow$$

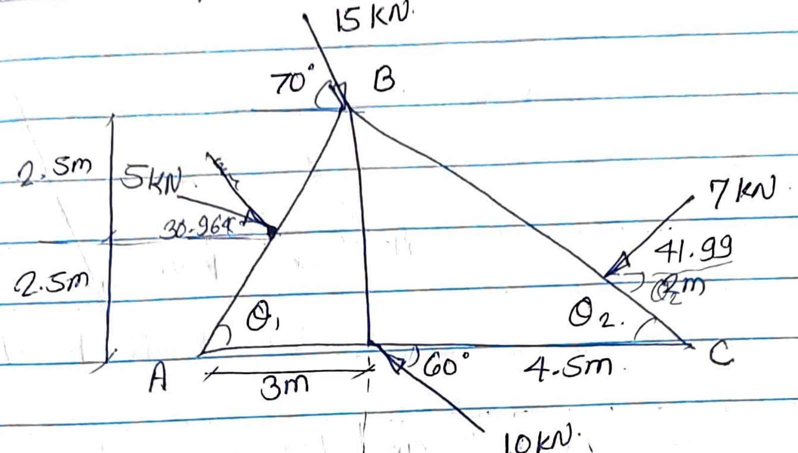
$$R = \sqrt{108.79^2 + 90.96^2}$$
$$= 141.80\,N.$$
$$\theta = \tan^{-1}\frac{90.96}{108.79} = 39.89^\circ$$

$$\Sigma M_0 = -84\times 2.5 + 123\times 2.5 - 55\times 2.5 + 79\times 2.5$$
$$- 50\times 2.5\cos 40 + 60\times 2.5\sin 40$$
$$= 158.16\,N.m \circlearrowleft$$
$$\Sigma M_0 = R\times d$$

$$d = \frac{158.16}{141.80} = 1.12\,m$$



A triangular plate ABC is subjected to four co-planar forces as shown in fig. Find the resultant completely & locate its position with respect to point A.



$\Sigma F_x = \cancel{5 \cos}$

$\theta_1 = \tan^{-1} \frac{5}{3} = 59.036$

$\theta_2 = \tan^{-1} \frac{5}{4.5} = 48.012$

$\Sigma F_x = 5 \cos 30.964 + 15 \cos 70 - 10 \cos 60 - 7 \cos 41.99$

$= -0.78 \text{ kN} \leftarrow$

$\Sigma F_y = -5 \sin 30.964 - 15 \sin 70 + 10 \sin 60 - 7 \sin 41.99$

$= -12.69 \text{ kN} \downarrow$

$R = \sqrt{(0.78)^2 + (12.69)^2}$

$= 12.71 \text{ kN}$

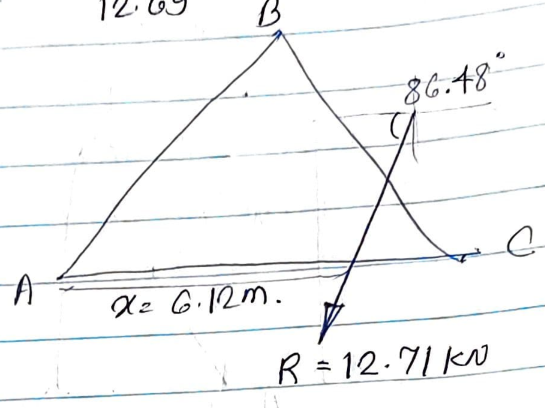
$\theta = \tan^{-1} \frac{12.69}{0.78} = 86.48°$

$\Sigma M_{@A} = -5 \cos 30.96 \times 2.5 - 5 \sin 30.96 \times 1.5 - 15 \cos 70 \times 5$
$- 15 \sin 70 \times 3 + 10 \sin 60 \times 3 + 7 \cos 41.99 \times 2 \sin 48.012$
$- 7 \sin 41.99 \times (7.5 - 2 \cos 48.012)$

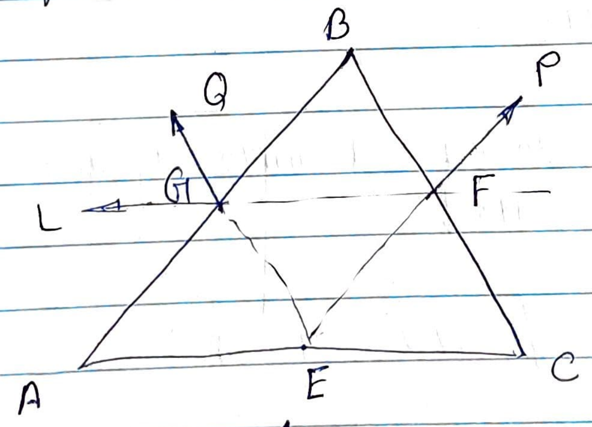
$= -77.66 \text{ kN.m}$

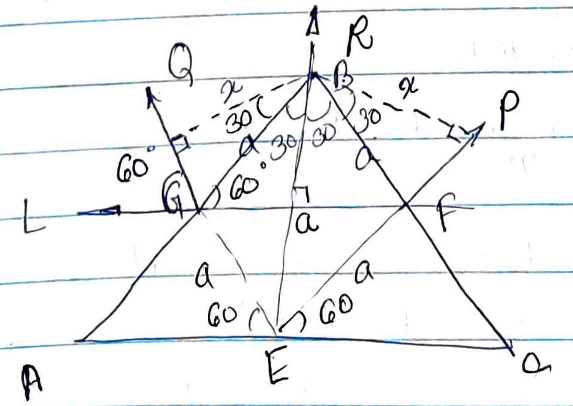
By Varignon's theorem.

$$x = \frac{\Sigma M_A}{\Sigma F_y} = \frac{77.66}{12.69} = 6.12m$$



* Three forces P, Q and L are acting on an equilateral triangular plate ABC as shown in fig. The resultant of 100N is known to pass vertically up from point B. Find the magnitudes of the three forces such the moment of the three forces about B is zero. Points E, F, G are the centres of the sides of the triangular plate.





As resultant is vertical $\therefore \Sigma F_x = 0$

$$\Sigma F_x = P\cos 60 - Q\cos 60 - L = 0$$

$$\therefore P\cos 60 - Q\cos 60 = L \quad \text{--- (1)}$$

$$\Sigma F_y = P\sin 60 + Q\sin 60 = R$$

$$P\sin 60 + Q\sin 60 = 100 \quad \text{--- (2)}$$

We know $\Sigma M_{@B} = 0$.

$$\Sigma M_{@B} = P(a\cos 30) - Q(a\cos 30) - L(a\cos 30) = 0$$

$$(P - Q - L)(a\cos 30) = 0$$

$$\therefore P - Q = L \quad \text{--- (3)}$$

Put (3) in (1)

$$P\cos 60 - Q\cos 60 = P - Q$$

$$0.5P - 0.5Q = P - Q$$

$$\therefore P = Q \quad \text{--- (4)}$$

$\therefore$ Put (4) in (2)

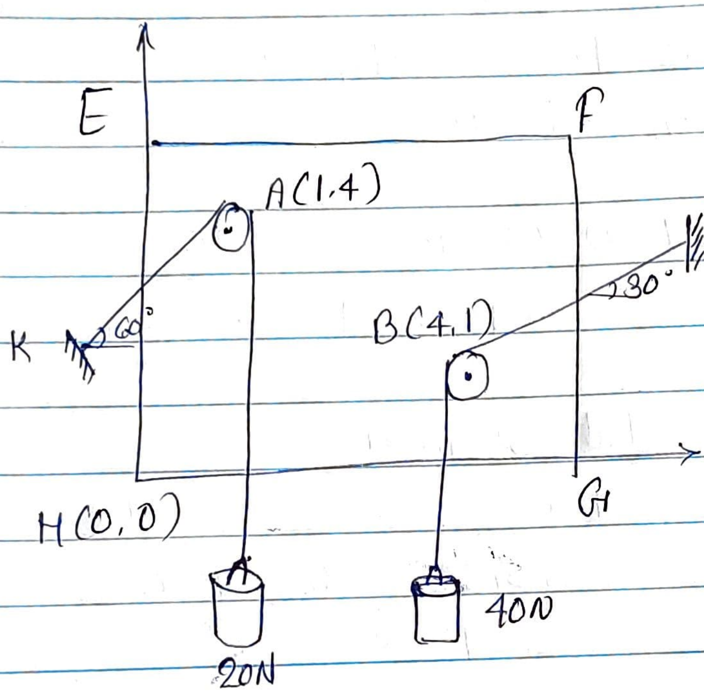
$$P\sin 60 + P\sin 60 = 100$$

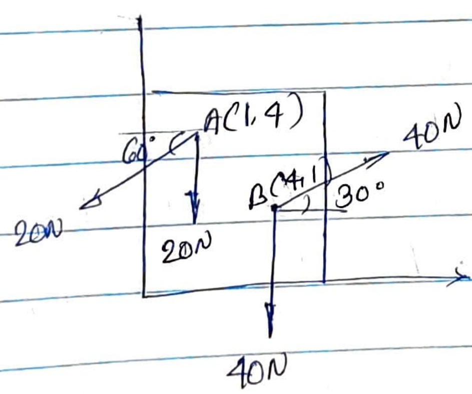
$$P(\sin 60 + \sin 60) = 100$$

$$\therefore P = \frac{100}{2\sin 60} = 57.735N$$

$$\boxed{P = Q = 57.735N; \quad L = 0}$$

* A fixed square board EFGIH carries two pulley A & B which carry load of 20N & 40N respectively with the help of cables fixed at point K & J as shown in fig. The diameter of each pulleys is 400mm. With reference to xy-axis the co-ordinates of centre of pulleys are A(1, 4) & B(4, 1) m. Find
(i) Magnitude of resultant force on the board.
(ii) position x-axis intercept, y-axis intercept of the resultant force.





$$\Sigma F_x = 40\cos 30 - 20\cos 60 = 24.64\ N$$

$$\Sigma F_y = -40 - 20 - 20\sin 60 + 40\sin 30$$

$$= -57.32N$$

$$R = \sqrt{24.64^2 + 57.32^2} = 62.39\ N.$$

$$\alpha = \tan^{-1}\left(\frac{57.32}{24.64}\right) = 66.74^\circ$$

$\Sigma M_{@H} = -20 \times 1 - 40 \times 4 + 20 \cos 60 \times 4 - 20 \sin 60 \times 1$

$- 40 \cos 30 \times 1 + 40 \sin 30 \times 4$

$= -111.96 \text{ N.m}$.

By Varignon's theorem.

$x = \frac{\Sigma M_{@H}}{\Sigma F_y} = \frac{111.96}{57.32}$

$x = 1.95 \text{m}$.

$y = \frac{\Sigma M_{@H}}{\Sigma F_x} = \frac{111.96}{24.64}$

$= 4.54 \text{m}$.