Total No. of Questions : 9]

SEAT No. :

**P6492**

[Total No. of Pages : 4

**[5868]-109**

**First Year Engineering**

**ENGINEERING MATHEMATICS - II**

**(2019 Pattern) (Semester - I & III) (107008)**

*Time : 2½ Hours]* *[Max. Marks : 70*

***Instructions to the candidates:***

1) *Q.No. 1 is compulsory.*
2) *Solve Q.2 or Q.3, Q.4 or Q.5, Q.6 or Q.7, Q.8, or Q.9.*
3) *Neat diagrams must be drawn whenever necessary.*
4) *Figures to the right indicate full marks.*
5) *Use of electronic pocket calculator is allowed.*
6) *Assume suitable data if necessary.*

***Q1)*** Write the correct option for the following multiple choice questions.

a) $\int_0^{\frac{\pi}{2}} \cos^6 x =$ **[2]**

i) $\frac{5}{16}$ ii) $\frac{5\pi}{32}$

iii) $\frac{16\pi}{10}$ iv) $\frac{5\pi}{48}$

b) The curve $y^2(x-a) = x^2(2a-x)$ is **[2]**

i) Symmetric about X - axis and net passing through origin
ii) Symmetric about Y - axis and net passing through origin
iii) Symmetric about X - axis and passing through origin
iv) Symmetric about Y - axis and passing through origin

c) The value of double integral $\int_0^1\int_0^1 \frac{1}{\sqrt{1-x^2}\sqrt{1-y^2}} dx\, dy$ is **[2]**

i) $\frac{\pi}{2}$ ii) $\frac{\pi^2}{2}$

iii) $\frac{\pi^2}{4}$ iv) $\frac{\pi^2}{16}$

*P.T.O.*

d) The Centre (C) and radius ($r$) of the sphere $x^2 + y^2 + z^2 - 2y - 4z - 11 = 0$ are **[2]**

i) $C \equiv (0,1,2); r = 4$  ii) $C \equiv (0,-1,-2); r = 2$

iii) $C \equiv (0,2,4); r = 4$  iv) $C \equiv (0,1,2); r = 2$

e) The number of loops in the rose curve $r = a \cos 4\theta$ are **[1]**

i) 2  ii) 4

iii) 6  iv) 8

f) $\iint\limits_R dx\,dy$ represents **[1]**

i) Volume  ii) Centre of gravity

iii) Moment of inertia  iv) Area of region R

*Q2)* a) If $I_n = \int_{\pi/4}^{\pi/2} \cot^n \theta \, d\theta$ prove that $I_n = \frac{1}{n-1} - I_{n-2}$. **[5]**

b) Show that $\int_0^1 x^{m-1}\left(1-x^2\right)^{n-1} dx = \frac{1}{2}\beta\left(\frac{m}{2}, n\right)$. **[5]**

c) Prove that $\int_0^1 \frac{x^a - 1}{\log x} dx = \log(1+a), a \geq 0$. **[5]**

OR

*Q3)* a) If $I_n = \int_0^{\pi/2} x^n \sin x \, dx$ then prove that $I_n = n\left(\frac{\pi}{2}\right)^{n-1} - n(n-1)I_{n-2}$. **[5]**

b) Show that $\int_0^\infty e^{-h^2x^2} dx = \frac{\sqrt{\pi}}{2h}$. **[5]**

c) Show that **[5]**

$$\int_a^b e^{-x^2} dx = \frac{\sqrt{\pi}}{2}\left[erf(b) - erf(a)\right]$$

OR

*Q4)* a) Trace the curve $x^2y^2 = a^2(y^2 - x^2)$. **[5]**

b) Trace the curve $r = a(1 - \sin\theta)$. **[5]**

c) Find the whole length of the loop of the curve $3y^2 = x(x-1)^2$. **[5]**

OR

*Q5)* a) Trace the curve $y^2(2a - x) = x^3$. **[5]**

b) Trace the curve $r = a\cos 2\theta$. **[5]**

c) Trace the curve $x^{2/3} + y^{2/3} = a^{2/3}$. **[5]**

*Q6)* a) Prove that the two spheres $x^2 + y^2 + z^2 - 2x + 4y - 4z = 0$ and $x^2 + y^2 + z^2 + 10x + 2z + 10 = 0$ touch each other and find the co-ordinates of the point of contact. **[5]**

b) Find the equation of right circular cone whose vertex is $(1, -1, 2)$, axis is the line $\frac{x-1}{2} = \frac{y+1}{1} = \frac{z-2}{-2}$ and the semi-vertical angle $45°$. **[5]**

c) Find the equation of right circular cylinder of radius a whose axis passes through the origin and makes equal angles with the co-ordinate axes. **[5]**

OR

*Q7)* a) Show that the plane $x - 2y - 2z - 7 = 0$ touches the sphere $x^2 + y^2 + z^2 - 10y - 10z - 31 = 0$. Also find the point of contact. **[5]**

b) Find the equation of right circular cone with vertex at origin, axis the $Y$-axis and semi-vertical angle $30°$. **[5]**

c) Find the equation of right circular cylinder of radius $\sqrt{6}$ whose axis is the line $\frac{x}{1} = \frac{y}{-1} = \frac{z}{1}$. **[5]**

*Q8)* a) Change the order of integration and evaluate $\int_0^{\pi}\int_x^{\pi}\frac{\sin y}{y}\,dx\,dy$. **[5]**

b) Find the area of one loop of $r = a\sin 2\theta$. **[5]**

c) Find the moment of inertia of one loop of the lemniscate $r^2 = a^2\cos 2\theta$ about initial line. Given that $\rho = \frac{2m}{a^2}$, $m$ is the mass of loop of lemniscate. **[5]**

OR

*Q9)* a) Evaluate $\iint ydxdy$ over the region enclosed by the parabola $x^2 = y$, and the line $y = x + 2$. **[5]**

b) Evaluate $\iiint x^2 yzdxdydz$, throughout the volume bounded by the plane $x = 0, y = 0, z = 0$ $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$. **[5]**

c) Find the $y$- coordinate of the centre of gravity of the area bounded by $r = a\sin\theta$ and $r = 2a\sin\theta$. Given that the area bounded by these curves is $\frac{3\pi a^2}{4}$. **[5]**