Rehder: Mathematical Implementation Details

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1 Proportionality Constant

Equation 4 of the paper mentions a delta probability distribution which is proportional to the porduct of 4 distributions. (Two normal and the other two circular-normal) As given in paper:

$$\begin{aligned} p(\Delta x, \Delta y, \Delta z) \propto \exp&\left(\frac{-(\Delta x - \Delta t\nu \cos\psi)^2}{2\sigma_v^2}\right) \\ .\exp&\left(\frac{-(\Delta y - \Delta t\nu \sin\psi)^2}{2\sigma_v^2}\right) \\ &.\exp(k_{\Delta\psi}\cos\Delta_\psi) \\ &.\exp(k_v \cos\angle\Delta y, \Delta x - \psi) \end{aligned} \tag{1}$$

Let the proportionality constant be k, then from the PDFs of normal and circular normal distributions we have:

$$k = \frac{1}{8\pi^3\sigma_{vx}\sigma_{vy}I_0(k_{\Delta\psi})I_0(k_v)} \tag{2}$$

Where $I_0(.)$ is the 0-order modified Bessel function.
The final form of the Bessel equation turns out to be(assumption: $a_1 = 0$ which implies all odd terms die to zero):

$$I_0(x) = y_1(x) = a_0[1 + \sum_{m=1}^{\infty}\frac{(-1)^m x^{2m}}{2^{2m}(m!)^2}] \tag{3}$$

where y was substituted as: $y = \phi(r, x) = a_0x^r + \sum_{n=1}^{\infty} a_n x^{r+n}$ in the standard 0 order Bessel equation:

$$x^2\frac{\mathrm{d}^2y}{\mathrm{d}x^2} + x\frac{\mathrm{d}^1y}{\mathrm{d}x^1} + x^2y = 0 \tag{4}$$