Assignment 04:

Implement Gradient Descent Algorithm to find the local minima of a function.
For example, find the local minima of the function $y=(x+3)^2$ starting from the point x=2.

Aim *:* Implement Gradient Descent Algorithm

Sotware Requirements :

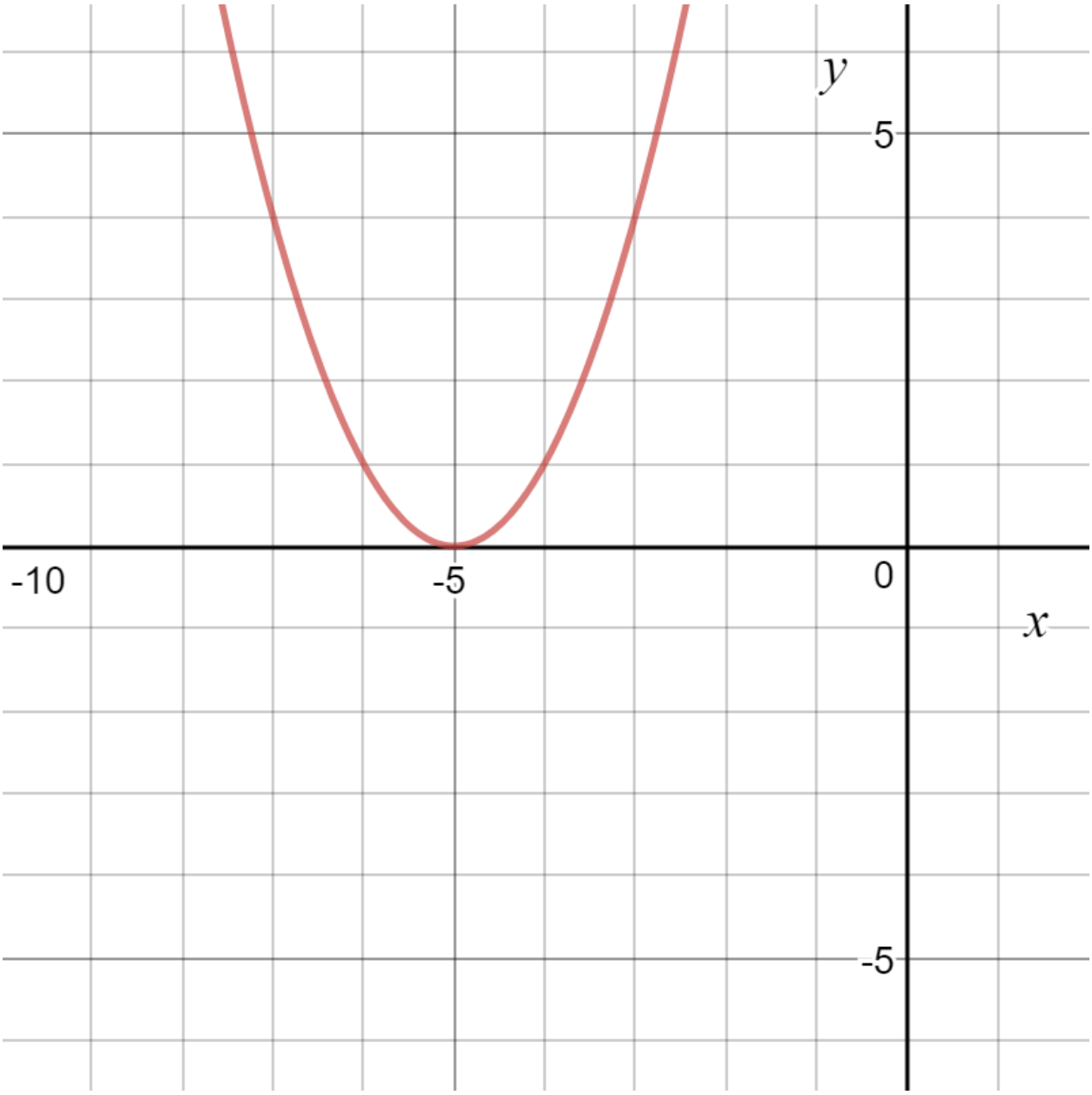
1. Fedora
2. Python 3.10 above
3. IDLE

Theory : gradient descent

It is an optimization algorithm to find the minimum of a function. We start with a random point on the function and move in the **negative direction** of the **gradient of the function** to reach the **local/global minima**.

Example by hand :

Question : Find the local minima of the function $y=(x+5)^2$ starting from the point x=3



Solution : We know the answer just by looking at the graph. $y = (x+5)^2$ reaches it's minimum value when $x = -5$ (i.e when $x=-5$, $y=0$). Hence $x=-5$ is the local and global minima of the function.

Now, let's see how to obtain the same numerically using gradient descent.

Step 1 : Initialize x =3. Then, find the gradient of the function, dy/dx = 2*(x+5).

Step 2 : Move in the direction of the negative of the gradient (Why?). But wait, how much to move? For that, we require a learning rate. Let us assume the **learning rate → 0.01**

Step 3 : Let's perform 2 iterations of gradient descent

Initialize Parameters :

$$X_0 = 3$$

$$Learning\ rate = 0.01$$

$$\frac{dy}{dx} = \frac{d}{dx}(x+5)^2 = 2 * (x+5)$$

Iteration 1 :

$$X_1 = X_0 - (learning\ rate) * (\frac{dy}{dx})$$

$$X_1 = 3 - (0.01) * \big(2 * (3+5)\big) = 2.84$$

Iteration 2 :

$$X_2 = X_1 - (learning\ rate) * (\frac{dy}{dx})$$

$$X_2 = 2.84 - (0.01) * \big(2 * (2.84+5)\big) = 2.6832$$

Step 4 : We can observe that the X value is slowly decreasing and should converge to -5 (the local minima). However, how many iterations should we perform?

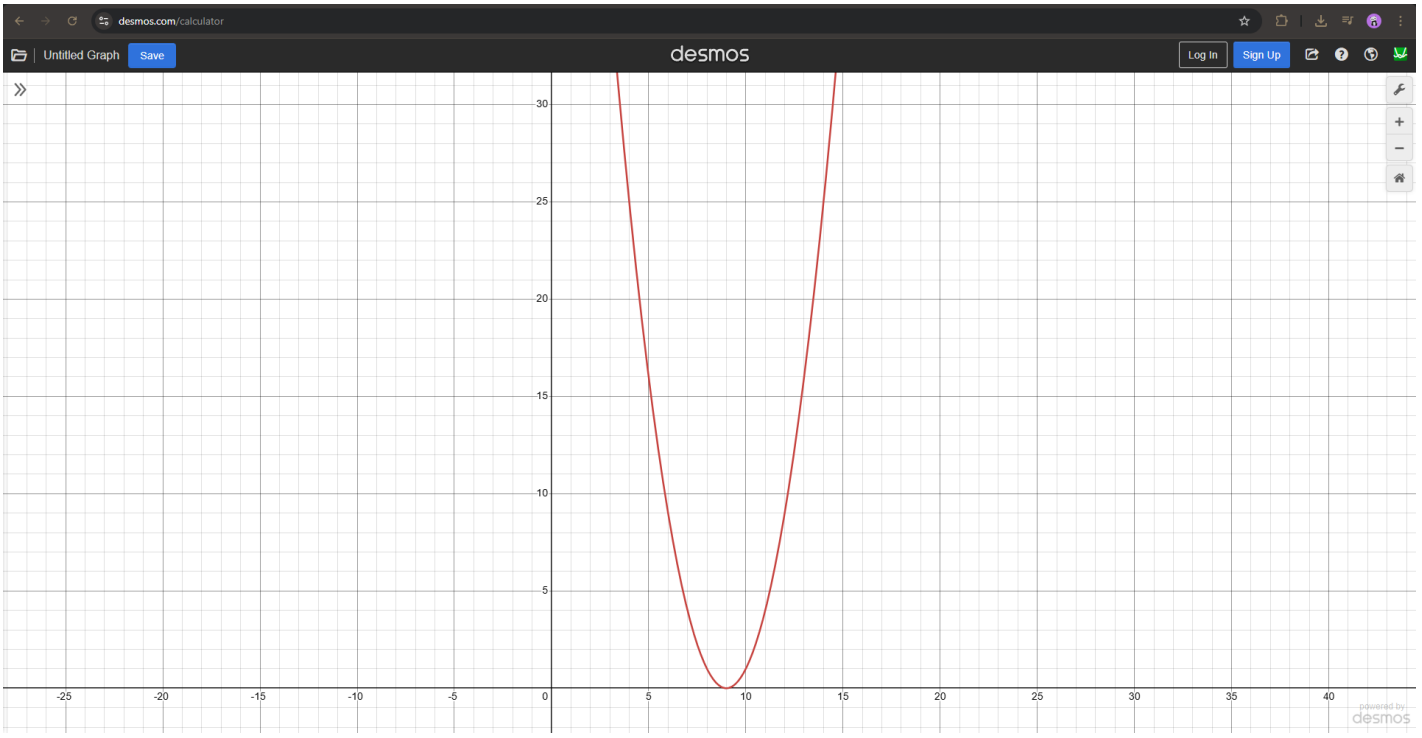
Let us set a precision variable in our algorithm which calculates the difference between two consecutive “x” values. If the difference between x values from 2 consecutive iterations is lesser than the precision we set, stop the algorithm!

Code and Output:

Graph : To draw the graph for $y=(x-9)^2$ go to

https://www.desmos.com/calculator

There write equation $y=(x-9)^2$ the graph is seen as follows



Conclusion: Gradient descent algorithm implemented on given function successfully.

Viva Questions

1. What is Gradient Descent

2. Where gradient Descent is applicable