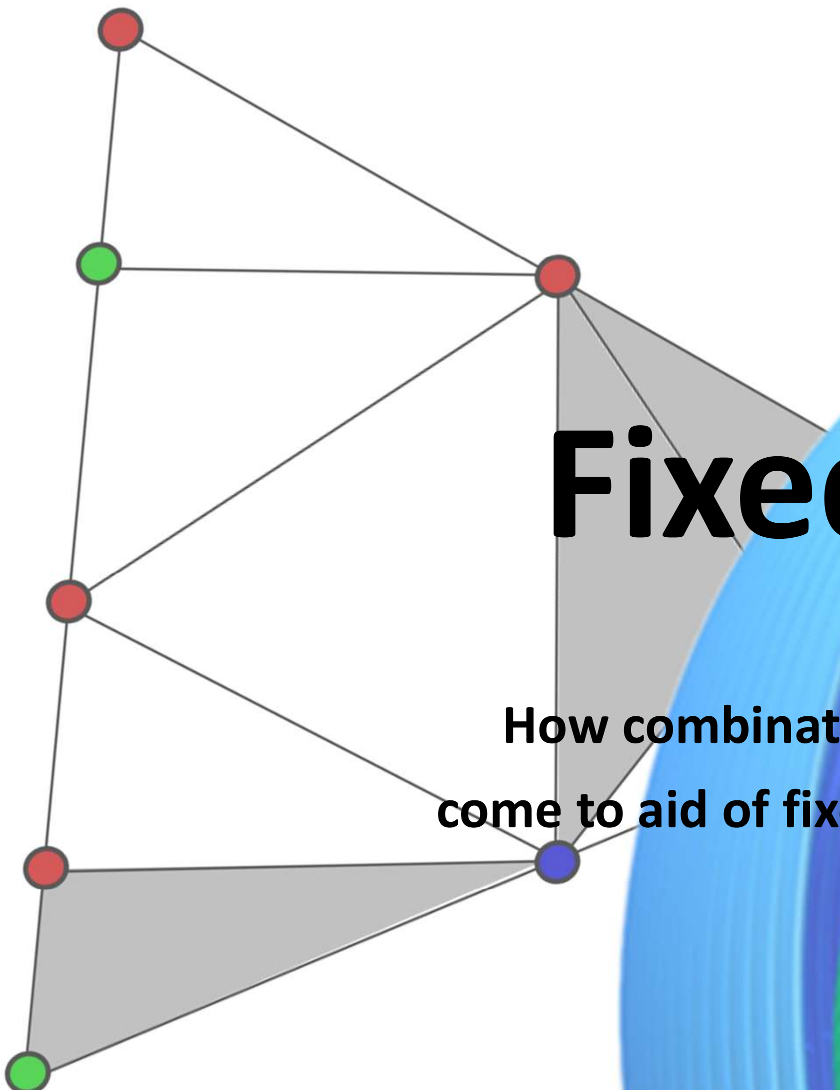
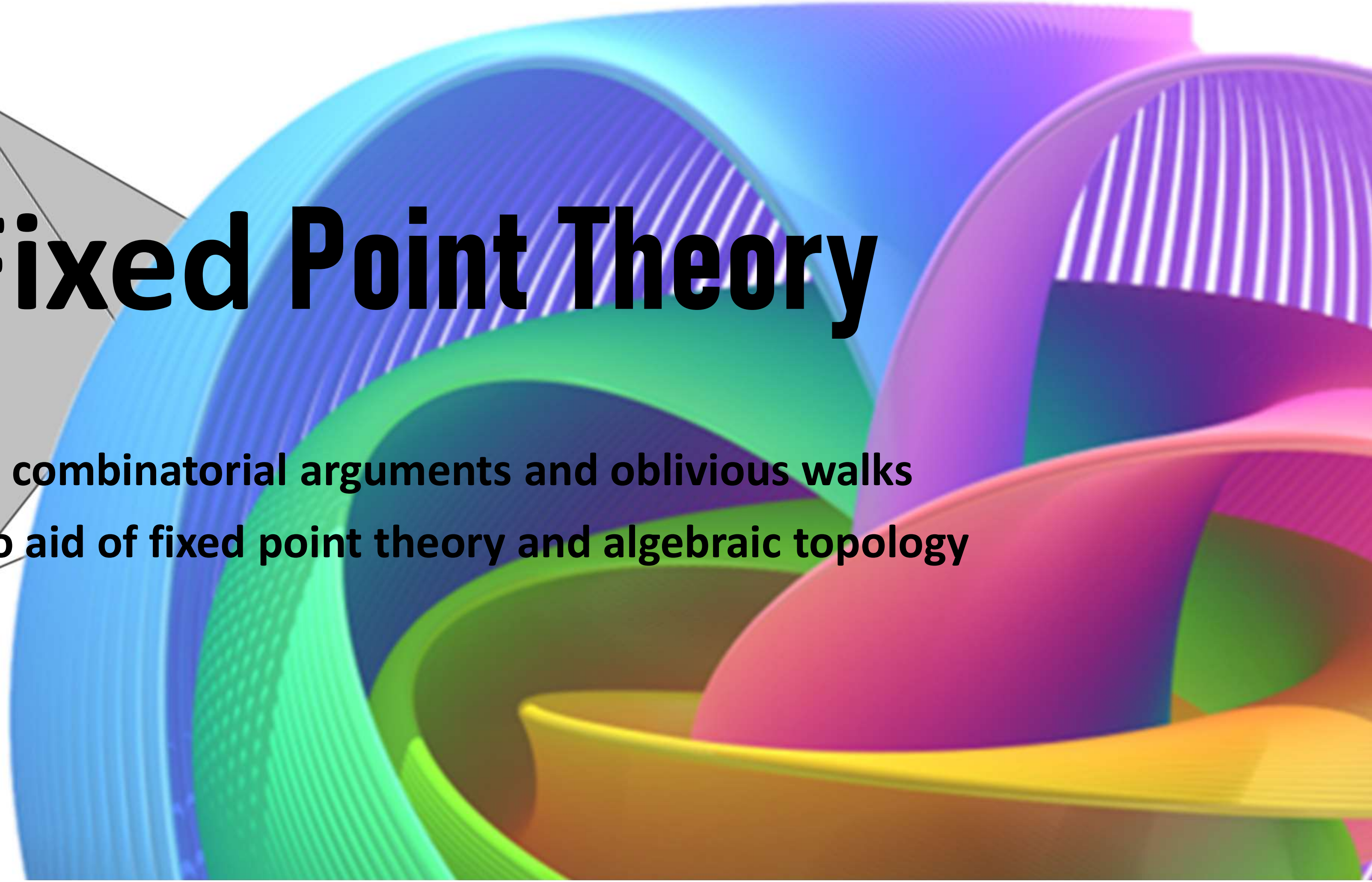




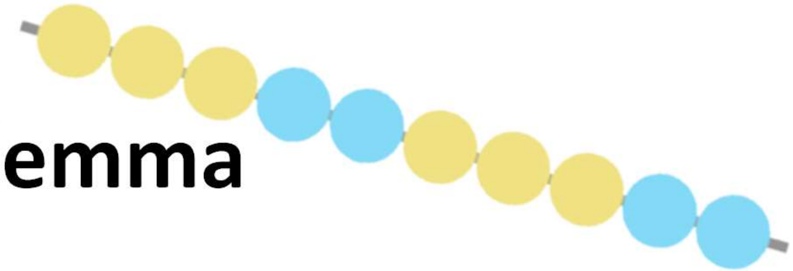
Fixed Point Theory

How combinatorial arguments and oblivious walks
come to aid of fixed point theory and algebraic topology

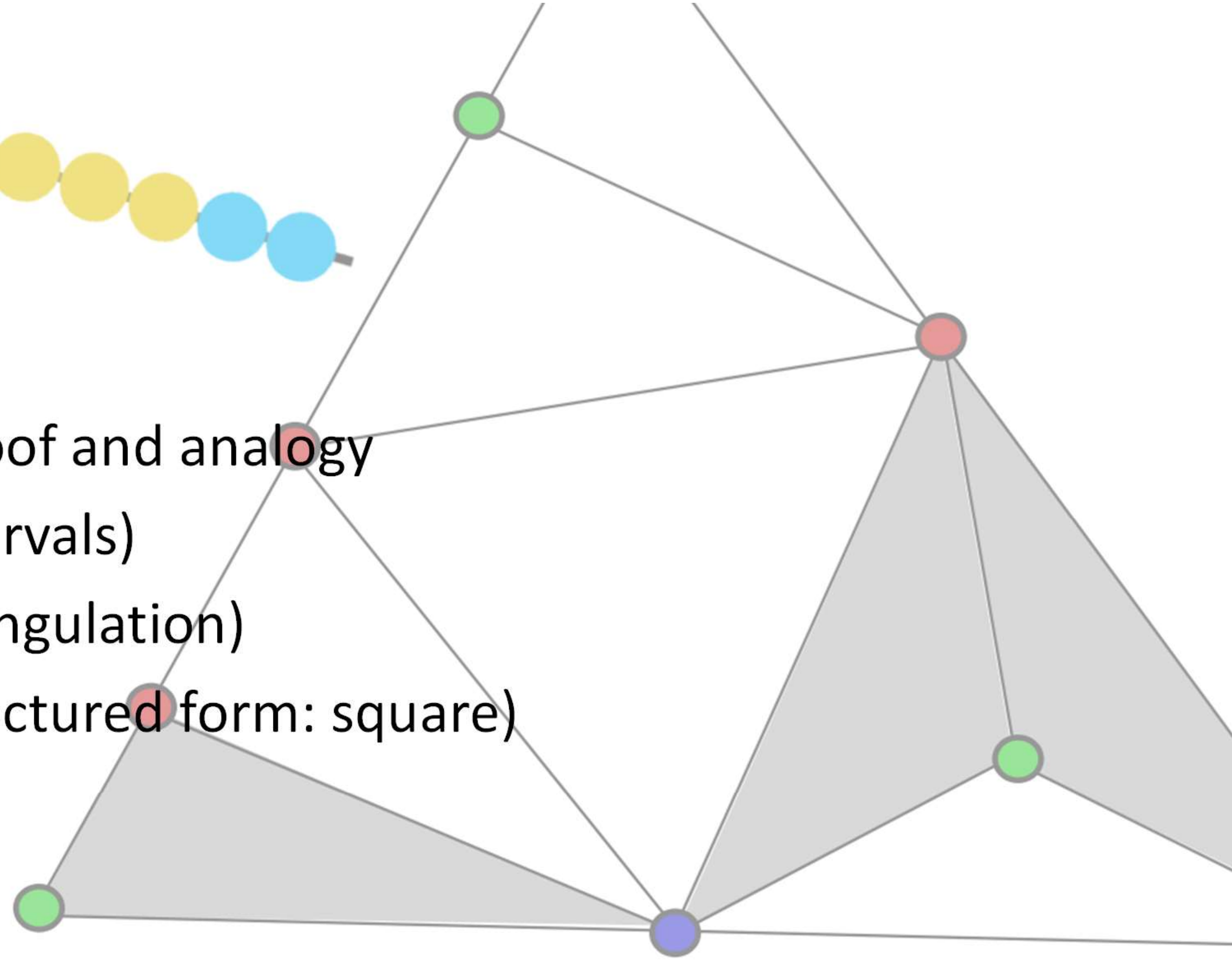
Fixed Point Theory: Background and applications

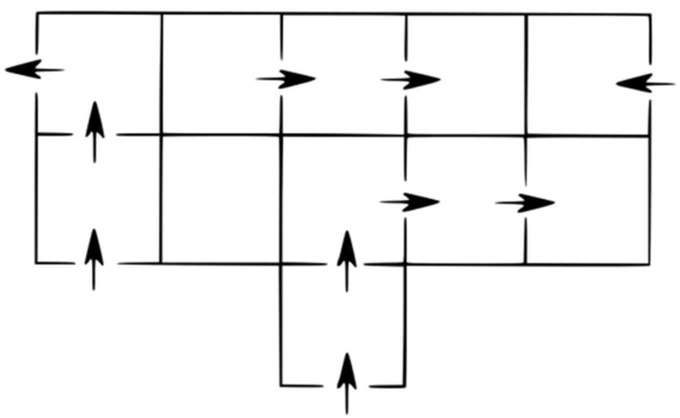
- Simple observation: Map of a country
- Fluid dynamics: Gin bottle conundrum
- Game Theory: Existence of Nash equilibrium
- Real Analysis: Continuous map $\boldsymbol{f}: \boldsymbol{R^n} \rightarrow \boldsymbol{R^n}$ with bounded displacement is onto
- Complex Analysis: Proof of fundamental theorem of algebra
- Algebraic Topology: Existence of a cross in opposing paths of a rectangle

Sperner lemma



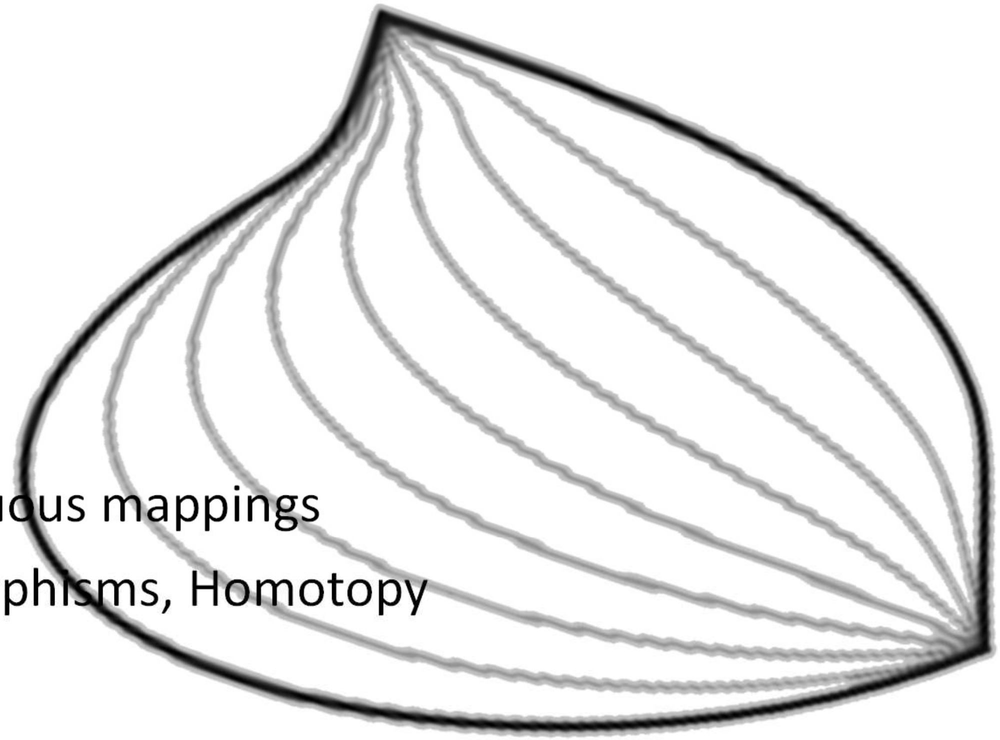
- Background and usage
- Hotel routing problem , proof and analogy
- One-dimensional case (intervals)
- Two-dimensional case (triangulation)
- Two-dimensional case (structured form: square)

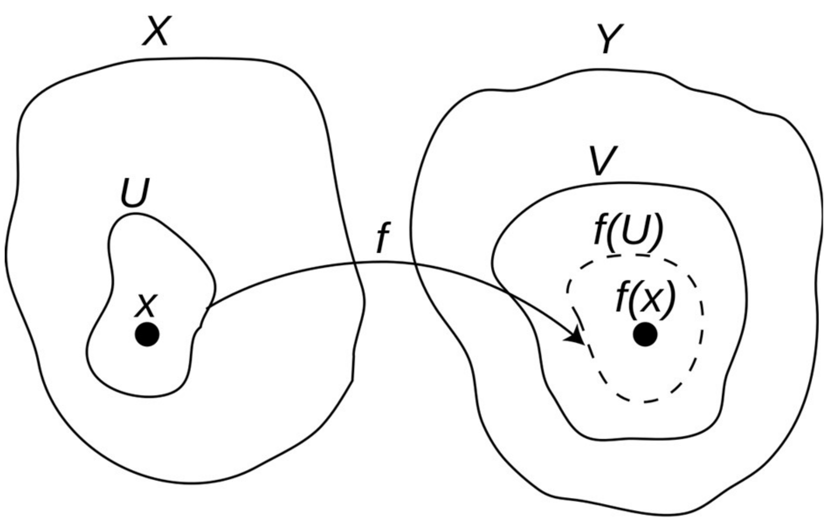


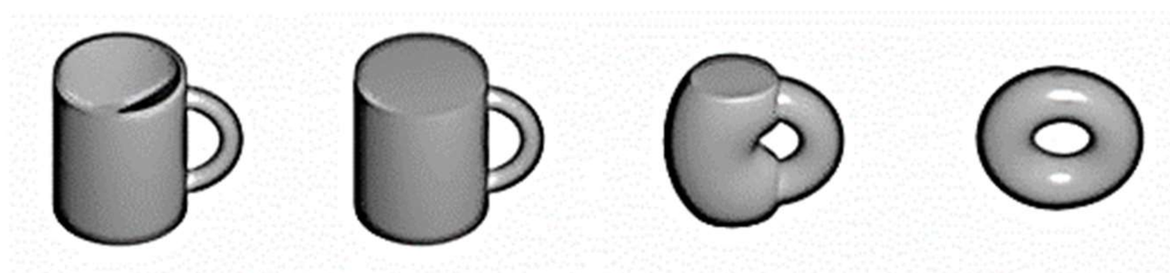


Preliminaries to BPFT

- Mappings between different spaces
- Injections, bijections and inverse maps
- Notion of continuity, composition of continuous mappings
- Algebraic Topology: Morphisms, Homeomorphisms, Homotopy

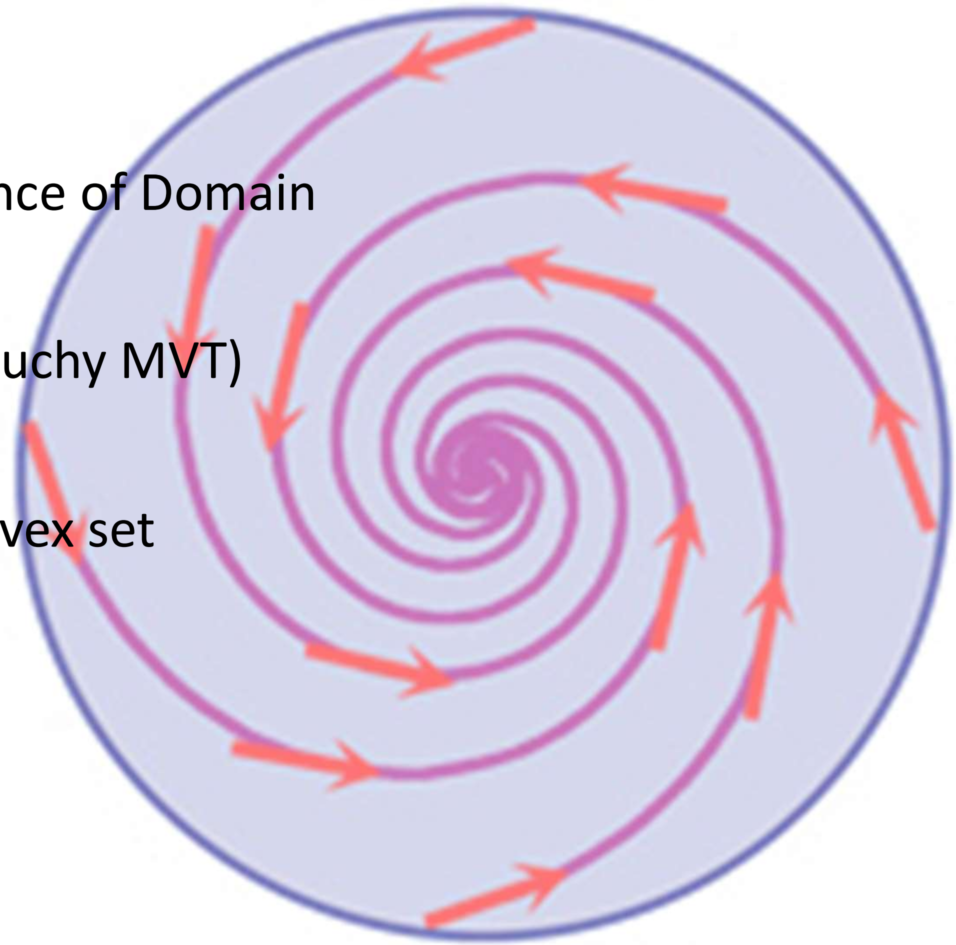






Brouwer Fixed Point Theorem

- Statement of the theorem
- Background & Motivation towards the Invariance of Domain
- One dimensional case: IVT, a pseudo-FPT
- Intuitive proof of IVT, generalizations (MVT, Cauchy MVT)
- Homeomorphism preserves FPT property
- Homeomorphism of S^1 with any bounded convex set
- Homeomorphism of S^1 with the unit square
- Homeomorphism of S^1 with R^2



Two-dimensional BFPT

- Sufficient to prove on unit square
- Assume the contrary!
- Arbitrary subdivision to squares & classifying vertices
- Coloring the subdivision vertices and using Sperner lemma
- Identifying the rainbow square as a vertex quadruple
- Notion of compactness and existence of convergent subsequence
- Limiting the rainbow squares to a point
- Prove that the limit point is the fixed point by assuming the contrary