WASHINGTON STATE
UNIVERSITY

Dynamic Programming for Power Systems

Aryan Ritwajeet Jha

Contents

- *Anamika Dubey and Surya Santoso (2015)* Electric Vehicle Charging on Residential Distribution Systems: Impacts and Mitigation. It is a harder dynamic programming problem.
- Problem Statements where Dynamic Programming is utilized.
  - Recursive Fibonacci Sequence with Memoization.
  - Shortest Path Problem with Memoization.

Electric Vehicle Charging on Residential Distribution Systems: Impacts and Mitigation

Anamika Dubey and Surya Santoso (2015)

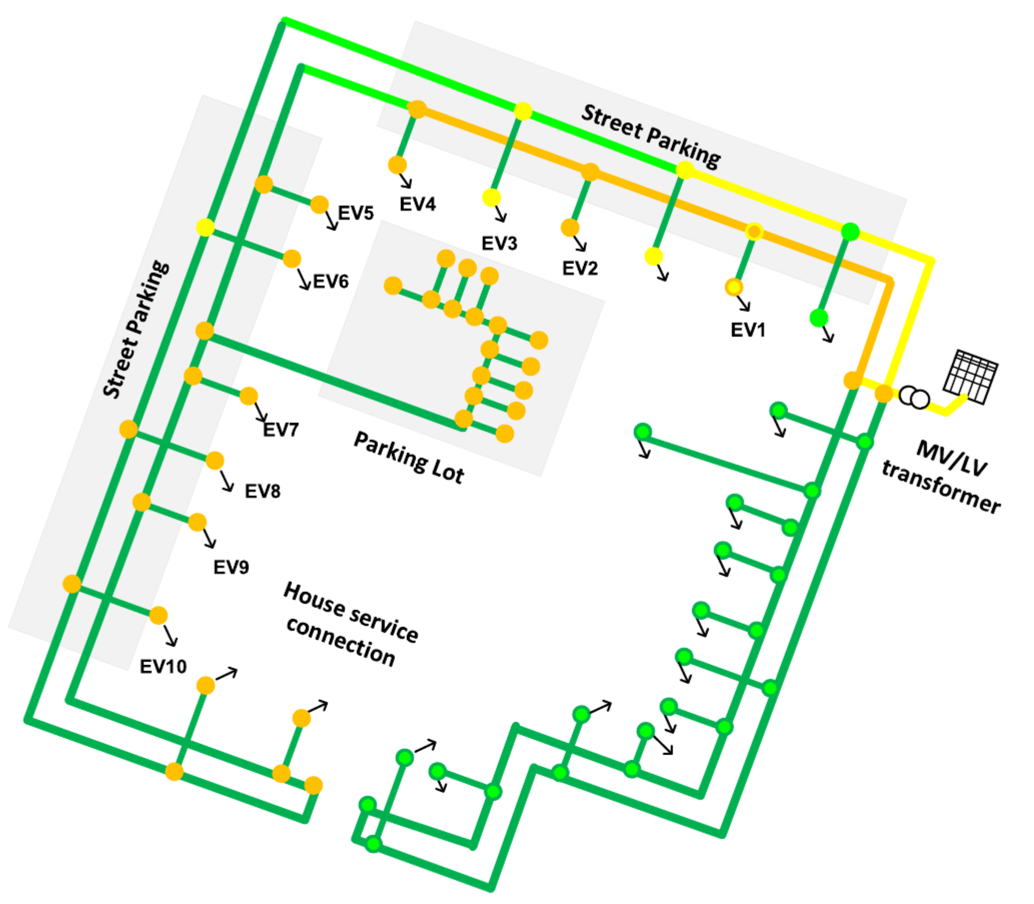

FIGURE 14. Distribution circuit loading for the distributed EV charging scenario.

EVs are plugged in during the night for charging.

Charging at multiple locations of the residential grid (a distribution network) at the same time causes voltage drops.

Such uncoordinated power consumption at the local scale can lead to grid problems.

Electric Vehicle Charging on Residential Distribution Systems: Impacts and Mitigation

Anamika Dubey and Surya Santoso (2015)

Context:

- ✓ Charge ***M*** vehicles (loads) during night-time from ***M*** charging stations (nodes).
  - Night-time = 6PM to 6AM.
- ✓ Charge them fully before the next morning.
  - They aren't leaving before 6AM, so no hard constraints on the available charging timings themselves.

$$V_{var}\left(t, \boldsymbol{Q}(t), \boldsymbol{P}(t)\right) = \sum_{i=1}^{M} \left(1 - V_i\left(t, \boldsymbol{Q}(t), \boldsymbol{P}(t)\right)\right) \quad (1)$$

$$J = \int_0^T V_{var}\left(t, \boldsymbol{Q}(t), \boldsymbol{P}(t)\right) dt \quad (2)$$

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$$\min\left(J\right)=\min\left(\int_0^T V_{var}\left(t,\boldsymbol{Q}\left(t\right),\boldsymbol{P}\left(t\right)\right)dt\right) \qquad (3)$$

Subject to

$$\dot{Q}_i\left(t\right)=P_i\left(t\right) \quad \forall i\in\{1,\ldots M\}$$

$$0\le P_i\left(t\right)\le P_i \quad \forall i\in\{1,\ldots M\} \qquad (4)$$

$$Q_i\left(0\right)=Q_{i,0} \quad \forall i\in\{1,\ldots M\}$$
$$Q_i\left(T\right)=E_i \quad \forall i\in\{1,\ldots M\} \qquad (5)$$

Electric Vehicle Charging on Residential Distribution Systems: Impacts and Mitigation
Anamika Dubey and Surya Santoso

- Objective:
  - minimize impact on residential grid voltage, for the whole night.
  - Design objective (constraint): Charge every EV fully by time *T.*

The problem formulation is given as follows:

$$f_t\left(\boldsymbol{Q}_t\right) = \min\left(V_{var}\left(t, \boldsymbol{Q}_t, \boldsymbol{P}_t\right) + f_{t+1}\left(\boldsymbol{Q}_{t+1}\right)\right) \quad t = 1, 2 \ldots T \tag{7}$$

Subject to

$$\boldsymbol{Q}_t = \boldsymbol{Q}_{t+1} - \boldsymbol{P}_t \Delta t \tag{8}$$

$$Q_i^0 \le Q_{t,i} \le E_i \quad \forall i \in 1, \ldots M$$

$$P_{t,i} = \begin{cases} 0 \\ P_i/2 & \forall i \in 1, \ldots M \\ P_i \end{cases}$$

$$Q_{T,i} = E_i \quad \forall i \in 1, \ldots M \tag{9}$$

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- Objective:
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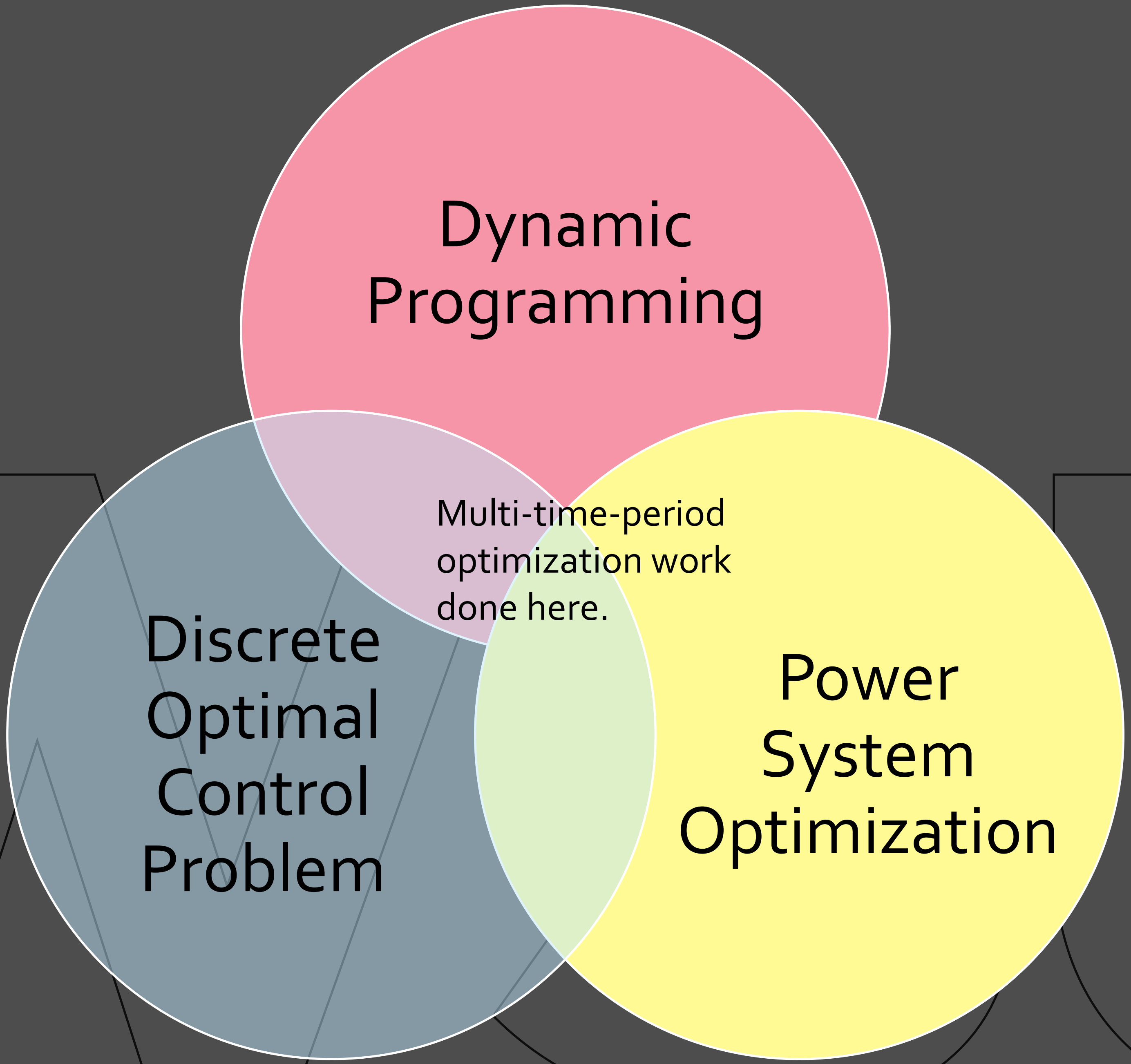
Design Formulation:

- Battery-charger power levels *P(t)* variable only in discrete-steps. (zero, half-of-max-capacity, max-capacity)
- *Power-flow-analysis also done in discrete time-steps.*
- *We're solving a discretized Optimal Control Problem.*
- *This is a 'harder' dynamic programming problem because of the boundary constraints.*
- *Dynamic Programming Successive Approximation Method? Need to study it in detail.*


Dynamic Programming
Multi-time-period optimization work done here.
Discrete Optimal Control Problem
Power System Optimization

Dynamic Programming

Bellman showed that a dynamic optimization problem in discrete time can be stated in a recursive, step-by-step form known as backward induction by writing down the relationship between the value function in one period and the value function in the next period. The relationship between these two value functions is called the "Bellman equation".

Richard E. Bellman

Article Talk

From Wikipedia, the free encyclopedia

Richard Ernest Bellman[3] (August 26, 1920 – March 19, 1984) was an American applied mathematician, who introduced dynamic programming in 1953, and made important contributions in other fields of mathematics, such as biomathematics. He founded the leading biomathematical journal Mathematical Biosciences.

```
const fib = (n) => {
  if (n <= 2) return 1;
  return fib(n - 1) + fib(n - 2);
};
```

Fibonacci Sequence using Dynamic Programming

coderbyte

```
const fib = (n) => {
  if (n <= 2) return 1;
  return fib(n - 1) + fib(n - 2);
};
```

fib(7) → 13

```javascript
const fib = (n) => {
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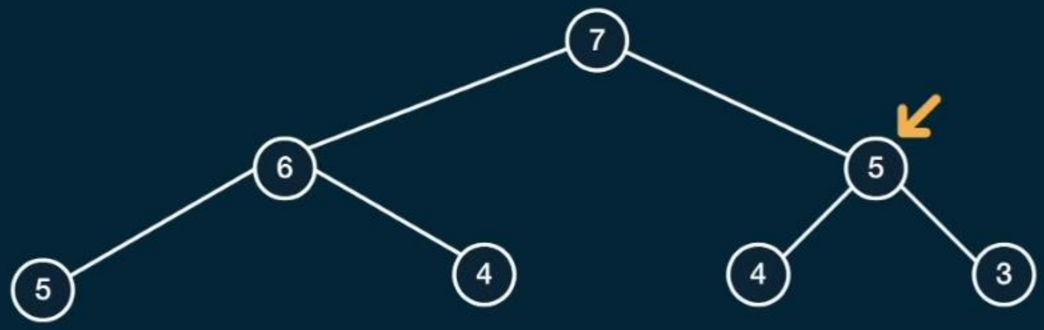

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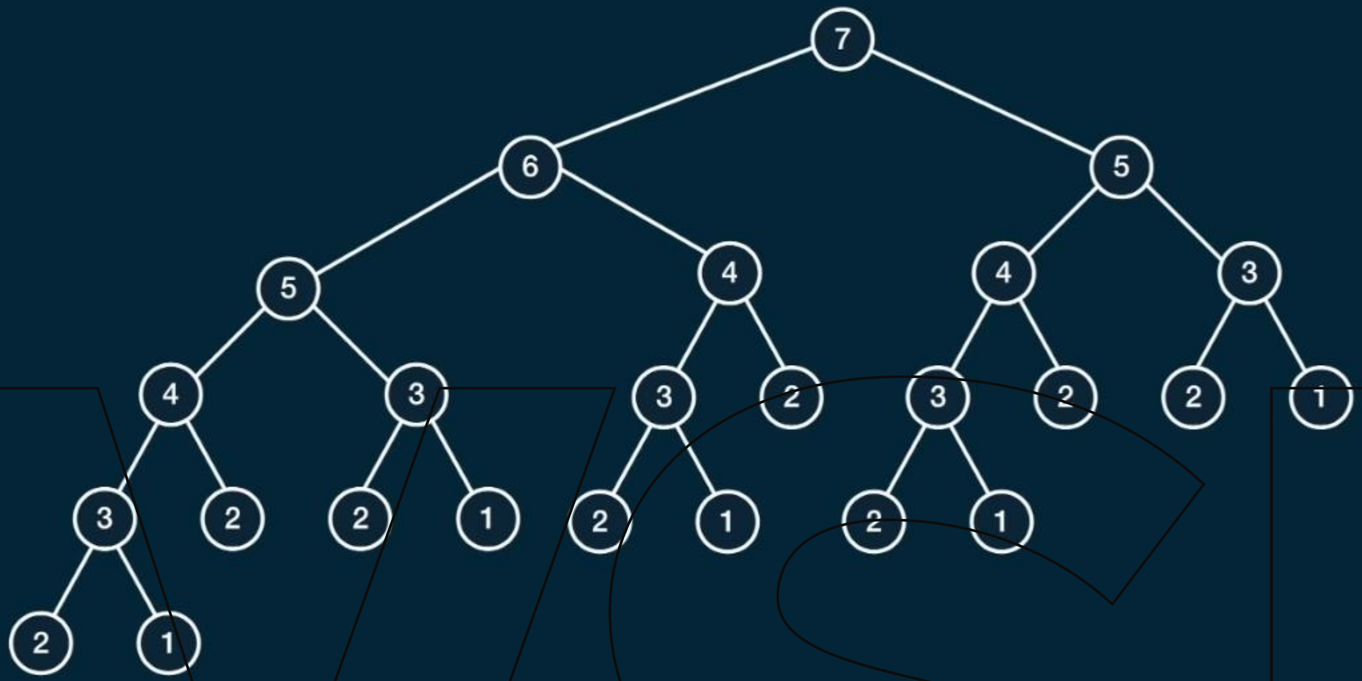

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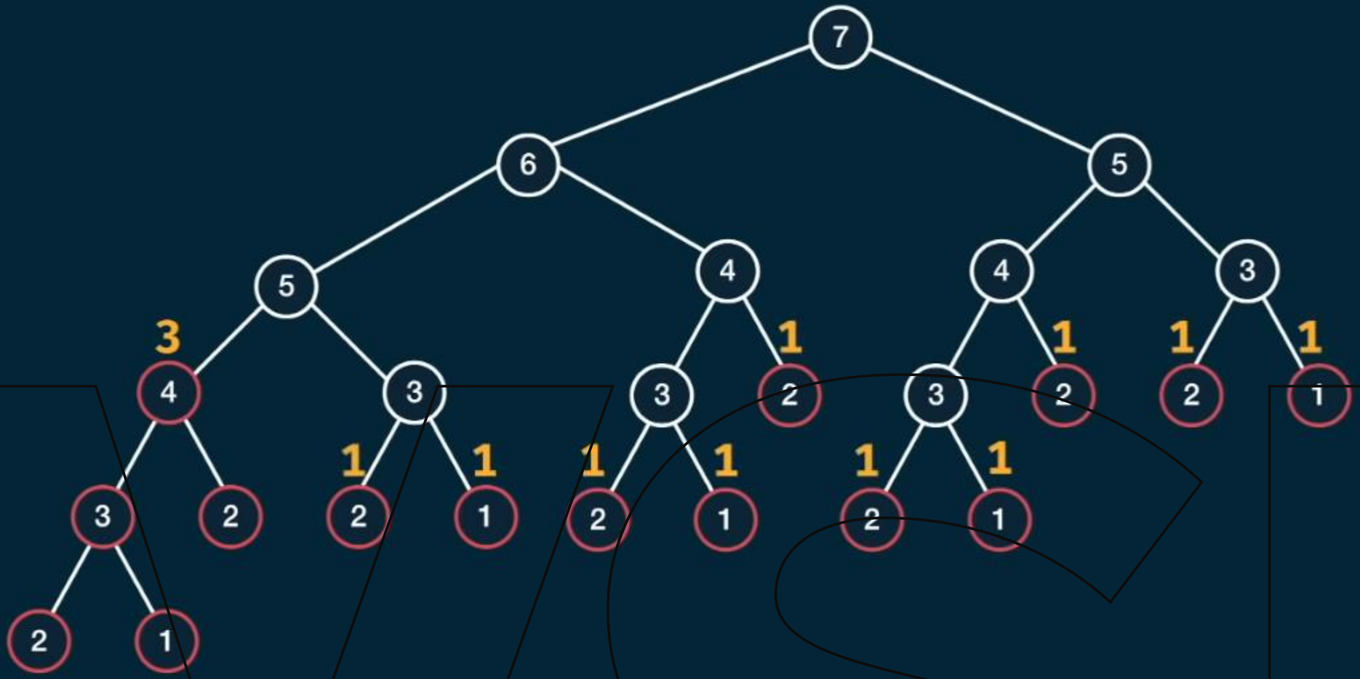

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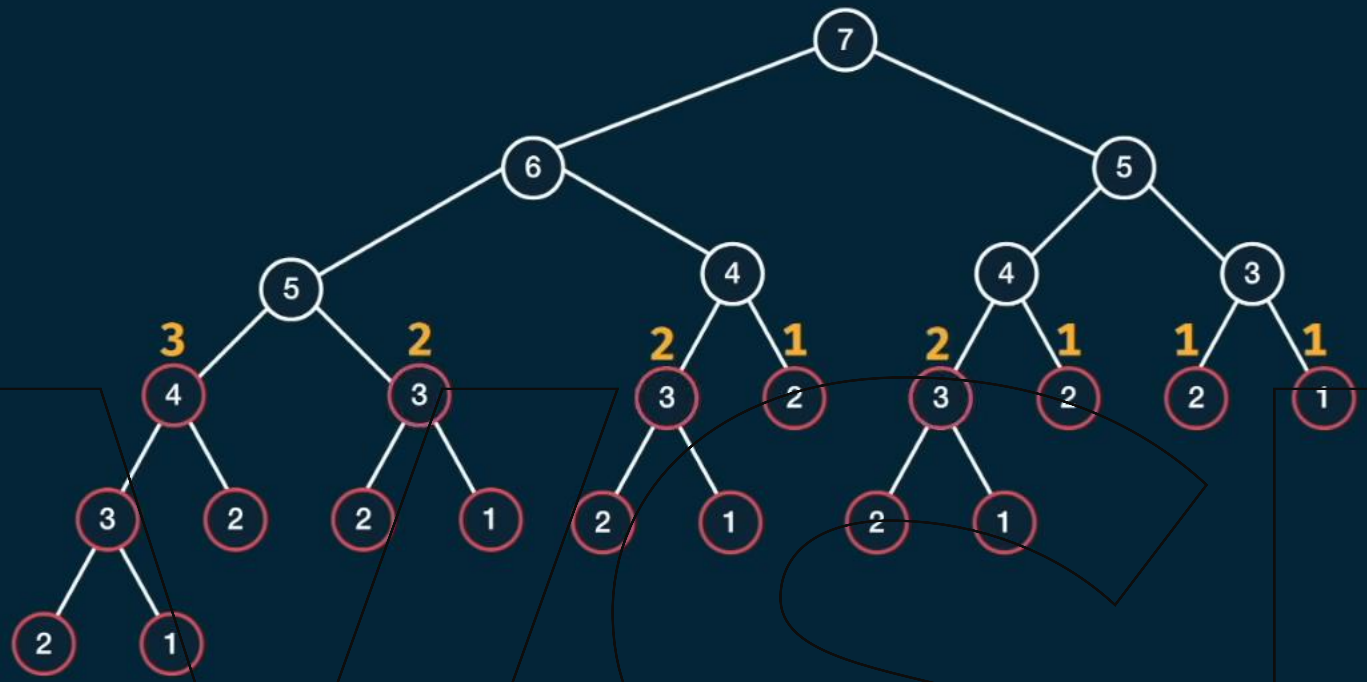

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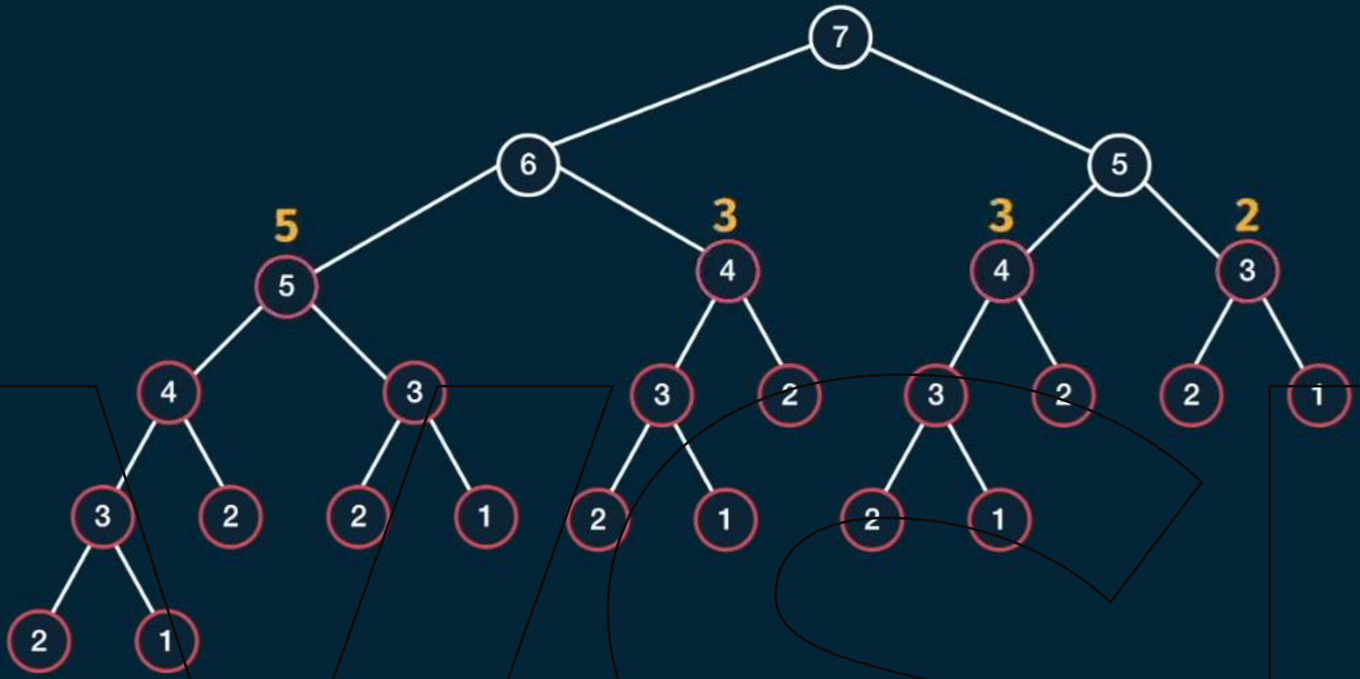

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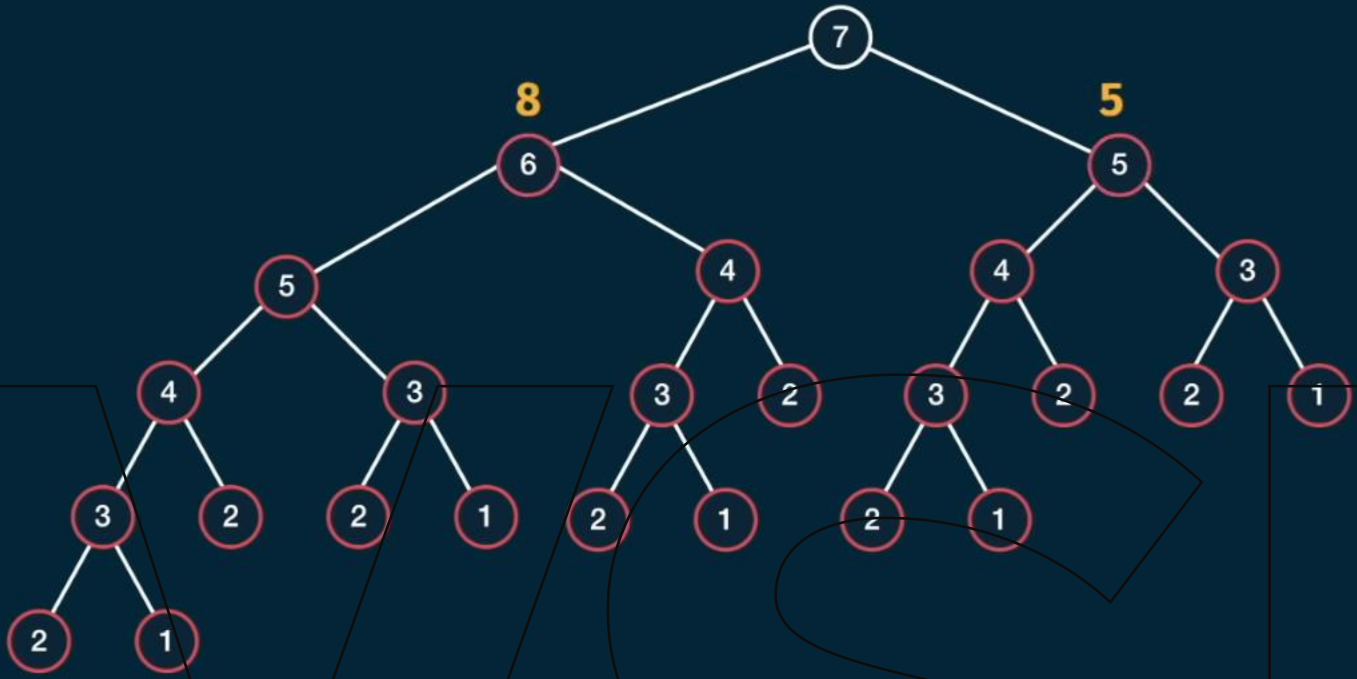

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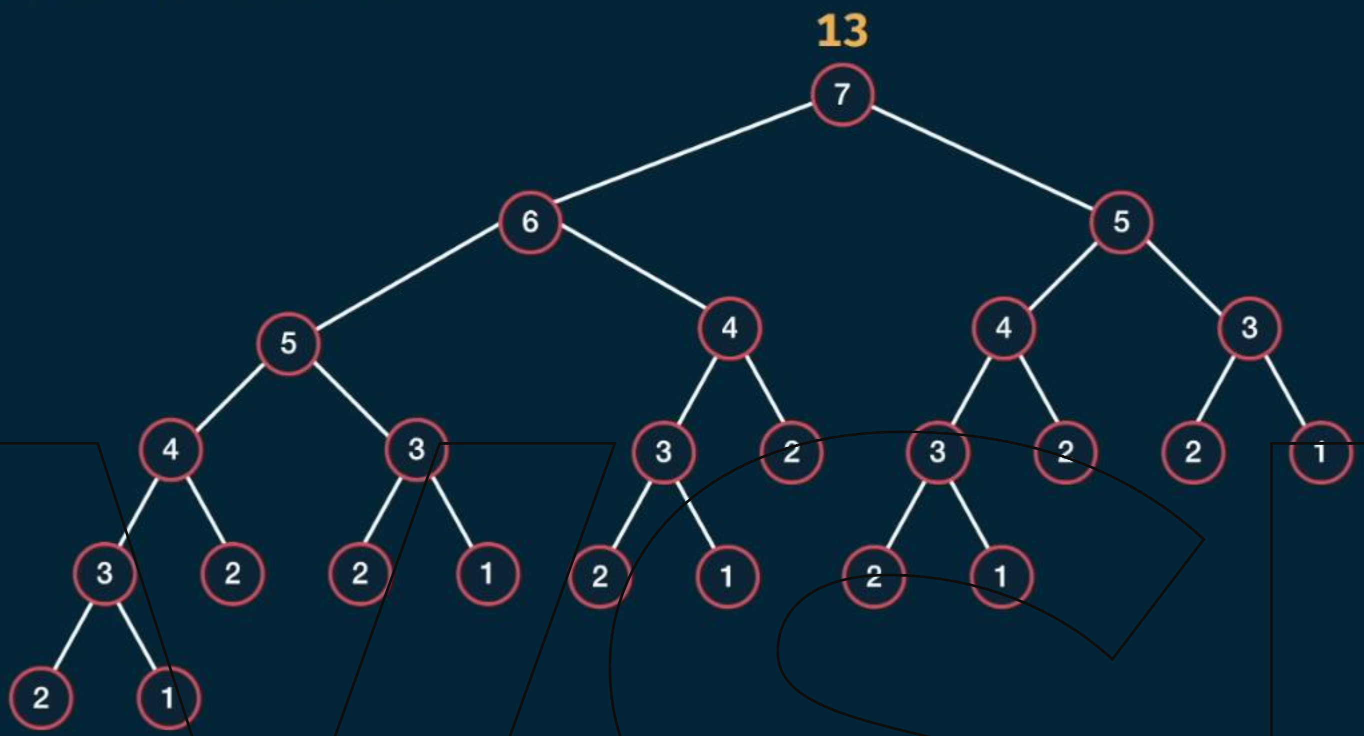
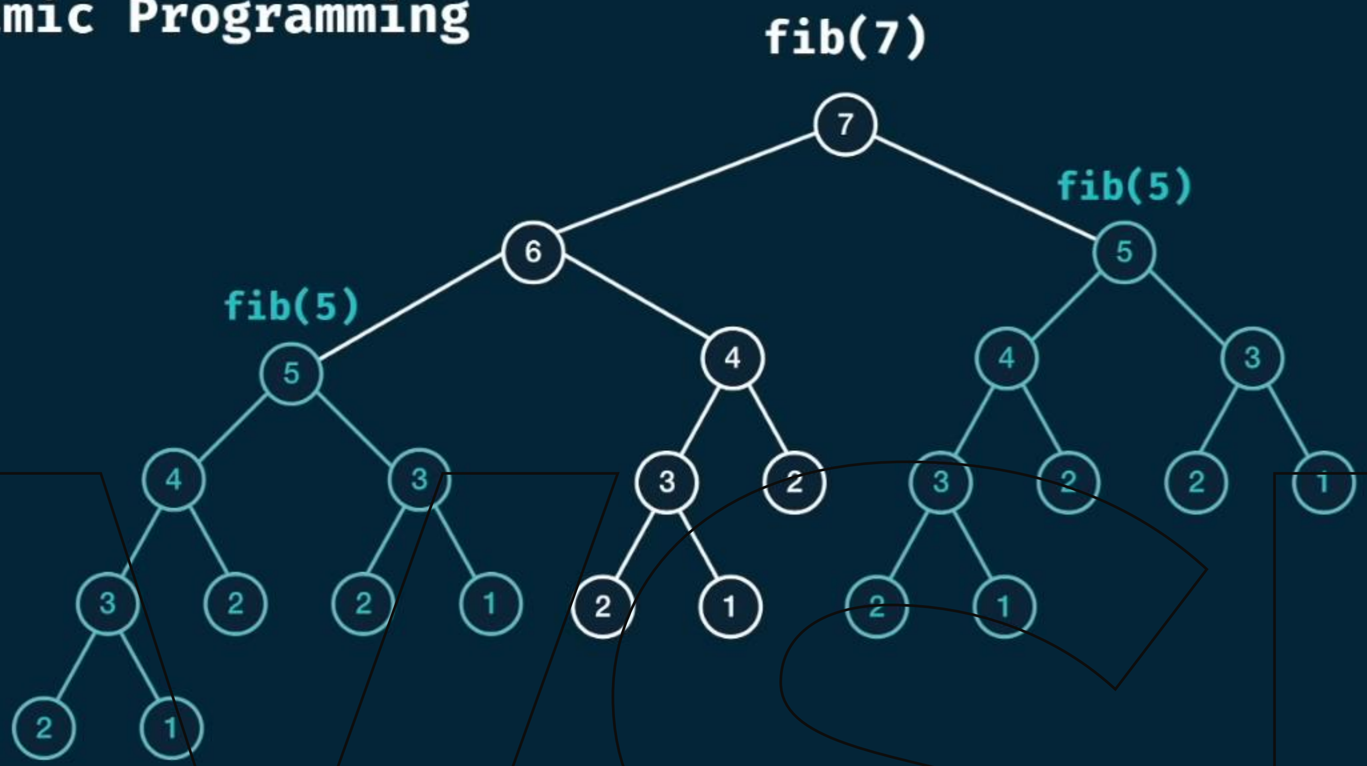
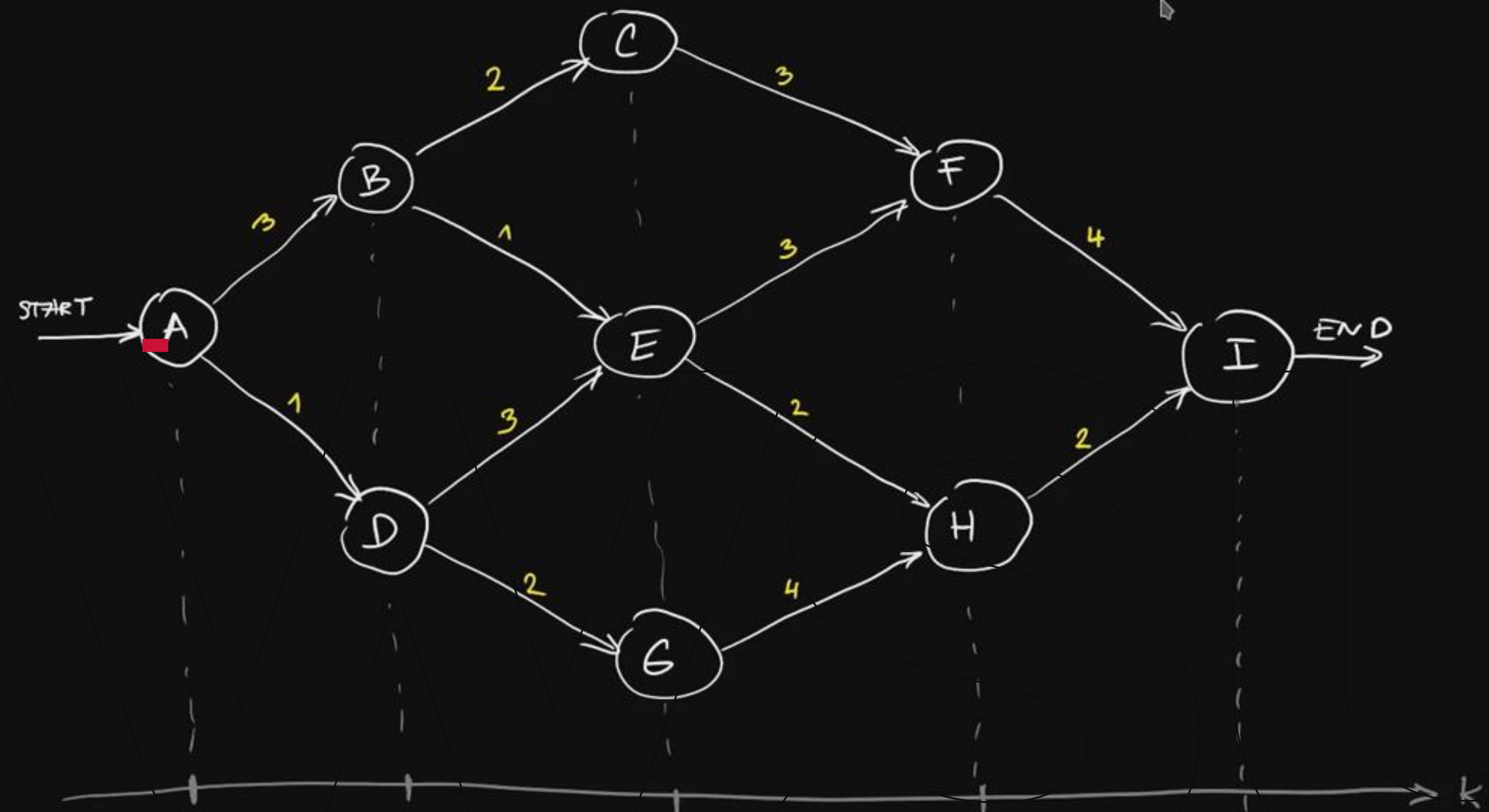


Dynamic Programming

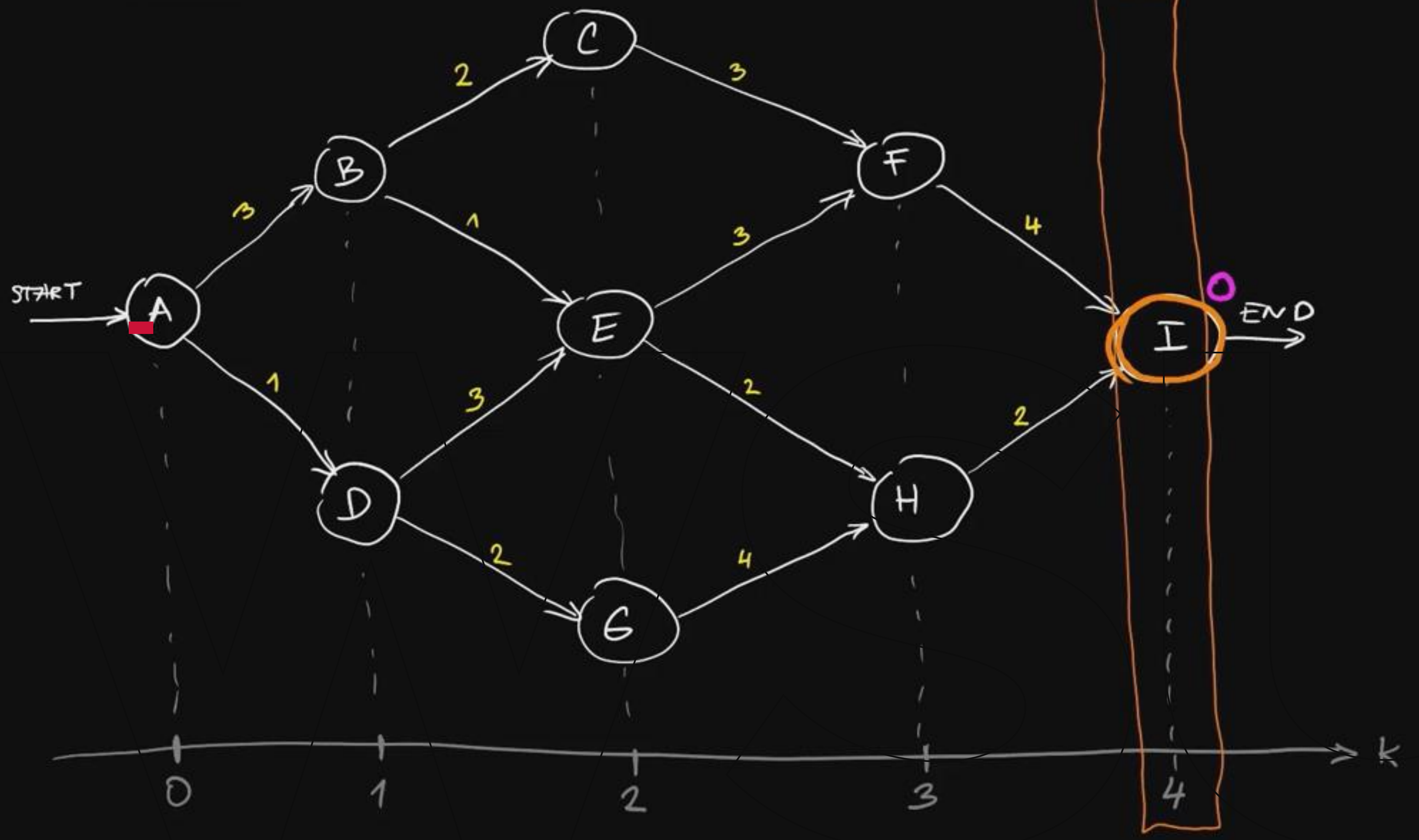


: shortest path through a graph

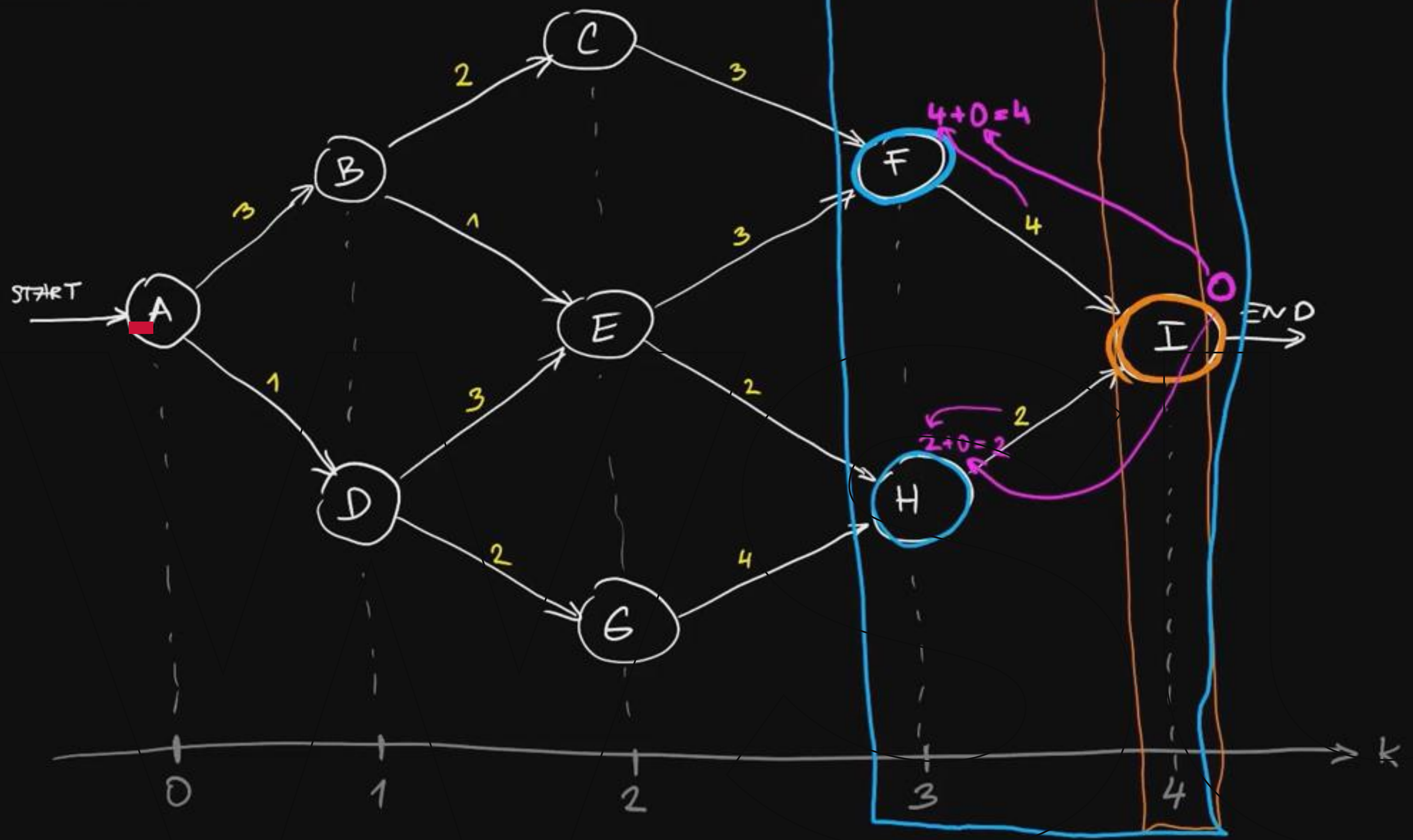


Shortest Path Problem using Dynamic Programming

: shortest path through a graph



: shortest path through a graph



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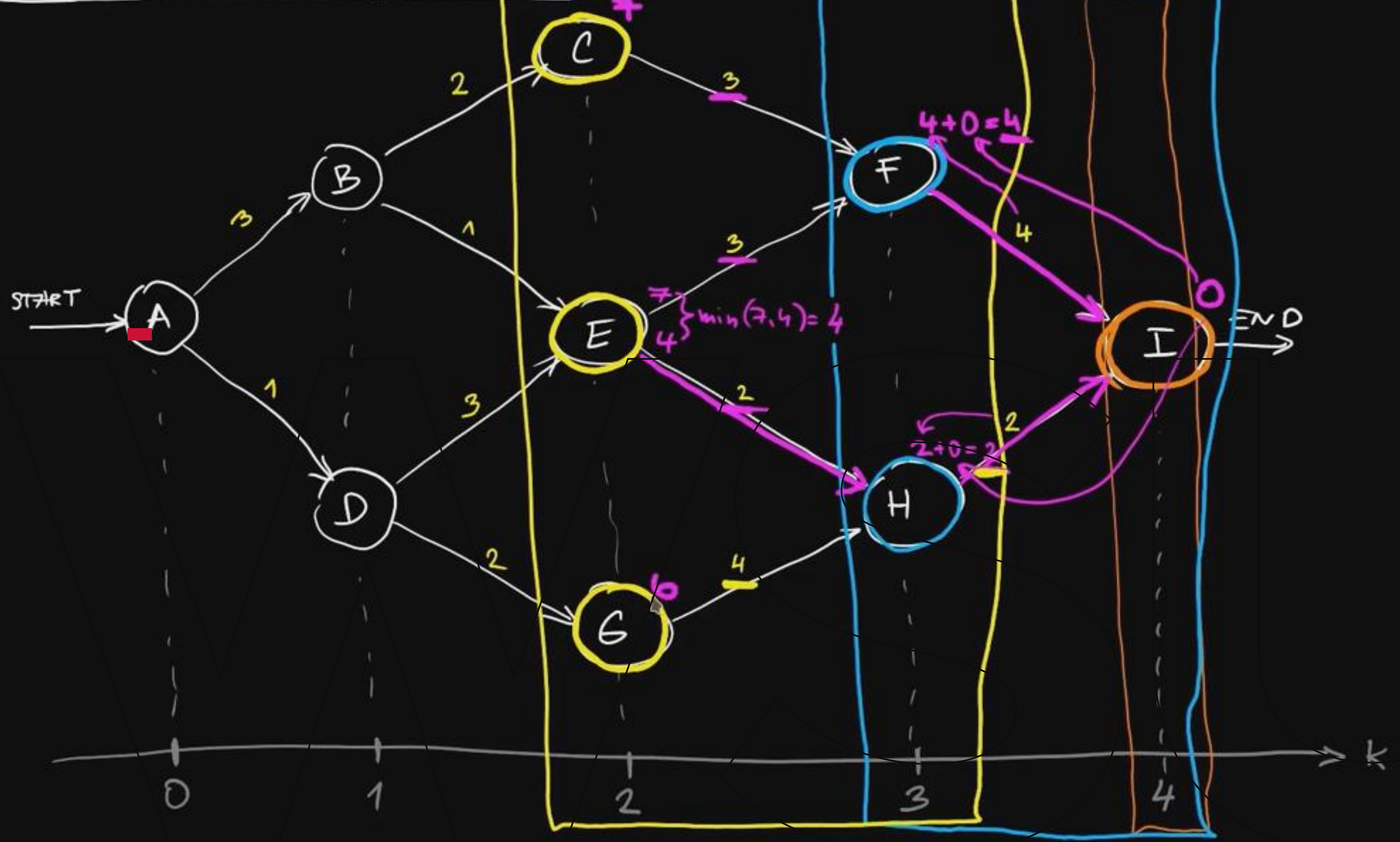

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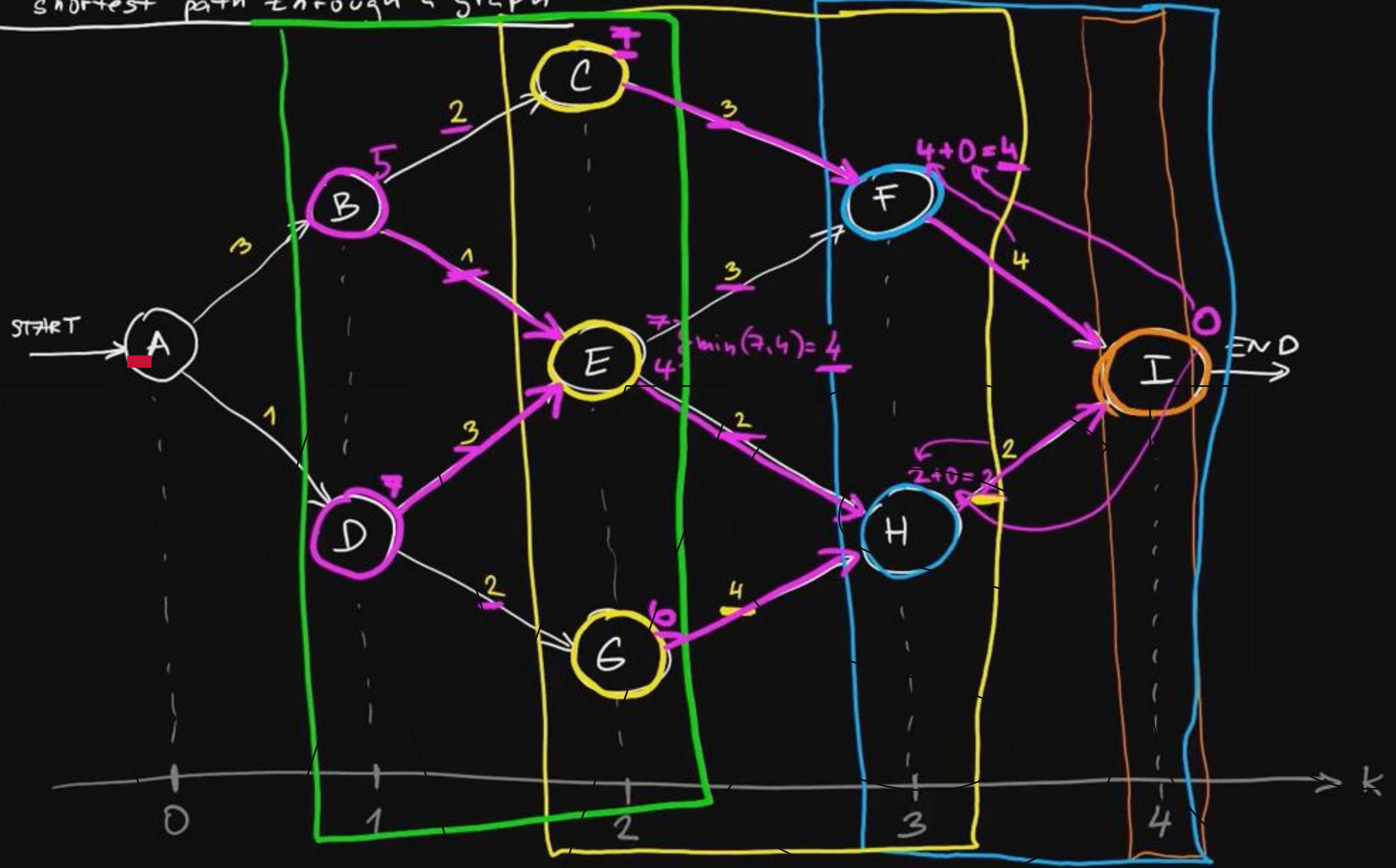

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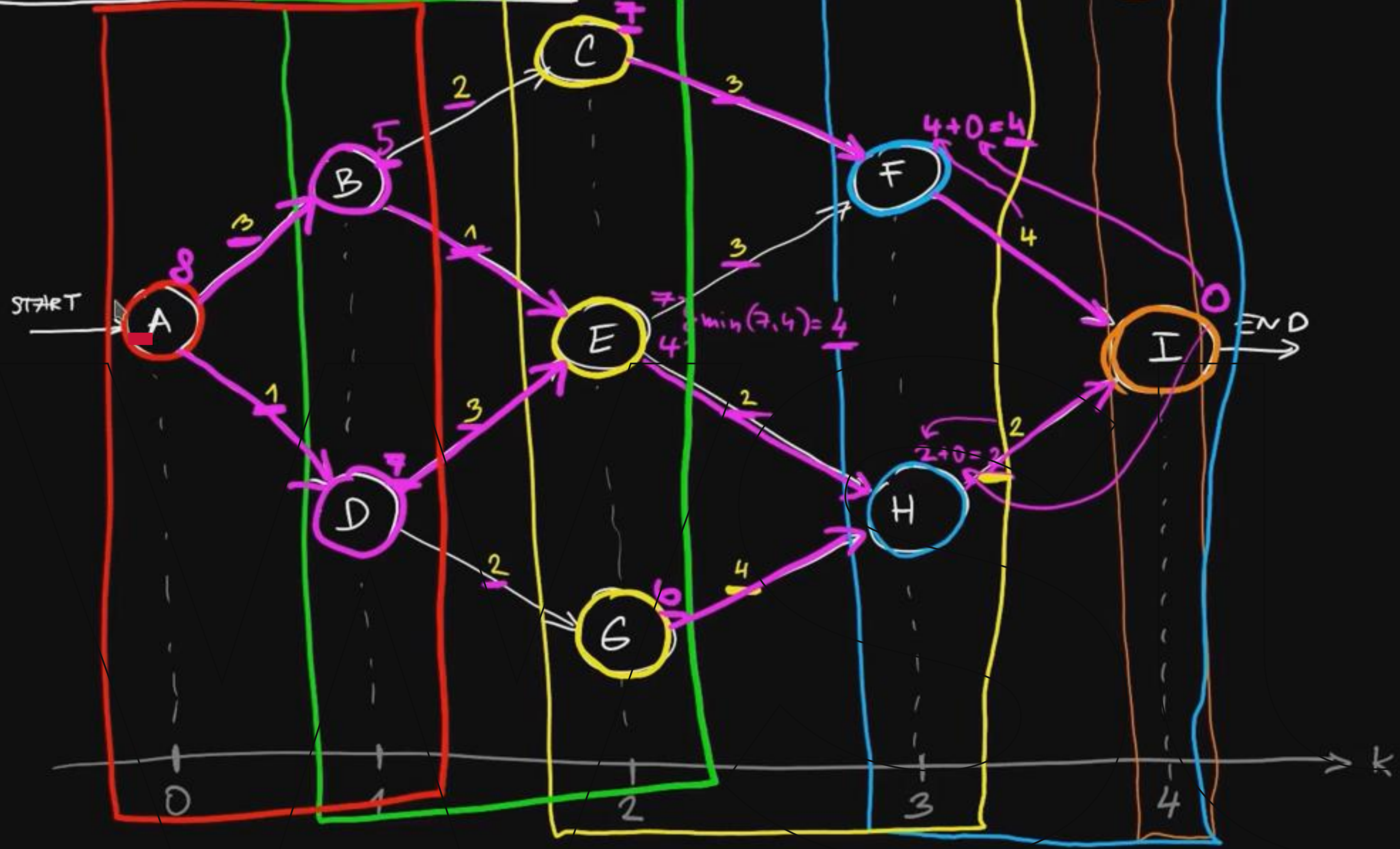

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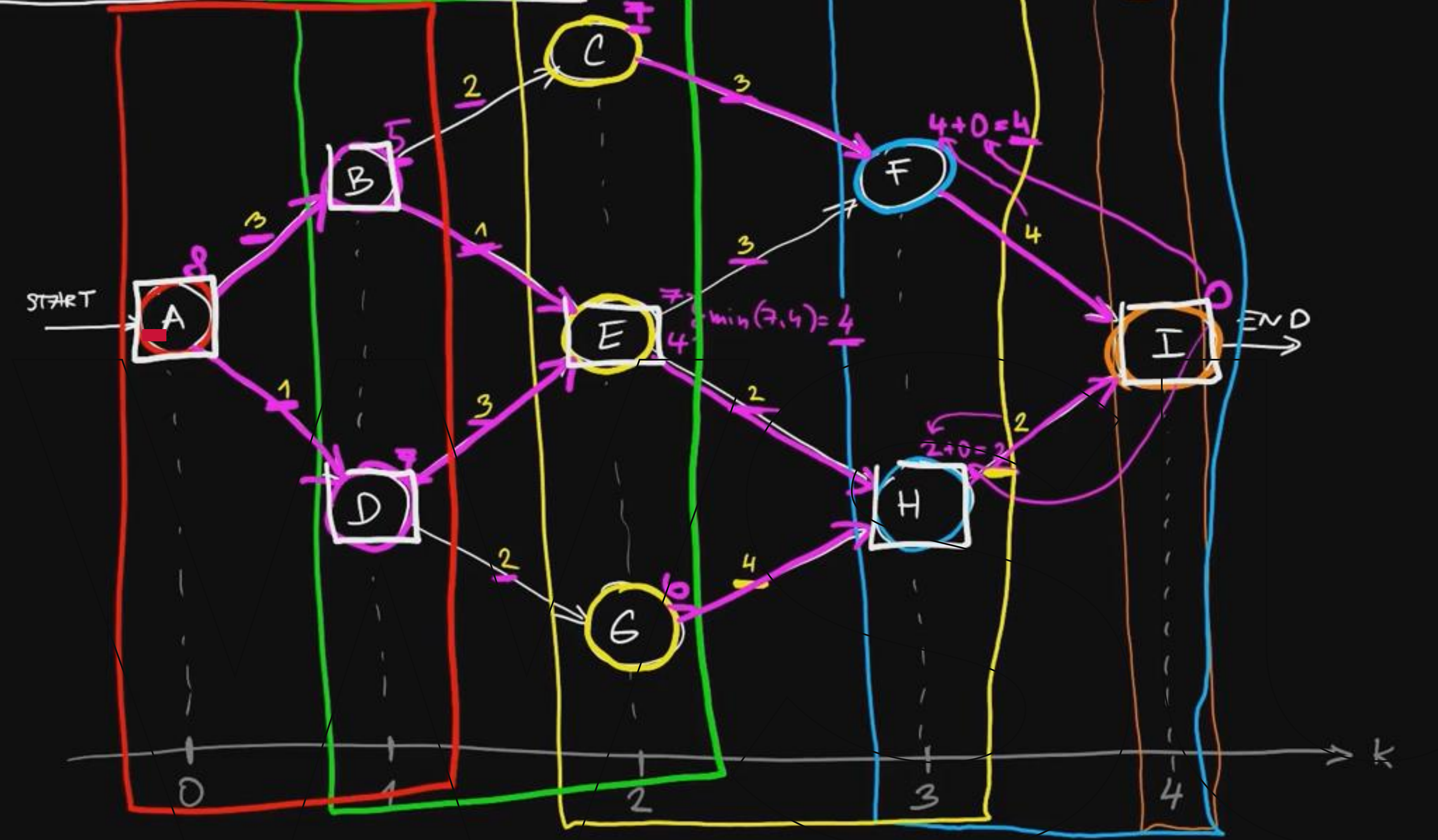

Do we even need a dynamic programming formulation in Power Systems?

Instead of using dynamic programming and optimization on

- N state variables (including memory elements like Battery SOC)
- C constraints
- - All for T time steps

Why not just form:

- N*T state variables (including Battery SOC)
- C*T constraints plus T additional constraints to maintain *continuity* for those memory elements. (Let's assume just one battery in our whole grid)
- All into one big 'steady-state-like' Optimization problem and plug it into a solver?

Do we even need a dynamic programming formulation in Power Systems?

Instead of using dynamic programming and optimization, why not just incorporate more state variables, more constraints (both by a factor of the number of time-steps: T) into one big 'steady-state-like' Optimization problem and plug it into a solver?

I do not have an answer for this now, I'd imagine that the time complexity will increase exponentially, but that's just my speculation.

References (not in order)

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7. *Johan Driesen et al (KU Leuven, 2009)* - The Impact of Charging Plug-In Hybrid Electric Vehicles on a Residential Distribution Grid