Branch Flow Model: Relaxations and Convexification

Aryan Ritwajeet Jha

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Contents

1 Branch Flow Model: Relaxations and Convexification

Table 1: Table describing the Branch Flow Model equations.

Equation #	Equation	Unknowns	Knowns	No. of Equations
13	$p_j = \Sigma P_{jk} + \Sigma(P_{ij} - r_{ij}l_{ij}) + g_j v_j$	$1 \times p_0$ $m \times P_{ij}$ $m \times l_{ij}$ $n \times v_j$	$n \times p_j$ $m \times r_{ij}$ $(n+1) \times g_j$ $1 \times v_0$	$(n+1)$
14	$q_j = \Sigma Q_{jk} + \Sigma(Q_{ij} - x_{ij}l_{ij}) + b_j v_j$	$1 \times q_0$ $m \times Q_{ij}$ $m \times l_{ij}$ $n \times v_j$	$n \times q_j$ $m \times x_{ij}$ $(n+1) \times b_j$ $1 \times v_0$	$(n+1)$
15	$v_j = v_i + (r_{ij}^2 + x_{ij}^2)l_{ij} - 2(r_{ij}P_{ij} + x_{ij}Q_{ij})$	$m \times P_{ij}$ $m \times Q_{ij}$ $m \times l_{ij}$ $n \times v_j$	$b \times r_{ij}$ $m \times x_{ij}$ $1 \times v_0$	m
16	$l_{ij} = \frac{P_{ij}^2 + Q_{ij}^2}{v_j}$	$m \times P_{ij}$ $m \times Q_{ij}$ $m \times l_{ij}$ $n \times v_j$	$1 \times v_0$	m
13 to 16		$1 \times p_0$ $1 \times q_0$ $m \times P_{ij}$ $m \times Q_{ij}$ $m \times l_{ij}$ $n \times v_j$	$n \times p_j$ $n \times q_j$ $m \times r_{ij}$ $m \times x_{ij}$ $(n+1) \times g_j$ $(n+1) \times b_j$ $1 \times v_0$	$2(n+1+m)$
		$2(n+1+m)$	$4n+2m+3$	$2(n+1+m)$

2 Radial Distribution Load Flow Using Conic Programming

All equations are for power flow from bus j to k:

$$P_{jk} = G_{jk}|V_j|^2 - G_{jk}|V_j||V_k|cos(\theta_{jk}) + B_{jk}|V_j||V_k|sin(\theta_{jk}) \quad (1)$$
$$Q_{jk} = B_{jk}|V_j|^2 - B_{jk}|V_j||V_k|cos(\theta_{jk}) - G_{jk}|V_j||V_k|sin(\theta_{jk}) \quad (2)$$

$$u_j = \frac{|V_j|^2}{\sqrt{2}}$$

$$R_{jk} = |V_j||V_k|cos(\theta_{jk})$$
$$j_{jk} = |V_j||V_k|sin(\theta_{jk})$$

$$P_{jk} = \sqrt{2}G_{jk}u_j - G_{jk}R_{jk} + B_{jk}I_{jk} \quad (3)$$
$$Q_{jk} = \sqrt{2}B_{jk}u_j - B_{jk}R_{jk} + G_{jk}I_{jk} \quad (4)$$
$$R_{jk}^2 + I_{jk}^2 = 2u_ju_k \quad (5)$$

Equation #	Equation	Unknowns	Knowns	No. of Equations
6	$p_{Lj} = -\sqrt{2}u_j\Sigma G_{jk} + \Sigma(G_{jk}R_{jk} - B_{jk}I_{jk})$	$1 \times p_0$ $m \times R_{jk}$ $m \times I_{jk}$ $n \times u_j$	$n \times p_{Lj}$ $m \times G_{jk}$ $m \times B_{jk}$ $1 \times u_0$	$(n+1)$
7	$q_{Lj} = \sqrt{2}u_j\Sigma B_{jk} + \Sigma(B_{jk}R_{jk} + G_{jk}I_{jk})$	$1 \times q_0$ $m \times R_{jk}$ $m \times I_{jk}$ $n \times u_j$	$n \times q_{Lj}$ $m \times G_{jk}$ $m \times B_{jk}$ $1 \times u_0$	$(n+1)$
5	$R_{jk}^2 + I_{jk}^2 = 2u_ju_k$	$m \times R_{jk}$ $m \times I_{jk}$ $n \times u_j$	-	m
5 to 7		$1 \times p_0$ $1 \times q_0$ $m \times R_{jk}$ $m \times I_{jk}$ $n \times u_j$	$n \times p_{Lj}$ $n \times q_{Lj}$ $m \times G_{jk}$ $m \times B_{jk}$ $1 \times u_0$	$2(n+1)+m$
		$n+2m+2$	$2n+2m+2$	$2(n+1)+m$
	Assuming radial distribution with $n = m$	$3n+2$	$4n+2$	$3n+2$

2.1 Conic Programming Formulation

Maximize ΣR_{jk} subject to

$$
\begin{aligned}
p_{Lj} &= -\sqrt{2}u_j \Sigma G_{jk} + \Sigma(G_{jk}R_{jk} - B_{jk}I_{jk}) && \text{for all } n+1 \text{ buses.}\\
q_{Lj} &= \sqrt{2}u_j \Sigma B_{jk} + \Sigma(B_{jk}R_{jk} + G_{jk}I_{jk}) && \text{for all } n+1 \text{ buses.}\\
R_{jk}^2 + I_{jk}^2 &<= 2u_j u_k && \text{for all } m \text{ lines.}\\
u_0 & && \text{known.}\\
p_{Lj}, q_{Lj} & && \text{known for all non-slack buses.}\\
R_{jk} &>= 0 && \text{for all } m \text{ lines.}
\end{aligned}
$$

3 To do next:

Specify the sigma limits (member elements of a summation.)

Maybe check out "On Implementing a Primal-Dual Interior-Point Method for Conic Quadratic Optimization"