$$
\begin{aligned}
J(x_0, 0) = &\min_{\substack{u_j \in U_j \\ j=0,1\ldots N-1}} \mathbb{E}\left[\sum_{j=0}^{N-1} \{L(x_j, u_j, j)\} + G(x_N, N)\right] \\
&s.t. \\
&x_{k+1} = f(x_j, u_j, j) + \sigma_{j+1}\xi_{j+1} \\
&j = 0, 1, \ldots N-1
\end{aligned} \tag{1}
$$

$$
J(x_N, N) = G(x_N, N) \tag{2}
$$

$$
J(x_k, k) = \min_{u_k \in U_k} \mathbb{E}\left[J(x_{k+1}, k+1) + L(x_k, u_k, k)\right] \tag{3}
$$

$$
\begin{aligned}
\bar{x}_{k+1} &= f(\bar{x}_k, \bar{u}_k, k) \\
\bar{x}_0 &= x_0
\end{aligned} \tag{4}
$$

$$
\begin{aligned}
f(x_k, u_k, k) &= f(\bar{x}_k, \bar{u}_k, k) + f_x \delta x_k + f_u \delta u_k \\
&\quad + \frac{1}{2} f_{xx} (\delta x_k)^2 + \frac{1}{2} f_{uu} (\delta u_k)^2 \\
&where \\
&\delta x_k = x_k - \bar{x}_k \\
&\delta u_k = u_k - \bar{u}_k
\end{aligned} \tag{5}
$$

$$
\begin{aligned}
\delta x_{k+1} &= f(x_k, u_k, k) + \sigma_{k+1}\xi_{k+1} - f(\bar{x}_k, \bar{u}_k, k) \\
\text{or, } \delta x_{k+1} &= f_x \delta x_k + f_u \delta u_k + \frac{1}{2} f_{xu} (\delta x_k)(\delta u_k) \\
&\quad + \frac{1}{2} f_{xx} (\delta x_k)^2 + \frac{1}{2} f_{uu} (\delta u_k)^2 + \sigma_{k+1}\xi_{k+1}
\end{aligned} \tag{6}
$$

Similarly,

$$
\begin{aligned}
L(x_k, u_k, k) &= L(\bar{x}_k, \bar{u}_k, k) + L_x \delta x_k + L_u \delta u_k \\
&\quad + \frac{1}{2} L_{xx} (\delta x_k)^2 + \frac{1}{2} L_{uu} (\delta u_k)^2
\end{aligned} \tag{7}
$$

and,

$$
J(x_k, k) = J(\bar{x}_k, k) + \delta x + \frac{1}{2} J_{xx} (\delta x_{k+1})^2 \tag{8}
$$