$$J(x_0, 0) = \min_{\substack{u_j \in U_j \\ j=0,1\ldots N-1}} \mathbb{E}\left[\sum_{j=0}^{N-1}\{L(x_j, u_j, j)\} + G(x_N, N)\right]$$

$$s.t. \tag{1}$$

$$x_{k+1} = f(x_j, u_j, j) + \sigma_{j+1}\xi_{j+1}$$
$$j = 0, 1, \ldots N-1$$

$$J(x_N, N) = G(x_N, N) \tag{2}$$

$$J(x_k, k) = \min_{u_k \in U_k} \mathbb{E}\left[J(x_{k+1}, k+1) + L(x_k, u_k, k)\right] \tag{3}$$

$$\bar{x}_{k+1} = f(\bar{x}_k, \bar{u}_k, k) \tag{4}$$
$$\bar{x}_0 = x_0$$

$$f(x_k, u_k, k) = f(\bar{x}_k, \bar{u}_k, k) + f_x\delta x_k + f_u\delta u_k \tag{5}$$
$$+ \frac{1}{2}f_{xx}(\delta x_k)^2 + \frac{1}{2}f_{uu}(\delta u_k)^2$$

$$where$$
$$\delta x_k = x_k - \bar{x}_k$$
$$\delta u_k = u_k - \bar{u}_k$$

$$\delta x_{k+1} = f(x_k, u_k, k) + \sigma_{k+1}\xi_{k+1} - f(\bar{x}_k, \bar{u}_k, k)$$
$$or,\ \delta x_{k+1} = f_x\delta x_k + f_u\delta u_k + \frac{1}{2}f_{xu}(\delta x_k)(\delta u_k) \tag{6}$$
$$+ \frac{1}{2}f_{xx}(\delta x_k)^2 + \frac{1}{2}f_{uu}(\delta u_k)^2 + \sigma_{k+1}\xi_{k+1}$$

Similarly,

$$L(x_k, u_k, k) = L(\bar{x}_k, \bar{u}_k, k) + L_x\delta x_k + L_u\delta u_k$$
$$+ \frac{1}{2}L_{xx}(\delta x_k)^2 + \frac{1}{2}L_{uu}(\delta u_k)^2 \tag{7}$$

and,

$$J(x_k, k) = J(\bar{x}_k, k) + J_x\delta x_k + \frac{1}{2}J_{xx}(\delta x_{k+1})^2 \tag{8}$$