Problem 6

A simple dartboard is shown below:

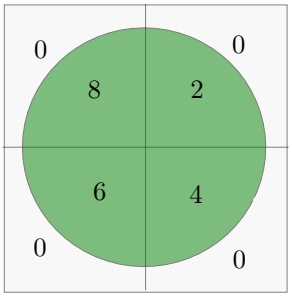

Figure 1: A dart board with given scores for hitting different regions.

Your favourite EE 507 instructor throws a dart at the board. He hits the board with probability 0.7 and misses the board with probability 0.3. Given that he hit the board, he is equally likely to hit any point on it. He receives a score of 0 if he misses the board, and receives the score shown if he hits the board.

a. What are the outcomes of this experiment? How many are there?

b. What are the events in this experiment? How many are there?

c. What is the probability that your instructor's score is more than 3?

d. Let's say we repeated the experiment twice, independently. What is the probability that the score is greater than 3 on the first try, and less than or equal to 3 on the second try?
Also what is the probability that the two scores are different?

Solution

Let the experiment of the instructor throwing the dart on the board be called E.

a. The outcomes can be listed as $\Omega = \{A_0, A_2, A_4, A_6, A_8\}$ where A_i refers to scoring i points with the dart. A_0 represents the dart missing the board and scoring zero points.
In total there are 5 outcomes for this experiment.

b. An event is a subset of the power set of the set of outcomes (Ω) of the experiment.

The power set of Ω is $\wp$:

$$\begin{aligned}\wp = \{&\{\phi\}, \{A_0\}, \{A_2\}, \cdots \{A_8\},\\ &\{A_0, A_2\}, \{A_0, A_4\} \cdots \{A_6, A_8\},\\ &\cdots \{A_0, A_2, A_4, A_6, A_8\}\}\end{aligned}$$

Say, a subset $\{A_0, A_4\}$ of $\wp$ represents the event {The dart misses the board OR The dart hits the board in the region with 4 points}.
There are $2^5 = 32$ events for this experiment.

c. $P(\text{Score} > 3) = P(\{A_4, A_6, A_8\})$.
Outcomes are always disjoint events, so we can use Axiom 3.
$P(\{A_4, A_6, A_8\}) = P(A_4) + P(A_6) + P(A_8) = 3 * 0.7/4 = 0.525$.

d. Repeated trials are independent trials. We can simply multiply the individual probabilities to obtain the final probability.
$P(\text{Score} > 3$ on the first trial AND Score < 3 on the second trials$) = P(\text{Score} > 3) * P(\text{Score} \leq 3) = 0.525(1 - 0.525) = 0.249375$

$P(\text{Both Scores Different}) = 1 - P(\text{Both Scores Same})$
$= 1 - P(\{\{A_0A_0\}, \{A_2A_2\}, \{A_4A_4\}, \{A_6A_6\}, \{A_8A_8\}\})$
$= 1 - [0.3^2 + 4 * \{\frac{0.7}{4}\}^2] = 0.7875$
