MULTI-PERIOD OPTIMAL POWER FLOW

By

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The members of the Committee appointed to examine the dissertation of ARYAN RITWAJEET JHA find it satisfactory and recommend that it be accepted.

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TBA

MULTI-PERIOD OPTIMAL POWER FLOW

Abstract

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TBA

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

Dedication

TBA

Chapter One

SOME FORMATTING EXAMPLES

1.1 Chapter one tittle section

Chapter Two

FIGURES AND TABLES

2.1 Examples of a figure

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APPENDIX

Appendix A

Branch Flow Model: Relaxations and Convexification

In [2] the authors came up the Relaxed Branch Flow Model, and showed that in the case of Tree/Radial networks, the Relaxed Model can solve for the unique optimal solution, including the bus angles, and in the case of weakly meshed networks, there is a mechanism for extracting the bus angles from the relaxed solution, to find out its unique solution, if it exists.

Legend for Table A.2:

TABLE A.1 Table describing the variables involved in the Branch Flow Model equations.

Symbol	Meaning
p_j, q_j	Real, Reactive Power flowing from bus j into the network.
P_{ij}, Q_{ij}	Real, Reactive Power flowing in branch $i \to j$ (sending-end).
I_{ij}, l_{ij}	Complex Current flowing in branch

Table A.2 Table describing the Branch Flow Model equations.

Equation #	Equation	Unknowns	Knowns	No. of Equations
13	$p_j = \Sigma P_{jk} + \Sigma(P_{ij} - r_{ij}l_{ij}) + g_j v_j$	$1 \times p_0$ $m \times P_{ij}$ $m \times l_{ij}$ $n \times v_j$	$n \times p_j$ $m \times r_{ij}$ $(n+1) \times g_j$ $1 \times v_0$	$(n+1)$
14	$q_j = \Sigma Q_{jk} + \Sigma(Q_{ij} - x_{ij}l_{ij}) + b_j v_j$	$1 \times q_0$ $m \times Q_{ij}$ $m \times l_{ij}$ $n \times v_j$	$n \times q_j$ $m \times x_{ij}$ $(n+1) \times b_j$ $1 \times v_0$	$(n+1)$
15	$v_j = v_i + (r_{ij}^2 + x_{ij}^2)l_{ij} - 2(r_{ij}P_{ij} + x_{ij}Q_{ij})$	$m \times P_{ij}$ $m \times Q_{ij}$ $m \times l_{ij}$ $n \times v_j$	$b \times r_{ij}$ $m \times x_{ij}$ $1 \times v_0$	m
16	$l_{ij} = \frac{P_{ij}^2 + Q_{ij}^2}{v_j}$	$m \times P_{ij}$ $m \times Q_{ij}$ $m \times l_{ij}$ $n \times v_j$	$1 \times v_0$	m
13 to 16		$1 \times p_0$ $1 \times q_0$ $m \times P_{ij}$ $m \times Q_{ij}$ $m \times l_{ij}$ $n \times v_j$	$n \times p_j$ $n \times q_j$ $m \times r_{ij}$ $m \times x_{ij}$ $(n+1) \times g_j$ $(n+1) \times b_j$ $1 \times v_0$	$2(n+1+m)$
		$2(n+1+m)$	$4n+2m+3$	$2(n+1+m)$

Appendix B

Abstracts: Optimization-based Methods for solving MP-OPF

In [4], the authors use a two-step paradigm for solving the MP-OPF problem, by first solving for a more relaxed SOCP problem for the all of the time-steps in a horizon, and using the SOC values from its solution, solve for the NLP OPF problem for every time-step in parallel.

In [3], they prove that for 'realistic' systems, appending an additional 'complementarity' cost function to the original objective function, Simultaneous Charging and Discharging (SCD) in the optimal solution is avoided, and that the Mixed-Integer SOCP problem of AC-OPF with energy storage can be relaxed into a regular SOCP problem without violating the battery physics in the optimal solution.

Appendix C

Abstracts: Dynamic Programming Methods for solving MP-OPF

In [1], the authors use a Differential Dynamic Programming approach, which involved usage of Forward and Backward passes made over a sequence of time-steps, doing a back-and-forth between computation for one time-step, say t, by making some assumptions on any variables required from the next time-step $t+1$, and then updating the assumed values at t, once new values for the $t+1$ time-step have been made.

Appendix D

Abstracts: Differential Dynamic Programming

In [5], the authors lay out the framework on how Differential Dynamic Programming can be utilized to solve a Mutli-Stage Uncertain Optimal Control problem programmatically.

Appendix E

Models: Battery Model for Mutli-Period OPF

Table E.1 Description of Grid Parameters

Variable	Description
$\mathcal{G}$	Set of all nodes
$\mathcal{L}$	
$\mathcal{T}$	
$\cdot\sqcup$	

In [3, 4], the batteries are modelled using four state/control variables, which are:

Table E.2 Description of Battery Variables

Variable	Description	Dimension	Dimension pu
$B_{n,k}$	State of Charge (SOC) of Battery	$[kWh]$	$[pu\,h]$
$P^c_{n,k}$	Average Charging Power of the Battery during the k-th time interval.	$[kW]$	$[pu]$
$P^d_{n,k}$	Average Discharging Power of the Battery during the k-th time interval.	$[kW]$	$[pu]$
$q_{B_{n,k}}$	Average Reactive Power Output from the Battery Inverter during the k-th time interval.	$[kVAr]$	$[pu]$

$k \in \{0, 1, 2, \ldots, T/\Delta t\}$ The index of the discretized time intervals, where k represents the k-th time interval of duration Δt within the continuous time range $[0, T]$. The corresponding continuous time interval is $[(k-1)\Delta t, k\Delta t]$.

$n \in \mathcal{G}$ The node n is an element of the set of all nodes in the power grid $\mathcal{G}$.