Branch Flow Model: Relaxations and Convexification

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1 Branch Flow Model: Relaxations and Convexification

Equation #	Equation	Unknowns	Knowns	No. of Equations
13	$p_j = \Sigma P_{jk} + \Sigma(P_{ij} - r_{ij} l_{ij}) + g_j v_j$	$1 \times p_0$ $m \times P_{ij}$ $m \times l_{ij}$ $n \times v_j$	$n \times p_j$ $m \times r_{ij}$ $(n+1) \times g_j$ $1 \times v_0$	$(n+1)$
14	$q_j = \Sigma Q_{jk} + \Sigma(Q_{ij} - x_{ij} l_{ij}) + b_j v_j$	$1 \times q_0$ $m \times Q_{ij}$ $m \times l_{ij}$ $n \times v_j$	$n \times q_j$ $m \times x_{ij}$ $(n+1) \times b_j$ $1 \times v_0$	$(n+1)$
15	$v_j = v_i + (r_{ij}^2 + x_{ij}^2) l_{ij} - 2(r_{ij} P_{ij} + x_{ij} Q_{ij})$	$m \times P_{ij}$ $m \times Q_{ij}$ $m \times l_{ij}$ $n \times v_j$	$b \times r_{ij}$ $m \times x_{ij}$ $1 \times v_0$	m
16	$l_{ij} = \frac{P_{ij}^2 + Q_{ij}^2}{v_j}$	$m \times P_{ij}$ $m \times Q_{ij}$ $m \times l_{ij}$ $n \times v_j$	$1 \times v_0$	m
13 to 16		$1 \times p_0$ $1 \times q_0$ $m \times P_{ij}$ $m \times Q_{ij}$ $m \times l_{ij}$ $n \times v_j$	$n \times p_j$ $n \times q_j$ $m \times r_{ij}$ $m \times x_{ij}$ $(n+1) \times g_j$ $(n+1) \times b_j$ $1 \times v_0$	$2(n+1+m)$
		$2(n+1+m)$	$4n+2m+3$	

2 Radial Distribution Load Flow Using Conic Programming

All equations are for power flow from bus i to k:

$$P_{ik} = G_{ik}|V_i|^2 - G_{ik}|V_i||V_k|cos(\theta_{ik}) + B_{ik}|V_i||V_k|sin(\theta_{ik}) \quad (1)$$
$$Q_{ik} = B_{ik}|V_i|^2 - B_{ik}|V_i||V_k|cos(\theta_{ik}) - G_{ik}|V_i||V_k|sin(\theta_{ik}) \quad (2)$$

$$u_i = \frac{|V_i|^2}{\sqrt{2}}$$

$$R_{ik} = |V_i||V_k|cos(\theta_{ik})$$
$$I_{ik} = |V_i||V_k|sin(\theta_{ik})$$

$$P_{ik} = \sqrt{2}G_{ik}u_i - G_{ik}R_{ik} + B_{ik}I_{ik} \quad (3)$$
$$Q_{ik} = \sqrt{2}B_{ik}u_i - B_{ik}R_{ik} + G_{ik}I_{ik} \quad (4)$$
$$R_{ik}^2 + I_{ik}^2 = 2u_iu_k \quad (5)$$

Ignore Table below for now.

Equation #	Equation	Unknowns	Knowns	No. of Equations
6	$p_{Li} = \Sigma G_{jk} + \Sigma(P_{ij} - r_{ij}l_{ij}) + g_jv_j$	$1 \times p_0$ $m \times P_{ij}$ $m \times l_{ij}$ $n \times v_j$	$n \times p_j$ $m \times r_{ij}$ $(n+1) \times g_j$ $1 \times v_0$	$(n+1)$
7	$q_j = \Sigma Q_{jk} + \Sigma(Q_{ij} - x_{ij}l_{ij}) + b_jv_j$	$1 \times q_0$ $m \times Q_{ij}$ $m \times l_{ij}$ $n \times v_j$	$n \times q_j$ $m \times x_{ij}$ $(n+1) \times b_j$ $1 \times v_0$	$(n+1)$
5	$v_j = v_i + (r_{ij}^2 + x_{ij}^2)l_{ij} - 2(r_{ij}P_{ij} + x_{ij}Q_{ij})$	$m \times P_{ij}$ $m \times Q_{ij}$ $m \times l_{ij}$ $n \times v_j$	$b \times r_{ij}$ $m \times x_{ij}$ $1 \times v_0$	m
5 to 7		$1 \times p_0$ $1 \times q_0$ $m \times P_{ij}$ $m \times Q_{ij}$ $m \times l_{ij}$ $n \times v_j$	$n \times p_j$ $n \times q_j$ $m \times r_{ij}$ $m \times x_{ij}$ $(n+1) \times g_j$ $(n+1) \times b_j$ $1 \times v_0$	$2(n+1+m)$
		$2(n+1+m)$	$4n+2m+3$	