A Complex LASSO-Approach for Localizing Forced Oscillations in Power Systems

A brief reiteration

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Introduction

Natural Oscillations	Forced Oscillations
natural	forced
naturla	forced
natural	forced

Theory

State Space Representation

Express Power System Dynamics in State Space:

$$\begin{aligned} \underset{n\times 1}{\dot{\mathrm{x}}(t)} &= \underset{n\times n}{\mathrm{A}}\,\underset{n\times 1}{\mathrm{x}(t)} + \underset{n\times m}{\mathrm{B}}\,\underset{m\times 1}{\mathrm{u}(t)} \\ \underset{p\times 1}{\mathrm{y}(t)} &= \underset{p\times n}{\mathrm{C}}\,\underset{n\times 1}{\mathrm{x}(t)} \\ &\forall t \geq 0 \end{aligned} \tag{1}$$

$\underset{n\times 1}{\mathrm{x}(t)}$: internal state variables + controller variables vector

$\underset{m\times 1}{\mathrm{u}(t)}$: forced oscillation vector

Forced Oscillation Vector

Express Forced Oscillations based on locations of origin and signal composition:

$$\underset{m\times 1}{\mathbf{u}(t)} = \underset{m\times 1}{\begin{bmatrix} u_1(t) \\ u_2(t) \\ \vdots \\ u_m(t) \end{bmatrix}} = \underset{m\times 1}{\begin{bmatrix} \sum_{l=1}^{M_1} a_{1,l} \sin(\omega_{2,l} t + \phi_{1,l}) \\ \sum_{l=1}^{M_2} a_{2,l} \sin(\omega_{2,l} t + \phi_{2,l}) \\ \vdots \\ \sum_{l=1}^{M_m} a_{m,l} \sin(\omega_{m,l} t + \phi_{m,l}) \end{bmatrix}} \tag{2}$$

$a_{r,l} \geq 0$

$\omega_{r,l} = 2\pi f \geq 0$

(r, l) refer to the l^{th} sinusoid at the r^{th} location

Sparsity Assumption

Let $\boldsymbol{a_r}(t)$ be the vector representation of the sinusoids at the r^{th} location.

$$\underset{1\times M_r}{\boldsymbol{a_r}(t)} = \underset{1\times M_r}{\begin{bmatrix} a_{r,1}\sin(\omega_{r,1}t+\phi_{r,1}) & \dots & a_{r,M_r}\sin(\omega_{r,M_r}t+\phi_{r,M_r}) \end{bmatrix}}$$

Assumption 1: Both $\boldsymbol{u}(t)$ and $\boldsymbol{a_r}(t)$ are sparse vectors. That implies most locations are FO free and for the locations that do have FO, the number of distinct sinusoidal components of the local FO is small.

$$||\boldsymbol{u}(t)||_0 << m$$
$$||\boldsymbol{a_r}(t)||_0 << M_r$$

Discrete State Space Representation

Most PMUs sample at $30Hz$ or $60Hz$. So the sampling time period is $T = 1/30s$ or $1/60s$.

Let $\underset{n\times n}{\boldsymbol{A_d}} = \exp{(AT)}$ and $\underset{n\times m}{\boldsymbol{B_d}} = \{\int_0^T \exp{(A(T-s))}ds\}\boldsymbol{B}$.

Let $k = 0, 1, \ldots$ and define $\mathbf{x}[k] \triangleq \mathbf{x}(kT)$, $\mathbf{u}[k] \triangleq \mathbf{u}(kT)$ and $\mathbf{y}[k] \triangleq \mathbf{y}(kT)$. Suppose that $\mathbf{u}(t)$ is piece-wise constant during $kT \leq t \leq (k+1)T$. Then

$$\begin{aligned} \underset{n\times 1}{\boldsymbol{x}[k+1]} &= \underset{n\times n}{\boldsymbol{A_d}}\underset{n\times 1}{\boldsymbol{x}[k]} + \underset{n\times m}{\boldsymbol{B_d}}\underset{m\times 1}{\boldsymbol{u}[k]} \\ \underset{p\times 1}{\boldsymbol{y}[k]} &= \underset{p\times n}{\boldsymbol{C}}\underset{n\times 1}{\boldsymbol{x}[k]} \end{aligned} \tag{3}$$

We assume $\mathbf{x}[0] = 0$ as our focus is only on inputs triggered by FOs.

Transfer Function from Discrete State Space Representation

Let $\underset{p\times m}{H_z} = \underset{p\times n}{C}\,\underset{n\times n}{[zI - A_d]}^{-1}\underset{n\times m}{B_d}$.

From standard Linear System Analysis, we may conclude that
$\underset{p\times 1}{Y[z]} = \underset{p\times m}{H[z]}\underset{m\times 1}{U[z]}$
where $Y[z]$, $U[z]$ are the Z-Transforms of $y[n]$ and $u[n]$.
Replacing z with $\exp(j\Omega)$, where $\Omega \in (0, 2\pi)$, we get the DTFT representation for the system:

$$\underset{p\times 1}{Y[\Omega]} = \underset{p\times m}{H[\Omega]}\underset{m\times 1}{U[\Omega]} \tag{4}$$

Transfer Function of Forced Oscillation Vector

Applying DTFT on $\boldsymbol{u}(t)$ to get $\boldsymbol{U}[\Omega]$:

$$\underset{m\times 1}{\mathrm{U}(\Omega)} = \underset{m\times 1}{\begin{bmatrix} \sum_{l=1}^{M_1} a_{1,l}\{\exp(-j\phi_{1,l})\delta(\Omega+\omega_{1,l}) + \exp(j\phi_{1,l})\delta(\Omega-\omega_{1,l})\} \\ \sum_{l=1}^{M_2} a_{2,l}[\exp(-j\phi_{2,l})\delta(\Omega+\omega_{2,l}) + \{\exp(j\phi_{2,l})\delta(\Omega-\omega_{2,l})\} \\ \vdots \\ \sum_{l=1}^{M_m} a_{m,l}\{\exp(-j\phi_{m,l})\delta(\Omega+\omega_{m,l}) + \exp(j\phi_{m,l})\delta(\Omega-\omega_{m,l})\} \end{bmatrix}} \tag{5}$$

$\omega_{r,l}$ can be substituted by $\dfrac{1}{T}\tilde{\omega}_{r,l}$

Introduction to My Work

Transient vs Steady State Stability

Transient Stability	Steady State Stability
A sudden, out-of-trend, high magnitude change in a state variable(s) causes blackouts.	Accumulation of several seemingly minor trends in state variables over time, ultimately leading to a critical point where a small change could cause blackouts.
Chief parameters of concern are ROCOF, frequency nadir, steady-state frequency deviation.	Autocorrelation and covariance are some of the commonly used parameters for prognosis.
Inertia is a fundamental parameter here.	Inertia plays a minor role here.

Bifurcations and Critical Slowing Down

Bifurcation: A qualitative change in the 'motion' of a dynamical System due to a quantitative change in one of its parameters. Serious bifurcations, called Critical Bifurcations, cause the system to become unstable from stable.

Bifurcations and Critical Slowing Down

Critical Slowing Down: Dynamical Systems exhibit early statistical warning signs before collapsing:

- Increased recovery times from perturbations.
- Increased signal variance from the mean trajectory.
- Increased flicker and asymmetry in the signal

The above three properties can be identified by increasing variance and autocorrelation in time-series measurements taken from the system.

Procedure

Procedure

- Accessed a bunch of real-world frequency time-series data and plotted their:
    - bulk distribution (pdf)
    - auto-correlation curves

  in order to demonstrate that:
    - grid frequencies often exhibit significant deviation from gaussianity

  autocorrelation can be a reasonable indication of grid steady-state stability.
- Obtained explanation for the *signature dynamics* of each grid.
- Then implemented and tested the IEEE 9 Bus System in PSSE for symptoms of Critical Slowing Down as an Early Warning System.

Results

Results

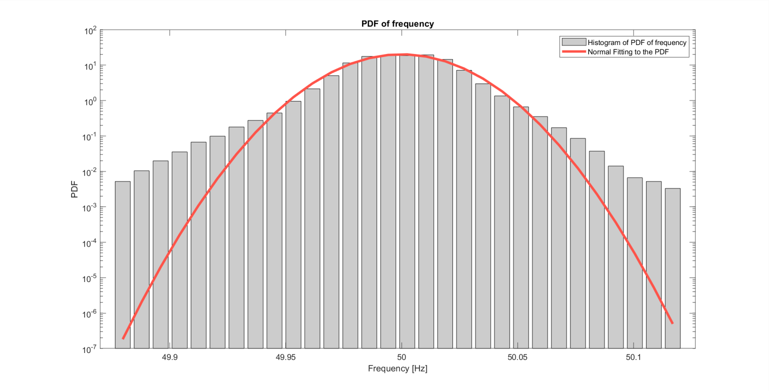

Figure 1: Continental European Grid frequency PDF: Heavier tails than a Gaussian Distribution.

Results

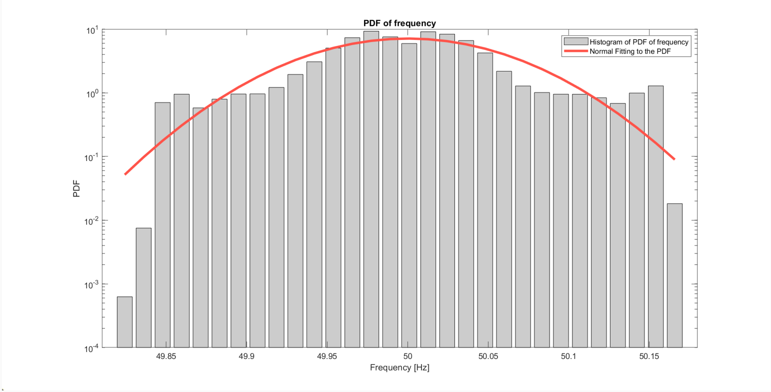

Figure 2: Mallorcan (an islanded Spanish grid) frequency pdf

Results

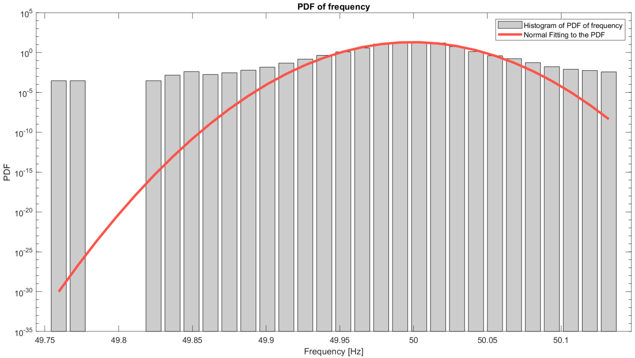

Figure 3: French grid frequency pdf including a blackout

Results

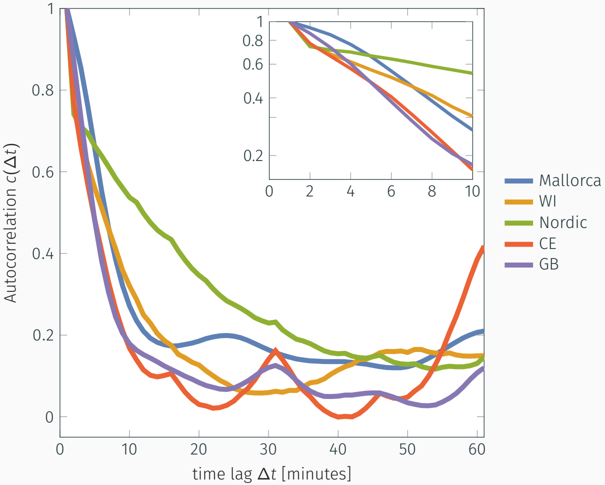

Figure 4: Autocorrelation decay of different synchronous regions.

Results

Table 1: Inverse-correlation values for different grids

Grid name	Inverse-correlation value T^{-1} [min^{-1}]
Mallorca	0.0654
Western Interconnection	0.0498
Nordic	0.0235
Continental Europe	0.0829
Great Britain	0.0879

Figure 5: Inverse correlation time is proportional to the damping constant of the grid.