

A Complex LASSO-Approach for Localizing Forced Oscillations in Power Systems

A brief reiteration

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Introduction

Forced Oscillations in Power Systems

Natural Oscillations	Forced Oscillations
natural	forced
natural	forced
natural	forced



Theory

State Space Representation

Express Power System Dynamics in State Space:

$$\begin{aligned}\dot{\mathbf{x}}(t) &= \underset{n \times 1}{\mathbf{A}} \underset{n \times n}{\mathbf{x}}(t) + \underset{n \times m}{\mathbf{B}} \underset{m \times 1}{\mathbf{u}}(t) \\ \mathbf{y}(t) &= \underset{p \times 1}{\mathbf{C}} \underset{p \times n}{\mathbf{x}}(t) \\ \forall t &\geq 0\end{aligned}\tag{1}$$

$\mathbf{x}(t)$: internal state variables + controller variables vector
 $n \times 1$

$\mathbf{u}(t)$: forced oscillation vector
 $m \times 1$



Forced Oscillation Vector

Express Forced Oscillations based on locations of origin and signal composition:

$$\underset{m \times 1}{\mathbf{u}(t)} = \underset{m \times 1}{\begin{bmatrix} u_1(t) \\ u_2(t) \\ \vdots \\ u_m(t) \end{bmatrix}} = \underset{m \times 1}{\begin{bmatrix} \sum_{l=1}^{M_1} a_{1,l} \sin(\omega_{2,l}t + \phi_{1,l}) \\ \sum_{l=1}^{M_2} a_{2,l} \sin(\omega_{2,l}t + \phi_{2,l}) \\ \vdots \\ \sum_{l=1}^{M_m} a_{m,l} \sin(\omega_{m,l}t + \phi_{m,l}) \end{bmatrix}} \quad (2)$$

$$a_{r,l} \geq 0$$

$$\omega_{r,l} = 2\pi f \geq 0$$

(r, l) refer to the l^{th} sinusoid at the r^{th} location



Sparsity Assumption

Let $\mathbf{a}_r(t)$ be the vector representation of the sinusoids at the r^{th} location.

$$\underset{1 \times M_r}{\mathbf{a}_r(t)} = \left[\underset{1 \times M_r}{a_{r,1} \sin(\omega_{r,1}t + \phi_{r,1})} \quad \dots \quad a_{r,M_r} \sin(\omega_{r,M_r}t + \phi_{r,M_r}) \right]$$

Assumption 1: Both $\mathbf{u}(t)$ and $\mathbf{a}_r(t)$ are sparse vectors. That implies most locations are FO free and for the locations that do have FO, the number of distinct sinusoidal components of the local FO is small.

$$\|\mathbf{u}(t)\|_0 \ll m$$

$$\|\mathbf{a}_r(t)\|_0 \ll M_r$$



Discrete State Space Representation

Most PMUs sample at 30Hz or 60Hz. So the sampling time period is $T = 1/30\text{s}$ or $1/60\text{s}$.

Let $\underset{n \times n}{A_d} = \exp(AT)$ and $\underset{n \times m}{B_d} = \{\int_0^T \exp(A(T-s))ds\}B$.

Let $k = 0, 1, \dots$ and define $\mathbf{x}[k] \triangleq \mathbf{x}(kT)$, $\mathbf{u}[k] \triangleq \mathbf{u}(kT)$ and $\mathbf{y}[k] \triangleq \mathbf{y}(kT)$. Suppose that $\mathbf{u}(t)$ is piece-wise constant during $kT \leq t \leq (k+1)T$. Then

$$\begin{aligned} \underset{n \times 1}{\mathbf{x}[k+1]} &= \underset{n \times n}{A_d} \underset{n \times 1}{\mathbf{x}[k]} + \underset{n \times m}{B_d} \underset{m \times 1}{\mathbf{u}[k]} \\ \underset{p \times 1}{\mathbf{y}[k]} &= \underset{p \times n}{C} \underset{n \times 1}{\mathbf{x}[k]} \end{aligned} \tag{3}$$

We assume $\mathbf{x}[0] = 0$ as our focus is only on inputs triggered by FOs.



Transfer Function from Discrete State Space Representation

$$\text{Let } H_z = \underset{p \times m}{C} \underset{p \times n}{[zI - A_d]}^{-1} \underset{n \times m}{B_d}.$$

From standard Linear System Analysis, we may conclude that

$$\underset{p \times 1}{Y[z]} = \underset{p \times m}{H[z]} \underset{m \times 1}{U[z]}$$

where $Y[z]$, $U[z]$ are the Z-Transforms of $y[n]$ and $u[n]$.

Replacing z with $\exp(j\Omega)$, where $\Omega \in (0, 2\pi)$, we get the DTFT representation for the system:

$$\underset{p \times 1}{Y[\Omega]} = \underset{p \times m}{H[\Omega]} \underset{m \times 1}{U[\Omega]} \quad (4)$$



Transfer Function of Forced Oscillation Vector

Applying DTFT on $u(t)$ to get $U[\Omega]$:

$$\underset{m \times 1}{U(\Omega)} = j\pi \begin{bmatrix} \sum_{l=1}^{M_1} a_{1,l} \{ \exp(-j\phi_{1,l})\delta(\Omega + \omega_{1,l}) - \exp(j\phi_{1,l})\delta(\Omega - \omega_{1,l}) \} \\ \sum_{l=1}^{M_2} a_{2,l} \{ \exp(-j\phi_{2,l})\delta(\Omega + \omega_{2,l}) - \exp(j\phi_{2,l})\delta(\Omega - \omega_{2,l}) \} \\ \vdots \\ \sum_{l=1}^{M_m} a_{m,l} \{ \exp(-j\phi_{m,l})\delta(\Omega + \omega_{m,l}) - \exp(j\phi_{m,l})\delta(\Omega - \omega_{m,l}) \} \end{bmatrix} \quad (5)$$

$\omega_{r,l}$ can be substituted by $\frac{1}{T}\tilde{\omega}_{r,l}$



Introduction to My Work

Transient vs Steady State Stability

Transient Stability

A sudden, out-of-trend, high magnitude change in a state variable(s) causes blackouts.

Chief parameters of concern are ROCOF, frequency nadir, steady-state frequency deviation.

Inertia is a fundamental parameter here.

Steady State Stability

Accumulation of several seemingly minor trends in state variables over time, ultimately leading to a **critical point** where a small change could cause blackouts.

Autocorrelation and covariance are some of the commonly used parameters for prognosis.

Inertia plays a minor role here.



Bifurcations and Critical Slowing Down

Bifurcation: A qualitative change in the 'motion' of a dynamical System due to a quantitative change in one of its parameters. Serious bifurcations, called **Critical Bifurcations**, cause the system to become unstable from stable.



Bifurcations and Critical Slowing Down

Critical Slowing Down: Dynamical Systems exhibit early statistical warning signs before collapsing:

- Increased recovery times from perturbations.
- Increased signal variance from the mean trajectory.
- Increased flicker and asymmetry in the signal

The above three properties can be identified by increasing variance and autocorrelation in time-series measurements taken from the system.



Procedure

- Accessed a bunch of real-world frequency time-series data and plotted their:

- bulk distribution (pdf)
- auto-correlation curves

in order to demonstrate that:

- grid frequencies often exhibit significant deviation from gaussianity

autocorrelation can be a reasonable indication of grid steady-state stability.

- Obtained explanation for the *signature dynamics* of each grid.
- Then implemented and tested the IEEE 9 Bus System in PSSE for symptoms of Critical Slowing Down as an Early Warning System.



Results

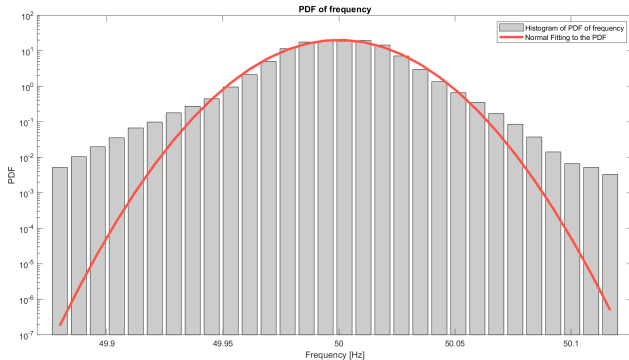


Figure 1: Continental European Grid frequency PDF: Heavier tails than a Gaussian Distribution.



Results

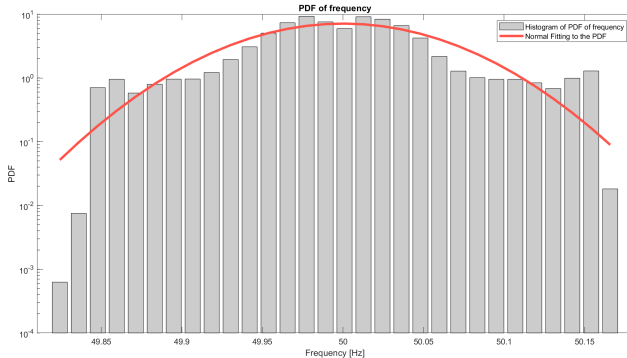


Figure 2: Mallorcan (an islanded Spanish grid) frequency pdf



Results

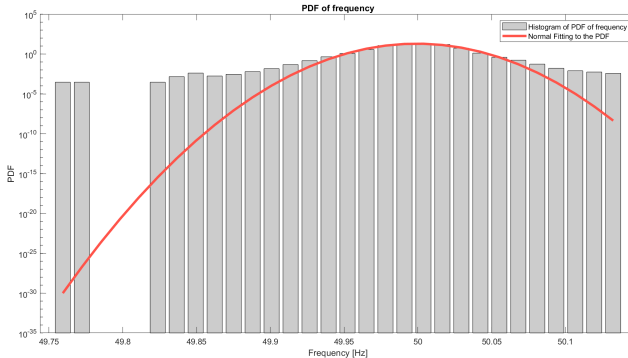


Figure 3: French grid frequency pdf including a blackout



Results

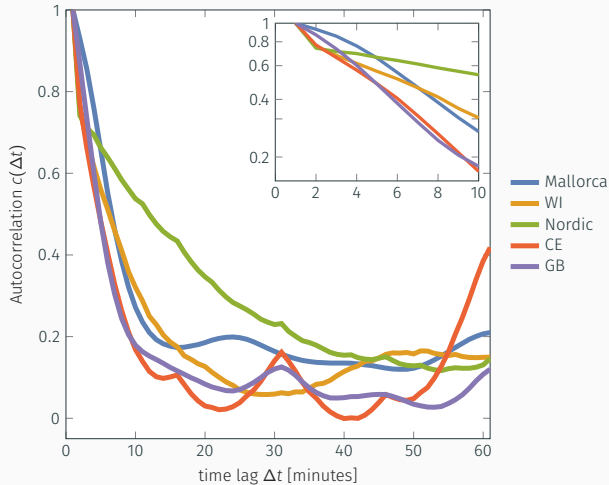


Figure 4: Autocorrelation decay of different synchronous regions.



Table 1: Inverse-correlation values for different grids

Grid name	Inverse-correlation value T^{-1} [min^{-1}]
Mallorca	0.0654
Western Interconnection	0.0498
Nordic	0.0235
Continental Europe	0.0829
Great Britain	0.0879

Figure 5: Inverse correlation time is proportional to the damping constant of the grid.

