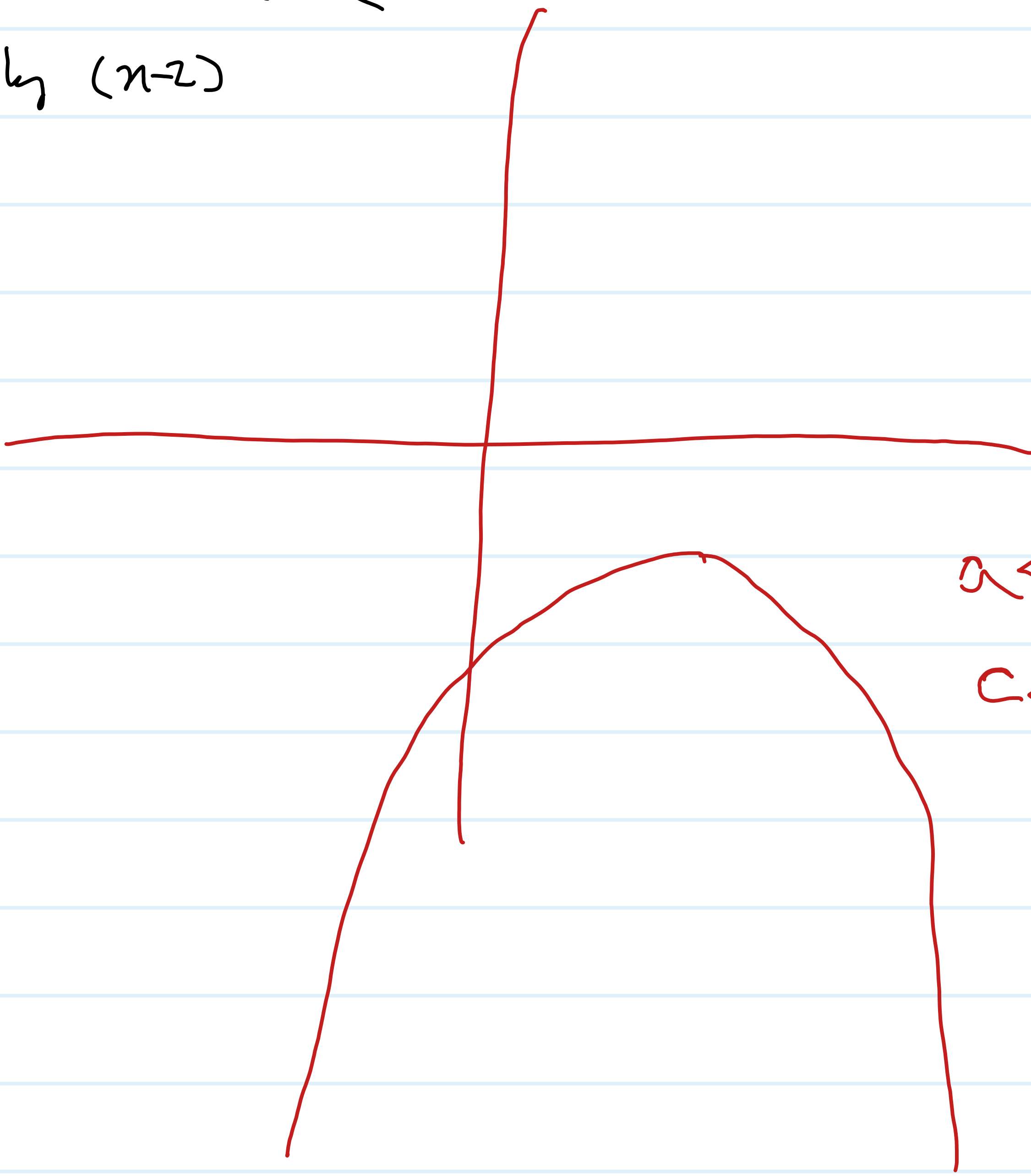
$x^3 - 4x^2 + 5x - 2$

is div by $(x-2)$



$a < 0$

$c < 0$

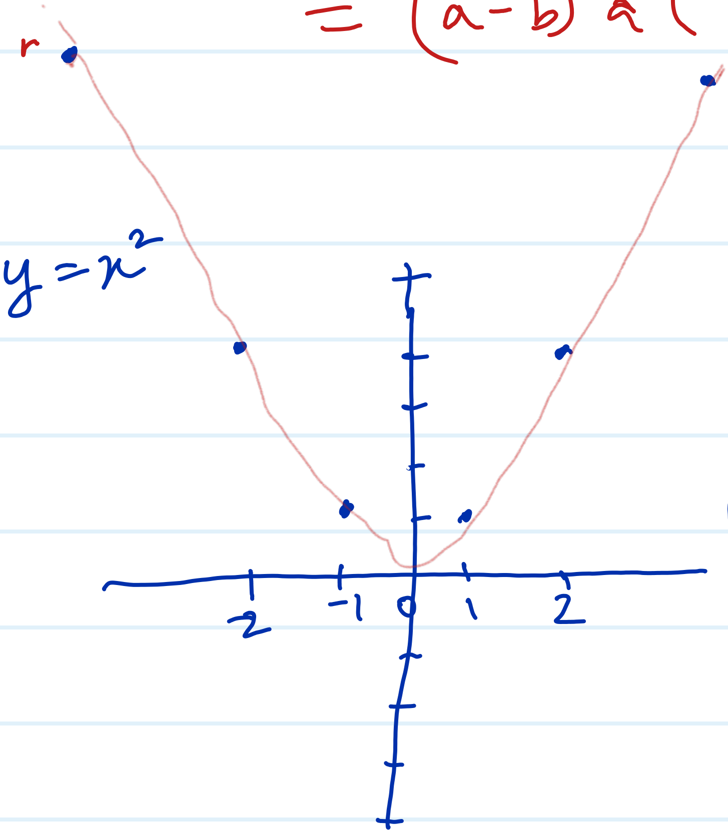
(1)

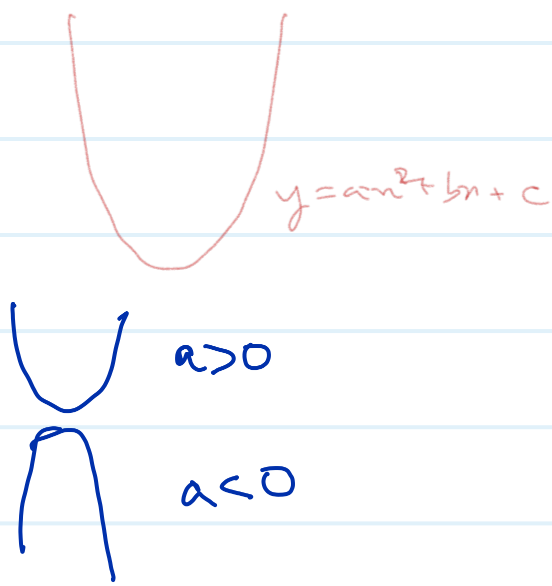
$\boxed{5}$ $px^3 + qx^2 + rx + s = 0$

$$\frac{-s}{p} = x_1 x_2 x_3$$

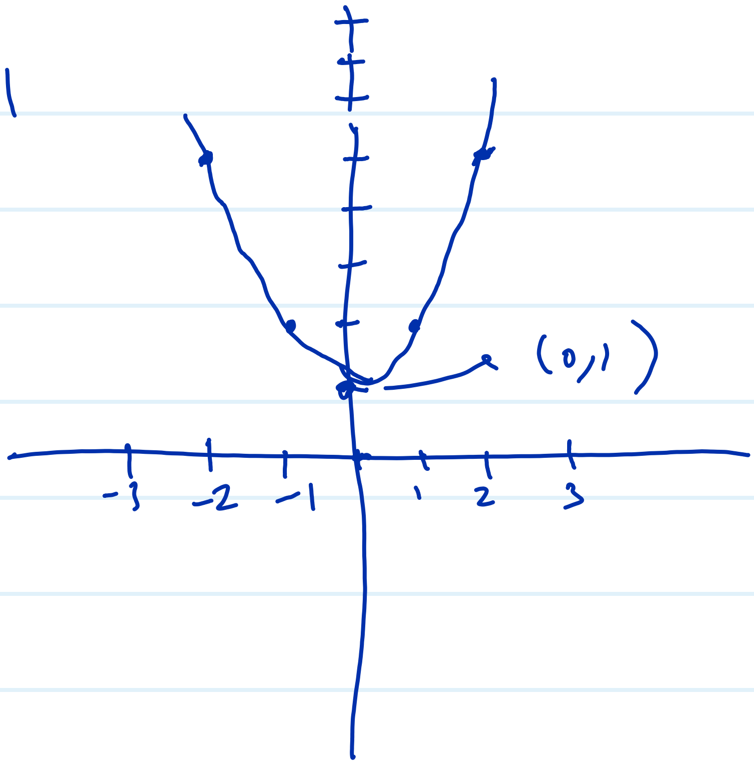
$$= (a-b)a(a+b)$$

$\boxed{4.}$ $y = x^2$

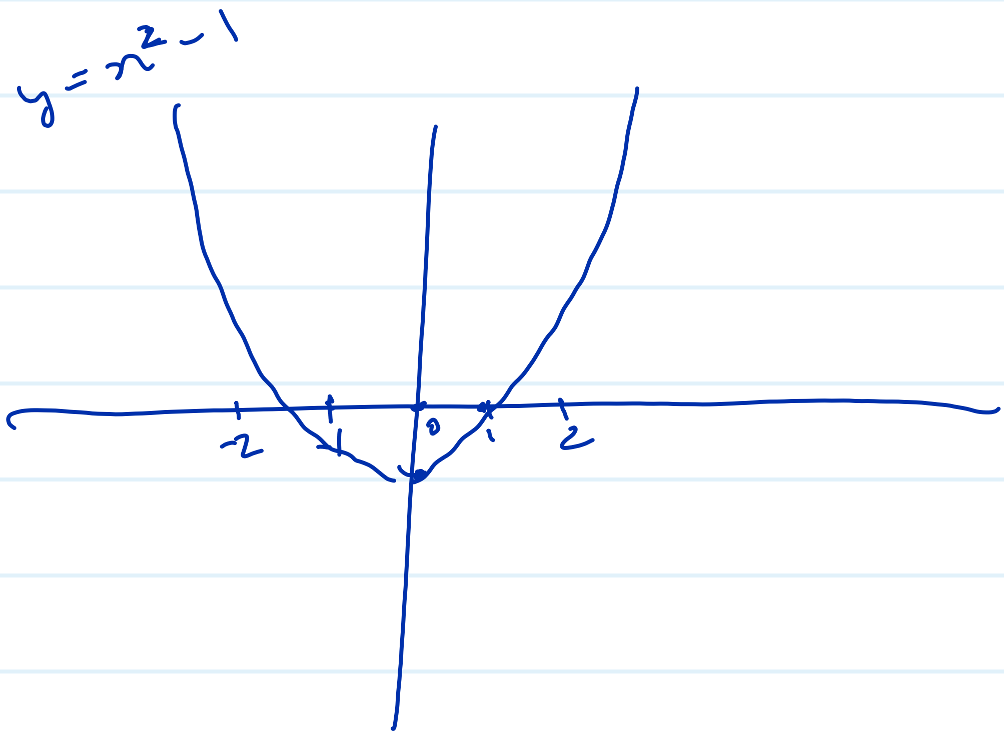




$y = x^2 + 1$

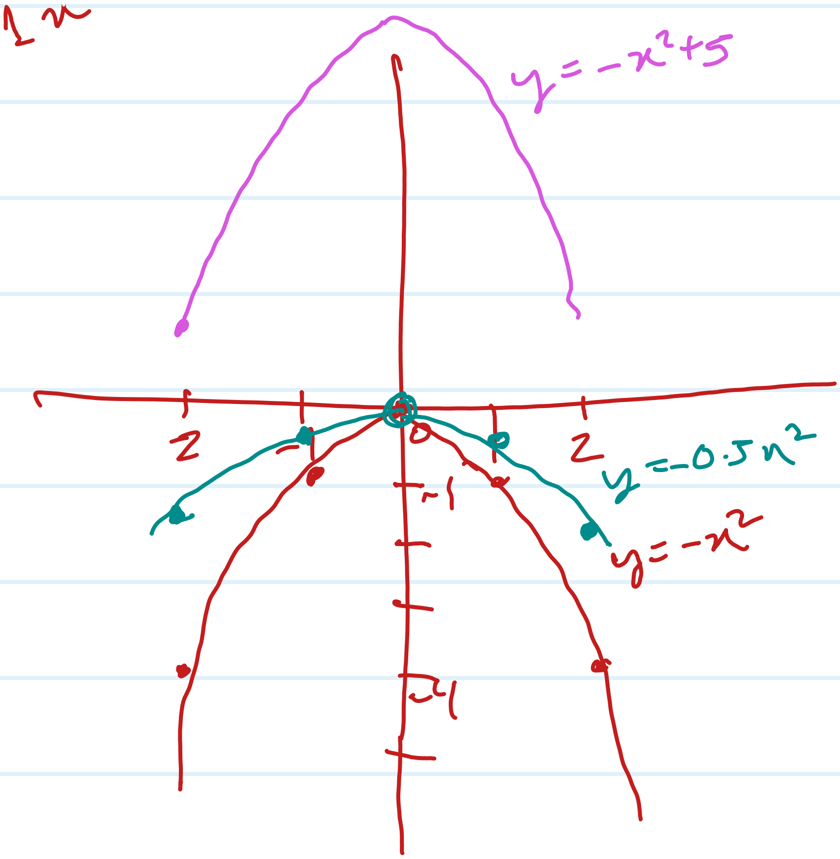


$y = x^2 - 1$



$y = -1x^2$

$y = -0.5x^2$



$$y = ax^2 + bx + c$$

$1.\ (x-x_1)^2$

or $\frac{y}{a} = \underbrace{x^2 + \frac{b}{a}x}_{} + \frac{c}{a}$

$2.\ (x-x_1)(x-x_2)$

$(x-3)(x-5)$

or $\boxed{\frac{y}{a} = \left(x + \frac{b}{2a}\right)^2 - \left(\frac{b}{2a}\right)^2 + \frac{c}{a}}$

$(x-7)^2$

$x^2 - 14x + 49$

$\{(x-7)^2 - 49\}$

$$x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2$$

$x = -\frac{b}{2a} \Rightarrow (\ )^2 = 0$

$\min \{(x-7)^2 + 8\}$

or $\frac{y}{a} = \left(x + \frac{b}{2a}\right)^2 + \left\{\frac{c}{a} - \left(\frac{b}{2a}\right)^2\right\}$

$\min (x^2 - 14x + 57)$

arg min. $\frac{y}{a}$: $x = -\frac{b}{2a}$

$$\left(\frac{y}{a}\right)_{min} = \left\{\frac{c}{a} - \left(\frac{b}{2a}\right)^2\right\}$$

or $y = a\left(x+\frac{b}{2a}\right)^2 + \left\{c - \frac{b^2}{4a}\right\}$

arg min. y : $x = -\frac{b}{2a}$

$y_{min.} = \left\{c - \frac{b^2}{4a}\right\}$

$y = ax^2 + \cancel{b}x + c$

or $\boxed{y = a\left(x+\frac{b}{2a}\right)^2 + \left(c - \frac{b^2}{4a}\right)}$

if $y = 0$

then $0 = a\left(x+\frac{b}{2a}\right)^2 + \left(c - \frac{b^2}{4a}\right)$

or $\frac{b^2}{4a} - c = a\left(x+\frac{b}{2a}\right)^2$

or $\frac{b^2}{4a^2} - \frac{c}{a} = \left(x+\frac{b}{2a}\right)^2$

$\sqrt{64} = \pm 8$

$x^2 - 64 = 0$

$x = \pm 8$

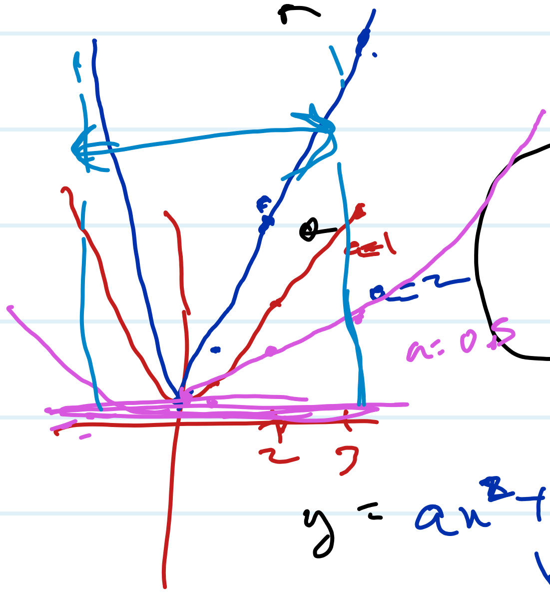
or $\pm\sqrt{\frac{b^2}{4a^2} - \frac{c}{a}} = \left(x + \frac{b}{2a}\right)$

or $x = -\frac{b}{2a} \pm \sqrt{\frac{b^2}{4a^2} - \frac{c}{a}}$

or $x = -\frac{b}{2a} \pm \sqrt{\left(\frac{1}{4a^2}\right)\left\{b^2 - 4ac\right\}}$

or $x = -\frac{b}{2a} \pm \frac{1}{2a}\sqrt{b^2 - 4ac}$

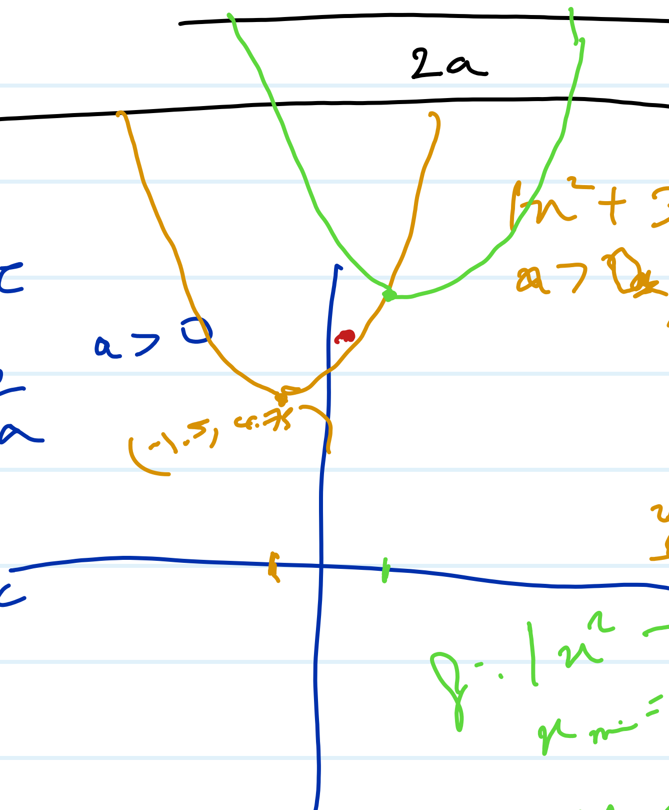
$$\boxed{x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$$



$y = ax^2 + bx + c$

$x = -\frac{b}{2a}$

$a > 0$



$1x^2 + 3x + 7$

$a > 0$

$x_{min} = -\frac{b}{2a}$

$= -\frac{3}{2}$

$y = \frac{19}{4}$

$f: 1x^2 - 3x + 9$

$x_{min} = \frac{3}{2}$

$y = \frac{27}{4} = 6.75$

$y\left(x = -\frac{b}{2a}\right)$

$y_1 = a \times \left(-\frac{b}{2a}\right)^2 + b\left(-\frac{b}{2a}\right) + c$

$y = \frac{b^2}{4a} - \frac{b^2}{2a} + c$

$$\boxed{y = \frac{-b^2}{4a} + c}$$

$f: -2x^2 + x + 5$

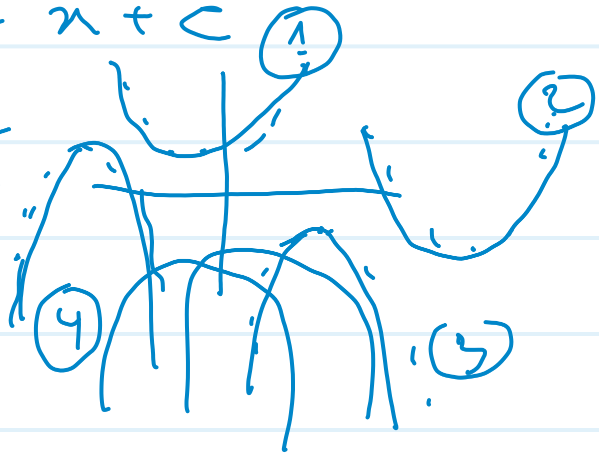
$-a\left(x + \frac{b}{2a}\right)^2 + \left(\ldots\right.$

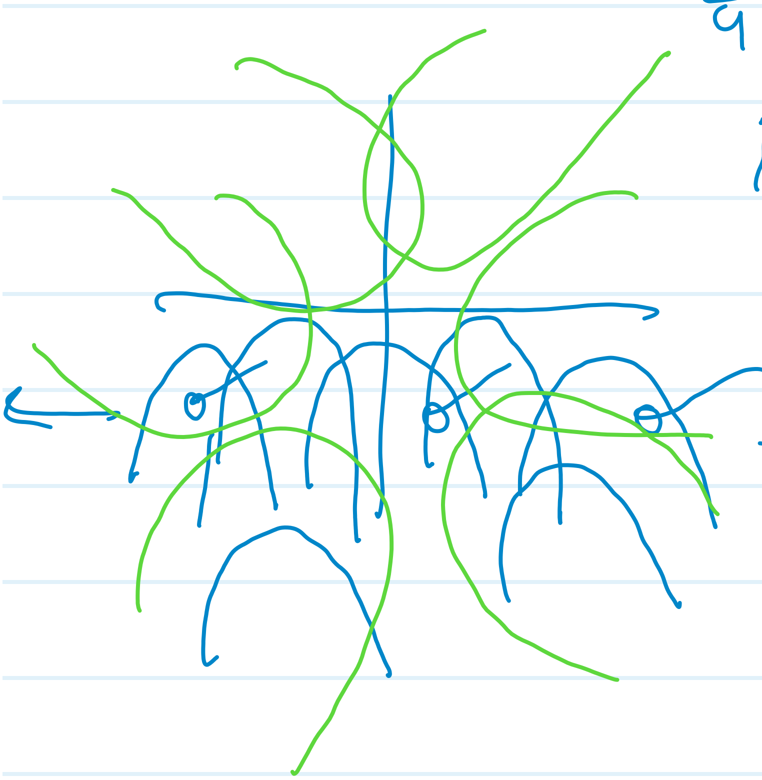
$x = -\frac{b}{2a}$

$x = \frac{1}{4}$ $\quad y = \frac{41}{8} = 5.25$

4. $f(x) = ax^2 + x + c$

$ac > \frac{1}{4}$



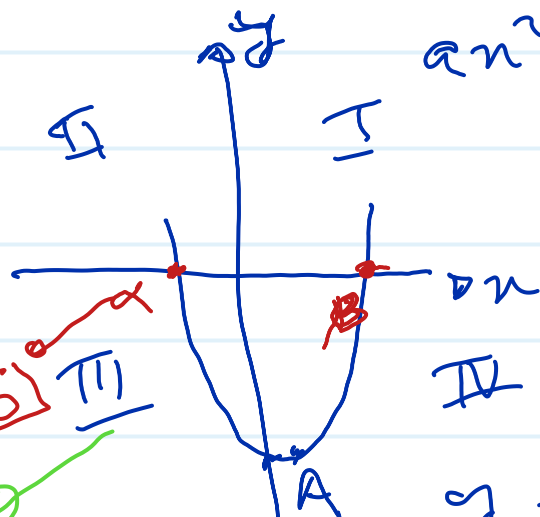


$\rightarrow \quad a < 0$

$ac > \frac{1}{4} \Rightarrow c < 0$

$\boxed{(2)\ a<0, c<0}$ Ans

C) 1.



$ax^2 + bx + c$

$a > 0 \ \exists\ \min = A$

$x_n = \frac{-b}{2a} > 0$

$\Rightarrow \boxed{b < 0}$

$\underline{y} = \frac{-b^2}{4a} + c < 0$

$\Rightarrow (-) + c < 0$

$\Rightarrow c$ ☹ $\in$

$\alpha . \beta = \frac{c}{a}$

$\frac{c}{a} < 0$

$a > 0 \Rightarrow \boxed{c < 0}$

(1) $a > 0$ ✓

(2) $c < 0$ ✓

(3) $a > 0$ and $c > 0$

(4) $b > 0$ ✗

$(1, -1, \frac{1}{8})$

$b = -1$

$a = 1$

$\underline{y} = \frac{-1}{4} + c$

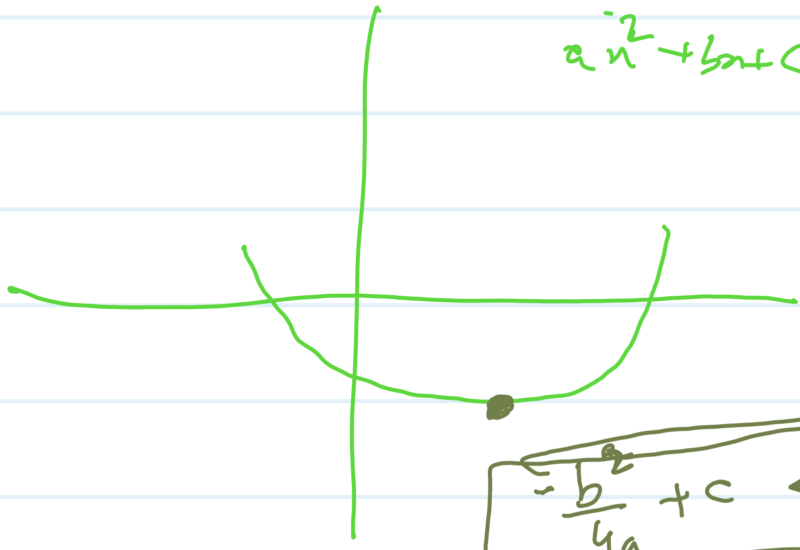
$c \in (-\infty, \frac{1}{4})$

$c = \frac{1}{8}$

$\boxed{(1)(2)}$ Ans

C. $\boxed{2}$

ax^2+bx+c



$\boxed{a>0}$

$-\frac{b}{2a}>0 \Rightarrow \boxed{b<0}$

$\alpha\beta = \frac{c}{a} < 0 \Rightarrow \boxed{c<0}$

$\boxed{-\frac{b^2}{4a}+c<0}$

(1) $\frac{-b}{c}+\frac{a}{c}$ ○ $\underbrace{(-)+(-)}_{(-)}$

(2) $\frac{a^2c+b^2c}{ab}$ ✗ $\frac{(+)(-)+(+)(-)}{(+)(-)} = (+)$

(3) abc ✗

(4) $ab+ac-bc$ ○ (+)

$(-)\quad(-)-(+)$

$(-)+(-)+(-)=(-)$

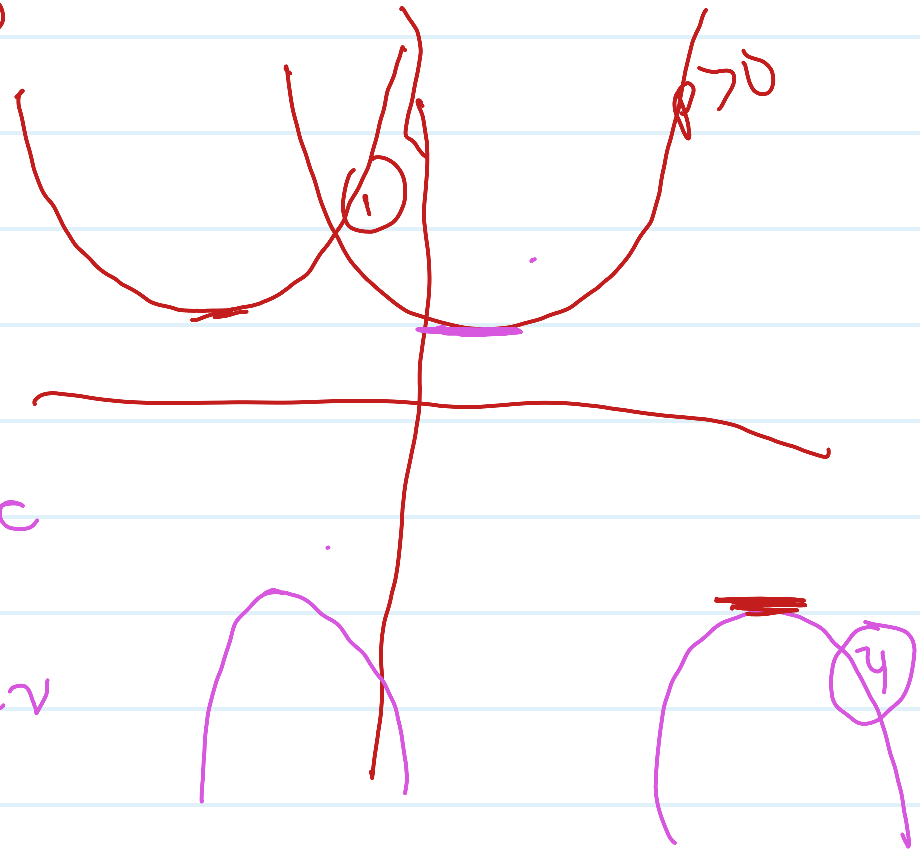
$ab+ac+bc$

$(-)\quad(-)\quad(+)\quad(-)$

$\boxed{(1),(4)}$ Ans

$\boxed{A}$ $\boxed{4}$ $f: px^2 + qx + r$ $\quad y = px^2 + qx + r$

$f.\ p > 0$



p, q, r

$~~x~~\ x^* = \frac{-b}{2a}$

$y^* = \frac{-b^2}{4a} + c$

$\Rightarrow y^* = \frac{-q^2}{4p} + r$

$p y^* > 0$

$p\left(\frac{-q^2}{4p} + r\right) > 0$

$\Rightarrow -\frac{q^2}{4} + rp > 0$

A. $\boxed{4}$ $\boxed{q^2 < 4pr}$ Ans

MCQ

B $\boxed{5}$ $x^3 - 3x^2 + 2$

$\alpha\beta\gamma = \frac{-2}{1} = -2$

$\alpha + \beta + \gamma = \frac{-(-3)}{1} = 3$

$\alpha\beta + \beta\gamma + \gamma\alpha = \frac{0}{1} = 0$

$x^2(x-1) - 2x(x-1) - 2(x-1)$

$(x^2 - 2x - 2)(x-1)$

$\alpha, \gamma = \frac{2 \pm \sqrt{4+8}}{2}$ $\quad \boxed{\beta = 1}$

$\boxed{\alpha, \gamma = 1 \pm \sqrt{3}}$

$\alpha + \beta + \gamma = 3$

$\alpha \cdot \beta\gamma = -2$

$\boxed{(2) \quad a = 1, \; b = \pm\sqrt{3}}$ Ans

$\alpha\beta + \beta\gamma + \gamma\alpha$

$= \beta(\alpha + \gamma) + \gamma\alpha$

$= 2 - 2$

$= 0$

B

$\boxed{6}$ $x^3 - 3px^2 + qx - r = 0$

$x_1, x_2, x_3 = \alpha - \beta, \alpha, \alpha + \beta$

$$\sum x_i = -\frac{(-3p)}{1} = 3p$$

$$= 3\alpha \Rightarrow \boxed{\alpha = p}$$

$$\sum x_i x_j = \frac{(q)}{1} = q$$

$$= \alpha^2 - \beta^2 + 2\alpha^2$$

$$= 3\alpha^2 - \beta^2$$

$$\sum x_i x_j x_k = -\frac{(-r)}{1} = r$$

$$= (\alpha^2 - \beta^2)\alpha$$

$3\alpha^2 - \beta^2 = q$

$\Rightarrow 3p^2 - \beta^2 = q$

$\Rightarrow \beta = \sqrt{3p^2 - q}$

$r = (p^2 - (3p^2 - q))p$

$\Rightarrow r = (q - 2p^2)p$

$\Rightarrow r = pq - 2p^3$

$\Rightarrow 2p^3 = pq - r$

$\boxed{(1)\ 2p^3 = pq - r}$ Ans

B

$\boxed{7.}$

$$\overbrace{(x^3 - 4x^2 + 3x - 2)}^{p(x)} \div g(x)$$

$$= (x-2), -2x$$

$\therefore$ $$(p(x) + 2x) \div (x-2) = g(x)$$

$\therefore$ $p(x) + 2x$ is fully divisible by $x-2$ $\therefore$

$$x^3 - 4x^2 + 5x - 2$$

$$x^2(x-2) - 2x(x-2) + 1(x-2)$$

$\therefore$ $$g(x) = (x^2 - 2x + 1) = (x-1)^2$$

$\boxed{(1)\ x^2 - 2x + 1}$ Ans

B

$\boxed{8}$, $f(x) = 1x^3 + ax^2 + bx + c$

$f(1) = f(2) = 0$

$f(4) = f(0)$

$f(0) = c$

$f(1) = 1 + a + b + c = 0$

$f(2) = 8 + 4a + 2b + c = 0$

$f(4) = 64 + 16a + 4b + c$

$64 + 16a + 4b = 0$

$\boxed{16 + 4a + b = 0}$ (i)

$\boxed{7 + 3a + b = 0}$ (ii)

$9 + a = 0 \Rightarrow \boxed{a = -9}$

$\boxed{b = 20}$

$\boxed{(1)\ a = -9, b = 20, c = -12}$ Ans

$\boxed{c = -12}$

B

$\boxed{9}$ $1x^3 - 5x^2 + 2x - 10$

$y = (\alpha+\beta+\gamma)\left(\frac{1}{\alpha}+\frac{1}{\beta}+\frac{1}{\gamma}\right)$

$\Rightarrow y = (\alpha+\beta+\gamma)\frac{\Sigma\alpha\beta}{\alpha\beta\gamma}$

$\Rightarrow y = \frac{\left(\frac{-b}{a}\right)\cdot\left(\frac{c}{a}\right)}{\left(\frac{-d}{a}\right)}$ $\Rightarrow$ $\boxed{y = \frac{bc}{ad} = 1}$

$\boxed{(4)\ 1}$ Ans

$\boxed{c/3}$ MSQ

$f: 1x^2 - mx + n$ ⟨ 4, $\beta = ?$

$n \in Z^+$, $n \% 7 = 0$, $n < 50$ → $n = 7, 14, 21, 28, 35, 42, 49$

$m = \text{Prime } Z^+$

$g: 1x^2 - mx + s$ ⟨ 9, $\gamma = ?$

$16 - 4m + n = 0$

$m = \frac{n+16}{4}$

n	✓?
7	×
14	×
21	×
28	✓ (m = 11 ✓)
35	×
42	×
49	×

$m = 28$

$\Rightarrow \gamma = m - 9$

$\Rightarrow \boxed{\gamma = 19}$

$\boxed{s = 9\gamma = 171}$

$\boxed{\begin{array}{l}(2)\ m = 11 \\ (3)\ n = 28\end{array}}$ ✓

$\boxed{(2)\ (3)}$ Ans

$\boxed{D \mid 1}$

A: False?

Mult. inv. of $p(x) = \frac{1}{p(x)}$

(it does not have to be a polynomial)

B: True

$\frac{1}{x}$ is not polynomial

no options?

$\boxed{D \mid 2}$

PTO

$\boxed{B|G}$

BG

$ax^3+bx^2+cx+d=0 < \alpha, \beta, \gamma$

- $\alpha+\beta+\gamma = -\frac{b}{a}$
- $\alpha\beta+\beta\gamma+\gamma\alpha = \frac{c}{a}$
- $\alpha\beta\gamma = -\frac{d}{a}$

$ax^2+bx+c=0 < \alpha, \beta$

- $\alpha+\beta = -\frac{b}{a}$
- $\alpha\beta = \frac{c}{a}$

$(-1)^n \times \frac{?}{a}$

n → # elements in a product

- SOPs: Sum of Products.

$f = x^3 - 3px^2 + qx - r \leqslant$ $a \to \alpha-\beta$, $b \to \alpha$, $c \to \alpha+\beta$

(i) $a+b+c = -\frac{(-3p)}{1} = 3p$

(ii) $\Sigma ab = \frac{q}{1} = q$

(iii) $abc = -\frac{(-r)}{1} = r$

(iii) $(\alpha-\beta)\,\alpha\,(\alpha+\beta) = r$

or $(\alpha^2-\beta^2)\alpha = r$

(i) $(\alpha-\beta)+\alpha+(\alpha+\beta) = 3p$

or $3\alpha = 3p \Rightarrow \boxed{\alpha = p}$

(ii) $(\alpha-\beta)\alpha + \alpha\cdot(\alpha+\beta) + (\alpha-\beta)(\alpha+\beta) = q$

$\Rightarrow (\alpha^2-\beta^2) + \alpha(2\alpha) = q$

$\Rightarrow 3\alpha^2 - \beta^2 = q$

$\Rightarrow \boxed{3p^2 - \beta^2 = q}$

$\Rightarrow \boxed{\beta^2 = 3p^2 - q}$

$(\alpha^2-\beta^2)\alpha = r$

$(p^2-\beta^2)p = r$

$(q-2p^2)p = r$

$\Rightarrow \boxed{pq - r = 2p^3}$

$\boxed{(1)\ 2p^3 = pq - r}$ Ans

$\boxed{B|7}$

$p = gq + r \Rightarrow gq = p - r$

$x^3 - 4x^2 + 5x - 2$

is div by $(x-2)$

$$\boxed{\begin{aligned} p(x) &= d(x)\cdot q(x) + r(x) \\ &\text{degree}(d(x)) \le \text{degree}(p(x)) \\ &\text{degree}(r(x)) < \text{degree}(d(x)) \end{aligned}}$$

$$\underbrace{p(x) - r(x)} = d(x) \cdot q(x)$$

$$p'(x) = d(x)\, q(x) \quad x_1, x_2, x_3, \ldots$$

$p'(x)$ is div. by $q(x)$.

putting $x = \alpha$ on both sides:

$$p'(x_1) = \underbrace{d(x_1) \cdot \cancel{q(x_1)}^{\,0}}_{0}$$

$\Downarrow$

LHS $= p'(x_1) = 0$

$$x^3 - 4x^2 + 5x - 2 \xrightarrow{x=2} 0$$

is div by $(x-2)$

$$x^2 (x-2) - 2x(x-2) + 1(x-2) = 0$$

or $(x^2 - 2x + 1)(x-2) = 0$

or $q(x) = x^2 - 2x + 1$

$\boxed{(1)\ x^2 - 2x + 1}$ Ans

$\boxed{B18}$ Attempt

$\boxed{B19}$ $(\alpha + \beta + \gamma)\left(\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}\right)$