



# **Distributed Optimization in Distribution Systems**



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# Distributed Optimization in Distribution Systems

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- Current problem statement:
  - Perform Distributed Optimal Power Flow on a Single-phase power distribution network.
- Desired objective(s):
  - Incorporate battery storage into the algorithm.
  - Convert the code (currently in MATLAB) into Python/Julia.
  - Later: Incorporate battery storage into a three-phase network.

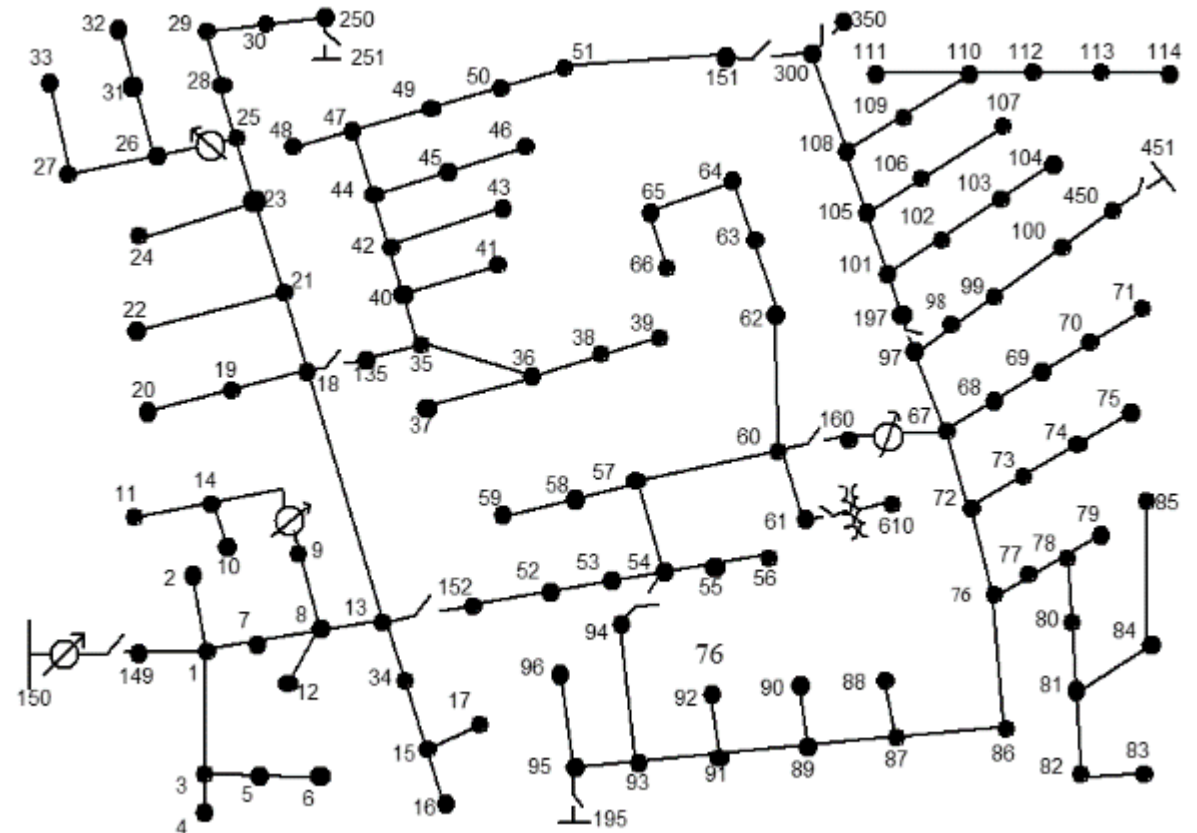
*This is based on the previous work by:*

- [Rabayet Sadnan and Anamika Dubey \(2022\) Distributed Computing for Scalable Optimal Power Flow in Large Radial Electric Power Distribution Systems with Distributed Energy Resources](#)
- [Rabayet Sadnan and Anamika Dubey \(2021\) Distributed Optimization Using Reduced Network Equivalents for Radial Power Distribution Systems](#)



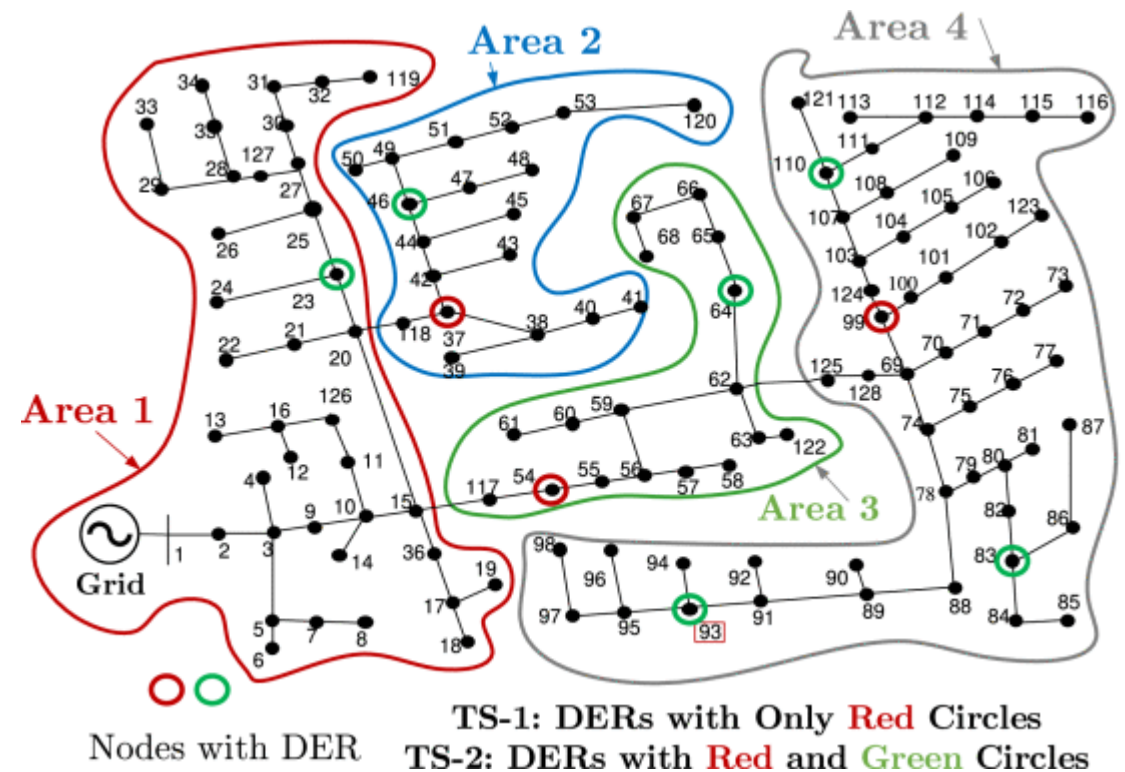
# Distributed Optimization in Distribution Systems

- **Current problem statement:**
  - Single-phase power distribution network (Here, the IEEE123 Node Test System)



# Distributed Optimization in Distribution Systems

- **Current problem statement:**
  - Divide the network into several areas in order to solve for their Optimal Power Flow solutions separately, with exchange of boundary variables after every 'macro-iteration'.

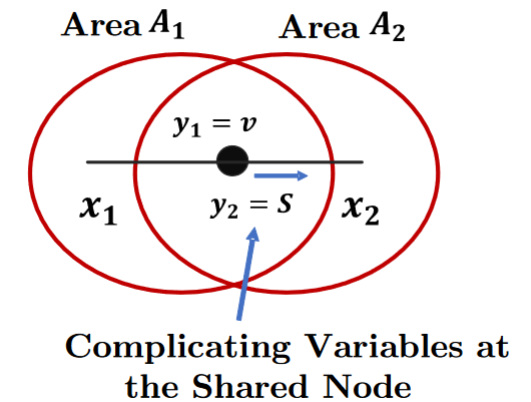


# Distributed Optimization in Distribution Systems

## • Current problem statement:

- Perform a 'Distributed OPF' for minimizing line losses for each area. After every macro-iteration, exchange border variables between connected areas.

$$\begin{aligned}
 (\mathbf{D1}) \quad & \min f_m = \sum_{\{ij\} \in \mathcal{E}_m} l_{ij} r_{ij} \\
 \text{s.t.} \quad & P_{ij} - r_{ij} l_{ij} - p_{Lj} + p_{Dj} = \sum_{k:j \rightarrow k} P_{jk} \\
 & Q_{ij} - x_{ij} l_{ij} - q_{Lj} + q_{Dj} = \sum_{k:j \rightarrow k} Q_{jk} \\
 & v_j = v_i - 2(r_{ij} P_{ij} + x_{ij} Q_{ij}) + (r_{ij}^2 + x_{ij}^2) l_{ij} \\
 & v_o = v_{o'}; \\
 & P_{jk} = p_{k'} \quad ; \forall \{jk\} \in \mathcal{E}_m, \text{ where } k \in C_h \\
 & Q_{jk} = q_{k'} \quad ; \forall \{jk\} \in \mathcal{E}_m, \text{ where } k \in C_h \\
 & -\sqrt{S_{DRj}^2 - p_{Dj}^2} \leq q_{Dj} \leq \sqrt{S_{DRj}^2 - p_{Dj}^2} \\
 & \underline{V}^2 \leq v_j \leq \bar{V}^2 \quad ; \forall j \in \mathcal{N}_m \\
 & l_{ij} \leq \left(I_{ij}^{rated}\right)^2 \quad ; \forall \{ij\} \in \mathcal{E}_m
 \end{aligned}$$



# Some test runs over different area divisions

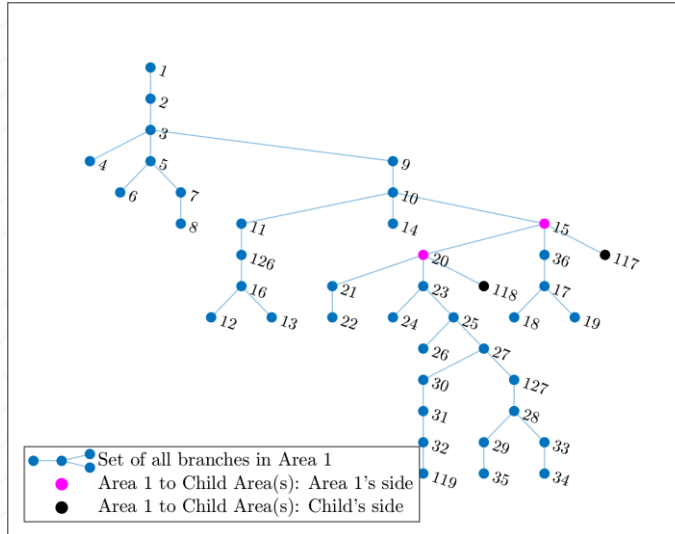
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The Distributed Optimization Algorithm was run on the IEEE 123 Node Test Feeder, which was divided into three, four and five Areas in separate runs.

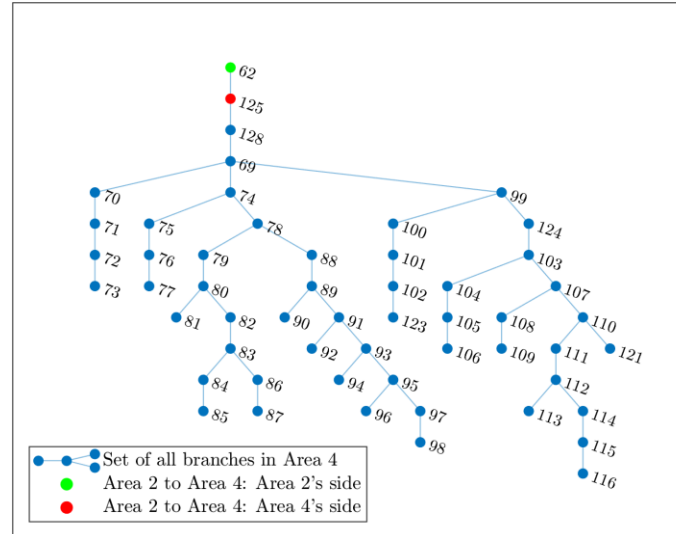
This was to check if the results of the algorithm are consistent irrespective of the number of areas the system was divided into.



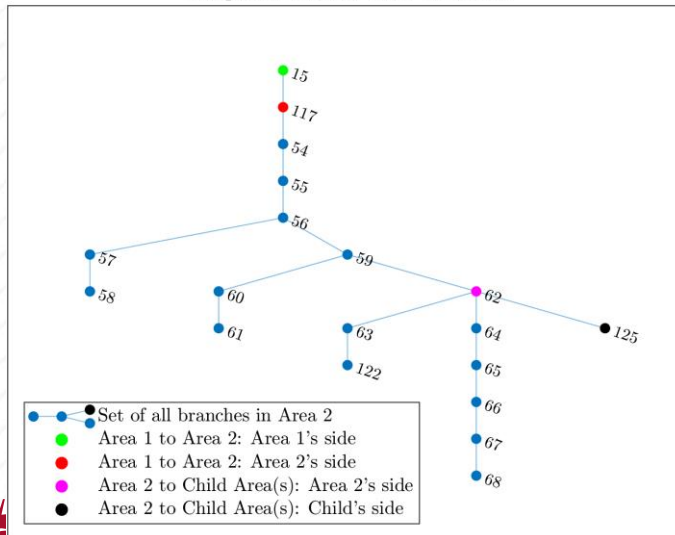
Graph for Area 1 with 41 nodes



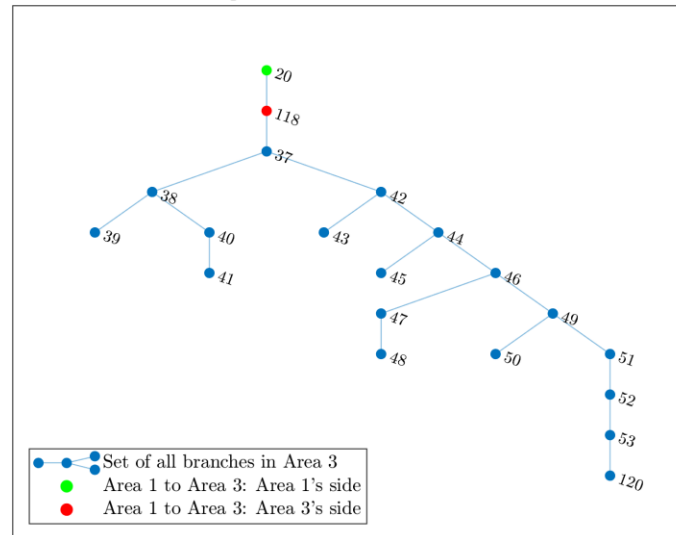
Graph for Area 4 with 54 nodes



Graph for Area 2 with 19 nodes



Graph for Area 3 with 20 nodes

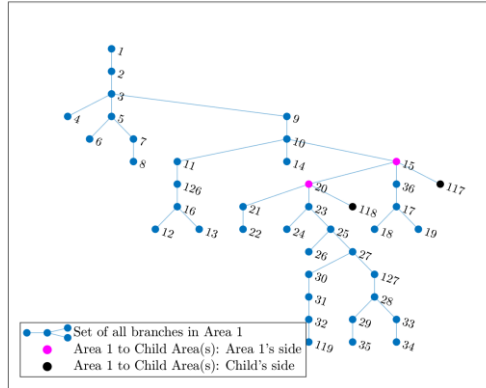


# 4 Area System

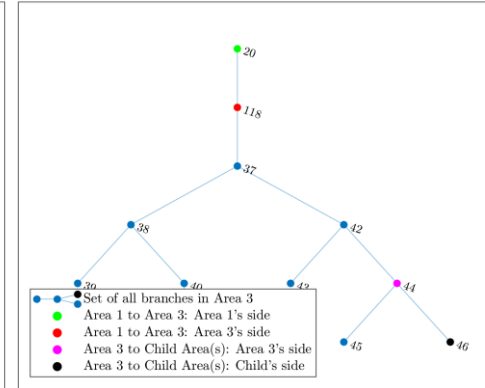


# 5 Area System

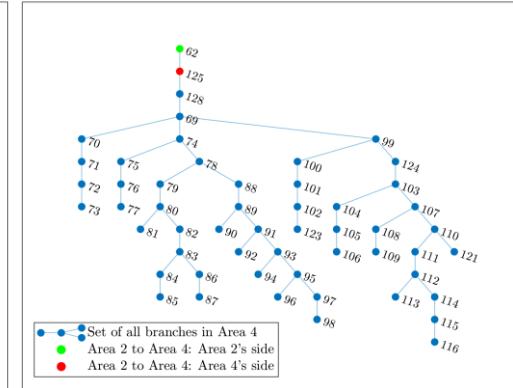
Graph for Area 1 with 41 nodes



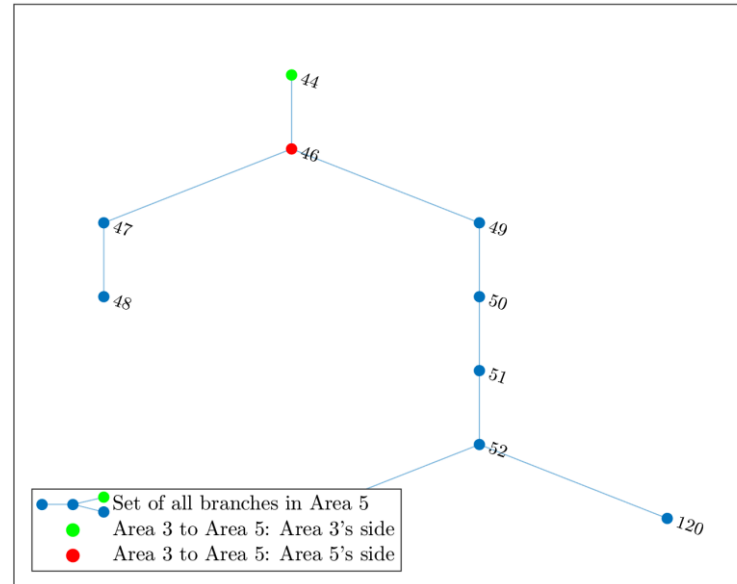
Graph for Area 3 with 12 nodes



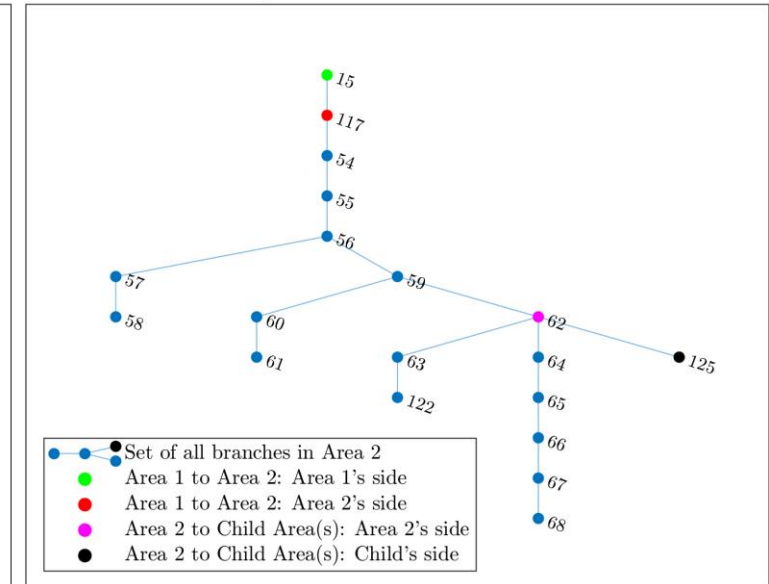
Graph for Area 4 with 54 nodes



Graph for Area 5 with 10 nodes

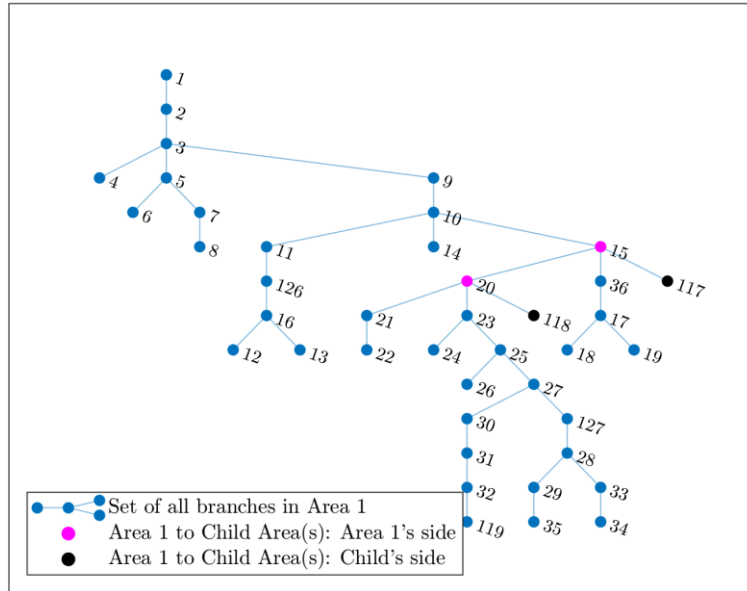


Graph for Area 2 with 19 nodes

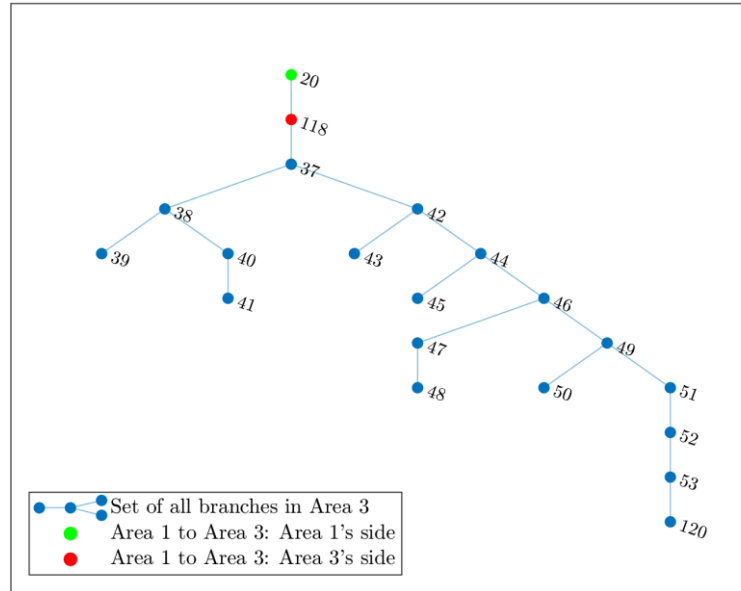




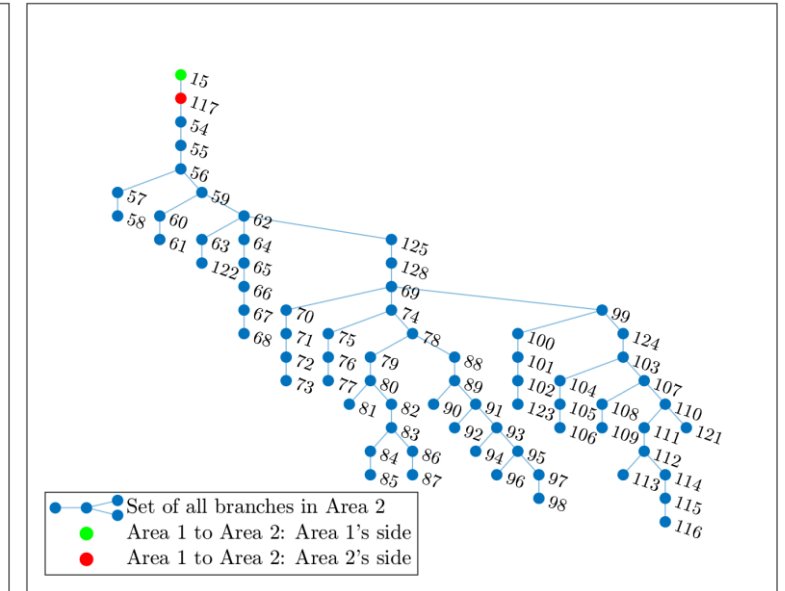
Graph for Area 1 with 41 nodes



Graph for Area 3 with 20 nodes



Graph for Area 2 with 71 nodes



# 3 Area System

# Some Test Runs over Different Area Divisions

- Optimal line losses match for all sets of areas.

- 1 Area (Centralized OPF):
  - Line Loss = 12.094kW
  - Substation Power = 768.17kW
- 5 Area:
  - Line Loss = 12.18kW
  - Substation Power = 768.24kW
- 4 Area:
  - Line Loss = 12.18kW
  - Substation Power = 768.24kW
- 3 Area:
  - Line Loss = 12.09kW
  - Substation Power = 768.16kW



# Next Objectives

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Desired objective(s):

Incorporate battery storage into the algorithm using Multi-Period Optimization.

Convert the code (currently in MATLAB) into Python/Julia.

Later: Incorporate battery storage into a three-phase network.

