

MULTI-PERIOD OPTIMAL POWER FLOW

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TBA

MULTI-PERIOD OPTIMAL POWER FLOW

Abstract

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Dedication

TBA

Chapter One

SOME FORMATTING EXAMPLES

1.1 Chapter one tittle section

Chapter Two

FIGURES AND TABLES

2.1 Examples of a figure

REFERENCES

- [1] Aayushya Agarwal and Larry Pileggi. “Large Scale Multi-Period Optimal Power Flow With Energy Storage Systems Using Differential Dynamic Programming”. In: *IEEE Trans. Power Syst.* 37.3 (Sept. 2021), pp. 1750–1759. ISSN: 1558-0679. DOI: [10.1109/TPWRS.2021.3115636](https://doi.org/10.1109/TPWRS.2021.3115636).
- [2] Masoud Farivar and Steven H. Low. “Branch flow model: Relaxations and convexification”. In: *2012 IEEE 51st IEEE Conference on Decision and Control (CDC)* (Dec. 2012), pp. 3672–3679. ISSN: 0743-1546. DOI: [10.1109/CDC.2012.6425870](https://doi.org/10.1109/CDC.2012.6425870).
- [3] Nawaf Nazir and Mads Almassalkhi. “Receding-Horizon Optimization of Unbalanced Distribution Systems with Time-Scale Separation for Discrete and Continuous Control Devices”. In: *2018 Power Systems Computation Conference (PSCC)* (June 2018), pp. 1–7. DOI: [10.23919/PSCC.2018.8442555](https://doi.org/10.23919/PSCC.2018.8442555).
- [4] Nawaf Nazir, Pavan Racherla, and Mads Almassalkhi. “Optimal multi-period dispatch of distributed energy resources in unbalanced distribution feeders”. In: *arXiv* (June 2019). DOI: [10.48550/arXiv.1906.04108](https://doi.org/10.48550/arXiv.1906.04108). eprint: [1906.04108](https://arxiv.org/abs/1906.04108).
- [5] Xiaodong Qian and Yuanguo Zhu. “Differential Dynamic Programming for Multistage Uncertain Optimal Control”. In: *2014 Seventh International Joint Conference on Computational Sciences and Optimization*. IEEE, July 2014, pp. 88–92. DOI: [10.1109/CSO.2014.25](https://doi.org/10.1109/CSO.2014.25).

APPENDIX

Appendix A

Branch Flow Model: Relaxations and Convexification

In [2] the authors came up the Relaxed Branch Flow Model, and showed that in the case of Tree/Radial networks, the Relaxed Model can solve for the unique optimal solution, including the bus angles, and in the case of weakly meshed networks, there is a mechanism for extracting the bus angles from the relaxed solution, to find out its unique solution, if it exists.

Legend for Table A.2:

TABLE A.1 Table describing the variables involved in the Branch Flow Model equations.

Symbol	Meaning
p_j, q_j	Real, Reactive Power flowing from bus j into the network.
P_{ij}, Q_{ij}	Real, Reactive Power flowing in branch $i \rightarrow j$ (sending-end).
I_{ij}, l_{ij}	Complex Current flowing in branch

TABLE A.2 Table describing the Branch Flow Model equations.

Equation #	Equation	Unknowns	Knowns	No. of Equations
13	$p_j = \Sigma P_{jk} + \Sigma(P_{ij} - r_{ij}l_{ij}) + g_jv_j$	$1 \times p_0$	$n \times p_j$	$(n + 1)$
		$m \times P_{ij}$	$m \times r_{ij}$	
		$m \times l_{ij}$	$(n + 1) \times g_j$	
		$n \times v_j$	$1 \times v_0$	
14	$q_j = \Sigma Q_{jk} + \Sigma(Q_{ij} - x_{ij}l_{ij}) + b_jv_j$	$1 \times q_0$	$n \times q_j$	$(n + 1)$
		$m \times Q_{ij}$	$m \times x_{ij}$	
		$m \times l_{ij}$	$(n + 1) \times b_j$	
		$n \times v_j$	$1 \times v_0$	
15	$v_j = v_i + (r_{ij}^2 + x_{ij}^2)l_{ij} - 2(r_{ij}P_{ij} + x_{ij}Q_{ij})$	$m \times P_{ij}$	$b \times r_{ij}$	m
		$m \times Q_{ij}$	$m \times x_{ij}$	
		$m \times l_{ij}$	$1 \times v_0$	
		$n \times v_j$		
16	$l_{ij} = \frac{P_{ij}^2 + Q_{ij}^2}{v_j}$	$m \times P_{ij}$		m
		$m \times Q_{ij}$	$1 \times v_0$	
		$m \times l_{ij}$		
		$n \times v_j$		
13 to 16		$1 \times p_0$	$n \times p_j$	$2(n + 1 + m)$
		$1 \times q_0$	$n \times q_j$	
		$m \times P_{ij}$	$m \times r_{ij}$	
		$m \times Q_{ij}$	$m \times x_{ij}$	
		$m \times l_{ij}$	$(n + 1) \times g_j$	
		$n \times v_j$	$(n + 1) \times b_j$	
			$1 \times v_0$	
		$2(n + 1 + m)$	$4n + 2m + 3$	$2(n + 1 + m)$

Appendix B

Abstracts: Optimization-based Methods for solving MP-OPF

In [4], the authors use a two-step paradigm for solving the MP-OPF problem, by first solving for a more relaxed SOCP problem for the all of the time-steps in a horizon, and using the SOC values from its solution, solve for the NLP OPF problem for every time-step in parallel.

In [3], they prove that for 'realistic' systems, appending an additional 'complementarity' cost function to the original objective function, Simultaneous Charging and Discharging (SCD) in the optimal solution is avoided, and that the Mixed-Integer SOCP problem of AC-OPF with energy storage can be relaxed into a regular SOCP problem without violating the battery physics in the optimal solution.

Appendix C

Abstracts: Dynamic Programming

Methods for solving MP-OPF

In [1], the authors use a Differential Dynamic Programming approach, which involved usage of Forward and Backward passes made over a sequence of time-steps, doing a back-and-forth between computation for one time-step, say t , by making some assumptions on any variables required from the next time-step $t + 1$, and then updating the assumed values at t , once new values for the $t + 1$ time-step have been made.

Appendix D

Abstracts: Differential Dynamic Programming

In [5], the authors lay out the framework on how Differential Dynamic Programming can be utilized to solve a Mutli-Stage Uncertain Optimal Control problem programmatically.

Appendix E

Models: Battery Model for Mutli-Period OPF

In [3, 4], the batteries are modelled using four state/control variables, which are:

TABLE E.1 Description of Battery Variables

Variable	Description	Dimension	Dimension pu
$B_{n,k}$	State of Charge (SOC) of Battery	$[kWh]$	$[pu\ h]$
$P_{n,k}^c$	Average Charging Power of the Battery during the k -th time interval.	$[kW]$	$[pu]$
$P_{n,k}^d$	Average Discharging Power of the Battery during the k -th time interval.	$[kW]$	$[pu]$
$q_{B_{n,k}}$	Average Reactive Power Output from the Battery Inverter during the k -th time interval.	$[kVAr]$	$[pu]$