

EE 507 Random Processes

Homework 01

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Problem 1

You pick two cards at random (without replacement) from a standard deck of 52 cards. Please answer the following questions:

- What is the probability that both cards show the same value (e.g. both are 4's or both are Kings.)
- Given that the two cards have the same color, what is the probability that both cards show the same value? Is this probability larger or smaller than the probability in part a? Conceptually, why does this make sense?
- Are the events that the two cards have the same color and the two cards have the same value independent?

Solution

Experiment S : Two cards are drawn at random without replacement from a standard deck of 52 cards.

Let event $A = \{\text{"Both cards are of the same value"}\}$.

Let event $B = \{\text{"Both cards are of the same colour"}\}$.

So $AB = \{\text{"Both cards are of the same value and of the same colour"}\}$

a.

$$\begin{aligned} P(A) &= \frac{52 * 3}{52 * 51} \\ \text{or, } P(A) &= \frac{1}{17} \\ \text{or, } P(A) &\approx 0.0588 \end{aligned} \tag{1}$$

b.

$$P(A|B) = \frac{P(AB)}{P(B)} \quad (2)$$

$$\text{But, } P(B) = \frac{52 * 25}{52 * 51}$$

$$\text{or, } P(B) = \frac{25}{51}$$

$$\text{or, } P(B) \approx 0.4902 \quad (3)$$

$$\text{or, } P(AB) = \frac{52 * 1}{52 * 51}$$

$$\text{or, } P(AB) = \frac{1}{51}$$

$$\text{or, } P(AB) \approx 0.0196 \quad (4)$$

Thus, Using eqs. (1) to (4), we get:

$$P(A|B) = \frac{1}{25}$$

$$\text{or, } P(A|B) = 0.040 \quad (5)$$

This probability is smaller (approximately two-thirds of) than the probability computed in part a). Conceptually, this makes sense as the be restricting the outcomes to only those cards with the same colour, the total number of outcomes were reduced by around half ($52 * 25$ from $52 * 51$) but the number of successful outcomes reduced to a third of the original ($52 * 1$ from $52 * 3$).

- c. In part b), it was seen that the knowledge of event B happening altered our knowledge (decreased the probability of happening) of event A (eq. (5) compared to eq. (1)). This violates the condition of independence of events A and B as $P(A|B) \neq P(A)$.
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Problem 2

Solution

Problem 3

Solution

Problem 4

Solution

Problem 5

Solution

Problem 6

Solution

Problem 7

Solution
