$$J(x_0, 0) = \min_{\substack{u_j \in U_j \\ j=0,1...N-1}} \mathbb{E} \left[\sum_{j=0}^{N-1} \{L(x_j, u_j, j)\} + G(x_N, N) \right]$$
$$s.t. \tag{1}$$
$$x_{k+1} = f(x_j, u_j, j) + \sigma_{j+1}\xi_{j+1}$$
$$j = 0, 1, \ldots N-1$$

$$J(x_N, N) = G(x_N, N) \tag{2}$$
$$J(x_k, k) = \min_{u_k \in U_k} \mathbb{E}[J(x_{k+1}, k+1) + L(x_k, u_k, k)] \tag{3}$$

$$\bar{x}_{k+1} = f(\bar{x}_k, \bar{u}_k, k) \tag{4}$$
$$\bar{x}_0 = x_0$$
$$\tag{5}$$

$$f(x_k, u_k, k) = f(\bar{x}_k, \bar{u}_k, k) + f_x \delta x_k + f_u \delta u_k \tag{6}$$
$$+ \frac{1}{2} f_{xx}(\delta x_k)^2 + \frac{1}{2} f_{uu}(\delta u_k)^2$$
$$where$$
$$\delta x_k = x_k - \bar{x}_k$$
$$\delta u_k = u_k - \bar{u}_k$$