

MULTI-PERIOD OPTIMAL POWER FLOW

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To the Faculty of Washington State University:

The members of the Committee appointed to examine the dissertation of ARYAN RIT-
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MULTI-PERIOD OPTIMAL POWER FLOW

Abstract

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TABLE OF CONTENTS

	Page
ACKNOWLEDGMENT	iii
ABSTRACT	iv
LIST OF TABLES	vi
LIST OF FIGURES	vii
CHAPTER	
1 SOME FORMATTING EXAMPLES	1
1.1 Chapter one tittle section	1
2 FIGURES AND TABLES	2
2.1 Examples of a figure	2
REFERENCES	3
APPENDIX	
A Branch Flow Model: Relaxations and Convexification	5
B Abstracts: Optimization-based Methods for solving MP-OPF	7
C Abstracts: Dynamic Programming Methods for solving MP-OPF	8

LIST OF TABLES

A.1	Table describing the variables involved in the Branch Flow Model equations.	5
A.2	Table describing the Branch Flow Model equations.	6

LIST OF FIGURES

Dedication

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Chapter One

SOME FORMATTING EXAMPLES

1.1 Chapter one tittle section

Chapter Two

FIGURES AND TABLES

2.1 Examples of a figure

REFERENCES

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APPENDIX

Appendix A

Branch Flow Model: Relaxations and Convexification

In [2] the authors came up the Relaxed Branch Flow Model, and showed that in the case of Tree/Radial networks, the Relaxed Model can solve for the unique optimal solution, including the bus angles, and in the case of weakly meshed networks, there is a mechanism for extracting the bus angles from the relaxed solution, to find out its unique solution, if it exists.

Legend for Table A.2:

TABLE A.1 Table describing the variables involved in the Branch Flow Model equations.

Symbol	Meaning
p_j, q_j	Real, Reactive Power flowing from bus j into the network.
P_{ij}, Q_{ij}	Real, Reactive Power flowing in branch $i \rightarrow j$ (sending-end).
I_{ij}, l_{ij}	Complex Current flowing in branch

TABLE A.2 Table describing the Branch Flow Model equations.

Equation #	Equation	Unknowns	Knowns	No. of Equations
13	$p_j = \Sigma P_{jk} + \Sigma(P_{ij} - r_{ij}l_{ij}) + g_jv_j$	$1 \times p_0$	$n \times p_j$	$(n + 1)$
		$m \times P_{ij}$	$m \times r_{ij}$	
		$m \times l_{ij}$	$(n + 1) \times g_j$	
		$n \times v_j$	$1 \times v_0$	
14	$q_j = \Sigma Q_{jk} + \Sigma(Q_{ij} - x_{ij}l_{ij}) + b_jv_j$	$1 \times q_0$	$n \times q_j$	$(n + 1)$
		$m \times Q_{ij}$	$m \times x_{ij}$	
		$m \times l_{ij}$	$(n + 1) \times b_j$	
		$n \times v_j$	$1 \times v_0$	
15	$v_j = v_i + (r_{ij}^2 + x_{ij}^2)l_{ij} - 2(r_{ij}P_{ij} + x_{ij}Q_{ij})$	$m \times P_{ij}$	$b \times r_{ij}$	m
		$m \times Q_{ij}$	$m \times x_{ij}$	
		$m \times l_{ij}$	$1 \times v_0$	
		$n \times v_j$		
16	$l_{ij} = \frac{P_{ij}^2 + Q_{ij}^2}{v_j}$	$m \times P_{ij}$		m
		$m \times Q_{ij}$	$1 \times v_0$	
		$m \times l_{ij}$		
		$n \times v_j$		
13 to 16		$1 \times p_0$	$n \times p_j$	$2(n + 1 + m)$
		$1 \times q_0$	$n \times q_j$	
		$m \times P_{ij}$	$m \times r_{ij}$	
		$m \times Q_{ij}$	$m \times x_{ij}$	
		$m \times l_{ij}$	$(n + 1) \times g_j$	
		$n \times v_j$	$(n + 1) \times b_j$	
			$1 \times v_0$	
		$2(n + 1 + m)$	$4n + 2m + 3$	$2(n + 1 + m)$

Appendix B

Abstracts: Optimization-based Methods for solving MP-OPF

In [4], the authors use a two-step paradigm for solving the MP-OPF problem, by first solving for a more relaxed SOCP problem for the all of the time-steps in a horizon, and using the SOC values from its solution, solve for the NLP OPF problem for every time-step in parallel.

In [3], they prove that for 'realistic' systems, appending an additional 'complementarity' cost function to the original objective function, Simultaneous Charging and Discharging (SCD) in the optimal solution is avoided, and that the Mixed-Integer SOCP problem of AC-OPF with energy storage can be relaxed into a regular SOCP problem without violating the battery physics in the optimal solution.

Appendix C

Abstracts: Dynamic Programming

Methods for solving MP-OPF

In [1], the authors use a Differential Dynamic Programming approach, which involved usage of Forward and Backward passes made over a sequence of time-steps, doing a back-and-forth between computation for one time-step, say t , by making some assumptions on any variables required from the next time-step $t + 1$, and then updating the assumed values at t , once new values for the $t + 1$ time-step have been made.