Data Analysis for Predicting Instabilities in Power Systems

A THESIS

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Chapters for the Thesis

Chapter 1

Introduction

Unlike transient faults in a power grid which can generally be attributed to a sudden but tangible anomaly (corrective outcomes of protection mechanisms, sudden failure of a generator or transformer, line faults), steady state instabilities may remain undetected until they accumulate over time to manifest as a major upset to the grid [1] and/or make the grid less robust/more susceptible to collapses [2]. The causes of these disturbances is often an accruing of stochastic perturbations in the state variables of the grids when it is already stressed by an increased power demand [3]. The accrual of stochastic perturbations in turn can be caused due to various physical phenomena, including measurement noises, distributed renewable generation fluctuations [4], sudden gaps in power demand and supply due to power trading [2], strict operational frequency deadbands [5, 6], etc.

While modeling every significant possible source of stochastic disturbance can be difficult or perhaps even outright impossible, at least their detection can be made through model free data-driven statistical analysis, enabling early detection of grid stability problems for a timely course-corrective action [2, 7, 8].

Bifurcation Theory [3, 9–11] helps explain the erratic functioning of stressed dynamical systems such as the power grid, and the theory of Critical Slowing Down [12] lists tangible quantitative analysis tools which can help us detect an impending 'bifurcation' (blackout) in the power grid.

In this thesis, we first investigate various real-world grid frequency time series archives on their robustness against minor disturbances and any kind of long-standing stability problems in them, through the use of bulk distribution probability density functions and autocorrelation decay plots. We refer to this analysis as Offline/Postmortem analysis as the input data used is sampled for a long period (several months or years). Next, we investigate the effectiveness of two statistical parameters computed in real-time listed out as per the theory of Critical Slowing Down, namely Autocorrelation and Variance as Early Warning

Sign Indicators of an approaching bifurcation in a power grid. We label this analysis as the Online/Real-time analysis as the input data here is only in instantaneously available from a stream of PMU data.

Chapter 2

Literature Review and Objectives

For offline analysis of the grids, archived frequency time-series data of several real-world grids was downloaded from these websites and/or papers: [13–21]. Schafer et al's paper [2] was referred to for the analysis of these grids. Their paper analyzes how almost all grids show a significant level of deviation from the commonly used assumption that power demand fluctuations follow the Ornstein-Uhlenbeck's Process in which the state variable follows the Gaussian distribution around a nominal mean and a bounded standard deviation. They also explain some of the causes of detected instabilities for various power grids, including measurement noise and energy trading fluctuations.

Most of the recent literature analyzing the effects of Critical Slowing Down on various dynamical systems including but not limited to: power grids, ecological population dynamics, predator-prey ecosystems, prediction of epileptic seizures in patients, climate systems, financial markets, prediction of conversion of vegetation area into deserts, and so on, credit the review made by Scheffer et al in [12]. The paper lists out systems in which Critical Slowing Down has been observed and provides accessible mathematical explanations behind its working, such as why is there an increase of autocorrelation and variance of state variables of real-world physical processes as the system approaches a 'critical bifurcation'. The mathematical term has also been called appropriately as 'critical transition' or 'tipping point' by the author of [22], whose paper explains various normal forms of bifurcations (Fold, Hopf, Saddle Node, Transcritical, Pitchfork) via the concept of fast-slow stochastic systems. For developing a working understanding of bifurcation theory, university lecture notes by [9] and books [10, 11] were utilized.

For real-time/online analysis, authors in [8] have utilized PSAT [23] to simulate a steadily stressed power grid and have demonstrated that the computation of autocorrelation of detrended bus voltages and the computation of variance of detrended line currents can function as reliable Early Warning Signs of increasing instability. The detrending is required in order to filter any measurement noise from the data, which may skew the computed statistical parameters towards bogus values. Adeen et al's paper [4] simulated several Stochastic Differential Equations based on the Ornstein-Uhlenbeck's Process with different values of α (autocorrelation coefficient) and analyzed their Fourier Spectrums to conclude that an increased autocorrelation does in fact lead to a greater amplitude of noise and therefore a higher risk of instability in a power system. Authors in [7] tested various power grids which were driven towards bifurcation and demonstrated that an increase of autocorrelation and variance values of bus voltages (tested in simulation) and grid frequency (tested on the time-series data measured at the Bonneville Power Administration minutes before the blackout of 10 August 1996) can reliably predict the impending bifurcation early enough for mitigating actions to be taken by the grid operator.

For simulation implementations in PSSE 34.3, the community run website run by Jervis Whitley [24] was very helpful.

This thesis aims to highlight how statistical analysis can help predict/observe/detect steady state instabilities in power grids through both offline and online studies while making the least number of assumptions about the grid models themselves due to its data-driven approach. Statistical analysis can detect both lingering instability causing agents in the grids through Offline/Postmortem Analysis as well as predict any impending blackout/'bifurcation' in the grid through Online/Real-time Analysis. Typical power grid state variables such as Bus Voltages, Line Currents/MVAs and Grid Frequencies obtainable from a stream of PMU data may be used as inputs for such data-driven analysis. Tools used for Offline/Postmortem Analysis are visual inspection of bulk-distribution PDFs and estimating grid damping constants from autocorrelation decay curves. Tools used for Online/Real-time Analysis involve computing fixed-lag autocorrelation and variance of the filtered detrended fluctuations.

All simulations were done in Siemens PSSE 34.3 in conjunction with Python 2.7 (for writing automation scripts). All data analysis was conducted in MATLAB 2022a. A working implementation for anyone interested may be downloaded via [Simulation, Offline Analysis, Online Analysis]

Chapter 3

Theory

Detrended Fluctuation Analysis: A method of analyzing real-world time series for self-affinity, i.e. how correlated a signal's future value is to it's past values. Say, $x(t)$ is a signal from a natural process, then in order to detrend it, it can first be passed via a low-pass filter `LPF` to obtain $\texttt{LPF}(x(t))$ and the resultant signal be subtracted from the original in order to obtain the detrended version of the natural process signal $\tilde{x}(t)$ or $d(x(t))$:

$$\tilde{x}(t) \;\texttt{ or }\; d(x(t)) = x(t) - \texttt{LPF}(x(t)) \tag{1}$$

Autocorrelation function: A statistical measure of the correlation of a state variable with a time-lagged version of itself.

$$c(x(t), \tau) = \int_{-\infty}^{\infty} x(t)x(t+\tau)dt \tag{2}$$

In terms of discrete time functions, autocorrelation may be expressed as:

$$c(x[n], \tau) = \sum_{-\infty}^{\infty} x[n]x[n+\tau] \tag{3}$$

It may be noted that the autocorrelation function of any time-varying variable has two degrees of freedom, the first being continuous time t (or instance n for discrete time functions) and the second parameter being the lag τ, i.e. the time duration by which the time-varying variable is displaced/lagged against itself for performing the autocorrelation.

Since in this thesis the function is used in two ways where only one of the parameters is varying (with the other being kept constant) each time, two variations of the autocorrelation function with individual names are specified here:

Fixed Time Autocorrelation: Autocorrelation function computed over a snapshot of a time-varying variable over a fixed time window. The usage of Fixed Time Autocorrelation can be seen in the Offline Analysis chapter of this thesis.

$$c(\tau) = \frac{\sum_{1}^{W_{total}} x[n]x[n+\tau]}{\sum_{1}^{W_{total}} x[n]^2} \tag{4}$$

In this thesis, the window length W_{total} ranges from months to years.

Fixed Lag Autocorrelation: Autocorrelation function computed over a window running over a continuously generated stream of data in which the value of the lag τ is fixed. The usage of Fixed Lag Autocorrelation can be seen in the Online Analysis of this thesis.

$$c(t)|_{\tau=\tau_{fixed}} = \frac{\sum_{i}^{i+W} x[n]x[n+\tau_{fixed}]}{\sum_{i}^{i+W} x[n]^2} \tag{5}$$

In this thesis, the window length used is $W = 15$ seconds. The value of τ used is $\tau_{fixed} = 1$ second.

Bifurcation Theory: We'll be using the concepts Bifurcations and Critical Bifurcations to explain why a small yet steady change in the parameters of a dynamical system (such as the power grid) can remain inconspicuous only to, upon reaching a 'tipping' point or 'Critical Transition', manifest as a sudden major upset to the 'motion' of the dynamical system. We use the words Bifurcations and Critical Bifurcations almost interchangeably, although only Critical Bifurcations are significant enough to alter the dynamics of a system from stable to unstable.

Critical Slowing Down: The theory of Critical Slowing Down applies on dynamical systems on the verge of 'tipping' or 'bifurcation' and how they show warning signs before breaking down or descending into instability. These warning signs such as an 'increased time to settle', 'increased autocorrelation and variance of fluctuations', etc. [12] can be statistically analyzed to predict the onset of such a bifurcation for a given system. Autocorrelation $c(t, \tau)$ of any detrended physical/natural signal $\tilde{x}(t)$ or $d(x(t))$, should decrease exponentially as the time-lag τ is increased.

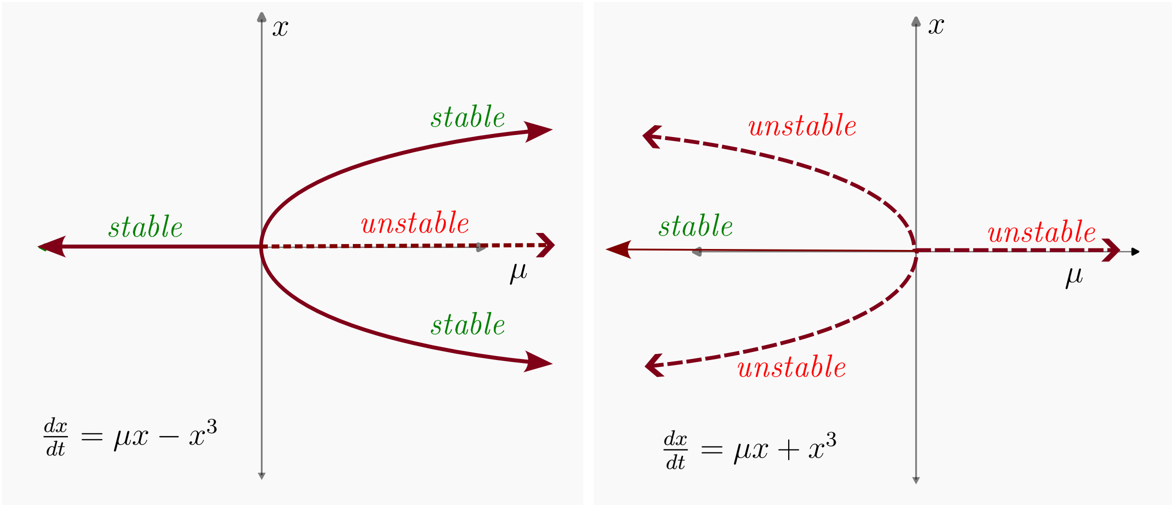

Figure 1: Bifurcation diagram for the normal form of the Pitchfork Bifurcation $\frac{dx}{dt} = \mu x - x^3$. For $\mu \leq 0$, the only stable equilibrium solution for the dynamical system is $x = 0$. Upon reaching the critical 'tipping' point $\mu = 0$, $x = 0$ no longer remains a stable equilibrium path for the dynamical system and instead two different stable paths emerge: $x = \sqrt{+\mu}$ and $x = -\sqrt{\mu}$. Such a critical point, in which one stable path bifurcates into at least two distinct paths is called a bifurcation point and the phenomenon is known as 'Critical Bifurcation'.

Chapter 4

Offline Analysis

Various frequency time-series archives for a diverse set of real-world grids were obtained and analyzed by plotting their bulk distribution probability density functions and autocorrelation decay functions. The data for most European and US grids was conveniently curated by the authors of [13, 14]. For the other regions of the world, [15, 16] had the data for the Tokyo grid, [17, 18] for the Nordic grids, [19, 20] for Continental European grid and [21] for the UK Grid. All time series were collected at sampling rates between 0.5 seconds (Tokyo) and 10 seconds (Continental Europe).

Here is a table of sampling times and total duration of times over which the frequency time series archive were obtained.

The plotted bulk distribution PDFs visually revealed insights including any deadbands [5, 6] mandated in their grid operation, their skewness, thickness of their tails, etc. A quantitative study of their moments like kurtosis and skewness was not conducted as was

Table 1: Grid-wise sampling data

Grid	Nominal Frequency	Sampling time	Total Sampled Duration	Presented in Fixed Time Autocorrelation Plot?
Continental European (CE)	50Hz	1s	1 year (2019)	Yes
Nordic	50Hz	0.5s	2 years (2018 and 2019)	Yes
Great Britain (GB)	50Hz	0.5s	2 years (2019 and 2020)	Yes
Mallorcan (Spain)	50Hz	1s	3 months (Oct to Dec 2019)	Yes
Western Interconnection (US-WI)	60Hz	1s	7 days (in May 2019)	Yes
Texas (US-TX)	60Hz	1s	3 days (in May 2019)	No
Tokyo	50Hz	1s	5 months (Jan, July, Aug, Oct, Dec 2020)	No
France (RTE)	50Hz	10s	1 year (2019)	No
Indian (NRLDC)	50Hz	30s	5 days (3 days in 2019 and 2 days in 2020)	No

done in [2].

In the plotted autocorrelation decay curves, all grids showed exponentially decreasing autocorrelations $c(\tau)$ for smaller values of τ. In order to confirm if the decrease was indeed exponential with a grid-specific decay constant also called as the inverse-time correlation or ${t_{corr}}^{-1}$, semi-log graphs ($\log(c(\tau))$ vs τ) were also plotted. The decay constants were computed by calculating the slopes of the semi-log graphs and it was found that the grids which were bigger, more robust and showed bulk distribution PDFs which were less-deviating from Gaussian distributions, had greater values of inverse-time correlation ${t_{corr}}^{-1}$. This parameter can also be construed as α, the relative damping strength of the grid for small oscillations and can be an indicator of the overall robustness of the grid.

Chapter 5

Online Analysis

On similar lines as [7, 8], we're interested in testing if symptoms of Critical Slowing Down can be detected by a real-time/online analysis of the state variables of a power grid. In other words, we're interested in checking if computing the autocorrelation and variance of real-time PMU data processed over a running window can provide us with Early Warning Signs of an impending instability.

Bifurcation Theory states that a small change in system parameters, such the governor reference power for a generator (P_{Gen}) at certain points, can lead to major upsets in the stability of the power grid. We ran a simulation in which a system was purposefully stressed (via a near constant linear load increment) as time progressed but many restrictions/safety mechanisms were lifted with the aim of singling-out the cause of bifurcation to a change in P_{Gen}(s), in order to best demonstrate that the proposed statistical mechanisms (computing autocorrelations and variances) function well as Early Warning Signs even for slow and steady variations of loads, and not just for sudden changes in state variables caused due to reactionary corrective protection mechanisms or the machines not being given 'free-range' for chasing load increments due to specified safety limits on maximum allowed generated powers. Below is the set of special conditions used for the simulation of the IEEE 9 Bus system:

1. The three load points of the system (Buses 5, 6 and 8) were linearly increased in time, at a rate of $\Delta P\%$ per minute plus a small white noise component $\mathcal{N}(0, \sigma_v)$, with every increment happening at Δt time intervals.

$$P_{L_i}(t + \Delta t) = P_{L_i}(t) * \left(1 + \frac{\Delta P_{L_i}}{100}\right) + \mathcal{N}(0, \sigma_v) \tag{6}$$

   Here, we assigned ΔP_L values randomly between $8 - 12\%$ for every load bus, $\sigma_v = 0.01$ and $\Delta t = 0.1$ seconds.

2. Simulation ODE solver solves for the new state variables for the system every 0.01 seconds. This means that the simulation output can be likened to a stream of PMU data whose sampling rate is 100 Hz.

3. Protection mechanisms were disabled. No remedial/corrective action was taken for any drop in bus voltages/grid frequency or any increase in line currents/MVAs.

4. 'Dummy' governors were placed on the three generators (at buses 1, 2 and 3) which could respond instantly to load changes by changing the set reference generation powers P_{Gen}(s) with zero time lag.

5. The generator limits for $P_{Gen_{MAX}}$, $Q_{Gen_{MAX}}$, etc. were removed. Thus the generators had complete freedom to 'chase' the load increments at the load buses, including factoring in the extra line-losses.

It should be noted that while autocorrelation was used in both online/real-time and offline/postmortem analyses, the two usages were different in:

- their mode of procuring and processing input data (a running window of an incoming stream of data vs previously stored months/years worth of time series),
- the degrees of freedom allowed for its two parameter variables (which out of t and τ is allowed to be constant),
- their theoretically expected output data (autocorrelation is should decrease exponentially with respect to time lag τ but increase with time t if that the system is being progressively stressed with time)

Chapter 6

Summary and Conclusions

Offline/Postmortem Analysis

- Frequency time-series for months/years of data obtained from various real-world grids were converted into probability distribution function plots and autocorrelation decay plots ($c(\tau)$ vs τ plots).
- Visual inspection of the probability distribution function plots provided many insights into the presence of long-standing steady-state instabilities in the grid as well as the grid's resilience against any additional instability causing agents. Generally the PDFs of the more robust grids such as the RTE (France) and Continental European grids were mostly Gaussian except that they had heavier tails, whereas the smaller or island grids, such as the Mallorcan (Spain) grid had multiple peaks, skewed distributions and thus an overall visible deviation from Gaussianity which explains their higher susceptibility to steady-state deviations and thus a greater degree of vulnerability to grid failures.

- For most grids, the autocorrelation functions exponentially decayed with respect to time lag τ for smaller values of τ but certain grids showed significant deviation from the expected norm. For example the Continental European and UK grids showed a spike in autocorrelation decay function at time lags of every 15 minutes. This spike, which indicates an inherent instability causing agent in the grid systems, can be attributed to their 15 minute power trading intervals. Unlike the amount of transacted power which is suddenly varied every 15 minutes, the power grids, being dynamical systems cannot instantly adjust to the new power settings and thus the sudden imbalance of supply and demand leads to transients in the grid state variables.

- Autocorrelation decay curves of other grids (Nordic, Japan, US-Western Interconnection) initially decreased exponentially but later followed between a very slowly decaying or almost constant curve with respect to τ. This can be attributed to measurement noise in the frequency detection.

- From the initial exponential decay of the curves, semi-log graphs were plotted and their inverse correlation times ${t_{corr}}^{-1}$ were obtained. As per the Ornstein-Uhlenbeck Process this inverse correlation time can be likened to the damping constant α of the grids. As per our theoretical expectations, the bigger and more robust grids had higher values of α compared to the smaller, islanded grids.

Online/Real-time Analysis

- The IEEE 9 Bus System was progressively stressed in a time-domain simulation until 'bifurcation' was achieved [7]. In terms of implementation, 'bifurcation' was concluded to have taken place when the simulation solver could no longer converge to a solution without violating convergence thresholds. PSSE 34.3 simply calls out this occurrence as 'Network Not Converged'.

- The bus voltages were detrended with the help of a low pass filter, and their variance σ^2 as well as autocorrelations $c(t, \tau)$ with a fixed time lag $\tau = 1$ instance were computed over a running window.

- A new statistical parameter, called the Modified Kendall's τ Correlation Coefficient (MKTCC) was employed to check if the increase in the autocorrelations and variances was statistically significant. The reason for using a modified version of the normally used Kendall's τ Correlation Coefficient was to accommodate for the degree of certainty/confidence in predicting the correlation apart from the absolute value of correlation itself.

- Both autocorrelation and variance were found to be appropriate Early Warning Sign Indicators for an impending bifurcation, predicting the event minutes earlier.

Chapter 7

Future Work

The grid analyzed for online/real-time analysis should be bigger, in order to demonstrate spatial variation in the early warning sign indicators for different buses/areas and for singling out areas which are more vulnerable to steady state instabilities.

Despite the successful application of statistical analysis to detect symptoms of Critical Slowing Down in various phenomena [12], autocorrelation and variance are not certain indicators for the same, at least by themselves [25]. In order to tackle that, statistical parameters other than autocorrelation and variance can be investigated for their feasibility as Early Warning Signal indicators. Even for the same statistical indicators, changing the length of the running window, time lag τ, sampling rate etc. can have a significant effect on their effectiveness. Thus an 'optimal' set of parameters could be researched for, which may be different for different grids, but shouldn't vary for a particular grid once computed. On similar lines, grid state variables other than bus voltages, line current/MVAs, grid frequencies may be investigated.

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