SCALABLE MULTI-PERIOD OPTIMAL POWER FLOW ALGORITHMS FOR ACTIVE DISTRIBUTION SYSTEMS

By

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SCALABLE MULTI-PERIOD OPTIMAL POWER FLOW

ALGORITHMS FOR

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Abstract

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TBA

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Dedication

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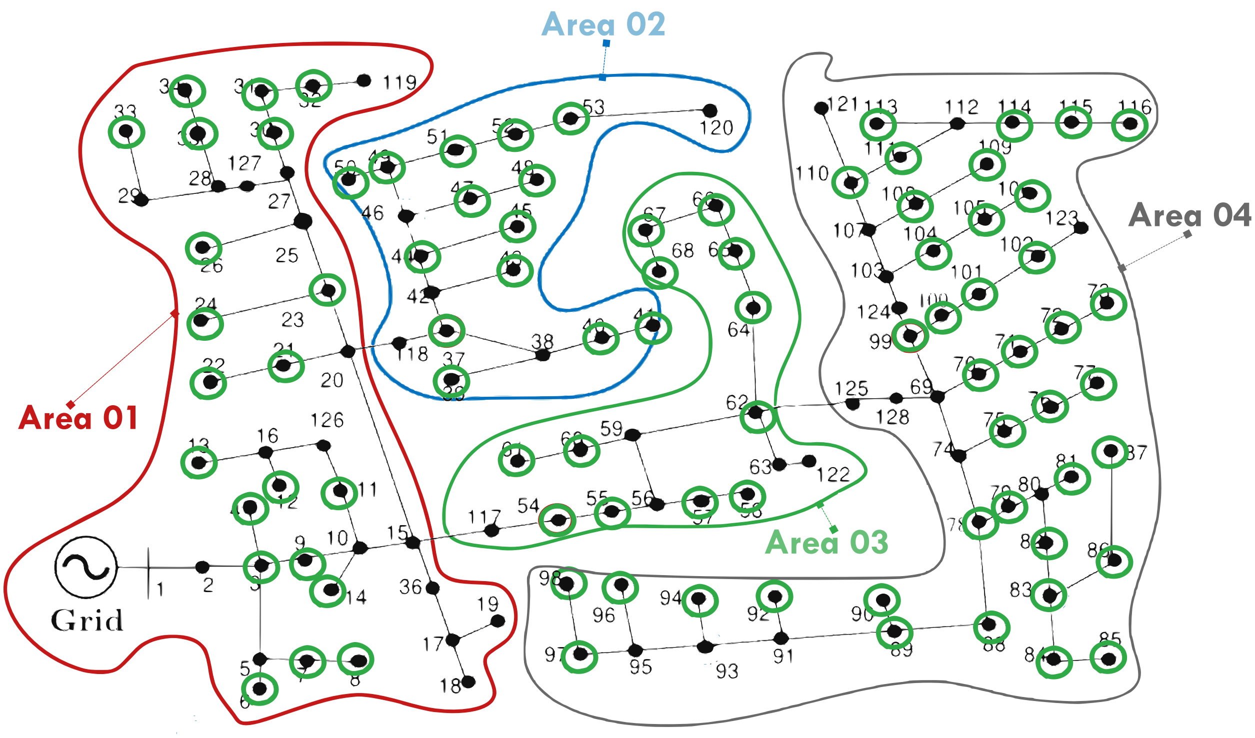

Figure 1 IEEE 123 Node System divided into four areas with DER nodes labelled.

Chapter One

FIGURES AND TABLES

1.1 Examples of a figure

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APPENDIX

Appendix A

Branch Flow Model: Relaxations and Convexification

In [2] the authors came up the Relaxed Branch Flow Model, and showed that in the case of Tree/Radial networks, the Relaxed Model can solve for the unique optimal solution, including the bus angles, and in the case of weakly meshed networks, there is a mechanism for extracting the bus angles from the relaxed solution, to find out its unique solution, if it exists.

Legend for Table A.2:

Table A.1 Table describing the variables involved in the Branch Flow Model equations.

Symbol	Meaning
p_j, q_j	Real, Reactive Power flowing from bus j into the network.
P_{ij}, Q_{ij}	Real, Reactive Power flowing in branch (i, j) (sending-end).
I_{ij}, l_{ij}	Complex Current flowing in branch

TABLE A.2 Table describing the Branch Flow Model equations.

Equation #	Equation	Unknowns	Knowns	No. of Equations
13	$p_j = \Sigma P_{jk} + \Sigma(P_{ij} - r_{ij}l_{ij}) + g_j v_j$	$1 \times p_0$ $m \times P_{ij}$ $m \times l_{ij}$ $n \times v_j$	$n \times p_j$ $m \times r_{ij}$ $(n+1) \times g_j$ $1 \times v_0$	$(n+1)$
14	$q_j = \Sigma Q_{jk} + \Sigma(Q_{ij} - x_{ij}l_{ij}) + b_j v_j$	$1 \times q_0$ $m \times Q_{ij}$ $m \times l_{ij}$ $n \times v_j$	$n \times q_j$ $m \times x_{ij}$ $(n+1) \times b_j$ $1 \times v_0$	$(n+1)$
15	$v_j = v_i + (r_{ij}^2 + x_{ij}^2)l_{ij} - 2(r_{ij}P_{ij} + x_{ij}Q_{ij})$	$m \times P_{ij}$ $m \times Q_{ij}$ $m \times l_{ij}$ $n \times v_j$	$b \times r_{ij}$ $m \times x_{ij}$ $1 \times v_0$	m
16	$l_{ij} = \frac{P_{ij}^2 + Q_{ij}^2}{v_j}$	$m \times P_{ij}$ $m \times Q_{ij}$ $m \times l_{ij}$ $n \times v_j$	$1 \times v_0$	m
13 to 16		$1 \times p_0$ $1 \times q_0$ $m \times P_{ij}$ $m \times Q_{ij}$ $m \times l_{ij}$ $n \times v_j$	$n \times p_j$ $n \times q_j$ $m \times r_{ij}$ $m \times x_{ij}$ $(n+1) \times g_j$ $(n+1) \times b_j$ $1 \times v_0$	$2(n+1+m)$
		$2(n+1+m)$	$4n+2m+3$	$2(n+1+m)$

Appendix B

Abstracts: Optimization-based Methods for solving MP-OPF

In [5], the authors use a two-step paradigm for solving the MP-OPF problem, by first solving for a more relaxed SOCP problem for the all of the time-steps in a horizon, and using the SOC values from its solution, solve for the NLP OPF problem for every time-step in parallel.‘’

In [4], they prove that for ’realistic’ systems, appending an additional ‘complementarity’ cost function to the original objective function, Simultaneous Charging and Discharging (SCD) in the optimal solution is avoided, and that the Mixed-Integer SOCP problem of AC-OPF with energy storage can be relaxed into a regular SOCP problem without violating the battery physics in the optimal solution.

In [8], the authors present a linear approximation based on Taylor’s second order terms of a grid with various components associated with nonlinear and mixed-integer decision variables, and use a *Successive Linear Approximation* algorithm to tighthen the relaxation to capture the nonlinearity of the original system.

Original Problem: Mixed-Integer Nonlinear Optimization Model - Full Horizon

$$\min_{q^t_{D_j},P^t_{c_j},P_{d_j}{}^t,q^t_{B_j}} \quad \sum_{t=1}^{T} \sum_{(i,j)\in\mathcal{L}} (r_{ij} l^t_{ij}) \tag{B.1}$$

s.t.

$$\sum_{(j,k)\in\mathcal{L}} P^t_{jk} = P^t_{ij} + (p^t_{D0_j} - p^t_{Dcurt_j}) + (P^t_{cj} + P^t_{dj}) - (p^t_{L0j} + p^t_{Lodj} - p^t_{Ludj}) \tag{B.2}$$

$$\sum_{(j,k)\in\mathcal{L}} Q^t_{jk} = Q^t_{ij} + (q^t_{D_j}) + () - \big(q^t_{L0j} + \tan(\psi^t_j)(p^t_{Lodj} - p^t_{Ludj})\big) \tag{B.3}$$

$$P^t_{jk} = g_{jk}(V^t_j)^2 - g_{jk}V^t_jV^t_k\cos(\theta^t_{jk}) - b_{jk}V^t_jV^t_k\sin(\theta^t_{jk}) \tag{B.4}$$

$$Q^t_{jk} = -b_{jk}(V^t_j)^2 + b_{jk}V^t_jV^t_k\cos(\theta^t_{jk}) - g_{jk}V^t_jV^t_k\sin(\theta^t_{jk}) \tag{B.5}$$

$$\big[(V^t_j)^2 + (V^t_k)^2 - 2V^t_jV^t_k\cos(\theta^t_{jk})(g^2_{jk} + b^2_{jk})\big] \in [0, |\overline{I}_{jk}|^2] \tag{B.6}$$

$$B^t_j{=}B^{t-1}_j + z\Delta t\eta_c P^t_{c_j} - (1-z)\Delta t\frac{1}{\eta_d}P^t_{d_j} \tag{B.7}$$

$$B^0_j = 0.5(soc_{max} + soc_{min})E_{Rated} = 0.625E_{Rated} \tag{B.8}$$

$$B^T_j = B^0_j \tag{B.9}$$

$$where, \tag{B.10}$$

$$(i,j) : \text{Branch connecting nodes } i \text{ and } j \tag{B.11}$$

$$p^t_j = p_{Dj}{}^t - p_{Lj}{}^t \tag{B.12}$$

$$q^t_j = -q_{Lj}{}^t \tag{B.13}$$

$$t = \{1, 2, \ldots T\} \tag{B.14}$$

$$z = \{0, 1\} \tag{B.15}$$

Appendix C

Abstracts: Dynamic Programming Methods for solving MP-OPF

In [1], the authors use a Differential Dynamic Programming approach, which involved usage of Forward and Backward passes made over a sequence of time-steps, doing a back-and-forth between computation for one time-step, say t, by making some assumptions on any variables required from the next time-step $t+1$, and then updating the assumed values at t, once new values for the $t+1$ time-step have been made.

Appendix D

Abstracts: Differential Dynamic Programming

In [6], the authors lay out the framework on how Differential Dynamic Programming can be utilized to solve a Mutli-Stage Uncertain Optimal Control problem programmatically.

$$
\begin{aligned}
J(x_0, 0) = & \min_{\substack{u_j \in U_j \\ j=0,1\ldots N-1}} \mathbb{E}\left[\sum_{j=0}^{N-1}\{L(x_j, u_j, j)\} + G(x_N, N)\right] \\
& s.t. \\
& x_{k+1} = f(x_j, u_j, j) + \sigma_{j+1}\xi_{j+1} \\
& j = 0, 1, \ldots N-1
\end{aligned} \tag{D.1}
$$

$$
J(x_N, N) = G(x_N, N) \tag{D.2}
$$

$$
J(x_k, k) = \min_{u_k \in U_k} \mathbb{E}\left[J(x_{k+1}, k+1) + L(x_k, u_k, k)\right] \tag{D.3}
$$

$$
\begin{aligned}
\bar{x}_{k+1} &= f(\bar{x}_k, \bar{u}_k, k) \\
\bar{x}_0 &= x_0
\end{aligned} \tag{D.4}
$$

$$
\begin{aligned}
f(x_k, u_k, k) = f(\bar{x}_k, \bar{u}_k, k) &+ f_x \delta x_k + f_u \delta u_k \\
&+ \frac{1}{2} f_{xx}(\delta x_k)^2 + \frac{1}{2} f_{uu}(\delta u_k)^2 \\
where & \\
\delta x_k &= x_k - \bar{x}_k \\
\delta u_k &= u_k - \bar{u}_k
\end{aligned} \tag{D.5}
$$

$$
\begin{aligned}
\delta x_{k+1} &= f(x_k, u_k, k) + \sigma_{k+1}\xi_{k+1} - f(\bar{x}_k, \bar{u}_k, k) \\
\text{or, } \delta x_{k+1} &= f_x \delta x_k + f_u \delta u_k + \frac{1}{2} f_{xu}(\delta x_k)(\delta u_k) \qquad \text{(D.6)} \\
&+ \frac{1}{2} f_{xx}(\delta x_k)^2 + \frac{1}{2} f_{uu}(\delta u_k)^2 + \sigma_{k+1}\xi_{k+1}
\end{aligned}
$$

Similarly,

$$
\begin{aligned}
L(x_k, u_k, k) &= L(\bar{x}_k, \bar{u}_k, k) + L_x \delta x_k + L_u \delta u_k \\
&+ \frac{1}{2} L_{xx}(\delta x_k)^2 + \frac{1}{2} L_{uu}(\delta u_k)^2 \qquad \text{(D.7)}
\end{aligned}
$$

and,

$$
J(x_k, k) = J(\bar{x}_k, k) + J_x \delta x_k + \frac{1}{2} J_{xx}(\delta x_{k+1})^2 \qquad \text{(D.8)}
$$

Appendix E

Models: Battery Model for Multi-Period OPF

All subscripts j for a variable imply the node in the power grid (for node j). All superscripts t refer to the time period number t.

TABLE E.1 Description of Grid Parameters

Variable	Description
$\mathcal{N}$	Set of all nodes. $\mathcal{N} = \{1, 2, \ldots n\}$
$\mathcal{L}$	Set of all branches. $\mathcal{L} = \{1, 2, \ldots l\} = \{(i, k)\} \subset (\mathcal{N} \times \mathcal{N})$.
$\mathcal{D}$	Set of all nodes containing DERs. $\mathcal{D} \subset \mathcal{N}$
$\mathcal{B}$	Set of all nodes containing storage. $\mathcal{B} \subset \mathcal{N}$
Δt	Duration of a single time period. Here $\Delta t = 15$ min. $= 0.25$ h.
T	Prediction Horizon Duration. Total number of time-intervals solved for as part of one instance of MP-OPF.

Table E.2 Description of Branch Flow Model Variables

Variable	Description
p_j^t	Fixed Real Power Generation minus Fixed Real Power Load. Here, $p_j^t = {p_{D}}_j{}^t - {p_{L}}_j{}^t$. A known (predicted) value $\forall\, t, j$.
q_j^t	Fixed Reactive Power Generation minus Fixed Reactive Power Load. Here, $q_j^t = -{q_{L}}_j{}^t$. A known (predicted) value $\forall\, t, j$.
$p_{D_j}^t$	Real Power Generated by DERs
$p_{L_j}^t$	Real Power Demand
$q_{L_j}^t$	Reactive Power Demand

Table E.3 Values, Lower Bounds and Upper Bounds on BFM Variables

Variable	Value or Limits	Description
$P_{DER_{Max}}$	$\sim [2, 20]\,$kW	P_{Rated} of corresponding DER.
$\lvert q_{B_{Max}} \rvert$	$\sim [1, 10]\,$kVAr	$q_{D_{Rated}}$ of corresponding DER.
V_{min}	$0.95\, pu$	
V_{Max}	$1.05\, pu$	
v	$[V_{min}^2, V_{Max}^2]$	Squared magnitude of nodal voltage

In [4, 5], the batteries are modelled using four state/control variables, which are:

Table E.4 Description of Battery Variables

Variable	Description	Dimension	Dimension pu
$B_{n,t}$	State of Charge (SOC) of Battery after the t-th time interval.	$[kWh]$	$[pu\,h]$
$P^c_{n,t}$	Average Charging Power of the Battery during the t-th time interval.	$[kW]$	$[pu]$
$P^d_{n,t}$	Average Discharging Power of the Battery during the t-th time interval.	$[kW]$	$[pu]$
$q_{B_{n,t}}$	Average Reactive Power Output from the Battery Inverter during the t-th time interval.	$[kVAr]$	$[pu]$

where,

$t \in \{1, 2, \ldots, T\}$ The index of the discretized time intervals, where t represents the t-th time interval of duration Δt.

$n \in \mathcal{N}$ The node n is an element of the set of all nodes in the power grid $\mathcal{N}$. Note that n can be used both as an iterator or as the total number of nodes in the grid (i.e. the cardinality of $\mathcal{N}$), and its meaning should be obvious from context.

Table E.5 Values, Lower Bounds and Upper Bounds on Battery Variables (Part 1/2)

Variable	Value or Limits	Description
P_{Max}	$\sim [2, 20]\,\mathrm{kW}$	P_{Rated} of corresponding DER.
$\lvert q_{B_{Max}} \rvert$	$\sim [1, 10]\,\mathrm{kVAr}$	$q_{D_{Rated}}$ of corresponding DER.
P_d, P_c	$[0, P_{Max}]$	
q_B	$[-q_{B_{Max}}, q_{B_{Max}}]$	Currently linearly modeled (is actually quadratic)
E_{Rated}	$P_{Max} \times 4$ h	4 h of one-way Charging/Discharging at Maximum Power
B	$[0.30E_{Rated}, 0.95E_{Rated}]$	2.4 h of one-way Charging/Discharging at Maximum Power
B_0	$0.625E_{Rated}$	Batteries start with an SOC value in the middle of their SOC range.
η_d, η_c	0.95	

TABLE E.6 Values, Lower Bounds and Upper Bounds on Battery Variables (Part 2/2)

Variable	Value or Limits	Description
α	1e−3	Coefficient of auxiliary objective function penalizing SCD. Value depends on the magnitude of the loss term in the objective function
γ	50	Coefficient of auxiliary objective function penalizing deviation of final SOC value from a reference.

[1] A note on α: Too big a value of α would reduce both P_c and P_d terms to zero, whereas too small a value would not penalize SCD, causing physically infeasible solutions.

Optimization Equations

Original Problem: Mixed-Integer Nonlinear Optimization Model - Full Horizon

$$\min_{q_{D_j}^t, P_{c_j}^t, P_{d_j}{}^t, q_{B_j}^t} \quad \sum_{t=1}^{T} \sum_{(i,j)\in\mathcal{L}} (r_{ij} l_{ij}^t) \tag{E.1}$$

s.t.

$$p_j^t = \sum_{(j,k)\in\mathcal{L}} P_{jk}^t - \sum_{(i,j)\in\mathcal{L}} \{P_{ij}^t - r_{ij} l_{ij}^t\} - P_{d_j}^t + P_{c_j}^t \tag{E.2}$$

$$q_j^t = \sum_{(j,k)\in\mathcal{L}} Q_{jk}^t - \sum_{(i,j)\in\mathcal{L}} \{Q_{ij}^t - x_{ij} l_{ij}^t\} - q_{D_j}^t - q_{B_j}^t \tag{E.3}$$

$$v_j^t = v_i^t + \{r_{ij}^2 + x_{ij}^2\} l_{ij}^t - 2(r_{ij} P_{ij}^t + x_{ij} Q_{ij}^t) \tag{E.4}$$

$$l_{ij}^t = \frac{(P_{ij}^t)^2 + (Q_{ij}^t)^2}{v_i^t} \tag{E.5}$$

$$B_j^t = B_j^{t-1} + z\Delta t \eta_c P_{c_j}^t - (1-z)\Delta t \frac{1}{\eta_d} P_{d_j}^t \tag{E.6}$$

$$B_j^0 = 0.5(soc_{max} + soc_{min})E_{Rated} = 0.625 E_{Rated} \tag{E.7}$$

$$B_j^T = B_j^0 \tag{E.8}$$

$$where, \tag{E.9}$$

$$(i,j) : \text{Branch connecting nodes } i \text{ and } j \tag{E.10}$$

$$p_j^t = p_{Dj}{}^t - p_{Lj}{}^t \tag{E.11}$$

$$q_j^t = -q_{Lj}{}^t \tag{E.12}$$

$$t = \{1, 2, \ldots T\} \tag{E.13}$$

$$z = \{0, 1\} \tag{E.14}$$

(Integer Constraint Relaxed) Naive Brute Force Full Optimization Model - Full Horizon

Network Constraints

$$p_j^t = \sum_{(j,k)\in\mathcal{L}} P_{jk}^t - \sum_{(i,j)\in\mathcal{L}} \left\{P_{ij}^t - r_{ij}l_{ij}^t\right\} - P_{d_j}^t + P_{c_j}^t \tag{E.15}$$

$$q_j^t = \sum_{(j,k)\in\mathcal{L}} Q_{jk}^t - \sum_{(i,j)\in\mathcal{L}} \left\{Q_{ij}^t - x_{ij}l_{ij}^t\right\} - q_{D_j}^t - q_{B_j}^t \tag{E.16}$$

$$v_j^t = v_i^t + \left\{r_{ij}^2 + x_{ij}^2\right\} l_{ij}^t - 2(r_{ij}P_{ij}^t + x_{ij}Q_{ij}^t) \tag{E.17}$$

$$l_{ij}^t = \frac{(P_{ij}^t)^2 + (Q_{ij}^t)^2}{v_i^t} \tag{E.18}$$

$$where, \tag{E.19}$$

$$(i,j) : \text{Branch connecting nodes } i \text{ and } j \tag{E.20}$$

$$p_j^t = {p_{Dj}}^t - {p_{Lj}}^t \tag{E.21}$$

$$q_j^t = -{q_{Lj}}^t \tag{E.22}$$

$$t = \{1, 2, \dots T\} \tag{E.23}$$

where,

P_{ij}^t : Average Real Power Flow in Branch (i,j) during time-step t

$$v_j^t \in [0.95^2, 1.05^2] \tag{E.24}$$

$$l_{ij}^t \in [0, I_{Rated_{ij}}^2] \tag{E.25}$$

$$P_{Subs}^t \in [0, P_{SubsPeak}] \tag{E.26}$$

Storage Modelling

$$B_j^t = B_j^{t-1} + \Delta t \eta_c P_{c_j}^t - \Delta t \frac{1}{\eta_d} P_{d_j}^t \tag{E.27}$$

$$B_j^T = B_j^0 \tag{E.28}$$

$$B_j^0 = 0.625 E_{Rated,j} \tag{E.29}$$

where,

$P_{c,j}^t$: Average Charging Power of Battery j during time-step t

$P_{d,j}^t$: Average Discharging Power of Battery j during time-step t

B_j^t : SOC of Battery j at the end of t time-steps

q_{Bj}^t : Average Reactive Power of Battery j during time-step t

B_j^0 : SOC of Battery j at the beginning of the horizon

$P_{Rated,j}$: Rated Charging/Discharging Power of Battery j

$E_{Rated,j}$: Rated Maximum SOC of Battery j

$S_{Rated,j}$: Rated Apparent Power of Inverter at Battery j

$$soc_{min}, soc_{max} = 0.30, 0.95 \tag{E.30}$$

$$\eta_c, \eta_d = 0.95, 0.95 \tag{E.31}$$

$$P_{cj}^t, P_{dj}^t \in [0, P_{Rated,j}] \tag{E.32}$$

$$P_{cj}^t \cdot P_{dj}^t = 0 \tag{E.33}$$

$$S_{Rated,j} = 1.2 P_{Rated,j} \tag{E.34}$$

$$(P_{dj}^t - P_{cj}^t)^2 + (q_{Bj}^t)^2 \leq (S_{Rated,j})^2 \tag{E.35}$$

$$B_j^t \in [soc_{min}, soc_{max}] * E_{Rated,j} \tag{E.36}$$

$$q_{Bj}^t \in [-\sqrt{0.44}, \sqrt{0.44}] * S_{Rated,j} \tag{E.37}$$

$$t = \{1, 2, \ldots T\} \tag{E.38}$$

Solar PV Modelling

$$p^t_{D,j} : \text{Average Generated Real Power of DER } j \text{ during time-step } t$$

$$q^t_{Dj} : \text{Average Reactive Power of DER } j \text{ during time-step } t$$

$$S_{Rated,j} : \text{Rated Apparent Power of Inverter at DER } j$$

$$S_{Rated,j} = 1.2 P_{Rated,j} \tag{E.39}$$

$$(p^t_{Dj})^2 + (q^t_{Dj})^2 \leq (S_{Rated,j})^2 \tag{E.40}$$

$$q^t_{Dj} \in [-\sqrt{0.44}, \sqrt{0.44}] * S_{Rated,j} \tag{E.41}$$

$$t = \{1, 2, \ldots T\}$$

Objective Function Examples

Minimizing Line Losses with soft constraint on SCD

$$\begin{aligned} \min_{q^t_{D_j}, P^t_{c_j}, P_{d_j}{}^t, q^t_{B_j}} \quad & \sum_{t=1}^{T} \sum_{(i,j)\in\mathcal{L}} (r_{ij} l^t_{ij}) \\ & + \alpha \sum_{t=1}^{T} \sum_{j\in\mathcal{B}} \left\{ (1-\eta_c) P^t_{c_j} + \left(\frac{1}{\eta_d} - 1\right) P^t_{d_j} \right\} \end{aligned} \tag{E.42}$$

s.t.

Minimizing Line Losses with peak-shaving soft constraint on SCD

$$\begin{aligned} \min_{q^t_{D_j}, P^t_{c_j}, P_{d_j}{}^t, q^t_{B_j}} \quad & \sum_{t=1}^{T} \sum_{(i,j)\in\mathcal{L}} (r_{ij} l^t_{ij}) \\ & + \alpha \sum_{t=1}^{T} \sum_{j\in\mathcal{B}} \left\{ (1-\eta_c) P^t_{c_j} + \left(\frac{1}{\eta_d} - 1\right) P^t_{d_j} \right\} \end{aligned} \tag{E.43}$$

s.t.

Minimizing Total Substation Real Power with soft constraint on SCD

$$\min_{q_{D_j}^t, P_{c_j}^t, {P_{d_j}}^t, q_{B_j}^t} \quad \sum_{t=1}^{T} P_{Subs}^t \tag{E.44}$$
$$+ \alpha \sum_{t=1}^{T} \sum_{j \in \mathcal{B}} \left\{ (1-\eta_c) P_{c_j}^t + \left(\frac{1}{\eta_d} - 1 \right) P_{d_j}^t \right\}$$

s.t.

Minimizing Total Cost of Substation Real Power with soft constraint on SCD

$$\min_{q_{D_j}^t, P_{c_j}^t, {P_{d_j}}^t, q_{B_j}^t} \quad \sum_{t=1}^{T} P_{Subs}^t * cost(t) \tag{E.45}$$
$$+ \alpha \sum_{t=1}^{T} \sum_{j \in \mathcal{B}} \left\{ (1-\eta_c) P_{c_j}^t + \left(\frac{1}{\eta_d} - 1 \right) P_{d_j}^t \right\}$$

s.t.

Minimizing Line Losses with soft constraints on Terminal SOC and SCD

$$\min_{q_{D_j}^t, P_{c_j}^t, {P_{d_j}}^t, q_{B_j}^t} \quad \sum_{t=1}^{T} \sum_{(i,j) \in \mathcal{L}} (r_{ij} l_{ij}^t) \tag{E.46}$$
$$+ \alpha \sum_{t=1}^{T} \sum_{j \in \mathcal{B}} \left\{ (1-\eta_c) P_{c_j}^t + \left(\frac{1}{\eta_d} - 1 \right) P_{d_j}^t \right\}$$
$$+ \gamma \sum_{j \in \mathcal{B}} \left\{ \left(B_j^T - B_{ref_j} \right)^2 \right\}$$

s.t.

$$\min_{q^t_{D_j}, P^t_{c_j}, {P_{d_j}}^t, q^t_{B_j}} \quad \sum_{t=1}^{T} \sum_{(i,j)\in\mathcal{L}} (r_{ij} l^t_{ij}) \tag{E.47}$$

$$+ \alpha \sum_{t=1}^{T} \sum_{j\in\mathcal{B}} \left\{ (1-\eta_c) P^t_{c_j} + \left(\frac{1}{\eta_d} - 1 \right) P^t_{d_j} \right\} \tag{E.48}$$

$$+ \gamma \sum_{j\in\mathcal{B}} \left\{ \left(B^T_j - B_{ref_j} \right)^2 \right\} \tag{E.49}$$

s.t.

$$p^t_j = \sum_{(j,k)\in\mathcal{L}} P^t_{jk} - \sum_{(i,j)\in\mathcal{L}} \left\{ P^t_{ij} - r_{ij} l^t_{ij} \right\} - P^t_{d_j} + P^t_{c_j} \tag{E.50}$$

$$q^t_j = \sum_{(j,k)\in\mathcal{L}} Q^t_{jk} - \sum_{(i,j)\in\mathcal{L}} \left\{ Q^t_{ij} - x_{ij} l^t_{ij} \right\} - q^t_{D_j} - q^t_{B_j} \tag{E.51}$$

$$v^t_j = v^t_i + \left\{ r^2_{ij} + x^2_{ij} \right\} l^t_{ij} - 2(r_{ij} P^t_{ij} + x_{ij} Q^t_{ij}) \tag{E.52}$$

$$l^t_{ij} = \frac{(P^t_{ij})^2 + (Q^t_{ij})^2}{v^t_i} \tag{E.53}$$

$$B^t_j = B^{t-1}_j + \Delta t \eta_c P^t_{c_j} - \Delta t \frac{1}{\eta_d} P^t_{d_j} \tag{E.54}$$

$$B^0_j = 0.5(soc_{max} + soc_{min}) E_{Rated} = 0.625 E_{Rated} \tag{E.55}$$

$$where, \tag{E.56}$$

$$(i,j) : \text{Branch connecting nodes } i \text{ and } j \tag{E.57}$$

$$p^t_j = {p_{D_j}}^t - {p_{L_j}}^t \tag{E.58}$$

$$q^t_j = -{q_{L_j}}^t \tag{E.59}$$

$$P^t_{Subs} <= P^t_{SubsPeak} \tag{E.60}$$

$$t = \{1, 2, \ldots T\} \tag{E.61}$$

Previous Simulations

Step 2: Full Optimization Model - Single Time Step Greedy Approach

$$\min_{q^t_{D_j}, P^t_{c_j}, {P_{dj}}^t, q^t_{B_j}} \quad \sum_{(i,j)\in\mathcal{L}} (r_{ij} l^t_{ij}) + \alpha \sum_{j\in\mathcal{B}} \left\{ (1-\eta_c) P^t_{c_j} + \left(\frac{1}{\eta_d} - 1 \right) P^t_{d_j} \right\} \tag{E.62}$$

s.t.

$$p^t_j = \sum_{(j,k)\in\mathcal{L}} P^t_{jk} - \sum_{(i,j)\in\mathcal{L}} \left\{ P^t_{ij} - r_{ij} l^t_{ij} \right\} - P^t_{d_j} + P^t_{c_j} \tag{E.63}$$

$$q^t_j = \sum_{(j,k)\in\mathcal{L}} Q^t_{jk} - \sum_{(i,j)\in\mathcal{L}} \left\{ Q^t_{ij} - x_{ij} l^t_{ij} \right\} - q^t_{D_j} - q^t_{B_j} \tag{E.64}$$

$$v^t_j = v^t_i + \left\{ r^2_{ij} + x^2_{ij} \right\} l^t_{ij} - 2(r_{ij} P^t_{ij} + x_{ij} Q^t_{ij}) \tag{E.65}$$

$$l^t_{ij} = \frac{(P^t_{ij})^2 + (Q^t_{ij})^2}{v^t_i} \tag{E.66}$$

$$B^t_j = B^{t-1}_j + \Delta t \eta_c P^t_{c_j} - \Delta t \frac{1}{\eta_d} P^t_{d_j} \tag{E.67}$$

$$B^0_j = 0.5(soc_{max} + soc_{min}) E_{Rated} = 0.625 E_{Rated} \tag{E.68}$$

$$where, \tag{E.69}$$

$$(i,j) : \text{Branch connecting nodes } i \text{ and } j \tag{E.70}$$

$$p^t_j = {p_{Dj}}^t - {p_{Lj}}^t \tag{E.71}$$

$$q^t_j = -{q_{Lj}}^t \tag{E.72}$$

$$t = \{1, 2, \ldots T\} \tag{E.73}$$

Step 1b: Initialisation Lossless Optimization Model WITH Batteries - Single Time Step Greedy Approach

$$\min_{q^t_{D_j}, P^t_{c_j}, P_{d_j}{}^t, q^t_{B_j}} \quad \sum_{(i,j)\in\mathcal{L}} \left\{ r_{ij} \frac{(P^t_{ij})^2 + (Q^t_{ij})^2}{v^t_{0i}} \right\} + \alpha \sum_{j\in\mathcal{B}} \left\{ (1-\eta_c) P^t_{c_j} + \left(\frac{1}{\eta_d} - 1 \right) P^t_{d_j} \right\} \tag{E.74}$$

s.t.

$$p^t_j = \sum_{(j,k)\in\mathcal{L}} P^t_{jk} - \sum_{(i,j)\in\mathcal{L}} \left(P^t_{ij}\right) - P^t_{d_j} + P^t_{c_j} \tag{E.75}$$

$$q^t_j = \sum_{(j,k)\in\mathcal{L}} Q^t_{jk} - \sum_{(i,j)\in\mathcal{L}} \left(Q^t_{ij}\right) - q^t_{D_j} - q^t_{B_j} \tag{E.76}$$

$$v^t_{0j} = v^t_{0i} - 2(r_{ij} P^t_{ij} + x_{ij} Q^t_{ij}) \tag{E.77}$$

$$B^t_j = B^{t-1}_j + \Delta t \eta_c P^t_{c_j} - \Delta t \frac{1}{\eta_d} P^t_{d_j} \tag{E.78}$$

$$B^0_j = 0.5(soc_{max} + soc_{min}) E_{Rated} = 0.625 E_{Rated} \tag{E.79}$$

$$where, \tag{E.80}$$

$$(i,j) : \text{Branch connecting nodes } i \text{ and } j \tag{E.81}$$

$$p^t_j = p_{D_j}{}^t - p_{L_j}{}^t \tag{E.82}$$

$$q^t_j = -q_{L_j}{}^t \tag{E.83}$$

$$t = \{1, 2, \dots T\} \tag{E.84}$$

Step 1a: Initialisation Lossless Optimization Model WITHOUT Batteries - Single Time Step Greedy Approach

$$\min_{P^t_{ij},Q^t_{ij},v^t_j,q^t_{D_j}} \quad 0 \tag{E.85}$$

s.t.

$$p^t_j = \sum_{(j,k)\in\mathcal{L}} P^t_{jk} - \sum_{(i,j)\in\mathcal{L}} P^t_{ij} \tag{E.86}$$

$$q^t_j = \sum_{(j,k)\in\mathcal{L}} Q^t_{jk} - \sum_{(i,j)\in\mathcal{L}} Q^t_{ij} \tag{E.87}$$

$$v^t_j = v^t_i - 2(r_{ij}P^t_{ij} + x_{ij}Q^t_{ij}) \tag{E.88}$$

$$where, \tag{E.89}$$

$$(i,j) : \text{Branch connecting nodes } i \text{ and } j \tag{E.90}$$

$$p^t_j = {p_{Dj}}^t - {p_{Lj}}^t \tag{E.91}$$

$$q^t_j = -{q_{Lj}}^t \tag{E.92}$$

$$t = \{1, 2, \dots T\} \tag{E.93}$$

A simple metric called P_{Save} gives an indication of the effect of power generated by batteries and DERs which offset substation power (indirectly flowing it via its parent area if it is not directly connected to the substation).

Its formula is as shown:

$$P_{Save} = 100\% * \left(\frac{\sum_{j\in\mathcal{B}} \left(P_{d_j} - P_{c_j}\right)}{P_{12} + \sum_{j\in\mathcal{D}} P_{DER_j} + \sum_{j\in\mathcal{B}} \left(P_{d_j} - P_{c_j}\right)} \right) \tag{E.94}$$

Here are some key definitions used in this thesis, which have been aligned as per NERC's/NREL's technical documents [3, 7]:

- Inverted-Based Resources (IBR) [7]

- Grid Following Interters (GFL): Depend on an external constant voltage source to generate current [3]. They *track* an AC Voltage Waveform

- Grid Forming Inverters (GFM): Generate their own constant voltage [3]. They *generate* their own AC Voltage waveform. Actually have been in use in off-grid power systems for decades. NERC Inverter-based Resource Performance Working Group (IRPWG) proposed a unified definition: "An inverter that maintains a constant voltage phasor in the transient and sub-transient time frames"

- Short Circuit Ratio (SCR): (for a bus on the grid, potentially where an IBR is to be connected) Ratio of the total fault current (power) at the bus to the rated current (power) of the generator to be connected at the point. An indicator for the 'grid strength' at the point. An SCR value greated that 5 is considered healthy, and a value of less than 3 is considered weak. For a weak point with low SCR, EMT studies are usually conducted by the grid operators to detect transient issues which may not be caught by conventional Dyanmic Studies.

  $$SCR_{POI} = \frac{SCMVA_{POI}}{MW_{VER}}$$

  Here, VER denotes a Variable Energy Resources, like a PV.

- Weighted Short Circuit Ratio (WSCR): (for a grid with many buses where IBRs are potentially to be connected) A weighted average of the SCRs across the whole grid, to denote a relative 'grid strength'.

  $$WSCR = \frac{\Sigma_j^N SCMVA_j * P_{RatedMWj}}{(\Sigma_j^N P_{RatedMWj})^2}$$