Problem 7

Let's say an experiment has three outcomes.

a. How many events can be defined for this experiment?

Now we say we repeat the experiment twice (once more), independently.

b. How many outcomes does the combined experiment have?

c. How may events can be defined for the combined experiment?

d. How many events for the combined experiment can not be derived from the cartesian products for the individual experiments? Please give an example.

Solution

Let the experiment be called E_1 which has the three outcomes be A, B and C i.e. $\Omega = \{\text{A, B, C}\}$

a. # Events $= 2^n$ where n is the cardinality of the sample space Ω.
Here $n = 3$ and therefore # Events $= 2^3 = 8$.

Let the combined experiment be called E_2. The outcomes of this combined experiment will be represented by the new sample space $\Omega_2 = \{\text{AA, AB, AC, BA, BB, BC, CA, CB, CC}\}$.

b. # Outcomes = Cardinality ($n_2 = n^2$) of the sample space Ω_2.
Here $n_2 = 3^2 = 9$.

c. # Events $= 2^{n_2}$
Here $n_2 = 9$ and therefore # Events $= 2^9 = 512$.

d. # Events of the combined experiment which can be defined from the cartesian products of the individual experiments $= 2^n * 2^n = 2^{2n} = 2^6$.
Total # Events of the combined experiment $= 2^{n^2} = 2^9$.
Therefore, # Events which cannot be defined from the cartesian products of the individual experiments $= 2^{n^2} - 2^{2n} = 2^9 - 2^6 = 512 - 64 = 448$.

Examples for an event of E_2 which can be formed from the cartesian products of the events of E_1:

$$\{A\} \times \{A\} = \{AA\}$$
$$\{A, B, C\} \times \{A\} = \{AA, BA, CA\}$$
$$\{A, C\} \times \{B\} = \{AB, CB\}$$

Examples for events of E_2 which can NOT be formed from the cartesian products of the events of E_1:

$$\{AB, BC, CA\}$$
$$\{AA, CB\}$$
$$\{BC, CB\}$$
