A Complex LASSO-Approach for Localizing Forced Oscillations in Power Systems

A brief reiteration

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Introduction

Forced Oscillations in Power Systems

Natural Oscillations	Forced Oscillations
Resulting from ‘natural’ interaction between dynamic devices of a power system	Caused by an ‘external’ perturbation.
PSS and other control strategies typically employed to counter them	Detection and removal of the ‘external’ source can be necessary in some cases.
Example of a source can be Inter-area oscillations	Example of a source can be A mis-tuned controller associated with a cement mill or a generator

Theory

State Space Representation

Express Power System Dynamics in State Space:

$$\begin{aligned}\underset{n\times 1}{\dot{\mathrm{x}}(t)} &= \underset{n\times n}{\mathrm{A}}\,\underset{n\times 1}{\mathrm{x}(t)} + \underset{n\times m}{\mathrm{B}}\,\underset{m\times 1}{\mathrm{u}(t)} \\ \underset{p\times 1}{\mathrm{y}(t)} &= \underset{p\times n}{\mathrm{C}}\,\underset{n\times 1}{\mathrm{x}(t)} \\ &\forall t \geq 0\end{aligned} \tag{1}$$

$\underset{n\times 1}{\mathrm{x}(t)}$: internal state variables + controller variables vector

$\underset{m\times 1}{\mathrm{u}(t)}$: forced oscillation vector

Forced Oscillation Vector

Express Forced Oscillations based on locations of origin and signal composition:

$$\underset{m\times 1}{\mathbf{u}(t)} = \underset{m\times 1}{\begin{bmatrix} u_1(t) \\ u_2(t) \\ \vdots \\ u_m(t) \end{bmatrix}} = \underset{m\times 1}{\begin{bmatrix} \sum_{l=1}^{M_1} a_{1,l} \sin(\omega_{2,l} t + \phi_{1,l}) \\ \sum_{l=1}^{M_2} a_{2,l} \sin(\omega_{2,l} t + \phi_{2,l}) \\ \vdots \\ \sum_{l=1}^{M_m} a_{m,l} \sin(\omega_{m,l} t + \phi_{m,l}) \end{bmatrix}} \tag{2}$$

$a_{r,l} \geq 0$

$\omega_{r,l} = 2\pi f \geq 0$

(r, l) refer to the l^{th} sinusoid at the r^{th} location

Sparsity Assumption

Let $\boldsymbol{a_r}(t)$ be the vector representation of the sinusoids at the r^{th} location.

$$\underset{1\times M_r}{\boldsymbol{a_r}(t)} = \underset{1\times M_r}{\begin{bmatrix} a_{r,1}\sin(\omega_{r,1}t+\phi_{r,1}) & \dots & a_{r,M_r}\sin(\omega_{r,M_r}t+\phi_{r,M_r}) \end{bmatrix}}$$

Assumption 1: Both $\boldsymbol{u}(t)$ and $\boldsymbol{a_r}(t)$ are sparse vectors. That implies most locations are FO free and for the locations that do have FO, the number of distinct sinusoidal components of the local FO is small.

$$||\boldsymbol{u}(t)||_0 << m$$
$$||\boldsymbol{a_r}(t)||_0 << M_r$$

Discrete State Space Representation

Most PMUs sample at $30Hz$ or $60Hz$. So the sampling time period is $T = 1/30s$ or $1/60s$.

Let $\underset{n\times n}{\boldsymbol{A_d}} = \exp(AT)$ and $\underset{n\times m}{\boldsymbol{B_d}} = \{\int_0^T \exp(A(T-s))ds\}\boldsymbol{B}$.

Let $k = 0, 1, \ldots$ and define $\mathbf{x}[k] \triangleq \mathbf{x}(kT)$, $\mathbf{u}[k] \triangleq \mathbf{u}(kT)$ and $\mathbf{y}[k] \triangleq \mathbf{y}(kT)$. Suppose that $\mathbf{u}(t)$ is piece-wise constant during $kT \leq t \leq (k+1)T$. Then

$$\begin{aligned} \underset{n\times 1}{\boldsymbol{x}[k+1]} &= \underset{n\times n}{\boldsymbol{A_d}}\underset{n\times 1}{\boldsymbol{x}[k]} + \underset{n\times m}{\boldsymbol{B_d}}\underset{m\times 1}{\boldsymbol{u}[k]} \\ \underset{p\times 1}{\boldsymbol{y}[k]} &= \underset{p\times n}{\boldsymbol{C}}\underset{n\times 1}{\boldsymbol{x}[k]} \end{aligned} \tag{3}$$

We assume $\mathbf{x}[0] = 0$ as our focus is only on inputs triggered by FOs.

Transfer Function from Discrete State Space Representation

Let $\underset{p\times m}{H_z} = \underset{p\times n}{C}\left[zI - \underset{n\times n}{A_d}\right]^{-1}\underset{n\times m}{B_d}$.

From standard Linear System Analysis, we may conclude that
$\underset{p\times 1}{Y[z]} = \underset{p\times m}{H[z]}\underset{m\times 1}{U[z]}$
where $Y[z]$, $U[z]$ are the Z-Transforms of $y[n]$ and $u[n]$.
Replacing z with $\exp(j\Omega)$, where $\Omega \in (0, 2\pi)$, we get the DTFT representation for the system:

$$\underset{p\times 1}{Y[\Omega]} = \underset{p\times m}{H[\Omega]}\underset{m\times 1}{U[\Omega]} \tag{4}$$

Transfer Function of Forced Oscillation Vector

Applying DTFT on $\boldsymbol{u}(t)$ to get $\boldsymbol{U}[\Omega]$:

$$\underset{m\times 1}{\mathrm{U}(\Omega)} = j\pi \underset{m\times 1}{\begin{bmatrix} \sum_{l=1}^{M_1} a_{1,l}\{\exp\left(-j\phi_{1,l}\right)\delta(\Omega+\omega_{1,l}) - \exp\left(j\phi_{1,l}\right)\delta(\Omega-\omega_{1,l})\} \\ \sum_{l=1}^{M_2} a_{2,l}\{\exp\left(-j\phi_{2,l}\right)\delta(\Omega+\omega_{2,l}) - \exp\left(j\phi_{2,l}\right)\delta(\Omega-\omega_{2,l})\} \\ \vdots \\ \sum_{l=1}^{M_m} a_{m,l}\{\exp\left(-j\phi_{m,l}\right)\delta(\Omega+\omega_{m,l}) - \exp\left(j\phi_{m,l}\right)\delta(\Omega-\omega_{m,l})\} \end{bmatrix}} \tag{5}$$

$$\omega_{r,l} \text{ can be substituted by } \frac{1}{T}\tilde{\omega}_{r,l}$$

Introduction to My Work

Transient vs Steady State Stability

Transient Stability	Steady State Stability
A sudden, out-of-trend, high magnitude change in a state variable(s) causes blackouts.	Accumulation of several seemingly minor trends in state variables over time, ultimately leading to a critical point where a small change could cause blackouts.
Chief parameters of concern are ROCOF, frequency nadir, steady-state frequency deviation.	Autocorrelation and covariance are some of the commonly used parameters for prognosis.
Inertia is a fundamental parameter here.	Inertia plays a minor role here.

Bifurcations and Critical Slowing Down

Bifurcation: A qualitative change in the 'motion' of a dynamical System due to a quantitative change in one of its parameters. Serious bifurcations, called Critical Bifurcations, cause the system to become unstable from stable.

Bifurcations and Critical Slowing Down

Critical Slowing Down: Dynamical Systems exhibit early statistical warning signs before collapsing:

- Increased recovery times from perturbations.
- Increased signal variance from the mean trajectory.
- Increased flicker and asymmetry in the signal

The above three properties can be identified by increasing variance and autocorrelation in time-series measurements taken from the system.

Procedure

Procedure

- Accessed a bunch of real-world frequency time-series data and plotted their:
  - bulk distribution (pdf)
  - auto-correlation curves

  in order to demonstrate that:
  - grid frequencies often exhibit significant deviation from gaussianity

  autocorrelation can be a reasonable indication of grid steady-state stability.
- Obtained explanation for the *signature dynamics* of each grid.
- Then implemented and tested the IEEE 9 Bus System in PSSE for symptoms of Critical Slowing Down as an Early Warning System.

Results

Results: Real World Data

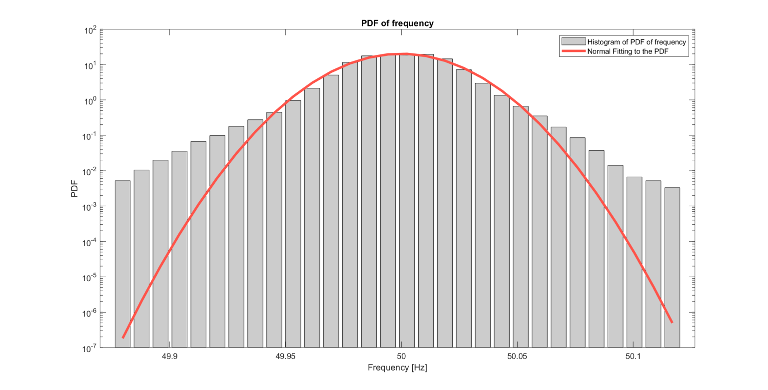

Figure 1: Continental European Grid frequency PDF: Heavier tails than a Gaussian Distribution.

Results: Real World Data

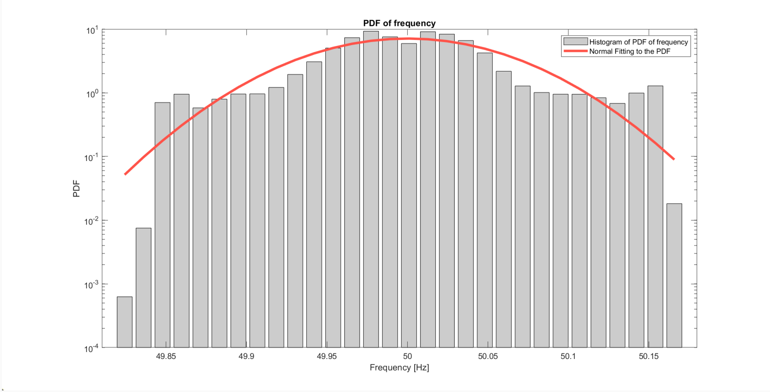

Figure 2: Mallorcan (an islanded Spanish grid) frequency pdf

Results: Real World Data

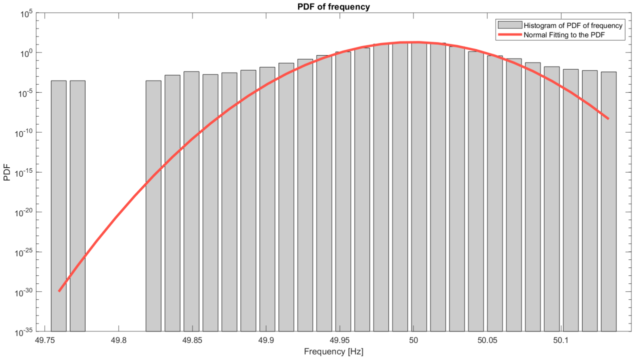

Figure 3: French grid frequency pdf including a blackout

Results: Real World Data

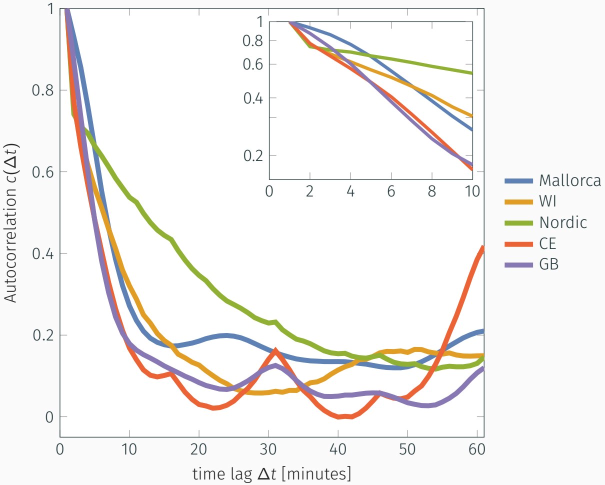

Figure 4: Autocorrelation decay of different synchronous regions.

Results: Real World Data

Table 1: Inverse-correlation values for different grids

Grid name	Inverse-correlation value T^{-1} $[min^{-1}]$
Mallorca	0.0654
Western Interconnection	0.0498
Nordic	0.0235
Continental Europe	0.0829
Great Britain	0.0879

Figure 5: Inverse correlation time is proportional to the damping constant of the grid.

Results: PSSE Simulation

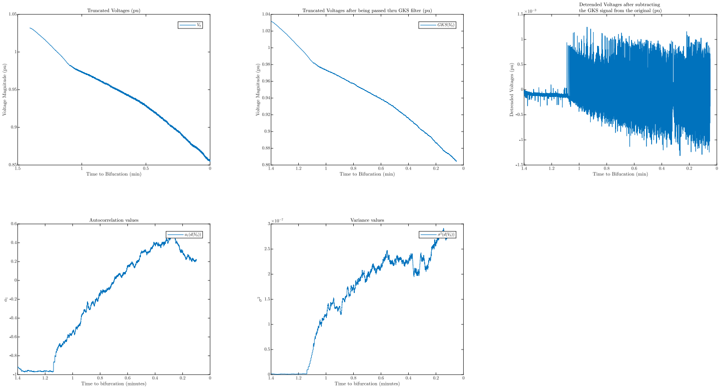

Figure 6: Analysis of a bus voltage for the IEEE 9 Bus system vs a constant load increment.