Report
Assignment-1

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1 Motivation

Comparison of Eigenvalue Decomposition and Singular value Decomposition when performed on an image in view of image compression.

2 EVD

EVD of a matrix M is decomposition of it into product of 3 matrices such that middle matrix is diagonal Matrix, first and last matrices are inverses of each other.The decomposition looks like $M = PDP^{-1}$ where D is the diagonal matrix(This contains eigenvalues).The columns of P are Eigen vectors.
We notice that order of columns can be changed as long as D and P^{-1} are changed. So, we change the order of columns as in decreasing order of magnitude of entries of D.Then we take first k columns to reconstruct the matrix.
$M_{recons} = P[:, : k] * D[: k, : k] * P^{-1}[: k, :]$.

3 SVD

SVD of a matrix(M of dimension $m * n$ is decomposition of it into product of 3 matrices $M = U * \Sigma * V^*$, where U is an $m * m$ complex unitary matrix,Σ is an $m * n$ rectangular diagonal matrix with non-negative real numbers and V is an $n * n$ complex unitary matrix.
In case of our image, M is real and hence U and V are orthogonal matrices. The diagonal entries of Σ are called singular values.
$M_{recons} = U[:, : k] * \Sigma[: k, : k] * V^*[: k, :]$

4 Experimental results

- After performing both EVD and SVD on the image for various values of k, The plot of frobenius norm of error image vs k is in Figure 1.

- EVD and SVD reconstruction Images and corresponding error images for various values of k are found in Figure 2.

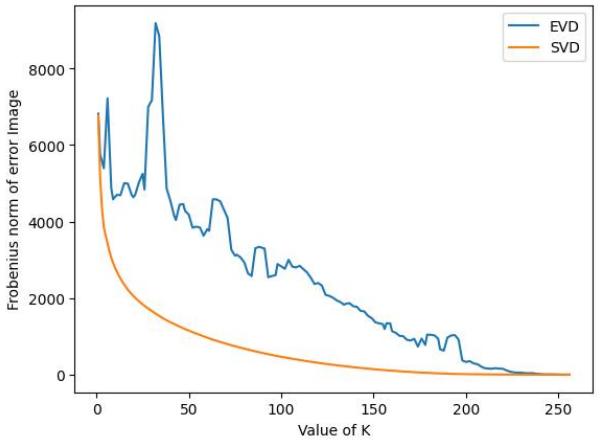

Figure 1: Frobenius Norm vs k plot

5 Inferences

1. From the graph we can notice that Frobenius norm of error image decreases with increase in number of features.

2. We can also conclude that,with increase in number of features, error in SVD decreases much more rapidly than with EVD and also the graph is smooth and concave up in case of SVD. Concave up indicates that decrease in error decreases with increase in features indicating most features are captured by first few columns (Making it suitable for compression).

3. When looking at the images, though all the images converge to original image when features are large, SVD converges much more rapidly than EVD. Also there are less disturbances or noises in case of SVD than EVD.

4. Looking at image compression point of view, SVD is a better choice than EVD from the fact that at k = 30 itself SVD has captured most of the essence where as EVD still had random dots.

5. Overall, SVD did a good job compared to EVD in getting all the essence with less value of k. Image compression will be better with SVD than EVD.

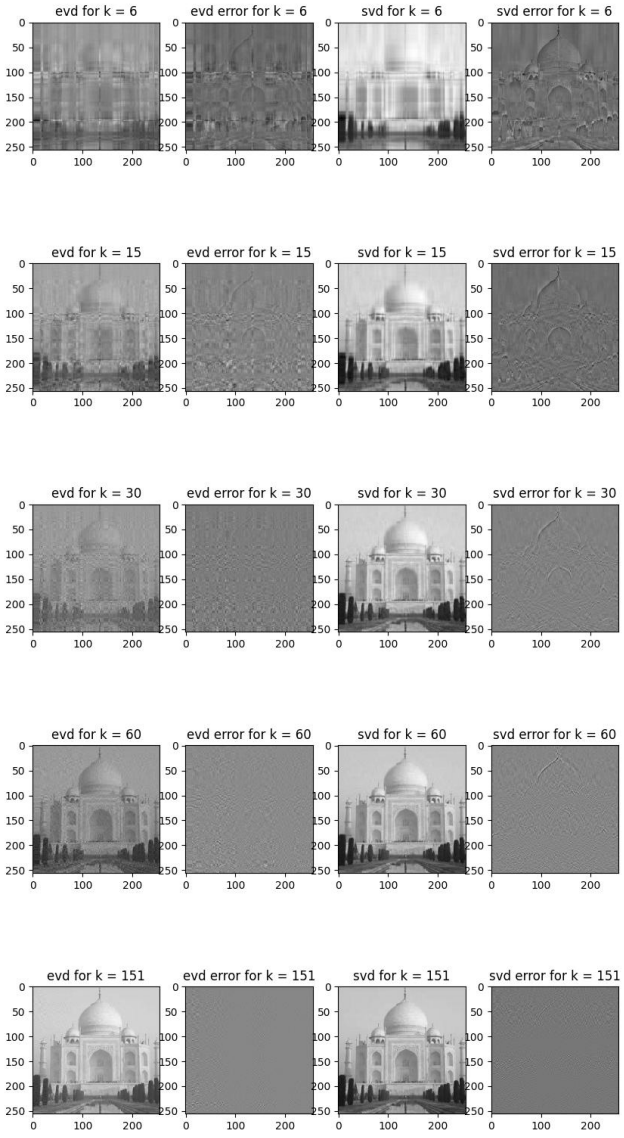

Figure 2: Images for various k