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DIGITAL COMMUNICATION LAB (EC39001)

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Introduction

This experiment involves generating Pseudo Noise(PN) Sequence, and studying its properties. PN sequence is generated by a combinational circuit and shift register.

Key Objectives

For this experiment, out key objectives are:

- Generate and observe sequence from the combinational circuits with respective polynomials:
  - $x^4 + x^2 + 1$. *(Reducible)*
  - $x^4 + x^3 + x^2 + x + 1$. *(Irreducible)*
  - $x^4 + x^3 + 1$. *(Irreducible)*
- Calculate and Observe the Autocorrelation function of PN Sequences.
- Study the properties of PN Sequence.

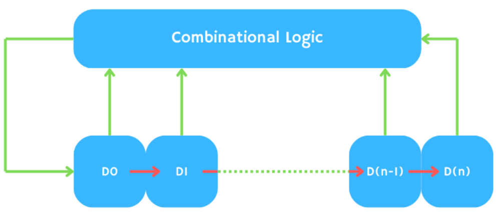
Circuit Components Used

The components used for this experiment were:

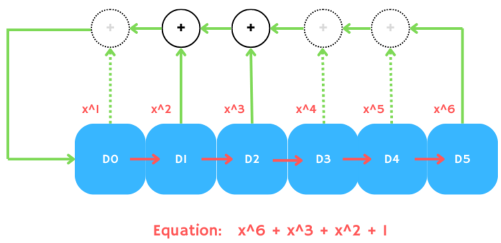
- IC 74LS95B Shift Register
- IC 7486 Quad 2 input ExOR
- IC 7427 Triple 3 input NOR
- IC 7432 Quad 2 input OR
- 4 Resistors
- 4 LEDs

Theory

Sequences can be generated by using Shift Registers/Flip Flops and Feedback Combinational Circuit processing the bit before it is fed as input into the Shift Register. A general circuit for that can be seen below



(a) Block Circuit Diagram



(b) Circuit Diagram for a polynomial

Figure 1: General Circuit Diagram

The combinational logic block usually consists of XORs as they are linear operations. With that in mind, let us now look at how polynomials can define circuits in figure (b).

Pseudo Noise(PN) Sequence, as the name suggests, is a pseudo random sequence of bits, It is called pseudo because it has a finite periodicity. For example, a circuit with n flip flops can generate a PN sequence of **length 2^n - 1**. After that the sequence repeats itself, therefore periodic. However as we will see in the Results section, we can use PN Sequence as a random sequence as it's auto correlation function is a bunch of dirac deltas repeated at every time step = the sequence's periodicity.

Let us now look at some properties of PN Sequence:

- **Run Length**
  A "run" is defined as a subsequence of identical symbols(0s or 1s) within a period. In PN Sequences, about half the runs are of legnth 1, about a quater of runs are of the length 2 and so on.
- **Balance Property**
  In PN Sequence, number of 1s is 1 greater than number of 0s. This arises from the fact that 00...0 is not present in the state of PN Sequence as it is a lock state.
- **Correlation Property**
  Autocorrelation has already been mentioned and will be seen shortly. Crosscorrelation between two different PN Sequences is always 0.

We can determine *just by looking at the polynomials* whether or not the circuit will produce a PN Sequence. More on that in the Dicussion section. However we study the effect of Reducible/Irreducibility of polynomial on whether or not the circuit generates a PN Sequence.

Results

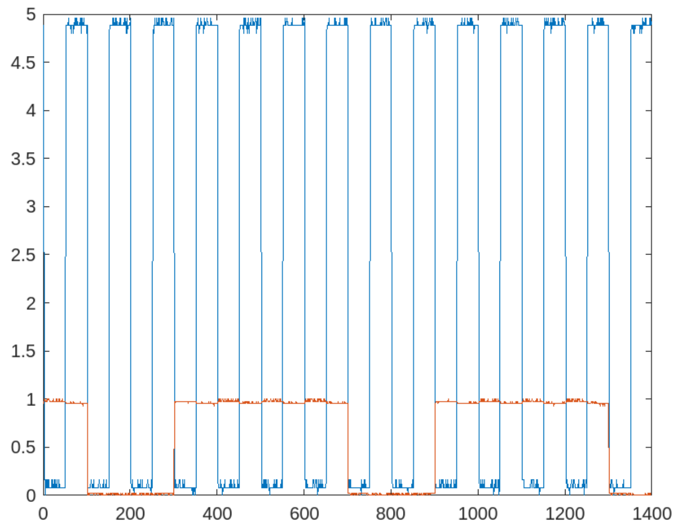


Figure 2: Sequence generated by $x^4 + x^2 + 1$

This circuit generates the sequence

$$\longrightarrow 0 \longrightarrow 0 \longrightarrow 1 \longrightarrow 1 \longrightarrow 1 \longrightarrow 1 \longrightarrow$$

This is not a PN Sequence as it is non Maximal Sequence and doesn't satisfy properties like Run Length, Balance Property. Let us take a look at its Autocorrelation Function:

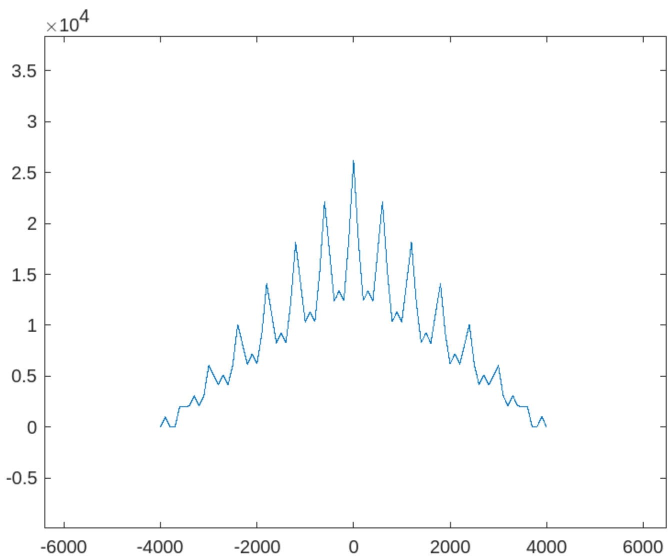


Figure 3: Autocorrelation of sequence generated by $x^4 + x^2 + 1$

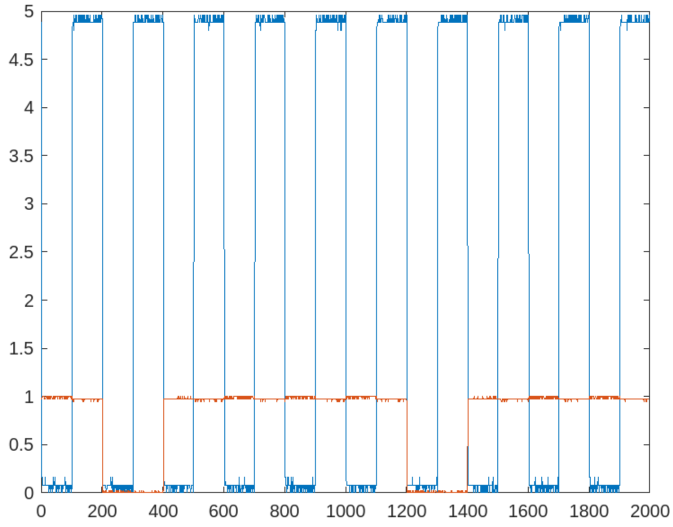


Figure 4: Sequence generated by $x^4 + x^3 + x^2 + x + 1$

This circuit generates the sequence

$$\longrightarrow 0 \longrightarrow 1 \longrightarrow 1 \longrightarrow 1 \longrightarrow 1 \longrightarrow$$

This is not a PN Sequence as it is non Maximal Sequence and doesn't satisfy properties like Run Length, Balance Property. Let us take a look at its Autocorrelation Function:

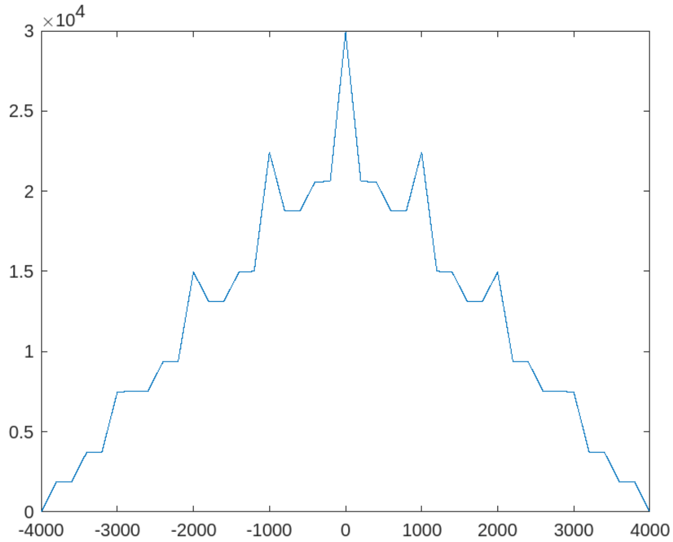


Figure 5: Autocorrelation of sequence generated by $x^4 + x^3 + x^2 + x + 1$

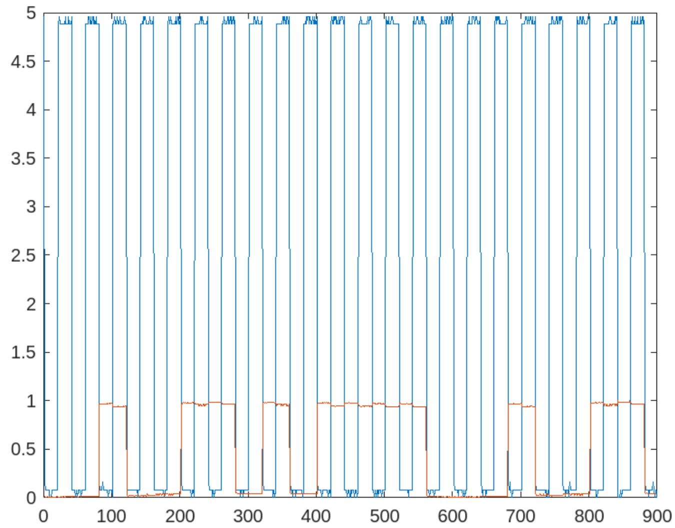


Figure 6: Sequence generated by $x^4 + x^3 + 1$

This circuit generates the sequence

$$\longrightarrow 0 \longrightarrow 1 \longrightarrow 0 \longrightarrow 1 \longrightarrow 1 \longrightarrow 1 \longrightarrow 1 \longrightarrow 0 \longrightarrow 0 \longrightarrow 0 \longrightarrow 1 \longrightarrow 0 \longrightarrow 0 \longrightarrow 1 \longrightarrow 1$$

This is a PN Sequence as it is Maximal Sequence and satisfies all properties like Run Length, Balance Property. Let us take a look at its Autocorrelation Function:

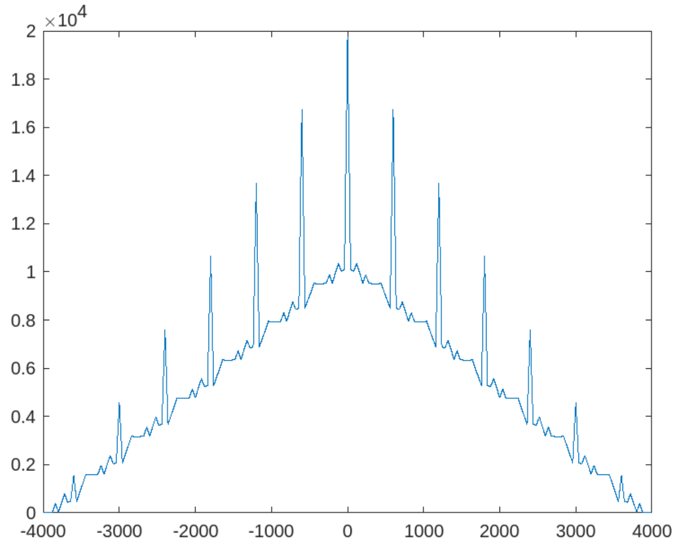


Figure 7: Autocorrelation of sequence generated by $x^4 + x^3 + 1$

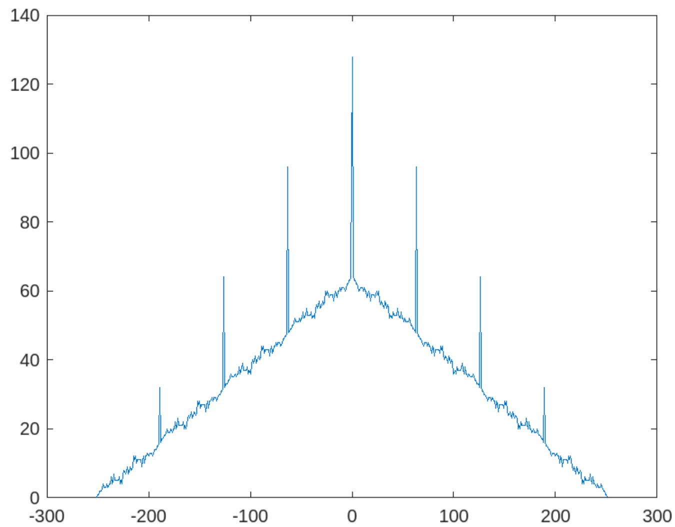


Figure 8: Autocorrelation of sequence generated by $x^6 + x + 1$

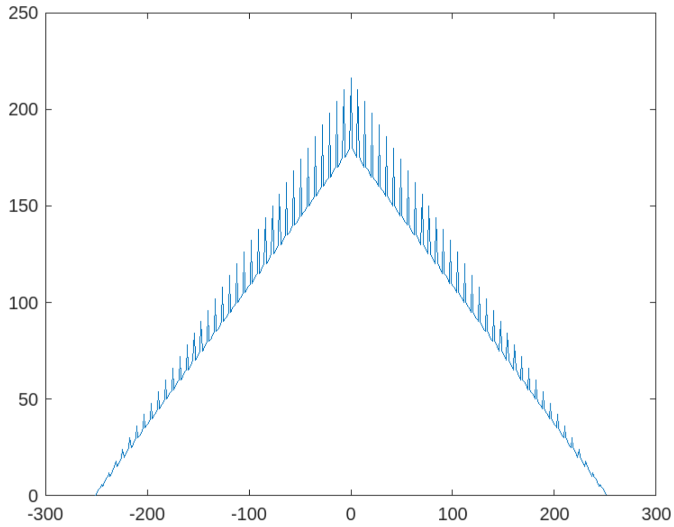


Figure 9: Autocorrelation of sequence generated by $x^6 + x^5 + x^4 + x^3 + x^2 + x + 1$

As we can see, the spikes in 2nd figure much less spaced than the 1st figure. This is because 2nd polynomial doesn't generate a PN Sequence and therefore it is non maximal length. So the peaks repeat faster in the 2nd figure than the 1st figure.

Discussion

From this experiment, our key takeaways were:

- Which polynomials produce PN Sequence?

  We have noticed that **Reducible** Polynomials certainly **do not produce PN Sequences**. However, **Irreducible** Polynomials *do not necessarily produce PN Sequences*. So which polynomials produce PN Sequence?

- Primitive Polynomials

  Polynomials which are irreducible and the taps are co prime to each other, and there are even number of them, are called Primitive Polynomials. Primitive Polynomials produce PN Sequences, as we have already seen with $x^4 + x^3 + 1$.

- Lock States in Sequence Generators

  Since we are using XOR as the only operation, 0 is a lockstate as it is a linear operation. Once all the flip flops reach to 0 state simultaneously, the state remains locked,i.e, doesn't change further. In order to avoid lock state we also add a condition to check that when all the flip flops are at 0, the output of Combinational Circuit should be 1.

- Autocorrelation Curves Linear Slope

  Autocorrelation Curves for Sequences follow a pattern consisting of repetitive spikes at $t = 0$ and every integer multiple of Time Period(T). T is the period/length of the sequence. At other places the value should be constant. However here we notice that the peaks of spike is reducing as we move away from 0 and the value at other places is also linearly reducing instead of staying constant. This is due to the finite nature of the sequence, with maximum length on either side when at 0. Therefore value at 0 is the maximum. If the samples were hypothetically infinite in number, we wouldn't have observed this kind of linear fall instead of constant values.

- Autocorrelation Curves

  PN Sequences have steep peaks, whereas the sequences generated in Figure 2, 4 and 8 are not as sharp. This is because they are not PN Sequences. PN Sequences act like noise in the time frame $t = 0$ - $t = T - 1$, where T is the period of the sequence. Since noise has a dirac delta as autocorrelation, autocorrelation of PN Sequence is a repeated dirac deltas.

Conclusion

In this experiment, we learnt and observed PN Sequence, circuits that generate them, its properties and importance.