EXERCISE 7.1

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Choose the correct answer from the given four options in the following questions:

1. The distance of the point P (2, 3) from the x-axis is

(A) 2 (B) 3 (C) 1 (D) 5

Solution:

(B) 3

We know that,

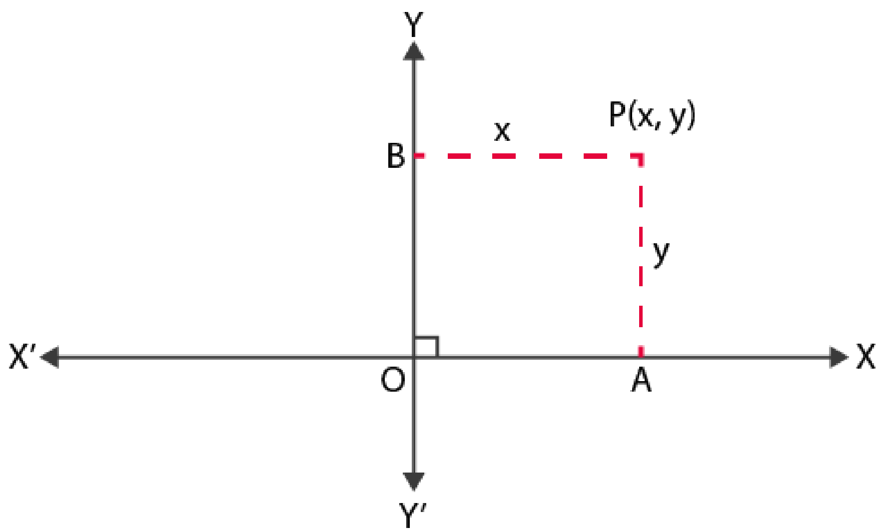
(x, y) is a point on the Cartesian plane in first quadrant.

Then,

x = Perpendicular distance from Y - axis and

y = Perpendicular distance from X – axis

Therefore, the perpendicular distance from X-axis = y coordinate = 3



2. The distance between the points A (0, 6) and B (0, –2) is

(A) 6 (B) 8 (C) 4 (D) 2

Solution:

(B) 8

Distance formula: $d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$

According to the question,

We have,

$x_1 = 0, x_2 = 0$

$y_1 = 6, y_2 = - 2$

$d^2 = (0 – 0)^2 + (- 2 – 6)^2$

$d= \sqrt{((0)^2+ (-8)^2)}$

$d = \sqrt{64}$

d = 8 units

Therefore, the distance between A (0, 6) and B (0, 2) is 8

3. The distance of the point P (–6, 8) from the origin is

(A) 8 **(B) 2√7** **(C) 10** **(D) 6**

Solution: (c) 10

Distance formula: $d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$

According to the question,

We have;

$x_1 = -6, x_2 = 0$

$y_2 = 8, y_2 = 0$

$d^2 = [0 - (-6)]^2 + [0 - 8]^2$

$d = \sqrt{((0-(-6))^2 + (0-8)^2}$

$d = \sqrt{((6)^2 + (-8)^2)}$

$d = \sqrt{(36 + 64)}$

$d = \sqrt{100}$

$d = 10$

Therefore, the distance between P (- 6, 8) and origin O (0, 0) is 10

4. The distance between the points (0, 5) and (–5, 0) is

(A) 5 **(B) 5√2** **(C) 2√5** **(D) 10**

Solution: (B) 5√2

Distance formula: $d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$

According to the question,

We have;

$x_1 = 0, x_2 = -5$

$y_2 = 5, y_2 = 0$

$d^2 = ((-5) - 0)^2 + (0 - 5)^2$

$d = \sqrt{(-5-0)^2 + (0-5)^2}$

$d = \sqrt{((-5)^2 + (-5)^2)}$

$d = \sqrt{(25 + 25)}$

$d = \sqrt{50} = 5\sqrt{2}$

So the distance between (0, 5) and (- 5, 0) = $5\sqrt{2}$

5. AOBC is a rectangle whose three vertices are vertices A (0, 3), O (0, 0) and B (5, 0). The length of its diagonal is

(A) 5 **(B) 3** **(C) √34** **(D) 4**

Solution: (C) √34

The three vertices are: A = (0, 3), O = (0, 0), B = (5, 0)

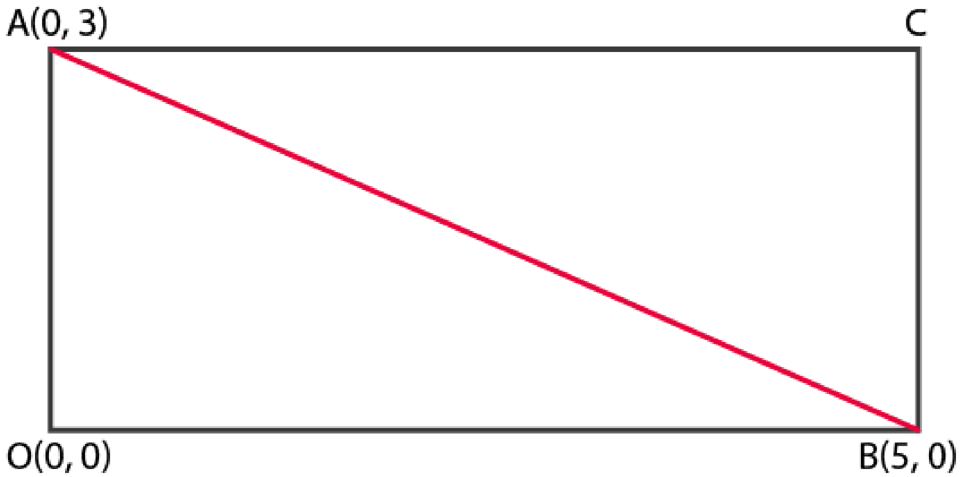
We know that, the diagonals of a rectangle are of equal length,

Length of the diagonal AB = Distance between the points A and B

Distance formula: $d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$

According to the question,

We have;



$x_1 = 0$, $x_2 = 5$

$y_2 = 3$, $y_2 = 0$

$d^2 = (5 - 0)^2 + (0 - 3)^2$

$d = \sqrt{((5-0)^2 + (0-3)^2)}$

$d = \sqrt{(25 + 9)} = \sqrt{34}$

Distance between A (0, 3) and B (5, 0) is $\sqrt{34}$

Therefore, the length of its diagonal is $\sqrt{34}$

6. The perimeter of a triangle with vertices (0, 4), (0, 0) and (3, 0) is

(A) 5 (B) 12 (C) 11 (D) 7+ $\sqrt{5}$

Solution:

(B) 12

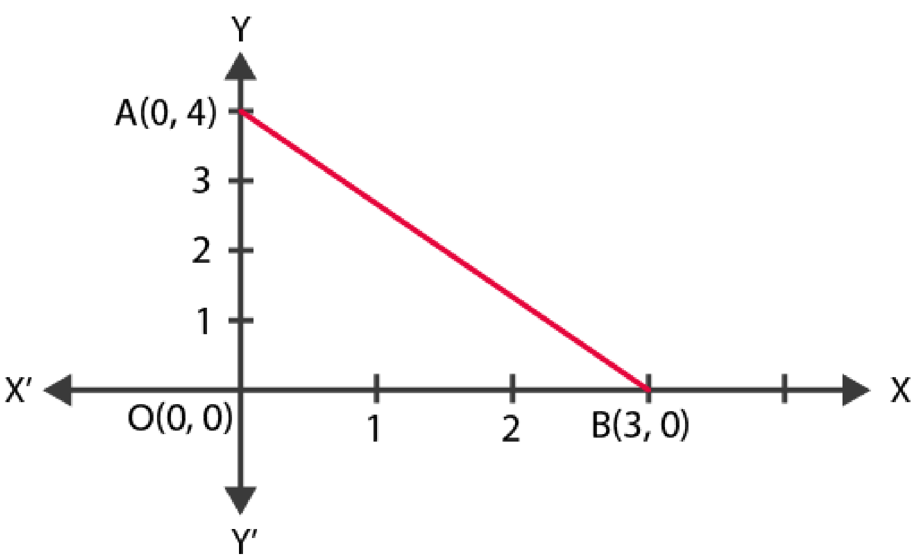
The vertices of a triangle are (0, 4), (0, 0) and (3, 0).

Now, perimeter of ΔAOB = Sum of the length of all its sides:

= distance between (OA+OB+AB)

Distance between the points (x_1, y_1) and (x_2, y_2) is given by,

$d = \sqrt{((x_2 - x_1)^2 + (y_2 - y_1)^2)}$



To find:

Distance between A(0, 4) and O(0, 0) + Distance between O(0, 0) and B(3, 0) + Distance between A(0, 4) and B(3, 0)

$$= \sqrt{(0-0)^2+(0-4)^2} + \sqrt{(3-0)^2+(0-0)^2} + \sqrt{(3-0)^2+(0-4)^2}$$

$$= \sqrt{0+16} + \sqrt{9+0} + \sqrt{(3)^2+(4)^2}$$

$$= 4+3+\sqrt{9+16}$$

$$= 7+\sqrt{25} = 7+5 = 12$$

Therefore, the required perimeter of triangle is 12

7. The area of a triangle with vertices A (3, 0), B (7, 0) and C (8, 4) is

(A) 14 (B) 28 (C) 8 (D) 6

Solution:

(c) 8

Vertices of the triangle are,

A (x_1, y_1) = (3, 0)

B (x_2, y_2) = (7, 0)

C (x_3, y_3) = (8, 4)

Area of triangle $= \left| \frac{1}{2}[x_1(y_2-y_3) + x_2(y_3-y_1) + x_3(y_1-y_2)] \right|$

$= \left| \frac{1}{2}[3(0-4) + 7(4-0) + 8(0-0)] \right|$

$= \left| \frac{1}{2}[-12 + 28 + 0] \right|$

$= \left| \frac{1}{2}[16] \right|$

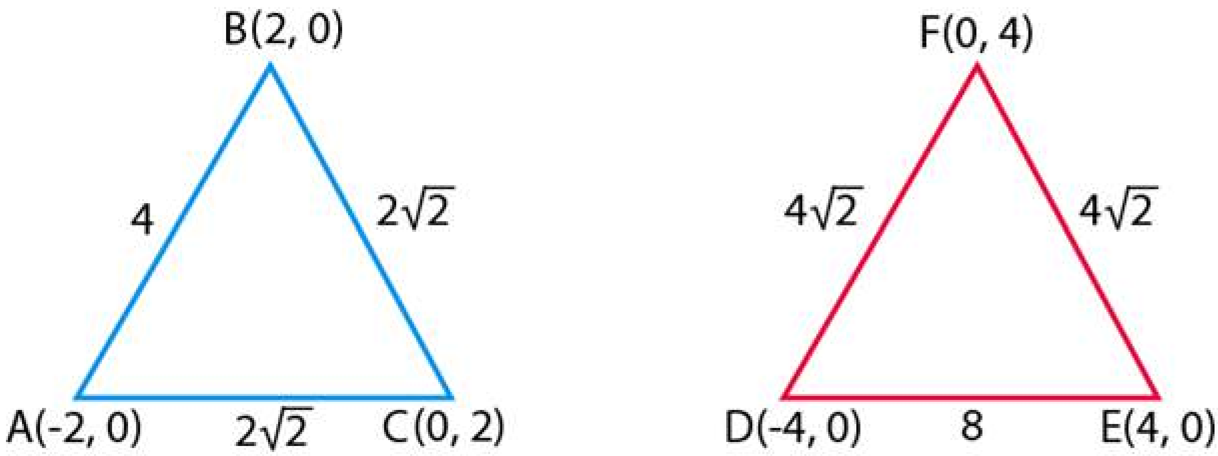
$= 8$

Therefore, the area of ΔABC is 8.

EXERCISE 7.2 PAGE NO: 80

State whether the following statements are true or false. Justify your answer.
1. △ABC with vertices A (–2, 0), B (2, 0) and C (0, 2) is similar to △DEF with vertices D (–4, 0) E (4, 0) and F (0, 4).
Solution:

True.
Justification:



Using distance formula, $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
We can find,

$$AB = \sqrt{(2+2)^2 + 0} = \sqrt{16} = 4$$

$$BC = \sqrt{(0-2)^2 + (2-0)^2} = \sqrt{8} = 2\sqrt{2}$$

$$CA = \sqrt{(-2-0)^2 + (0-2)^2} = \sqrt{8} = 2\sqrt{2}$$

$$DE = \sqrt{(4+4)^2 + 0} = \sqrt{64} = 8$$

$$EF = \sqrt{(0-4)^2 + (4-0)^2} = \sqrt{32} = 4\sqrt{2}$$

$$FD = \sqrt{(-4-0)^2 + (0-4)^2} = \sqrt{32} = 4\sqrt{2}$$

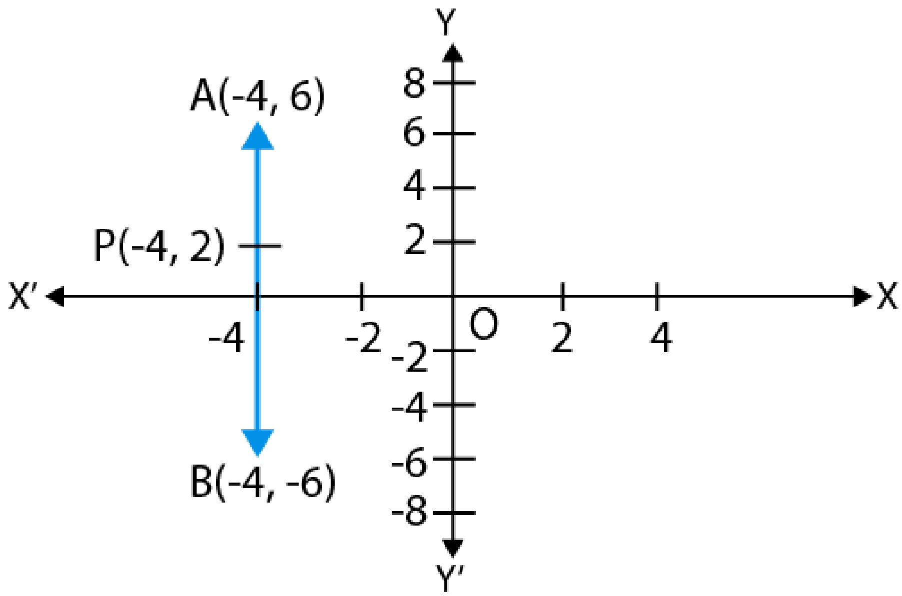
$$\therefore \quad \frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{FD} = \frac{1}{2} \quad \Rightarrow \quad \Delta ABC \sim \Delta DEF$$

Hence, triangle ABC and DEF are similar.

2. Point P (– 4, 2) lies on the line segment joining the points A (– 4, 6) and B (– 4, – 6).
Solution:

True.
Justification:
Plotting the points P(- 4, 2), A (- 4, 6) and B (- 4, - 6) on a graph paper and connecting

the points we get the graph,



Hence, from the graph it is clear that, point P (- 4, 2) lies on the line segment joining the points A (- 4, 6) and B (- 4, - 6),

3. The points (0, 5), (0, –9) and (3, 6) are collinear.
Solution:

False

Justification:

The points are collinear if area of a triangle formed by its points is equals to the zero.
Given,
$x_1 = 0$, $x_2 = 0$, $x_3 = 3$ and
$y_1 = 5$, $y_2 = - 9$, $y_3 = 6$

$\because$ Area of triangle $= \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

$\Delta = \frac{1}{2}[0(- 9 - 6) + 0(6 - 5) + 4(5 + 9)]$

$\Delta = \frac{1}{2}(0 + 0 + 3 \times 14$

$\Delta = 42/2 = 21 \neq 0$

From the above equation, it is clear that the points are not collinear.

4. Point P (0, 2) is the point of intersection of y–axis and perpendicular bisector of line segment joining the points A (–1, 1) and B (3, 3).
Solution:

False

Justification:

We know that, the points lying on perpendicular bisector of the line segment joining the two points is equidistant from the two points.
i.e., PA should be equals to the PB.

Using distance formula,
$d = \sqrt{((x_2 - x_1)^2 + (y_2 - y_1)^2)}$

$$PA = \sqrt{[-4-(4)]^2 + (6-2)^2}$$

$$PA = \sqrt{(0)^2 + (4)^2} = 4$$

$$PB = \sqrt{[-4-4]^2 + (-6-2)^2}$$

$$PB = \sqrt{0^2 + (-8)^2} = 8$$

$\because PA \neq PB$

5. Points A (3, 1), B (12, –2) and C (0, 2) cannot be the vertices of a triangle.
Solution:

True.

Justification:

Coordinates of A = (x_1, y_1) = (3, 1)
Coordinates of B = (x_2, y_2) = (12, - 2)
Coordinates of C = (x_3, y_3) = (0, 2)
Area of ΔABC = Δ = $1/2\ [x_1 (y_2 - y_3) + x_2 (y_3 - y_1) + x_3 (y_1 - y_2)]$
Δ = ½ [3 - (2 - 2) + 12(2 - 1) + 0{1 - (- 2)}]
Δ = ½ [3(- 4) + 12(1) + 0]
Δ = ½ (- 12 + 12) =0
Area of ΔABC = 0
Since, the points A (3, 1), B (12, - 2) and C (0, 2) are collinear.
Therefore, the points A (3, 1), B (12, - 2) and C (0, 2) can't be the vertices of a triangle.

6. Points A (4, 3), B (6, 4), C (5, –6) and D (–3, 5) are the vertices of a parallelogram.
Solution:

False

Justification:

The given points are A (4, 3), B (6, 4), C (5, –6) and D (–3, 5)

Finding the distance between A and B

$$AB = \sqrt{(6-4)^2 + (4-3)^2} = \sqrt{2^2 + 1^2} = \sqrt{5}$$

Finding the distance between B and C

$$BC = \sqrt{(5-6)^2 + (-6-4)^2}$$

$$BC = \sqrt{(-1)^2 + (-10)^2}$$

$$BC = \sqrt{1 + 100} = \sqrt{101}$$

Finding the distance between C and D

$$CD = \sqrt{(-3-5)^2 + (5+6)^2}$$

$$CD = \sqrt{(-8)^2 + (11)^2}$$

$$CD = \sqrt{64 + 121}$$

$$CD = \sqrt{185}$$

Finding the distance between D and A

$$DA = \sqrt{(4+3)^2 + (3-5)^2}$$

$$DA = \sqrt{7^2 + (-2)^2}$$

$$DA = \sqrt{49 + 4} = \sqrt{53}$$

Since the distances are different, we can conclude that the points are not the vertices of a parallelogram.

EXERCISE 7.3 PAGE NO: 83

1. Name the type of triangle formed by the points A (–5, 6), B (–4, –2) and C (7, 5).
Solution:

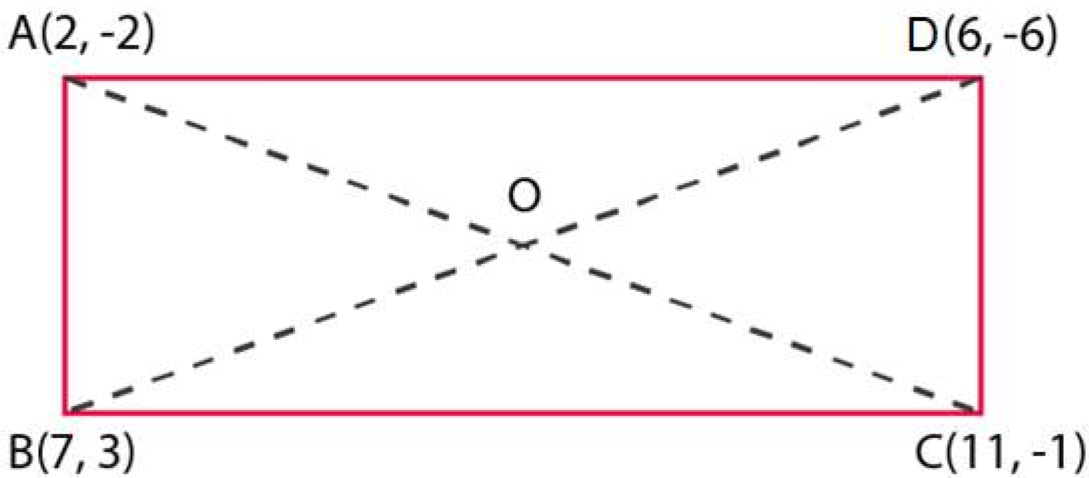
The points are A (–5, 6), B (–4, –2) and C (7, 5)
Using distance formula,
$d = \sqrt{((x_2 - x_1)^2 + (y_2 - y_1)^2)}$
$AB = \sqrt{((-4+5)^2 + (-2-6)^2)}$
$= \sqrt{1+64}$
$=\sqrt{65}$
$BC=\sqrt{((7+4)^2 + (5+2)^2)}$
$=\sqrt{121 + 49}$
$=\sqrt{170}$
$AC=\sqrt{((7+5)^2 + (5-6)^2)}$
$=\sqrt{144 + 1}$
$=\sqrt{145}$
Since all sides are of different length, ABC is a scalene triangle.

2. Find the points on the *x*–axis which are at a distance of $2\sqrt{5}$ from the point (7, –4). How many such points are there?
Solution:

Let coordinates of the point=(x, 0)(given that the point lies on x axis)
$x_1=7.\ y_1=-4$
$x_2=x.\ y_2=0$
Distance $=\sqrt{(x_2-x_1)^2+ (y_2-y_1)^2}$
According to the question,
$2\sqrt{5}=\sqrt{(x-7)^2+ (0-(-4))^2}$
Squaring L.H.S and R.H.S
$20=x^2+49-14x+16$
$20=x^2+65-14x$
$0=x^2-14x+45$
$0=x^2-9x-5x+45$
$0=x(x-9)-5(x-9)$
$0=(x-9)\ (x-5)$
$x-9 =0.\ x-5= 0$
$x=9$ or $x=5$
Therefore, coordinates of points.....(9,0)or(5,0)

3. What type of a quadrilateral do the points A (2, –2), B (7, 3), C (11, –1) and D (6, –6) taken in that order, form?
Solution:

The points are A (2, –2), B (7, 3), C (11, –1) and D (6, –6)



Using distance formula,

d = √ ((x2 - x1)2 + (y2 - y1)2)

$AB = \sqrt{(7-2)^2 + (3+2)^2}$

$= \sqrt{(5)^2 + (5)^2}$

$= \sqrt{25 + 25}$

$= \sqrt{50}$

$= 5\sqrt{2}$

$BC = \sqrt{(11-7)^2 + (-1-3)^2}$

$= \sqrt{(4)^2 + (-4)^2}$

$= \sqrt{16 + 16}$

$= \sqrt{32}$

$= 4\sqrt{2}$

CD = $\sqrt{(6-11)^2 + (-6+1)^2}$

$= \sqrt{(-5)^2 + (-5)^2}$

$= \sqrt{25 + 25}$

$= \sqrt{50}$

$= 5\sqrt{2}$

DA = $\sqrt{(2-6)^2 + (-2+6)^2}$

$= \sqrt{(-4)^2 + (4)^2}$

$= \sqrt{16 + 16}$

$= \sqrt{32}$

$= 4\sqrt{2}$

Finding diagonals AC and BD, we get,

AC = $\sqrt{(11-2)^2 + (-1+2)^2}$

$= \sqrt{(9)^2 + (1)^2}$

$= \sqrt{81 + 1}$

$= \sqrt{82}$

And BD = $\sqrt{(6-7)^2 + (-6-3)^2}$

$= \sqrt{(-1)^2 + (-9)^2}$

$= \sqrt{1 + 81}$

$= \sqrt{82}$

The Quadrilateral formed is rectangle.

4. Find the value of *a*, if the distance between the points A (–3, –14) and B (*a*, –5) is 9 units.

Solution:

Distance between two points (x_1,y_1) (x_2,y_2) is :

$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

Distance between A (–3, –14) and B (a, –5) is :

$=\sqrt{[(a+3)^2+(-5+14)^2]} =9$

Squaring on L.H.S and R.H.S.

$(a+3)^2+81=81$

$(a+3)^2=0$

$(a+3)(a+3)=0$

$a+3=0$

$a= -3$

5. Find a point which is equidistant from the points A (–5, 4) and B (–1, 6)? How many such points are there?

Solution:

Let the point be P

According to the question,

P is equidistant from A (–5, 4) and B (–1, 6)

Then the point P $= ((x_1+x_2)/2, (y_1+y_2)/2)$

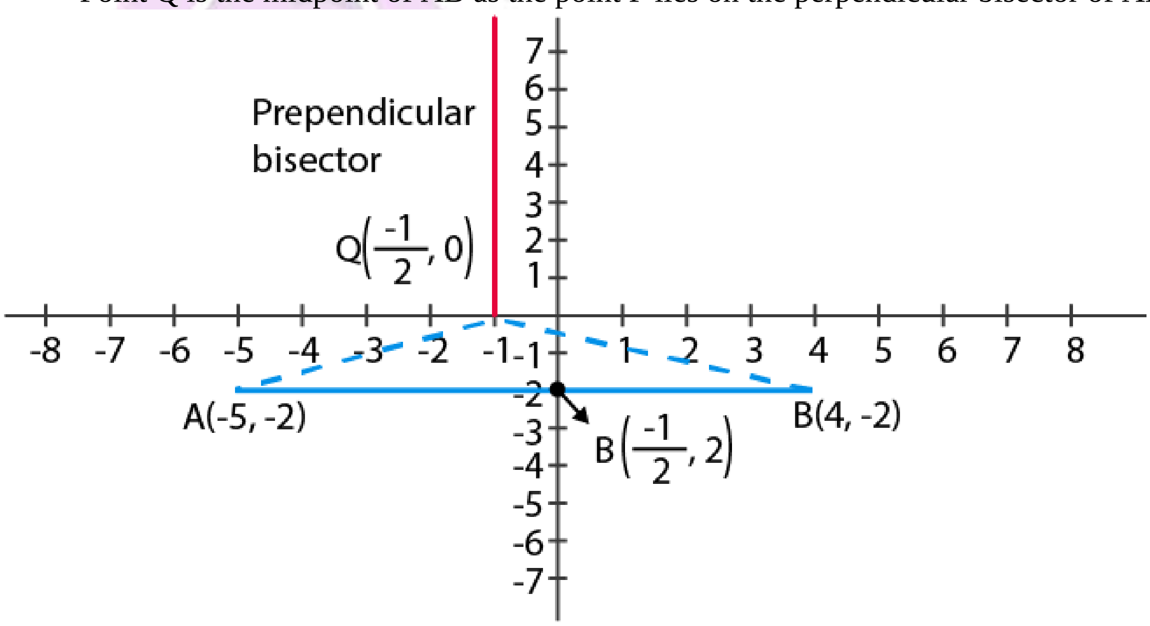
$= ((-5-1)/2, (6+4)/2)$

$= (-3 , 5)$

6. Find the coordinates of the point Q on the x–axis which lies on the perpendicular bisector of the line segment joining the points A (–5, –2) and B(4, –2). Name the type of triangle formed by the points Q, A and B.

Solution:

Point Q is the midpoint of AB as the point P lies on the perpendicular bisector of AB.



By mid point formula:

$(x_1 + x_2)/2 = (-5+4)/2$

$= -½$

$x = -½$

Given that, P lies on x axis, so y=0

P(x,y)= (-½ , 0)

Therefore, it is an isosceles triangle

7. Find the value of m if the points (5, 1), (–2, –3) and (8, 2m) are collinear.

Solution:

The points A(5, 1), B(–2, –3) and C(8, 2m) are collinear.

i.e., Area of ΔABC = 0

$½ [x_1 (y_2 - y_3) + x_2 (y_3 - y_1) + x_3 (y_1 - y_2)]=0$

$½ [5(-3 - 2m) + (- 2)(2m - 1) + 8(1 - (- 3))]=0$

$½ (-15 – 10m – 4m + 2 + 32) = 0$

$½ (-14m + 19) = 0$

m = 19/14

8. If the point A (2, – 4) is equidistant from P (3, 8) and Q (–10, y), find the values of y. Also find distance PQ.

Solution:

Given points are $A(2, -4)$, $P(3, 8)$ and $Q(-10, y)$

According to the question,

$$PA = QA$$

$$\sqrt{(2-3)^2+(-4-8)^2} = \sqrt{(2+10)^2+(-4-y)^2}$$

$$\sqrt{(-1)^2+(-12)^2} = \sqrt{(12)^2+(4+y)^2}$$

$$\sqrt{1+144} = \sqrt{144+16+y^2+8y}$$

$$\sqrt{145} = \sqrt{160+y^2+8y}$$

On squaring both sides, we get

$$145 = 160 + y^2 + 8y$$

$$y^2 + 8y + 160 - 145 = 0$$

$$y^2 + 8y + 15 = 0$$

$$y^2 + 5y + 3y + 15 = 0$$

$$y(y+5) + 3(y+5) = 0$$

$$\Rightarrow \quad (y+5)(y+3) = 0$$

$$\Rightarrow \quad y+5 = 0 \quad \Rightarrow \quad y = -5$$

$$\text{and} \quad y+3 = 0 \quad \Rightarrow \quad y = -3$$

$\therefore \quad y = -3, -5$

Now, $\quad PQ = \sqrt{(-10-3)^2 + (y-8)^2}$

For $y = -3 \quad PQ = \sqrt{(-13)^2 + (-3-8)^2} = \sqrt{169+121} = \sqrt{290}$ units

and for $y = -5 \quad PQ = \sqrt{(-13)^2 + (-5-8)^2} = \sqrt{169+169} = \sqrt{338}$ units

Hence, values of y are -3 and -5, $PQ = \sqrt{290}$ and $\sqrt{338}$

9. Find the area of the triangle whose vertices are (–8, 4), (–6, 6) and (–3, 9).
Solution:

Given vertices are:

$(x_1, y_1) = (-8, 4)$
$(x_2, y_2) = (-6, 6)$
$(x_3, y_3) = (-3, 9)$

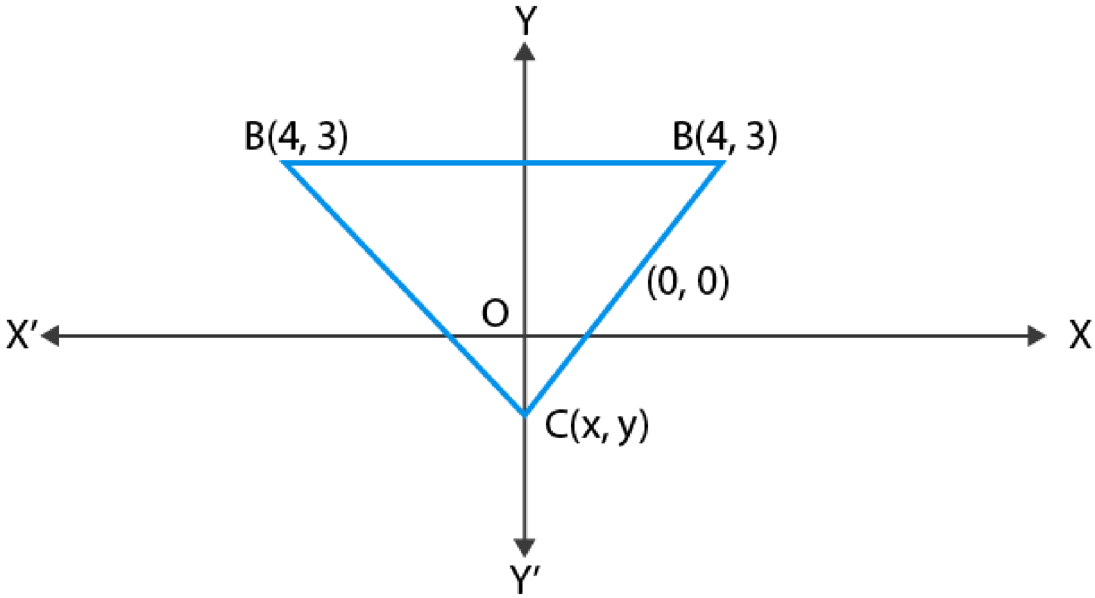
Area of triangle $= (½) (x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2))$
$= (½) (-8(6 - 9) + -6(9 - 4) + -3(4 - 6))$
$= (½) (-8(-3) + -6(5) + -3(-2))$
$= (½) (24 - 30 + 6)$
$= (½) (30 - 30)$
$= 0$ units.

10. In what ratio does the x–axis divide the line segment joining the points (– 4, – 6) and (–1, 7)? Find the coordinates of the point of division.
Solution:

Let the ratio in which x-axis divides the line segment joining (–4, –6) and (–1, 7) = 1: k.
Then,
x-coordinate becomes $(-1 - 4k) / (k + 1)$
y-coordinate becomes $(7 - 6k) / (k + 1)$
Since P lies on x-axis, y coordinate = 0
$(7 - 6k) / (k + 1) = 0$
$7 - 6k = 0$
$k = 6/7$
Now, $m_1 = 6$ and $m_2 = 7$
By using section formula,
$x = (m_1x_2 + m_2x_1)/(m_1 + m_2)$
$= (6(-1) + 7(-4))/(6 + 7)$
$= (- 6 -28)/13$
$= -34/13$
So, now
$y = (6(7) + 7(-6))/(6 + 7)$
$= (42 - 42)/13$
$= 0$
Hence, the coordinates of P are (-34/13, 0)

EXERCISE 7.4 PAGE NO: 85

1. If (– 4, 3) and (4, 3) are two vertices of an equilateral triangle, find the coordinates of the third vertex, given that the origin lies in the interior of the triangle.
Solution:



Let the vertices be (x,y)
Distance between (x,y) & (4,3) is = $\sqrt{((x-4)^2 + (y-3)^2)}$......(1)
Distance between (x,y) & (-4,3) is = $\sqrt{((x+4)^2 + (y-3)^2)}$......(2)
Distance between (4,3) &(-4,3) is $=\sqrt{((4+4)^2 + (3-3)^2)} = \sqrt{(8)^2}=8$
According to the question,
Equation (1)=(2)

$(x-4)^2=(x+4)^2$
$x^2-8x+16=x^2+8x+16$
$16x=0$
$x=0$

Also, equation (1)=8
$(x-4)^2+(y-3)^2=64$......... (3)
Substituting the value of x in (3)
Then $(0-4)^2+(y-3)^2=64$

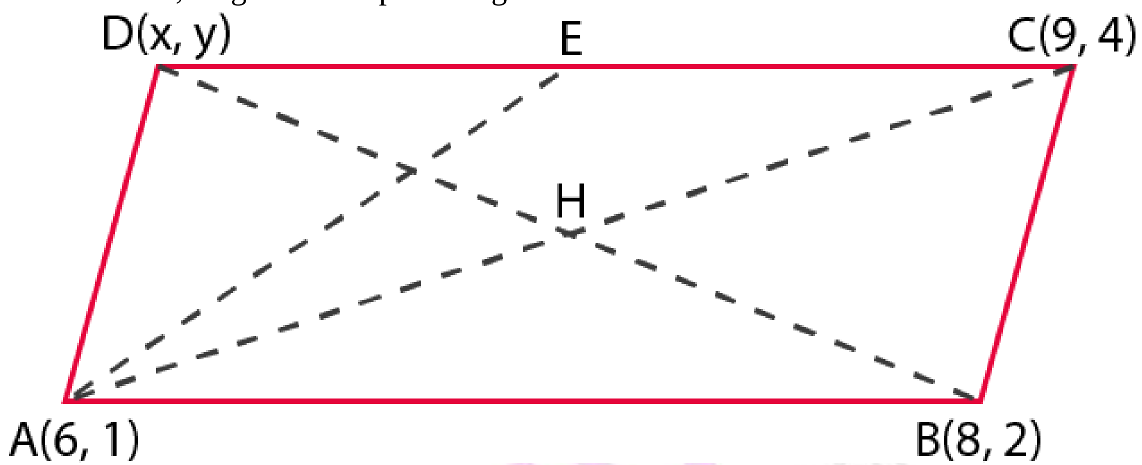
$(y-3)^2=64-16$
$(y-3)^2=48$
$y-3=(\pm)4\sqrt{3}$
$y=3(\pm)\, 4\sqrt{3}$

Neglect $y = 3+4\sqrt{3}$ as if $y = 3+4\sqrt{3}$ then origin cannot interior of triangle
Therefore, the third vertex = $(0, 3-4\sqrt{3})$

2. A (6, 1), B (8, 2) and C (9, 4) are three vertices of a parallelogram ABCD. If E is the midpoint of DC, find the area of △ ADE.

Solution:

According to the question,
The three vertices of a parallelogram ABCD are A (6, 1), B (8, 2) and C (9, 4)
Let the fourth vertex of parallelogram = (x, y),
We know that, diagonals of a parallelogram bisect each other



Since, mid - point of a line segment joining the points (x_1, y_1) and (x_2, y_2) is given by,

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Mid - point of BD = Mid - point of AC

$$\left(\frac{8+x}{2}, \frac{2+y}{2}\right) = \left(\frac{6+9}{2}, \frac{1+4}{2}\right)$$

$$\left(\frac{8+x}{2}, \frac{2+y}{2}\right) = \left(\frac{15}{2}, \frac{5}{2}\right)$$

So, we have,

$$\frac{8+x}{2} = \frac{15}{2}$$

$8 + x = 15$

$x = 7$

And,

$$\frac{2+y}{2} = \frac{5}{2}$$

$2 + y = 5 \rightarrow y = 3$

So, fourth vertex of a parallelogram is D (7, 3)

Now,

Mid - point of side

$$DC = \left(\frac{7+9}{2}, \frac{3+4}{2}\right)$$

$$E = \left(8, \frac{7}{2}\right)$$

∵ Area of ΔABC with vertices (x_1, y_1), (x_2, y_2) and (x_3, y_3);

$= \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

∴ Area of ΔADE with vertices A (6, 1), D (7, 3) and E (8, (7/2))

$$\Delta = \frac{1}{2}\left[6\left(3-\frac{7}{2}\right) + 7\left(\frac{7}{2}-1\right) + 8(1-3)\right]$$
$$= \frac{1}{2}\left[6 \times \left(\frac{-1}{2}\right) + 7\left(\frac{5}{2}\right) + 8(-2)\right]$$
$$= \frac{1}{2}\left(\frac{35}{2} - 19\right)$$
$$= \frac{1}{2}\left(\frac{-3}{2}\right)$$

= - ¾ but area can't be negative

Hence, the required area of ΔADE is ¾ sq. units

3. The points A (x_1, y_1), B (x_2, y_2) and C (x_3 y_3) are the vertices of ABC.

(i) The median from A meets BC at D. Find the coordinates of the point D.

(ii) Find the coordinates of the point P on AD such that AP : PD = 2 : 1

(iii) Find the coordinates of points Q and R on medians BE and CF, respectively such that BQ : QE = 2 : 1 and CR : RF = 2 : 1

(iv) What are the coordinates of the centroid of the triangle ABC?

Solution:

According to the question,

The vertices of ΔABC = A, B and C

Coordinates of A, B and C = A(x_1, y_1), B(x_2, y_2), C(x_3, y_3)

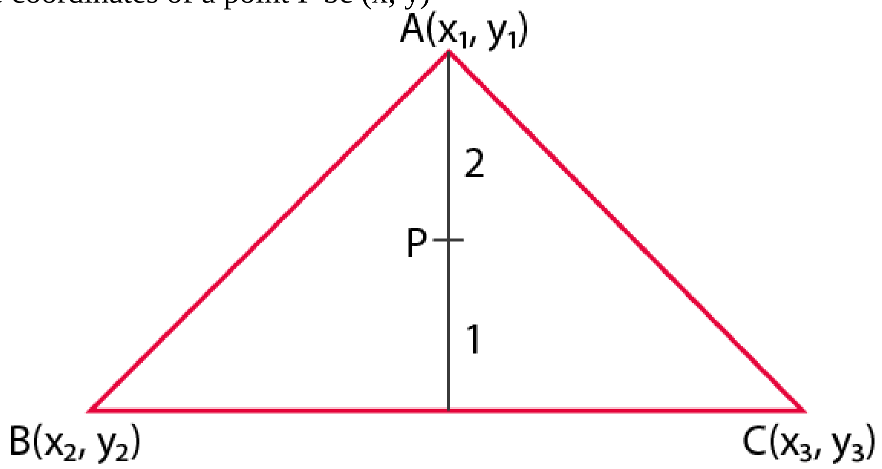
(i) As per given information D is the mid - point of BC and it bisect the line into two equal parts.

Coordinates of the mid - point of BC;

$$BC - \left(\frac{x_2 + x_3}{2}, \frac{y_2 + y_3}{2}\right)$$

$$\Rightarrow D = \left(\frac{x_2 + x_3}{2}, \frac{y_2 + y_3}{2}\right)$$

(ii) Let the coordinates of a point P be (x, y)



Given,

The ratio in which the point P(x, y), divide the line joining,

A(x_1, y_1) and D((x_2+x_3)/2 , (y_2+y_3)/2)) = 2:1

Then,

Coordinates of P =

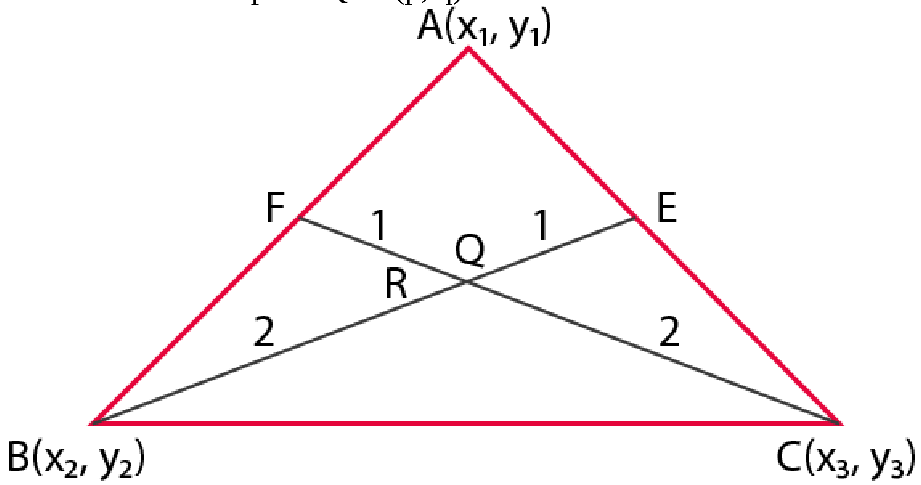
$$\left[\frac{2\times\left(\frac{x_2+x_3}{2}\right)+1\times x_1}{2+1},\frac{2\times\left(\frac{y_2+y_3}{2}\right)+1\times y_1}{2+1}\right]$$

By using internal section formula;

$$=\left(\frac{m_1x_2+m_2x_1}{m_1+m_2},\frac{m_1y_2+m_2y_1}{m_1+m_2}\right)$$

$$=\left(\frac{x_2+x_3+x_1}{3},\frac{y_2+y_3+y_1}{3}\right)$$

(iii) Let the coordinates of a point Q be (p, q)



Given,

The point Q (p, q),

Divide the line joining B(x_2, y_2) and E$\left(\frac{x_1+x_3}{2},\frac{y_1+y_3}{2}\right)$ in the ratio 2:1,

Then,

Coordinates of Q =

$$\left[\frac{2\times\left(\frac{x_1+x_3}{2}\right)+1\times x_2}{2+1},\frac{2\times\left(\frac{y_1+y_3}{2}\right)+1\times y_2}{2+1}\right]$$

$$=\left(\frac{x_1+x_2+x_3}{3},\frac{y_1+y_2+y_3}{3}\right)$$

Since, BE is the median of side CA, So BE divides AC in to two equal parts.

$\therefore$ mid - point of AC = Coordinate of E;

$$E=\left(\frac{x_1+x_3}{2},\frac{y_1+y_3}{2}\right)$$

So, the required coordinate of point Q;

$$Q=\left(\frac{x_1+x_2+x_3}{3},\frac{y_1+y_2+y_3}{3}\right)$$

Now,

Let the coordinates of a point E be (α, β)

Given,

Point R (α, β) divide the line joining C(x3, y3) and F $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$ in the ratio 2:1,

Then the coordinates of R;

$= \left[\frac{2\times\left(\frac{x_1+x_2}{2}\right)+1\times x_3}{2+1}, \frac{2\times\left(\frac{y_1+y_2}{2}\right)+1\times y_3}{2+1}\right]$

$= \left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$

Since, CF is the median of side AB.

So, CF divides AB in to two equal parts.

∴ mid - point of AB = Coordinate of F;

F = $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

So, the required coordinate of point R;

$= \left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$

(iv) Coordinate of the centroid of the ΔABC;

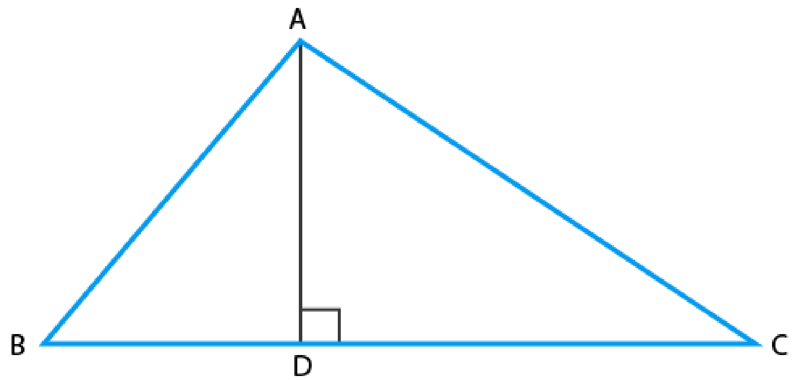
$= \left(\frac{\text{Sum of all coordinates of all vertices}}{3}, \frac{\text{Sum of all coordinates of all vertices}}{3}\right)$

$= \left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$

EXERCISE 6.1 PAGE NO: 60

1. In figure, if ∠BAC =90° and AD⊥BC. Then,

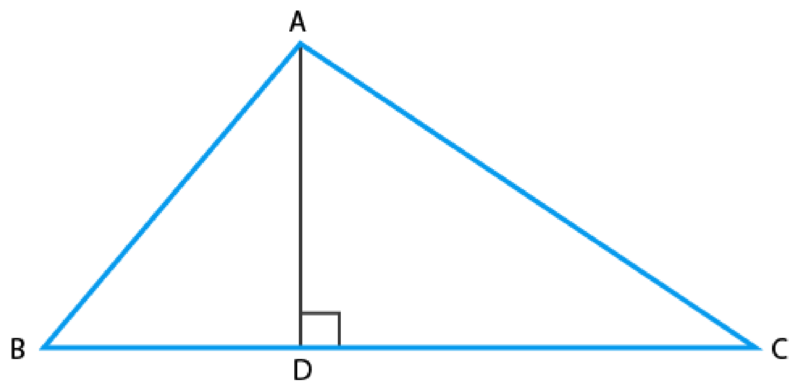
(a) BD.CD = BZC²   (b) AB.AC = BC²   (c) BD.CD=AD²   (d) AB.AC =AD²



Solution:

c) BD.CD=AD²

Explanation:



From ΔADB and ΔADC,
According to the question, we have,
∠D = ∠D = 90° (∵ AD ⊥ BC)
∠DBA = ∠DAC [each angle = 90°- ∠C]
Using AAA similarity criteria,
ΔADB ~ ΔADC
BD/AD = AD/CD
BD.CD = AD^2

2. If the lengths of the diagonals of rhombus are 16 cm and 12 cm. Then, the length of the sides of the rhombus is

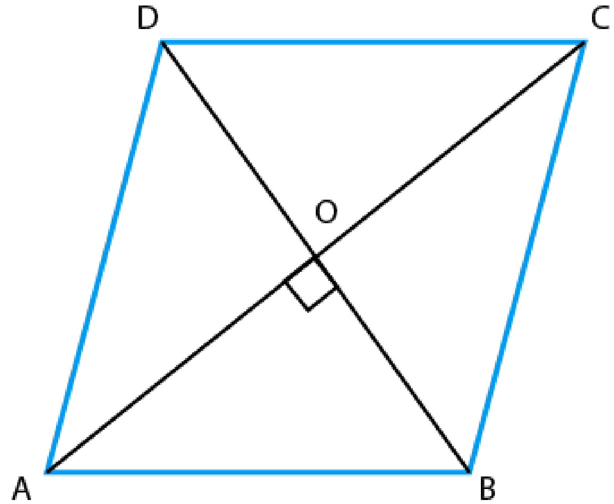
(a) 9 cm   (b) 10 cm   (c) 8 cm   (d) 20 cm

Solution:

(b) 10 cm

Explanation:

We know that,
A rhombus is a simple quadrilateral whose four sides are of same length and diagonals are perpendicular bisector of each other.
According to the question, we get,



According to the question,
AC = 16 cm and BD = 12 cm
$\angle AOB = 90°$
$\because$ AC and BD bisects each other
AO = ½ AC and BO = ½ BD
Then, we get,
AO = 8 cm and BO = 6 cm
Now, in right angled ΔAOB,
Using the Pythagoras theorem,
We have,
$AB^2 = AO^2 + OB^2$
$AB^2 = 8^2 + 6^2 = 64 + 36 = 100$
$\therefore AB = \sqrt{100} = 10$ cm
We know that the four sides of a rhombus are equal.
Therefore, we get,
one side of rhombus = 10 cm.

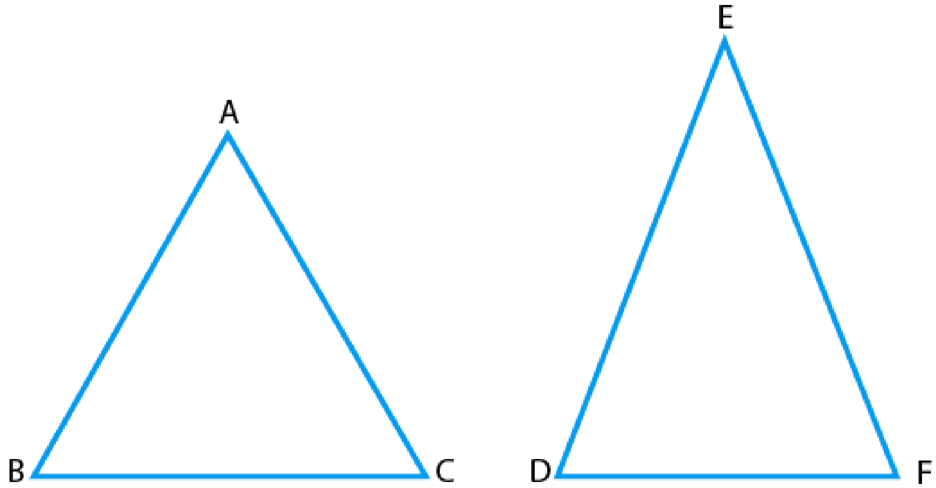
3. If ΔABC ~ ΔEDF and ΔABC is not similar to ΔDEF, then which of the following is not true?

(a) BC · EF = AC · FD **(b) AB · EF = AC · DE**
(c) BC · DE = AB · EF **(d) BC · DE = AB · FD**

Solution:

(c) BC · DE = AB · EF

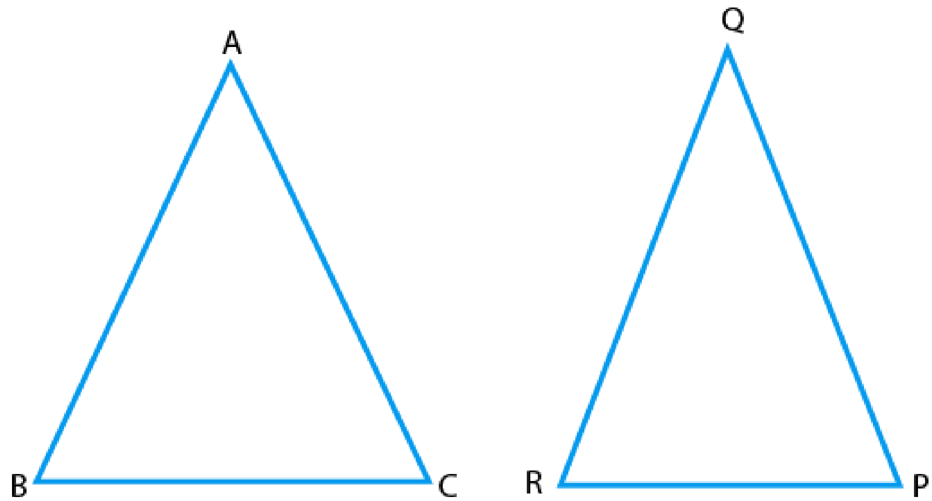
Explanation:
We know that,
If sides of one triangle are proportional to the side of the other triangle, and the corresponding angles are also equal, then the triangles are similar by SSS similarity.



So, ΔABC ~ ΔEDF
Using similarity property,
AB/ED = BC/DF = AC/EF
Taking AB/ED = BC/DF, we get
AB/ED = BC/DF
AB.DF = ED.BC
So, option (d) BC · DE = AB · FD is true
Taking BC/DF = AC/EF, we get
BC/DF = AC/EF
⇒ BC.EF = AC.DF
So, option (a) BC · EF = AC · FD is true
Taking AB/ED = AC/EF, we get,
AB/ED = AC/EF
AB.EF = ED.AC
So, option (b) AB · EF = AC · DE is true.

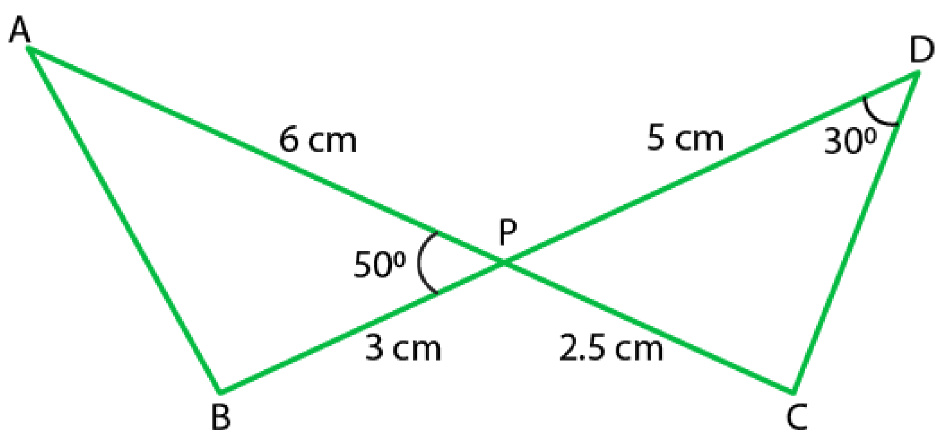
4. If in two Δ PQR,AB/QR = BC/PR = CA/PQ, then
(a)Δ PQR~Δ CAB **(b) Δ PQR ~ Δ ABC**
(c)Δ CBA ~ Δ PQR **(d) Δ BCA ~ Δ PQR**

Solution:
(a)Δ PQR~Δ CAB
<u>Explanation:</u>



From ΔABC and ΔPQR, we have,
AB/QR = BC/PR = CA/PQ
If sides of one triangle are proportional to the side of the other triangle, and their corresponding angles are also equal, then both the triangles are similar by SSS similarity.
Therefore, we have,
Δ PQR~Δ CAB

5. In figure, two line segments AC and BD intersect each other at the point P such that PA = 6 cm, PB = 3 cm, PC = 2.5 cm, PD = 5 cm, ∠APB = 50° and ∠CDP = 30°. Then, ∠PBA is equal to
(a) 50° **(b) 30°** **(c) 60°** **(d) 100°**



Solution:
(d) 100°
Explanation:
From ΔAPB and ΔCPD,
∠APB = ∠CPD = 50° (since they are vertically opposite angles)

AP/PD = 6/5 … (i)
Also, BP/CP = 3/2.5
Or BP/CP = 6/5 … (ii)
From equations (i) and (ii),
We get,
AP/PD = BP/CP
So, ΔAPB ~ ΔDPC [using SAS similarity criterion]
∴ ∠A = ∠D = 30° [since, corresponding angles of similar triangles]
Since, Sum of angles of a triangle = 180°,
In ΔAPB,
∠A + ∠B + ∠APB = 180°
So, 30° + ∠B + 50° = 180°
Then, ∠B = 180° - (50° + 30°)
∠B = 180 – 80° = 100°
Therefore, ∠PBA = 100°

EXERCISE 6.2

1. Is the triangle with sides 25 cm, 5 cm and 24 cm a right triangle? Give reason for your answer.
Solution:

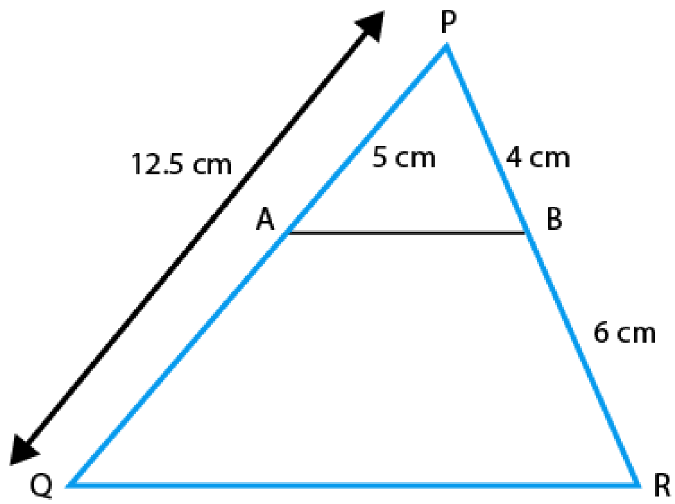
False
According to the question,
Let us assume that,
A = 25 cm
B = 5 cm
C = 24 cm
Now, Using Pythagoras Theorem,
We have,
$A^2 = B^2 + C^2$
$B^2 + C^2 = (5)^2 + (24)^2$
$B^2 + C^2 = 25 + 576$
$B^2 + C^2 = 601$
$A^2 = 600$
$600 \neq 601$
$A^2 \neq B^2 + C^2$
Since the sides does not satisfy the property of Pythagoras theorem, triangle with sides 25cm, 5cm and 24cm is not a right triangle.

2. It is given that ΔDEF ~ ΔRPQ. Is it true to say that ∠D = ∠R and ∠F = ∠P ? Why?
Solution:

False
We know that,
Corresponding angles are equal in similar triangles.
So, we get,
$\angle D = \angle R$
$\angle E = \angle P$
$\angle F = \angle Q$

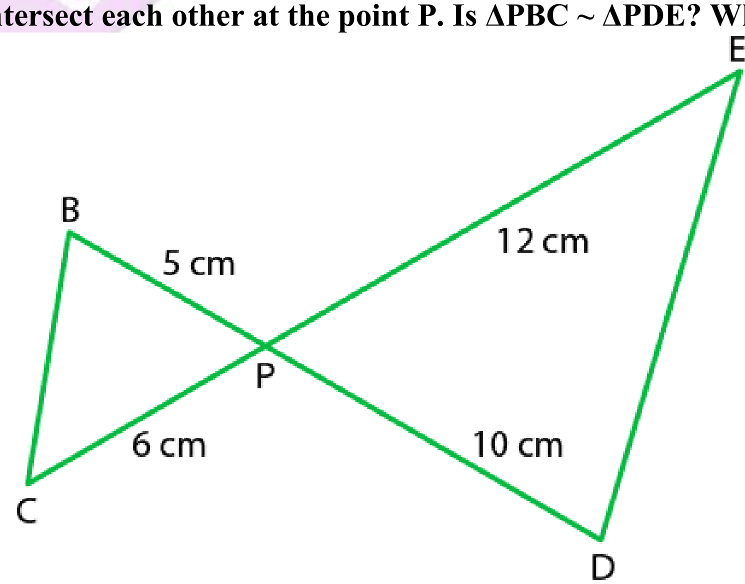
3. A and B are respectively the points on the sides PQ and PR of a ΔPQR such that PQ = 12.5 cm, PA = 5 cm, BR = 6 cm and PB = 4 cm. Is AB || QR? Give reason for your answer.
Solution:

True
According to the question,



PQ = 12.5 cm
PA = 5 cm
BR = 6 cm
PB = 4 cm
Then,
QA = QP – PA = 12.5 – 5 = 7.5 cm
So,
PA/AQ = 5/7.5 = 50/75 = 2/3 … (i)
PB/BR = 4/6 = 2/3 … (ii)
Form Equations (i) and (ii).
PA/AQ = PB/BR
We know that, if a line divides any two sides of a triangle in the same ratio, then the line is parallel to the third side.
Therefore,
AB || QR.

4. In figure, BD and CE intersect each other at the point P. Is ΔPBC ~ ΔPDE? Why?



Solution:

True

In ΔPBC and ΔPDE,

∠BPC = ∠EPD [vertically opposite angles]

PB/PD = 5/10 = ½ … (i)

PC/PE = 6/12 = ½ … (ii)

From equation (i) and (ii),

We get,

PB/PD = PC/PE

Since, ∠BPC of ΔPBC = ∠EPD of ΔPDE and the sides including these.

Then, by SAS similarity criteria

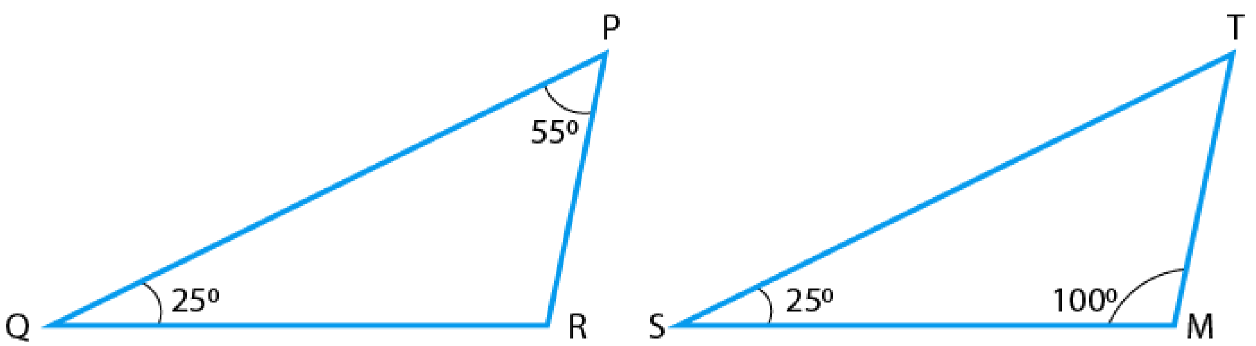
ΔPBC ~ ΔPDE

5. In ΔPQR and ΔMST, ∠P = 55°, ∠Q =25°, ∠M = 100° and ∠S = 25°. Is ΔQPR ~ ΔTSM? Why?

Solution:

We know that,

Sum of the three angles of a triangle = 180°.



Then, from ΔPQR,

∠P + ∠Q + ∠R = 180°

55° + 25° + ∠R = 180°

So, we get,

∠R = 180° - (55° + 25°) = 180°- 80° = 100°

Similarly, from ΔTSM,

∠T + ∠S + ∠M = 180°

∠T + ∠25° + 100° = 180°

So, we get,

∠T = 180°- (∠25° + 100°)

∠T = 180° - 125° = 55°

In ΔPQR and ΔTSM,

We have,

∠P = ∠T,

∠Q = ∠S

∠R = ∠M

Hence, ΔPQR ~ ΔTSM

Since, all corresponding angles are equal,

ΔQPR is similar to ΔTSM,

6. Is the following statement true? Why?

"Two quadrilaterals are similar, if their corresponding angles are equal".

Solution:

False

Two quadrilaterals cannot be similar, if only their corresponding angles are equal

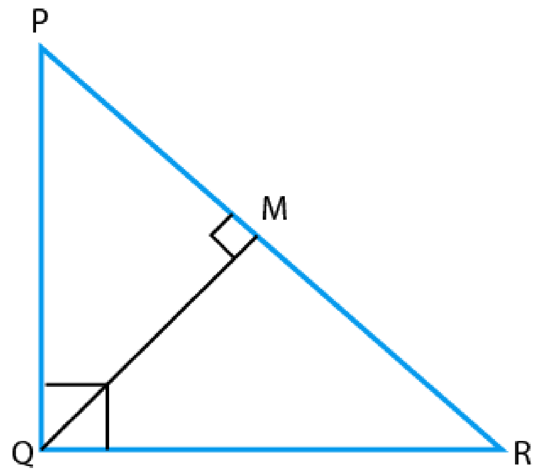
EXERCISE 6.3

PAGE NO: 66

1. In a ΔPQR, $PR^2 – PQ^2 = QR^2$ and M is a point on side PR such that QM ⊥ PR. Prove that $QM^2 = PM \times MR$.

Solution:

According to the question,



In ΔPQR,

$PR^2 = QR^2$ and QM⊥PR

Using Pythagoras theorem, we have,

$PR^2 = PQ^2 + QR^2$

ΔPQR is right angled triangle at Q.

From ΔQMR and ΔPMQ, we have,

∠M = ∠M

∠MQR = ∠QPM [= 90°-∠R]

So, using the AAA similarity criteria,

We have,

ΔQMR ~ ΔPMQ

Also, we know that,

Area of triangles = ½ × base × height

So, by property of area of similar triangles,

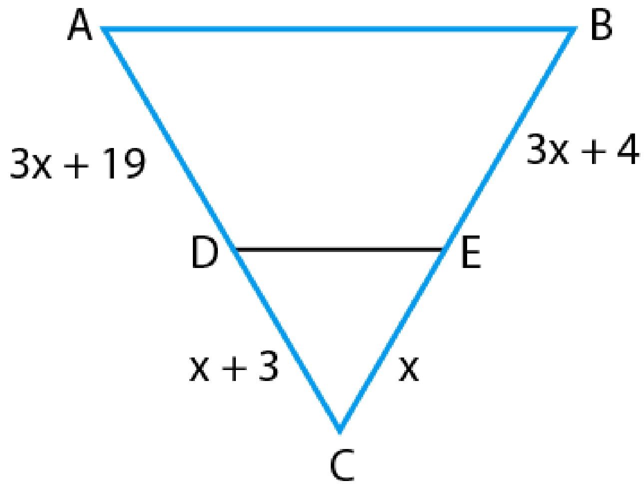
$$\frac{ar(\Delta QMR)}{ar(PMQ)} = \frac{(QM)^2}{(PM)^2}$$

$$\frac{ar(\Delta QMR)}{ar(PMQ)} = \frac{\frac{1}{2} \times RM \times QM}{\frac{1}{2} \times PM \times QM}$$

$$\frac{ar(\Delta QMR)}{ar(PMQ)} = \frac{(QM)^2}{(PM)^2}$$

$QM^2 = PM \times RM$
Hence proved.

2. Find the value of x for which DE||AB in given figure.

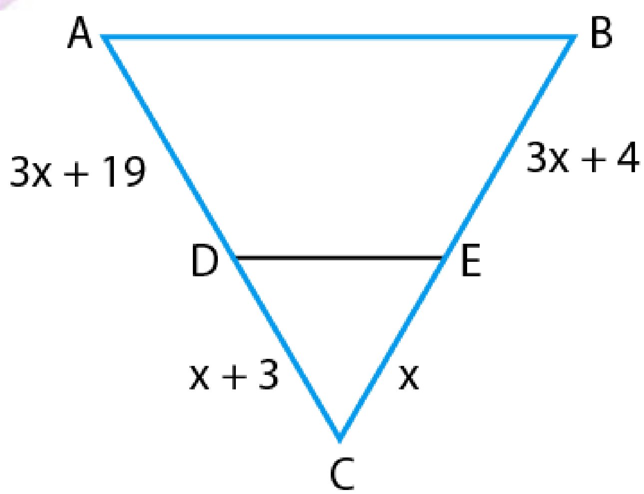


Solution:

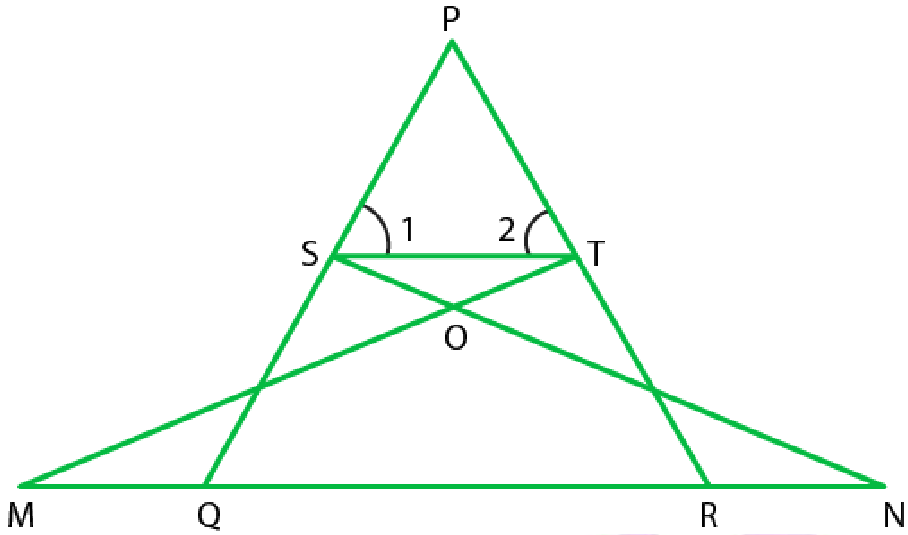
According to the question,
DE || AB
Using basic proportionality theorem,
CD/AD = CE/BE
∴ If a line is drawn parallel to one side of a triangle such that it intersects the other sides at distinct points, then, the other two sides are divided in the same ratio.
Hence, we can conclude that, the line drawn is equal to the third side of the triangle.

$$\frac{x+3}{3x+19} = \frac{x}{3x+4}$$

$(x + 3)(3x + 4) = x(3x + 19)$
$3x^2 + 4x + 9x + 12 = 3x^2 + 19x$
$19x - 13x = 12$
$6x = 12$
$\therefore x = 12/6 = 2$



3. In figure, if ∠1 =∠2 and ΔNSQ = ΔMTR, then prove that ΔPTS ~ ΔPRQ.



Solution:

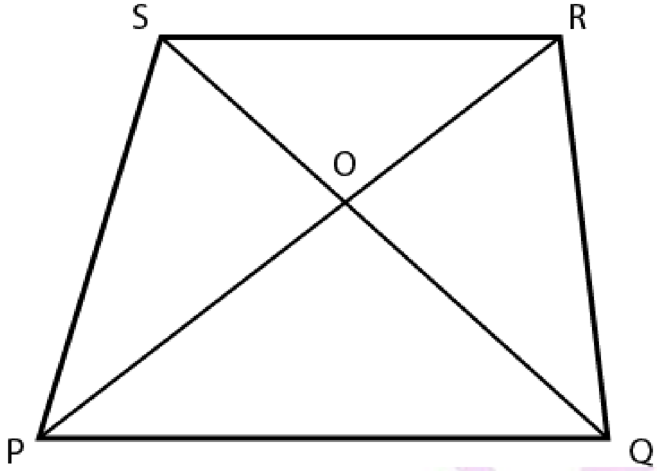
According to the question,
Δ NSQ ≅ ΔMTR
∠1 = ∠2
Since,
ΔNSQ = ΔMTR
So,
SQ = TR ….(i)
Also,
∠1 = ∠2 ⇒ PT = PS….(ii)
[Since, sides opposite to equal angles are also equal]
From Equation (i) and (ii).
PS/SQ = PT/TR
⇒ ST || QR
By converse of basic proportionality theorem, If a line is drawn parallel to one side of a triangle to intersect the other sides in distinct points, the other two sides are divided in the same ratio.
∴ ∠1 = PQR
And
∠2 = ∠PRQ
In ΔPTS and ΔPRQ.
∠P = ∠P [Common angles]
∠1 = ∠PQR (proved)
∠2 = ∠PRQ (proved)
∴ ΔPTS - ΔPRQ
[By AAA similarity criteria]
Hence proved.

4. Diagonals of a trapezium PQRS intersect each other at the point 0, PQ || RS and PQ = 3 RS. Find the ratio of the areas of Δ POQ and Δ ROS.
Solution:

According to the question,

PQRS is a trapezium in which PQ∥RS and PQ = 3RS

PQ/RS = 3/1 = 3 …(i)



In ΔPOQ and ΔROS,

∠SOR = ∠QOP [vertically opposite angles]

∠SRP = ∠RPQ [alternate angles]

∴ ΔPOQ ~ ΔROS [by AAA similarity criterion]

By property of area of similar triangle,

$$\frac{Ar(\Delta POQ)}{ar(SOR)} = \frac{(PQ)^2}{(RS)^2} = \left(\frac{PQ}{RS}\right)^2 = \left(\frac{3}{1}\right)^2$$

$$\Rightarrow \frac{ar(\Delta POQ)}{ar(SOR)} = \frac{9}{1}$$

Therefore, the required ratio = 9:1.

5. In figure, if AB || DC and AC, PQ intersect each other at the point O. Prove that OA.CQ = 0C.AP.

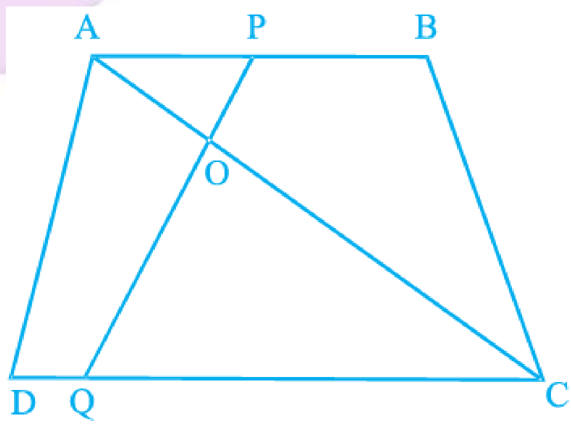


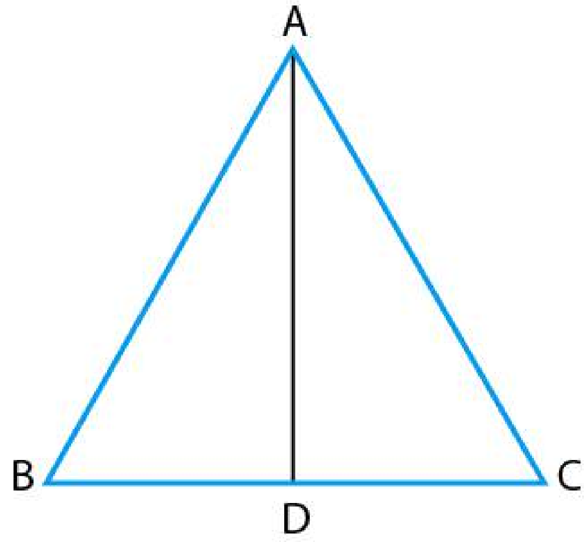
Fig. 6.10

Solution:

According to the question,

AC and PQ intersect each other at the point O and AB||DC.
From ΔAOP and ΔCOQ,
∠AOP = ∠COQ [Since they are vertically opposite angles]
∠APO = ∠CQO [since, AB||DC and PQ is transversal, the angles are alternate angles]
∴ ΔAOP ~ ΔCOQ [using AAA similarity criterion]
Then, since, corresponding sides are proportional
We have,
OA/OC = AP/CQ
OA × CQ = OC × AP
Hence Proved.

6. Find the altitude of an equilateral triangle of side 8 cm.
Solution:
Let ABC be an equilateral triangle of side 8 cm
AB = BC = CA = 8 cm. (all sides of an equilateral triangle is equal)



Draw altitude AD which is perpendicular to BC.
Then, D is the mid-point of BC.
∴ BD = CD = ½
BC = 8/2 = 4 cm
Now, by Pythagoras theorem
$AB^2 = AD^2 + BD^2$
$\Rightarrow (8)^2 = AD^2 + (4)^2$
$\Rightarrow 64 = AD^2 + 16$
$\Rightarrow AD = \sqrt{48} = 4\sqrt{3}$ cm.
Hence, altitude of an equilateral triangle is $4\sqrt{3}$ cm.

7. If ΔABC ~ ΔDEF, AB = 4 cm, DE = 6, EF = 9 cm and FD = 12 cm, then find the perimeter of ΔABC.
Solution:
According to the question,
AB = 4 cm,

DE = 6 cm
EF = 9 cm
FD = 12 cm
Also,
ΔABC ~ ΔDEF
We have,

$$\frac{AB}{ED} = \frac{BC}{EF} = \frac{AC}{DF}$$

$$\frac{4}{6} = \frac{BC}{9} = \frac{AC}{12}$$

By taking first two terms, we have

$$\frac{4}{6} = \frac{BC}{9}$$

$$BC = \frac{(4 \times 9)}{6} = 6\,\text{cm}$$

And by taking last two terms, we have,

$$\frac{BC}{9} = \frac{AC}{12}$$

$$\frac{6}{9} = \frac{AC}{12}$$

$$AC = \frac{6 \times 12}{9} = 8\,\text{cm}$$

Now,
Perimeter of ΔABC = AB + BC + AC
= 4 + 6 + 8 = 18 cm
Thus, the perimeter of the triangle is 18 cm.

EXERCISE 6.4

PAGE NO: 73

1. In Fig. 6.16, if ∠A = ∠C, AB = 6 cm, BP = 15 cm, AP = 12 cm and CP = 4 cm, then find the lengths of PD and CD.

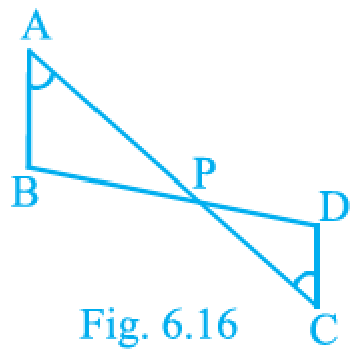


Fig. 6.16

Solution:

According to the question,
∠A = ∠C,
AB = 6 cm, BP = 15 cm,
AP = 12 cm
CP = 4 cm
From ΔAPB and ΔCPD,
∠A = ∠C
∠APB = ∠CPD [vertically opposite angles]
∴ By AAA similarity criteria,
ΔAPD ~ ΔCPD
Using basic proportionality theorem,

$$\frac{AP}{CP} = \frac{PB}{PD} = \frac{AB}{CD}$$

$$\frac{12}{4} = \frac{15}{PD} = \frac{6}{CD}$$

Considering AP/CP = PB/PD, we get,

$$\frac{12}{4} = \frac{15}{PD}$$

$$PD = \frac{15 \times 4}{12} = \frac{60}{12} = 5\text{cm}$$

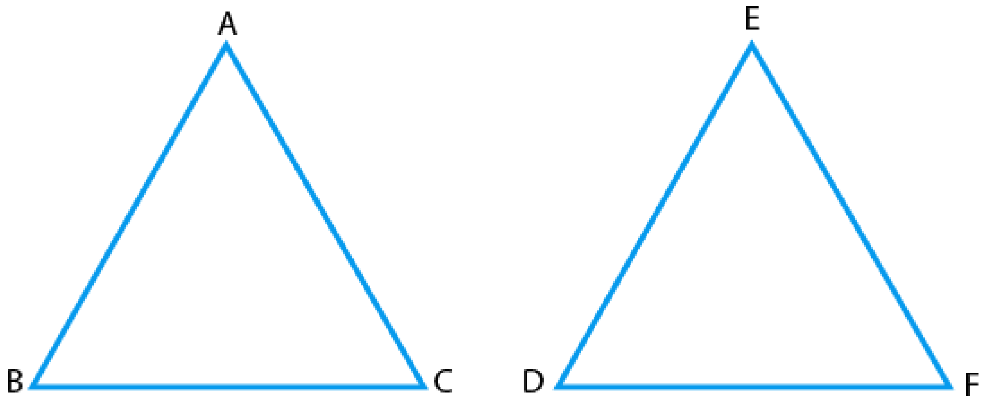
Considering, AP/CP = AB/CD

$$CD = \frac{(6 \times 4)}{12} = 2 \text{ cm}$$

Therefore,
Length of PD = 5 cm

Length of CD = 2 cm

2. It is given that Δ ABC ~ Δ EDF such that AB = 5 cm, AC = 7 cm, DF= 15 cm and DE = 12 cm. Find the lengths of the remaining sides of the triangles.
Solution:



According to the question,
ΔABC ~ ΔEDF
From property of similar triangle,
We know that, corresponding sides of ΔABC and ΔEDF are in the same ratio.
AB/ED = AC/EF = BC/DF ….(i)
According to the question,
AB = 5cm, AC = 7cm
DF = 15cm and DE = 12cm
Substituting these values in Equation (i), we get,

$$\frac{5}{12} = \frac{7}{EF} = \frac{BC}{15}$$

On taking 5/12 = 7/EF, we get,

$$\frac{5}{12} = \frac{7}{EF}$$

$$EF = \frac{12 \times 7}{5} = 16.8\text{cm}$$

On taking 5/12 = BC/15, we get,

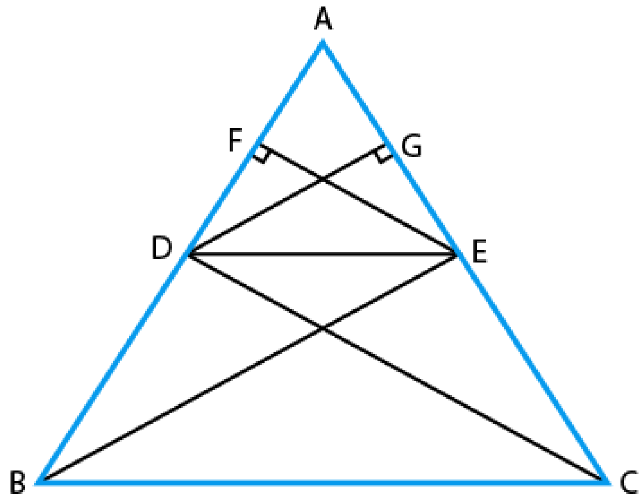
$$\frac{5}{12} = \frac{BC}{15}$$

$$BC = \frac{5 \times 15}{12} = 6.25 \text{ cm}$$

Hence, lengths of the remaining sides of the triangles are EF = 16.8 cm and BC = 6.25 cm

3. Prove that if a line is drawn parallel to one side of a triangle to intersect the other two sides, then the two sides are divided in the same ratio.
Solution:

Let a ΔABC in which a line DE parallel to BC intersects AB at D and AC at E.
To prove DE divides the two sides in the same ratio.
AD/DB = AE/EC



Construction:
Join BE, CD
Draw EF ⊥ AB and DG ⊥ AC.
We know that,
Area of triangle = ½ × base × height
Then,

$$\frac{ar(\Delta ADE)}{ar(\Delta BDE)} = \frac{\frac{1}{2} \times AD \times EF}{\frac{1}{2} \times DB \times EF}$$

$$\frac{ar(\Delta ADE)}{ar(\Delta BDE)} = \frac{AD}{DB} \quad ...(i)$$

$$\frac{ar(\Delta ADE)}{ar(\Delta DEC)} = \frac{\frac{1}{2} \times AE \times GD}{\frac{1}{2} \times EC \times GD}$$

$$\frac{ar(\Delta ADE)}{ar(\Delta DEC)} = \frac{AE}{EC} \quad ...(ii)$$

Since,
ΔBDE and ΔDEC lie between the same parallel DE and BC and are on the same base DE.
We have,
area (ΔBDE) = area(ΔDEC) …..(iii)
From Equation (i), (ii) and (iii),
We get,
AD/DB = AE/EC
Hence proved.

4. In Fig 6.17, if PQRS is a parallelogram and AB||PS, then prove that OC||SR.

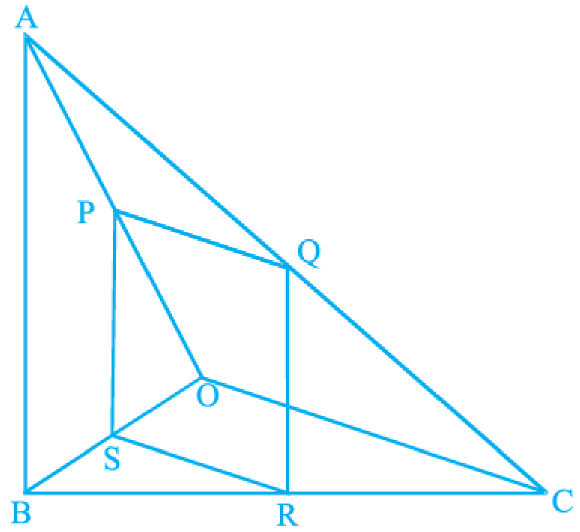
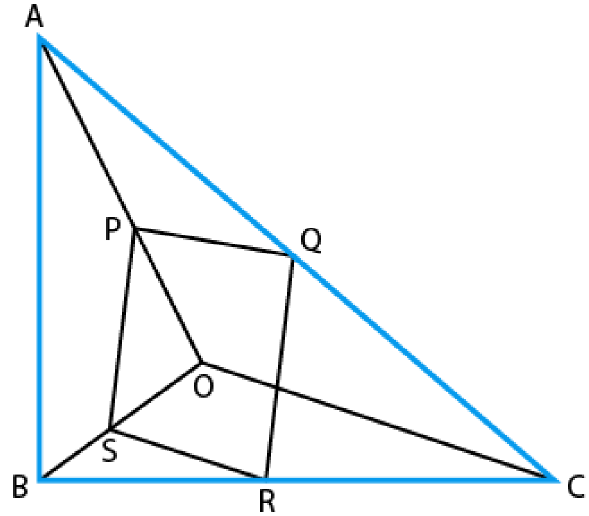


Fig. 6.17

Solution:

According to the question,
PQRS is a parallelogram,
Therefore, PQ || SR and PS || QR.
Also given, AB || PS.



To prove:

OC || SR

From ΔOPS and OAB,

PS||AB

∠POS = ∠AOB [common angle]

∠OSP = ∠OBA [corresponding angles]

ΔOPS ~ ΔOAB [by AAA similarity criteria]

Then,

Using basic proportionality theorem,

We get,

PS/AB = OS/OB …(i)

From ΔCQR and ΔCAB,

QR || PS || AB

∠QCR = ∠ACB [common angle]

∠CRQ = ∠CBA [corresponding angles]

ΔCQR ~ ΔCAB

Then, by basic proportionality theorem

$$\frac{QR}{AB} = \frac{CR}{CB}$$

$$\frac{PS}{AB} = \frac{CR}{CB} \quad \text{...(ii)}$$

[PS ≅ QR Since, PQRS is a parallelogram,]

From Equation (i) and (ii),

$$\frac{OS}{OB} = \frac{CR}{CB}$$

$$\frac{OB}{OS} = \frac{CB}{CR}$$

Subtracting 1 from L.H.S and R.H.S, we get,

$$\frac{OB}{OS} - 1 = \frac{CB}{CR} - 1$$

$$\frac{OB - OS}{OS} = \frac{(CB - CR)}{CR}$$

$$\frac{BS}{OS} = \frac{BR}{CR}$$

SR || OC [By converse of basic proportionality theorem]

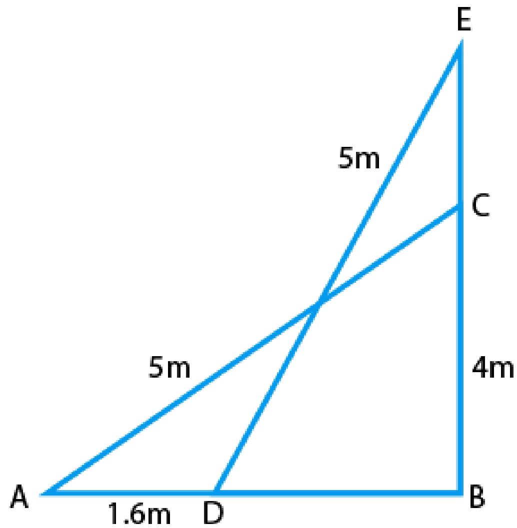
Hence proved.

5. A 5 m long ladder is placed leaning towards a vertical wall such that it reaches the wall at a point 4 m high. If the foot of the ladder is moved 1.6 m towards the wall, then find the distance by which the top of the ladder would slide upwards on the wall.

Solution:

Let the length of the ladder = AC = 5 m

Let the height of the wall on which ladder is placed = BC = 4m.

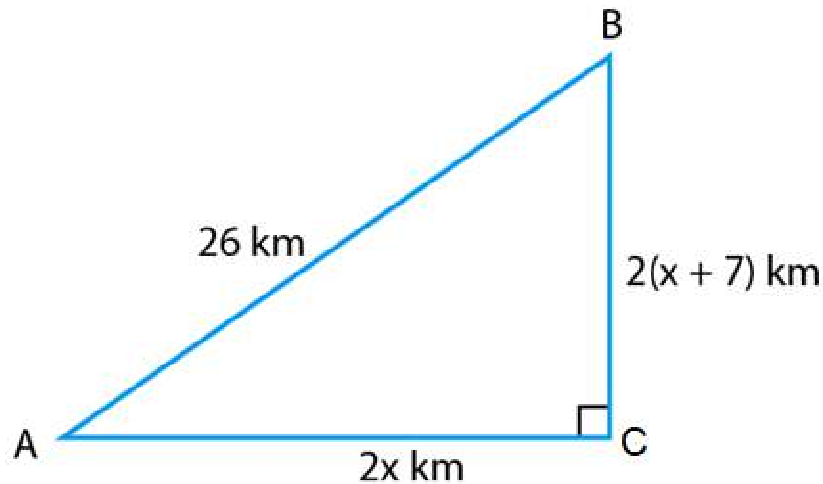


From right angled ΔEBD,
Using the Pythagoras Theorem,
$ED^2 = EB^2 + BD^2$
$(5)^2 = (EB)^2 + (14)^2$ [BD = 1.4]
$25 = (EB)^2 + 1.96$
$(EB)^2 = 25 – 1.96 = 23.04$
$EB = \sqrt{23.04} = 4.8$
Now, we have,
EC = EB – BC = 4.8 – 4 = 0.8
Hence, the top of the ladder would slide upwards on the wall by a distance of 0.8 m.

6. For going to a city B from city A, there is a route via city C such that AC⊥CB, AC = 2 x km and CB = 2 (x + 7) km. It is proposed to construct a 26 km highway which directly connects the two cities A and B. Find how much distance will be saved in reaching city B from city A after the construction of the highway.

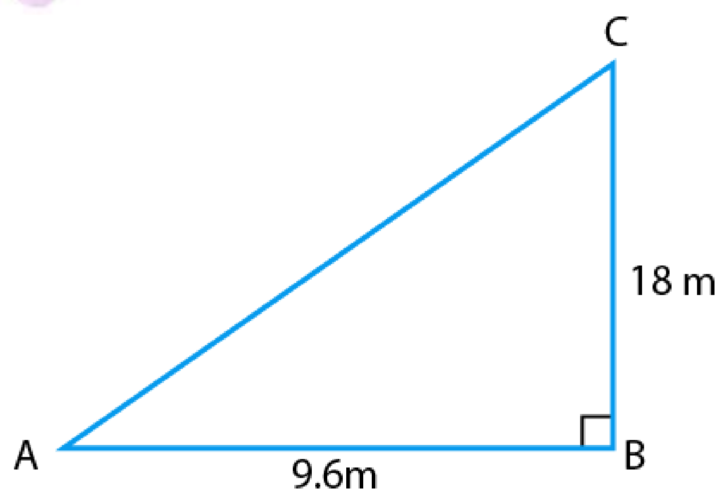
Solution:

According to the question,
AC⊥CB,
AC = 2x km,
CB = 2 (x + 7) km and AB = 26 km
Thus, we get Δ ACB right angled at C.
Now, from ΔACB,
Using Pythagoras Theorem,
$AB^2 = AC^2 + BC^2$
$\Rightarrow (26)^2 = (2x)^2 + \{2(x + 7)\}^2$
$\Rightarrow 676 = 4x^2 + 4(x^2 + 196 + 14x)$
$\Rightarrow 676 = 4x^2 + 4x^2 + 196 + 56x$
$\Rightarrow 676 = 8^2 + 56x + 196$
$\Rightarrow 8x^2 + 56x – 480 = 0$



Dividing the equation by 8, we get,
$x^2 + 7x - 60 = 0$
$x^2 + 12x - 5x - 60 = 0$
$x(x + 12)-5(x + 12) = 0$
$(x + 12)(x - 5) = 0$
$\therefore$ x = -12 or x = 5
Since the distance can't be negative, we neglect x = -12
$\therefore$ x = 5
Now,
AC = 2x = 10km
BC = 2(x + 7) = 2(5 + 7) = 24 km
Thus, the distance covered to city B from city A via city C = AC + BC
AC + BC = 10 + 24
= 34 km
Distance covered to city B from city A after the highway was constructed = BA = 26 km
Therefore, the distance saved = 34 – 26 = 8 km.

7. A flag pole 18 m high casts a shadow 9.6 m long. Find the distance of the top of the pole from the far end of the shadow.
Solution:



Let MN = 18 m be the flag pole and its shadow be LM = 9.6 m.
The distance of the top of the pole, N from the far end, L of the shadow is LN.
In right angled ΔLMN,
$LN^2 = LM^2 + MN^2$ [by Pythagoras theorem]
$\Rightarrow LN^2 = (9.6)^2 + (18)^2$
$\Rightarrow LN^2 = 9.216 + 324$
$\Rightarrow LN^2 = 416.16$
$\therefore LN = \sqrt{416.16} = 20.4$ m
Hence, the required distance is 20.4 m

EXERCISE 5.1 PAGE NO: 45

Choose the correct answer from the given four options in the following questions:

1. In an AP, if d = –4, n = 7, a_n = 4, then a is

(A) 6 (B) 7 (C) 20 (D) 28

Solution:

(D) 28

Explanation:

We know that nth term of an AP is

$a_n = a + (n - 1)d$

where,

a = first term

a_n is nth term

d is the common difference

According to the question,

$4 = a + (7 - 1)(- 4)$

$4 = a – 24$

$a = 24 + 4 = 28$

2. In an AP, if a = 3.5, d = 0, n = 101, then a_n will be

(A) 0 (B) 3.5 (C) 103.5 (D) 104.5

Solution:

(B) 3.5

Explanation:

We know that nth term of an AP is

$a_n = a + (n - 1)d$

Where,

a = first term

a_n is nth term

d is the common difference

$a_n = 3.5 + (101 - 1)0$

$= 3.5$

(Since, d = 0, it's a constant A.P)

3. The list of numbers – 10, – 6, – 2, 2,... is

(A) an AP with d = – 16

(B) an AP with d = 4

(C) an AP with d = – 4

(D) not an AP

Solution:

(B) an AP with d = 4

Explanation:

According to the question,

$a_1 = - 10$

$a_2 = - 6$

$a_3 = -2$
$a_4 = 2$
$a_2 - a_1 = 4$
$a_3 - a_2 = 4$
$a_4 - a_3 = 4$
$a_2 - a_1 = a_3 - a_2 = a_4 - a_3 = 4$
Therefore, it's an A.P with d = 4

4. The 11th term of the AP: –5, (–5/2), 0, 5/2, ...is
(A) –20 (B) 20 (C) –30 (D) 30

Solution:

(B) 20

Explanation:
First term, a = - 5
Common difference,
d = 5 – (-5/2) = 5/2
n = 11
We know that the nth term of an AP is
$a_n = a + (n - 1)d$
Where,
a = first term
a_n is nth term
d is the common difference
$a_{11} = -5 + (11 - 1)(5/2)$
$a_{11} = -5 + 25 = 20$

5. The first four terms of an AP, whose first term is –2 and the common difference is –2, are
(A) – 2, 0, 2, 4
(B) – 2, 4, – 8, 16
(C) – 2, – 4, – 6, – 8
(D) – 2, – 4, – 8, –16

Solution:

(C) – 2, – 4, – 6, – 8

Explanation:
First term, a = - 2
Second Term, d = - 2
$a_1 = a = -2$
We know that the nth term of an AP is
$a_n = a + (n - 1)d$
Where,
a = first term
a_n is nth term
d is the common difference
Hence, we have,
$a_2 = a + d = -2 + (-2) = -4$
Similarly,

$a_3 = -6$
$a_4 = -8$
So the A.P is
- 2, - 4, - 6, - 8

6. The 21st term of the AP whose first two terms are –3 and 4 is
(A) 17 **(B) 137** **(C) 143** **(D) –143**

Solution:

(B) 137

Explanation:
First two terms of an AP are a = - 3 and $a_2 = 4$.
We know, nth term of an AP is
$a_n = a + (n - 1)d$
Where,
a = first term
a_n is nth term
d is the common difference
$a_2 = a + d$
$4 = -3 + d$
$d = 7$
Common difference, d = 7
$a_{21} = a + 20d$
$= -3 + (20)(7)$
$= 137$

7. If the 2nd term of an AP is 13 and the 5th term is 25, what is its 7^{th} term?
(A) 30 **(B) 33** **(C) 37** **(D) 38**

Solution:

(B) 33

Explanation:
We know that the nth term of an AP is
$a_n = a + (n - 1)d$
Where,
a = first term
a_n is nth term
d is the common difference
$a_2 = a + d = 13$ …..(1)
$a_5 = a + 4d = 25$ …… (2)
From equation (1) we have,
$a = 13 – d$
Using this in equation (2), we have
$13 - d + 4d = 25$
$13 + 3d = 25$
$3d = 12$
$d = 4$
$a = 13 - 4 = 9$

$a_7 = a + 6d$
$= 9 + 6(4)$
$= 9 + 24 = 33$

8. Which term of the AP: 21, 42, 63, 84... is 210?

(A) 9^th^ **(B) 10^th^** **(C) 11^th^** **(D) 12^th^**

Solution:

(B) 10^th^

Explanation:

Let nth term of the given AP be 210.
According to question,
first term, a = 21
common difference, d = 42 – 21 = 21 and $a_n = 210$
We know that the nth term of an AP is
$a_n = a + (n - 1)d$
Where,
a = first term
a_n is nth term
d is the common difference
210 = 21 + (n - 1)21
189 = (n - 1)21
n - 1 = 9
n = 10
So, 10th term of an AP is 210.

9. If the common difference of an AP is 5, then what is $a_{18} – a_{13}$?

(A) 5 **(B) 20** **(C) 25** **(D) 30**

Solution:

(C) 25

Explanation:

Given, the common difference of AP i.e., d = 5
Now,
As we know, nth term of an AP is
$a_n = a + (n - 1)d$
where a = first term
a_n is nth term
d is the common difference
$a_{18} - a_{13} = a + 17d – (a + 12d)$
$= 5d$
$= 5(5)$
$= 25$

EXERCISE 5.2

1. Which of the following form an AP? Justify your answer.

(i) –1, –1, –1, –1,...

Solution:

We have $a_1 = -1$, $a_2 = -1$, $a_3 = -1$ and $a_4 = -1$

$a_2 - a_1 = 0$

$a_3 - a_2 = 0$

$a_4 - a_3 = 0$

Clearly, the difference of successive terms is same, therefore given list of numbers from an AP.

(ii) 0, 2, 0, 2,...

Solution:

We have $a_1 = 0$, $a_2 = 2$, $a_3 = 0$ and $a_4 = 2$

$a_2 - a_1 = 2$

$a_3 - a_2 = -2$

$a_4 - a_3 = 2$

Clearly, the difference of successive terms is not same, therefore given list of numbers does not form an AP.

(iii) 1, 1, 2, 2, 3, 3...

Solution:

We have $a_1 = 1$, $a_2 = 1$, $a_3 = 2$ and $a_4 = 2$

$a_2 - a_1 = 0$

$a_3 - a_2 = 1$

Clearly, the difference of successive terms is not same, therefore given list of numbers does not form an AP.

(iv) 11, 22, 33...

Solution:

We have $a_1 = 11$, $a_2 = 22$ and $a_3 = 33$

$a_2 - a_1 = 11$

$a_3 - a_2 = 11$

Clearly, the difference of successive terms is same, therefore given list of numbers form an AP.

(v) 1/2,1/3,1/4, ...

Solution:

We have $a_1 = ½$, $a_2 = 1/3$ and $a_3 = ¼$

$a_2 - a_1 = -1/6$

$a_3 - a_2 = -1/12$

Clearly, the difference of successive terms is not same, therefore given list of numbers does not form an AP.

(vi) $2, 2^2, 2^3, 2^4, \ldots$
Solution:

We have $a_1 = 2$, $a_2 = 2^2$, $a_3 = 2^3$ and $a_4 = 2^4$
$a_2 - a_1 = 2^2 - 2 = 4 - 2 = 2$
$a_3 - a_2 = 2^3 - 2^2 = 8 - 4 = 4$
Clearly, the difference of successive terms is not same, therefore given list of numbers does not form an AP.

(vii) $\sqrt{3}, \sqrt{12}, \sqrt{27}, \sqrt{48}, \ldots$
Solution:

We have,
$a_1 = \sqrt{3}$, $a_2 = \sqrt{12}$, $a_3 = \sqrt{27}$ and $a_4 = \sqrt{48}$
$a_2 - a_1 = \sqrt{12} - \sqrt{3} = 2\sqrt{3} - \sqrt{3} = \sqrt{3}$
$a_3 - a_2 = \sqrt{27} - \sqrt{12} = 3\sqrt{3} - 2\sqrt{3} = \sqrt{3}$
$a_4 - a_3 = \sqrt{48} - \sqrt{27} = 4\sqrt{3} - 3\sqrt{3} = \sqrt{3}$
Clearly, the difference of successive terms is same, therefore given list of numbers from an AP.

2. Justify whether it is true to say that –1, -3/2, –2, 5/2,... forms an AP as $a_2 - a_1 = a_3 - a_2$.
Solution:

False
$a_1 = -1$, $a_2 = -3/2$, $a_3 = -2$ and $a_4 = 5/2$
$a_2 - a_1 = -3/2 - (-1) = -\frac{1}{2}$
$a_3 - a_2 = -2 - (-3/2) = -\frac{1}{2}$
$a_4 - a_3 = 5/2 - (-2) = 9/2$
Clearly, the difference of successive terms in not same, all though, $a_2 - a_1 = a_3 - a_2$ but $a_4 - a_3 \neq a_3 - a_2$ therefore it does not form an AP.

3. For the AP: –3, –7, –11, ..., can we find directly $a_{30} - a_{20}$ without actually finding a_{30} and a_{20}? Give reasons for your answer.
Solution:

True
Given
First term, a = - 3
Common difference, $d = a_2 - a_1 = -7 - (-3) = -4$
$a_{30} - a_{20} = a + 29d - (a + 19d)$
$= 10d$
$= -40$
It is so because difference between any two terms of an AP is proportional to common difference of that AP

4. Two APs have the same common difference. The first term of one AP is 2 and that of the other is 7. The difference between their 10th terms is the same as the difference between their 21st

terms, which is the same as the difference between any two corresponding terms. Why?
Solution:

Suppose there are two AP's with first terms a and A
And their common differences are d and D respectively
Suppose n be any term
$a_n = a + (n - 1)d$
$A_n = A + (n - 1)D$
As common difference is equal for both AP's
We have D = d
Using this we have
$A_n - a_n = a + (n - - 1)d - [A + (n - 1)D]$
$= a + (n - 1)d - A - (n - 1)d$
$= a – A$
As a - A is a constant value
Therefore, difference between any corresponding terms will be equal to a - A.

EXERCISE 5.3 PAGE NO: 51

1. Match the APs given in column A with suitable common differences given in column B.

Column A		Column B	
(A_1)	**2, – 2, – 6, –10,...**	**(B_1)**	**2/3**
(A_2)	**a = –18, n = 10, a_n = 0**	**(B_2)**	**– 5**
(A_3)	**a = 0, a_{10} = 6**	**(B_3)**	**4**
(A_4)	**a_2 = 13, a_4 =3**	**(B_4)**	**– 4**
		(B_5)	**2**
		(B_6)	**1/2**
		(B_7)	**5**

Solution:

(A_1) AP is 2, - 2, - 6, - 10, ….
So common difference is simply
$a_2 - a_1 = - 2 - 2 = - 4 = (B_3)$

(A_2) Given
First term, a = - 18
No of terms, n = 10
Last term, $a_n = 0$
By using the nth term formula
$a_n = a + (n - 1)d$
$0 = - 18 + (10 - 1)d$
$18 = 9d$
$d = 2 = (B_5)$

(A_3) Given
First term, a = 0
Tenth term, $a_{10} = 6$
By using the nth term formula
$a_n = a + (n - 1)d$
$a_{10} = a + 9d$
$6 = 0 + 9d$
$d = 2/3 = (B_6)$

(A_4) Let the first term be a and common difference be d
Given that
$a_2 = 13$
$a_4 = 3$
$a_2 - a_4 = 10$
$a + d - (a + 3d) = 10$
$d - 3d = 10$
$- 2d = 10$
$d = - 5= (B_1)$

2. Verify that each of the following is an AP, and then write its next three terms.

(i) 0, 1/4, 1/2, 3/4,...

Solution:

Here,

$a_1 = 0$

$a_2 = ¼$

$a_3 = ½$

$a_4 = ¾$

$a_2 – a_1 = ¼ - 0 = ¼$

$a_3 – a_2 = ½ - ¼ = ¼$

$a_4 – a_3 = ¾ - ½ = ¼$

Since, difference of successive terms are equal,

Hence, 0, 1/4, 1/2, 3/4... is an AP with common difference ¼.

Therefore, the next three term will be,

¾ + ¼ , ¾ + 2(¼), ¾ + 3(¼)

1, 5/4 , 3/2

(ii) 5, 14/3, 13/3, 4...

Solution:

Here,

$a_1 = 5$

$a_2 = 14/3$

$a_3 = 13/3$

$a_4 = 4$

$a_2 – a_1 = 14/3 – 5 = -1/3$

$a_3 – a_2 = 13/3 – 14/3 = -1/3$

$a_4 – a_3 = 4 – 13/3 = -1/3$

Since, difference of successive terms are equal,

Hence, 5, 14/3, 13/3, 4... is an AP with common difference -1/3.

Therefore, the next three term will be,

4 + (-1/3), 4 + 2(-1/3), 4 + 3(-1/3)

11/3 , 10/3, 3

(iii) √3 , 2√3, 3√3,...

Solution:

Here,

$a_1 = \sqrt{3}$

$a_2 = 2\sqrt{3}$

$a_3 = 3\sqrt{3}$

$a_4 = 4\sqrt{3}$

$a_2 – a_1 = 2\sqrt{3} – \sqrt{3} = \sqrt{3}$

$a_3 – a_2 = 3\sqrt{3} – 2\sqrt{3}= \sqrt{3}$

$a_4 – a_3 = 4\sqrt{3} – 3\sqrt{3}= \sqrt{3}$

Since, difference of successive terms are equal,

Hence, √3 , 2√3, 3√3,... is an AP with common difference √3.

Therefore, the next three term will be,

$4\sqrt{3} + \sqrt{3}, 4\sqrt{3} + 2\sqrt{3}, 4\sqrt{3} + 3\sqrt{3}$
$5\sqrt{3}, 6\sqrt{3}, 7\sqrt{3}$

(iv) $a + b, (a + 1) + b, (a + 1) + (b + 1), ...$
Solution:

Here
$a_1 = a + b$
$a_2 = (a + 1) + b$
$a_3 = (a + 1) + (b + 1)$
$a_2 - a_1 = (a + 1) + b - (a + b) = 1$
$a_3 - a_2 = (a + 1) + (b + 1) - (a + 1) - b = 1$
Since, difference of successive terms are equal,
Hence, $a + b, (a + 1) + b, (a + 1) + (b + 1), ...$ is an AP with common difference 1.
Therefore, the next three term will be,
$(a + 1) + (b + 1) + 1, (a + 1) + (b + 1) + 1(2), (a + 1) + (b + 1) + 1(3)$
$(a + 2) + (b + 1), (a + 2) + (b + 2), (a + 3) + (b + 2)$

(v) $a, 2a + 1, 3a + 2, 4a + 3,...$
Solution:

Here $a_1 = a$
$a_2 = 2a + 1$
$a_3 = 3a + 2$
$a_4 = 4a + 3$
$a_2 - a_1 = (2a + 1) – (a) = a + 1$
$a_3 - a_2 = (3a + 2) – (2a + 1) = a + 1$
$a_4 - a_3 = (4a + 3) – (3a+2) = a + 1$
Since, difference of successive terms are equal,
Hence, $a, 2a + 1, 3a + 2, 4a + 3,...$ is an AP with common difference a+1.
Therefore, the next three term will be,
$4a + 3 +(a + 1), 4a + 3 + 2(a + 1), 4a + 3 + 3(a + 1)$
$5a + 4, 6a + 5, 7a + 6$

3. Write the first three terms of the APs when a and d are as given below:

(i) $a = 1/2, d = -1/6$
(ii) $a = –5, d = –3$
(iii) $a = 2, d = 1/\sqrt{2}$

Solution:

(i) $a = 1/2, d = -1/6$
We know that,
First three terms of AP are :
$a, a + d, a + 2d$
$½, ½ + (-1/6), ½ + 2 (-1/6)$
$½, 1/3, 1/6$

(ii) $a = –5, d = –3$
We know that,

First three terms of AP are :
a, a + d, a + 2d
-5, - 5 + 1 (- 3), - 5 + 2 (- 3)
- 5, - 8, - 11

(iii) $a = \sqrt{2}$, $d = 1/\sqrt{2}$
We know that,
First three terms of AP are :
a, a + d, a + 2d
$\sqrt{2}$, $\sqrt{2}+1/\sqrt{2}$, $\sqrt{2}+2/\sqrt{2}$
$\sqrt{2}$, $3/\sqrt{2}$, $4/\sqrt{2}$

4. Find a, b and c such that the following numbers are in AP: a, 7, b, 23, c.
Solution:
For a, 7, b, 23, c… to be in AP
it has to satisfy the condition,
$a_5 - a_4 = a_4 - a_3 = a_3 - a_2 = a_2 - a_1 = d$
Where d is thecommon difference
7 - a = b - 7 = 23 - b = c – 23 …(1)
Let us equate,
b - 7 = 23 – b
2b = 30
b = 15 (eqn 1)
And,
7 - a = b – 7
From eqn 1
7 - a = 15 – 7
a = - 1
And,
c - 23 = 23 – b
c - 23 = 23 – 15
c - 23 = 8
c = 31
So a = - 1
b = 15
c = 31
Then, we can say that, the sequence - 1, 7, 15, 23, 31 is an AP

5. Determine the AP whose fifth term is 19 and the difference of the eighth term from the thirteenth term is 20.
Solution:
We know that,
The first term of an AP = a
And, the common difference = d.
According to the question,
5^{th} term, $a_5 = 19$

Using the n^{th} term formula,
$a_n = a + (n - 1)d$
We get,
$a + 4d = 19$
$a = 19 - 4d$ …(1)
Also,
13^{th} term - 8^{th} term = 20
$a + 12d - (a + 7d) = 20$
$5d = 20$
$d = 4$
Substituting d = 4 in equation 1,
We get,
$a = 19 – 4(4)$
$a = 3$
Then, the AP becomes,
3, 3 + 4/, 3 + 2(4),...
3, 7, 11,...

EXERCISE 5.4 PAGE NO: 56

1. The sum of the first five terms of an AP and the sum of the first seven terms of the same AP is 167. If the sum of the first ten terms of this AP is 235, find the sum of its first twenty terms.

Solution:

We know that, in an A.P.,
First term = a
Common difference = d
Number of terms of an AP = n
According to the question,
We have,
$S_5 + S_7 = 167$
Using the formula for sum of n terms,
$S_n = (n/2)\ [2a + (n-1)d]$
So, we get,
$(5/2)\ [2a + (5-1)d] + (7/2)[2a + (7-1)d] = 167$
$5(2a + 4d) + 7(2a + 6d) = 334$
$10a + 20d + 14a + 42d = 334$
$24a + 62d = 334$
$12a + 31d = 167$
$12a = 167 - 31d$ …(1)
We have,
$S_{10} = 235$
$(10/2)\ [2a + (10-1)d] = 235$
$5[\ 2a + 9d] = 235$
$2a + 9d = 47$
Multiplying L.H.S and R.H.S by 6,
We get,
$12a + 54d = 282$
From equation (1)
$167 - 31d + 54d = 282$
$23d = 282 – 167$
$23d = 115$
$d = 5$
Substituting the value of d = 5 in equation (1)
$12a = 167 - 31(5)$
$12a = 167 – 155$
$12a = 12$
$a = 1$
We know that,
$S_{20} = (n/2)\ [2a + (20 – 1)d]$
$= 20/(2[2(1) + 19\ (5)])$
$= 10[\ 2 + 95]$
$= 970$
Therefore, the sum of first 20 terms is 970.

2. Find the

(i) Sum of those integers between 1 and 500 which are multiples of 2 as well as of 5.
(ii) Sum of those integers from 1 to 500 which are multiples of 2 as well as of 5 .
(iii) Sum of those integers from 1 to 500 which are multiples of 2 or 5.
[Hint (iii): These numbers will be: multiples of 2 + multiples of 5 – multiples of 2 as well as of 5]

Solution:

(i) Sum of those integers between 1 and 500 which are multiples of 2 as well as of 5.

We know that,
Multiples of 2 as well as of 5 = LCM of (2, 5) = 10
Multiples of 2 as well as of 5 between 1 and 500 = 10, 20, 30…, 490.
Hence,
We can conclude that 10, 20, 30…, 490 is an AP with common difference, d = 10
First term, a = 10
Let the number of terms in this AP = n
Using n^{th} term formula,
$a_n = a + (n - 1)d$
$490 = 10 + (n - 1)10$
$480 = (n - 1)10$
$n - 1 = 48$
$n = 49$
Sum of an AP,
$S_n = (n/2)\ [a + a_n]$, here a_n is the last term, which is given]
$= (49/2) \times [10 + 490]$
$= (49/2) \times [500]$
$= 49 \times 250$
$= 12250$
Therefore, sum of those integers between 1 and 500 which are multiples of 2 as well as of 5 = 12250

(ii) Sum of those integers from 1 to 500 which are multiples of 2 as well as of 5.

We know that,
Multiples of 2 as well as of 5 = LCM of (2, 5) = 10
Multiples of 2 as well as of 5 from 1 and 500 = 10, 20, 30…, 500.
Hence,
We can conclude that 10, 20, 30…, 500 is an AP with common difference, d = 10
First term, a = 10
Let the number of terms in this AP = n
Using n^{th} term formula,
$a_n = a + (n - 1)d$
$500 = 10 + (n - 1)10$
$490 = (n - 1)10$
$n - 1 = 49$
$n = 50$
Sum of an AP,
$S_n = (n/2)\ [\ a + a_n]$, here a_n is the last term, which is given]

= (50/2) ×[10+500]
= 25× [10 + 500]
= 25(510)
= 12750

Therefore, sum of those integers from 1 to 500 which are multiples of 2 as well as of 5= 12750

(iii) Sum of those integers from 1 to 500 which are multiples of 2 or 5.

We know that,
Multiples of 2 or 5 = Multiple of 2 + Multiple of 5 – Multiple of LCM (2, 5)
Multiples of 2 or 5 = Multiple of 2 + Multiple of 5 – Multiple of LCM (10)
Multiples of 2 or 5 from 1 to 500 = List of multiple of 2 from 1 to 500 + List of multiple of 5 from 1 to 500 - List of multiple of 10 from 1 to 500
= (2, 4, 6… 500) + (5, 10, 15… 500) - (10, 20, 30… 500)
Required sum = sum(2, 4, 6,…, 500) + sum(5, 10, 15,…, 500) - sum(10, 20, 30,., 500)
Consider the first series,
2, 4, 6, …., 500
First term, a = 2
Common difference, d = 2
Let n be no of terms
$a_n = a + (n - 1)d$
$500 = 2 + (n - 1)^2$
$498 = (n - 1)^2$
n - 1 = 249
n = 250
Sum of an AP, $S_n = (n/2) [a + a_n]$
Let the sum of this AP be S_1,
$S_1 = S_{250} = (250/2) \times[2+500]$
$S_1 = 125(502)$
$S_1 = 62750$ … (1)

Consider the second series,
5, 10, 15, …., 500
First term, a = 5
Common difference, d = 5
Let n be no of terms
By nth term formula
$a_n = a + (n - 1)d$
500 = 5 + (n - 1)
495 = (n - 1)5
n - 1 = 99
n = 100
Sum of an AP, $S_n = (n/2) [a + a_n]$
Let the sum of this AP be S_2,
$S_2 = S_{100} = (100/2) \times[5+500]$
$S_2 = 50(505)$

$S_2 = 25250$ … (2)

Consider the third series,

10, 20, 30, …., 500

First term, a = 10

Common difference, d = 10

Let n be no of terms

$a_n = a + (n - 1)d$

$500 = 10 + (n - 1)10$

$490 = (n - 1)10$

$n - 1 = 49$

$n = 50$

Sum of an AP, $S_n = (n/2) [a + a_n]$

Let the sum of this AP be S_3,

$S_3 = S_{50} = (50/2) \times [2+510]$

$S_3 = 25(510)$

$S_3 = 12750$ … (3)

Therefore, the required Sum, $S = S_1 + S_2 - S_3$

$S = 62750 + 25250 – 12750$

$= 75250$

3. The eighth term of an AP is half its second term and the eleventh term exceeds one third of its fourth term by 1. Find the 15th term.

Solution:

We know that,

First term of an AP = a

Common difference of AP = d

n^{th} term of an AP, $a_n = a + (n - 1)d$

According to the question,

$a_s = ½ a_2$

$2a_8 = a_2$

$2(a + 7d) = a + d$

$2a + 14d = a + d$

$a = - 13d$ …(1)

Also,

$a_{11} = 1/3\ a_4 + 1$

$3(a + 10d) = a + 3d + 3$

$3a + 30d = a + 3d + 3$

$2a + 27d = 3$

Substituting a = -13d in the equation,

$2 (- 13d) + 27d = 3$

$d = 3$

Then,

$a = - 13(3)= - 39$

Now,

$a_{15} = a + 14d$
$= -39 + 14(3)$
$= -39 + 42$
$= 3$
So 15^{th} term is 3.

4. An AP consists of 37 terms. The sum of the three middle most terms is 225 and the sum of the last three is 429. Find the AP.

Solution:

We know that,
First term of an AP = a
Common difference of AP = d
n^{th} term of an AP, $a_n = a + (n - 1)d$
Since, n = 37 (odd),
Middle term will be $(n+1)/2 = 19^{th}$ term
Thus, the three middle most terms will be,
18^{th}, 19^{th} and 20^{th} terms
According to the question,
$a_{18} + a_{19} + a_{20} = 225$
Using $a_n = a + (n - 1)d$
$a + 17d + a + 18d + a + 19d = 225$
$3a + 54d = 225$
$3a = 225 - 54d$
$a = 75 - 18d$ … (1)
Now, we know that last three terms will be 35^{th}, 36^{th} and 37^{th} terms.
According to the question,
$a_{35} + a_{36} + a_{37} = 429$
$a + 34d + a + 35d + a + 36d = 429$
$3a + 105d = 429$
$a + 35d = 143$
Substituting $a = 75 - 18d$ from equation 1,
$75 - 18d + 35d = 143$ [using eqn1]
$17d = 68$
$d = 4$
Then,
$a = 75 - 18(4)$
$a = 3$
Therefore, the AP is a, a + d, a + 2d….
i.e. 3, 7, 11….

5. Find the sum of the integers between 100 and 200 that are

(i) divisible by 9

(ii) not divisible by 9

[Hint (ii): These numbers will be: Total numbers – Total numbers divisible by 9]

Solution:

(i) The number between 100 and 200 which is divisible by 9 = 108, 117, 126, …198

Let the number of terms between 100 and 200 which is divisible by 9 = n

$a_n = a + (n - 1)d$

$198 = 108 + (n - 1)9$

$90 = (n - 1)9$

$n - 1 = 10$

$n = 11$

Sum of an AP = $S_n = (n/2) [a + a_n]$

$S_n = (11/2) \times [108 + 198]$

$= (11/2) \times 306$

$= 11(153)$

$= 1683$

(ii) Sum of the integers between 100 and 200 which is not divisible by 9 = (sum of total numbers between 100 and 200) – (sum of total numbers between 100 and 200 which is divisible by 9)

Sum, $S = S_1 - S_2$

Here,

S_1 = sum of AP 101, 102, 103, - - - , 199

S_2 = sum of AP 108, 117, 126, - - - , 198

For AP 101, 102, 103, - - - , 199

First term, a = 101

Common difference, d = 199

Number of terms = n

Then,

$a_n = a + (n - 1)d$

$199 = 101 + (n - 1)1$

$98 = (n - 1)$

$n = 99$

Sum of an AP = $S_n = (n/2) [a + a_n]$

Sum of this AP,

$S_1 = (99/2) \times [199 + 101]$

$= (99/2) \times 300$

$= 99(150)$

$= 14850$

For AP 108, 117, 126, - - - - , 198

First term, a = 108

Common difference, d = 9

Last term, $a_n = 198$

Number of terms = n

Then,

$a_n = a + (n - 1)d$

$198 = 108 + (n - 1)9$

$10 = (n - 1)$

$n = 11$

Sum of an AP = $S_n = (n/2)\,[a + a_n]$
Sum of this AP,

$S_2 = (11/2) \times [108 + 198]$
$= (11/2) \times (306)$
$= 11(153)$
$= 1683$

Substituting the value of S_1 and S_2 in the equation, $S = S_1 - S_2$

$S = S_1 + S_2$
$= 14850 – 1683$
$= 13167$

EXERCISE 4.1 PAGE NO: 36

Choose the correct answer from the given four options in the following questions:

1. Which of the following is a quadratic equation?

(A) $x^2 + 2x + 1 = (4 - x)^2 + 3$ **(B)** $-2x^2 = (5 - x)(2x-(2/5))$

(C) $(k + 1) x^2 + (3/2) x = 7$, **where** $k = -1$ **(D)** $x^3 - x^2 = (x - 1)^3$

Solution:

(D) $x^3 - x^2 = (x - 1)^3$

Explanation:

The standard form of a quadratic equation is given by,

$ax^2 + bx + c = 0, a \neq 0$

(A) Given, $x^2 + 2x + 1 = (4 - x)^2 + 3$

$x^2 + 2x + 1 = 16 - 8x + x^2 + 3$

$10x - 18 = 0$

which is not a quadratic equation.

(B) Given, $-2x^2 = (5 - x) (2x - 2/5)$

$-2x^2 = 10x - 2x^2 - 2 + 2/5x$

$52x - 10 = 0$

which is not a quadratic equation.

(C) Given, $(k + 1) x^2 + 3/2 x = 7$, where $k = -1$

$(-1 + 1) x^2 + 3/2 x = 7$

$3x - 14 = 0$

which is not a quadratic equation.

(D) Given, $x^3 - x^2 = (x - 1)^3$

$x^3 - x^2 = x^3 - 3x^2 + 3x - 1$

$2x^2 - 3x + 1 = 0$

which represents a quadratic equation.

2. Which of the following is not a quadratic equation?

(A) $2(x - 1)^2 = 4x^2 - 2x + 1$ **(B)** $2x - x^2 = x^2 + 5$

(C) $(\sqrt{2}x + \sqrt{3})^2 + x^2 = 3x^2 - 5x$ **(D)** $(x^2 + 2x)^2 = x^4 + 3 + 4x^3$

Solution:

(D) $(x^2 + 2x)^2 = x^4 + 3 + 4x^3$

A quadratic equation is represented by the form,

$ax^2 + bx + c = 0, a \neq 0$

(A) Given, $2(x - 1)^2 = 4x^2 - 2x + 1$

$2(x^2 - 2x + 1) = 4x^2 - 2x + 1$

$2x^2 + 2x - 1 = 0$

which is a quadratic equation.

(B) Given, $2x - x^2 = x^2 + 5$
$2x^2 - 2x + 5 = 0$
which is a quadratic equation.

(C) Given, $(\sqrt{2}x + \sqrt{3})^2 = 3x^2 - 5x$
$2x^2 + 2\sqrt{6}x + 3 = 3x^2 - 5x$
$x^2 - (5 + 2\sqrt{6})x - 3 = 0$
which is a quadratic equation.

(D) Given, $(x^2 + 2x)^2 = x^4 + 3 + 4x^2$
$x^4 + 4x^3 + 4x^2 = x^4 + 3 + 4x^2$
$4x^3 - 3 = 0$
which is a cubic equation and not a quadratic equation.

3. Which of the following equations has 2 as a root?

(A) $x^2 - 4x + 5 = 0$ **(B) $x^2 + 3x - 12 = 0$**
(C) $2x^2 - 7x + 6 = 0$ **(D) $3x^2 - 6x - 2 = 0$**

Solution:

(C) $2x^2 - 7x + 6 = 0$
If 2 is a root then substituting the value 2 in place of x should satisfy the equation.
(A) Given,
$x^2 - 4x + 5 = 0$
$(2)^2 - 4(2) + 5 = 1 \neq 0$
So, x = 2 is not a root of $x^2 - 4x + 5 = 0$
(B) Given, $x^2 + 3x - 12 = 0$
$(2)^2 + 3(2) - 12 = -2 \neq 0$
So, x = 2 is not a root of $x^2 + 3x - 12 = 0$
(C) Given, $2x^2 - 7x + 6 = 0$
$2(2)^2 - 7(2) + 6 = 0$
Here, x = 2 is a root of $2x^2 - 7x + 6 = 0$
(D) Given, $3x^2 - 6x - 2 = 0$
$3(2)^2 - 6(2) - 2 = -2 \neq 0$
So, x = 2 is not a root of $3x^2 - 6x - 2 = 0$

4. If ½ is a root of the equation $x^2 + kx - 5/4 = 0$, then the value of k is

(A) 2 **(B) – 2**
(C) ¼ **(D) ½**

Solution:

(A) 2
If ½ is a root of the equation
$x^2 + kx - 5/4 = 0$ then, substituting the value of ½ in place of x should give us the value of k.
Given, $x^2 + kx - 5/4 = 0$ where, x = ½
$(½)^2 + k(½) - (5/4) = 0$
$(k/2) = (5/4) - ¼$
k = 2

5. Which of the following equations has the sum of its roots as 3?

(A) $2x^2 – 3x + 6 = 0$ **(B) $–x^2 + 3x – 3 = 0$**

(C) $\sqrt{2}x^2 – 3/\sqrt{2}x+1=0$ **(D) $3x^2 – 3x + 3 = 0$**

Solution:

(B) $–x^2 + 3x – 3 = 0$

The sum of the roots of a quadratic equation $ax^2 + bx + c = 0$, $a \neq 0$ is given by,

Coefficient of x / coefficient of $x^2 = –(b/a)$

(A) Given, $2x^2 - 3x + 6 = 0$

Sum of the roots $= - b/a = -(-3/2) = 3/2$

(B) Given, $-x^2 + 3x – 3 = 0$

Sum of the roots $= - b/a = -(3/-1) = 3$

(C) Given, $\sqrt{2}x^2 – 3/\sqrt{2}x+1=0$

$2x^2 – 3x + \sqrt{2} = 0$

Sum of the roots $= - b/a = -(-3/2) = 3/2$

(D) Given, $3x^2 - 3x + 3 = 0$

Sum of the roots $= - b/a = -(-3/3) = 1$

EXERCISE 4.2 PAGE NO: 38

1. State whether the following quadratic equations have two distinct real roots. Justify your answer.

(i) $x^2 - 3x + 4 = 0$
(ii) $2x^2 + x - 1 = 0$
(iii) $2x^2 - 6x + 9/2 = 0$
(iv) $3x^2 - 4x + 1 = 0$
(v) $(x + 4)^2 - 8x = 0$
(vi) $(x - \sqrt{2})^2 - 2(x + 1) = 0$
(vii) $\sqrt{2}\, x^2 - (3/\sqrt{2})x + 1/\sqrt{2} = 0$
(viii) $x (1 - x) - 2 = 0$
(ix) $(x - 1) (x + 2) + 2 = 0$
(x) $(x + 1) (x - 2) + x = 0$

Solution:

(i)

The equation $x^2 - 3x + 4 = 0$ has no real roots.
$D = b^2 - 4ac$
$= (-3)^2 - 4(1)(4)$
$= 9 - 16 < 0$
Hence, the roots are imaginary.

(ii)

The equation $2x^2 + x - 1 = 0$ has two real and distinct roots.
$D = b^2 - 4ac$
$= 1^2 - 4(2)(-1)$
$= 1 + 8 > 0$
Hence, the roots are real and distinct.

(iii)

The equation $2x^2 - 6x + (9/2) = 0$ has real and equal roots.
$D = b^2 - 4ac$
$= (-6)^2 - 4(2)(9/2)$
$= 36 - 36 = 0$
Hence, the roots are real and equal.

(iv)

The equation $3x^2 - 4x + 1 = 0$ has two real and distinct roots.
$D = b^2 - 4ac$
$= (-4)^2 - 4(3)(1)$
$= 16 - 12 > 0$
Hence, the roots are real and distinct.

(v)

The equation $(x + 4)^2 - 8x = 0$ has no real roots.

Simplifying the above equation,
$x^2 + 8x + 16 - 8x = 0$
$x^2 + 16 = 0$
$D = b^2 - 4ac$
$= (0) - 4(1)(16) < 0$
Hence, the roots are imaginary.

(vi)

The equation $(x - \sqrt{2})^2 – \sqrt{2}(x+1)=0$ has two distinct and real roots.
Simplifying the above equation,
$x^2 – 2\sqrt{2}x + 2 – \sqrt{2}x – \sqrt{2} = 0$
$x^2 – \sqrt{2}(2+1)x + (2 – \sqrt{2}) = 0$
$x^2 – 3\sqrt{2}x + (2 – \sqrt{2}) = 0$
$D = b^2 - 4ac$
$= (– 3\sqrt{2})^2 – 4(1)(2 – \sqrt{2})$
$= 18 – 8 + 4\sqrt{2} > 0$
Hence, the roots are real and distinct.

(vii)

The equation $\sqrt{2}x^2 – 3x/\sqrt{2} + ½ = 0$ has two real and distinct roots.
$D = b^2 - 4ac$
$= (- 3/\sqrt{2})^2 – 4(\sqrt{2})(½)$
$= (9/2) - 2\sqrt{2} > 0$
Hence, the roots are real and distinct.

(viii)

The equation $x (1 - x) – 2 = 0$ has no real roots.
Simplifying the above equation,
$x^2 – x + 2 = 0$
$D = b^2 - 4ac$
$= (-1)^2 - 4(1)(2)$
$= 1 - 8 < 0$
Hence, the roots are imaginary.

(ix)

The equation $(x - 1)(x + 2) + 2 = 0$ has two real and distinct roots.
Simplifying the above equation,
$x^2 – x + 2x – 2 + 2 = 0$
$x^2 + x = 0$
$D = b^2 - 4ac$
$= 1^2 - 4(1)(0)$
$= 1 - 0 > 0$
Hence, the roots are real and distinct.

(x)

The equation $(x + 1)(x - 2) + x = 0$ has two real and distinct roots.

Simplifying the above equation,
$x^2 + x – 2x – 2 + x = 0$
$x^2 – 2 = 0$
$D = b^2 - 4ac$
$= (0)^2 - 4(1)(-2)$
$= 0 + 8 > 0$
Hence, the roots are real and distinct.

2. Write whether the following statements are true or false. Justify your answers.

(i) Every quadratic equation has exactly one root.
(ii) Every quadratic equation has at least one real root.
(iii) Every quadratic equation has at least two roots.
(iv) Every quadratic equations has at most two roots.
(v) If the coefficient of x2 and the constant term of a quadratic equation have opposite signs, then the quadratic equation has real roots.
(vi) If the coefficient of x2 and the constant term have the same sign and if the coefficient of x term is zero, then the quadratic equation has no real roots.

Solution:

(i) False. For example, a quadratic equation $x^2 – 9 = 0$ has two distinct roots – 3 and 3.
(ii) False. For example, equation $x^2 + 4 = 0$ has no real root.
(iii) False. For example, a quadratic equation $x^2 – 4x + 4 = 0$ has only one root which is 2.
(iv) True, because every quadratic polynomial has almost two roots.
(v) True, because in this case discriminant is always positive.
For example, in $ax^2+ bx + c = 0$, as a and c have opposite sign, $ac < 0$
$\Rightarrow$ Discriminant $= b^2 – 4ac > 0$.
(vi) True, because in this case discriminant is always negative.
For example, in $ax^2+ bx + c = 0$, as $b = 0$, and a and c have same sign then $ac > 0$
$\Rightarrow$ Discriminant $= b^2 – 4ac = – 4ac < 0$

3. A quadratic equation with integral coefficient has integral roots. Justify your answer.

Solution:

No, a quadratic equation with integral coefficients may or may not have integral roots.
Justification
Consider the following equation,
$8x^2 - 2x – 1 = 0$
The roots of the given equation are ½ and - ¼ which are not integers.
Hence, a quadratic equation with integral coefficient might or might not have integral roots.

EXERCISE 4.3

1. Find the roots of the quadratic equations by using the quadratic formula in each of the following:

(i) $2x^2 - 3x - 5 = 0$
(ii) $5x^2 + 13x + 8 = 0$
(iii) $-3x^2 + 5x + 12 = 0$
(iv) $-x^2 + 7x - 10 = 0$
(v) $x^2 + 2\sqrt{2}x - 6 = 0$
(vi) $x^2 - 3\sqrt{5}x + 10 = 0$
(vii) $(\frac{1}{2})x^2 - \sqrt{11}x + 1 = 0$

Solution:

The quadratic formula for finding the roots of quadratic equation $ax^2 + bx + c = 0$, $a \neq 0$ is given by,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

(i) $2x^2 - 3x - 5 = 0$

$$\therefore x = \frac{-(-3) \pm \sqrt{3^2 - 4(2)(-5)}}{2(2)}$$
$$= \frac{3 \pm \sqrt{49}}{4}$$
$$= \frac{3 \pm 7}{4} = \frac{5}{2}, -1$$

(ii) $5x^2 + 13x + 8 = 0$

$$\therefore x = \frac{-13 \pm \sqrt{(-13)^2 - 4(5)(8)}}{2(5)}$$
$$= \frac{-13 \pm \sqrt{9}}{10}$$
$$= \frac{-13 \pm 3}{10} = -1, -\frac{8}{5}$$

(iii) $-3x^2 + 5x + 12 = 0$

$$\therefore x = \frac{-5 \pm \sqrt{5^2 - 4(-3)(12)}}{2(-3)}$$
$$= \frac{-5 \pm \sqrt{169}}{-6}$$
$$= \frac{5 \pm 13}{6} = 3, -\frac{4}{3}$$

(iv) $-x^2 + 7x - 10 = 0$

$$\therefore\ x = \frac{-7 \pm \sqrt{(-7)^2 - 4(-1)(-10)}}{2(-1)}$$

$$= \frac{-7 \pm \sqrt{9}}{-2}$$

$$= \frac{7 \pm 3}{2} = \mathbf{5, 2}$$

(v) $x^2 + 2\sqrt{2}x - 6 = 0$

$$\therefore\ x = \frac{-2\sqrt{2} \pm \sqrt{\left(2\sqrt{2}\right)^2 - 4(1)(-6)}}{2(1)}$$

$$= \frac{-2\sqrt{2} \pm \sqrt{32}}{2}$$

$$= \frac{-2\sqrt{2} \pm 4\sqrt{2}}{2} = \mathbf{\sqrt{2}, -3\sqrt{2}}$$

(vi) $x^2 - 3\sqrt{5}x + 10 = 0$

$$\therefore\ x = \frac{-(-3\sqrt{5}) \pm \sqrt{\left(-3\sqrt{5}\right)^2 - 4(1)(10)}}{2(1)}$$

$$= \frac{3\sqrt{5} \pm \sqrt{5}}{2} = \mathbf{2\sqrt{5}, \sqrt{5}}$$

(vii) $(\frac{1}{2})x^2 - \sqrt{11}x + 1 = 0$

$$\therefore\ x = \frac{-\left(-\sqrt{11}\right) \pm \sqrt{\left(-\sqrt{11}\right)^2 - 4\left(\frac{1}{2}\right)(1)}}{2\left(\frac{1}{2}\right)}$$

$$= \frac{\sqrt{11} \pm \sqrt{9}}{1}$$

$$= \sqrt{11} \pm 3 = \mathbf{3 + \sqrt{11}, -3 + \sqrt{11}}$$

EXERCISE 4.4 PAGE NO: 42

1. Find a natural number whose square diminished by 84 is equal to thrice of 8 more than the given number.

Solution:

Let the natural number = 'x'.
According to the question,
We get the equation,
$x^2 - 84 = 3(x+8)$
$x^2 - 84 = 3x + 24$
$x^2 - 3x - 84 - 24 = 0$
$x^2 - 3x - 108 = 0$
$x^2 - 12x + 9x - 108 = 0$
$x(x - 12) + 9(x - 12) = 0$
$(x + 9)(x - 12)$
$\Rightarrow x = -9$ and $x = 12$
Since, natural numbers cannot be negative.
The number is 12.

2. A natural number, when increased by 12, equals 160 times its reciprocal. Find the number.

Solution:

Let the natural number = x
When the number increased by 12 = x + 12
Reciprocal of the number = 1/x
According to the question, we have,
x + 12 = 160 times of reciprocal of x
$x + 12 = 160/x$
$x(x + 12) = 160$
$x^2 + 12x - 160 = 0$
$x^2 + 20x - 8x - 160 = 0$
$x(x + 20) - 8(x + 20) = 0$
$(x + 20)(x - 8) = 0$
$x + 20 = 0$ or $x - 8 = 0$
$x = -20$ or $x = 8$
Since, natural numbers cannot be negative.
The required number = x = 8

3. A train, travelling at a uniform speed for 360 km, would have taken 48 minutes less to travel the same distance if its speed were 5 km/h more. Find the original speed of the train.

Solution:

Let original speed of train = x km/h
We know,
Time = distance/speed

According to the question, we have,
Time taken by train = 360/x hour
And, Time taken by train its speed increase 5 km/h = 360/(x + 5)
It is given that,
Time taken by train in first - time taken by train in 2nd case = 48 min = 48/60 hour
$360/x - 360/(x+5) = 48/60 = 4/5$
$360(1/x - 1/(x+5)) = 4/5$
$360 \times 5/4\ (5/(x^2+5x)) = 1$
$450 \times 5 = x^2 + 5x$
$x^2 + 5x - 2250 = 0$
$x = (-5 \pm \sqrt{(25+9000)})/2$
$= (-5 \pm \sqrt{(9025)})/2$
$= (-5 \pm 95)/2$
$= -50, 45$
But $x \neq -50$ because speed cannot be negative
So, x = 45 km/h
Hence, original speed of train = 45 km/h

4. If Zeba were younger by 5 years than what she really is, then the square of her age (in years) would have been 11 more than five times her actual age. What is her age now?
Solution:

Let Zeba's age = x
According to the question,
$(x-5)^2=11+5x$
$x^2+25-10x=11+5x$
$x^2-15x+14=0$
$x^2-14x-x+14=0$
$x(x-14)-1(x-14)=0$
$x=1$ or $x=14$
We have to neglect 1 as 5 years younger than 1 cannot happen.
Therefore, Zeba's present age = 14 years.

EXERCISE 3.1

Choose the correct answer from the given four options:

1. Graphically, the pair of equations

$6x – 3y + 10 = 0$

$2x – y + 9 = 0$

represents two lines which are

(A) intersecting at exactly one point (B) intersecting at exactly two points

(C) coincident (D) parallel.

Solution:

(D) Parallel

Explanation:

The given equations are,

6x-3y+10 = 0

dividing by 3

⇒ 2x-y+ 10/3= 0… (i)

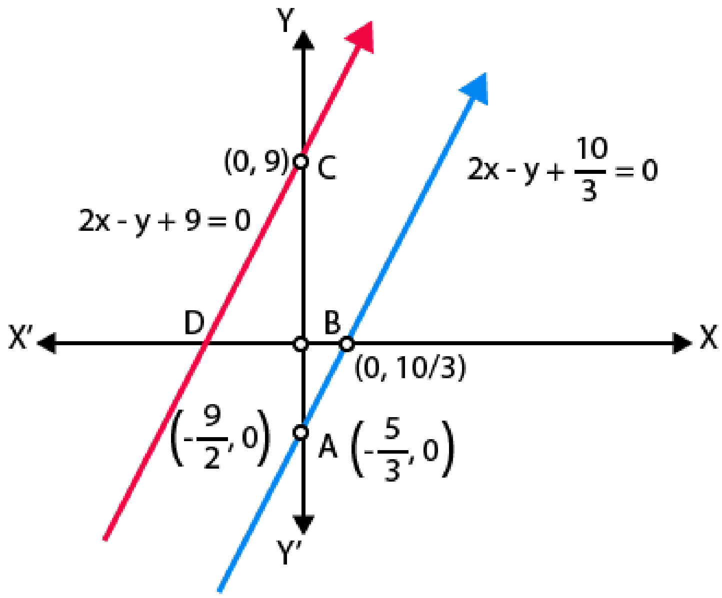
And 2x-y+9=0…(ii)

Table for 2x-y+ 10/3 = 0,

x	0	-5/3
y	10/3	0

Table for 2x-y+9=0

x	0	-9/2
y	9	0



Hence, the pair of equations represents two parallel lines.

2. The pair of equations $x + 2y + 5 = 0$ and $-3x - 6y + 1 = 0$ have

(A) a unique solution (B) exactly two solutions

(C) infinitely many solutions (D) no solution

Solution:

(D) no solution

Explanation:

The equations are:

$x + 2y + 5 = 0$

$-3x - 6y + 1 = 0$

$a_1 = 1$; $b_1 = 2$; $c_1 = 5$

$a_2 = -3$; $b_2 = -6$; $c_2 = 1$

$a_1/a_2 = -1/3$

$b_1/b_2 = -2/6 = -1/3$

$c_1/c_2 = 5/1 = 5$

Here,

$a_1/a_2 = b_1/b_2 \neq c_1/c_2$

Therefore, the pair of equations has no solution.

3. If a pair of linear equations is consistent, then the lines will be

(A) parallel (B) always coincident

(C) intersecting or coincident (D) always intersecting

Solution:

(C) intersecting or coincident

Explanation:

Conditions for a pair of linear equations to be consistent are:

Intersecting lines, having a unique solution,

$a_1/a_2 \neq b_1/b_2$

Coincident or dependent

$a_1/a_2 = b_1/b_2 = c_1/c_2$

4. The pair of equations *y* = 0 and *y* = –7 has

(A) one solution (B) two solutions

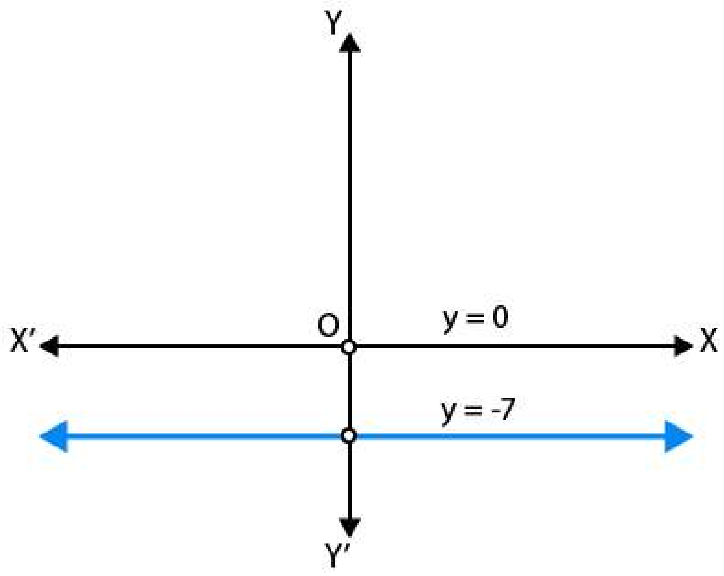
(C) infinitely many solutions (D) no solution

Solution:

(D) no solution

Explanation:

The given pair of equations are y = 0 and y = – 7.



Graphically, both lines are parallel and have no solution

5. The pair of equations *x* = *a* and *y* = *b* graphically represents lines which are

(A) parallel (B) intersecting at (*b, a*)

(C) coincident (D) intersecting at (*a, b*)

Solution:

(D) intersecting at (*a*, *b*)

Explanation:

Graphically in every condition,

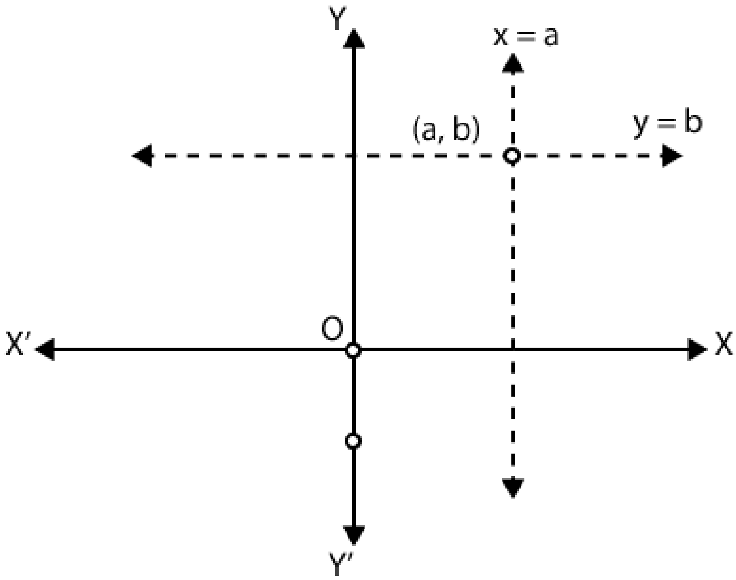
a, b>>0

a, b< 0

a>0, b< 0

a<0, b>0 but a = b≠ 0.

The pair of equations x = a and y = b graphically represents lines which are intersecting at (a, b).



Hence, in this case, two lines intersect at (a, b).

EXERCISE 3.2

1. Do the following pair of linear equations have no solution? Justify your answer.

(i) $2x + 4y = 3$

$12y + 6x = 6$

(ii) $x = 2y$

$y = 2x$

(iii) $3x + y – 3 = 0$

$2x + 2/3y = 2$

Solution:

The Condition for no solution = $a_1/a_2 = b_1/b_2 \neq c_1/c_2$ (parallel lines)

(i) Yes.

Given pair of equations are,

$2x+4y – 3 = 0$ and $6x + 12y – 6 = 0$

Comparing the equations with $ax+ by +c = 0$;

We get,

$a_1 = 2$, $b_1 = 4$, $c_1 = – 3$;

$a_2 = 6$, $b_2 = 12$, $c_2 = – 6$;

$a_1 /a_2 = 2/6 = 1/3$

$b_1 /b_2 = 4/12 = 1/3$

$c_1 /c_2 = – 3/ – 6 = ½$

Here, $a_1/a_2 = b_1/b_2 \neq c_1/c_2$, i.e parallel lines

Hence, the given pair of linear equations has no solution.

(ii) No.

Given pair of equations,

$x = 2y$ or $x – 2y = 0$

$y = 2x$ or $2x – y = 0$;

Comparing the equations with $ax+ by +c = 0$;

We get,

$a_1 = 1$, $b_1 = – 2$, $c_1 = 0$;

$a_2 = 2$, $b_2 = – 1$, $c_2 = 0$;

$a_1 /a_2 = ½$

$b_1 /b_2 = -2/-1 = 2$

Here, $a_1/a_2 \neq b_1/b_2$.

Hence, the given pair of linear equations has a unique solution.

(iii) No.

Given pair of equations,

$3x + y – 3 = 0$

$2x + 2/3\ y = 2$

Comparing the equations with $ax+ by +c = 0$;

We get,

$a_1 = 3$, $b_1 = 1$, $c_1 = – 3$;

$a_2 = 2$, $b_2 = 2/3$, $c_2 = – 2$;

$a_1 /a_2 = 2/6 = 3/2$

$b_1 /b_2 = 4/12 = 3/2$

$c_1 /c_2 = – 3/-2 = 3/2$

Here, $a_1/a_2 = b_1/b_2 = c_1/c_2$, i.e coincident lines

2. Do the following equations represent a pair of coincident lines? Justify your answer.

(i) $3x + 1/7y = 3$

$7x + 3y = 7$

(ii) $–2x – 3y = 1$

$6y + 4x = – 2$

(iii) $x/2 + y + 2/5 = 0$

$4x + 8y + 5/16 = 0$

Solution:

Condition for coincident lines,

$a_1/a_2 = b_1/b_2 = c_1/c_2$;

(i) No.

Given pair of linear equations are:

$3x + 1/7y = 3$

$7x + 3y = 7$

Comparing the above equations with $ax + by + c = 0$;

Here, $a_1 = 3$, $b_1 = 1/7$, $c_1 = – 3$;

And $a_2 = 7$, $b_2 = 3$, $c_2 = – 7$;

$a_1 /a_2 = 3/7$

$b_1 /b_2 = 1/21$

$c_1/c_2 = -3/-7 = 3/7$

Here, $a_1/a_2 \neq b_1/b_2$.

Hence, the given pair of linear equations has a unique solution.

(ii) Yes.

Given pair of linear equations.

$-2x - 3y - 1 = 0$ and $4x + 6y + 2 = 0$;

Comparing the above equations with $ax + by + c = 0$;

Here, $a_1 = -2$, $b_1 = -3$, $c_1 = -1$;

And $a_2 = 4$, $b_2 = 6$, $c_2 = 2$;

$a_1/a_2 = -2/4 = -½$

$b_1/b_2 = -3/6 = -½$

$c_1/c_2 = -½$

Here, $a_1/a_2 = b_1/b_2 = c_1/c_2$, i.e. coincident lines

Hence, the given pair of linear equations is coincident.

(iii) No.

Given pair of linear equations are

$x/2 + y + 2/5 = 0$

$4x + 8y + 5/16 = 0$

Comparing the above equations with $ax + by + c = 0$;

Here, $a_1 = ½$, $b_1 = 1$, $c_1 = 2/5$;

And $a_2 = 4$, $b_2 = 8$, $c_2 = 5/16$;

$a_1/a_2 = 1/8$

$b_1/b_2 = 1/8$

$c_1/c_2 = 32/25$

Here, $a_1/a_2 = b_1/b_2 \neq c_1/c_2$, i.e. parallel lines

Hence, the given pair of linear equations has no solution.

3. Are the following pair of linear equations consistent? Justify your answer.

(i) $-3x - 4y = 12$

$4y + 3x = 12$

(ii) $(3/5)x - y = ½$

$(1/5)x - 3y = 1/6$

(iii) $2ax + by = a$

1. ***ax* + 2*by* – 2*a* = 0; *a*, *b* ≠ 0**

(iv) $x + 3y = 11$

$2(2x + 6y) = 22$

Solution:

Conditions for pair of linear equations to be consistent are:

$a_1/a_2 \neq b_1/b_2$. [unique solution]

$a_1/a_2 = b_1/b_2 = c_1/c_2$ [coincident or infinitely many solutions]

(i) No.

The given pair of linear equations

$-3x - 4y - 12 = 0$ and $4y + 3x - 12 = 0$

Comparing the above equations with $ax + by + c = 0$;

We get,

$a_1 = -3$, $b_1 = -4$, $c_1 = -12$;

$a_2 = 3$, $b_2 = 4$, $c_2 = -12$;

$a_1/a_2 = -3/3 = -1$

$b_1/b_2 = -4/4 = -1$

$c_1/c_2 = -12/-12 = 1$

Here, $a_1/a_2 = b_1/b_2 \neq c_1/c_2$

Hence, the pair of linear equations has no solution, i.e., inconsistent.

(ii) Yes.

The given pair of linear equations

$(3/5)x - y = ½$

$(1/5)x - 3y = 1/6$

Comparing the above equations with $ax + by + c = 0$;

We get,

$a_1 = 3/5$, $b_1 = -1$, $c_1 = -½$;

$a_2 = 1/5$, $b_2 = 3$, $c_2 = -1/6$;

$a_1/a_2 = 3$

$b_1/b_2 = -1/-3 = 1/3$

$c_1/c_2 = 3$

Here, $a_1/a_2 \neq b_1/b_2$.

Hence, the given pair of linear equations has a unique solution, i.e., consistent.

(iii) Yes.

The given pair of linear equations –

$2ax + by - a = 0$ and $4ax + 2by - 2a = 0$

Comparing the above equations with $ax + by + c = 0$;

We get,

$a_1 = 2a$, $b_1 = b$, $c_1 = -a$;

$a_2 = 4a$, $b_2 = 2b$, $c_2 = -2a$;

$a_1/a_2 = ½$

$b_1/b_2 = ½$

$c_1/c_2 = ½$

Here, $a_1/a_2 = b_1/b_2 = c_1/c_2$

Hence, the given pair of linear equations has infinitely many solutions, i.e., consistent

(iv) No.

The given pair of linear equations

$x + 3y = 11$ and $2x + 6y = 11$

Comparing the above equations with $ax + by + c = 0$;

We get,

$a_1 = 1$, $b_1 = 3$, $c_1 = 11$

$a_2 = 2$, $b_2 = 6$, $c_2 = 11$

$a_1/a_2 = ½$

$b_1/b_2 = ½$

$c_1/c_2 = 1$

Here, $a_1/a_2 = b_1/b_2 \neq c_1/c_2$.

Hence, the given pair of linear equations has no solution.

EXERCISE 3.3

1. For which value(s) of λ , do the pair of linear equations

$\lambda x + y = \lambda^2$ and $x + \lambda y = 1$ have

(i) no solution?

(ii) infinitely many solutions?

(iii) a unique solution?

Solution:

The given pair of linear equations is

$\lambda x + y = \lambda^2$ and $x + \lambda y = 1$

$a_1 = \lambda$, $b_1 = 1$, $c_1 = -\lambda^2$

$a_2 = 1$, $b_2 = \lambda$, $c_2 = -1$

The given equations are;

$\lambda x + y - \lambda^2 = 0$

$x + \lambda y - 1 = 0$

Comparing the above equations with $ax + by + c = 0$;

We get,

$a_1 = \lambda$, $b_1 = 1$, $c_1 = -\lambda^2$;

$a_2 = 1$, $b_2 = \lambda$, $c_2 = -1$;

$a_1 / a_2 = \lambda/1$

$b_1 / b_2 = 1/\lambda$

$c_1 / c_2 = \lambda^2$

(i) For no solution,

$a_1/a_2 = b_1/b_2 \neq c_1/c_2$

i.e. $\lambda = 1/\lambda \neq \lambda^2$

so, $\lambda^2 = 1$;

and $\lambda^2 \neq \lambda$

Here, we take only $\lambda = -1$,

Since the system of linear equations has no solution.

(ii) For infinitely many solutions,

$a_1/a_2 = b_1/b_2 = c_1/c_2$

i.e. $\lambda = 1/\lambda = \lambda^2$

so $\lambda = 1/\lambda$ gives $\lambda = +1$;

$\lambda = \lambda^2$ gives $\lambda = 1,0$;

Hence satisfying both the equations

$\lambda = 1$ is the answer.

(iii) For a unique solution,

$a_1/a_2 \neq b_1/b_2$

so $\lambda \neq 1/\lambda$

hence, $\lambda^2 \neq 1$;

$\lambda \neq + 1$;

So, all real values of λ except $+1$.

2. For which value(s) of *k* will the pair of equations

***kx* + 3*y* = *k* – 3**

12*x* + *ky* = *k*

have no solution?

Solution:

The given pair of linear equations is

$kx + 3y = k – 3$ …(i)

$12x + ky = k$ …(ii)

On comparing the equations (i) and (ii) with $ax + by = c = 0$,

We get,

$a_1 = k$, $b_1 = 3$, $c_1 = -(k – 3)$

$a_2 = 12$, $b_2 = k$, $c_2 = – k$

Then,

$a_1 /a_2 = k/12$

$b_1 /b_2 = 3/k$

$c_1 /c_2 = (k-3)/k$

For no solution of the pair of linear equations,

$a_1/a_2 = b_1/b_2 \neq c_1/c_2$

$k/12 = 3/k \neq (k-3)/k$

Taking the first two parts, we get

$k/12 = 3/k$

$k^2 = 36$

$k = + 6$

Taking the last two parts, we get

$3/k \neq (k-3)/k$

$3k \neq k(k-3)$

$k^2 - 6k \neq 0$

so, $k \neq 0,6$

Therefore, the value of k for which the given pair of linear equations has no solution is $k = -6$.

3. For which values of *a* and *b* will the following pair of linear equations have infinitely many solutions?

$\boldsymbol{x + 2y = 1}$

$\boldsymbol{(a - b)x + (a + b)y = a + b - 2}$

Solution:

The given pair of linear equations are:

$x + 2y = 1$ …(i)

$(a-b)x + (a + b)y = a + b - 2$ …(ii)

On comparing with $ax + by = c = 0$ we get

$a_1 = 1$, $b_1 = 2$, $c_1 = -1$

$a_2 = (a - b)$, $b_2 = (a + b)$, $c_2 = -(a + b - 2)$

$a_1/a_2 = 1/(a-b)$

$b_1/b_2 = 2/(a+b)$

$c_1/c_2 = 1/(a+b-2)$

For infinitely many solutions of the pair of linear equations,

$a_1/a_2 = b_1/b_2 = c_1/c_2$(coincident lines)

so, $1/(a-b) = 2/(a+b) = 1/(a+b-2)$

Taking the first two parts,

$1/(a-b) = 2/(a+b)$

$a + b = 2(a - b)$

$a = 3b$ …(iii)

Taking the last two parts,

$2/(a+b) = 1/(a+b-2)$

$2(a + b - 2) = (a + b)$

$a + b = 4$ …(iv)

Now, put the value of a from Eq. (iii) in Eq. (iv), and we get

$3b + b = 4$

$4b = 4$

$b = 1$

Put the value of b in Eq. (iii), and we get

a = 3

So, the values (a,b) = (3,1) satisfy all the parts. Hence, the required values of a and b are 3 and 1, respectively, for which the given pair of linear equations has infinitely many solutions.

4. Find the value(s) of *p* in (i) to (iv) and *p* and *q* in (v) for the following pair of equations:

(i) $3x – y – 5 = 0$ and $6x – 2y – p = 0$, if the lines represented by these equations are parallel.

Solution:

Given pair of linear equations is

$3x – y – 5 = 0$ …(i)

$6x – 2y – p = 0$ …(ii)

On comparing with $ax + by + c = 0$ we get

We get,

$a_1 = 3$, $b_1 = – 1$, $c_1 = – 5$;

$a_2 = 6$, $b_2 = – 2$, $c_2 = – p$;

$a_1 /a_2 = 3/6 = ½$

$b_1 /b_2 = ½$

$c_1 /c_2 = 5/p$

Since the lines represented by these equations are parallel, then

$a_1/a_2 = b_1/b_2 \neq c_1/c_2$

Taking the last two parts, we get $½ \neq 5/p$

So, $p \neq 10$

Hence, the given pair of linear equations are parallel for all real values of p except 10.

(ii) $– x + py = 1$ and $px – y = 1$, if the pair of equations has no solution.

Solution:

Given pair of linear equations is

$– x + py = 1$ …(i)

$px – y – 1 = 0$ …(ii)

On comparing with $ax + by + c = 0$,

We get,

$a_1 = -1$, $b_1 = p$, $c_1 =- 1$;

$a_2 = p$, $b_2 = – 1$, $c_2 =- 1$;

$a_1 /a_2 = -1/p$

$b_1 /b_2 = – p$

$c_1 / c_2 = 1$

Since the lines equations have no solution, i.e., both lines are parallel to each other,

$a_1/a_2 = b_1/b_2 \neq c_1/c_2$

$-1/p = -p \neq 1$

Taking the last two parts, we get

$p \neq -1$

Taking the first two parts, we get

$p^2 = 1$

$p = + 1$

Hence, the given pair of linear equations has no solution for p = 1.

(iii) – 3*x* + 5*y* = 7 and 2*px* – 3*y* = 1, if the lines represented by these equations are intersecting at a unique point.

Solution:

Given, pair of linear equations is

$-3x + 5y = 7$

$2px - 3y = 1$

On comparing with $ax + by + c = 0$, we get

Here, $a_1 = -3$, $b_1 = 5$, $c_1 = -7$;

And $a_2 = 2p$, $b_2 = -3$, $c_2 = -1$;

$a_1 / a_2 = -3/ 2p$

$b_1 / b_2 = -5/3$

$c_1 / c_2 = 7$

Since the lines intersect at a unique point, i.e., it has a unique solution

$a_1/a_2 \neq b_1/b_2$

$-3/2p \neq -5/3$

$p \neq 9/10$

Hence, the lines represented by these equations intersect at a unique point for all real values of p except 9/10.

(iv) 2*x* + 3*y* – 5 = 0 and *px* – 6*y* – 8 = 0, if the pair of equations has a unique solution.

Solution:

Given, pair of linear equations is

$2x + 3y - 5 = 0$

$px - 6y - 8 = 0$

On comparing with $ax + by + c = 0$ we get

Here, $a_1 = 2$, $b_1 = 3$, $c_1 = -5$;

And $a_2 = p$, $b_2 = -6$, $c_2 = -8$;

$a_1/a_2 = 2/p$

$b_1/b_2 = -3/6 = -½$

$c_1/c_2 = 5/8$

Since the pair of linear equations has a unique solution,

$a_1/a_2 \neq b_1/b_2$

so $2/p \neq -½$

$p \neq -4$

Hence, the pair of linear equations has a unique solution for all values of p except – 4.

(v) 2*x* + 3*y* = 7 and 2*px* + *py* = 28 – *qy*, if the pair of equations has infinitely many solutions.

Solution:

Given pair of linear equations is

$2x + 3y = 7$

$2px + py = 28 - qy$

or $2px + (p + q)y - 28 = 0$

On comparing with $ax + by + c = 0$,

We get,

Here, $a_1 = 2$, $b_1 = 3$, $c_1 = -7$;

And $a_2 = 2p$, $b_2 = (p + q)$, $c_2 = -28$;

$a_1/a_2 = 2/2p$

$b_1/b_2 = 3/(p+q)$

$c_1/c_2 = ¼$

Since the pair of equations has infinitely many solutions i.e., both lines are coincident.

$a_1/a_2 = b_1/b_2 = c_1/c_2$

$1/p = 3/(p+q) = ¼$

Taking the first and third parts, we get

$p = 4$

Again, taking the last two parts, we get

$3/(p+q) = ¼$

$p + q = 12$

Since $p = 4$

So, $q = 8$

Here, we see that the values of p = 4 and q = 8 satisfy all three parts.

Hence, the pair of equations has infinitely many solutions for all values of p = 4 and q = 8.

5. Two straight paths are represented by the equations $x - 3y = 2$ and $-2x + 6y = 5$.Check whether the paths cross each other or not.

Solution:

Given linear equations are

$x - 3y - 2 = 0$ …(i)

$-2x + 6y - 5 = 0$ …(ii)

On comparing with ax + by c=0,

We get

$a_1 = 1$, $b_1 = -3$, $c_1 = -2$;

$a_2 = -2$, $b_2 = 6$, $c_2 = -5$;

$a_1/a_2 = -½$

$b_1/b_2 = -3/6 = -½$

$c_1/c_2 = 2/5$

i.e., $a_1/a_2 = b_1/b_2 \neq c_1/c_2$ [parallel lines]

Hence, two straight paths represented by the given equations never cross each other because they are parallel to each other.

6. Write a pair of linear equations which has the unique solution $x = -1$, $y = 3$. How many such pairs can you write?

Solution:

Condition for the pair of system to have a unique solution

$a_1/a_2 \neq b_1/b_2$

Let the equations be,

$a_1x + b_1y + c_1 = 0$

$a_2x + b_2y + c_2 = 0$

Since, x = – 1 and y = 3 is the unique solution of these two equations, then

It must satisfy the equations –

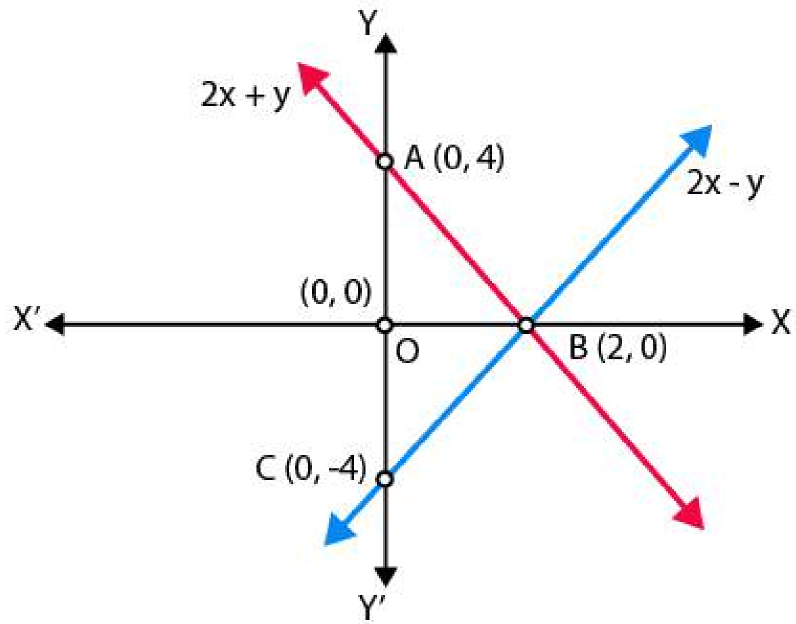
$a_1(-1) + b_1(3) + c_1 = 0$

$-a_1 + 3b_1 + c_1 = 0$ …(i)

and $a_2(-1) + b_2(3) + c_2 = 0$

$-a_2 + 3b_2 + c_2 = 0$ …(ii)

Since for the different values of a_1, b_1, c_1 and a_2, b_2, c_2 satisfy the Eqs. (i) and (ii),



Hence, infinitely many pairs of linear equations are possible.

7. If $2x + y = 23$ and $4x - y = 19$, find the values of $5y - 2x$ and y/x – 2.

Solution:

Given equations are

2x + y = 23 …(i)

4x – y = 19 …(ii)

On adding both equations, we get

6x = 42

So, x = 7

Put the value of x in Eq. (i), and we get

2(7) + y = 23

y = 23 – 14

so, y = 9

Hence 5y – 2x = 5(9) – 2(7) = 45 – 14 = 31

y/x – 2 = 9/7 -2 = -5/7

Hence, the values of (5y – 2x) and y/x – 2 are 31 and -5/7 respectively.

8. Find the values of *x* and *y* in the following rectangle [see Fig. 3.2].

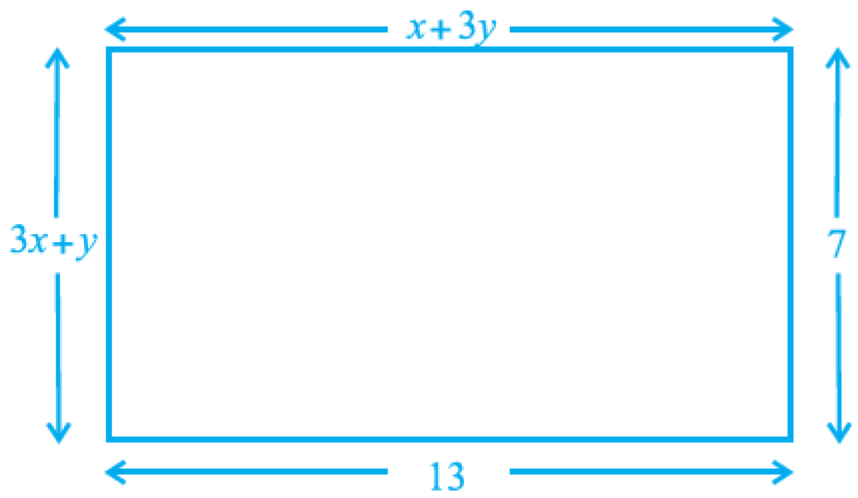


Fig. 3.2

Solution:

Using the property of a rectangle,

We know that,

Lengths are equal,

i.e., CD = AB

Hence, $x + 3y = 13$ …(i)

Breadth are equal,

i.e., AD = BC

Hence, $3x + y = 7$ …(ii)

On multiplying Eq. (ii) by 3 and then subtracting Eq. (i),

We get,

$8x = 8$

So, $x = 1$

On substituting $x = 1$ in Eq. (i),

We get,

$y = 4$

Therefore, the required values of x and y are 1 and 4, respectively.

EXERCISE 3.4

1. Graphically, solve the following pair of equations:

$2x + y = 6$

$2x – y + 2 = 0$

Find the ratio of the areas of the two triangles formed by the lines representing these equations with the *x*-axis and the lines with the *y*-axis.

Solution:

Given equations are 2x + y = 6 and 2x – y + 2 = 0

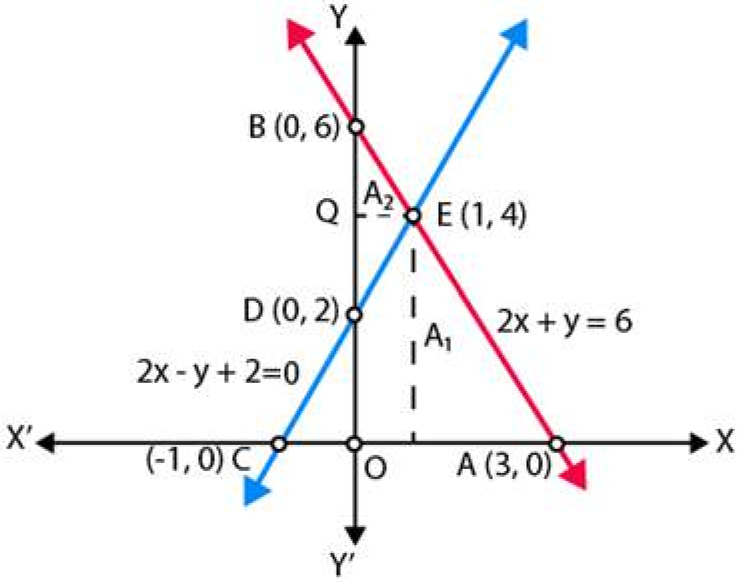
Table for equation 2x + y – 6 = 0, for x = 0, y = 6, for y = 0, x = 3.

x	0	3
y	6	0

Table for equation 2x – y + 2 = 0, for x = 0, y = 2, for y = 0,x = – 1

x	0	-1
y	2	0

Let A_1 and A_2 represent the areas of triangles ACE and BDE, respectively.



Let the area of triangle formed with x-axis = T_1

T_1 = Area of △ACE = ½ × AC × PE

T_1 = ½ × 4 × 4 = 8

And area of triangle formed with y-axis = T_2

T_1 = Area of △BDE = ½ × BD × QE

T_1 = ½ × 4 × 1 = 2

T_1:T_2 = 8:2 = 4:1

Hence, the pair of equations intersect graphically at point E(1,4)

i.e., x = 1 and y = 4.

2. Determine, graphically, the vertices of the triangle formed by the lines

y = x, 3y = x, x + y = 8

Solution:

Given linear equations are

y = x …(i)

3y = x …(ii)

and x + y = 8 …(iii)

Table for line y = x,

x	0	1	2
y	0	1	2

Table for line x = 3y,

x	0	3	6
y	0	1	2

Table for line x + y = 8

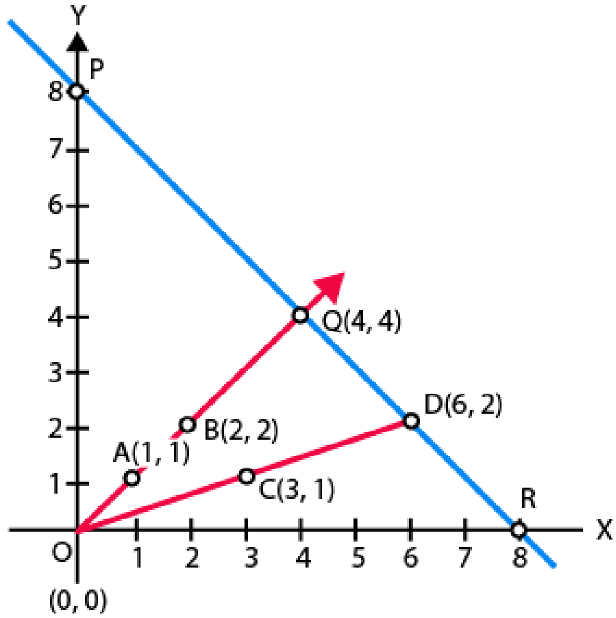
x	0	4	8

y	8	4	0

Plotting the points A (1, 1), B(2,2), C (3, 1), D (6, 2), we get the straight lines AB and CD.

Similarly, plotting the points P (0, 8), Q(4, 4) and R(8, 0), we get the straight line PQR.

AB and CD intersect the line PR on Q and D, respectively.



So, △OQD is formed by these lines. Hence, the vertices of the △OQD formed by the given lines are O(0, 0), Q(4, 4) and D(6,2).

3. Draw the graphs of the equations $x = 3$, $x = 5$ and $2x – y – 4 = 0$. Also, find the area of the quadrilateral formed by the lines and the x–axis.

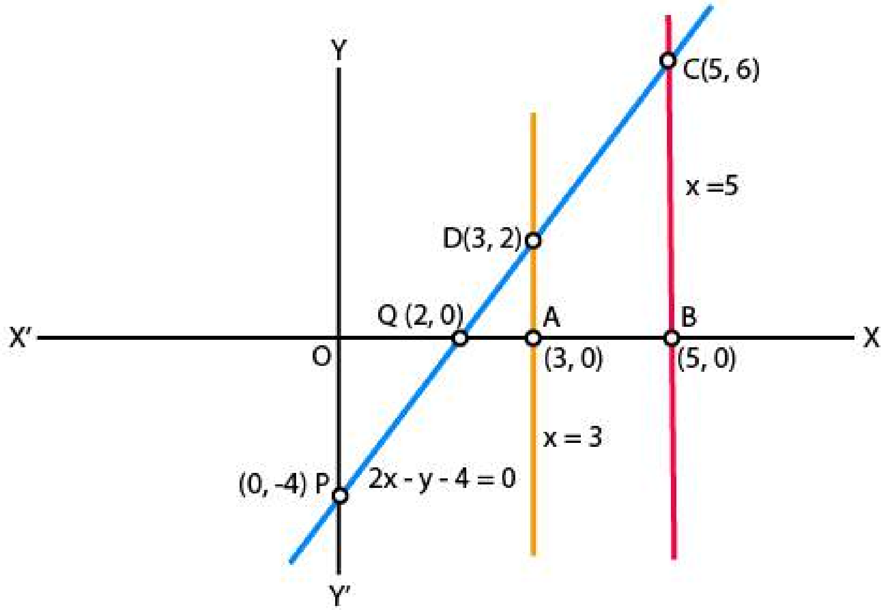
Solution:

Given equation of lines x = 3, x = 5 and 2x-y-4 = 0.

Table for line 2x – y – 4 = 0

x	0	2
y	-4	0

Plotting the graph, we get,



From the graph, we get,

AB = OB-OA = 5-3 = 2

AD = 2

BC = 6

Thus, quadrilateral ABCD is a trapezium, then,

Area of quadrilateral ABCD = ½ × (distance between parallel lines) = ½ × (AB) × (AD + BC)

= 8 sq units

4. The cost of 4 pens and 4 pencil boxes is Rs. 100. Three times the cost of a pen is Rs. 15 more than the cost of a pencil box. Form the pair of linear equations for the above situation. Find the cost of a pen and a pencil box.

Solution:

Let the cost of a pen and a pencil box be Rs x and Rs y, respectively.

According to the question,

$4x + 4y = 100$

Or $x + y = 25$ …(i)

$3x = y + 15$

Or $3x-y = 15$ …(ii)

On adding Equation (i) and (ii), we get,

$4x = 40$

So, $x = 10$

Substituting $x = 10$, in Eq. (i), we get y = 25-10 = 15

Hence, the cost of a pen = Rs. 10

The cost of a pencil box = Rs. 15

5. Determine, algebraically, the vertices of the triangle formed by the lines

$3x - y = 3$

$2x - 3y = 2$

$x + 2y = 8$

Solution:

3x – y = 2 …(i)

2x -3y = 2 …(ii)

x + 2y = 8 …(iii)

Let the equations of the line (i), (ii) and (iii) represent the side of a ΔABC.

On solving (i) and (ii),

We get,

[First, multiply Eq. (i) by 3 in Eq. (i) and then subtract]
(9x-3y)-(2x-3y) = 9-2

7x = 7

x = 1

Substituting x=1 in Eq. (i), we get

3×1-y = 3

y = 0

So, the coordinate of point B is (1, 0)

On solving lines (ii) and (iii),

We get,

[First, multiply Eq. (iii) by 2 and then subtract]
(2x + 4y)-(2x-3y) = 16-2

7y = 14

y = 2

Substituting y=2 in Eq. (iii), we get

x + 2 × 2 = 8

x + 4 = 8

x = 4

Hence, the coordinate of point C is (4, 2).

On solving lines (iii) and (i),

We get,

[First, multiply in Eq. (i) by 2 and then add]
$(6x-2y) + (x + 2y) = 6 + 8$

$7x = 14$

$x = 2$

Substituting $x=2$ in Eq. (i), we get

$3 \times 2 - y = 3$

$y = 3$

So, the coordinate of point A is (2, 3).

Hence, the vertices of the ΔABC formed by the given lines are as follows,

A (2, 3), B (1, 0) and C (4, 2).

6. Ankita travels 14 km to her home partly by rickshaw and partly by bus. She takes half an hour if she travels 2 km by rickshaw and the remaining distance by bus. On the other hand, if she travels 4 km by rickshaw and the remaining distance by bus, she takes 9 minutes longer. Find the speed of the rickshaw and of the bus.

Solution:

Let the speed of the rickshaw and the bus be x and y km/h, respectively.

Now, she has taken time to travel 2 km by rickshaw, $t_1 = (2/x)$ hr

Speed = distance/ time

She has taken time to travel the remaining distance i.e., $(14 - 2) = 12$km

By bus $t_2 = (12/y)$ hr

By the first condition,

$t_1 + t_2 = ½ = (2/x) + (12/y)$ … (i)

Now, she has taken time to travel 4 km by rickshaw, $t_3 = (4/x)$ hr

and she has taken time to travel the remaining distance i.e., $(14 - 4) = 10$km, by bus $= t_4 = (10/y)$ hr

By the second condition,

$t_3 + t_4 = ½ + 9/60 = ½ + 3/20$

$(4/x) + (10/y) = (13/20)$ …(ii)

Let $(1/x) = u$ and $(1/y) = v$

Then equations (i) and (ii) become

$2u + 12v = ½$ …(iii)

$4u + 10v = 13/20$…(iv)

[First, multiply Eq. (iii) by 2 and then subtract]
$(4u + 24v) - (4u + 10v) = 1 - 13/20$

$14v = 7/20$

$v = 1/40$

Substituting the value of v in Eq. (iii),

$2u + 12(1/40) = \frac{1}{2}$

$2u = 2/10$

$u = 1/10$

$x = 1/u = 10$km/hr

$y = 1/v = 40$km/hr

Hence, the speed of rickshaw = 10 km/h

And the speed of the bus = 40 km/h.

EXERCISE 2.1 PAGE NO: 9

Choose the correct answer from the given four options in the following questions:

1. If one of the zeroes of the quadratic polynomial $(k–1)x^2 + kx + 1$ is –3, then the value of k is

(A) 4/3 **(B) -4/3**

(C) 2/3 **(D) -2/3**

Solution:

(A) 4/3

Explanation:

According to the question,

-3 is one of the zeros of quadratic polynomial $(k-1)x^2+kx+1$

Substituting -3 in the given polynomial,

$(k-1)(-3)^2+k(-3)+1=0$

$(k-1)9+k(-3)+1 = 0$

$9k-9-3k+1=0$

$6k-8=0$

$k=8/6$

Therefore, $k=4/3$

Hence, **option (A)** is the correct answer.

2. A quadratic polynomial, whose zeroes are –3 and 4, is

(A) $x^2 – x + 12$ **(B) $x^2 + x + 12$**

(C) $(x^2/2)-(x/2)-6$ **(D) $2x^2 + 2x –24$**

Solution:

(C) $(x^2/2)-(x/2)-6$

Explanation:

Sum of zeroes, $\alpha+ \beta= -3+4 =1$

Product of Zeroes, $\alpha\beta = -3\times 4 = -12$

Therefore, the quadratic polynomial becomes,

x^2- (sum of zeroes)x+(product of zeroes)

$= x^2- (\alpha+ \beta)x+(\alpha\beta)$

$= x^2 - (1)x + (-12)$

$= x^2 - x -12$

Divide by 2, we get

$= x^2/2 - x/2 - 12/2$

$= x^2/2 - x/2 - 6$

Hence, **option (C)** is the correct answer.

3. If the zeroes of the quadratic polynomial $x^2 + (a + 1)x + b$ are 2 and –3, then

(A) $a = –7, b = –1$ **(B) $a = 5, b = –1$**

(C) $a = 2, b = – 6$ **(D) $a = 0, b = – 6$**

Solution:

(D) $a = 2, b = – 6$

Explanation:

According to the question,

$x^2 + (a+1)x + b$

Given that, the zeroes of the polynomial = 2 and -3,
When x = 2
$2^2 + (a+1)(2) + b = 0$
$4 + 2a+2 + b = 0$
$6 + 2a+b = 0$
$2a+b = -6$ ----- (1)
When x = -3,
$(-3)^2 + (a+1)(-3) + b = 0$
$9 - 3a-3 + b = 0$
$6 - 3a+b = 0$
$-3a+b = -6$ ----- (2)
Subtracting equation (2) from (1)
$2a+b - (-3a+b) = -6-(-6)$
$2a+b+3a-b = -6+6$
$5a = 0$
$a = 0$

Substituting the value of 'a' in equation (1), we get,
$2a + b = -6$
$2(0) +b = -6$
$b = -6$

Hence, **option (D)** is the correct answer.

4. The number of polynomials having zeroes as –2 and 5 is

(A) 1 **(B) 2**
(C) 3 **(D) more than 3**

Solution:

(D) more than 3

Explanation:

According to the question,
The zeroes of the polynomials = -2 and 5
We know that the polynomial is of the form,
$p(x) = ax^2 + bx + c.$
Sum of the zeroes = - (coefficient of x) ÷ coefficient of x^2 i.e.
Sum of the zeroes = - b/a
$- 2 + 5 = - b/a$
$3 = - b/a$
$b = - 3$ and $a = 1$
Product of the zeroes = constant term ÷ coefficient of x^2 i.e.
Product of zeroes = c/a
$(- 2)5 = c/a$
$- 10 = c$
Substituting the values of a, b and c in the polynomial $p(x) = ax^2 + bx + c$.
We get, $x^2 - 3x - 10$
Therefore, we can conclude that x can take any value.

Hence, **option (D)** is the correct answer.

5. Given that one of the zeroes of the cubic polynomial $ax^3 + bx^2 + cx + d$ is zero, the product of the other two zeroes is

(A) (–c/a) **(B) c/a**

(C) 0 **(D) (–b/a)**

Solution:

(B) (c/a)

Explanation:

According to the question,

We have the polynomial,

$ax^3 + bx^2 + cx + d$

We know that,

Sum of product of roots of a cubic equation is given by c/a

It is given that one root = 0

Now, let the other roots be α, β

So, we get,

$\alpha\beta + \beta(0) + (0)\alpha = c/a$

$\alpha\beta = c/a$

Hence the product of other two roots is c/a

Hence, **option (B)** is the correct answer

EXERCISE 2.2 PAGE NO: 11

1. Answer the following and justify:

(i) Can $x^2 – 1$ be the quotient on division of $x^6 + 2x^3 + x – 1$ by a polynomial in x of degree 5?

Solution:

No, x^2 - 1 cannot be the quotient on division of $x^6 + 2x^3 + x - 1$ by a polynomial in x of degree 5.

Justification:

When a degree 6 polynomial is divided by degree 5 polynomial,
The quotient will be of degree 1.
Assume that (x^2 - 1) divides the degree 6 polynomial with and the quotient obtained is degree 5 polynomial (1)
According to our assumption,
(degree 6 polynomial) = (x^2 - 1)(degree 5 polynomial) + r(x) [Since, (a = bq + r)]
= (degree 7 polynomial) + r(x) [Since, (x^2 term × x^5 term = x^7 term)]
= (degree 7 polynomial)
From the above equation, it is clear that, our assumption is contradicted.
x^2 - 1 cannot be the quotient on division of $x^6 + 2x^3 + x - 1$ by a polynomial in x of degree 5
Hence Proved.

(ii) What will the quotient and remainder be on division of $ax^2 + bx + c$ by $px^3 + qx^2 + rx + s, p \neq 0$?

Solution:

Degree of the polynomial $px^3 + qx^2 + rx + s$ is 3
Degree of the polynomial $ax^2 + bx + c$ is 2
Here, degree of $px^3 + qx^2 + rx + s$ is greater than degree of the $ax^2 + bx + c$
Therefore, the quotient would be zero,
And the remainder would be the dividend = $ax^2 + bx + c$.

(iii) If on division of a polynomial $p(x)$ by a polynomial $g(x)$, the quotient is zero, what is the relation between the degrees of $p(x)$ and $g(x)$?

Solution:

We know that,
p(x)= g(x) × q(x)+r(x)
According to the question,
q(x) =0
When q(x)=0, then r(x) is also = 0
So, now when we divide p(x) by g(x),
Then p(x) should be equal to zero
Hence, the relation between the degrees of p (x) and g (x) is the degree p(x)<degree g(x)

(iv) If on division of a non-zero polynomial $p(x)$ by a polynomial $g(x)$, the remainder is zero, what is the relation between the degrees of $p(x)$ and $g(x)$?

Solution:

In order to divide p(x) by g(x)

We know that,
Degree of p(x) > degree of g(x)
or
Degree of p(x)= degree of g(x)
Therefore, we can say that,
The relation between the degrees of *p* (*x*) and *g* (*x*) is degree of p(x) $\geq$ degree of g(x)

(v) Can the quadratic polynomial $x^2 + kx + k$ have equal zeroes for some odd integer $k > 1$?
Solution:

A Quadratic Equation will have equal roots if it satisfies the condition:

$b^2 - 4ac = 0$

Given equation is $x^2 + kx + k = 0$

$a = 1, b = k, x = k$

Substituting in the equation we get,

$k^2 - 4 (1) (k) = 0$

$k^2 - 4k = 0$

$k (k - 4) = 0$

$k = 0 , k = 4$

But in the question, it is given that k is greater than 1.
Hence the value of k is 4 if the equation has common roots.
Hence if the value of k = 4, then the equation ($x^2 + kx + k$) will have equal roots.

EXERCISE 2.3 PAGE NO: 12

Find the zeroes of the following polynomials by factorisation method.

1. $4x^2 – 3x – 1$

Solution:

$4x^2 – 3x – 1$

Splitting the middle term, we get,

$4x^2-4x+1x-1$

Taking the common factors out, we get,

4x(x-1) +1(x-1)

On grouping, we get,

(4x+1)(x-1)

So, the zeroes are,

4x+1= 0⇒ 4x=-1 ⇒x= (-1/4)

(x-1) = 0 ⇒ x=1

Therefore, zeroes are (-1/4) and 1

Verification:

Sum of the zeroes = - (coefficient of x) ÷ coefficient of x^2

$\alpha + \beta = - b/a$

1 - 1/4 = - (- 3)/4 = ¾

Product of the zeroes = constant term ÷ coefficient of x^2

$\alpha\ \beta = c/a$

1(- 1/4) = - ¼

- 1/4 = - 1/4

2. $3x^2 + 4x – 4$

Solution:

$3x^2 + 4x – 4$

Splitting the middle term, we get,

$3x^2 + 6x - 2x – 4$

Taking the common factors out, we get,

3x(x+2) -2(x+2)

On grouping, we get,

(x+2)(3x-2)

So, the zeroes are,

x+2=0 ⇒ x= -2

3x-2=0⇒ 3x=2⇒x=2/3

Therefore, zeroes are (2/3) and -2

Verification:

Sum of the zeroes = - (coefficient of x) ÷ coefficient of x^2

$\alpha + \beta = - b/a$

- 2 + (2/3) = - (4)/3

= - 4/3 = - 4/3

Product of the zeroes = constant term ÷ coefficient of x^2

$\alpha\ \beta = c/a$
Product of the zeroes = (- 2) (2/3) = - 4/3

3. $5t^2 + 12t + 7$
Solution:

$5t^2 + 12t + 7$
Splitting the middle term, we get,
$5t^2 + 5t + 7t + 7$
Taking the common factors out, we get,
5t (t+1) +7(t+1)
On grouping, we get,
(t+1)(5t+7)
So, the zeroes are,
$t+1=0 \Rightarrow y= -1$
$5t+7=0 \Rightarrow 5t=-7 \Rightarrow t=-7/5$
Therefore, zeroes are (-7/5) and -1
Verification:
Sum of the zeroes = - (coefficient of x) ÷ coefficient of x^2
$\alpha + \beta = - b/a$
(- 1) + (- 7/5) = - (12)/5
= - 12/5 = - 12/5
Product of the zeroes = constant term ÷ coefficient of x^2
$\alpha\ \beta = c/a$
(- 1)(- 7/5) = 7/5
7/5 = 7/5

4. $t^3 – 2t^2 – 15t$
Solution:

$t^3 – 2t^2 – 15t$
Taking t common, we get,
$t (t^2 -2t -15)$
Splitting the middle term of the equation $t^2 -2t -15$, we get,
$t(t^2 -5t + 3t -15)$
Taking the common factors out, we get,
t (t (t-5) +3(t-5)
On grouping, we get,
t (t+3)(t-5)
So, the zeroes are,
t=0
$t+3=0 \Rightarrow t= -3$
$t -5=0 \Rightarrow t=5$
Therefore, zeroes are 0, 5 and -3
Verification:
Sum of the zeroes = - (coefficient of x^2) ÷ coefficient of x^3
$\alpha + \beta + \gamma = - b/a$

(0) + (- 3) + (5) = - (- 2)/1
= 2 = 2
Sum of the products of two zeroes at a time = coefficient of x ÷ coefficient of x^3
αβ + βγ + αγ = c/a
(0)(- 3) + (- 3) (5) + (0) (5) = - 15/1
= - 15 = - 15
Product of all the zeroes = - (constant term) ÷ coefficient of x^3
αβγ = - d/a
(0)(- 3)(5) = 0
0 = 0

5. $2x^2 + (7/2)x + 3/4$
Solution:

$2x^2 + (7/2)x + 3/4$
The equation can also be written as,
$8x^2+14x+3$
Splitting the middle term, we get,
$8x^2+12x+2x+3$
Taking the common factors out, we get,
4x (2x+3) +1(2x+3)
On grouping, we get,
(4x+1)(2x+3)
So, the zeroes are,
4x+1=0 ⇒ x = -1/4
2x+3=0 ⇒ x = -3/2
Therefore, zeroes are -1/4 and -3/2
Verification:
Sum of the zeroes = - (coefficient of x) ÷ coefficient of x^2
α + β = - b/a
(- 3/2) + (- 1/4) = - (7)/4
= - 7/4 = - 7/4
Product of the zeroes = constant term ÷ coefficient of x^2
α β = c/a
(- 3/2)(- 1/4) = (3/4)/2
3/8 = 3/8

EXERCISE 2.4 PAGE NO: 14

1. For each of the following, find a quadratic polynomial whose sum and product respectively of the zeroes are as given. Also find the zeroes of these polynomials by factorisation.

(i) (–8/3), 4/3
(ii) 21/8, 5/16
(iii) -2√3, -9
(iv) (-3/(2√5)), -½

Solution:

(i) Sum of the zeroes = - 8/3
Product of the zeroes = 4/3
P(x) = x^2 - (sum of the zeroes) + (product of the zeroes)
Then, P(x)= x^2 - (-8x)/3 + 4/3
P(x)= $3x^2 + 8x + 4$
Using splitting the middle term method,
$3x^2 + 8x + 4 = 0$
$3x^2 + (6x + 2x) + 4 = 0$
$3x^2 + 6x + 2x + 4 = 0$
$3x(x + 2) + 2(x + 2) = 0$
$(x + 2)(3x + 2) = 0$
⇒ x = - 2, - 2/3

(ii) Sum of the zeroes = 21/8
Product of the zeroes = 5/16
P(x) = x^2 - (sum of the zeroes) + (product of the zeroes)
Then, P(x)= x^2 - 21x/8 + 5/16
P(x)= $16x^2 - 42x + 5$
Using splitting the middle term method,
$16x^2 - 42x + 5 = 0$
$16x^2 - (2x + 40x) + 5 = 0$
$16x^2 - 2x - 40x + 5 = 0$
$2x (8x - 1) - 5(8x - 1) = 0$
$(8x - 1)(2x - 5) = 0$
⇒ x = 1/8, 5/2

(iii) Sum of the zeroes = - 2√3
Product of the zeroes = - 9
P(x) = x^2 - (sum of the zeroes) + (product of the zeroes)
Then, P(x) = $x^2 - (-2\sqrt{3}x) - 9$
Using splitting the middle term method,
$x^2 + 2\sqrt{3}x - 9 = 0$
$x^2 + (3\sqrt{3}x - \sqrt{3}x) - 9 = 0$
$x^2 + 3\sqrt{3}x - \sqrt{3}x - 9 = 0$
$x(x + 3\sqrt{3}) - \sqrt{3}(x + 3\sqrt{3}) = 0$
$(x + 3\sqrt{3})(x - \sqrt{3}) = 0$

$\Rightarrow x = \sqrt{3}, -3\sqrt{3}$

(iv) Sum of the zeroes = $-3/2\sqrt{5}x$
Product of the zeroes = $-\frac{1}{2}$
$P(x) = x^2$ - (sum of the zeroes) + (product of the zeroes)
Then, $P(x)= x^2 \; -3/2\sqrt{5}x - \frac{1}{2}$
$P(x)= 2\sqrt{5}x^2 + 3x - \sqrt{5}$
Using splitting the middle term method,
$2\sqrt{5}x^2 + 3x - \sqrt{5} = 0$
$2\sqrt{5}x^2 + (5x - 2x) - \sqrt{5} = 0$
$2\sqrt{5}x^2 + 5x - 2x - \sqrt{5} = 0$
$\sqrt{5}x\,(2x + \sqrt{5}) - 1(2x + \sqrt{5}) = 0$
$(2x + \sqrt{5})(\sqrt{5}x - 1) = 0$
$\Rightarrow x = 1/\sqrt{5}, -\sqrt{5}/2$

2. Given that the zeroes of the cubic polynomial $x^3 - 6x^2 + 3x + 10$ are of the form a, $a + b$, $a + 2b$ for some real numbers a and b, find the values of a and b as well as the zeroes of the given polynomial.
Solution:

Given that a, a+b, a+2b are roots of given polynomial $x^3-6x^2+3x+10$
Sum of the roots $\Rightarrow$ a+2b+a+a+b = -coefficient of x^2/ coefficient of x^3
$\Rightarrow 3a+3b = -(-6)/1 = 6$
$\Rightarrow 3(a+b) = 6$
$\Rightarrow$ a+b = 2 --------- (1) b = 2-a

Product of roots $\Rightarrow$ (a+2b)(a+b)a = -constant/coefficient of x^3
$\Rightarrow (a+b+b)(a+b)a = -10/1$

Substituting the value of a+b=2 in it
$\Rightarrow (2+b)(2)a = -10$
$\Rightarrow (2+b)2a = -10$
$\Rightarrow (2+2-a)2a = -10$
$\Rightarrow (4-a)2a = -10$
$\Rightarrow 4a-a^2 = -5$
$\Rightarrow a^2-4a-5 = 0$
$\Rightarrow a^2-5a+a-5 = 0$
$\Rightarrow (a-5)(a+1) = 0$
a-5 = 0 or a+1 = 0
a = 5 a = -1

a = 5, -1 in (1) a+b = 2

When a = 5, 5+b=2 $\Rightarrow$ b=-3
a = -1, -1+b=2 $\Rightarrow$ b= 3

$\therefore$ If a=5 then b= -3

or
If a= -1 then b=3

3. Given that √2 is a zero of the cubic polynomial $6x^3 + \sqrt{2}\,x^2 - 10x - 4\sqrt{2}$, find its other two zeroes.
Solution:

Given, √2 is one of the zero of the cubic polynomial.
Then, (x-√2) is one of the factor of the given polynomial p(x) = $6x^3+\sqrt{2}x^2-10x-4\sqrt{2}$.
So, by dividing p(x) by x-√2

$$\begin{array}{r|l} & 6x^2+7\sqrt{2}x+4 \\ \hline (x-\sqrt{2}) & 6x^3+\sqrt{2}x^2-10x-4\sqrt{2} \\ & 6x^3-6\sqrt{2}x^2 \\ & -\quad + \\ \hline & \quad 7\sqrt{2}x^2-10x-4\sqrt{2} \\ & \quad 7\sqrt{2}x^2-14x \\ & \quad -\quad\quad + \\ \hline & \quad\quad 4x-4\sqrt{2} \\ & \quad\quad 4x-4\sqrt{2} \\ \hline & \quad\quad\quad 0 \end{array}$$

$6x^3+\sqrt{2}x^2-10x-4\sqrt{2} = (x-\sqrt{2})(6x^2+7\sqrt{2}x+4)$
By splitting the middle term,
We get,
$(x-\sqrt{2})(6x^2+4\sqrt{2}x+3\sqrt{2}x+4)$
$= (x-\sqrt{2})[2x(3x+2\sqrt{2})+\sqrt{2}(3x+2\sqrt{2})]$
$= (x-\sqrt{2})(2x+\sqrt{2})(3x+2\sqrt{2})$

To get the zeroes of p(x),
Substitute p(x)= 0
$(x-\sqrt{2})(2x+\sqrt{2})(3x+2\sqrt{2}) = 0$
$x = \sqrt{2}$, $x = -\sqrt{2}/2$, $x = -2\sqrt{2}/3$
which is equal to,
$x = \sqrt{2}$, $x = -\sqrt{2}/2$, $x = -2\sqrt{2}/3$
Hence, the other two zeroes of p(x) are $-\sqrt{2}/2$ and $-2\sqrt{2}/3$

EXERCISE 1.1 PAGE NO: 2

Choose the correct answer from the given four options in the following questions:

1. For some integer m, every even integer is of the form:

(A) m (B) m +1

(C) 2m (D) 2m+1

Solution:

(C)2m

Explanation:

Even integers are those integers which are divisible by 2.

Hence, we can say that every integer which is a multiple of 2 must be an even integer.

Therefore, let us conclude that,

for an integer 'm', every even integer must be of the form

$2 \times m = 2m$.

Hence, **option (C)** is the correct answer.

2. For some integer q, every odd integer is of the form

(A) q (B) q +1

(C) 2g (D) 2q +1

Solution:

(D) 2q+1

Explanation:

Odd integers are those integers which are not divisible by 2.

Hence, we can say that every integer which is a multiple of 2 must be an even integer, while 1 added to every integer which is multiplied by 2 is an odd integer.

Therefore, let us conclude that,

for an integer 'q', every odd integer must be of the form

$(2 \times q)+1 = 2q+1$.

Hence, **option (D)** is the correct answer.

3. $n^2 - 1$ is divisible by 8, if n is

(A) an integer (B) a natural number

(C) an odd integer (D) an even integer

Solution:

(C) an odd integer

Explanation:

Let $x = n^2 - 1$

In the above equation, n can be either even or odd.

Let us assume that n= even.

So, when n = even i.e., n = 2k, where k is an integer,

We get,

$\Rightarrow x = (2k)^2-1$

$\Rightarrow x = 4k^2 - 1$

At k = -1, $x = 4(-1)^2 - 1 = 4 - 1 = 3$, is not divisible by 8.
At k = 0, $x = 4(0)^2 - 1 = 0 - 1 = -1$, is not divisible by 8
Let us assume that n= odd:
So, when n = odd i.e., n = 2k + 1, where k is an integer,
We get,
$\Rightarrow x = 2k + 1$
$\Rightarrow x = (2k+1)^2 - 1$
$\Rightarrow x = 4k^2 + 4k + 1 - 1$
$\Rightarrow x = 4k^2 + 4k$
$\Rightarrow x = 4k(k+1)$
At k = -1, x = 4(-1)(-1+1) = 0 which is divisible by 8.
At k = 0, x = 4(0)(0+1) = 0 which is divisible by 8 .
At k = 1, x = 4(1)(1+ 1) = 8 which is divisible by 8.

From the above two observation, we can conclude that, if n is odd, if n odd, n^2-1 is divisible by 8.
Hence, **option (C)** is the correct answer.

4. If the HCF of 65 and 117 is expressible in the form 65m – 117, then the value of m is
(A) 4 **(B) 2**
(C) 1 **(D) 3**

Solution:
(B) 2
Explanation:
Let us find the HCF of 65 and 117,

117 = 1×65 + 52

65 = 1× 52 + 13

52 = 4 ×13 + 0

Hence, we get the HCF of 65 and 117 = 13.

According to the question,

65m - 117 = 13

65m = 117+13 = 130

∴ m =130/65 = 2

Hence, **option (B)** is the correct answer.

5. The largest number which divides 70 and 125, leaving remainders 5 and 8, respectively, is
(A) 13 **(B) 65**
(C) 875 **(D) 1750**

Solution:
(A) 13
Explanation:
According to the question,
We have to find the largest number which divides 70 and 125, leaving remainders 5 and 8.

This can be also written as,
To find the largest number which exactly divides (70 - 5), and (125 - 8)
The largest number that divides 65 and 117 is also the Highest Common Factor of 65 and 117
Therefore, the required number is the HCF of 65 and 117

Factors of 65 = 1, 5, 13, 65
Factors of 117 = 1, 3, 9, 13, 39, 117
Common Factors = 1, 13
Highest Common factor (HCF) = 13
i.e., the largest number which divides 70 and 125, leaving remainders 5 and 8, respectively = 13
Hence, **option (A)** is the correct answer.

EXERCISE 1.2 PAGE NO: 4

1. Write whether every positive integer can be of the form 4q + 2, where q is an integer. Justify your answer.

Solution:

No, every positive integer cannot be of the form 4q + 2, where q is an integer.

Justification:

All the numbers of the form 4q + 2, where 'q' is an integer, are even numbers which are not divisible by '4'.

For example,

When q=1,

4q+2 = 4(1) + 2= 6.

When q=2,

4q+2 = 4(2) + 2= 10

When q=0,

4q+2 = 4(0) + 2= 2 and so on.

So, any number which is of the form 4q+2 will give only even numbers which are not multiples of 4.

Hence, every positive integer **cannot** be written in the form 4q+2

2. "The product of two consecutive positive integers is divisible by 2". Is this statement true or false? Give reasons.

Solution:

Yes, the statement "the product of two consecutive positive integers is divisible by 2" is true.

Justification:

Let the two consecutive positive integers = a, a + 1

According to Euclid's division lemma,

We have,

$a = bq + r$, where $0 \leq r < b$

For b = 2, we have $a = 2q + r$, where $0 \leq r < 2$ ... (i)

Substituting r = 0 in equation (i),

We get,

a = 2q, is divisible by 2.

a + 1 = 2q + 1, is not divisible by 2.

Substituting r = 1 in equation (i),

We get,

a = 2q + 1, is not divisible by 2.

a + 1 = 2q + 1+1 = 2q + 2, is divisible by 2.

Thus, we can conclude that, for $0 \leq r < 2$, one out of every two consecutive integers is divisible by 2. So, the product of the two consecutive positive numbers will also be even.

Hence, the statement "product of two consecutive positive integers is divisible by 2" is true.

3. "The product of three consecutive positive integers is divisible by 6". Is this statement true or false? Justify your answer.

Solution:

Yes, the statement "the product of three consecutive positive integers is divisible by 6" is true.

Justification:

Consider the 3 consecutive numbers 2, 3, 4

$(2 \times 3 \times 4)/6 = 24/6 = 4$

Now, consider another 3 consecutive numbers 4, 5, 6

$(4 \times 5 \times 6)/6 = 120/6 = 20$

Now, consider another 3 consecutive numbers 7, 8, 9

$(7 \times 8 \times 9)/6 = 504/6 = 84$

Hence, the statement "product of three consecutive positive integers is divisible by 6" is true.

4. Write whether the square of any positive integer can be of the form 3m + 2, where m is a natural number. Justify your answer.

Solution:

No, the square of any positive integer cannot be written in the form 3m + 2 where m is a natural number

Justification:

According to Euclid's division lemma,

A positive integer 'a' can be written in the form of bq + r

a = bq + r, where b, q and r are any integers,

For b = 3

a = 3(q) + r, where, r can be an integers,

For r = 0, 1, 2, 3..........

3q + 0, 3q + 1, 3q + 2, 3q + 3....... are positive integers,

$(3q)^2 = 9q^2 = 3(3q^2) = 3m$ (where $3q^2 = m$)

$(3q+1)^2 = (3q+1)^2 = 9q^2+1+6q = 3(3q^2+2q) +1 = 3m + 1$ (Where, $m = 3q^2+2q$)

$(3q+2)^2 = (3q+2)^2 = 9q^2+4+12q = 3(3q^2+4q) +4 = 3m + 4$ (Where, $m = 3q^2+2q$)

$(3q+3)^2 = (3q+3)^2 = 9q^2+9+18q = 3(3q^2+6q) +9 = 3m + 9$ (Where, $m = 3q^2+2q$)

Hence, there is no positive integer whose square can be written in the form 3m + 2 where m is a natural number.

5. A positive integer is of the form 3q + 1, q being a natural number. Can you write its square in any form other than 3m + 1, i.e., 3m or 3m + 2 for some integer m? Justify your answer.

Solution:

No.

Justification:

Consider the positive integer 3q + 1, where q is a natural number.

$(3q + 1)2 = 9q^2 + 6q + 1$

$= 3(3q^2 + 2q) + 1$

$= 3m + 1$, (where m is an integer which is equal to $3q^2 + 2q$.

Thus $(3q + 1)^2$ cannot be expressed in any other form apart from 3m + 1.

EXERCISE 1.3 PAGE NO: 6

1. Show that the square of any positive integer is either of the form 4q or 4q + 1 for some integer q.

Solution:

According to Euclid's division lemma,

$a=bq+r$

According to the question,

When b = 4.

$a = 4k + r, 0 \leq r < 4$

When r = 0, we get, a = 4k

$a^2 = 16k^2 = 4(4k^2) = 4q$, where $q = 4k^2$

When r = 1, we get, a = 4k + 1

$a^2 = (4k + 1)^2 = 16k^2 + 1 + 8k = 4(4k + 2) + 1 = 4q + 1$, where $q = k(4k + 2)$

When r = 2, we get, a = 4k + 2

$a^2 = (4k + 2)^2 = 16k^2 + 4 + 16k = 4(4k^2 + 4k + 1) = 4q$, where $q = 4k^2 + 4k + 1$

When r = 3, we get, a = 4k + 3

$a^2 = (4k + 3)^2 = 16k^2 + 9 + 24k = 4(4k^2 + 6k + 2) + 1$

$= 4q + 1$, where $q = 4k^2 + 6k + 2$

Therefore, the square of any positive integer is either of the form 4q or 4q + 1 for some integer q.

Hence Proved.

2. Show that cube of any positive integer is of the form 4m, 4m + 1 or 4m + 3, for some integer m.

Solution:

Let a be any positive integer and b = 4.

According to Euclid Division Lemma,

$a = bq + r \quad [0 \leq r < b]$

$a = 3q + r \quad [0 \leq r < 4]$

According to the question, the possible values of r are,

r = 0, r = 1, r = 2, r = 3

When r = 0,

$a = 4q + 0$

$a = 4q$

Taking cubes on LHS and RHS,

We have,

$a^3 = (4q)^3$

$a^3 = 4\ (16q^3)$

$a^3 = 4m \quad$ [where m is an integer = $16q^3$]

When r = 1,

$a = 4q + 1$

Taking cubes on LHS and RHS,

We have,

$a^3 = (4q + 1)^3$

$a^3 = 64q^3 + 1^3 + 3 \times 4q \times 1\ (4q + 1)$

$a^3 = 64q^3 + 1 + 48q^2 + 12q$

$a^3 = 4 (16q^3 + 12q^2 + 3q) + 1$
$a^3 = 4m + 1$ [where m is an integer $= 16q^3 + 12q^2 + 3q$]
When r = 2,
$a = 4q + 2$
Taking cubes on LHS and RHS,
We have,
$a^3 = (4q + 2)^3$
$a^3 = 64q^3 + 2^3 + 3 \times 4q \times 2 (4q + 2)$
$a^3 = 64q^3 + 8 + 96q^2 + 48q$
$a^3 = 4 (16q^3 + 2 + 24q^2 + 12q)$
$a^3 = 4m$ [where m is an integer $=16q^3 + 2 + 24q^2 + 12q$]
When r = 3,
$a = 4q + 3$
Taking cubes on LHS and RHS,
We have,
$a^3 = (4q + 3)^3$
$a^3 = 64q^3 + 27 + 3 \times 4q \times 3 (4q + 3)$
$a^3 = 64q^3 + 24 + 3 + 144q^2 + 108q$
$a^3 = 4 (16q^3 + 36q^2 + 27q + 6) + 3$
$a^3 = 4m + 3$ [where m is an integer $=16q^3 + 36q^2 + 27q + 6$]
Hence, the cube of any positive integer is in the form of 4m, 4m+1 or 4m+3.

3. Show that the square of any positive integer cannot be of the form 5q + 2 or 5q + 3 for any integer q.
Solution:
Let the positive integer = a
According to Euclid's division lemma,
$a = bm + r$
According to the question, b = 5
$a = 5m + r$
So, r= 0, 1, 2, 3, 4
When r = 0, a = 5m.
When r = 1, a = 5m + 1.
When r = 2, a = 5m + 2.
When r = 3, a = 5m + 3.
When r = 4, a = 5m + 4.
Now,
When a = 5m
$a^2 = (5m)^2 = 25m^2$
$a^2 = 5(5m^2) = 5q$, where $q = 5m^2$
When a = 5m + 1
$a^2 = (5m + 1)^2 = 25m^2 + 10 m + 1$
$a^2 = 5 (5m^2 + 2m) + 1 = 5q + 1$, where $q = 5m^2 + 2m$
When a = 5m + 2
$a^2 = (5m + 2)^2$
$a^2 = 25m^2 + 20m + 4$

$a^2 = 5(5m^2 + 4m) + 4$
$a^2 = 5q + 4$ where $q = 5m^2 + 4m$
When $a = 5m + 3$
$a^2 = (5m + 3)^2 = 25m^2 + 30m + 9$
$a^2 = 5(5m^2 + 6m + 1) + 4$
$a^2 = 5q + 4$ where $q = 5m^2 + 6m + 1$
When $a = 5m + 4$
$a^2 = (5m + 4)^2 = 25m^2 + 40m + 16$
$a^2 = 5(5m^2 + 8m + 3) + 1$
$a^2 = 5q + 1$ where $q = 5m^2 + 8m + 3$
Therefore, square of any positive integer cannot be of the form $5q + 2$ or $5q + 3$.
Hence Proved.

4. Show that the square of any positive integer cannot be of the form 6m + 2 or 6m + 5 for any integer m.
Solution:

Let the positive integer = a
According to Euclid's division algorithm,
$a = 6q + r$, where $0 \le r < 6$
$a^2 = (6q + r)^2 = 36q^2 + r^2 + 12qr$ [$\because (a+b)^2 = a^2 + 2ab + b^2$]
$a^2 = 6(6q^2 + 2qr) + r^2$ ...(i), where, $0 \le r < 6$
When r = 0, substituting r = 0 in Eq.(i), we get
$a^2 = 6(6q^2) = 6m$, where, $m = 6q^2$ is an integer.
When r = 1, substituting r = 1 in Eq.(i), we get
$a^2 = 6(6q^2 + 2q) + 1 = 6m + 1$, where, $m = (6q^2 + 2q)$ is an integer.
When r = 2, substituting r = 2 in Eq(i), we get
$a^2 = 6(6q^2 + 4q) + 4 = 6m + 4$, where, $m = (6q^2 + 4q)$ is an integer.
When r = 3, substituting r = 3 in Eq.(i), we get
$a^2 = 6(6q^2 + 6q) + 9 = 6(6q^2 + 6a) + 6 + 3$
$a^2 = 6(6q^2 + 6q + 1) + 3 = 6m + 3$, where, $m = (6q + 6q + 1)$ is integer.
When r = 4, substituting r = 4 in Eq.(i) we get
$a^2 = 6(6q^2 + 8q) + 16$
$= 6(6q^2 + 8q) + 12 + 4$
$\Rightarrow a^2 = 6(6q^2 + 8q + 2) + 4 = 6m + 4$, where, $m = (6q^2 + 8q + 2)$ is integer.
When r = 5, substituting r = 5 in Eq.(i), we get
$a^2 = 6(6q^2 + 10q) + 25 = 6(6q^2 + 10q) + 24 + 1$
$a^2 = 6(6q^2 + 10q + 4) + 1 = 6m + 1$, where, $m = (6q^2 + 10q + 4)$ is integer. Hence, the square of any positive integer cannot be of the form $6m + 2$ or $6m + 5$ for any integer m.

Hence Proved

5. Show that the square of any odd integer is of the form 4q + 1, for some integer q.
Solution:

Let a be any odd integer and b = 4.
According to Euclid's algorithm,
$a = 4m + r$ for some integer $m \ge 0$

And r = 0,1,2,3 because $0 \le r < 4$.
So, we have that,
a = 4m or 4m + 1 or 4m + 2 or 4m + 3 So, a = 4m + 1 or 4m + 3
We know that, a cannot be 4m or 4m + 2, as they are divisible by 2.

$(4m + 1)^2 = 16m^2 + 8m + 1$
$= 4(4m^2 + 2m) + 1$
$= 4q + 1$, where q is some integer and $q = 4m^2 + 2m$.
$(4m + 3)^2 = 16m^2 + 24m + 9$
$= 4(4m^2 + 6m + 2) + 1$
$= 4q + 1$, where q is some integer and $q = 4m^2 + 6m + 2$

Therefore, Square of any odd integer is of the form 4q + 1, for some integer q.
Hence Proved.

6. If n is an odd integer, then show that $n^2 – 1$ is divisible by 8.
Solution:

We know that any odd positive integer n can be written in form $4q + 1$ or $4q + 3$.
So, according to the question,
When $n = 4q + 1$,
Then $n^2 - 1 = (4q + 1)^2 - 1 = 16q^2 + 8q + 1 - 1 = 8q(2q + 1)$, is divisible by 8.
When $n = 4q + 3$,
Then $n^2 - 1 = (4q + 3)^2 - 1 = 16q^2 + 24q + 9 - 1 = 8(2q^2 + 3q + 1)$, is divisible by 8.
So, from the above equations, it is clear that, if n is an odd positive integer $n^2 - 1$ is divisible by 8.
Hence Proved.

7. Prove that if x and y are both odd positive integers, then $x^2 + y^2$ is even but not divisible by 4.
Solution:

Let the two odd positive numbers x and y be 2k + 1 and 2p + 1, respectively
i.e., $x^2 + y^2 = (2k + 1)^2 +(2p + 1)^2$
$= 4k^2 + 4k + 1 + 4p^2 + 4p + 1$
$= 4k^2 + 4p^2 + 4k + 4p + 2$
$= 4 (k^2 + p^2 + k + p) + 2$

Thus, the sum of square is even the number is not divisible by 4
Therefore, if x and y are odd positive integer, then $x^2 + y^2$ is even but not divisible by four.
Hence Proved

EXERCISE 1.4 PAGE NO: 7

1. Show that the cube of a positive integer of the form 6q + r, q is an integer and r = 0, 1, 2, 3, 4, 5 is also of the form 6m + r.

Solution:

6q + r is a positive integer, where q is an integer and r = 0, 1, 2, 3, 4, 5
Then, the positive integers are of the form 6q, 6q+1, 6q+2, 6q+3, 6q+4 and 6q+5.
Taking cube on L.H.S and R.H.S,
For 6q,
$(6q)^3 = 216\ q^3 = 6(36q)^3 + 0$
$= 6m + 0$, (where m is an integer $= (36q)^3$)
For 6q+1,
$(6q+1)^3 = 216q^3 + 108q^2 + 18q + 1$
$= 6(36q^3 + 18q^2 + 3q) + 1$
$= 6m + 1$, (where m is an integer $= 36q^3 + 18q^2 + 3q$)
For 6q+2,
$(6q+2)^3 = 216q^3 + 216q^2 + 72q + 8$
$= 6(36q^3 + 36q^2 + 12q + 1) +2$
$= 6m + 2$, (where m is an integer $= 36q^3 + 36q^2 + 12q + 1$)
For 6q+3,
$(6q+3)^3 = 216q^3 + 324q^2 + 162q + 27$
$= 6(36q^3 + 54q^2 + 27q + 4) + 3$
$= 6m + 3$, (where m is an integer $= 36q^3 + 54q^2 + 27q + 4$)
For 6q+4,
$(6q+4)^3 = 216q^3 + 432q^2 + 288q + 64$
$= 6(36q^3 + 72q^2 + 48q + 10) + 4$
$= 6m + 4$, (where m is an integer $= 36q^3 + 72q^2 + 48q + 10$)
For 6q+5,
$(6q+5)^3 = 216q^3 + 540q^2 + 450q + 125$
$= 6(36q^3 + 90q^2 + 75q + 20) + 5$
$= 6m + 5$, (where m is an integer $= 36q^3 + 90q^2 + 75q + 20$)
Hence, the cube of a positive integer of the form 6q + r, q is an integer and r = 0, 1, 2, 3, 4, 5 is also of the form 6m + r.

2. Prove that one and only one out of n, n + 2 and n + 4 is divisible by 3, where n is any positive integer.

Solution:

According to Euclid's division Lemma,
Let the positive integer = n
And b=3
n =3q+r, where q is the quotient and r is the remainder
$0 \leq r < 3$ implies remainders may be 0, 1 and 2
Therefore, n may be in the form of 3q, 3q+1, 3q+2
When n=3q
n+2=3q+2

$n+4=3q+4$
Here n is only divisible by 3
When $n = 3q+1$
$n+2=3q=3$
$n+4=3q+5$
Here only $n+2$ is divisible by 3
When $n=3q+2$
$n+2=3q+4$
$n+4=3q+2+4=3q+6$
Here only $n+4$ is divisible by 3
So, we can conclude that one and only one out of n, $n + 2$ and $n + 4$ is divisible by 3.
Hence Proved