Question 1.1:

What is the force between two small charged spheres having charges of 2×10^{-7} C and 3×10^{-7} C placed 30 cm apart in air?

Answer

Repulsive force of magnitude 6×10^{-3} N

Charge on the first sphere, $q_1 = 2 \times 10^{-7}$ C

Charge on the second sphere, $q_2 = 3 \times 10^{-7}$ C

Distance between the spheres, $r = 30$ cm $= 0.3$ m

Electrostatic force between the spheres is given by the relation,

$$F = \frac{q_1 q_2}{4\pi \in_0 r^2}$$

Where, $\in_0$ = Permittivity of free space

$$\frac{1}{4\pi \in_0} = 9 \times 10^9 \ \mathrm{N\,m^2\,C^{-2}}$$

$$F = \frac{9 \times 10^9 \times 2 \times 10^{-7} \times 3 \times 10^{-7}}{(0.3)^2} = 6 \times 10^{-3}\ \mathrm{N}$$

Hence, force between the two small charged spheres is 6×10^{-3} N. The charges are of same nature. Hence, force between them will be repulsive.

Question 1.2:

The electrostatic force on a small sphere of charge 0.4 μC due to another small sphere of charge − 0.8 μC in air is 0.2 N. (a) What is the distance between the two spheres? (b) What is the force on the second sphere due to the first?

Answer

Electrostatic force on the first sphere, $F = 0.2$ N

Charge on this sphere, $q_1 = 0.4\ \mu\text{C} = 0.4 \times 10^{-6}$ C

Charge on the second sphere, $q_2 = -0.8\ \mu\text{C} = -0.8 \times 10^{-6}$ C

Electrostatic force between the spheres is given by the relation,

$$F = \frac{q_1 q_2}{4\pi \in_0 r^2}$$

Where, $\in_0$ = Permittivity of free space

$$\text{And, } \frac{1}{4\pi \in_0} = 9 \times 10^9\ \text{N m}^2\ \text{C}^{-2}$$

$$\begin{aligned} r^2 &= \frac{q_1 q_2}{4\pi \in_0 F} \\ &= \frac{0.4 \times 10^{-6} \times 8 \times 10^{-6} \times 9 \times 10^9}{0.2} \\ &= 144 \times 10^{-4} \end{aligned}$$

$$r = \sqrt{144 \times 10^{-4}} = 0.12\ \text{m}$$

The distance between the two spheres is 0.12 m.

Both the spheres attract each other with the same force. Therefore, the force on the second sphere due to the first is 0.2 N.

Question 1.3:

Check that the ratio $ke^2/G\, m_e m_p$ is dimensionless. Look up a Table of Physical Constants and determine the value of this ratio. What does the ratio signify?

Answer

The given ratio is $\frac{ke^2}{Gm_e m_p}$.

Where,

G = Gravitational constant

Its unit is N m^2 kg^{-2}.

m_e and m_p = Masses of electron and proton.

Their unit is kg.

e = Electric charge.

Its unit is C.

$$k = \text{A constant}$$
$$= \frac{1}{4\pi \in_0}$$

$\in_0$ = Permittivity of free space

Its unit is N m^2 C^{-2}.

$$\text{Therefore, unit of the given ratio } \frac{ke^2}{Gm_e m_p} = \frac{\left[\text{Nm}^2\ \text{C}^{-2}\right]\left[\text{C}^{-2}\right]}{\left[\text{Nm}^2\ \text{kg}^{-2}\right]\left[\text{kg}\right]\left[\text{kg}\right]}$$
$$= \text{M}^0\text{L}^0\text{T}^0$$

Hence, the given ratio is dimensionless.

$e = 1.6 \times 10^{-19}$ C

G = 6.67×10^{-11} N m^2 kg^{-2}

$m_e = 9.1 \times 10^{-31}$ kg

$m_p = 1.66 \times 10^{-27}$ kg

Hence, the numerical value of the given ratio is

$$\frac{ke^2}{Gm_e m_p} = \frac{9\times10^9\times\left(1.6\times10^{-19}\right)^2}{6.67\times10^{-11}\times9.1\times10^{-3}\times1.67\times10^{-22}} \approx 2.3\times10^{39}$$

This is the ratio of electric force to the gravitational force between a proton and an electron, keeping distance between them constant.

Question 1.4:

Explain the meaning of the statement 'electric charge of a body is quantised'.

Why can one ignore quantisation of electric charge when dealing with macroscopic i.e., large scale charges?

Answer

Electric charge of a body is quantized. This means that only integral (1, 2, …., n) number of electrons can be transferred from one body to the other. Charges are not transferred in fraction. Hence, a body possesses total charge only in integral multiples of electric charge.

In macroscopic or large scale charges, the charges used are huge as compared to the magnitude of electric charge. Hence, quantization of electric charge is of no use on macroscopic scale. Therefore, it is ignored and it is considered that electric charge is continuous.

Question 1.5:

When a glass rod is rubbed with a silk cloth, charges appear on both. A similar phenomenon is observed with many other pairs of bodies. Explain how this observation is consistent with the law of conservation of charge.

Answer

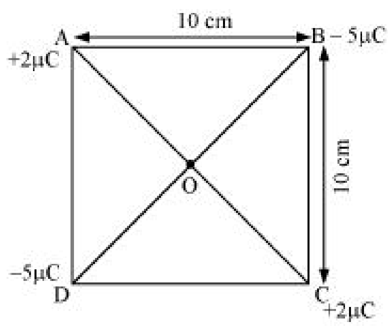
Rubbing produces charges of equal magnitude but of opposite nature on the two bodies because charges are created in pairs. This phenomenon of charging is called charging by friction. The net charge on the system of two rubbed bodies is zero. This is because equal amount of opposite charges annihilate each other. When a glass rod is rubbed with a silk cloth, opposite natured charges appear on both the bodies. This phenomenon is in consistence with the law of conservation of energy. A similar phenomenon is observed with many other pairs of bodies.

Question 1.6:

Four point charges $q_A = 2$ µC, $q_B = −5$ µC, $q_C = 2$ µC, and $q_D = −5$ µC are located at the corners of a square ABCD of side 10 cm. What is the force on a charge of 1 µC placed at the centre of the square?

Answer

The given figure shows a square of side 10 cm with four charges placed at its corners. O is the centre of the square.



Where,

(Sides) AB = BC = CD = AD = 10 cm

(Diagonals) AC = BD = $10\sqrt{2}$ cm

AO = OC = DO = OB = $5\sqrt{2}$ cm

A charge of amount 1µC is placed at point O.

Force of repulsion between charges placed at corner A and centre O is equal in magnitude but opposite in direction relative to the force of repulsion between the charges placed at corner C and centre O. Hence, they will cancel each other. Similarly, force of attraction between charges placed at corner B and centre O is equal in magnitude but opposite in direction relative to the force of attraction between the charges placed at corner D and centre O. Hence, they will also cancel each other. Therefore, net force caused by the four charges placed at the corner of the square on 1 µC charge at centre O is zero.

Question 1.7:

An electrostatic field line is a continuous curve. That is, a field line cannot have sudden breaks. Why not?

Explain why two field lines never cross each other at any point?

Answer

An electrostatic field line is a continuous curve because a charge experiences a continuous force when traced in an electrostatic field. The field line cannot have sudden breaks because the charge moves continuously and does not jump from one point to the other.

If two field lines cross each other at a point, then electric field intensity will show two directions at that point. This is not possible. Hence, two field lines never cross each other.

Question 1.8:

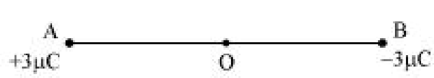
Two point charges $q_A = 3$ μC and $q_B = -3$ μC are located 20 cm apart in vacuum.

What is the electric field at the midpoint O of the line AB joining the two charges?

If a negative test charge of magnitude 1.5×10^{-9} C is placed at this point, what is the force experienced by the test charge?

Answer

The situation is represented in the given figure. O is the mid-point of line AB.



Distance between the two charges, AB = 20 cm

$\therefore$AO = OB = 10 cm

Net electric field at point O = E

Electric field at point O caused by +3μC charge,

$E_1 = \dfrac{3\times10^{-6}}{4\pi\epsilon_0(\text{AO})^2} = \dfrac{3\times10^{-6}}{4\pi\epsilon_0\left(10\times10^{-2}\right)^2}$ N/C along OB

Where,

ϵ_0 = Permittivity of free space

$$\frac{1}{4\pi\epsilon_0} = 9\times10^{9}\ \text{N m}^2\ \text{C}^{-2}$$

Magnitude of electric field at point O caused by −3μC charge,

$E_2 = \left|\dfrac{-3\times10^{-6}}{4\pi\epsilon_0(\text{OB})^2}\right| = \dfrac{3\times10^{-6}}{4\pi\epsilon_0\left(10\times10^{-2}\right)^2}$ N/C along OB

$$\therefore E = E_1 + E_2$$

$$= 2\times\left[\left(9\times10^{9}\right)\times\frac{3\times10^{-6}}{\left(10\times10^{-2}\right)^2}\right]$$ [Since the values of E_1 and E_2 are same, the value is multiplied with 2]

= 5.4×10^{6} N/C along OB

Therefore, the electric field at mid-point O is 5.4×10^{6} N C^{-1} along OB.

A test charge of amount 1.5×10^{-9} C is placed at mid-point O.

$q = 1.5 \times 10^{-9}$ C

Force experienced by the test charge = F

$\therefore F = qE$

$= 1.5 \times 10^{-9} \times 5.4 \times 10^{6}$

$= 8.1 \times 10^{-3}$ N

The force is directed along line OA. This is because the negative test charge is repelled by the charge placed at point B but attracted towards point A.

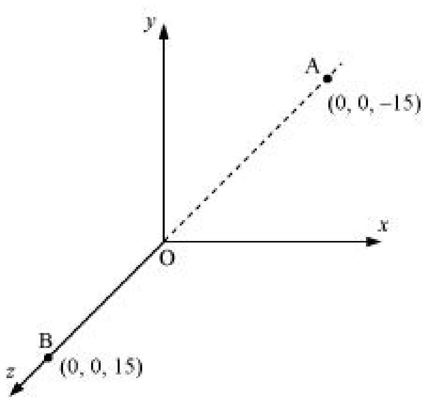
Therefore, the force experienced by the test charge is 8.1×10^{-3} N along OA.

Question 1.9:

A system has two charges $q_A = 2.5 \times 10^{-7}$ C and $q_B = -2.5 \times 10^{-7}$ C located at points A: (0, 0, − 15 cm) and B: (0, 0, + 15 cm), respectively. What are the total charge and electric dipole moment of the system?

Answer

Both the charges can be located in a coordinate frame of reference as shown in the given figure.



At A, amount of charge, $q_A = 2.5 \times 10^{-7}$C

At B, amount of charge, $q_B = -2.5 \times 10^{-7}$ C

Total charge of the system,

$q = q_A + q_B$

$= 2.5 \times 10^{7}$ C $- 2.5 \times 10^{-7}$ C

$= 0$

Distance between two charges at points A and B,

$d = 15 + 15 = 30$ cm $= 0.3$ m

Electric dipole moment of the system is given by,

$p = q_A \times d = q_B \times d$

$= 2.5 \times 10^{-7} \times 0.3$

$= 7.5 \times 10^{-8}$ C m along positive z-axis

Therefore, the electric dipole moment of the system is 7.5×10^{-8} C m along positive z–axis.

Question 1.10:

An electric dipole with dipole moment 4×10^{-9} C m is aligned at 30° with the direction of a uniform electric field of magnitude 5×10^{4} N C^{-1}. Calculate the magnitude of the torque acting on the dipole.

Answer

Electric dipole moment, $p = 4 \times 10^{-9}$ C m

Angle made by p with a uniform electric field, $\theta = 30°$

Electric field, $E = 5 \times 10^{4}$ N C^{-1}

Torque acting on the dipole is given by the relation,

$\tau = pE \sin\theta$

$$= 4\times10^{-9}\times5\times10^{4}\times\sin 30$$
$$= 20\times10^{-5}\times\frac{1}{2}$$
$$= 10^{-4}\ \text{N m}$$

Therefore, the magnitude of the torque acting on the dipole is 10^{-4} N m.

Question 1.11:

A polythene piece rubbed with wool is found to have a negative charge of 3×10^{-7} C.

Estimate the number of electrons transferred (from which to which?)

Is there a transfer of mass from wool to polythene?

Answer

When polythene is rubbed against wool, a number of electrons get transferred from wool to polythene. Hence, wool becomes positively charged and polythene becomes negatively charged.

Amount of charge on the polythene piece, $q = -3 \times 10^{-7}$ C

Amount of charge on an electron, $e = -1.6 \times 10^{-19}$ C

Number of electrons transferred from wool to polythene = n

n can be calculated using the relation,

$q = ne$

$$n = \frac{q}{e}$$

$$= \frac{-3 \times 10^{-7}}{-1.6 \times 10^{-19}}$$

$= 1.87 \times 10^{12}$

Therefore, the number of electrons transferred from wool to polythene is 1.87×10^{12}.

Yes.

There is a transfer of mass taking place. This is because an electron has mass,

$m_e = 9.1 \times 10^{-3}$ kg

Total mass transferred to polythene from wool,

$m = m_e \times n$

$= 9.1 \times 10^{-31} \times 1.85 \times 10^{12}$

$= 1.706 \times 10^{-18}$ kg

Hence, a negligible amount of mass is transferred from wool to polythene.

Question 1.12:

Two insulated charged copper spheres A and B have their centers separated by a distance of 50 cm. What is the mutual force of electrostatic repulsion if the charge on each is 6.5×10^{-7} C? The radii of A and B are negligible compared to the distance of separation.

What is the force of repulsion if each sphere is charged double the above amount, and the distance between them is halved?

Answer

Charge on sphere A, q_A = Charge on sphere B, $q_B = 6.5 \times 10^{-7}$ C

Distance between the spheres, r = 50 cm = 0.5 m

Force of repulsion between the two spheres,

$$F = \frac{q_A q_B}{4\pi \epsilon_0 r^2}$$

Where,

ϵ_0 = Free space permittivity

$$\frac{1}{4\pi \epsilon_0} = 9 \times 10^9 \text{ N m}^2 \text{ C}^{-2}$$

$$\therefore F = \frac{9 \times 10^9 \times \left(6.5 \times 10^{-7}\right)^2}{(0.5)^2}$$

$= 1.52 \times 10^{-2}$ N

Therefore, the force between the two spheres is 1.52×10^{-2} N.

After doubling the charge, charge on sphere A, q_A = Charge on sphere B, $q_B = 2 \times 6.5 \times 10^{-7}$ C $= 1.3 \times 10^{-6}$ C

The distance between the spheres is halved.

$\therefore \quad r = \frac{0.5}{2} = 0.25 \text{ m}$

Force of repulsion between the two spheres,

$$F = \frac{q_A q_B}{4\pi \epsilon_0 r^2}$$

$$= \frac{9\times10^9 \times 1.3\times10^{-6} \times 1.3\times10^{-6}}{(0.25)^2}$$

$= 16 \times 1.52 \times 10^{-2}$

$= 0.243$ N

Therefore, the force between the two spheres is 0.243 N.

Question 1.13:

Suppose the spheres A and B in Exercise 1.12 have identical sizes. A third sphere of the same size but uncharged is brought in contact with the first, then brought in contact with the second, and finally removed from both. What is the new force of repulsion between A and B?

Answer

Distance between the spheres, A and B, $r = 0.5$ m

Initially, the charge on each sphere, $q = 6.5 \times 10^{-7}$ C

When sphere A is touched with an uncharged sphere C, $\frac{q}{2}$ amount of charge from A will transfer to sphere C. Hence, charge on each of the spheres, A and C, is $\frac{q}{2}$.

When sphere C with charge $\frac{q}{2}$ is brought in contact with sphere B with charge q, total charges on the system will divide into two equal halves given as,

$$\frac{\frac{q}{2}+q}{2}=\frac{3q}{4}$$

Each sphere will each half. Hence, charge on each of the spheres, C and B, is $\frac{3q}{4}$.

Force of repulsion between sphere A having charge $\frac{q}{2}$ and sphere B having charge $\frac{3q}{4}$ =

$$\frac{\frac{q}{2}\times\frac{3q}{4}}{4\pi\in_0 r^2}=\frac{3q^2}{8\times 4\pi\in_0 r^2}$$

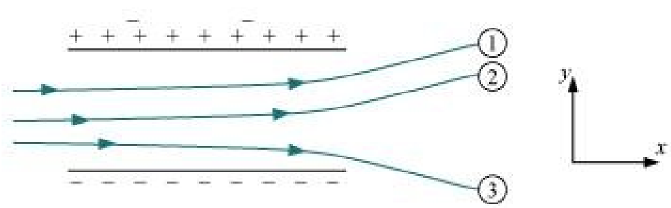
$$=9\times10^9\times\frac{3\times\left(6.5\times10^{-7}\right)^2}{8\times\left(0.5\right)^2}$$

$$=5.703\ \times\ 10^{-3}\ \text{N}$$

Therefore, the force of attraction between the two spheres is 5.703 × 10^{-3} N.

Question 1.14:

Figure 1.33 shows tracks of three charged particles in a uniform electrostatic field. Give the signs of the three charges. Which particle has the highest charge to mass ratio?



Answer

Opposite charges attract each other and same charges repel each other. It can be observed that particles 1 and 2 both move towards the positively charged plate and repel away from the negatively charged plate. Hence, these two particles are negatively charged. It can also be observed that particle 3 moves towards the negatively charged plate and repels away from the positively charged plate. Hence, particle 3 is positively charged.

The charge to mass ratio (emf) is directly proportional to the displacement or amount of deflection for a given velocity. Since the deflection of particle 3 is the maximum, it has the highest charge to mass ratio.

Question 1.15:

Consider a uniform electric field **E** = 3×10^3 îN/C. (a) What is the flux of this field through a square of 10 cm on a side whose plane is parallel to the *yz* plane? (b) What is the flux through the same square if the normal to its plane makes a 60° angle with the *x*-axis?

Answer

Electric field intensity, $\vec{E} = 3 \times 10^3 \hat{i}$ N/C

Magnitude of electric field intensity, $\left|\vec{E}\right| = 3 \times 10^3$ N/C

Side of the square, s = 10 cm = 0.1 m

Area of the square, $A = s^2 = 0.01 \text{ m}^2$

The plane of the square is parallel to the *y*-*z* plane. Hence, angle between the unit vector normal to the plane and electric field, $\theta = 0°$

Flux (Φ) through the plane is given by the relation,

$\Phi = \left|\vec{E}\right| A \cos\theta$

$= 3 \times 10^3 \times 0.01 \times \cos 0°$

$= 30 \text{ N m}^2/\text{C}$

Plane makes an angle of 60° with the *x*-axis. Hence, $\theta = 60°$

Flux, $\Phi = \left|\vec{E}\right| A \cos\theta$

$= 3 \times 10^3 \times 0.01 \times \cos 60°$

$= 30 \times \frac{1}{2} = 15$ N m^2/C

Question 1.16:

What is the net flux of the uniform electric field of Exercise 1.15 through a cube of side 20 cm oriented so that its faces are parallel to the coordinate planes?

Answer

All the faces of a cube are parallel to the coordinate axes. Therefore, the number of field lines entering the cube is equal to the number of field lines piercing out of the cube. As a result, net flux through the cube is zero.

Question 1.17:

Careful measurement of the electric field at the surface of a black box indicates that the net outward flux through the surface of the box is 8.0×10^3 N m^2/C. (a) What is the net charge inside the box? (b) If the net outward flux through the surface of the box were zero, could you conclude that there were no charges inside the box? Why or Why not?

Answer

Net outward flux through the surface of the box, $\Phi = 8.0 \times 10^3$ N m^2/C

For a body containing net charge q, flux is given by the relation,

$$\phi = \frac{q}{\epsilon_0}$$

ϵ_0 = Permittivity of free space

$= 8.854 \times 10^{-12}$ N^{-1}C^2 m^{-2}

$q = \epsilon_0 \Phi$

$= 8.854 \times 10^{-12} \times 8.0 \times 10^{3}$

$= 7.08 \times 10^{-8}$

$= 0.07$ μC

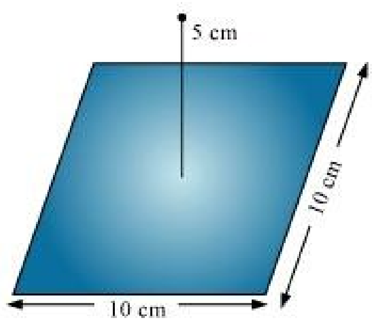
Therefore, the net charge inside the box is 0.07 μC.

No

Net flux piercing out through a body depends on the net charge contained in the body. If net flux is zero, then it can be inferred that net charge inside the body is zero. The body may have equal amount of positive and negative charges.

Question 1.18:

A point charge +10 μC is a distance 5 cm directly above the centre of a square of side 10 cm, as shown in Fig. 1.34. What is the magnitude of the electric flux through the square? (*Hint:* Think of the square as one face of a cube with edge 10 cm.)



Answer

The square can be considered as one face of a cube of edge 10 cm with a centre where charge q is placed. According to Gauss's theorem for a cube, total electric flux is through all its six faces.

$$\phi_{\text{Total}} = \frac{q}{\epsilon_0}$$

Hence, electric flux through one face of the cube i.e., through the square, $\phi = \dfrac{\phi_{\text{Total}}}{6}$

$$= \frac{1}{6}\frac{q}{\epsilon_0}$$

Where,

ϵ_0 = Permittivity of free space

$= 8.854 \times 10^{-12}\ \mathrm{N^{-1}C^2\,m^{-2}}$

$q = 10\ \mu\mathrm{C} = 10 \times 10^{-6}\ \mathrm{C}$

$$\therefore \phi = \frac{1}{6} \times \frac{10\times10^{-6}}{8.854\times10^{-12}}$$

$= 1.88 \times 10^{5}\ \mathrm{N\,m^2\,C^{-1}}$

Therefore, electric flux through the square is $1.88 \times 10^{5}\ \mathrm{N\,m^2\,C^{-1}}$.

Question 1.19:

A point charge of 2.0 μC is at the centre of a cubic Gaussian surface 9.0 cm on edge. What is the net electric flux through the surface?

Answer

Net electric flux (Φ_{Net}) through the cubic surface is given by,

$$\phi_{\text{Net}} = \frac{q}{\epsilon_0}$$

Where,

ϵ_0 = Permittivity of free space

$= 8.854 \times 10^{-12}\ \mathrm{N^{-1}C^2\,m^{-2}}$

q = Net charge contained inside the cube = 2.0 μC = 2×10^{-6} C

$\therefore \phi_{Net} = \frac{2 \times 10^{-6}}{8.854 \times 10^{-12}}$

$= 2.26 \times 10^5$ N m^2 C^{-1}

The net electric flux through the surface is 2.26×10^5 N m^{2}C^{-1}.

Question 1.20:

A point charge causes an electric flux of -1.0×10^3 Nm^2/C to pass through a spherical Gaussian surface of 10.0 cm radius centered on the charge. (a) If the radius of the Gaussian surface were doubled, how much flux would pass through the surface? (b) What is the value of the point charge?

Answer

Electric flux, $\Phi = -1.0 \times 10^3$ N m^2/C

Radius of the Gaussian surface,

$r = 10.0$ cm

Electric flux piercing out through a surface depends on the net charge enclosed inside a body. It does not depend on the size of the body. If the radius of the Gaussian surface is doubled, then the flux passing through the surface remains the same i.e., -10^3 N m^2/C.

Electric flux is given by the relation,

$$\phi = \frac{q}{\epsilon_0}$$

Where,

q = Net charge enclosed by the spherical surface

ϵ_0 = Permittivity of free space = 8.854×10^{-12} N^{-1}C^2 m^{-2}

$\therefore q = \phi \epsilon_0$

$= -1.0 \times 10^3 \times 8.854 \times 10^{-12}$

$= -8.854 \times 10^{-9}$ C

$= -8.854$ nC

Therefore, the value of the point charge is −8.854 nC.

Question 1.21:

A conducting sphere of radius 10 cm has an unknown charge. If the electric field 20 cm from the centre of the sphere is 1.5×10^3 N/C and points radially inward, what is the net charge on the sphere?

Answer

Electric field intensity (*E*) at a distance (*d*) from the centre of a sphere containing net charge *q* is given by the relation,

$$E = \frac{q}{4\pi \epsilon_0 d^2}$$

Where,

q = Net charge = 1.5×10^3 N/C

d = Distance from the centre = 20 cm = 0.2 m

ϵ_0 = Permittivity of free space

And, $\frac{1}{4\pi \epsilon_0} = 9 \times 10^9$ N m^2 C^{-2}

$\therefore q = E\left(4\pi \epsilon_0\right)d^2$

$$= \frac{1.5 \times 10^3 \times (0.2)^2}{9 \times 10^9}$$

$= 6.67 \times 10^9$ C

$= 6.67$ nC

Therefore, the net charge on the sphere is 6.67 nC.

Question 1.22:

A uniformly charged conducting sphere of 2.4 m diameter has a surface charge density of 80.0 µC/m^2. (a) Find the charge on the sphere. (b) What is the total electric flux leaving the surface of the sphere?

Answer

Diameter of the sphere, d = 2.4 m

Radius of the sphere, r = 1.2 m

Surface charge density, σ = 80.0 µC/m^2 = 80 × 10^{-6} C/m^2

Total charge on the surface of the sphere,

Q = Charge density × Surface area

$= \sigma \times 4\pi r^2$

= 80 × 10^{-6} × 4 × 3.14 × (1.2)^2

= 1.447 × 10^{-3} C

Therefore, the charge on the sphere is 1.447 × 10^{-3} C.

Total electric flux (ϕ_{Total}) leaving out the surface of a sphere containing net charge Q is given by the relation,

$$\phi_{\text{TOtal}} = \frac{Q}{\epsilon_0}$$

Where,

ϵ_0 = Permittivity of free space

= 8.854 × 10^{-12} N^{-1}C^2 m^{-2}

$Q = 1.447 \times 10^{-3}$ C

$$\phi_{\text{Total}} = \frac{1.44 \times 10^{-3}}{8.854 \times 10^{-12}}$$

$= 1.63 \times 10^{8}$ N C^{-1} m^{2}

Therefore, the total electric flux leaving the surface of the sphere is 1.63×10^{8} N C^{-1} m^{2}.

Question 1.23:

An infinite line charge produces a field of 9×10^{4} N/C at a distance of 2 cm. Calculate the linear charge density.

Answer

Electric field produced by the infinite line charges at a distance *d* having linear charge density λ is given by the relation,

$$E = \frac{\lambda}{2\pi \in_0 d}$$

$$\lambda = 2\pi \in_0 dE$$

Where,

$d = 2$ cm $= 0.02$ m

$E = 9 \times 10^{4}$ N/C

$\in_0$ = Permittivity of free space

$$\frac{1}{4\pi \in_0} = 9 \times 10^{9} \text{ N m}^2 \text{ C}^{-2}$$

$$\lambda = \frac{0.02 \times 9 \times 10^{4}}{2 \times 9 \times 10^{9}}$$

$= 10$ μC/m

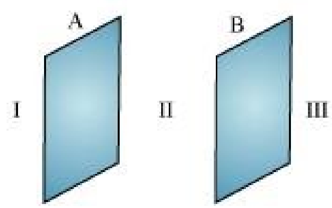
Therefore, the linear charge density is 10 μC/m.

Question 1.24:

Two large, thin metal plates are parallel and close to each other. On their inner faces, the plates have surface charge densities of opposite signs and of magnitude 17.0×10^{-22} C/m^2. What is **E**: (a) in the outer region of the first plate, (b) in the outer region of the second plate, and (c) between the plates?

<u>Answer</u>

The situation is represented in the following figure.



A and B are two parallel plates close to each other. Outer region of plate A is labelled as **I**, outer region of plate B is labelled as **III**, and the region between the plates, A and B, is labelled as **II.**

Charge density of plate A, $\sigma = 17.0 \times 10^{-22}$ C/m^2

Charge density of plate B, $\sigma = -17.0 \times 10^{-22}$ C/m^2

In the regions, **I** and **III**, electric field E is zero. This is because charge is not enclosed by the respective plates.

Electric field E in region **II** is given by the relation,

$$E = \frac{\sigma}{\epsilon_0}$$

Where,

ϵ_0 = Permittivity of free space = 8.854×10^{-12} N^{-1}C^2 m^{-2}

$\therefore \quad E = \frac{17.0\times10^{-22}}{8.854\times10^{-12}}$

$= 1.92 \times 10^{-10}$ N/C

Therefore, electric field between the plates is 1.92×10^{-10} N/C.

Question 1.25:

An oil drop of 12 excess electrons is held stationary under a constant electric field of 2.55×10^4 N C^{-1} in Millikan's oil drop experiment. The density of the oil is 1.26 g cm^{-3}. Estimate the radius of the drop. ($g = 9.81$ m s^{-2}; $e = 1.60 \times 10^{-19}$ C).

Answer

Excess electrons on an oil drop, $n = 12$

Electric field intensity, $E = 2.55 \times 10^4$ N C^{-1}

Density of oil, $\rho = 1.26$ gm/cm^3 = 1.26×10^3 kg/m^3

Acceleration due to gravity, g = 9.81 m s^{-2}

Charge on an electron, $e = 1.6 \times 10^{-19}$ C

Radius of the oil drop = r

Force (F) due to electric field E is equal to the weight of the oil drop (W)

$F = W$

$Eq = m$g

$Ene = \frac{4}{3}\pi r^3 \times \rho \times \mathrm{g}$

Where,

q = Net charge on the oil drop = ne

m = Mass of the oil drop

= Volume of the oil drop × Density of oil

$$= \frac{4}{3}\pi r^3 \times \rho$$

$$\therefore r = \sqrt[3]{\frac{3Ene}{4\pi\rho g}}$$

$$= \sqrt[3]{\frac{3\times 2.55\times 10^4 \times 12\times 1.6\times 10^{-19}}{4\times 3.14\times 1.26\times 10^3 \times 9.81}}$$

$$= \sqrt[3]{946.09\times 10^{-21}}$$

$$= 9.82\times 10^{-7} \text{ m}$$

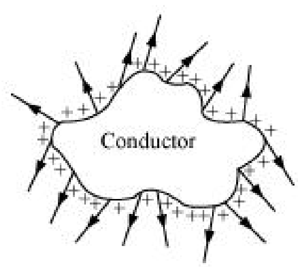
= 9.82×10^{-4} mm

Therefore, the radius of the oil drop is 9.82×10^{-4} mm.

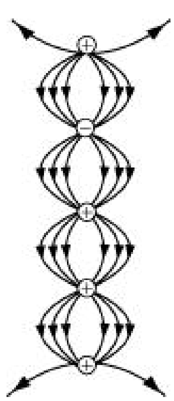
Question 1.26:

Which among the curves shown in Fig. 1.35 cannot possibly represent electrostatic field lines?

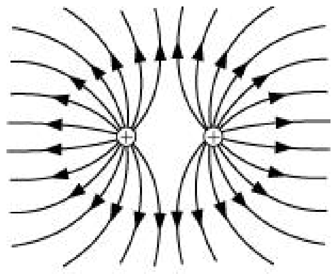
(a)



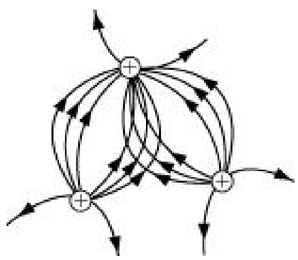
(b)



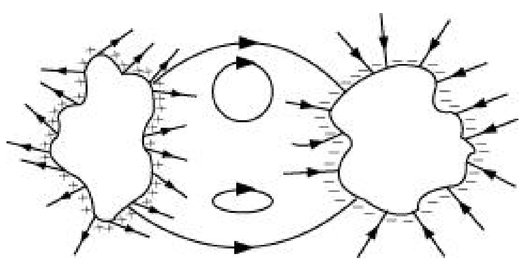
(c)



(d)



(e)



Answer

The field lines showed in (a) do not represent electrostatic field lines because field lines must be normal to the surface of the conductor.

The field lines showed in (b) do not represent electrostatic field lines because the field lines cannot emerge from a negative charge and cannot terminate at a positive charge.

The field lines showed in (c) represent electrostatic field lines. This is because the field lines emerge from the positive charges and repel each other.

The field lines showed in (d) do not represent electrostatic field lines because the field lines should not intersect each other.

The field lines showed in (e) do not represent electrostatic field lines because closed loops are not formed in the area between the field lines.

Question 1.27:

In a certain region of space, electric field is along the z-direction throughout. The magnitude of electric field is, however, not constant but increases uniformly along the positive *z*-direction, at the rate of 10^5 NC^{-1} per metre. What are the force and torque experienced by a system having a total dipole moment equal to 10^{-7} Cm in the negative *z*-direction?

Answer

Dipole moment of the system, $p = q \times dl = -10^{-7}$ C m

Rate of increase of electric field per unit length,

$$\frac{dE}{dl} = 10^{+5} \text{ N C}^{-1}$$

Force (F) experienced by the system is given by the relation,

$F = qE$

$$F = q\frac{dE}{dl} \times dl$$

$$= p \times \frac{dE}{dl}$$

$= -10^{-7} \times 10^{-5}$

$= -10^{-2}$ N

The force is -10^{-2} N in the negative z-direction i.e., opposite to the direction of electric field. Hence, the angle between electric field and dipole moment is 180°.

Torque (τ) is given by the relation,

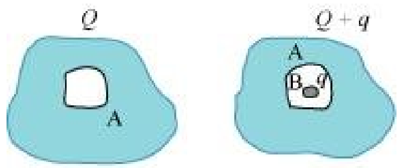
$\tau = pE \sin 180°$

$= 0$

Therefore, the torque experienced by the system is zero.

Question 1.28:

A conductor A with a cavity as shown in Fig. 1.36(a) is given a charge Q. Show that the entire charge must appear on the outer surface of the conductor. (b) Another conductor B with charge q is inserted into the cavity keeping B insulated from A. Show that the total charge on the outside surface of A is $Q + q$ [Fig. 1.36(b)]. (c) A sensitive instrument is to be shielded from the strong electrostatic fields in its environment. Suggest a possible way.



(a) (b)

Answer

Let us consider a Gaussian surface that is lying wholly within a conductor and enclosing the cavity. The electric field intensity E inside the charged conductor is zero.

Let q is the charge inside the conductor and $\in_0$ is the permittivity of free space.

According to Gauss's law,

Flux, $\phi = \overrightarrow{E}.\overrightarrow{ds} = \dfrac{q}{\in_0}$

Here, $E = 0$

$\dfrac{q}{\in_0} = 0$

$\because \in_0 \neq 0$

$\therefore q = 0$

Therefore, charge inside the conductor is zero.

The entire charge Q appears on the outer surface of the conductor.

The outer surface of conductor A has a charge of amount Q. Another conductor B having charge $+q$ is kept inside conductor A and it is insulated from A. Hence, a charge of amount $-q$ will be induced in the inner surface of conductor A and $+q$ is induced on the outer surface of conductor A. Therefore, total charge on the outer surface of conductor A is $Q + q$.

A sensitive instrument can be shielded from the strong electrostatic field in its environment by enclosing it fully inside a metallic surface. A closed metallic body acts as an electrostatic shield.

Question 1.29:

A hollow charged conductor has a tiny hole cut into its surface. Show that the electric field in the hole is $\left(\frac{\sigma}{2\epsilon_0}\right)\hat{n}$, where $\hat{n}$ is the unit vector in the outward normal direction, and σ is the surface charge density near the hole.

Answer

Let us consider a conductor with a cavity or a hole. Electric field inside the cavity is zero.

Let E is the electric field just outside the conductor, q is the electric charge, σ is the charge density, and ϵ_0 is the permittivity of free space.

Charge $|q| = \vec{\sigma} \times \vec{ds}$

According to Gauss's law,

$$\text{Flux, } \phi = \vec{E}.\vec{ds} = \frac{|q|}{\epsilon_0}$$

$$Eds = \frac{\vec{\sigma} \times \vec{ds}}{\epsilon_0}$$

$$\therefore E = \frac{\sigma}{\epsilon_0}\hat{n}$$

Therefore, the electric field just outside the conductor is $\frac{\sigma}{\epsilon_0}\hat{n}$. This field is a superposition of field due to the cavity (E') and the field due to the rest of the charged conductor (E'). These fields are equal and opposite inside the conductor, and equal in magnitude and direction outside the conductor.

$$\therefore E' + E' = E$$
$$E' = \frac{E}{2}$$
$$= \frac{\sigma}{2\epsilon_0}\hat{n}$$

Therefore, the field due to the rest of the conductor is $\frac{\sigma}{\epsilon_0}\hat{n}$.

Hence, proved.

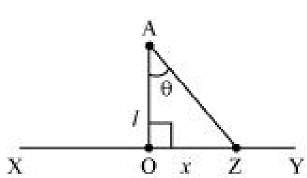
Question 1.30:

Obtain the formula for the electric field due to a long thin wire of uniform linear charge density λ without using Gauss's law. [*Hint:* Use Coulomb's law directly and evaluate the necessary integral.]

<u>Answer</u>

Take a long thin wire XY (as shown in the figure) of uniform linear charge density λ.

Consider a point A at a perpendicular distance *l* from the mid-point O of the wire, as shown in the following figure.



Let E be the electric field at point A due to the wire, XY.

Consider a small length element dx on the wire section with OZ = x

Let q be the charge on this piece.

$$\therefore q = \lambda dx$$

Electric field due to the piece,

$$dE = \frac{1}{4\pi \in_0} \frac{\lambda dx}{(\text{AZ})^2}$$

$$\text{However, } AZ = \sqrt{\left(l^2 + x^2\right)}$$

$$\therefore dE = \frac{\lambda dx}{4\pi \in_0 \left(l^2 + x^2\right)}$$

The electric field is resolved into two rectangular components. $dE\cos\theta$ is the perpendicular component and $dE\sin\theta$ is the parallel component.

When the whole wire is considered, the component $dE\sin\theta$ is cancelled.

Only the perpendicular component $dE\cos\theta$ affects point A.

Hence, effective electric field at point A due to the element dx is dE_1.

$$\therefore dE_1 = \frac{\lambda dx\cos\theta}{4\pi \in_0 \left(x^2 + l^2\right)} \qquad \ldots (1)$$

InΔAZO,

$$\tan\theta = \frac{x}{l}$$

$$x = l\tan\theta \qquad \ldots (2)$$

On differentiating equation (2), we obtain

$$\frac{dx}{d\theta} = l\sin^2\theta$$

$$dx = l\sin^2\theta\, d\theta \qquad \ldots (3)$$

From equation (2),

$$x^2 + l^2 = l^2 + l^2\tan^2\theta$$

$$\therefore l^2\left(1+\tan^2\theta\right)=l^2\sec^2\theta$$

$$x^2+l^2=l^2\sin^2\theta \qquad \ldots(4)$$

Putting equations (3) and (4) in equation (1), we obtain

$$\therefore dE_1=\frac{\lambda l\sec^2 d\theta}{4\pi\in_0 l^2\sec^2\theta}\times\cos\theta$$

$$\therefore dE_1=\frac{\lambda\cos\theta d\theta}{4\pi\in_0 l} \qquad \ldots(5)$$

The wire is so long that θ tends from $-\frac{\pi}{2}$ to $+\frac{\pi}{2}$.

By integrating equation (5), we obtain the value of field E_1 as,

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} dE_1=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\frac{\lambda}{4\pi\in_0 l}\cos\theta d\theta$$

$$E_1=\frac{\lambda}{4\pi\in_0 l}\left[\sin\theta\right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}}$$

$$=\frac{\lambda}{4\pi\in_0 l}\times 2$$

$$E_1=\frac{\lambda}{2\pi\in_0 l}$$

Therefore, the electric field due to long wire is $\frac{\lambda}{2\pi\in_0 l}$.

Question 1.31:

It is now believed that protons and neutrons (which constitute nuclei of ordinary matter) are themselves built out of more elementary units called quarks. A proton and a neutron consist of three quarks each. Two types of quarks, the so called ‘up’ quark (denoted by u) of charge (+2/3) *e*, and the ‘down’ quark (denoted by d) of charge (−1/3) *e*, together with electrons build up ordinary matter. (Quarks of other types have also been found which give rise to different unusual varieties of matter.) Suggest a possible quark composition of a proton and neutron.

Answer

A proton has three quarks. Let there be n up quarks in a proton, each having a charge of $+\frac{2}{3}e$.

Charge due to n up quarks $=\left(\frac{2}{3}e\right)n$

Number of down quarks in a proton = 3 − n

Each down quark has a charge of $-\frac{1}{3}e$.

Charge due to (3 − n) down quarks $=\left(-\frac{1}{3}e\right)(3-n)$

Total charge on a proton = + e

$$\therefore e=\left(\frac{2}{3}e\right)n+\left(-\frac{1}{3}e\right)(3-n)$$
$$e=\left(\frac{2ne}{3}\right)-e+\frac{ne}{3}$$
$$2e=ne$$
$$n=2$$

Number of up quarks in a proton, n = 2

Number of down quarks in a proton = 3 − n = 3 − 2 = 1

Therefore, a proton can be represented as ‘uud’.

A neutron also has three quarks. Let there be n up quarks in a neutron, each having a charge of $+\frac{3}{2}e$.

Charge on a neutron due to n up quarks $=\left(+\frac{3}{2}e\right)n$

Number of down quarks is 3 − n,each having a charge of $\left(-\frac{1}{3}\right)e$.

Charge on a neutron due to $(3-n)$ down quarks = $\left(-\frac{1}{3}e\right)(3-n)$

Total charge on a neutron = 0

$$0 = \left(\frac{2}{3}e\right)n + \left(-\frac{1}{3}e\right)(3-n)$$
$$0 = \frac{2}{3}en - e + \frac{ne}{3}$$
$$e = ne$$
$$n = 1$$

Number of up quarks in a neutron, $n = 1$

Number of down quarks in a neutron = $3 - n = 2$

Therefore, a neutron can be represented as 'udd'.

Question 1.32:

Consider an arbitrary electrostatic field configuration. A small test charge is placed at a null point (i.e., where **E** = 0) of the configuration. Show that the equilibrium of the test charge is necessarily unstable.

Verify this result for the simple configuration of two charges of the same magnitude and sign placed a certain distance apart.

Answer

Let the equilibrium of the test charge be stable. If a test charge is in equilibrium and displaced from its position in any direction, then it experiences a restoring force towards a null point, where the electric field is zero. All the field lines near the null point are directed inwards towards the null point. There is a net inward flux of electric field through a closed surface around the null point. According to Gauss's law, the flux of electric field through a surface, which is not enclosing any charge, is zero. Hence, the equilibrium of the test charge can be stable.

Two charges of same magnitude and same sign are placed at a certain distance. The mid-point of the joining line of the charges is the null point. When a test charged is displaced along the line, it experiences a restoring force. If it is displaced normal to the joining line, then the net force takes it away from the null point. Hence, the charge is unstable because stability of equilibrium requires restoring force in all directions.

Question 1.33:

A particle of mass *m* and charge (−*q*) enters the region between the two charged plates initially moving along *x*-axis with speed *vx* (like particle 1 in Fig. 1.33). The length of plate is *L* and an uniform electric field *E* is maintained between the plates. Show that the vertical deflection of the particle at the far edge of the plate is $qEL^2/(2m v_x^2)$.

Compare this motion with motion of a projectile in gravitational field discussed in Section 4.10 of Class XI Textbook of Physics.

Answer

Charge on a particle of mass $m = -q$

Velocity of the particle = v_x

Length of the plates = L

Magnitude of the uniform electric field between the plates = E

Mechanical force, F = Mass (m) × Acceleration (a)

$$a = \frac{F}{m}$$

However, electric force, $F = qE$

Therefore, acceleration, $a = \frac{qE}{m}$ ... (1)

Time taken by the particle to cross the field of length L is given by,

$$t = \frac{\text{Length of the plate}}{\text{Velocity of the particle}} = \frac{L}{v_x} \quad \text{... (2)}$$

In the vertical direction, initial velocity, $u = 0$

According to the third equation of motion, vertical deflection s of the particle can be obtained as,

$$s = ut + \frac{1}{2}at^2$$

$$s = 0 + \frac{1}{2}\left(\frac{qE}{m}\right)\left(\frac{L}{v_x}\right)^2$$

$$s = \frac{qEL^2}{2mV_x^2} \qquad \ldots (3)$$

Hence, vertical deflection of the particle at the far edge of the plate is

$qEL^2/\left(2mv_x^2\right)$. This is similar to the motion of horizontal projectiles under gravity.

Question 1.34:

Suppose that the particle in Exercise in 1.33 is an electron projected with velocity v_x= 2.0 × 10^6 m s^{-1}. If E between the plates separated by 0.5 cm is 9.1 × 10^2 N/C, where will the electron strike the upper plate? (| e | =1.6 × 10^{-19} C, m_e = 9.1 × 10^{-31} kg.)

Answer

Velocity of the particle, v_x = 2.0 × 10^6 m/s

Separation of the two plates, d = 0.5 cm = 0.005 m

Electric field between the two plates, E = 9.1 × 10^2 N/C

Charge on an electron, q = 1.6 × 10^{-19} C

Mass of an electron, m_e = 9.1 × 10^{-31} kg

Let the electron strike the upper plate at the end of plate L, when deflection is s.

Therefore,

$$s = \frac{qEL^2}{2mv_x^2}$$

$$L = \sqrt{\frac{2dmv_x^2}{qE}}$$

$$= \sqrt{\frac{2\times0.005\times9.1\times10^{-31}\times\left(2.0\times10^{6}\right)^2}{1.6\times10^{-10}\times9.1\times10^{2}}}$$

$$= \sqrt{0.025\times10^{-2}} = \sqrt{2.5\times10^{-4}}$$

$$= 1.6\times10^{-2}\ \text{m}$$

$$= 1.6\ \text{cm}$$

Therefore, the electron will strike the upper plate after travelling 1.6 cm.

Question 2.1:

Two charges 5×10^{-8} C and -3×10^{-8} C are located 16 cm apart. At what point(s) on the line joining the two charges is the electric potential zero? Take the potential at infinity to be zero.

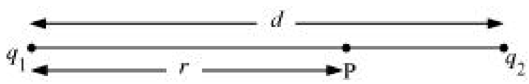
Answer

There are two charges,

$$q_1 = 5 \times 10^{-8}\,\text{C}$$
$$q_2 = -3 \times 10^{-8}\,\text{C}$$

Distance between the two charges, d = 16 cm = 0.16 m

Consider a point P on the line joining the two charges, as shown in the given figure.



r = Distance of point P from charge q_1

Let the electric potential (V) at point P be zero.

Potential at point P is the sum of potentials caused by charges q_1 and q_2 respectively.

$$\therefore V = \frac{q_1}{4\pi \in_0 r} + \frac{q_2}{4\pi \in_0 (d-r)} \quad \text{... (i)}$$

Where,

$\in_0$ = Permittivity of free space

For $V = 0$, equation (i) reduces to

$$\frac{q_1}{4\pi \in_0 r} = -\frac{q_2}{4\pi \in_0 (d-r)}$$

$$\frac{q_1}{r} = \frac{-q_2}{d-r}$$

$$\frac{5\times10^{-8}}{r} = -\frac{(-3\times10^{-8})}{(0.16-r)}$$

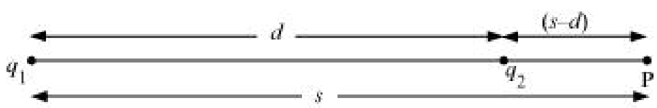
$$\frac{0.16}{r} - 1 = \frac{3}{5}$$

$$\frac{0.16}{r} = \frac{8}{5}$$

$$\therefore r = 0.1\,\text{m} = 10\,\text{cm}$$

Therefore, the potential is zero at a distance of 10 cm from the positive charge between the charges.

Suppose point P is outside the system of two charges at a distance *s* from the negative charge, where potential is zero, as shown in the following figure.



For this arrangement, potential is given by,

$$V = \frac{q_1}{4\pi \in_0 s} + \frac{q_2}{4\pi \in_0 (s-d)} \qquad \text{... (ii)}$$

For $V = 0$, equation (ii) reduces to

$$\frac{q_1}{4\pi \in_0 s} = -\frac{q_2}{4\pi \in_0 (s-d)}$$

$$\frac{q_1}{s} = \frac{-q_2}{s-d}$$

$$\frac{5\times10^{-8}}{s} = -\frac{(-3\times10^{-8})}{(s-0.16)}$$

$$1 - \frac{0.16}{s} = \frac{3}{5}$$

$$\frac{0.16}{s} = \frac{2}{5}$$

$$\therefore s = 0.4\,\text{m} = 40\,\text{cm}$$

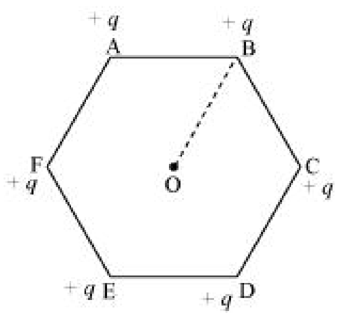
Therefore, the potential is zero at a distance of 40 cm from the positive charge outside the system of charges.

Question 2.2:

A regular hexagon of side 10 cm has a charge 5 µC at each of its vertices. Calculate the potential at the centre of the hexagon.

Answer

The given figure shows six equal amount of charges, q, at the vertices of a regular hexagon.



Where,

Charge, $q = 5\ \mu C = 5 \times 10^{-6}$ C

Side of the hexagon, l = AB = BC = CD = DE = EF = FA = 10 cm

Distance of each vertex from centre O, d = 10 cm

Electric potential at point O,

$$V = \frac{6 \times q}{4\pi \in_0 d}$$

Where,

$\in_0$ = Permittivity of free space

$$\frac{1}{4\pi \in_0} = 9\times10^9 \text{ N C}^{-2}\text{ m}^{-2}$$

$$\therefore V = \frac{6\times9\times10^9\times5\times10^{-6}}{0.1}$$

$$= 2.7\times10^6 \text{ V}$$

Therefore, the potential at the centre of the hexagon is 2.7×10^6 V.

Question 2.3:

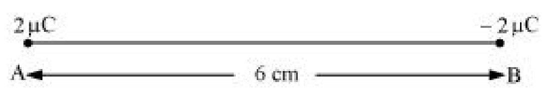
Two charges 2 μC and −2 μC are placed at points A and B 6 cm apart.

Identify an equipotential surface of the system.

What is the direction of the electric field at every point on this surface?

Answer

The situation is represented in the given figure.



An equipotential surface is the plane on which total potential is zero everywhere. This plane is normal to line AB. The plane is located at the mid-point of line AB because the magnitude of charges is the same.

The direction of the electric field at every point on this surface is normal to the plane in the direction of AB.

Question 2.4:

A spherical conductor of radius 12 cm has a charge of 1.6×10^{-7}C distributed uniformly on its surface. What is the electric field

Inside the sphere

Just outside the sphere

At a point 18 cm from the centre of the sphere?

Answer

Radius of the spherical conductor, r = 12 cm = 0.12 m

Charge is uniformly distributed over the conductor, $q = 1.6 \times 10^{-7}$ C

Electric field inside a spherical conductor is zero. This is because if there is field inside the conductor, then charges will move to neutralize it.

Electric field E just outside the conductor is given by the relation,

$$E = \frac{q}{4\pi \epsilon_0 r^2}$$

Where,

ϵ_0 = Permittivity of free space

$$\frac{1}{4\pi \epsilon_0} = 9\times10^9 \text{ N m}^2 \text{ C}^{-2}$$

$$\therefore E = \frac{1.6\times10^{-7}\times9\times10^{-9}}{(0.12)^2}$$

$$= 10^5 \text{ N C}^{-1}$$

Therefore, the electric field just outside the sphere is 10^5 N C^{-1}.

Electric field at a point 18 m from the centre of the sphere = E_1

Distance of the point from the centre, d = 18 cm = 0.18 m

$$E_1 = \frac{q}{4\pi \epsilon_0 d^2}$$

$$= \frac{9\times10^9\times1.6\times10^{-7}}{(18\times10^{-2})^2}$$

$$= 4.4\times10^4 \text{ N/C}$$

Therefore, the electric field at a point 18 cm from the centre of the sphere is

4.4×10^4 N/C .

Question 2.5:

A parallel plate capacitor with air between the plates has a capacitance of 8 pF (1pF = 10^{-12} F). What will be the capacitance if the distance between the plates is reduced by half, and the space between them is filled with a substance of dielectric constant 6?

Answer

Capacitance between the parallel plates of the capacitor, C = 8 pF

Initially, distance between the parallel plates was *d* and it was filled with air. Dielectric constant of air, $k = 1$

Capacitance, *C*, is given by the formula,

$$C = \frac{k \in_0 A}{d}$$
$$= \frac{\in_0 A}{d} \qquad \text{... (i)}$$

Where,

A = Area of each plate

$\in_0$ = Permittivity of free space

If distance between the plates is reduced to half, then new distance, $d' = \frac{d}{2}$

Dielectric constant of the substance filled in between the plates, $k' = 6$

Hence, capacitance of the capacitor becomes

$$C' = \frac{k' \in_0 A}{d'} = \frac{6 \in_0 A}{\frac{d}{2}} \qquad \text{... (ii)}$$

Taking ratios of equations (i) and (ii), we obtain

$$\begin{aligned} C' &= 2\times 6C \\ &= 12C \\ &= 12\times 8 = 96\ \text{pF} \end{aligned}$$

Therefore, the capacitance between the plates is 96 pF.

Question 2.6:

Three capacitors each of capacitance 9 pF are connected in series.

What is the total capacitance of the combination?

What is the potential difference across each capacitor if the combination is connected to a 120 V supply?

Answer

Capacitance of each of the three capacitors, $C = 9$ pF

Equivalent capacitance (C') of the combination of the capacitors is given by the relation,

$$\begin{aligned} \frac{1}{C'} &= \frac{1}{C}+\frac{1}{C}+\frac{1}{C} \\ &= \frac{1}{9}+\frac{1}{9}+\frac{1}{9} = \frac{3}{9} = \frac{1}{3} \\ \therefore C' &= 3\mu\text{F} \end{aligned}$$

Therefore, total capacitance of the combination is $3\mu\text{F}$.

Supply voltage, $V = 100$ V

Potential difference (V') across each capacitor is equal to one-third of the supply voltage.

$$\therefore V' = \frac{V}{3} = \frac{120}{3} = 40\ \text{V}$$

Therefore, the potential difference across each capacitor is 40 V.

Question 2.7:

Three capacitors of capacitances 2 pF, 3 pF and 4 pF are connected in parallel.

What is the total capacitance of the combination?

Determine the charge on each capacitor if the combination is connected to a 100 V supply.

Answer

Capacitances of the given capacitors are

$$C_1 = 2 \text{ pF}$$
$$C_2 = 3 \text{ pF}$$
$$C_3 = 4 \text{ pF}$$

For the parallel combination of the capacitors, equivalent capacitor C' is given by the algebraic sum,

$$C' = 2 + 3 + 4 = 9 \text{ pF}$$

Therefore, total capacitance of the combination is 9 pF.

Supply voltage, V = 100 V

The voltage through all the three capacitors is same = V = 100 V

Charge on a capacitor of capacitance C and potential difference V is given by the relation,

$q = VC$ … (i)

For C = 2 pF,

$$\text{Charge} = VC = 100 \times 2 = 200 \text{ pC} = 2 \times 10^{-10} \text{ C}$$

For C = 3 pF,

$$\text{Charge} = VC = 100 \times 3 = 300 \text{ pC} = 3 \times 10^{-10} \text{ C}$$

For C = 4 pF,

$$\text{Charge} = VC = 100 \times 4 = 200\ \text{pC} = 4 \times 10^{-10}\ \text{C}$$

Question 2.8:

In a parallel plate capacitor with air between the plates, each plate has an area of 6×10^{-3} m^2 and the distance between the plates is 3 mm. Calculate the capacitance of the capacitor. If this capacitor is connected to a 100 V supply, what is the charge on each plate of the capacitor?

Answer

Area of each plate of the parallel plate capacitor, $A = 6 \times 10^{-3}\ m^2$

Distance between the plates, $d = 3\ mm = 3 \times 10^{-3}\ m$

Supply voltage, $V = 100$ V

Capacitance C of a parallel plate capacitor is given by,

$$C = \frac{\epsilon_0 A}{d}$$

Where,

ϵ_0 = Permittivity of free space

$= 8.854 \times 10^{-12}\ N^{-1}\ m^{-2}\ C^{-2}$

$$\therefore C = \frac{8.854\times10^{-12}\times6\times10^{-3}}{3\times10^{-3}}$$
$$=17.71\times10^{-12}\,\mathrm{F}$$
$$=17.71\ \mathrm{pF}$$

Potential V is related with the charge q and capacitance C as

$$V = \frac{q}{C}$$
$$\therefore q = VC$$
$$=100\times17.71\times10^{-12}$$
$$=1.771\times10^{-9}\ \mathrm{C}$$

Therefore, capacitance of the capacitor is 17.71 pF and charge on each plate is 1.771×10^{-9} C.

Question 2.9:

Explain what would happen if in the capacitor given in Exercise 2.8, a 3 mm thick mica sheet (of dielectric constant = 6) were inserted between the plates,

While the voltage supply remained connected.

After the supply was disconnected.

Answer

Dielectric constant of the mica sheet, $k = 6$

Initial capacitance, $C = 1.771 \times 10^{-11}$ F

New capacitance, $C' = kC = 6\times1.771\times10^{-11} = 106$ pF

Supply voltage, $V = 100$ V

New charge, $q' = C'V = 6\times1.771\times10^{-9} = 1.06\times10^{-8}\,\mathrm{C}$

Potential across the plates remains 100 V.

Dielectric constant, $k = 6$

Initial capacitance, $C = 1.771 \times 10^{-11}$ F

New capacitance, $C' = kC = 6 \times 1.771 \times 10^{-11} = 106$ pF

If supply voltage is removed, then there will be no effect on the amount of charge in the plates.

Charge = 1.771×10^{-9} C

Potential across the plates is given by,

$$\begin{aligned}\therefore V' &= \frac{q}{C'}\\ &= \frac{1.771 \times 10^{-9}}{106 \times 10^{-12}}\\ &= 16.7 \text{ V}\end{aligned}$$

Question 2.10:

A 12 pF capacitor is connected to a 50V battery. How much electrostatic energy is stored in the capacitor?

Answer

Capacitor of the capacitance, $C = 12$ pF $= 12 \times 10^{-12}$ F

Potential difference, $V = 50$ V

Electrostatic energy stored in the capacitor is given by the relation,

$$\begin{aligned}E &= \frac{1}{2}CV^2\\ &= \frac{1}{2} \times 12 \times 10^{-12} \times (50)^2\\ &= 1.5 \times 10^{-8} \text{ J}\end{aligned}$$

Therefore, the electrostatic energy stored in the capacitor is 1.5×10^{-8} J.

Question 2.11:

A 600 pF capacitor is charged by a 200 V supply. It is then disconnected from the supply and is connected to another uncharged 600 pF capacitor. How much electrostatic energy is lost in the process?

Answer

Capacitance of the capacitor, $C = 600$ pF

Potential difference, $V = 200$ V

Electrostatic energy stored in the capacitor is given by,

$$E = \frac{1}{2}CV^2$$
$$= \frac{1}{2}\times\left(600\times10^{-12}\right)\times\left(200\right)^2$$
$$= 1.2\times10^{-5}\ \text{J}$$

If supply is disconnected from the capacitor and another capacitor of capacitance $C = 600$ pF is connected to it, then equivalent capacitance (C') of the combination is given by,

$$\frac{1}{C'} = \frac{1}{C}+\frac{1}{C}$$
$$= \frac{1}{600}+\frac{1}{600} = \frac{2}{600} = \frac{1}{300}$$
$$\therefore C' = 300\ \text{pF}$$

New electrostatic energy can be calculated as

$$E' = \frac{1}{2}\times C'\times V^2$$
$$= \frac{1}{2}\times300\times\left(200\right)^2$$
$$= 0.6\times10^{-5}\ \text{J}$$

$$\text{Loss in electrostatic energy} = E - E'$$
$$= 1.2\times10^{-5} - 0.6\times10^{-5}$$
$$= 0.6\times10^{-5}$$
$$= 6\times10^{-6}\ \text{J}$$

Therefore, the electrostatic energy lost in the process is 6×10^{-6} J.

Question 2.12:

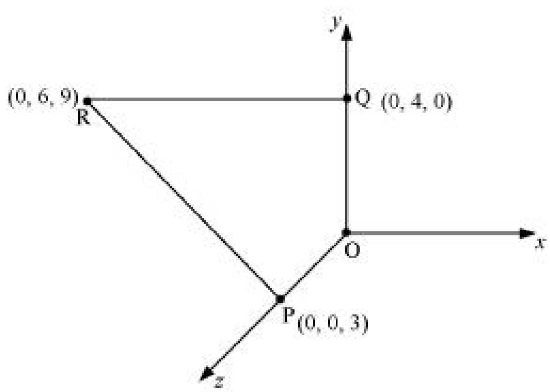
A charge of 8 mC is located at the origin. Calculate the work done in taking a small charge of -2×10^{-9} C from a point P (0, 0, 3 cm) to a point Q (0, 4 cm, 0), via a point R (0, 6 cm, 9 cm).

Answer

Charge located at the origin, q = 8 mC= 8×10^{-3} C

Magnitude of a small charge, which is taken from a point P to point R to point Q, $q_1 = -2 \times 10^{-9}$ C

All the points are represented in the given figure.



Point P is at a distance, d_1 = 3 cm, from the origin along z-axis.

Point Q is at a distance, d_2 = 4 cm, from the origin along y-axis.

Potential at point P, $V_1 = \dfrac{q}{4\pi \in_0 \times d_1}$

Potential at point Q, $V_2 = \dfrac{q}{4\pi \in_0 d_2}$

Work done (W) by the electrostatic force is independent of the path.

$$\therefore W = q_1 \left[V_2 - V_1\right]$$
$$= q_1 \left[\frac{q}{4\pi \in_0 d_2} - \frac{q}{4\pi \in_0 d_1}\right]$$
$$= \frac{qq_1}{4\pi \in_0}\left[\frac{1}{d_2} - \frac{1}{d_1}\right] \qquad \ldots (i)$$

Where, $\dfrac{1}{4\pi \in_0} = 9\times10^9 \text{ N m}^2 \text{ C}^{-2}$

$$\therefore W = 9\times10^9 \times 8\times10^{-3} \times \left(-2\times10^{-9}\right)\left[\frac{1}{0.04} - \frac{1}{0.03}\right]$$
$$= -144\times10^{-3} \times \left(\frac{-25}{3}\right)$$
$$= 1.27 \text{ J}$$

Therefore, work done during the process is 1.27 J.

Question 2.13:

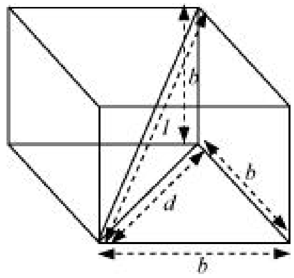
A cube of side b has a charge q at each of its vertices. Determine the potential and electric field due to this charge array at the centre of the cube.

Answer

Length of the side of a cube = b

Charge at each of its vertices = q

A cube of side b is shown in the following figure.



d = Diagonal of one of the six faces of the cube

$$d^2 = \sqrt{b^2 + b^2} = \sqrt{2b^2}$$
$$d = b\sqrt{2}$$

l = Length of the diagonal of the cube

$$l^2 = \sqrt{d^2 + b^2}$$
$$= \sqrt{\left(\sqrt{2}b\right)^2 + b^2} = \sqrt{2b^2 + b^2} = \sqrt{3b^2}$$
$$l = b\sqrt{3}$$

$r = \frac{l}{2} = \frac{b\sqrt{3}}{2}$ is the distance between the centre of the cube and one of the eight vertices

The electric potential (V) at the centre of the cube is due to the presence of eight charges at the vertices.

$$V = \frac{8q}{4\pi \in_0}$$
$$= \frac{8q}{4\pi \in_0 \left(b\frac{\sqrt{3}}{2}\right)}$$
$$= \frac{4q}{\sqrt{3}\pi \in_0 b}$$

Therefore, the potential at the centre of the cube is $\frac{4q}{\sqrt{3}\pi \in_0 b}$.

The electric field at the centre of the cube, due to the eight charges, gets cancelled. This is because the charges are distributed symmetrically with respect to the centre of the cube. Hence, the electric field is zero at the centre.

Question 2.14:

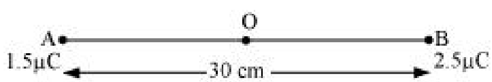
Two tiny spheres carrying charges 1.5 μC and 2.5 μC are located 30 cm apart. Find the potential and electric field:

at the mid-point of the line joining the two charges, and

at a point 10 cm from this midpoint in a plane normal to the line and passing through the mid-point.

Answer

Two charges placed at points A and B are represented in the given figure. O is the mid-point of the line joining the two charges.



Magnitude of charge located at A, q_1 = 1.5 μC

Magnitude of charge located at B, q_2 = 2.5 μC

Distance between the two charges, d = 30 cm = 0.3 m

Let V_1 and E_1 are the electric potential and electric field respectively at O.

V_1 = Potential due to charge at A + Potential due to charge at B

$$V_1 = \frac{q_1}{4\pi \in_0 \left(\frac{d}{2}\right)} + \frac{q_2}{4\pi \in_0 \left(\frac{d}{2}\right)} = \frac{1}{4\pi \in_0 \left(\frac{d}{2}\right)}(q_1 + q_2)$$

Where,

$\in_0$ = Permittivity of free space

$$\frac{1}{4\pi \in_0} = 9\times10^9 \text{ NC}^2 \text{ m}^{-2}$$

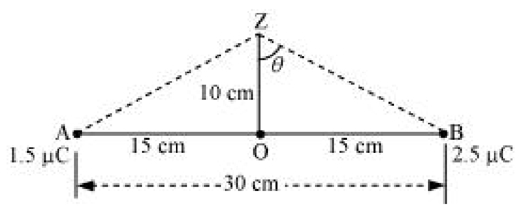
$$\therefore V_1 = \frac{9\times10^9\times10^{-6}}{\left(\frac{0.30}{2}\right)}(2.5+1.5) = 2.4\times10^5 \text{ V}$$

E_1 = Electric field due to q_2 − Electric field due to q_1

$$= \frac{q_2}{4\pi \in_0 \left(\frac{d}{2}\right)^2} - \frac{q_1}{4\pi \in_0 \left(\frac{d}{2}\right)^2}$$
$$= \frac{9\times10^9}{\left(\frac{0.30}{2}\right)^2} \times 10^6 \times (2.5 - 1.5)$$
$$= 4\times10^5 \text{ V m}^{-1}$$

Therefore, the potential at mid-point is 2.4×10^5 V and the electric field at mid-point is 4×10^5 V m^{-1}. The field is directed from the larger charge to the smaller charge.

Consider a point Z such that normal distanceOZ = 10 cm = 0.1 m, as shown in the following figure.



V_2 and E_2 are the electric potential and electric field respectively at Z.

It can be observed from the figure that distance,

$$\text{BZ} = \text{AZ} = \sqrt{(0.1)^2 + (0.15)^2} = 0.18 \text{ m}$$

V_2= Electric potential due to A + Electric Potential due to B

$$= \frac{q_1}{4\pi \in_0 (\text{AZ})} + \frac{q_1}{4\pi \in_0 (\text{BZ})}$$
$$= \frac{9\times10^9 \times 10^{-6}}{0.18}(1.5 + 2.5)$$
$$= 2\times10^5 \text{ V}$$

Electric field due to q at Z,

$$E_\text{A} = \frac{q_1}{4\pi \in_0 (\text{AZ})^2}$$
$$= \frac{9\times10^9 \times 1.5\times10^{-6}}{(0.18)^2}$$
$$= 0.416\times10^6 \text{ V/m}$$

Electric field due to q_2 at Z,

$$E_{\mathrm{B}} = \frac{q_2}{4\pi \in_0 (\mathrm{BZ})^2}$$
$$= \frac{9\times10^9 \times 2.5\times10^{-6}}{(0.18)^2}$$
$$= 0.69\times10^6 \text{ V m}^{-1}$$

The resultant field intensity at Z,

$$E = \sqrt{E_{\mathrm{A}}^2 + E_{\mathrm{B}}^2 + 2E_{\mathrm{A}}E_{\mathrm{B}}\cos 2\theta}$$

Where, $2\theta$is the angle, $\angle AZ$ B

From the figure, we obtain

$$\cos\theta = \frac{0.10}{0.18} = \frac{5}{9} = 0.5556$$
$$\theta = \cos^{-1} 0.5556 = 56.25$$
$$\therefore 2\theta = 112.5^{\circ}$$
$$\cos 2\theta = -0.38$$
$$E = \sqrt{\left(0.416\times10^6\right)^2 \times \left(0.69\times10^6\right)^2 + 2\times0.416\times0.69\times10^{12}\times(-0.38)}$$
$$= 6.6\times10^5 \text{ V m}^{-1}$$

Therefore, the potential at a point 10 cm (perpendicular to the mid-point) is 2.0×10^5 V and electric field is 6.6×10^5 V m^{-1}.

Question 2.15:

A spherical conducting shell of inner radius $r1$ and outer radius $r2$ has a charge Q.

A charge q is placed at the centre of the shell. What is the surface charge density on the inner and outer surfaces of the shell?

Is the electric field inside a cavity (with no charge) zero, even if the shell is not spherical, but has any irregular shape? Explain.

Answer

Charge placed at the centre of a shell is $+q$. Hence, a charge of magnitude $-q$ will be induced to the inner surface of the shell. Therefore, total charge on the inner surface of the shell is $-q$.

Surface charge density at the inner surface of the shell is given by the relation,

$$\sigma_1 = \frac{\text{Total charge}}{\text{Inner surface area}} = \frac{-q}{4\pi r_1^2} \qquad \ldots (i)$$

A charge of $+q$ is induced on the outer surface of the shell. A charge of magnitude Q is placed on the outer surface of the shell. Therefore, total charge on the outer surface of the shell is $Q + q$. Surface charge density at the outer surface of the shell,

$$\sigma_2 = \frac{\text{Total charge}}{\text{Outer surface area}} = \frac{Q+q}{4\pi r_2^2} \qquad \ldots (ii)$$

Yes

The electric field intensity inside a cavity is zero, even if the shell is not spherical and has any irregular shape. Take a closed loop such that a part of it is inside the cavity along a field line while the rest is inside the conductor. Net work done by the field in carrying a test charge over a closed loop is zero because the field inside the conductor is zero. Hence, electric field is zero, whatever is the shape.

Question 2.16:

Show that the normal component of electrostatic field has a discontinuity from one side of a charged surface to another given by

$$\left(\overrightarrow{E_2} - \overrightarrow{E_1}\right).\hat{n} = \frac{\sigma}{\epsilon_0}$$

Where $\hat{n}$ is a unit vector normal to the surface at a point and σ is the surface charge density at that point. (The direction of $\hat{n}$ is from side 1 to side 2.) Hence show that just outside a conductor, the electric field is σ $\hat{n}/\epsilon_0$

Show that the tangential component of electrostatic field is continuous from one side of a charged surface to another. [Hint: For (a), use Gauss's law. For, (b) use the fact that work done by electrostatic field on a closed loop is zero.]

Answer

Electric field on one side of a charged body is E_1 and electric field on the other side of the same body is E_2. If infinite plane charged body has a uniform thickness, then electric field due to one surface of the charged body is given by,

$$\overrightarrow{E_1} = -\frac{\sigma}{2\in_0}\hat{n} \qquad \ldots (i)$$

Where,

$\hat{n}$ = Unit vector normal to the surface at a point

σ = Surface charge density at that point

Electric field due to the other surface of the charged body,

$$\overrightarrow{E_2} = -\frac{\sigma}{2\in_0}\hat{n} \qquad \ldots (ii)$$

Electric field at any point due to the two surfaces,

$$\overrightarrow{E_2} - \overrightarrow{E_1} = \frac{\sigma}{2\in_0}\hat{n} + \frac{\sigma}{2\in_0}\hat{n} = \frac{\sigma}{\in_0}\hat{n}$$

$$\left(\overrightarrow{E_2} - \overrightarrow{E_1}\right).\hat{n} = \frac{\sigma}{\in_0} \qquad \ldots (iii)$$

Since inside a closed conductor, $\overrightarrow{E_1} = 0$,

$$\therefore \quad \vec{E} = \overrightarrow{E_2} = -\frac{\sigma}{2\in_0}\hat{n}$$

Therefore, the electric field just outside the conductor is $\frac{\sigma}{\in_0}\hat{n}$.

When a charged particle is moved from one point to the other on a closed loop, the work done by the electrostatic field is zero. Hence, the tangential component of electrostatic field is continuous from one side of a charged surface to the other.

Question 2.17:

A long charged cylinder of linear charged density λ is surrounded by a hollow co-axial conducting cylinder. What is the electric field in the space between the two cylinders?

Answer

Charge density of the long charged cylinder of length L and radius r is λ.

Another cylinder of same length surrounds the pervious cylinder. The radius of this cylinder is R.

Let E be the electric field produced in the space between the two cylinders.

Electric flux through the Gaussian surface is given by Gauss's theorem as,

$$\phi = E\left(2\pi d\right)L$$

Where, d = Distance of a point from the common axis of the cylinders

Let q be the total charge on the cylinder.

It can be written as

$$\therefore \phi = E\left(2\pi dL\right) = \frac{q}{\in_0}$$

Where,

q = Charge on the inner sphere of the outer cylinder

$\in_0$ = Permittivity of free space

$$E\left(2\pi dL\right) = \frac{\lambda L}{\in_0}$$

$$E = \frac{\lambda}{2\pi \in_0 d}$$

Therefore, the electric field in the space between the two cylinders is $\frac{\lambda}{2\pi \in_0 d}$.

Question 2.18:

In a hydrogen atom, the electron and proton are bound at a distance of about 0.53 Å:

Estimate the potential energy of the system in eV, taking the zero of the potential energy at infinite separation of the electron from proton.

What is the minimum work required to free the electron, given that its kinetic energy in the orbit is half the magnitude of potential energy obtained in (a)?

What are the answers to (a) and (b) above if the zero of potential energy is taken at 1.06 Å separation?

Answer

The distance between electron-proton of a hydrogen atom, $d = 0.53\ \text{Å}$

Charge on an electron, $q_1 = -1.6 \times 10^{-19}$ C

Charge on a proton, $q_2 = +1.6 \times 10^{-19}$ C

Potential at infinity is zero.

Potential energy of the system, *p-e* = Potential energy at infinity − Potential energy at distance, *d*

$$= 0 - \frac{q_1 q_2}{4\pi \epsilon_0 d}$$

Where,

ϵ_0 is the permittivity of free space

$$\frac{1}{4\pi \epsilon_0} = 9 \times 10^9 \text{ Nm}^2 \text{ C}^{-2}$$

$$\therefore \text{Potential energy} = 0 - \frac{9 \times 10^9 \times \left(1.6 \times 10^{-19}\right)^2}{0.53 \times 10^{10}} = -43.7 \times 10^{-19} \text{ J}$$

Since $1.6 \times 10^{-19}\text{ J} = 1\text{ eV}$,

$$\therefore \text{ Potential energy} = -43.7 \times 10^{-19} = \frac{-43.7 \times 10^{-19}}{1.6 \times 10^{-19}} = -27.2 \text{ eV}$$

Therefore, the potential energy of the system is −27.2 eV.

Kinetic energy is half of the magnitude of potential energy.

$$\text{Kinetic energy} = \frac{1}{2}\times(-27.2) = 13.6\text{ eV}$$

Total energy = 13.6 − 27.2 = 13.6 eV

Therefore, the minimum work required to free the electron is 13.6 eV.

When zero of potential energy is taken, $d_1 = 1.06\,\text{Å}$

∴Potential energy of the system = Potential energy at d_1 − Potential energy at d

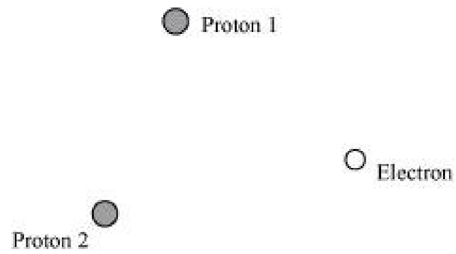
$$= \frac{q_1 q_2}{4\pi \in_0 d_1} - 27.2\text{ eV}$$
$$= \frac{9\times10^9\times\left(1.6\times10^{-19}\right)^2}{1.06\times10^{-10}} - 27.2\text{ eV}$$
$$= 21.73\times10^{-19}\text{ J} - 27.2\text{ eV}$$
$$= 13.58\text{ eV} - 27.2\text{ eV}$$
$$= -13.6\text{ eV}$$

Question 2.19:

If one of the two electrons of a H_2 molecule is removed, we get a hydrogen molecular ion H_2^+. In the ground state of an H_2^+, the two protons are separated by roughly 1.5 Å, and the electron is roughly 1 Å from each proton. Determine the potential energy of the system. Specify your choice of the zero of potential energy.

Answer

The system of two protons and one electron is represented in the given figure.



Charge on proton 1, $q_1 = 1.6 \times 10^{-19}$ C

Charge on proton 2, $q_2 = 1.6 \times 10^{-19}$ C

Charge on electron, $q_3 = -1.6 \times 10^{-19}$ C

Distance between protons 1 and 2, $d_1 = 1.5 \times 10^{-10}$ m

Distance between proton 1 and electron, $d_2 = 1 \times 10^{-10}$ m

Distance between proton 2 and electron, $d_3 = 1 \times 10^{-10}$ m

The potential energy at infinity is zero.

Potential energy of the system,

$$V = \frac{q_1 q_2}{4\pi \epsilon_0 d_1} + \frac{q_2 q_3}{4\pi \epsilon_0 d_3} + \frac{q_3 q_1}{4\pi \epsilon_0 d_2}$$

$$\text{Substituting } \frac{1}{4\pi \epsilon_0} = 9 \times 10^9 \text{ N m}^2 \text{ C}^{-2}, \text{ we obtain}$$

$$V = \frac{9 \times 10^9 \times 10^{-19} \times 10^{-19}}{10^{-10}} \left[-(16)^2 + \frac{(1.6)^2}{1.5} + -(1.6)^2 \right]$$

$$= -30.7 \times 10^{-19} \text{ J}$$

$$= -19.2 \text{ eV}$$

Therefore, the potential energy of the system is −19.2 eV.

Question 2.20:

Two charged conducting spheres of radii *a* and *b* are connected to each other by a wire. What is the ratio of electric fields at the surfaces of the two spheres? Use the result obtained to explain why charge density on the sharp and pointed ends of a conductor is higher than on its flatter portions.

Answer

Let a be the radius of a sphere A, Q_A be the charge on the sphere, and C_A be the capacitance of the sphere. Let b be the radius of a sphere B, Q_B be the charge on the sphere, and C_B be the capacitance of the sphere. Since the two spheres are connected with a wire, their potential (V) will become equal.

Let $E_A$be the electric field of sphere A and E_B be the electric field of sphere B. Therefore, their ratio,

$$\frac{E_A}{E_B} = \frac{Q_A}{4\pi \in_0 \times a_2} \times \frac{b^2 \times 4\pi \in_0}{Q_B}$$

$$\frac{E_A}{E_B} = \frac{Q_A}{Q_B} \times \frac{b^2}{a^2} \qquad \text{... (1)}$$

However, $\dfrac{Q_A}{Q_B} = \dfrac{C_A V}{C_B V}$

And, $\dfrac{C_A}{C_B} = \dfrac{a}{b}$

$$\therefore \frac{Q_A}{Q_B} = \frac{a}{b} \qquad \text{... (2)}$$

Putting the value of (2) in (1), we obtain

$$\therefore \frac{E_A}{E_B} = \frac{a}{b}\frac{b^2}{a^2} = \frac{b}{a}$$

Therefore, the ratio of electric fields at the surface is $\dfrac{b}{a}$.

Question 2.21:

Two charges $-q$ and $+q$ are located at points (0, 0, $-$ a) and (0, 0, a), respectively.

What is the electrostatic potential at the points?

Obtain the dependence of potential on the distance r of a point from the origin when r/a >> 1.

How much work is done in moving a small test charge from the point (5, 0, 0) to (−7, 0, 0) along the *x*-axis? Does the answer change if the path of the test charge between the same points is not along the *x*-axis?

Answer

Zero at both the points

Charge − q is located at (0, 0, − a) and charge + q is located at (0, 0, a). Hence, they form a dipole. Point (0, 0, z) is on the axis of this dipole and point (x, y, 0) is normal to the axis of the dipole. Hence, electrostatic potential at point (x, y, 0) is zero. Electrostatic potential at point (0, 0, z) is given by,

$$V = \frac{1}{4\pi \in_0}\left(\frac{q}{z-a}\right) + \frac{1}{4\pi \in_0}\left(-\frac{q}{z+a}\right)$$
$$= \frac{q(z+a-z+a)}{4\pi \in_0 (z^2-a^2)}$$
$$= \frac{2qa}{4\pi \in_0 (z^2-a^2)} = \frac{p}{4\pi \in_0 (z^2-a^2)}$$

Where,

$\in_0$ = Permittivity of free space

p = Dipole moment of the system of two charges = $2qa$

Distance r is much greater than half of the distance between the two charges. Hence, the potential (V) at a distance r is inversely proportional to square of the distance i.e.,
$$V \propto \frac{1}{r^2}$$

Zero

The answer does not change if the path of the test is not along the *x*-axis.

A test charge is moved from point (5, 0, 0) to point (−7, 0, 0) along the *x*-axis. Electrostatic potential (V_1) at point (5, 0, 0) is given by,

$$V_1 = \frac{-q}{4\pi\in_0}\frac{1}{\sqrt{(5-0)^2+(-a)^2}} + \frac{q}{4\pi\in_0}\frac{1}{(5-0)^2+a^2}$$

$$= \frac{-q}{4\pi\in_0\sqrt{25^2+a^2}} + \frac{q}{4\pi\in_0\sqrt{25+a^2}}$$

$$= 0$$

Electrostatic potential, V_2, at point (− 7, 0, 0) is given by,

$$V_2 = \frac{-q}{4\pi\in_0}\frac{1}{\sqrt{(-7)^2+(-a)^2}} + \frac{q}{4\pi\in_0}\frac{1}{\sqrt{(-7)^2+(a)^2}}$$

$$= \frac{-q}{4\pi\in_0\sqrt{49+a^2}} + \frac{q}{4\pi\in_0}\frac{1}{\sqrt{49+a^2}}$$

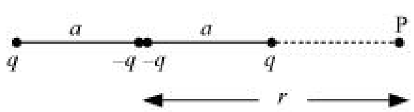
$$= 0$$

Hence, no work is done in moving a small test charge from point (5, 0, 0) to point (−7, 0, 0) along the *x*-axis.

The answer does not change because work done by the electrostatic field in moving a test charge between the two points is independent of the path connecting the two points.

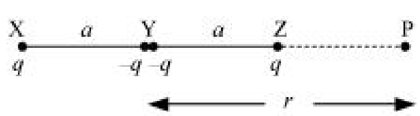
Question 2.22:

Figure 2.34 shows a charge array known as an *electric quadrupole*. For a point on the axis of the quadrupole, obtain the dependence of potential on r for $r/a >> 1$, and contrast your results with that due to an electric dipole, and an electric monopole (i.e., a single charge).



Answer

Four charges of same magnitude are placed at points X, Y, Y, and Z respectively, as shown in the following figure.



A point is located at P, which is r distance away from point Y.

The system of charges forms an electric quadrupole.

It can be considered that the system of the electric quadrupole has three charges.

Charge $+q$ placed at point X

Charge $-2q$ placed at point Y

Charge $+q$ placed at point Z

XY = YZ = a

YP = r

PX = $r + a$

PZ = $r - a$

Electrostatic potential caused by the system of three charges at point P is given by,

$$\begin{aligned}
V &= \frac{1}{4\pi \epsilon_0}\left[\frac{q}{\mathrm{XP}} - \frac{2q}{\mathrm{YP}} + \frac{q}{\mathrm{ZP}}\right] \\
&= \frac{1}{4\pi \epsilon_0}\left[\frac{q}{r+a} - \frac{2q}{r} + \frac{q}{r-a}\right] \\
&= \frac{q}{4\pi \epsilon_0}\left[\frac{r(r-a) - 2(r+a)(r-a) + r(r+a)}{r(r+a)(r-a)}\right] \\
&= \frac{q}{4\pi \epsilon_0}\left[\frac{r^2 - ra - 2r^2 + 2a^2 + r^2 + ra}{r(r^2 - a^2)}\right] = \frac{q}{4\pi \epsilon_0}\left[\frac{2a^2}{r(r^2 - a^2)}\right] \\
&= \frac{2qa^2}{4\pi \epsilon_0 r^3\left(1 - \frac{a^2}{r^2}\right)}
\end{aligned}$$

Since $\frac{r}{a} >> 1$,

$$\therefore \frac{a}{r} << 1$$

$\frac{a^2}{r^2}$ is taken as negligible.

$$\therefore V = \frac{2qa^2}{4\pi \epsilon_0 r^3}$$

It can be inferred that potential, $V \propto \frac{1}{r^3}$

However, it is known that for a dipole, $V \propto \frac{1}{r^2}$

And, for a monopole, $V \propto \frac{1}{r}$

Question 2.23:

An electrical technician requires a capacitance of 2 µF in a circuit across a potential difference of 1 kV. A large number of 1 µF capacitors are available to him each of which can withstand a potential difference of not more than 400 V. Suggest a possible arrangement that requires the minimum number of capacitors.

Answer

Total required capacitance, $C = 2$ µF

Potential difference, $V = 1$ kV = 1000 V

Capacitance of each capacitor, $C_1 = 1$µF

Each capacitor can withstand a potential difference, $V_1 = 400$ V

Suppose a number of capacitors are connected in series and these series circuits are connected in parallel (row) to each other. The potential difference across each row must be 1000 V and potential difference across each capacitor must be 400 V. Hence, number of capacitors in each row is given as

$$\frac{1000}{400} = 2.5$$

Hence, there are three capacitors in each row.

Capacitance of each row $= \frac{1}{1+1+1} = \frac{1}{3}\ \mu F$

Let there are n rows, each having three capacitors, which are connected in parallel. Hence, equivalent capacitance of the circuit is given as

$$\frac{1}{3}+\frac{1}{3}+\frac{1}{3}+\ldots\ldots\ldots\ldots n \text{ terms}$$
$$=\frac{n}{3}$$

However, capacitance of the ciruit is given as 2 μF.

$$\therefore \frac{n}{3}=2$$
$$n=6$$

Hence, 6 rows of three capacitors are present in the circuit. A minimum of 6 × 3 i.e., 18 capacitors are required for the given arrangement.

Question 2.24:

What is the area of the plates of a 2 F parallel plate capacitor, given that the separation between the plates is 0.5 cm? [You will realize from your answer why ordinary capacitors are in the range of µF or less. However, electrolytic capacitors do have a much larger capacitance (0.1 F) because of very minute separation between the conductors.]

Answer

Capacitance of a parallel capacitor, $V = 2$ F

Distance between the two plates, $d = 0.5$ cm $= 0.5 \times 10^{-2}$ m

Capacitance of a parallel plate capacitor is given by the relation,

$$C=\frac{\epsilon_0 A}{d}$$
$$A=\frac{Cd}{\epsilon_0}$$

Where,

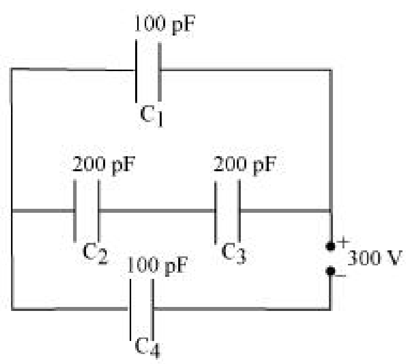
ϵ_0 = Permittivity of free space = 8.85×10^{-12} $C^2 N^{-1} m^{-2}$

$$\therefore A = \frac{2 \times 0.5 \times 10^{-2}}{8.85 \times 10^{-12}} = 1130 \text{ km}^2$$

Hence, the area of the plates is too large. To avoid this situation, the capacitance is taken in the range of μF.

Question 2.25:

Obtain the equivalent capacitance of the network in Fig. 2.35. For a 300 V supply, determine the charge and voltage across each capacitor.



Answer

Capacitance of capacitor C_1 is 100 pF.

Capacitance of capacitor C_2 is 200 pF.

Capacitance of capacitor C_3 is 200 pF.

Capacitance of capacitor C_4 is 100 pF.

Supply potential, V = 300 V

Capacitors C_2 and C_3 are connected in series. Let their equivalent capacitance be C'.

$$\therefore \frac{1}{C'} = \frac{1}{200} + \frac{1}{200} = \frac{2}{200}$$
$$C' = 100 \text{ pF}$$

Capacitors C_1 and C' are in parallel. Let their equivalent capacitance be C''.

$$\therefore C'' = C' + C_1$$
$$= 100 + 100 = 200 \text{ pF}$$

C'' and C_4 are connected in series. Let their equivalent capacitance be C.

$$\therefore \frac{1}{C} = \frac{1}{C''} + \frac{1}{C_4}$$
$$= \frac{1}{200} + \frac{1}{100} = \frac{2+1}{200}$$
$$C = \frac{200}{3} \text{ pF}$$

Hence, the equivalent capacitance of the circuit is $\frac{200}{3}$ pF.

Potential difference across $C'' = V''$

Potential difference across $C_4 = V_4$

$$\therefore V'' + V_4 = V = 300 \text{ V}$$

Charge on C_4 is given by,

$Q_4 = CV$

$$= \frac{200}{3} \times 10^{-12} \times 300$$
$$= 2 \times 10^{-8} \text{ C}$$

$$\therefore V_4 = \frac{Q_4}{C_4}$$
$$= \frac{2 \times 10^{-8}}{100 \times 10^{-12}} = 200 \text{ V}$$

$\therefore$ Voltage across C_1 is given by,

$$V_1 = V - V_4$$
$$= 300 - 200 = 100 \text{ V}$$

Hence, potential difference, V_1, across C_1 is 100 V.

Charge on C_1 is given by,

$$\begin{aligned} Q_1 &= C_1V_1 \\ &= 100\times10^{-12}\times100 \\ &= 10^{-8}\ \text{C} \end{aligned}$$

C_2 and C_3 having same capacitances have a potential difference of 100 V together. Since C_2 and C_3 are in series, the potential difference across C_2 and C_3 is given by,

$V_2 = V_3 = 50$ V

Therefore, charge on C_2 is given by,

$$\begin{aligned} Q_2 &= C_2V_2 \\ &= 200\times10^{-12}\times50 \\ &= 10^{-8}\text{C} \end{aligned}$$

And charge on C_3 is given by,

$$\begin{aligned} Q_3 &= C_3V_3 \\ &= 200\times10^{-12}\times50 \\ &= 10^{-8}\text{C} \end{aligned}$$

Hence, the equivalent capacitance of the given circuit is $\frac{200}{3}$ pF with,

$$\begin{aligned} &Q_1 = 10^{-8}\ \text{C}, && V_1 = 100\ \text{V} \\ &Q_2 = 10^{-8}\ \text{C}, && V_2 = 50\ \text{V} \\ &Q_3 = 10^{-8}\ \text{C}, && V_3 = 50\ \text{V} \\ &Q_4 = 2\times10^{-8}\ \text{C}, && V_4 = 200\ \text{V} \end{aligned}$$

Question 2.26:

The plates of a parallel plate capacitor have an area of 90 cm^2 each and are separated by 2.5 mm. The capacitor is charged by connecting it to a 400 V supply.

How much electrostatic energy is stored by the capacitor?

View this energy as stored in the electrostatic field between the plates, and obtain the energy per unit volume u. Hence arrive at a relation between u and the magnitude of electric field E between the plates.

Answer

Area of the plates of a parallel plate capacitor, A = 90 cm^2 = 90 × 10^{-4} m^2

Distance between the plates, d = 2.5 mm = 2.5 × 10^{-3} m

Potential difference across the plates, V = 400 V

Capacitance of the capacitor is given by the relation,

$$C = \frac{\epsilon_0 A}{d}$$

Electrostatic energy stored in the capacitor is given by the relation, $E_1 = \frac{1}{2} C V^2$

$$= \frac{1}{2} \frac{\epsilon_0 A}{d} V^2$$

Where,

ϵ_0 = Permittivity of free space = 8.85 × 10^{-12} C^2 N^{-1} m^{-2}

$$\therefore E_1 = \frac{1 \times 8.85 \times 10^{-12} \times 90 \times 10^{-4} \times (400)^2}{2 \times 2.5 \times 10^{-3}} = 2.55 \times 10^{-6} \text{ J}$$

Hence, the electrostatic energy stored by the capacitor is 2.55×10^{-6} J.

Volume of the given capacitor,

$$\begin{aligned} V' &= A \times d \\ &= 90 \times 10^{-4} \times 25 \times 10^{-3} \\ &= 2.25 \times 10^{-4} \text{ m}^3 \end{aligned}$$

Energy stored in the capacitor per unit volume is given by,

$$u = \frac{E_1}{V'}$$

$$= \frac{2.55\times10^{-6}}{2.25\times10^{-4}} = 0.113\ \mathrm{J\,m^{-3}}$$

Again, $u = \frac{E_1}{V'}$

$$= \frac{\frac{1}{2}CV^2}{Ad} = \frac{\frac{\epsilon_0 A}{2d}V^2}{Ad} = \frac{1}{2}\epsilon_0\left(\frac{V}{d}\right)^2$$

Where,

$\frac{V}{d}$ = Electric intensity = E

$$\therefore u = \frac{1}{2}\epsilon_0 E^2$$

Question 2.27:

A 4 μF capacitor is charged by a 200 V supply. It is then disconnected from the supply, and is connected to another uncharged 2 μF capacitor. How much electrostatic energy of the first capacitor is lost in the form of heat and electromagnetic radiation?

Answer

Capacitance of a charged capacitor, $C_1 = 4\mu\mathrm{F} = 4\times10^{-6}\ \mathrm{F}$

Supply voltage, $V_1 = 200$ V

Electrostatic energy stored in C_1 is given by,

$$E_1 = \frac{1}{2}C_1V_1^2$$

$$= \frac{1}{2}\times4\times10^{-6}\times(200)^2$$

$$= 8\times10^{-2}\ \mathrm{J}$$

Capacitance of an uncharged capacitor, $C_2 = 2\mu\mathrm{F} = 2\times10^{-6}\ \mathrm{F}$

When C_2 is connected to the circuit, the potential acquired by it is V_2.

According to the conservation of charge, initial charge on capacitor C_1 is equal to the final charge on capacitors, C_1 and C_2.

$$\therefore V_2(C_1 + C_2) = C_1V_1$$
$$V_2 \times (4+2) \times 10^{-6} = 4 \times 10^{-6} \times 200$$
$$V_2 = \frac{400}{3} \text{ V}$$

Electrostatic energy for the combination of two capacitors is given by,

$$E_2 = \frac{1}{2}(C_1 + C_2)V_2^2$$
$$= \frac{1}{2}(2+4) \times 10^{-6} \times \left(\frac{400}{3}\right)^2$$
$$= 5.33 \times 10^{-2} \text{ J}$$

Hence, amount of electrostatic energy lost by capacitor C_1

$= E_1 - E_2$

$= 0.08 - 0.0533 = 0.0267$

$= 2.67 \times 10^{-2}$ J

Question 2.28:

Show that the force on each plate of a parallel plate capacitor has a magnitude equal to (½) QE, where Q is the charge on the capacitor, and E is the magnitude of electric field between the plates. Explain the origin of the factor ½.

Answer

Let F be the force applied to separate the plates of a parallel plate capacitor by a distance of Δx. Hence, work done by the force to do so = $F\Delta x$

As a result, the potential energy of the capacitor increases by an amount given as $uA\Delta x$.

Where,

u = Energy density

A = Area of each plate

d = Distance between the plates

V = Potential difference across the plates

The work done will be equal to the increase in the potential energy i.e.,

$$F\Delta x = uA\Delta x$$
$$F = uA = \left(\frac{1}{2}\epsilon_0 E^2\right)A$$

Electric intensity is given by,

$$E = \frac{V}{d}$$
$$\therefore F = \frac{1}{2}\epsilon_0\left(\frac{V}{d}\right)EA = \frac{1}{2}\left(\epsilon_0 A\frac{V}{d}\right)E$$

However, capacitance, $C = \frac{\epsilon_0 A}{d}$

$$\therefore F = \frac{1}{2}(CV)E$$

Charge on the capacitor is given by,

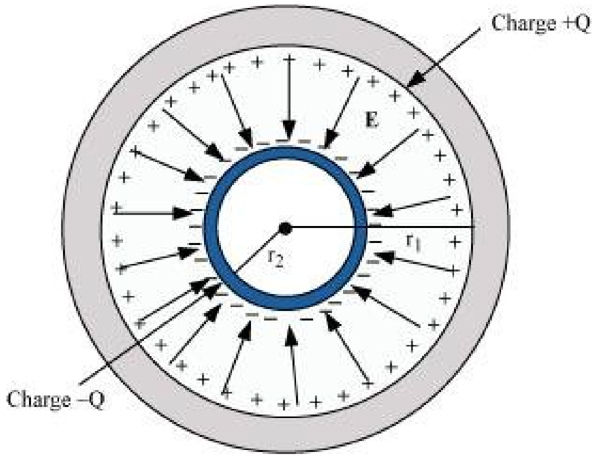
$Q = CV$

$$\therefore F = \frac{1}{2}QE$$

The physical origin of the factor, $\frac{1}{2}$, in the force formula lies in the fact that just outside the conductor, field is E and inside it is zero. Hence, it is the average value, $\frac{E}{2}$, of the field that contributes to the force.

Question 2.29:

A spherical capacitor consists of two concentric spherical conductors, held in position by suitable insulating supports (Fig. 2.36). Show



that the capacitance of a spherical capacitor is given by

$$C = \frac{4\pi \epsilon_0 r_1 r_2}{r_1 - r_2}$$

where r_1 and r_2 are the radii of outer and inner spheres, respectively.

Answer

Radius of the outer shell = r_1

Radius of the inner shell = r_2

The inner surface of the outer shell has charge $+Q$.

The outer surface of the inner shell has induced charge $-Q$.

Potential difference between the two shells is given by,

$$V = \frac{Q}{4\pi \epsilon_0 r_2} - \frac{Q}{4\pi \epsilon_0 r_1}$$

Where,

ϵ_0 = Permittivity of free space

$$V = \frac{Q}{4\pi \in_0}\left[\frac{1}{r_2} - \frac{1}{r_1}\right]$$
$$= \frac{Q(r_1 - r_2)}{4\pi \in_0 r_1 r_2}$$

Capacitance of the given system is given by,

$$C = \frac{\text{Charge}(Q)}{\text{Potential difference}(V)}$$
$$= \frac{4\pi \in_0 r_1 r_2}{r_1 - r_2}$$

Hence, proved.

Question 2.30:

A spherical capacitor has an inner sphere of radius 12 cm and an outer sphere of radius 13 cm. The outer sphere is earthed and the inner sphere is given a charge of 2.5 µC. The space between the concentric spheres is filled with a liquid of dielectric constant 32.

Determine the capacitance of the capacitor.

What is the potential of the inner sphere?

Compare the capacitance of this capacitor with that of an isolated sphere of radius 12 cm. Explain why the latter is much smaller.

Answer

Radius of the inner sphere, r_2 = 12 cm = 0.12 m

Radius of the outer sphere, r_1 = 13 cm = 0.13 m

Charge on the inner sphere, $q = 2.5\mu\text{C} = 2.5 \times 10^{-6}\text{C}$

Dielectric constant of a liquid, $\in_r = 32$

Capacitance of the capacitor is given by the relation,

$$C = \frac{4\pi \epsilon_0 \epsilon_r r_1 r_2}{r_1 - r_2}$$

Where,

ϵ_0 = Permittivity of free space = $8.85\times10^{-12}\ \text{C}^2\ \text{N}^{-1}\ \text{m}^{-2}$

$$\frac{1}{4\pi \epsilon_0} = 9\times10^9\ \text{N m}^2\,\text{C}^{-2}$$

$$\therefore C = \frac{32\times0.12\times0.13}{9\times10^9\times(0.13-0.12)}$$
$$\approx 5.5\times10^{-9}\ \text{F}$$

Hence, the capacitance of the capacitor is approximately $5.5\times10^{-9}\ \text{F}$.

Potential of the inner sphere is given by,

$$V = \frac{q}{C}$$
$$= \frac{2.5\times10^{-6}}{5.5\times10^{-9}} = 4.5\times10^2\ \text{V}$$

Hence, the potential of the inner sphere is $4.5\times10^2\ \text{V}$.

Radius of an isolated sphere, $r = 12 \times 10^{-2}$ m

Capacitance of the sphere is given by the relation,

$$C' = 4\pi \epsilon_0 r$$
$$= 4\pi\times8.85\times10^{-12}\times12\times10^{-12}$$
$$= 1.33\times10^{-11}\ \text{F}$$

The capacitance of the isolated sphere is less in comparison to the concentric spheres. This is because the outer sphere of the concentric spheres is earthed. Hence, the potential difference is less and the capacitance is more than the isolated sphere.

Question 2.31:

Answer carefully:

Two large conducting spheres carrying charges Q_1 and Q_2 are brought close to each other. Is the magnitude of electrostatic force between them exactly given by $Q_1 Q_2/4\pi \epsilon_0 r^2$, where r is the distance between their centres?

If Coulomb's law involved $1/r^3$ dependence (instead of $1/r^2$), would Gauss's law be still true?

A small test charge is released at rest at a point in an electrostatic field configuration. Will it travel along the field line passing through that point?

What is the work done by the field of a nucleus in a complete circular orbit of the electron? What if the orbit is elliptical?

We know that electric field is discontinuous across the surface of a charged conductor. Is electric potential also discontinuous there?

What meaning would you give to the capacitance of a single conductor?

Guess a possible reason why water has a much greater dielectric constant (= 80) than say, mica (= 6).

Answer

The force between two conducting spheres is not exactly given by the expression, $Q_1 Q_2/4\pi \epsilon_0 r^2$, because there is a non-uniform charge distribution on the spheres.

Gauss's law will not be true, if Coulomb's law involved $1/r^3$ dependence, instead of $1/r^2$, on r.

Yes,

If a small test charge is released at rest at a point in an electrostatic field configuration, then it will travel along the field lines passing through the point, only if the field lines are straight. This is because the field lines give the direction of acceleration and not of velocity.

Whenever the electron completes an orbit, either circular or elliptical, the work done by the field of a nucleus is zero.

No

Electric field is discontinuous across the surface of a charged conductor. However, electric potential is continuous.

The capacitance of a single conductor is considered as a parallel plate capacitor with one of its two plates at infinity.

Water has an unsymmetrical space as compared to mica. Since it has a permanent dipole moment, it has a greater dielectric constant than mica.

Question 2.32:

A cylindrical capacitor has two co-axial cylinders of length 15 cm and radii 1.5 cm and 1.4 cm. The outer cylinder is earthed and the inner cylinder is given a charge of 3.5 µC. Determine the capacitance of the system and the potential of the inner cylinder. Neglect end effects (i.e., bending of field lines at the ends).

Answer

Length of a co-axial cylinder, l = 15 cm = 0.15 m

Radius of outer cylinder, r_1 = 1.5 cm = 0.015 m

Radius of inner cylinder, r_2 = 1.4 cm = 0.014 m

Charge on the inner cylinder, q = 3.5 µC = 3.5×10^{-6} C

Capacitance of a co-axial cylinder of radii r_1 and r_2 is given by the relation,

$$C = \frac{2\pi \in_0 l}{\log_e \frac{r_1}{r_2}}$$

Where,

$\in_0$ = Permittivity of free space = $8.85 \times 10^{-12}\ \text{N}^{-1}\ \text{m}^{-2}\ \text{C}^2$

$$\therefore C = \frac{2\pi \times 8.85 \times 10^{-12} \times 0.15}{2.3026\ \log_{10}\left(\frac{0.15}{0.14}\right)}$$

$$= \frac{2\pi \times 8.85 \times 10^{-12} \times 0.15}{2.3026 \times 0.0299} = 1.2 \times 10^{-10}\ \text{F}$$

Potential difference of the inner cylinder is given by,

$$V = \frac{q}{C}$$
$$= \frac{3.5\times10^{-6}}{1.2\times10^{-10}} = 2.92\times10^{4}\ \text{V}$$

Question 2.33:

A parallel plate capacitor is to be designed with a voltage rating 1 kV, using a material of dielectric constant 3 and dielectric strength about 10^7 Vm^{-1}. (Dielectric strength is the maximum electric field a material can tolerate without breakdown, i.e., without starting to conduct electricity through partial ionisation.) For safety, we should like the field never to exceed, say 10% of the dielectric strength. What minimum area of the plates is required to have a capacitance of 50 pF?

Answer

Potential rating of a parallel plate capacitor, V = 1 kV = 1000 V

Dielectric constant of a material, $\epsilon_r = 3$

Dielectric strength = 10^7 V/m

For safety, the field intensity never exceeds 10% of the dielectric strength.

Hence, electric field intensity, E = 10% of $10^7 = 10^6$ V/m

Capacitance of the parallel plate capacitor, C = 50 pF = 50×10^{-12} F

Distance between the plates is given by,

$$d = \frac{V}{E}$$
$$= \frac{1000}{10^6} = 10^{-3}\ \text{m}$$

Capacitance is given by the relation,

$$C = \frac{\epsilon_0 \epsilon_r A}{d}$$

Where,

A = Area of each plate

$\in_0$ = Permittivity of free space = $8.85\times10^{-12}\ \text{N}^{-1}\,\text{C}^2\text{m}^{-2}$

$$\therefore A = \frac{Cd}{\in_0 \in_r}$$
$$= \frac{50\times10^{-12}\times10^{-3}}{8.85\times10^{-12}\times3} \approx 19\ \text{cm}^2$$

Hence, the area of each plate is about 19 cm^2.

Question 2.34:

Describe schematically the equipotential surfaces corresponding to

a constant electric field in the *z*-direction,

a field that uniformly increases in magnitude but remains in a constant (say, *z*) direction,

a single positive charge at the origin, and

a uniform grid consisting of long equally spaced parallel charged wires in a plane.

Answer

Equidistant planes parallel to the *x*-*y* plane are the equipotential surfaces.

Planes parallel to the *x*-*y* plane are the equipotential surfaces with the exception that when the planes get closer, the field increases.

Concentric spheres centered at the origin are equipotential surfaces.

A periodically varying shape near the given grid is the equipotential surface. This shape gradually reaches the shape of planes parallel to the grid at a larger distance.

Question 2.35:

In a Van de Graaff type generator a spherical metal shell is to be a 15×10^6 V electrode. The dielectric strength of the gas surrounding the electrode is 5×10^7 Vm^{-1}. What is the minimum radius of the spherical shell required? (You will learn from this exercise why one cannot build an electrostatic generator using a very small shell which requires a small charge to acquire a high potential.)

Answer

Potential difference, $V = 15 \times 10^6$ V

Dielectric strength of the surrounding gas = 5×10^7 V/m

Electric field intensity, E = Dielectric strength = 5×10^7 V/m

Minimum radius of the spherical shell required for the purpose is given by,

$$r = \frac{V}{E}$$
$$= \frac{15\times10^6}{5\times10^7} = 0.3 \text{ m} = 30 \text{ cm}$$

Hence, the minimum radius of the spherical shell required is 30 cm.

Question 2.36:

A small sphere of radius r_1 and charge q_1 is enclosed by a spherical shell of radius r_2 and charge q_2. Show that if q_1 is positive, charge will necessarily flow from the sphere to the shell (when the two are connected by a wire) no matter what the charge q_2 on the shell is.

Answer

According to Gauss's law, the electric field between a sphere and a shell is determined by the charge q_1 on a small sphere. Hence, the potential difference, V, between the sphere and the shell is independent of charge q_2. For positive charge q_1, potential difference V is always positive.

Question 2.37:

Answer the following:

The top of the atmosphere is at about 400 kV with respect to the surface of the earth, corresponding to an electric field that decreases with altitude. Near the surface of the earth, the field is about 100 Vm^{-1}. Why then do we not get an electric shock as we step out of our house into the open? (Assume the house to be a steel cage so there is no field inside!)

A man fixes outside his house one evening a two metre high insulating slab carrying on its top a large aluminium sheet of area $1m^2$. Will he get an electric shock if he touches the metal sheet next morning?

The discharging current in the atmosphere due to the small conductivity of air is known to be 1800 A on an average over the globe. Why then does the atmosphere not discharge itself completely in due course and become electrically neutral? In other words, what keeps the atmosphere charged?

What are the forms of energy into which the electrical energy of the atmosphere is dissipated during a lightning? (Hint: The earth has an electric field of about 100 Vm^{-1} at its surface in the downward direction, corresponding to a surface charge density = -10^{-9} C m^{-2}. Due to the slight conductivity of the atmosphere up to about 50 km (beyond which it is good conductor), about + 1800 C is pumped every second into the earth as a whole. The earth, however, does not get discharged since thunderstorms and lightning occurring continually all over the globe pump an equal amount of negative charge on the earth.)

Answer

We do not get an electric shock as we step out of our house because the original equipotential surfaces of open air changes, keeping our body and the ground at the same potential.

Yes, the man will get an electric shock if he touches the metal slab next morning. The steady discharging current in the atmosphere charges up the aluminium sheet. As a result, its voltage rises gradually. The raise in the voltage depends on the capacitance of the capacitor formed by the aluminium slab and the ground.

The occurrence of thunderstorms and lightning charges the atmosphere continuously. Hence, even with the presence of discharging current of 1800 A, the atmosphere is not discharged completely. The two opposing currents are in equilibrium and the atmosphere remains electrically neutral.

During lightning and thunderstorm, light energy, heat energy, and sound energy are dissipated in the atmosphere.

Question 3.1:

The storage battery of a car has an emf of 12 V. If the internal resistance of the battery is 0.4Ω, what is the maximum current that can be drawn from the battery?

Answer

Emf of the battery, $E = 12$ V

Internal resistance of the battery, $r = 0.4\ \Omega$

Maximum current drawn from the battery = I

According to Ohm's law,

$$E = Ir$$

$$I = \frac{E}{r}$$

$$= \frac{12}{0.4} = 30 \text{ A}$$

The maximum current drawn from the given battery is 30 A.

Question 3.2:

A battery of emf 10 V and internal resistance 3 Ω is connected to a resistor. If the current in the circuit is 0.5 A, what is the resistance of the resistor? What is the terminal voltage of the battery when the circuit is closed?

Answer

Emf of the battery, $E = 10$ V

Internal resistance of the battery, $r = 3\ \Omega$

Current in the circuit, $I = 0.5$ A

Resistance of the resistor = R

The relation for current using Ohm's law is,

$$I = \frac{E}{R + r}$$

$$R + r = \frac{E}{I}$$

$$= \frac{10}{0.5} = 20\ \Omega$$

$$\therefore R = 20 - 3 = 17\ \Omega$$

Terminal voltage of the resistor = V

According to Ohm's law,

$V = IR$

$= 0.5 \times 17$

$= 8.5$ V

Therefore, the resistance of the resistor is 17 Ω and the terminal voltage is

8.5 V.

Question 3.3:

Three resistors 1 Ω, 2 Ω, and 3 Ω are combined in series. What is the total resistance of the combination?

If the combination is connected to a battery of emf 12 V and negligible internal resistance, obtain the potential drop across each resistor.

Answer

Three resistors of resistances 1 Ω, 2 Ω, and 3 Ω are combined in series. Total resistance of the combination is given by the algebraic sum of individual resistances.

Total resistance = 1 + 2 + 3 = 6 Ω

Current flowing through the circuit = I

Emf of the battery, E = 12 V

Total resistance of the circuit, R = 6 Ω

The relation for current using Ohm's law is,

$$I = \frac{E}{R}$$
$$= \frac{12}{6} = 2 \text{ A}$$

Potential drop across 1 Ω resistor = V_1

From Ohm's law, the value of V_1 can be obtained as

$V_1 = 2 \times 1 = 2$ V … (i)

Potential drop across 2 Ω resistor = V_2

Again, from Ohm's law, the value of V_2 can be obtained as

$V_2 = 2 \times 2 = 4$ V … (ii)

Potential drop across 3 Ω resistor = V_3

Again, from Ohm's law, the value of V_3 can be obtained as

$V_3 = 2 \times 3 = 6$ V … (iii)

Therefore, the potential drop across 1 Ω, 2 Ω, and 3 Ω resistors are 2 V, 4 V, and 6 V respectively.

Question 3.4:

Three resistors 2 Ω, 4 Ω and 5 Ω are combined in parallel. What is the total resistance of the combination?

If the combination is connected to a battery of emf 20 V and negligible internal resistance, determine the current through each resistor, and the total current drawn from the battery.

Answer

There are three resistors of resistances,

$R_1 = 2\ \Omega$, $R_2 = 4\ \Omega$, and $R_3 = 5\ \Omega$

They are connected in parallel. Hence, total resistance (R) of the combination is given by,

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$
$$= \frac{1}{2} + \frac{1}{4} + \frac{1}{5} = \frac{10+5+4}{20} = \frac{19}{20}$$
$$\therefore R = \frac{20}{19}\ \Omega$$

Therefore, total resistance of the combination is $\frac{20}{19}\ \Omega$.

Emf of the battery, $V = 20$ V

Current (I_1) flowing through resistor R_1 is given by,

$$I_1 = \frac{V}{R_1}$$
$$= \frac{20}{2} = 10\ \text{A}$$

Current (I_2) flowing through resistor R_2 is given by,

$$I_2 = \frac{V}{R_2}$$
$$= \frac{20}{4} = 5\ \text{A}$$

Current (I_3) flowing through resistor R_3 is given by,

$$I_3 = \frac{V}{R_3}$$
$$= \frac{20}{5} = 4\ \text{A}$$

Total current, $I = I_1 + I_2 + I_3 = 10 + 5 + 4 = 19$ A

Therefore, the current through each resister is 10 A, 5 A, and 4 A respectively and the total current is 19 A.

Question 3.5:

At room temperature (27.0 °C) the resistance of a heating element is 100 Ω. What is the temperature of the element if the resistance is found to be 117 Ω, given that the temperature coefficient of the material of the resistor is $1.70 \times 10^{-4}\ °C^{-1}$

Answer

Room temperature, $T = 27°C$

Resistance of the heating element at T, $R = 100\ \Omega$

Let T_1 is the increased temperature of the filament.

Resistance of the heating element at T_1, $R_1 = 117\ \Omega$

Temperature co-efficient of the material of the filament,

$\alpha = 1.70 \times 10^{-4} °C^{-1}$

α is given by the relation,

$$\alpha = \frac{R_1 - R}{R(T_1 - T)}$$

$$T_1 - T = \frac{R_1 - R}{R\alpha}$$

$$T_1 - 27 = \frac{117 - 100}{100(1.7 \times 10^{-4})}$$

$$T_1 - 27 = 1000$$

$$T_1 = 1027\,°C$$

Therefore, at 1027°C, the resistance of the element is 117Ω.

Question 3.6:

A negligibly small current is passed through a wire of length 15 m and uniform cross-section 6.0×10^{-7} m^2, and its resistance is measured to be 5.0 Ω. What is the resistivity of the material at the temperature of the experiment?

<u>Answer</u>

Length of the wire, l =15 m

Area of cross-section of the wire, $a = 6.0 \times 10^{-7}$ m^2

Resistance of the material of the wire, $R = 5.0\ \Omega$

Resistivity of the material of the wire = ρ

Resistance is related with the resistivity as

$$R = \rho \frac{l}{A}$$
$$\rho = \frac{RA}{l}$$
$$= \frac{5 \times 6 \times 10^{-7}}{15} = 2 \times 10^{-7}\ \Omega\ \text{m}$$

Therefore, the resistivity of the material is $2 \times 10^{-7}\ \Omega$ m.

Question 3.7:

A silver wire has a resistance of 2.1 Ω at 27.5 °C, and a resistance of 2.7 Ω at 100 °C. Determine the temperature coefficient of resistivity of silver.

<u>Answer</u>

Temperature, $T_1 = 27.5$°C

Resistance of the silver wire at T_1, $R_1 = 2.1\ \Omega$

Temperature, $T_2 = 100°C$

Resistance of the silver wire at T_2, $R_2 = 2.7\ \Omega$

Temperature coefficient of silver = α

It is related with temperature and resistance as

$$\alpha = \frac{R_2 - R_1}{R_1(T_2 - T_1)}$$
$$= \frac{2.7 - 2.1}{2.1(100 - 27.5)} = 0.0039\ °C^{-1}$$

Therefore, the temperature coefficient of silver is $0.0039°C^{-1}$.

Question 3.8:

Aheating element using nichrome connected to a 230 V supply draws an initial current of 3.2 A which settles after a few seconds toa steady value of 2.8 A. What is the steady temperature of the heating element if the room temperature is 27.0 °C? Temperature coefficient of resistance of nichrome averaged over the temperature range involved is $1.70 \times 10^{-4}\ °C^{-1}$.

Answer

Supply voltage, $V = 230$ V

Initial current drawn, $I_1 = 3.2$ A

Initial resistance = R_1, which is given by the relation,

$$R_1 = \frac{V}{I}$$
$$= \frac{230}{3.2} = 71.87\ \Omega$$

Steady state value of the current, $I_2 = 2.8$ A

Resistance at the steady state = R_2, which is given as

$$R_2 = \frac{230}{2.8} = 82.14\ \Omega$$

Temperature co-efficient of nichrome, $\alpha = 1.70 \times 10^{-4}\ °C^{-1}$

Initial temperature of nichrome, T_1= 27.0°C

Study state temperature reached by nichrome = T_2

T_2 can be obtained by the relation for α,

$$\alpha = \frac{R_2 - R_1}{R_1 (T_2 - T_1)}$$

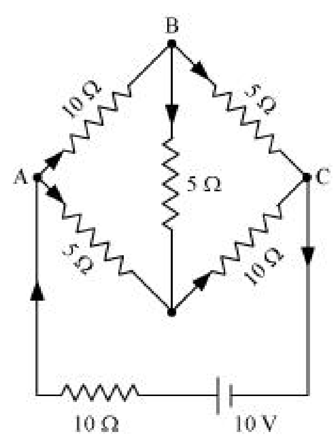
$$T_2 - 27\ °C = \frac{82.14 - 71.87}{71.87 \times 1.7 \times 10^{-4}} = 840.5$$

$$T_2 = 840.5 + 27 = 867.5\ °C$$

Therefore, the steady temperature of the heating element is 867.5°C

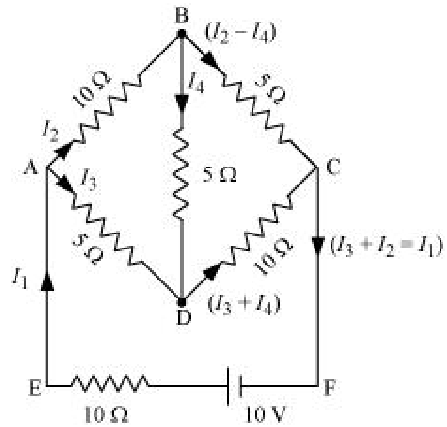
Question 3.9:

Determine the current in each branch of the network shown in fig 3.30:



Answer

Current flowing through various branches of the circuit is represented in the given figure.



I_1 = Current flowing through the outer circuit

I_2 = Current flowing through branch AB

I_3 = Current flowing through branch AD

$I_2 - I_4$ = Current flowing through branch BC

$I_3 + I_4$ = Current flowing through branch CD

I_4 = Current flowing through branch BD

For the closed circuit ABDA, potential is zero i.e.,

$10I_2 + 5I_4 - 5I_3 = 0$

$2I_2 + I_4 - I_3 = 0$

$I_3 = 2I_2 + I_4 \ldots (1)$

For the closed circuit BCDB, potential is zero i.e.,

$5(I_2 - I_4) - 10(I_3 + I_4) - 5I_4 = 0$

$5I_2 + 5I_4 - 10I_3 - 10I_4 - 5I_4 = 0$

$5I_2 - 10I_3 - 20I_4 = 0$

$I_2 = 2I_3 + 4I_4 \ldots (2)$

For the closed circuit ABCFEA, potential is zero i.e.,

$-10 + 10(I_1) + 10(I_2) + 5(I_2 - I_4) = 0$

$10 = 15I_2 + 10I_1 - 5I_4$

$3I_2 + 2I_1 - I_4 = 2$ … (3)

From equations (1) and (2), we obtain

$I_3 = 2(2I_3 + 4I_4) + I_4$

$I_3 = 4I_3 + 8I_4 + I_4$

$3I_3 = 9I_4$

$3I_4 = + I_3$ … (4)

Putting equation (4) in equation (1), we obtain

$I_3 = 2I_2 + I_4$

$4I_4 = 2I_2$

$I_2 = - 2I_4$ … (5)

It is evident from the given figure that,

$I_1 = I_3 + I_2$ … (6)

Putting equation (6) in equation (1), we obtain

$3I_2 + 2(I_3 + I_2) - I_4 = 2$

$5I_2 + 2I_3 - I_4 = 2$ … (7)

Putting equations (4) and (5) in equation (7), we obtain

$5(-2\ I_4) + 2(-\ 3\ I_4) - I_4 = 2$

$10I_4 - 6I_4 - I_4 = 2$

$17I_4 = - 2$

$$I_4 = \frac{-2}{17}\ \text{A}$$

Equation (4) reduces to

$I_3 = - 3(I_4)$

$$= -3\left(\frac{-2}{17}\right) = \frac{6}{17}\ \text{A}$$

$$I_2 = -2(I_4)$$

$$= -2\left(\frac{-2}{17}\right) = \frac{4}{17}\ \text{A}$$

$$I_2 - I_4 = \frac{4}{17} - \left(\frac{-2}{17}\right) = \frac{6}{17}\ \text{A}$$

$$I_3 + I_4 = \frac{6}{17} + \left(\frac{-2}{17}\right) = \frac{4}{17}\ \text{A}$$

$$I_1 = I_3 + I_2$$

$$= \frac{6}{17} + \frac{4}{17} = \frac{10}{17}\ \text{A}$$

Therefore, current in branch $\text{AB} = \frac{4}{17}\ \text{A}$

In branch BC = $\frac{6}{17}\ \text{A}$

In branch CD = $\frac{-4}{17}\ \text{A}$

In branch AD $= \frac{6}{17}\ \text{A}$

In branch BD = $\left(\frac{-2}{17}\right)\ \text{A}$

Total current = $\frac{4}{17} + \frac{6}{17} + \frac{-4}{17} + \frac{6}{17} + \frac{-2}{17} = \frac{10}{17}\ \text{A}$

Question 3.10:

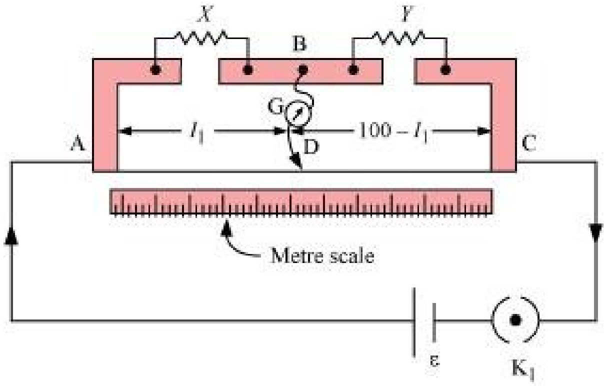
In a metre bridge [Fig. 3.27], the balance point is found to be at 39.5 cm from the end *A*, when the resistor *Y* is of 12.5 Ω. Determine the resistance of *X*. Why are the connections between resistors in a Wheatstone or meter bridge made of thick copper strips?

Determine the balance point of the bridge above if *X* and *Y* are interchanged.

What happens if the galvanometer and cell are interchanged at the balance point of the bridge? Would the galvanometer show any current?

Answer

A metre bridge with resistors X and Y is represented in the given figure.



Balance point from end A, $l_1 = 39.5$ cm

Resistance of the resistor $Y = 12.5\ \Omega$

Condition for the balance is given as,

$$\frac{X}{Y} = \frac{100 - l_1}{l_1}$$

$$X = \frac{100 - 39.5}{39.5} \times 12.5 = 8.2\ \Omega$$

Therefore, the resistance of resistor X is 8.2 Ω.

The connection between resistors in a Wheatstone or metre bridge is made of thick copper strips to minimize the resistance, which is not taken into consideration in the bridge formula.

If X and Y are interchanged, then l_1 and $100-l_1$ get interchanged.

The balance point of the bridge will be $100-l_1$ from A.

$100-l_1 = 100 - 39.5 = 60.5$ cm

Therefore, the balance point is 60.5 cm from A.

When the galvanometer and cell are interchanged at the balance point of the bridge, the galvanometer will show no deflection. Hence, no current would flow through the galvanometer.

Question 3.11:

A storage battery of emf 8.0 V and internal resistance 0.5 Ω is being charged by a 120 V dc supply using a series resistor of 15.5 Ω. What is the terminal voltage of the battery during charging? What is the purpose of having a series resistor in the charging circuit?

Answer

Emf of the storage battery, $E = 8.0$ V

Internal resistance of the battery, $r = 0.5\ \Omega$

DC supply voltage, $V = 120$ V

Resistance of the resistor, $R = 15.5\ \Omega$

Effective voltage in the circuit = V^1

R is connected to the storage battery in series. Hence, it can be written as

$V^1 = V - E$

$V^1 = 120 - 8 = 112$ V

Current flowing in the circuit = I, which is given by the relation,

$$I = \frac{V^1}{R+r}$$
$$= \frac{112}{15.5+5} = \frac{112}{16} = 7\text{ A}$$

Voltage across resistor R given by the product, $IR = 7 \times 15.5 = 108.5$ V

DC supply voltage = Terminal voltage of battery + Voltage drop across R

Terminal voltage of battery = 120 − 108.5 = 11.5 V

A series resistor in a charging circuit limits the current drawn from the external source. The current will be extremely high in its absence. This is very dangerous.

Question 3.12:

In a potentiometer arrangement, a cell of emf 1.25 V gives a balance point at 35.0 cm length of the wire. If the cell is replaced by another cell and the balance point shifts to 63.0 cm, what is the emf of the second cell?

Answer

Emf of the cell, $E_1 = 1.25$ V

Balance point of the potentiometer, $l_1 = 35$ cm

The cell is replaced by another cell of emf E_2.

New balance point of the potentiometer, $l_2 = 63$ cm

The balance condition is given by the relation,

$$\frac{E_1}{E_2} = \frac{l_1}{l_2}$$

$$E_2 = E_1 \times \frac{l_2}{l_1}$$

$$= 1.25 \times \frac{63}{35} = 2.25 \text{ V}$$

Therefore, emf of the second cell is 2.25V.

Question 3.13:

The number density of free electrons in a copper conductor estimated in Example 3.1 is 8.5×10^{28} m^{-3}. How long does an electron take to drift from one end of a wire 3.0 m long to its other end? The area of cross-section of the wire is 2.0×10^{-6} m^2 and it is carrying a current of 3.0 A.

Answer

Number density of free electrons in a copper conductor, $n = 8.5 \times 10^{28}\ \text{m}^{-3}$ Length of the copper wire, $l = 3.0$ m

Area of cross-section of the wire, $A = 2.0 \times 10^{-6}\ \text{m}^2$

Current carried by the wire, $I = 3.0$ A, which is given by the relation,

$I = nAeV_d$

Where,

e = Electric charge = 1.6×10^{-19} C

V_d = Drift velocity $= \dfrac{\text{Length of the wire}(l)}{\text{Time taken to cover } l(t)}$

$$I = nAe\frac{l}{t}$$

$$t = \frac{nAel}{I}$$

$$= \frac{3 \times 8.5 \times 10^{28} \times 2 \times 10^{-6} \times 1.6 \times 10^{-19}}{3.0}$$

$$= 2.7 \times 10^{4}\ \text{s}$$

Therefore, the time taken by an electron to drift from one end of the wire to the other is 2.7×10^4 s.

Question 3.14:

The earth's surface has a negative surface charge density of $10^{-9}\ \text{C m}^{-2}$. The potential difference of 400 kV between the top of the atmosphere and the surface results (due to the low conductivity of the lower atmosphere) in a current of only 1800 A over the entire globe. If there were no mechanism of sustaining atmospheric electric field, how much time (roughly) would be required to neutralise the earth's surface? (This never happens in practice because there is a mechanism to replenish electric charges, namely the continual thunderstorms and lightning in different parts of the globe). (Radius of earth = 6.37×10^6 m.)

Answer

Surface charge density of the earth, $\sigma = 10^{-9}$ C m^{-2}

Current over the entire globe, $I = 1800$ A

Radius of the earth, $r = 6.37 \times 10^{6}$ m

Surface area of the earth,

$A = 4\pi r^2$

$= 4\pi \times (6.37 \times 10^{6})^2$

$= 5.09 \times 10^{14}$ m^2

Charge on the earth surface,

$q = \sigma \times A$

$= 10^{-9} \times 5.09 \times 10^{14}$

$= 5.09 \times 10^{5}$ C

Time taken to neutralize the earth's surface = t

Current, $I = \frac{q}{t}$

$$t = \frac{q}{I}$$

$$= \frac{5.09 \times 10^{5}}{1800} = 282.77 \text{ s}$$

Therefore, the time taken to neutralize the earth's surface is 282.77 s.

Question 3.15:

Six lead-acid type of secondary cells each of emf 2.0 V and internal resistance 0.015 Ω are joined in series to provide a supply to a resistance of 8.5 Ω. What are the current drawn from the supply and its terminal voltage?

A secondary cell after long use has an emf of 1.9 V and a large internal resistance of 380 Ω. What maximum current can be drawn from the cell? Could the cell drive the starting motor of a car?

Answer

Number of secondary cells, $n = 6$

Emf of each secondary cell, $E = 2.0$ V

Internal resistance of each cell, $r = 0.015\ \Omega$

series resistor is connected to the combination of cells.

Resistance of the resistor, $R = 8.5\ \Omega$

Current drawn from the supply = I, which is given by the relation,

$$I = \frac{nE}{R + nr}$$
$$= \frac{6 \times 2}{8.5 + 6 \times 0.015}$$
$$= \frac{12}{8.59} = 1.39 \text{ A}$$

Terminal voltage, $V = IR = 1.39 \times 8.5 = 11.87$ A

Therefore, the current drawn from the supply is 1.39 A and terminal voltage is

11.87 A.

After a long use, emf of the secondary cell, $E = 1.9$ V

Internal resistance of the cell, $r = 380\ \Omega$

Hence, maximum current $= \frac{E}{r} = \frac{1.9}{380} = 0.005 \text{ A}$

Therefore, the maximum current drawn from the cell is 0.005 A. Since a large current is required to start the motor of a car, the cell cannot be used to start a motor.

Question 3.16:

Two wires of equal length, one of aluminium and the other of copper have the same resistance. Which of the two wires is lighter? Hence explain why aluminium wires are preferred for overhead power cables. ($\rho_{Al} = 2.63 \times 10^{-8}\ \Omega$ m, $\rho_{Cu} = 1.72 \times 10^{-8}\ \Omega$ m, Relative density of Al = 2.7, of Cu = 8.9.)

Answer

Resistivity of aluminium, $\rho_{Al} = 2.63 \times 10^{-8}\ \Omega$ m

Relative density of aluminium, $d_1 = 2.7$

Let l_1 be the length of aluminium wire and m_1 be its mass.

Resistance of the aluminium wire = R_1

Area of cross-section of the aluminium wire = A_1

Resistivity of copper, $\rho_{Cu} = 1.72 \times 10^{-8}\ \Omega$ m

Relative density of copper, $d_2 = 8.9$

Let l_2 be the length of copper wire and m_2 be its mass.

Resistance of the copper wire = R_2

Area of cross-section of the copper wire = A_2

The two relations can be written as

$$R_1 = \rho_1 \frac{l_1}{A_1} \quad \ldots (1)$$

$$R_2 = \rho_2 \frac{l_2}{A_2} \quad \ldots (2)$$

It is given that,

$$R_1 = R_2$$

$$\therefore \rho_1 \frac{l_1}{A_1} = \rho_2 \frac{l_2}{A_2}$$

And,

$$l_1 = l_2$$
$$\therefore \frac{\rho_1}{A_1} = \frac{\rho_2}{A_2}$$
$$\frac{A_1}{A_2} = \frac{\rho_1}{\rho_2}$$
$$= \frac{2.63 \times 10^{-8}}{1.72 \times 10^{-8}} = \frac{2.63}{1.72}$$

Mass of the aluminium wire,

m_1 = Volume × Density

$= A_1 l_1 \times d_1 = A_1\ l_1 d_1$ … (3)

Mass of the copper wire,

m_2 = Volume × Density

$= A_2 l_2 \times d_2 = A_2\ l_2 d_2$ … (4)

Dividing equation (3) by equation (4), we obtain

$$\frac{m_1}{m_2} = \frac{A_1 l_1 d_1}{A_2 l_2 d_2}$$
For $l_1 = l_2$,
$$\frac{m_1}{m_2} = \frac{A_1 d_1}{A_2 d_2}$$
For $\frac{A_1}{A_2} = \frac{2.63}{1.72}$,
$$\frac{m_1}{m_2} = \frac{2.63}{1.72} \times \frac{2.7}{8.9} = 0.46$$

It can be inferred from this ratio that m_1 is less than m_2. Hence, aluminium is lighter than copper.

Since aluminium is lighter, it is preferred for overhead power cables over copper.

Question 3.17:

What conclusion can you draw from the following observations on a resistor made of alloy manganin?

Current **A**	**Voltage** **V**	**Current** **A**	**Voltage** **V**
0.2	3.94	3.0	59.2
0.4	7.87	4.0	78.8
0.6	11.8	5.0	98.6
0.8	15.7	6.0	118.5
1.0	19.7	7.0	138.2
2.0	39.4	8.0	158.0

Answer

It can be inferred from the given table that the ratio of voltage with current is a constant, which is equal to 19.7. Hence, manganin is an ohmic conductor i.e., the alloy obeys Ohm's law. According to Ohm's law, the ratio of voltage with current is the resistance of the conductor. Hence, the resistance of manganin is 19.7 Ω.

Question 3.18:

Answer the following questions:

A steady current flows in a metallic conductor of non-uniform cross- section. Which of these quantities is constant along the conductor: current, current density, electric field, drift speed?

Is Ohm's law universally applicable for all conducting elements?

If not, give examples of elements which do not obey Ohm's law.

A low voltage supply from which one needs high currents must have very low internal resistance. Why?

A high tension (HT) supply of, say, 6 kV must have a very large internal resistance. Why?

Answer

When a steady current flows in a metallic conductor of non-uniform cross-section, the current flowing through the conductor is constant. Current density, electric field, and drift speed are inversely proportional to the area of cross-section. Therefore, they are not constant.

No, Ohm’s law is not universally applicable for all conducting elements. Vacuum diode semi-conductor is a non-ohmic conductor. Ohm’s law is not valid for it.

According to Ohm’s law, the relation for the potential is $V = IR$

Voltage (V) is directly proportional to current (I).

R is the internal resistance of the source.

$$I = \frac{V}{R}$$

If V is low, then R must be very low, so that high current can be drawn from the source.

In order to prohibit the current from exceeding the safety limit, a high tension supply must have a very large internal resistance. If the internal resistance is not large, then the current drawn can exceed the safety limits in case of a short circuit.

Question 3.19:

Choose the correct alternative:

Alloys of metals usually have (greater/less) resistivity than that of their constituent metals.

Alloys usually have much (lower/higher) temperature coefficients of resistance than pure metals.

The resistivity of the alloy manganin is nearly independent of/increases rapidly with increase of temperature.

The resistivity of a typical insulator (e.g., amber) is greater than that of a metal by a factor of the order of ($10^{22}/10^{3}$).

Answer

Alloys of metals usually have greater resistivity than that of their constituent metals.

Alloys usually have lower temperature coefficients of resistance than pure metals.

The resistivity of the alloy, manganin, is nearly independent of increase of temperature.

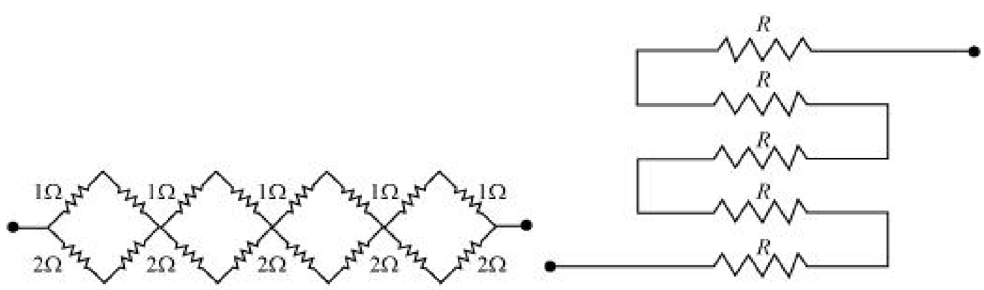
The resistivity of a typical insulator is greater than that of a metal by a factor of the order of 10^{22}.

Question 3.20:

Given n resistors each of resistance R, how will you combine them to get the (i) maximum (ii) minimum effective resistance? What is the ratio of the maximum to minimum resistance?

Given the resistances of 1 Ω, 2 Ω, 3 Ω, how will be combine them to get an equivalent resistance of (i) (11/3) Ω (ii) (11/5) Ω, (iii) 6 Ω, (iv) (6/11) Ω?

Determine the equivalent resistance of networks shown in Fig. 3.31.



(a) (b)

<u>Answer</u>

Total number of resistors = n

Resistance of each resistor = R

When n resistors are connected in series, effective resistance R_1 is the maximum, given by the product nR.

Hence, maximum resistance of the combination, $R_1 = nR$

When n resistors are connected in parallel, the effective resistance (R_2) is the minimum, given by the ratio $\frac{R}{n}$.

Hence, minimum resistance of the combination, $R_2 = \frac{R}{n}$

The ratio of the maximum to the minimum resistance is,

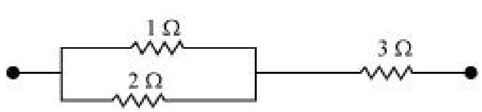
$$\frac{R_1}{R_2} = \frac{nR}{\frac{R}{n}} = n^2$$

The resistance of the given resistors is,

$R_1 = 1\ \Omega$, $R_2 = 2\ \Omega$, $R_3 = 3\ \Omega2$

Equivalent resistance, $R' = \frac{11}{3}\ \Omega$

Consider the following combination of the resistors.

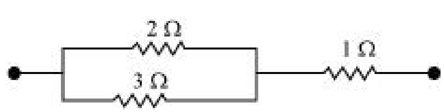


Equivalent resistance of the circuit is given by,

$$R' = \frac{2\times1}{2+1} + 3 = \frac{2}{3} + 3 = \frac{11}{3}\ \Omega$$

Equivalent resistance, $R' = \frac{11}{5}\ \Omega$

Consider the following combination of the resistors.



Equivalent resistance of the circuit is given by,

$$R' = \frac{2\times3}{2+3} + 1 = \frac{6}{5} + 1 = \frac{11}{5}\ \Omega$$

Equivalent resistance, $R' = 6\ \Omega$

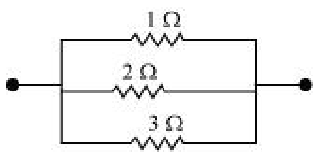
Consider the series combination of the resistors, as shown in the given circuit.

Equivalent resistance of the circuit is given by the sum,

$R' = 1 + 2 + 3 = 6\ \Omega$

Equivalent resistance, $R' = \frac{6}{11}\ \Omega$

Consider the series combination of the resistors, as shown in the given circuit.



Equivalent resistance of the circuit is given by,

$$R' = \frac{1\times2\times3}{1\times+2\times3+3\times1} = \frac{6}{11}\ \Omega$$

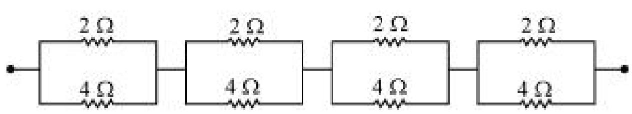
(a) It can be observed from the given circuit that in the first small loop, two resistors of resistance 1 Ω each are connected in series.

Hence, their equivalent resistance = (1+1) = 2 Ω

It can also be observed that two resistors of resistance 2 Ω each are connected in series.

Hence, their equivalent resistance = (2 + 2) = 4 Ω.

Therefore, the circuit can be redrawn as



It can be observed that 2 Ω and 4 Ω resistors are connected in parallel in all the four loops. Hence, equivalent resistance (R') of each loop is given by,

$$R' = \frac{2\times4}{2+4} = \frac{8}{6} = \frac{4}{3}\ \Omega$$

The circuit reduces to,

All the four resistors are connected in series.

Hence, equivalent resistance of the given circuit is $\frac{4}{3} \times 4 = \frac{16}{3}\ \Omega$

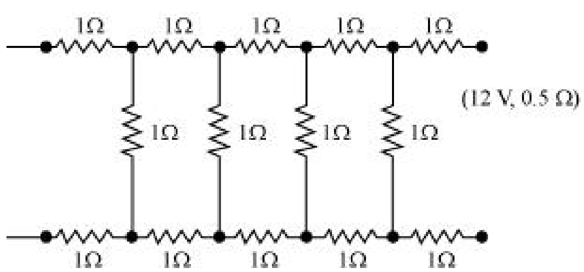
It can be observed from the given circuit that five resistors of resistance R each are connected in series.

Hence, equivalent resistance of the circuit = $R + R + R + R + R$

$= 5\ R$

Question 3.21:

Determine the current drawn from a 12 V supply with internal resistance 0.5 Ω by the infinite network shown in Fig. 3.32. Each resistor has 1 Ω resistance.



Answer

The resistance of each resistor connected in the given circuit, $R = 1\ \Omega$

Equivalent resistance of the given circuit = R'

The network is infinite. Hence, equivalent resistance is given by the relation,

$$\therefore R' = 2 + \frac{R'}{(R'+1)}$$

$$(R')^2 - 2R' - 2 = 0$$

$$R' = \frac{2 \pm \sqrt{4+8}}{2}$$

$$= \frac{2 \pm \sqrt{12}}{2} = 1 \pm \sqrt{3}$$

Negative value of R' cannot be accepted. Hence, equivalent resistance,

$$R' = \left(1+\sqrt{3}\right) = 1 + 1.73 = 2.73\ \Omega$$

Internal resistance of the circuit, $r = 0.5\ \Omega$

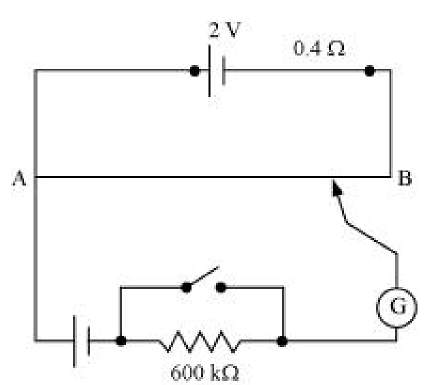
Hence, total resistance of the given circuit = 2.73 + 0.5 = 3.23 Ω

Supply voltage, $V = 12$ V

According to Ohm's Law, current drawn from the source is given by the ratio, $\frac{12}{3.23}$ = 3.72 A

Question 3.22:

Figure 3.33 shows a potentiometer with a cell of 2.0 V and internal resistance 0.40 Ω maintaining a potential drop across the resistor wire AB. A standard cell which maintains a constant emf of 1.02 V (for very moderate currents up to a few mA) gives a balance point at 67.3 cm length of the wire. To ensure very low currents drawn from the standard cell, a very high resistance of 600 kΩ is put in series with it, which is shorted close to the balance point. The standard cell is then replaced by a cell of unknown emf ε and the balance point found similarly, turns out to be at 82.3 cm length of the wire.



What is the value ε ?

What purpose does the high resistance of 600 kΩ have?

Is the balance point affected by this high resistance?

Is the balance point affected by the internal resistance of the driver cell?

Would the method work in the above situation if the driver cell of the potentiometer had an emf of 1.0 V instead of 2.0 V?

(f) Would the circuit work well for determining an extremely small emf, say of the order of a few mV (such as the typical emf of a thermo-couple)? If not, how will you modify the circuit?

Answer

Constant emf of the given standard cell, E_1 = 1.02 V

Balance point on the wire, l_1 = 67.3 cm

A cell of unknown emf, ε,replaced the standard cell. Therefore, new balance point on the wire, l = 82.3 cm

The relation connecting emf and balance point is,

$$\frac{E_1}{l_1} = \frac{\varepsilon}{l}$$

$$\varepsilon = \frac{l}{l_1} \times E_1$$

$$= \frac{82.3}{67.3} \times 1.02 = 1.247 \text{ V}$$

The value of unknown emfis 1.247 V.

The purpose of using the high resistance of 600 kΩ is to reduce the current through the galvanometer when the movable contact is far from the balance point.

The balance point is not affected by the presence of high resistance.

The point is not affected by the internal resistance of the driver cell.

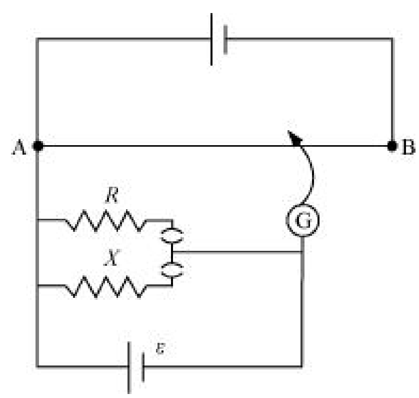
The method would not work if the driver cell of the potentiometer had an emf of 1.0 V instead of 2.0 V. This is because if the emf of the driver cell of the potentiometer is less than the emf of the other cell, then there would be no balance point on the wire.

The circuit would not work well for determining an extremely small emf. As the circuit would be unstable, the balance point would be close to end A. Hence, there would be a large percentage of error.

The given circuit can be modified if a series resistance is connected with the wire AB. The potential drop across AB is slightly greater than the emf measured. The percentage error would be small.

Question 3.23:

Figure 3.34 shows a potentiometer circuit for comparison of two resistances. The balance point with a standard resistor $R = 10.0\ \Omega$ is found to be 58.3 cm, while that with the unknown resistance X is 68.5 cm. Determine the value of X. What might you do if you failed to find a balance point with the given cell of emf ε?



Answer

Resistance of the standard resistor, $R = 10.0\ \Omega$

Balance point for this resistance, $l_1 = 58.3$ cm

Current in the potentiometer wire = i

Hence, potential drop across R, $E_1 = iR$

Resistance of the unknown resistor = X

Balance point for this resistor, $l_2 = 68.5$ cm

Hence, potential drop across X, $E_2 = iX$

The relation connecting emf and balance point is,

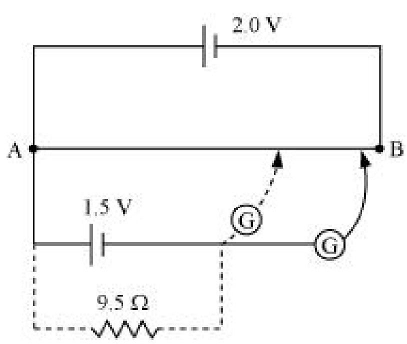
$$\frac{E_1}{E_2} = \frac{l_1}{l_2}$$
$$\frac{iR}{iX} = \frac{l_1}{l_2}$$
$$X = \frac{l_1}{l_2} \times R$$
$$= \frac{68.5}{58.3} \times 10 = 11.749\ \Omega$$

Therefore, the value of the unknown resistance, X, is 11.75 Ω.

If we fail to find a balance point with the given cell of emf, ε, then the potential drop across R and X must be reduced by putting a resistance in series with it. Only if the potential drop across R or X is smaller than the potential drop across the potentiometer wire AB, a balance point is obtained.

Question 3.24:

Figure 3.35 shows a 2.0 V potentiometer used for the determination of internal resistance of a 1.5 V cell. The balance point of the cell in open circuit is 76.3 cm. When a resistor of 9.5 Ω is used in the external circuit of the cell, the balance point shifts to 64.8 cm length of the potentiometer wire. Determine the internal resistance of the cell.



Answer

Internal resistance of the cell = r

Balance point of the cell in open circuit, l_1 = 76.3 cm

An external resistance (R) is connected to the circuit with R = 9.5 Ω

New balance point of the circuit, l_2 = 64.8 cm

Current flowing through the circuit = I

The relation connecting resistance and emf is,

$$r = \left(\frac{l_1 - l_2}{l_2}\right)R$$
$$= \frac{76.3 - 64.8}{64.8} \times 9.5 = 1.68\ \Omega$$

Therefore, the internal resistance of the cell is 1.68Ω.

Question 4.1:

A circular coil of wire consisting of 100 turns, each of radius 8.0 cm carries a current of 0.40 A. What is the magnitude of the magnetic field **B** at the centre of the coil?

Answer

Number of turns on the circular coil, $n = 100$

Radius of each turn, $r = 8.0$ cm = 0.08 m

Current flowing in the coil, $I = 0.4$ A

Magnitude of the magnetic field at the centre of the coil is given by the relation,

$$|\mathbf{B}| = \frac{\mu_0}{4\pi}\frac{2\pi nI}{r}$$

Where,

μ_0 = Permeability of free space

$= 4\pi \times 10^{-7}$ T m A^{-1}

$$|\mathbf{B}| = \frac{4\pi \times 10^{-7}}{4\pi} \times \frac{2\pi \times 100 \times 0.4}{0.08}$$
$$= 3.14 \times 10^{-4} \text{ T}$$

Hence, the magnitude of the magnetic field is 3.14×10^{-4} T.

Question 4.2:

A long straight wire carries a current of 35 A. What is the magnitude of the field **B** at a point 20 cm from the wire?

Answer

Current in the wire, I = 35 A

Distance of a point from the wire, r = 20 cm = 0.2 m

Magnitude of the magnetic field at this point is given as:

$$B = \frac{\mu_0}{4\pi}\frac{2I}{r}$$

Where,

μ_0 = Permeability of free space = $4\pi \times 10^{-7}$ T m A^{-1}

$$B = \frac{4\pi \times 10^{-7} \times 2 \times 35}{4\pi \times 0.2}$$
$$= 3.5 \times 10^{-5} \text{ T}$$

Hence, the magnitude of the magnetic field at a point 20 cm from the wire is 3.5×10^{-5} T.

Question 4.3:

A long straight wire in the horizontal plane carries a current of 50 A in north to south direction. Give the magnitude and direction of **B** at a point 2.5 m east of the wire.

<u>Answer</u>

Current in the wire, I = 50 A

A point is 2.5 m away from the East of the wire.

∴ Magnitude of the distance of the point from the wire, r = 2.5 m.

Magnitude of the magnetic field at that point is given by the relation, $B = \frac{\mu_0 2I}{4\pi r}$

Where,

μ_0 = Permeability of free space = $4\pi \times 10^{-7}$ T m A^{-1}

$$B = \frac{4\pi \times 10^{-7} \times 2 \times 50}{4\pi \times 2.5}$$
$$= 4 \times 10^{-6} \text{ T}$$

The point is located normal to the wire length at a distance of 2.5 m. The direction of the current in the wire is vertically downward. Hence, according to the Maxwell's right hand thumb rule, the direction of the magnetic field at the given point is vertically upward.

Question 4.4:

A horizontal overhead power line carries a current of 90 A in east to west direction. What is the magnitude and direction of the magnetic field due to the current 1.5 m below the line?

Answer

Current in the power line, I = 90 A

Point is located below the power line at distance, r = 1.5 m

Hence, magnetic field at that point is given by the relation,

$$B = \frac{\mu_0 2I}{4\pi r}$$

Where,

μ_0 = Permeability of free space = $4\pi \times 10^{-7}$ T m A^{-1}

$$B = \frac{4\pi \times 10^{-7} \times 2 \times 90}{4\pi \times 1.5} = 1.2 \times 10^{-5} \text{ T}$$

The current is flowing from East to West. The point is below the power line. Hence, according to Maxwell's right hand thumb rule, the direction of the magnetic field is towards the South.

Question 4.5:

What is the magnitude of magnetic force per unit length on a wire carrying a current of 8 A and making an angle of 30º with the direction of a uniform magnetic field of 0.15 T?

Answer

Current in the wire, $I = 8$ A

Magnitude of the uniform magnetic field, $B = 0.15$ T

Angle between the wire and magnetic field, $\theta = 30°$.

Magnetic force per unit length on the wire is given as:

$f = BI \sin\theta$

$= 0.15 \times 8 \times 1 \times \sin 30°$

$= 0.6 \text{ N m}^{-1}$

Hence, the magnetic force per unit length on the wire is 0.6 N m^{-1}.

Question 4.6:

A 3.0 cm wire carrying a current of 10 A is placed inside a solenoid perpendicular to its axis. The magnetic field inside the solenoid is given to be 0.27 T. What is the magnetic force on the wire?

Answer

Length of the wire, $l = 3$ cm $= 0.03$ m

Current flowing in the wire, $I = 10$ A

Magnetic field, $B = 0.27$ T

Angle between the current and magnetic field, $\theta = 90°$

Magnetic force exerted on the wire is given as:

$F = BIl\sin\theta$

$= 0.27 \times 10 \times 0.03 \sin 90°$

$= 8.1 \times 10^{-2}$ N

Hence, the magnetic force on the wire is 8.1×10^{-2} N. The direction of the force can be obtained from Fleming's left hand rule.

Question 4.7:

Two long and parallel straight wires A and B carrying currents of 8.0 A and 5.0 A in the same direction are separated by a distance of 4.0 cm. Estimate the force on a 10 cm section of wire A.

Answer

Current flowing in wire A, $I_A = 8.0$ A

Current flowing in wire B, $I_B = 5.0$ A

Distance between the two wires, $r = 4.0$ cm $= 0.04$ m

Length of a section of wire A, $l = 10$ cm $= 0.1$ m

Force exerted on length l due to the magnetic field is given as:

$$B = \frac{\mu_0 2\, I_A I_B l}{4\pi r}$$

Where,

μ_0 = Permeability of free space $= 4\pi \times 10^{-7}$ T m A^{-1}

$$B = \frac{4\pi \times 10^{-7} \times 2 \times 8 \times 5 \times 0.1}{4\pi \times 0.04}$$
$$= 2 \times 10^{-5} \text{ N}$$

The magnitude of force is 2×10^{-5} N. This is an attractive force normal to A towards B because the direction of the currents in the wires is the same.

Question 4.8:

A closely wound solenoid 80 cm long has 5 layers of windings of 400 turns each. The diameter of the solenoid is 1.8 cm. If the current carried is 8.0 A, estimate the magnitude of **B** inside the solenoid near its centre.

Answer

Length of the solenoid, l = 80 cm = 0.8 m

There are five layers of windings of 400 turns each on the solenoid.

∴ Total number of turns on the solenoid, $N = 5 \times 400 = 2000$

Diameter of the solenoid, D = 1.8 cm = 0.018 m

Current carried by the solenoid, I = 8.0 A

Magnitude of the magnetic field inside the solenoid near its centre is given by the relation,

$$B = \frac{\mu_0 N I}{l}$$

Where,

μ_0 = Permeability of free space = $4\pi \times 10^{-7}$ T m A^{-1}

$$B = \frac{4\pi \times 10^{-7} \times 2000 \times 8}{0.8}$$
$$= 8\pi \times 10^{-3} = 2.512 \times 10^{-2} \text{ T}$$

Hence, the magnitude of the magnetic field inside the solenoid near its centre is 2.512×10^{-2} T.

Question 4.9:

A square coil of side 10 cm consists of 20 turns and carries a current of 12 A. The coil is suspended vertically and the normal to the plane of the coil makes an angle of 30º with the direction of a uniform horizontal magnetic field of magnitude 0.80 T. What is the magnitude of torque experienced by the coil?

Answer

Length of a side of the square coil, l = 10 cm = 0.1 m

Current flowing in the coil, I = 12 A

Number of turns on the coil, n = 20

Angle made by the plane of the coil with magnetic field, $\theta = 30°$

Strength of magnetic field, B = 0.80 T

Magnitude of the magnetic torque experienced by the coil in the magnetic field is given by the relation,

$\tau = n\, BIA \sin\theta$

Where,

A = Area of the square coil

$\Rightarrow l \times l = 0.1 \times 0.1 = 0.01\ \text{m}^2$

$\therefore \tau = 20 \times 0.8 \times 12 \times 0.01 \times \sin 30°$

$= 0.96$ N m

Hence, the magnitude of the torque experienced by the coil is 0.96 N m.

Question 4.10:

Two moving coil meters, M_1 and M_2 have the following particulars:

$R_1 = 10\ \Omega$, $N_1 = 30$,

$A_1 = 3.6 \times 10^{-3}\ m^2$, $B_1 = 0.25$ T

$R_2 = 14\ \Omega$, $N_2 = 42$,

$A_2 = 1.8 \times 10^{-3}\ m^2$, $B_2 = 0.50$ T

(The spring constants are identical for the two meters).

Determine the ratio of (a) current sensitivity and (b) voltage sensitivity of M_2 and M_1.

Answer

For moving coil meter M_1:

Resistance, $R_1 = 10\ \Omega$

Number of turns, $N_1 = 30$

Area of cross-section, $A_1 = 3.6 \times 10^{-3}\ m^2$

Magnetic field strength, $B_1 = 0.25$ T

Spring constant $K_1 = K$

For moving coil meter M_2:

Resistance, $R_2 = 14\ \Omega$

Number of turns, $N_2 = 42$

Area of cross-section, $A_2 = 1.8 \times 10^{-3}\ m^2$

Magnetic field strength, $B_2 = 0.50$ T

Spring constant, $K_2 = K$

Current sensitivity of M_1 is given as:

$$I_{s1} = \frac{N_1 \ B_1 \ A_1}{K_1}$$

And, current sensitivity of M_2 is given as:

$$I_{s2} = \frac{N_2 \ B_2 \ A_2}{K_2}$$

$\therefore$ Ratio $\dfrac{I_{s2}}{I_{s1}} = \dfrac{N_2 \ B_2 \ A_2 \, K_1}{K_2 \, N_1 \ B_1 \ A_1}$

$$= \frac{42 \times 0.5 \times 1.8 \times 10^{-3} \times K}{K \times 30 \times 0.25 \times 3.6 \times 10^{-3}} = 1.4$$

Hence, the ratio of current sensitivity of M_2 to M_1 is 1.4.

Voltage sensitivity for M_2 is given as:

$$V_{s2} = \frac{N_2 \ B_2 \ A_2}{K_2 \ R_2}$$

And, voltage sensitivity for M_1 is given as:

$$V_{s1} = \frac{N_1 \ B_1 \ A_1}{K_1}$$

$\therefore$ Ratio $\dfrac{V_{s2}}{Vs1} = \dfrac{N_2 \ B_2 \ A_2 \, K_1 \, R_1}{K_2 \, R_2 \ N_1 \ B_1 \ A_1}$

$$= \frac{42 \times 0.5 \times 1.8 \times 10^{-3} \times 10 \times K}{K \times 14 \times 30 \times 0.25 \times 3.6 \times 10^{-3}} = 1$$

Hence, the ratio of voltage sensitivity of M_2 to M_1 is 1.

Question 4.11:

In a chamber, a uniform magnetic field of 6.5 G (1 G = 10^{-4} T) is maintained. An electron is shot into the field with a speed of 4.8×10^{6} m s^{-1} normal to the field. Explain why the path of the electron is a circle. Determine the radius of the circular orbit. ($e = 1.6 \times 10^{-19}$ C, $m_e = 9.1 \times 10^{-31}$ kg)

Answer

Magnetic field strength, $B = 6.5$ G $= 6.5 \times 10^{-4}$ T

Speed of the electron, $v = 4.8 \times 10^{6}$ m/s

Charge on the electron, $e = 1.6 \times 10^{-19}$ C

Mass of the electron, $m_e = 9.1 \times 10^{-31}$ kg

Angle between the shot electron and magnetic field, $\theta = 90°$

Magnetic force exerted on the electron in the magnetic field is given as:

$F = evB \sin\theta$

This force provides centripetal force to the moving electron. Hence, the electron starts moving in a circular path of radius r.

Hence, centripetal force exerted on the electron,

$$F_c = \frac{mv^2}{r}$$

In equilibrium, the centripetal force exerted on the electron is equal to the magnetic force i.e.,

$$F_c = F$$

$$\frac{mv^2}{r} = evB \sin\theta$$

$$r = \frac{mv}{Be\sin\theta}$$

$$= \frac{9.1\times10^{-31}\times4.8\times10^{6}}{6.5\times10^{-4}\times1.6\times10^{-19}\times\sin 90°}$$

$$= 4.2\times10^{-2} \text{ m } = 4.2 \text{ cm}$$

Hence, the radius of the circular orbit of the electron is 4.2 cm.

Question 4.12:

In Exercise 4.11 obtain the frequency of revolution of the electron in its circular orbit. Does the answer depend on the speed of the electron? Explain.

Answer

Magnetic field strength, $B = 6.5 \times 10^{-4}$ T

Charge of the electron, $e = 1.6 \times 10^{-19}$ C

Mass of the electron, $m_e = 9.1 \times 10^{-31}$ kg

Velocity of the electron, $v = 4.8 \times 10^{6}$ m/s

Radius of the orbit, $r = 4.2$ cm $= 0.042$ m

Frequency of revolution of the electron $= \nu$

Angular frequency of the electron $= \omega = 2\pi\nu$

Velocity of the electron is related to the angular frequency as:

$v = r\omega$

In the circular orbit, the magnetic force on the electron is balanced by the centripetal force. Hence, we can write:

$$evB = \frac{mv^2}{r}$$

$$eB = \frac{m}{r}(r\omega) = \frac{m}{r}(r2\pi\nu)$$

$$\nu = \frac{Be}{2\pi m}$$

This expression for frequency is independent of the speed of the electron.

On substituting the known values in this expression, we get the frequency as:

$$\nu = \frac{6.5\times10^{-4}\times1.6\times10^{-19}}{2\times3.14\times9.1\times10^{-31}}$$

$$= 18.2 \times 10^{6} \text{ Hz}$$

$$\approx 18 \text{ MHz}$$

Hence, the frequency of the electron is around 18 MHz and is independent of the speed of the electron.

Question 4.13:

A circular coil of 30 turns and radius 8.0 cm carrying a current of 6.0 A is suspended vertically in a uniform horizontal magnetic field of magnitude 1.0 T. The field lines make an angle of 60º with the normal of the coil. Calculate the magnitude of the counter torque that must be applied to prevent the coil from turning.

Would your answer change, if the circular coil in (a) were replaced by a planar coil of some irregular shape that encloses the same area? (All other particulars are also unaltered.)

Answer

Number of turns on the circular coil, $n = 30$

Radius of the coil, $r = 8.0$ cm $= 0.08$ m

Area of the coil $= \pi r^2 = \pi(0.08)^2 = 0.0201\ \text{m}^2$

Current flowing in the coil, $I = 6.0$ A

Magnetic field strength, $B = 1$ T

Angle between the field lines and normal with the coil surface,

$\theta = 60°$

The coil experiences a torque in the magnetic field. Hence, it turns. The counter torque applied to prevent the coil from turning is given by the relation,

$\tau = n\, IBA \sin\theta$ … (*i*)

$= 30 \times 6 \times 1 \times 0.0201 \times \sin 60°$

$= 3.133$ N m

It can be inferred from relation (*i*) that the magnitude of the applied torque is not dependent on the shape of the coil. It depends on the area of the coil. Hence, the answer would not change if the circular coil in the above case is replaced by a planar coil of some irregular shape that encloses the same area.

Question 4.14:

Two concentric circular coils X and Y of radii 16 cm and 10 cm, respectively, lie in the same vertical plane containing the north to south direction. Coil X has 20 turns and carries a current of 16 A; coil Y has 25 turns and carries a current of 18 A. The sense of the current in X is anticlockwise, and clockwise in Y, for an observer looking at the coils facing west. Give the magnitude and direction of the net magnetic field due to the coils at their centre.

Answer

Radius of coil X, r_1 = 16 cm = 0.16 m

Radius of coil Y, r_2 = 10 cm = 0.1 m

Number of turns of on coil X, n_1 = 20

Number of turns of on coil Y, n_2 = 25

Current in coil X, I_1 = 16 A

Current in coil Y, I_2 = 18 A

Magnetic field due to coil X at their centre is given by the relation,

$$B_1 = \frac{\mu_0 n_1 I_1}{2r_1}$$

Where,

μ_0 = Permeability of free space = $4\pi \times 10^{-7}\ \text{T m A}^{-1}$

$$\therefore B_1 = \frac{4\pi \times 10^{-7} \times 20 \times 16}{2 \times 0.16}$$
$$= 4\pi \times 10^{-4}\ \text{T}\ \ \text{(towards East)}$$

Magnetic field due to coil Y at their centre is given by the relation,

$$B_2 = \frac{\mu_0 n_2 I_2}{2r_2}$$
$$= \frac{4\pi \times 10^{-7} \times 25 \times 18}{2 \times 0.10}$$
$$= 9\pi \times 10^{-4} \text{ T (towards West)}$$

Hence, net magnetic field can be obtained as:

$$B = B_2 - B_1$$
$$= 9\pi \times 10^{-4} - 4\pi \times 10^{-4}$$
$$= 5\pi \times 10^{-4} \text{ T}$$
$$= 1.57 \times 10^{-3} \text{ T (towards West)}$$

Question 4.15:

A magnetic field of 100 G (1 G = 10^{-4} T) is required which is uniform in a region of linear dimension about 10 cm and area of cross-section about 10^{-3} m^2. The maximum current-carrying capacity of a given coil of wire is 15 A and the number of turns per unit length that can be wound round a core is at most 1000 turns m^{-1}. Suggest some appropriate design particulars of a solenoid for the required purpose. Assume the core is not ferromagnetic

Answer

Magnetic field strength, B = 100 G = 100 × 10^{-4} T

Number of turns per unit length, n = 1000 turns m^{-1}

Current flowing in the coil, I = 15 A

Permeability of free space, $\mu_0 = 4\pi \times 10^{-7}$ T m A^{-1}

Magnetic field is given by the relation,

$$B = \mu_0 n I$$

$$\therefore nI = \frac{B}{\mu_0}$$

$$= \frac{100 \times 10^{-4}}{4\pi \times 10^{-7}} = 7957.74$$

$$\approx 8000 \text{ A/m}$$

If the length of the coil is taken as 50 cm, radius 4 cm, number of turns 400, and current 10 A, then these values are not unique for the given purpose. There is always a possibility of some adjustments with limits.

Question 4.16:

For a circular coil of radius *R* and *N* turns carrying current *I*, the magnitude of the magnetic field at a point on its axis at a distance *x* from its centre is given by,

$$B = \frac{\mu_0 I R^2 N}{2\left(x^2 + R^2\right)^{\frac{3}{2}}}$$

Show that this reduces to the familiar result for field at the centre of the coil.

Consider two parallel co-axial circular coils of equal radius *R*, and number of turns *N*, carrying equal currents in the same direction, and separated by a distance *R*. Show that the field on the axis around the mid-point between the coils is uniform over a distance that is small as compared to *R*, and is given by,

$$B = 0.72 - \frac{\mu_0 BNI}{R}$$, approximately.

[Such an arrangement to produce a nearly uniform magnetic field over a small region is known as *Helmholtz coils*.]

Answer

Radius of circular coil = *R*

Number of turns on the coil = *N*

Current in the coil = *I*

Magnetic field at a point on its axis at distance x is given by the relation,

$$B = \frac{\mu_0 I R^2 N}{2\left(x^2 + R^2\right)^{\frac{3}{2}}}$$

Where,

μ_0 = Permeability of free space

If the magnetic field at the centre of the coil is considered, then $x = 0$.

$$\therefore B = \frac{\mu_0 I R^2 N}{2R^3} = \frac{\mu_0 I N}{2R}$$

This is the familiar result for magnetic field at the centre of the coil.

Radii of two parallel co-axial circular coils = R

Number of turns on each coil = N

Current in both coils = I

Distance between both the coils = R

Let us consider point Q at distance d from the centre.

Then, one coil is at a distance of $\frac{R}{2} + d$ from point Q.

$\therefore$ Magnetic field at point Q is given as:

$$B_1 = \frac{\mu_0 N I R^2}{2\left[\left(\frac{R}{2} + d\right)^2 + R^2\right]^{\frac{3}{2}}}$$

Also, the other coil is at a distance of $\frac{R}{2} - d$ from point Q.

$\therefore$ Magnetic field due to this coil is given as:

$$B_2 = \frac{\mu_0 N I R^2}{2\left[\left(\frac{R}{2} - d\right)^2 + R^2\right]^{\frac{3}{2}}}$$

Total magnetic field,

$$B = B_1 + B_2$$

$$= \frac{\mu_0 I R^2}{2}\left[\left\{\left(\frac{R}{2}-d\right)^2 + R^2\right\}^{-\frac{3}{2}} + \left\{\left(\frac{R}{2}+d\right)^2 + R^2\right\}^{-\frac{3}{2}}\right]$$

$$= \frac{\mu_0 I R^2}{2}\left[\left(\frac{5R^2}{4} + d^2 - Rd\right)^{-\frac{3}{2}} + \left(\frac{5R^2}{4} + d^2 + Rd\right)^{-\frac{3}{2}}\right]$$

$$= \frac{\mu_0 I R^2}{2} \times \left(\frac{5R^2}{4}\right)^{-\frac{3}{2}}\left[\left(1 + \frac{4}{5}\frac{d^2}{R^2} - \frac{4}{5}\frac{d}{R}\right)^{-\frac{3}{2}} + \left(1 + \frac{4}{5}\frac{d^2}{R^2} + \frac{4}{5}\frac{d}{R}\right)^{-\frac{3}{2}}\right]$$

For $d << R$, neglecting the factor $\frac{d^2}{R^2}$, we get:

$$\approx \frac{\mu_0 I R^2}{2} \times \left(\frac{5R^2}{4}\right)^{-\frac{3}{2}} \times \left[\left(1 - \frac{4d}{5R}\right)^{-\frac{3}{2}} + \left(1 + \frac{4d}{5R}\right)^{-\frac{3}{2}}\right]$$

$$\approx \frac{\mu_0 I R^2 N}{2R^3} \times \left(\frac{4}{5}\right)^{\frac{3}{2}}\left[1 - \frac{6d}{5R} + 1 + \frac{6d}{5R}\right]$$

$$B = \left(\frac{4}{5}\right)^{\frac{3}{2}} \frac{\mu_0 I N}{R} = 0.72\left(\frac{\mu_0 I N}{R}\right)$$

Hence, it is proved that the field on the axis around the mid-point between the coils is uniform.

Question 4.17:

A toroid has a core (non-ferromagnetic) of inner radius 25 cm and outer radius 26 cm, around which 3500 turns of a wire are wound. If the current in the wire is 11 A, what is the magnetic field (a) outside the toroid, (b) inside the core of the toroid, and (c) in the empty space surrounded by the toroid.

Answer

Inner radius of the toroid, r_1 = 25 cm = 0.25 m

Outer radius of the toroid, r_2 = 26 cm = 0.26 m

Number of turns on the coil, N = 3500

Current in the coil, I = 11 A

Magnetic field outside a toroid is zero. It is non-zero only inside the core of a toroid.

Magnetic field inside the core of a toroid is given by the relation,

$$B = \frac{\mu_0 NI}{l}$$

Where,

μ_0 = Permeability of free space = $4\pi \times 10^{-7}\ \mathrm{T\,m\,A^{-1}}$

l = length of toroid

$$\begin{aligned}
&= 2\pi\left[\frac{r_1 + r_2}{2}\right] \\
&= \pi(0.25 + 0.26) \\
&= 0.51\pi \\
\therefore B &= \frac{4\pi \times 10^{-7} \times 3500 \times 11}{0.51\pi} \\
&\approx 3.0 \times 10^{-2}\ \mathrm{T}
\end{aligned}$$

Magnetic field in the empty space surrounded by the toroid is zero.

Question 4.18:

Answer the following questions:

A magnetic field that varies in magnitude from point to point but has a constant direction (east to west) is set up in a chamber. A charged particle enters the chamber and travels undeflected along a straight path with constant speed. What can you say about the initial velocity of the particle?

A charged particle enters an environment of a strong and non-uniform magnetic field varying from point to point both in magnitude and direction, and comes out of it following a complicated trajectory. Would its final speed equal the initial speed if it suffered no collisions with the environment?

An electron travelling west to east enters a chamber having a uniform electrostatic field in north to south direction. Specify the direction in which a uniform magnetic field should be set up to prevent the electron from deflecting from its straight line path.

Answer

The initial velocity of the particle is either parallel or anti-parallel to the magnetic field. Hence, it travels along a straight path without suffering any deflection in the field.

Yes, the final speed of the charged particle will be equal to its initial speed. This is because magnetic force can change the direction of velocity, but not its magnitude.

An electron travelling from West to East enters a chamber having a uniform electrostatic field in the North-South direction. This moving electron can remain undeflected if the electric force acting on it is equal and opposite of magnetic field. Magnetic force is directed towards the South. According to Fleming's left hand rule, magnetic field should be applied in a vertically downward direction.

Question 4.19:

An electron emitted by a heated cathode and accelerated through a potential difference of 2.0 kV, enters a region with uniform magnetic field of 0.15 T. Determine the trajectory of the electron if the field (a) is transverse to its initial velocity, (b) makes an angle of 30º with the initial velocity.

Answer

Magnetic field strength, $B = 0.15$ T

Charge on the electron, $e = 1.6 \times 10^{-19}$ C

Mass of the electron, $m = 9.1 \times 10^{-31}$ kg

Potential difference, $V = 2.0$ kV $= 2 \times 10^3$ V

Thus, kinetic energy of the electron $= eV$

$$\Rightarrow eV = \frac{1}{2}mv^2$$

$$v = \sqrt{\frac{2eV}{m}} \qquad \ldots (1)$$

Where,

v = velocity of the electron

Magnetic force on the electron provides the required centripetal force of the electron. Hence, the electron traces a circular path of radius r.

Magnetic force on the electron is given by the relation,

$B\,ev$

Centripetal force $= \frac{mv^2}{r}$

$$\therefore Bev = \frac{mv^2}{r}$$

$$r = \frac{mv}{Be} \qquad \ldots (2)$$

From equations (1) and (2), we get

$$r = \frac{m}{Be}\left[\frac{2eV}{m}\right]^{\frac{1}{2}}$$

$$= \frac{9.1\times10^{-31}}{0.15\times1.6\times10^{-19}} \times \left(\frac{2\times1.6\times10^{-19}\times2\times10^{3}}{9.1\times10^{-31}}\right)^{\frac{1}{2}}$$

$$= 100.55\times10^{-5}$$

$$= 1.01\times10^{-3}\text{ m}$$

$$= 1\text{ mm}$$

Hence, the electron has a circular trajectory of radius 1.0 mm normal to the magnetic field.

When the field makes an angle θ of 30° with initial velocity, the initial velocity will be,

$$v_1 = v\sin\theta$$

From equation (2), we can write the expression for new radius as:

$$r_{1,} = \frac{mv_1}{Be}$$

$$= \frac{mv\sin\theta}{Be}$$

$$= \frac{9.1\times10^{-31}}{0.15\times1.6\times10^{-19}}\times\left[\frac{2\times1.6\times10^{-19}\times2\times10^{3}}{9\times10^{-31}}\right]^{\frac{1}{2}}\times\sin30°$$

$$= 0.5\times10^{-3}\ \text{m}$$

$$= 0.5\ \text{mm}$$

Hence, the electron has a helical trajectory of radius 0.5 mm along the magnetic field direction.

Question 4.20:

A magnetic field set up using Helmholtz coils (described in Exercise 4.16) is uniform in a small region and has a magnitude of 0.75 T. In the same region, a uniform electrostatic field is maintained in a direction normal to the common axis of the coils. A narrow beam of (single species) charged particles all accelerated through 15 kV enters this region in a direction perpendicular to both the axis of the coils and the electrostatic field. If the beam remains undeflected when the electrostatic field is 9.0×10^{-5} V m^{-1}, make a simple guess as to what the beam contains. Why is the answer not unique?

Answer

Magnetic field, B = 0.75 T

Accelerating voltage, V = 15 kV = 15×10^{3} V

Electrostatic field, $E = 9 \times 10^{5}$ V m^{-1}

Mass of the electron = m

Charge of the electron = e

Velocity of the electron = v

Kinetic energy of the electron = eV

$$\Rightarrow \frac{1}{2}mv^2 = eV$$

$$\therefore \frac{e}{m} = \frac{v^2}{2V} \qquad \text{... (1)}$$

Since the particle remains undeflected by electric and magnetic fields, we can infer that the electric field is balancing the magnetic field.

$$\therefore eE = evB$$

$$v = \frac{E}{B} \qquad \text{... (2)}$$

Putting equation (2) in equation (1), we get

$$\frac{e}{m} = \frac{1}{2}\frac{\left(\frac{E}{B}\right)^2}{V} = \frac{E^2}{2VB^2}$$

$$= \frac{\left(9.0\times10^5\right)^2}{2\times15000\times\left(0.75\right)^2} = 4.8\times10^7 \text{ C/kg}$$

This value of specific charge *e*/*m* is equal to the value of deuteron or deuterium ions. This is not a unique answer. Other possible answers are He^{++}, Li^{++}, etc.

Question 4.21:

A straight horizontal conducting rod of length 0.45 m and mass 60 g is suspended by two vertical wires at its ends. A current of 5.0 A is set up in the rod through the wires.

What magnetic field should be set up normal to the conductor in order that the tension in the wires is zero?

What will be the total tension in the wires if the direction of current is reversed keeping the magnetic field same as before? (Ignore the mass of the wires.) g = 9.8 m s^{-2}.

Answer

Length of the rod, *l* = 0.45 m

Mass suspended by the wires, $m = 60$ g $= 60 \times 10^{-3}$ kg

Acceleration due to gravity, g = 9.8 m/s^2

Current in the rod flowing through the wire, $I = 5$ A

Magnetic field (B) is equal and opposite to the weight of the wire i.e.,

$$BIl = mg$$
$$\therefore B = \frac{mg}{Il}$$
$$= \frac{60 \times 10^{-3} \times 9.8}{5 \times 0.45} = 0.26 \text{ T}$$

A horizontal magnetic field of 0.26 T normal to the length of the conductor should be set up in order to get zero tension in the wire. The magnetic field should be such that Fleming's left hand rule gives an upward magnetic force.

If the direction of the current is revered, then the force due to magnetic field and the weight of the wire acts in a vertically downward direction.

$\therefore$Total tension in the wire = $BIl + mg$

$$= 0.26 \times 5 \times 0.45 + \left(60 \times 10^{-3}\right) \times 9.8$$
$$= 1.176 \text{ N}$$

Question 4.22:

The wires which connect the battery of an automobile to its starting motor carry a current of 300 A (for a short time). What is the force per unit length between the wires if they are 70 cm long and 1.5 cm apart? Is the force attractive or repulsive?

Answer

Current in both wires, $I = 300$ A

Distance between the wires, $r = 1.5$ cm $= 0.015$ m

Length of the two wires, $l = 70$ cm $= 0.7$ m

Force between the two wires is given by the relation,

$$F = \frac{\mu_0 I^2}{2\pi r}$$

Where,

μ_0 = Permeability of free space = $4\pi \times 10^{-7}\ \mathrm{T\,m\,A^{-1}}$

$$\therefore F = \frac{4\pi \times 10^{-7} \times (300)^2}{2\pi \times 0.015}$$
$$= 1.2\ \mathrm{N/m}$$

Since the direction of the current in the wires is opposite, a repulsive force exists between them.

Question 4.23:

A uniform magnetic field of 1.5 T exists in a cylindrical region of radius10.0 cm, its direction parallel to the axis along east to west. A wire carrying current of 7.0 A in the north to south direction passes through this region. What is the magnitude and direction of the force on the wire if,

the wire intersects the axis,

the wire is turned from N-S to northeast-northwest direction,

the wire in the N-S direction is lowered from the axis by a distance of 6.0 cm?

Answer

Magnetic field strength, B = 1.5 T

Radius of the cylindrical region, r = 10 cm = 0.1 m

Current in the wire passing through the cylindrical region, I = 7 A

If the wire intersects the axis, then the length of the wire is the diameter of the cylindrical region.

Thus, $l = 2r = 0.2$ m

Angle between magnetic field and current, $\theta = 90°$

Magnetic force acting on the wire is given by the relation,

$F = BIl \sin \theta$

$= 1.5 \times 7 \times 0.2 \times \sin 90°$

$= 2.1$ N

Hence, a force of 2.1 N acts on the wire in a vertically downward direction.

New length of the wire after turning it to the Northeast-Northwest direction can be given as: :

$$l_1 = \frac{l}{\sin \theta}$$

Angle between magnetic field and current, $\theta = 45°$

Force on the wire,

$F = BIl_1 \sin \theta$

$$\begin{aligned} &= BIl \\ &= 1.5 \times 7 \times 0.2 \\ &= 2.1 \text{ N} \end{aligned}$$

Hence, a force of 2.1 N acts vertically downward on the wire. This is independent of angleθbecause $l \sin\theta$ is fixed.

The wire is lowered from the axis by distance, $d = 6.0$ cm

Let l_2 be the new length of the wire.

$$\begin{aligned} \therefore \left(\frac{l_2}{2}\right)^2 &= 4(d+r) \\ &= 4(10+6) = 4(16) \\ \therefore l_2 &= 8 \times 2 = 16 \text{ cm} = 0.16 \text{ m} \end{aligned}$$

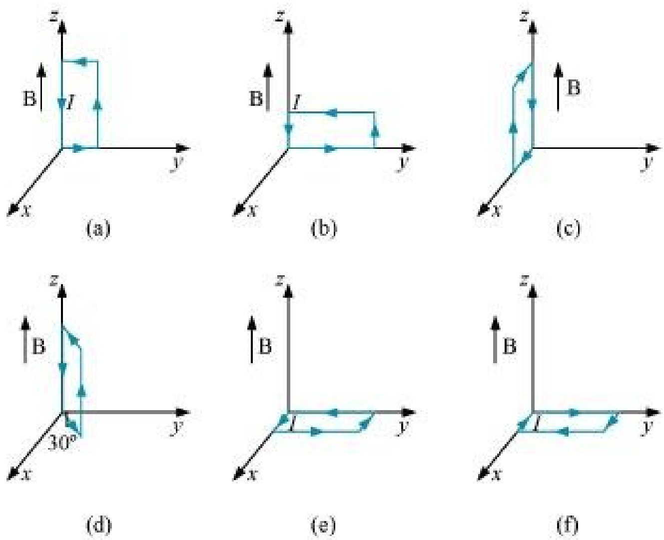
Magnetic force exerted on the wire,

$$F_2 = BIl_2$$
$$= 1.5 \times 7 \times 0.16$$
$$= 1.68 \text{ N}$$

Hence, a force of 1.68 N acts in a vertically downward direction on the wire.

Question 4.24:

A uniform magnetic field of 3000 G is established along the positive *z*-direction. A rectangular loop of sides 10 cm and 5 cm carries a current of 12 A. What is the torque on the loop in the different cases shown in Fig. 4.28? What is the force on each case? Which case corresponds to stable equilibrium?



Answer

Magnetic field strength, $B = 3000 \text{ G} = 3000 \times 10^{-4} \text{ T} = 0.3 \text{ T}$

Length of the rectangular loop, $l = 10$ cm

Width of the rectangular loop, $b = 5$ cm

Area of the loop,

$A = l \times b = 10 \times 5 = 50 \text{ cm}^2 = 50 \times 10^{-4} \text{ m}^2$

Current in the loop, $I = 12$ A

Now, taking the anti-clockwise direction of the current as positive and vise-versa:

Torque, $\vec{\tau} = I\vec{A}\times\vec{B}$

From the given figure, it can be observed that *A* is normal to the *y*-*z* plane and *B* is directed along the *z*-axis.

$$\therefore \tau = 12\times\left(50\times10^{-4}\right)\hat{i}\times0.3\hat{k}$$
$$= -1.8\times10^{-2}\,\hat{j}\ \text{N m}$$

The torque is 1.8×10^{-2} N m along the negative *y*-direction. The force on the loop is zero because the angle between *A* and *B* is zero.

This case is similar to case (a). Hence, the answer is the same as (a).

Torque $\tau = I\vec{A}\times\vec{B}$

From the given figure, it can be observed that *A* is normal to the *x*-*z* plane and *B* is directed along the *z*-axis.

$$\therefore \tau = -12\times\left(50\times10^{-4}\right)\hat{j}\times0.3\hat{k}$$
$$= -1.8\times10^{-2}\hat{i}\ \text{N m}$$

The torque is 1.8×10^{-2} N m along the negative *x* direction and the force is zero.

Magnitude of torque is given as:

$$|\tau| = IAB$$
$$= 12\times50\times10^{-4}\times0.3$$
$$= 1.8\times10^{-2}\ \text{N m}$$

Torque is 1.8×10^{-2} N m at an angle of 240° with positive *x* direction. The force is zero.

Torque $\tau = I\vec{A}\times\vec{B}$

$$= \left(50\times10^{-4}\times12\right)\hat{k}\times0.3\hat{k}$$
$$= 0$$

Hence, the torque is zero. The force is also zero.

Torque $\tau = I\vec{A}\times\vec{B}$

$$= \left(50 \times 10^{-4} \times 12\right) \hat{k} \times 0.3 \hat{k}$$
$$= 0$$

Hence, the torque is zero. The force is also zero.

In case (e), the direction of $I\vec{A}$ and $\vec{B}$ is the same and the angle between them is zero. If displaced, they come back to an equilibrium. Hence, its equilibrium is stable.

Whereas, in case (f), the direction of $I\vec{A}$ and $\vec{B}$ is opposite. The angle between them is 180°. If disturbed, it does not come back to its original position. Hence, its equilibrium is unstable.

Question 4.25:

A circular coil of 20 turns and radius 10 cm is placed in a uniform magnetic field of 0.10 T normal to the plane of the coil. If the current in the coil is 5.0 A, what is the

total torque on the coil,

total force on the coil,

average force on each electron in the coil due to the magnetic field?

(The coil is made of copper wire of cross-sectional area 10^{-5} m^2, and the free electron density in copper is given to be about 10^{29} m^{-3}.)

Answer

Number of turns on the circular coil, $n = 20$

Radius of the coil, $r = 10$ cm $= 0.1$ m

Magnetic field strength, $B = 0.10$ T

Current in the coil, $I = 5.0$ A

The total torque on the coil is zero because the field is uniform.

The total force on the coil is zero because the field is uniform.

Cross-sectional area of copper coil, $A = 10^{-5}$ m^2

Number of free electrons per cubic meter in copper, $N = 10^{29}$ /m^3

Charge on the electron, $e = 1.6 \times 10^{-19}$ C

Magnetic force, $F = Bev_d$

Where,

v_d = Drift velocity of electrons

$$= \frac{I}{NeA}$$

$$\therefore F = \frac{BeI}{NeA}$$

$$= \frac{0.10 \times 5.0}{10^{29} \times 10^{-5}} = 5 \times 10^{-25} \text{ N}$$

Hence, the average force on each electron is 5×10^{-25} N.

Question 4.26:

A solenoid 60 cm long and of radius 4.0 cm has 3 layers of windings of 300 turns each. A 2.0 cm long wire of mass 2.5 g lies inside the solenoid (near its centre) normal to its axis; both the wire and the axis of the solenoid are in the horizontal plane. The wire is connected through two leads parallel to the axis of the solenoid to an external battery which supplies a current of 6.0 A in the wire. What value of current (with appropriate sense of circulation) in the windings of the solenoid can support the weight of the wire? g = 9.8 m s^{-2}

Answer

Length of the solenoid, L = 60 cm = 0.6 m

Radius of the solenoid, r = 4.0 cm = 0.04 m

It is given that there are 3 layers of windings of 300 turns each.

$\therefore$ Total number of turns, $n = 3 \times 300 = 900$

Length of the wire, $l = 2$ cm = 0.02 m

Mass of the wire, $m = 2.5$ g = 2.5×10^{-3} kg

Current flowing through the wire, $i = 6$ A

Acceleration due to gravity, g = 9.8 m/s^2

Magnetic field produced inside the solenoid, $B = \frac{\mu_0 n I}{L}$

Where,

μ_0 = Permeability of free space = $4\pi \times 10^{-7}\ \text{T m A}^{-1}$

I = Current flowing through the windings of the solenoid

Magnetic force is given by the relation,

$$F = Bil$$
$$= \frac{\mu_0 n I}{L} il$$

Also, the force on the wire is equal to the weight of the wire.

$$\therefore mg = \frac{\mu_0 n I i l}{L}$$
$$I = \frac{mgL}{\mu_0 n i l}$$
$$= \frac{2.5 \times 10^{-3} \times 9.8 \times 0.6}{4\pi \times 10^{-7} \times 900 \times 0.02 \times 6} = 108 \text{ A}$$

Hence, the current flowing through the solenoid is 108 A.

Question 4.27:

A galvanometer coil has a resistance of 12 Ω and the metre shows full scale deflection for a current of 3 mA. How will you convert the metre into a voltmeter of range 0 to 18 V?

Answer

Resistance of the galvanometer coil, $G = 12\ \Omega$

Current for which there is full scale deflection, $I_g = 3\ \text{mA} = 3 \times 10^{-3}\ \text{A}$

Range of the voltmeter is 0, which needs to be converted to 18 V.

$\therefore V = 18\ \text{V}$

Let a resistor of resistance R be connected in series with the galvanometer to convert it into a voltmeter. This resistance is given as:

$$R = \frac{V}{I_g} - \text{G}$$

$$= \frac{18}{3 \times 10^{-3}} - 12 = 6000 - 12 = 5988\ \Omega$$

Hence, a resistor of resistance $5988\ \Omega$ is to be connected in series with the galvanometer.

Question 4.28:

A galvanometer coil has a resistance of $15\ \Omega$ and the metre shows full scale deflection for a current of 4 mA. How will you convert the metre into an ammeter of range 0 to 6 A?

Answer

Resistance of the galvanometer coil, $G = 15\ \Omega$

Current for which the galvanometer shows full scale deflection,

$I_g = 4\ \text{mA} = 4 \times 10^{-3}\ \text{A}$

Range of the ammeter is 0, which needs to be converted to 6 A.

$\therefore$ Current, $I = 6\ \text{A}$

A shunt resistor of resistance S is to be connected in parallel with the galvanometer to convert it into an ammeter. The value of S is given as:

$$S=\frac{I_g G}{I-I_g}$$

$$=\frac{4\times10^{-3}\times15}{6-4\times10^{-3}}$$

$$S=\frac{6\times10^{-2}}{6-0.004}=\frac{0.06}{5.996}$$

$$\approx 0.01\ \Omega = 10\ \text{m}\Omega$$

Hence, a $10\ \text{m}\Omega$ shunt resistor is to be connected in parallel with the galvanometer.

Question 5.1:

Answer the following questions regarding earth's magnetism:

A vector needs three quantities for its specification. Name the three independent quantities conventionally used to specify the earth's magnetic field.

The angle of dip at a location in southern India is about 18º.

Would you expect a greater or smaller dip angle in Britain?

If you made a map of magnetic field lines at Melbourne in Australia, would the lines seem to go into the ground or come out of the ground?

In which direction would a compass free to move in the vertical plane point to, if located right on the geomagnetic north or south pole?

The earth's field, it is claimed, roughly approximates the field due to a dipole of magnetic moment 8 × 1022 J T^{-1} located at its centre. Check the order of magnitude of this number in some way.

(f) Geologists claim that besides the main magnetic N-S poles, there are several local poles on the earth's surface oriented in different directions. How is such a thing possible at all?

Answer

The three independent quantities conventionally used for specifying earth's magnetic field are:

Magnetic declination,

Angle of dip, and

Horizontal component of earth's magnetic field

(b) The angle of dip at a point depends on how far the point is located with respect to the North Pole or the South Pole. The angle of dip would be greater in Britain (it is about 70°) than in southern India because the location of Britain on the globe is closer to the magnetic North Pole.

(c) It is hypothetically considered that a huge bar magnet is dipped inside earth with its north pole near the geographic South Pole and its south pole near the geographic North Pole.

Magnetic field lines emanate from a magnetic north pole and terminate at a magnetic south pole. Hence, in a map depicting earth's magnetic field lines, the field lines at Melbourne, Australia would seem to come out of the ground.

(d) If a compass is located on the geomagnetic North Pole or South Pole, then the compass will be free to move in the horizontal plane while earth's field is exactly vertical to the magnetic poles. In such a case, the compass can point in any direction.

(e) Magnetic moment, $M = 8 \times 10^{22}$ J T^{-1}

Radius of earth, $r = 6.4 \times 10^{6}$ m

Magnetic field strength, $B = \frac{\mu_0 M}{4\pi r^3}$

Where,

μ_0 = Permeability of free space = $4\pi \times 10^{-7}$ Tm A^{-1}

$$\therefore B = \frac{4\pi \times 10^{-7} \times 8 \times 10^{22}}{4\pi \times \left(6.4 \times 10^{6}\right)^{3}} = 0.3 \text{ G}$$

This quantity is of the order of magnitude of the observed field on earth.

(f) Yes, there are several local poles on earth's surface oriented in different directions. A magnetised mineral deposit is an example of a local N-S pole.

Question 5.2:

Answer the following questions:

The earth's magnetic field varies from point to point in space.

Does it also change with time? If so, on what time scale does it change appreciably?

The earth's core is known to contain iron. Yet geologists do not regard this as a source of the earth's magnetism. Why?

The charged currents in the outer conducting regions of the earth's core are thought to be responsible for earth's magnetism. What might be the 'battery' (i.e., the source of energy) to sustain these currents?

The earth may have even reversed the direction of its field several times during its history of 4 to 5 billion years. How can geologists know about the earth's field in such distant past?

The earth’s field departs from its dipole shape substantially at large distances (greater than about 30,000 km). What agencies may be responsible for this distortion?

(f) Interstellar space has an extremely weak magnetic field of the order of 10−12 T. Can such a weak field be of any significant consequence? Explain.

[**Note:** Exercise 5.2 is meant mainly to arouse your curiosity. Answers to some questions above are tentative or unknown. Brief answers wherever possible are given at the end. For details, you should consult a good text on geomagnetism.]

Answer

Earth’s magnetic field changes with time. It takes a few hundred years to change by an appreciable amount. The variation in earth’s magnetic field with the time cannot be neglected.

(b) Earth’s core contains molten iron. This form of iron is not ferromagnetic. Hence, this is not considered as a source of earth’s magnetism.

(c) Theradioactivity in earth’s interior is the source of energy that sustains the currents in the outer conducting regions of earth’s core. These charged currents are considered to be responsible for earth’s magnetism.

(d) Earth reversed the direction of its field several times during its history of 4 to 5 billion years. These magnetic fields got weakly recorded in rocks during their solidification. One can get clues about the geomagnetic history from the analysis of this rock magnetism.

(e) Earth’s field departs from its dipole shape substantially at large distances (greater than about 30,000 km) because of the presence of the ionosphere. In this region, earth’s field gets modified because of the field of single ions. While in motion, these ions produce the magnetic field associated with them.

(f) An extremely weak magnetic field can bend charged particles moving in a circle. This may not be noticeable for a large radius path. With reference to the gigantic interstellar space, the deflection can affect the passage of charged particles.

Question 5.3:

A short bar magnet placed with its axis at 30º with a uniform external magnetic field of 0.25 T experiences a torque of magnitude equal to 4.5×10^{-2} J. What is the magnitude of magnetic moment of the magnet?

Answer

Magnetic field strength, $B = 0.25$ T

Torque on the bar magnet, $T = 4.5 \times 10^{-2}$ J

Angle between the bar magnet and the external magnetic field, $\theta = 30°$

Torque is related to magnetic moment (M) as:

$T = MB \sin \theta$

$$\therefore M = \frac{T}{B \sin \theta}$$

$$= \frac{4.5 \times 10^{-2}}{0.25 \times \sin 30°} = 0.36 \text{ J T}^{-1}$$

Hence, the magnetic moment of the magnet is 0.36 J T^{-1}.

Question 5.4:

A short bar magnet of magnetic moment m = 0.32 J T^{-1} is placed in a uniform magnetic field of 0.15 T. If the bar is free to rotate in the plane of the field, which orientation would correspond to its (a) stable, and (b) unstable equilibrium? What is the potential energy of the magnet in each case?

Answer

Moment of the bar magnet, $M = 0.32$ J T^{-1}

External magnetic field, $B = 0.15$ T

(a) The bar magnet is aligned along the magnetic field. This system is considered as being in stable equilibrium. Hence, the angle θ, between the bar magnet and the magnetic field is 0°.

Potential energy of the system $= -MB\cos\theta$

$$= -0.32 \times 0.15 \cos 0°$$
$$= -4.8 \times 10^{-2}\ \text{J}$$

(b) The bar magnet is oriented 180° to the magnetic field. Hence, it is in unstable equilibrium.

$\theta = 180°$

Potential energy $= -MB\cos\theta$

$$= -0.32 \times 0.15 \cos 180°$$
$$= 4.8 \times 10^{-2}\ \text{J}$$

Question 5.5:

A closely wound solenoid of 800 turns and area of cross section $2.5 \times 10^{-4}\ \text{m}^2$ carries a current of 3.0 A. Explain the sense in which the solenoid acts like a bar magnet. What is its associated magnetic moment?

Answer

Number of turns in the solenoid, $n = 800$

Area of cross-section, $A = 2.5 \times 10^{-4}\ \text{m}^2$

Current in the solenoid, $I = 3.0$ A

A current-carrying solenoid behaves as a bar magnet because a magnetic field develops along its axis, i.e., along its length.

The magnetic moment associated with the given current-carrying solenoid is calculated as:

$M = n\,I\,A$

$= 800 \times 3 \times 2.5 \times 10^{-4}$

$= 0.6 \text{ J T}^{-1}$

Question 5.6:

If the solenoid in Exercise 5.5 is free to turn about the vertical direction and a uniform horizontal magnetic field of 0.25 T is applied, what is the magnitude of torque on the solenoid when its axis makes an angle of 30° with the direction of applied field?

Answer

Magnetic field strength, $B = 0.25$ T

Magnetic moment, $M = 0.6 \text{ T}^{-1}$

The angle θ, between the axis of the solenoid and the direction of the applied field is 30°.

Therefore, the torque acting on the solenoid is given as:

$$\begin{aligned}\tau &= MB\sin\theta \\ &= 0.6 \times 0.25 \sin 30^\circ \\ &= 7.5 \times 10^{-2} \text{ J}\end{aligned}$$

Question 5.7:

A bar magnet of magnetic moment 1.5 J T^{-1} lies aligned with the direction of a uniform magnetic field of 0.22 T.

What is the amount of work required by an external torque to turn the magnet so as to align its magnetic moment: (i) normal to the field direction, (ii) opposite to the field direction?

What is the torque on the magnet in cases (i) and (ii)?

Answer

(a) Magnetic moment, $M = 1.5$ J T^{-1}

Magnetic field strength, $B = 0.22$ T

(i) Initial angle between the axis and the magnetic field, $\theta_1 = 0°$

Final angle between the axis and the magnetic field, $\theta_2 = 90°$

The work required to make the magnetic moment normal to the direction of magnetic field is given as:

$$\begin{aligned} W &= -MB\left(\cos\theta_2 - \cos\theta_1\right) \\ &= -1.5 \times 0.22\left(\cos 90° - \cos 0°\right) \\ &= -0.33\left(0 - 1\right) \\ &= 0.33 \text{ J} \end{aligned}$$

(ii) Initial angle between the axis and the magnetic field, $\theta_1 = 0°$

Final angle between the axis and the magnetic field, $\theta_2 = 180°$

The work required to make the magnetic moment opposite to the direction of magnetic field is given as:

$$\begin{aligned} W &= -MB\left(\cos\theta_2 - \cos\theta_1\right) \\ &= -1.5 \times 0.22\left(\cos 180 - \cos 0°\right) \\ &= -0.33\left(-1 - 1\right) \\ &= 0.66 \text{ J} \end{aligned}$$

(b) For case (i): $\theta = \theta_2 = 90°$

$\therefore$Torque, $\tau = MB\sin\theta$

$$\begin{aligned} &= 1.5 \times 0.22 \sin 90° \\ &= 0.33 \text{ J} \end{aligned}$$

For case (ii): $\theta = \theta_2 = 180°$

$\therefore$Torque, $\tau = MB\sin\theta$

$$= MB\sin 180° = 0 \text{ J}$$

Question 5.8:

A closely wound solenoid of 2000 turns and area of cross-section 1.6×10^{-4} m^2, carrying a current of 4.0 A, is suspended through its centre allowing it to turn in a horizontal plane.

What is the magnetic moment associated with the solenoid?

What is the force and torque on the solenoid if a uniform horizontal magnetic field of 7.5×10^{-2} T is set up at an angle of 30º with the axis of the solenoid?

Answer

Number of turns on the solenoid, $n = 2000$

Area of cross-section of the solenoid, $A = 1.6 \times 10^{-4}$ m^2

Current in the solenoid, $I = 4$ A

(a) The magnetic moment along the axis of the solenoid is calculated as:

$M = nAI$

$= 2000 \times 1.6 \times 10^{-4} \times 4$

$= 1.28$ Am^2

(b) Magnetic field, $B = 7.5 \times 10^{-2}$ T

Angle between the magnetic field and the axis of the solenoid, $\theta = 30°$

Torque, $\tau = MB\sin\theta$

$$= 1.28 \times 7.5 \times 10^{-2} \sin 30°$$
$$= 4.8 \times 10^{-2} \text{ Nm}$$

Since the magnetic field is uniform, the force on the solenoid is zero. The torque on the solenoid is 4.8×10^{-2} Nm.

Question 5.9:

A circular coil of 16 turns and radius 10 cm carrying a current of 0.75 A rests with its plane normal to an external field of magnitude 5.0×10^{-2} T. The coil is free to turn about an axis in its plane perpendicular to the field direction. When the coil is turned slightly and released, it oscillates about its stable equilibrium with a frequency of $2.0\ s^{-1}$. What is the moment of inertia of the coil about its axis of rotation?

Answer

Number of turns in the circular coil, $N = 16$

Radius of the coil, $r = 10$ cm $= 0.1$ m

Cross-section of the coil, $A = \pi r^2 = \pi \times (0.1)^2\ m^2$

Current in the coil, $I = 0.75$ A

Magnetic field strength, $B = 5.0 \times 10^{-2}$ T

Frequency of oscillations of the coil, $v = 2.0\ s^{-1}$

$\therefore$Magnetic moment, $M = NIA = NI\pi r^2$

$= 16 \times 0.75 \times \pi \times (0.1)^2$

$= 0.377\ J\ T^{-1}$

Frequency is given by the relation:

$$v = \frac{1}{2\pi}\sqrt{\frac{MB}{I}}$$

Where,

I = Moment of inertia of the coil

$$\therefore I = \frac{MB}{4\pi^2\nu^2}$$

$$= \frac{0.377 \times 5 \times 10^{-2}}{4\pi^2 \times (2)^2}$$

$$= 1.19 \times 10^{-4} \text{ kg m}^2$$

Hence, the moment of inertia of the coil about its axis of rotation is $1.19 \times 10^{-4} \text{ kg m}^2$.

Question 5.10:

A magnetic needle free to rotate in a vertical plane parallel to the magnetic meridian has its north tip pointing down at 22º with the horizontal. The horizontal component of the earth's magnetic field at the place is known to be 0.35 G. Determine the magnitude of the earth's magnetic field at the place.

Answer

Horizontal component of earth's magnetic field, B_H = 0.35 G

Angle made by the needle with the horizontal plane = Angle of dip = $\delta = 22°$

Earth's magnetic field strength = B

We can relate B and B_H as:

$$B_H = B\cos\theta$$

$$\therefore B = \frac{B_H}{\cos\delta}$$

$$= \frac{0.35}{\cos 22°} = 0.377 \text{ G}$$

Hence, the strength of earth's magnetic field at the given location is 0.377 G.

Question 5.11:

At a certain location in Africa, a compass points 12º west of the geographic north. The north tip of the magnetic needle of a dip circle placed in the plane of magnetic meridian points 60º above the horizontal. The horizontal component of the earth's field is measured to be 0.16 G. Specify the direction and magnitude of the earth's field at the location.

Answer

Angle of declination, $\theta = 12°$

Angle of dip, $\delta = 60°$

Horizontal component of earth's magnetic field, $B_H = 0.16$ G

Earth's magnetic field at the given location = B

We can relate B and B_H as:

$$B_H = B\cos\delta$$

$$\therefore B = \frac{B_H}{\cos\delta}$$

$$= \frac{0.16}{\cos 60°} = 0.32 \text{ G}$$

Earth's magnetic field lies in the vertical plane, 12° West of the geographic meridian, making an angle of 60° (upward) with the horizontal direction. Its magnitude is 0.32 G.

Question 5.12:

A short bar magnet has a magnetic moment of 0.48 J T^{-1}. Give the direction and magnitude of the magnetic field produced by the magnet at a distance of 10 cm from the centre of the magnet on (a) the axis, (b) the equatorial lines (normal bisector) of the magnet.

Answer

Magnetic moment of the bar magnet, $M = 0.48\ \mathrm{J\ T^{-1}}$

Distance, $d = 10$ cm = 0.1 m

The magnetic field at distance d, from the centre of the magnet on the axis is given by the relation:

$$B = \frac{\mu_0}{4\pi}\frac{2M}{d^3}$$

Where,

μ_0 = Permeability of free space = $4\pi \times 10^{-7}\ \mathrm{Tm\ A^{-1}}$

$$\therefore B = \frac{4\pi \times 10^{-7} \times 2 \times 0.48}{4\pi \times (0.1)^3}$$

$$= 0.96 \times 10^{-4}\ \mathrm{T} = 0.96\ \mathrm{G}$$

The magnetic field is along the S − N direction.

The magnetic field at a distance of 10 cm (i.e., $d = 0.1$ m) on the equatorial line of the magnet is given as:

$$B = \frac{\mu_0 \times M}{4\pi \times d^3}$$

$$= \frac{4\pi \times 10^{-7} \times 0.48}{4\pi (0.1)^3}$$

$$= 0.48\ \mathrm{G}$$

The magnetic field is along the N − S direction.

Question 5.13:

A short bar magnet placed in a horizontal plane has its axis aligned along the magnetic north-south direction. Null points are found on the axis of the magnet at 14 cm from the centre of the magnet. The earth's magnetic field at the place is 0.36 G and the angle of dip is zero. What is the total magnetic field on the normal bisector of the magnet at the same distance as the null–point (i.e., 14 cm) from the centre of the magnet? (At *null points*, field due to a magnet is equal and opposite to the horizontal component of earth's magnetic field.)

Answer

Earth's magnetic field at the given place, $H = 0.36$ G

The magnetic field at a distance *d*, on the axis of the magnet is given as:

$$B_1 = \frac{\mu_0}{4\pi}\frac{2M}{d^3} = H \qquad \ldots(i)$$

Where,

μ_0 = Permeability of free space

M = Magnetic moment

The magnetic field at the same distance *d*, on the equatorial line of the magnet is given as:

$$B_2 = \frac{\mu_0 M}{4\pi d^3} = \frac{H}{2} \qquad \text{[Using equation } (i)\text{]}$$

Total magnetic field, $B = B_1 + B_2$

$$= H + \frac{H}{2}$$

$$= 0.36 + 0.18 = 0.54 \text{ G}$$

Hence, the magnetic field is 0.54 G in the direction of earth's magnetic field.

Question 5.14:

If the bar magnet in exercise 5.13 is turned around by 180º, where will the new null points be located?

Answer

The magnetic field on the axis of the magnet at a distance d_1 = 14 cm, can be written as:

$$B_1 = \frac{\mu_0 2M}{4\pi (d_1)^3} = H \qquad \dots (1)$$

Where,

M = Magnetic moment

μ_0 = Permeability of free space

H = Horizontal component of the magnetic field at d_1

If the bar magnet is turned through 180°, then the neutral point will lie on the equatorial line.

Hence, the magnetic field at a distance d_2, on the equatorial line of the magnet can be written as:

$$B_2 = \frac{\mu_0 M}{4\pi (d_2)^3} = H \qquad \dots (2)$$

Equating equations (1) and (2), we get:

$$\frac{2}{(d_1)^3} = \frac{1}{(d_2)^3}$$

$$\left(\frac{d_2}{d_1}\right)^3 = \frac{1}{2}$$

$$\therefore d_2 = d_1 \times \left(\frac{1}{2}\right)^{\frac{1}{3}}$$

$$= 14 \times 0.794 = 11.1 \text{ cm}$$

The new null points will be located 11.1 cm on the normal bisector.

Question 5.15:

A short bar magnet of magnetic moment 5.25×10^{-2} J T^{-1} is placed with its axis perpendicular to the earth's field direction. At what distance from the centre of the magnet, the resultant field is inclined at 45º with earth's field on

its normal bisector and (b) its axis. Magnitude of the earth's field at the place is given to be 0.42 G. Ignore the length of the magnet in comparison to the distances involved.

Answer

Magnetic moment of the bar magnet, $M = 5.25 \times 10^{-2}$ J T^{-1}

Magnitude of earth's magnetic field at a place, $H = 0.42$ G $= 0.42 \times 10^{-4}$ T

The magnetic field at a distance R from the centre of the magnet on the normal bisector is given by the relation:

$$B = \frac{\mu_0 M}{4\pi R^3}$$

Where,

μ_0 = Permeability of free space = $4\pi \times 10^{-7}$ Tm A^{-1}

When the resultant field is inclined at 45° with earth's field, $B = H$

$$\therefore \frac{\mu_0 M}{4\pi R^3} = H = 0.42 \times 10^{-4}$$

$$R^3 = \frac{\mu_0 M}{0.42 \times 10^{-4} \times 4\pi}$$

$$= \frac{4\pi \times 10^{-7} \times 5.25 \times 10^{-2}}{4\pi \times 0.42 \times 10^{-4}} = 12.5 \times 10^{-5}$$

$$\therefore R = 0.05 \text{ m} = 5 \text{ cm}$$

The magnetic field at a distanced R' from the centre of the magnet on its axis is given as:

$$B' = \frac{\mu_0 2M}{4\pi R^3}$$

The resultant field is inclined at 45° with earth's field.

$$\therefore B' = H$$

$$\frac{\mu_0 2M}{4\pi (R')^3} = H$$

$$(R')^3 = \frac{\mu_0 2M}{4\pi \times H}$$

$$= \frac{4\pi \times 10^{-7} \times 2 \times 5.25 \times 10^{-2}}{4\pi \times 0.42 \times 10^{-4}} = 25 \times 10^{-5}$$

$$\therefore R' = 0.063 \text{ m} = 6.3 \text{ cm}$$

Question 5.16:

Answer the following questions:

Why does a paramagnetic sample display greater magnetisation (for the same magnetising field) when cooled?

Why is diamagnetism, in contrast, almost independent of temperature?

If a toroid uses bismuth for its core, will the field in the core be (slightly) greater or (slightly) less than when the core is empty?

Is the permeability of a ferromagnetic material independent of the magnetic field? If not, is it more for lower or higher fields?

Magnetic field lines are always nearly normal to the surface of a ferromagnet at every point. (This fact is analogous to the static electric field lines being normal to the surface of a conductor at every point.) Why?

(f) Would the maximum possible magnetisation of a paramagnetic sample be of the same order of magnitude as the magnetization of a ferromagnet?

Answer

(a) Owing to therandom thermal motion of molecules, the alignments of dipoles get disrupted at high temperatures. On cooling, this disruption is reduced. Hence, a paramagnetic sample displays greater magnetisation when cooled.

(b) The induced dipole moment in a diamagnetic substance is always opposite to the magnetising field. Hence, the internal motion of the atoms (which is related to the temperature) does not affect the diamagnetism of a material.

(c) Bismuth is a diamagnetic substance. Hence, a toroid with a bismuth core has a magnetic field slightly greater than a toroid whose core is empty.

(d) The permeability of ferromagnetic materials is not independent of the applied magnetic field. It is greater for a lower field and vice versa.

(e) The permeability of a ferromagnetic material is not less than one. It is always greater than one. Hence, magnetic field lines are always nearly normal to the surface of such materials at every point.

(f) The maximum possible magnetisation of a paramagnetic sample can be of the same order of magnitude as the magnetisation of a ferromagnet. This requires high magnetising fields for saturation.

Question 5.17:

Answer the following questions:

Explain qualitatively on the basis of domain picture the irreversibility in the magnetisation curve of a ferromagnet.

The hysteresis loop of a soft iron piece has a much smaller area than that of a carbon steel piece. If the material is to go through repeated cycles of magnetisation, which piece will dissipate greater heat energy?

'A system displaying a hysteresis loop such as a ferromagnet, is a device for storing memory?' Explain the meaning of this statement.

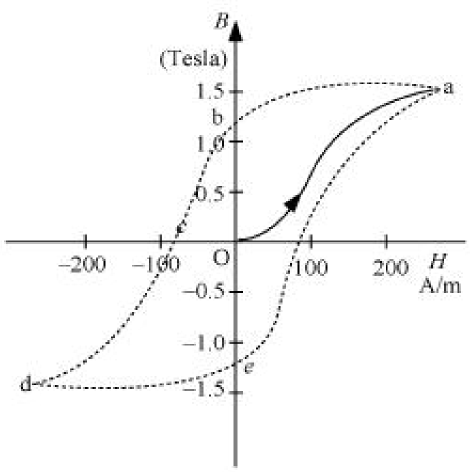
What kind of ferromagnetic material is used for coating magnetic tapes in a cassette player, or for building 'memory stores' in a modern computer?

A certain region of space is to be shielded from magnetic fields.

Suggest a method.

Answer

The hysteresis curve (*B*-*H* curve) of a ferromagnetic material is shown in the following figure.



It can be observed from the given curve that magnetisation persists even when the external field is removed. This reflects the irreversibility of a ferromagnet.

(b) The dissipated heat energy is directly proportional to the area of a hysteresis loop. A carbon steel piece has a greater hysteresis curve area. Hence, it dissipates greater heat energy.

(c) The value of magnetisation is memory or record of hysteresis loop cycles of magnetisation. These bits of information correspond to the cycle of magnetisation. Hysteresis loops can be used for storing information.

(d) Ceramic is used for coating magnetic tapes in cassette players and for building memory stores in modern computers.

(e)A certain region of space can be shielded from magnetic fields if it is surrounded by soft iron rings. In such arrangements, the magnetic lines are drawn out of the region.

Question 5.18:

A long straight horizontal cable carries a current of 2.5 A in the direction 10º south of west to 10° north of east. The magnetic meridian of the place happens to be 10º west of the geographic meridian. The earth's magnetic field at the location is 0.33 G, and the angle of dip is zero. Locate the line of neutral points (ignore the thickness of the cable). (At *neutral points*, magnetic field due to a current-carrying cable is equal and opposite to the horizontal component of earth's magnetic field.)

Answer

Current in the wire, $I = 2.5$ A

Angle of dip at the given location on earth, $\delta = 0°$

Earth's magnetic field, $H = 0.33$ G $= 0.33 \times 10^{-4}$ T

The horizontal component of earth's magnetic field is given as:

$H_H = H \cos \delta$

$= 0.33 \times 10^{-4} \times \cos 0° = 0.33 \times 10^{-4}$ T

The magnetic field at the neutral point at a distance R from the cable is given by the relation:

$$H_H = \frac{\mu_0 I}{2\pi R}$$

Where,

μ_0 = Permeability of free space = $4\pi \times 10^{-7}\,\text{Tm A}^{-1}$

$$\therefore R = \frac{\mu_0 I}{2\pi H_H}$$

$$= \frac{4\pi \times 10^{-7} \times 2.5}{2\pi \times 0.33 \times 10^{-4}} = 15.15 \times 10^{-3}\ \text{m} = 1.51\ \text{cm}$$

Hence, a set of neutral points parallel to and above the cable are located at a normal distance of 1.51 cm.

Question 5.19:

A telephone cable at a place has four long straight horizontal wires carrying a current of 1.0 A in the same direction east to west. The earth's magnetic field at the place is 0.39 G, and the angle of dip is 35º. The magnetic declination is nearly zero. What are the resultant magnetic fields at points 4.0 cm below the cable?

Answer

Number of horizontal wires in the telephone cable, $n = 4$

Current in each wire, $I = 1.0$ A

Earth's magnetic field at a location, $H = 0.39\ \text{G} = 0.39 \times 10^{-4}\ \text{T}$

Angle of dip at the location, $\delta = 35°$

Angle of declination, $\theta \sim 0°$

For a point 4 cm below the cable:

Distance, $r = 4\ \text{cm} = 0.04\ \text{m}$

The horizontal component of earth's magnetic field can be written as:

$H_h = H\cos\delta - B$

Where,

B = Magnetic field at 4 cm due to current I in the four wires

$$= 4 \times \frac{\mu_0 I}{2\pi r}$$

μ_0 = Permeability of free space = $4\pi \times 10^{-7}$ Tm A^{-1}

$$\therefore B = 4 \times \frac{4\pi \times 10^{-7} \times 1}{2\pi \times 0.04}$$

$= 0.2 \times 10^{-4}$ T = 0.2 G

$\therefore H_h = 0.39 \cos 35° - 0.2$

$= 0.39 \times 0.819 - 0.2 \approx 0.12$ G

The vertical component of earth's magnetic field is given as:

$H_v = H\sin\delta$

$= 0.39 \sin 35° = 0.22$ G

The angle made by the field with its horizontal component is given as:

$$\theta = \tan^{-1}\frac{H_v}{H_h}$$
$$= \tan^{-1}\frac{0.22}{0.12} = 61.39°$$

The resultant field at the point is given as:

$$H_1 = \sqrt{(H_v)^2 + (H_h)^2}$$
$$= \sqrt{(0.22)^2 + (0.12)^2} = 0.25 \text{ G}$$

For a point 4 cm above the cable:

Horizontal component of earth's magnetic field:

$H_h = H\cos\delta + B$

$= 0.39 \cos 35° + 0.2 = 0.52$ G

Vertical component of earth's magnetic field:

$H_v = H\sin\delta$

$= 0.39 \sin 35° = 0.22$ G

Angle, $\theta = \tan^{-1}\dfrac{H_v}{H_h} = \tan^{-1}\dfrac{0.22}{0.52} = 22.9°$

And resultant field:

$$H_2 = \sqrt{(H_v)^2 + (H_h)^2}$$
$$= \sqrt{(0.22)^2 + (0.52)^2} = 0.56 \text{ T}$$

Question 5.20:

A compass needle free to turn in a horizontal plane is placed at the centre of circular coil of 30 turns and radius 12 cm. The coil is in a vertical plane making an angle of 45º with the magnetic meridian. When the current in the coil is 0.35 A, the needle points west to east.

Determine the horizontal component of the earth's magnetic field at the location.

The current in the coil is reversed, and the coil is rotated about its vertical axis by an angle of 90º in the anticlockwise sense looking from above. Predict the direction of the needle. Take the magnetic declination at the places to be zero.

Answer

Number of turns in the circular coil, $N = 30$

Radius of the circular coil, $r = 12$ cm $= 0.12$ m

Current in the coil, $I = 0.35$ A

Angle of dip, $\delta = 45°$

The magnetic field due to current I, at a distance r, is given as:

$$B = \frac{\mu_0 2\pi NI}{4\pi r}$$

Where,

μ_0 = Permeability of free space = $4\pi \times 10^{-7}$ Tm A^{-1}

$$\therefore B = \frac{4\pi \times 10^{-7} \times 2\pi \times 30 \times 0.35}{4\pi \times 0.12}$$

= 5.49×10^{-5} T

The compass needle points from West to East. Hence, the horizontal component of earth's magnetic field is given as:

$B_H = B\sin\delta$

= $5.49 \times 10^{-5} \sin 45° = 3.88 \times 10^{-5}$ T = 0.388 G

When the current in the coil is reversed and the coil is rotated about its vertical axis by an angle of 90 º, the needle will reverse its original direction. In this case, the needle will point from East to West.

Question 5.21:

A magnetic dipole is under the influence of two magnetic fields. The angle between the field directions is 60º, and one of the fields has a magnitude of 1.2×10^{-2} T. If the dipole comes to stable equilibrium at an angle of 15º with this field, what is the magnitude of the other field?

Answer

Magnitude of one of the magnetic fields, $B_1 = 1.2 \times 10^{-2}$ T

Magnitude of the other magnetic field = B_2

Angle between the two fields, $\theta = 60°$

At stable equilibrium, the angle between the dipole and field B_1, $\theta_1 = 15°$

Angle between the dipole and field B_2, $\theta_2 = \theta - \theta_1 = 60° - 15° = 45°$

At rotational equilibrium, the torques between both the fields must balance each other.

$\therefore$Torque due to field B_1 = Torque due to field B_2

$MB_1 \sin\theta_1 = MB_2 \sin\theta_2$

Where,

M = Magnetic moment of the dipole

$$\therefore B_2 = \frac{B_1 \sin\theta_1}{\sin\theta_2}$$

$$= \frac{1.2 \times 10^{-2} \times \sin 15^\circ}{\sin 45^\circ} = 4.39 \times 10^{-3} \text{ T}$$

Hence, the magnitude of the other magnetic field is 4.39×10^{-3} T.

Question 5.22:

A monoenergetic (18 keV) electron beam initially in the horizontal direction is subjected to a horizontal magnetic field of 0.04 G normal to the initial direction. Estimate the up or down deflection of the beam over a distance of 30 cm (m_e= 9.11 × 10^{-19} C). [**Note:** *Data* in this exercise are so chosen that the answer will give you an idea of the effect of earth's magnetic field on the motion of the electron beam from the electron gun to the screen in a TV set.]

Answer

Energy of an electron beam, E = 18 keV = 18 × 10^3 eV

Charge on an electron, e = 1.6 × 10^{-19} C

E = 18 × 10^3 × 1.6 × 10^{-19} J

Magnetic field, B = 0.04 G

Mass of an electron, m_e = 9.11 × 10^{-19} kg

Distance up to which the electron beam travels, d = 30 cm = 0.3 m

We can write the kinetic energy of the electron beam as:

$$E = \frac{1}{2}mv^2$$

$$v = \sqrt{\frac{2E}{m}}$$

$$= \sqrt{\frac{2 \times 18 \times 10^3 \times 1.6 \times 10^{-19} \times 10^{-15}}{9.11 \times 10^{-31}}} = 0.795 \times 10^8 \text{ m/s}$$

The electron beam deflects along a circular path of radius, *r*.

The force due to the magnetic field balances the centripetal force of the path.

$$BeV = \frac{mv^2}{r}$$

$$\therefore r = \frac{mv}{Be}$$

$$= \frac{9.11 \times 10^{-31} \times 0.795 \times 10^8}{0.4 \times 10^{-4} \times 1.6 \times 10^{-19}} = 11.3 \text{ m}$$

Let the up and down deflection of the electron beam be $x = r(1 - \cos\theta)$

Where,

θ = Angle of declination

$$\sin\theta = \frac{d}{r}$$

$$= \frac{0.3}{11.3}$$

$$\theta = \sin^{-1}\frac{0.3}{11.3} = 1.521°$$

And $x = 11.3(1 - \cos 1.521°)$

$$= 0.0039 \text{ m} = 3.9 \text{ mm}$$

Therefore, the up and down deflection of the beam is 3.9 mm.

Question 5.23:

A sample of paramagnetic salt contains 2.0×10^{24} atomic dipoles each of dipole moment 1.5×10^{-23} J T^{-1}. The sample is placed under a homogeneous magnetic field of 0.64 T, and cooled to a temperature of 4.2 K. The degree of magnetic saturation achieved is equal to 15%. What is the total dipole moment of the sample for a magnetic field of 0.98 T and a temperature of 2.8 K? (Assume Curie's law)

Answer

Number of atomic dipoles, $n = 2.0 \times 10^{24}$

Dipole moment of each atomic dipole, $M = 1.5 \times 10^{-23}$ J T^{-1}

When the magnetic field, $B_1 = 0.64$ T

The sample is cooled to a temperature, $T_1 = 4.2°K$

Total dipole moment of the atomic dipole, $M_{tot} = n \times M$

$= 2 \times 10^{24} \times 1.5 \times 10^{-23}$

$= 30$ J T^{-1}

Magnetic saturation is achieved at 15%.

Hence, effective dipole moment, $M_1 = \frac{15}{100} \times 30 = 4.5 \text{ J T}^{-1}$

When the magnetic field, $B_2 = 0.98$ T

Temperature, $T_2 = 2.8°K$

Its total dipole moment = M_2

According to Curie's law, we have the ratio of two magnetic dipoles as:

$$\frac{M_2}{M_1} = \frac{B_2}{B_1} \times \frac{T_1}{T_2}$$

$$\therefore M_2 = \frac{B_2 T_1 M_1}{B_1 T_2}$$

$$= \frac{0.98 \times 4.2 \times 4.5}{2.8 \times 0.64} = 10.336 \text{ J T}^{-1}$$

Therefore, 10.336 J T^{-1} is the total dipole moment of the sample for a magnetic field of 0.98 T and a temperature of 2.8 K.

Question 5.24:

A Rowland ring of mean radius 15 cm has 3500 turns of wire wound on a ferromagnetic core of relative permeability 800. What is the magnetic field **B** in the core for a magnetising current of 1.2 A?

Answer

Mean radius of a Rowland ring, r = 15 cm = 0.15 m

Number of turns on a ferromagnetic core, N = 3500

Relative permeability of the core material, $\mu_r = 800$

Magnetising current, I = 1.2 A

The magnetic field is given by the relation:

$$B = \frac{\mu_r \mu_0 IN}{2\pi r}$$

Where,

μ_0 = Permeability of free space = $4\pi \times 10^{-7}$ Tm A^{-1}

$$B = \frac{800 \times 4\pi \times 10^{-7} \times 1.2 \times 3500}{2\pi \times 0.15} = 4.48 \text{ T}$$

Therefore, the magnetic field in the core is 4.48 T.

Question 5.25:

The magnetic moment vectors μ_s and μ_l associated with the intrinsic spin angular momentum **S** and orbital angular momentum **l**, respectively, of an electron are predicted by quantum theory (and verified experimentally to a high accuracy) to be given by:

$\mu_s = -(e/m)\,\mathbf{S}$,

$\mu_l = -(e/2m)\mathbf{l}$

Which of these relations is in accordance with the result expected *classically*? Outline the derivation of the classical result.

Answer

The magnetic moment associated with the intrinsic spin angular momentum ( ) is given as

The magnetic moment associated with the orbital angular momentum ( ) is given as

For current i and area of cross-section A, we have the relation:

Where,

e = Charge of the electron

r = Radius of the circular orbit

T = Time taken to complete one rotation around the circular orbit of radius r

Angular momentum, $l = mvr$

Where,

m = Mass of the electron

v = Velocity of the electron

Dividing equation (1) by equation (2), we get:

Therefore, of the two relations, is in accordance with classical physics.