REAL NUMBERS

(A) Main Concepts and Results

- Euclid's Division Lemma : Given two positive integers a and b, there exist unique integers q and r satisfying $a = bq + r$, $0 \leq r < b$.
- Euclid's Division Algorithm to obtain the HCF of two positive integers, say c and d, $c > d$.

  Step 1 : Apply Euclid's division lemma to c and d, to find whole numbers q and r, such that $c = dq + r$, $0 \leq r < d$.

  Step 2 : If $r = 0$, d is the HCF of c and d. If $r \neq 0$, apply the division lemma to d and r.

  Step 3 : Continue the process till the remainder is zero. The divisor at this stage will be the required HCF.
- Fundamental Theorem of Arithmetic : Every composite number can be expressed as a product of primes, and this expression (factorisation) is unique, apart from the order in which the prime factors occur.
- Let p be a prime number. If p divides a^2, then p divides a, where a is a positive integer.
- $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$ are irrational numbers.
- The sum or difference of a rational and an irrational number is irrational.
- The product or quotient of a non-zero rational number and an irrational number is irrational.
- For any two positive integers a and b, HCF (a, b) × LCM $(a, b) = a \times b$.

- Let $x = \frac{p}{q}$, p and q are co-prime, be a rational number whose decimal expansion terminates. Then, the prime factorisation of q is of the form $2^m.5^n$; m, n are non-negative integers.
- Let $x = \frac{p}{q}$ be a rational number such that the prime factorisation of q is not of the form $2^m.5^n$; m, n being non-negative integers. Then, x has a non-terminating repeating decimal expansion.

(B) Multiple Choice Questions

Choose the correct answer from the given four options:

Sample Question 1 : The decimal expansion of the rational number $\frac{33}{2^2.5}$ will terminate after

(A) one decimal place (B) two decimal places

(C) three decimal places (D) more than 3 decimal places

Solution : Answer (B)

Sample Question 2 : Euclid's division lemma states that for two positive integers a and b, there exist unique integers q and r such that $a = bq + r$, where r must satisfy

(A) $1 < r < b$ (B) $0 < r \le b$

(C) $0 \le r < b$ (D) $0 < r < b$

Solution : Answer (C)

EXERCISE 1.1

Choose the correct answer from the given four options in the following questions:

1. For some integer m, every even integer is of the form

(A) m (B) $m + 1$

(C) $2m$ (D) $2m + 1$

2. For some integer q, every odd integer is of the form

(A) q (B) $q + 1$

(C) $2q$ (D) $2q + 1$

3. $n^2 - 1$ is divisible by 8, if n is

(A) an integer (B) a natural number

(C) an odd integer (D) an even integer

4. If the HCF of 65 and 117 is expressible in the form $65m - 117$, then the value of m is

(A) 4 (B) 2

(C) 1 (D) 3

5. The largest number which divides 70 and 125, leaving remainders 5 and 8, respectively, is

(A) 13 (B) 65

(C) 875 (D) 1750

6. If two positive integers a and b are written as

$a = x^3y^2$ and $b = xy^3$; x, y are prime numbers, then HCF (a, b) is

(A) xy (B) xy^2 (C) x^3y^3 (D) x^2y^2

7. If two positive integers p and q can be expressed as

$p = ab^2$ and $q = a^3b$; a, b being prime numbers, then LCM (p, q) is

(A) ab (B) a^2b^2 (C) a^3b^2 (D) a^3b^3

8. The product of a non-zero rational and an irrational number is

(A) always irrational (B) always rational

(C) rational or irrational (D) one

9. The least number that is divisible by all the numbers from 1 to 10 (both inclusive) is

(A) 10 (B) 100 (C) 504 (D) 2520

10. The decimal expansion of the rational number $\frac{14587}{1250}$ will terminate after:

(A) one decimal place (B) two decimal places

(C) three decimal places (D) four decimal places

(C) Short Answer Questions with Reasoning

Sample Question 1: The values of the remainder r, when a positive integer a is divided by 3 are 0 and 1 only. Justify your answer.

Solution : No.

According to Euclid's division lemma,

$$a = 3q + r, \text{ where } 0 \leq r < 3$$

and r is an integer. Therefore, the values of r can be 0, 1 or 2.

Sample Question 2: Can the number 6^n, n being a natural number, end with the digit 5? Give reasons.

Solution : No, because $6^n = (2 \times 3)^n = 2^n \times 3^n$, so the only primes in the factorisation of 6^n are 2 and 3, and not 5.

Hence, it cannot end with the digit 5.

EXERCISE 1.2

1. Write whether every positive integer can be of the form $4q + 2$, where q is an integer. Justify your answer.

2. "The product of two consecutive positive integers is divisible by 2". Is this statement true or false? Give reasons.

3. "The product of three consecutive positive integers is divisible by 6". Is this statement true or false"? Justify your answer.

4. Write whether the square of any positive integer can be of the form $3m + 2$, where m is a natural number. Justify your answer.

5. A positive integer is of the form $3q + 1$, q being a natural number. Can you write its square in any form other than $3m + 1$, i.e., $3m$ or $3m + 2$ for some integer m? Justify your answer.

6. The numbers 525 and 3000 are both divisible only by 3, 5, 15, 25 and 75. What is HCF (525, 3000)? Justify your answer.

7. Explain why $3 \times 5 \times 7 + 7$ is a composite number.

8. Can two numbers have 18 as their HCF and 380 as their LCM? Give reasons.

9. Without actually performing the long division, find if $\frac{987}{10500}$ will have terminating or non-terminating (repeating) decimal expansion. Give reasons for your answer.

10. A rational number in its decimal expansion is 327.7081. What can you say about the prime factors of q, when this number is expressed in the form $\frac{p}{q}$? Give reasons.

(D) Short Answer Questions

Sample Question 1: Using Euclid's division algorithm, find which of the following pairs of numbers are co-prime:

(i) 231, 396 (ii) 847, 2160

Solution : Let us find the HCF of each pair of numbers.

(i) $396 = 231 \times 1 + 165$

$231 = 165 \times 1 + 66$

$165 = 66 \times 2 + 33$

$66 = 33 \times 2 + 0$

Therefore, HCF = 33. Hence, numbers are not co-prime.

(ii) $2160 = 847 \times 2 + 466$

$847 = 466 \times 1 + 381$

$466 = 381 \times 1 + 85$

$381 = 85 \times 4 + 41$

$85 = 41 \times 2 + 3$

$41 = 3 \times 13 + 2$

$3 = 2 \times 1 + 1$

$2 = 1 \times 2 + 0$

Therefore, the HCF = 1. Hence, the numbers are co-prime.

Sample Question 2: Show that the square of an odd positive integer is of the form $8m + 1$, for some whole number m.

Solution: Any positive odd integer is of the form $2q + 1$, where q is a whole number.

Therefore, $(2q + 1)^2 = 4q^2 + 4q + 1 = 4q\,(q + 1) + 1,$ (1)

$q\ (q + 1)$ is either 0 or even. So, it is $2m$, where m is a whole number.

Therefore, $(2q + 1)^2 = 4.2\,m + 1 = 8\,m + 1.$ [From (1)]

Sample Question 3: Prove that $\sqrt{2}+\sqrt{3}$ is irrational.

Solution : Let us suppose that $\sqrt{2}+\sqrt{3}$ is rational. Let $\sqrt{2}+\sqrt{3} = a$, where a is rational.

Therefore, $\sqrt{2} = a - \sqrt{3}$

Squaring on both sides, we get

$$2 = a^2 + 3 - 2a\sqrt{3}$$

Therefore, $\sqrt{3} = \dfrac{a^2 + 1}{2a}$, which is a contradiction as the right hand side is a rational number while $\sqrt{3}$ is irrational. Hence, $\sqrt{2} + \sqrt{3}$ is irrational.

EXERCISE 1.3

1. Show that the square of any positive integer is either of the form $4q$ or $4q + 1$ for some integer q.

2. Show that cube of any positive integer is of the form $4m$, $4m + 1$ or $4m + 3$, for some integer m.

3. Show that the square of any positive integer cannot be of the form $5q + 2$ or $5q + 3$ for any integer q.

4. Show that the square of any positive integer cannot be of the form $6m + 2$ or $6m + 5$ for any integer m.

5. Show that the square of any odd integer is of the form $4q + 1$, for some integer q.

6. If n is an odd integer, then show that $n^2 - 1$ is divisible by 8.

7. Prove that if x and y are both odd positive integers, then $x^2 + y^2$ is even but not divisible by 4.

8. Use Euclid's division algorithm to find the HCF of 441, 567, 693.

9. Using Euclid's division algorithm, find the largest number that divides 1251, 9377 and 15628 leaving remainders 1, 2 and 3, respectively.

10. Prove that $\sqrt{3} + \sqrt{5}$ is irrational.

11. Show that 12^n cannot end with the digit 0 or 5 for any natural number n.

12. On a morning walk, three persons step off together and their steps measure 40 cm, 42 cm and 45 cm, respectively. What is the minimum distance each should walk so that each can cover the same distance in complete steps?

13. Write the denominator of the rational number $\frac{257}{5000}$ in the form $2^m \times 5^n$, where m, n are non-negative integers. Hence, write its decimal expansion, without actual division.

14. Prove that $\sqrt{p} + \sqrt{q}$ is irrational, where p, q are primes.

(E) Long Answer Questions

Sample Question 1 : Show that the square of an odd positive integer can be of the form $6q + 1$ or $6q + 3$ for some integer q.

Solution : We know that any positive integer can be of the form $6m$, $6m + 1$, $6m + 2$, $6m + 3$, $6m + 4$ or $6m + 5$, for some integer m.

Thus, an odd positive integer can be of the form 6m + 1, 6m + 3, or 6m + 5 Thus we have:

$(6\,m + 1)^2 = 36\,m^2 + 12\,m + 1 = 6\,(6\,m^2 + 2\,m) + 1 = 6\,q + 1$, q is an integer

$(6\,m + 3)^2 = 36\,m^2 + 36\,m + 9 = 6\,(6\,m^2 + 6\,m + 1) + 3 = 6\,q + 3$, q is an integer

$(6\,m + 5)^2 = 36\,m^2 + 60\,m + 25 = 6\,(6\,m^2 + 10\,m + 4) + 1 = 6\,q + 1$, q is an integer.

Thus, the square of an odd positive integer can be of the form $6q + 1$ or $6q + 3$.

EXERCISE 1.4

1. Show that the cube of a positive integer of the form $6q + r$, q is an integer and $r = 0, 1, 2, 3, 4, 5$ is also of the form $6m + r$.

2. Prove that one and only one out of n, $n + 2$ and $n + 4$ is divisible by 3, where n is any positive integer.

3. Prove that one of any three consecutive positive integers must be divisible by 3.

4. For any positive integer n, prove that $n^3 - n$ is divisible by 6.

5. Show that one and only one out of n, $n + 4$, $n + 8$, $n + 12$ and $n + 16$ is divisible by 5, where n is any positive integer.

[**Hint:** Any positive integer can be written in the form $5q$, $5q+1$, $5q+2$, $5q+3$, $5q+4$].

CHAPTER 2

POLYNOMIALS

(A) Main Concepts and Results

- Geometrical meaning of zeroes of a polynomial: The zeroes of a polynomial $p(x)$ are precisely the x-coordinates of the points where the graph of $y = p(x)$ intersects the x-axis.
- Relation between the zeroes and coefficients of a polynomial: If α and β are the zeroes of a quadratic polynomial $ax^2 + bx + c$, then $\alpha + \beta = -\frac{b}{a}$, $\alpha\beta = \frac{c}{a}$.
- If α, β and γ are the zeroes of a cubic polynomial $ax^3 + bx^2 + cx + d$, then $\alpha+\beta+\gamma = -\frac{b}{a}$, $\alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a}$ and $\alpha\beta\gamma = \frac{-d}{a}$.
- The division algorithm states that given any polynomial $p(x)$ and any non-zero polynomial $g(x)$, there are polynomials $q(x)$ and $r(x)$ such that $p(x) = g(x)\, q(x) + r(x)$, where $r(x) = 0$ or degree $r(x) <$ degree $g(x)$.

(B) Multiple Choice Questions

Choose the correct answer from the given four options:

Sample Question 1: If one zero of the quadratic polynomial $x^2 + 3x + k$ is 2, then the value of k is

(A) 10 (B) –10 (C) 5 (D) –5

Solution : Answer (B)

Sample Question 2: Given that two of the zeroes of the cubic polynomial $ax^3 + bx^2 + cx + d$ are 0, the third zero is

(A) $\frac{-b}{a}$ (B) $\frac{b}{a}$ (C) $\frac{c}{a}$ (D) $-\frac{d}{a}$

Solution : Answer (A). [**Hint:** Because if third zero is α, sum of the zeroes $= \alpha + 0 + 0 = \frac{-b}{a}$]

EXERCISE 2.1

Choose the correct answer from the given four options in the following questions:

1. If one of the zeroes of the quadratic polynomial $(k–1)\, x^2 + k\,x + 1$ is –3, then the value of k is

(A) $\frac{4}{3}$ (B) $\frac{-4}{3}$ (C) $\frac{2}{3}$ (D) $\frac{-2}{3}$

2. A quadratic polynomial, whose zeroes are –3 and 4, is

(A) $x^2 – x + 12$ (B) $x^2 + x + 12$

(C) $\frac{x^2}{2} - \frac{x}{2} - 6$ (D) $2x^2 + 2x –24$

3. If the zeroes of the quadratic polynomial $x^2 + (a + 1)\, x + b$ are 2 and –3, then

(A) $a = –7, b = –1$ (B) $a = 5, b = –1$

(C) $a = 2, b = – 6$ (D) $a = 0, b = – 6$

4. The number of polynomials having zeroes as –2 and 5 is

(A) 1 (B) 2 (C) 3 (D) more than 3

5. Given that one of the zeroes of the cubic polynomial $ax^3 + bx^2 + cx + d$ is zero, the product of the other two zeroes is

(A) $-\frac{c}{a}$ (B) $\frac{c}{a}$ (C) 0 (D) $-\frac{b}{a}$

6. If one of the zeroes of the cubic polynomial $x^3 + ax^2 + bx + c$ is –1, then the product of the other two zeroes is

(A) $b – a + 1$ (B) $b – a – 1$ (C) $a – b + 1$ (D) $a – b –1$

7. The zeroes of the quadratic polynomial $x^2 + 99x + 127$ are

(A) both positive (B) both negative

(C) one positive and one negative (D) both equal

8. The zeroes of the quadratic polynomial $x^2 + kx + k$, $k \neq 0$,

(A) cannot both be positive (B) cannot both be negative

(C) are always unequal (D) are always equal

9. If the zeroes of the quadratic polynomial $ax^2 + bx + c$, $c \neq 0$ are equal, then

(A) c and a have opposite signs (B) c and b have opposite signs

(C) c and a have the same sign (D) c and b have the same sign

10. If one of the zeroes of a quadratic polynomial of the form x^2+ax+b is the negative of the other, then it

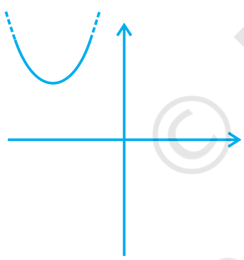
(A) has no linear term and the constant term is negative.

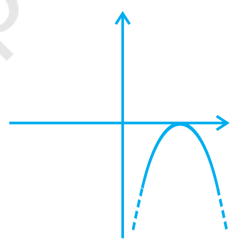
(B) has no linear term and the constant term is positive.

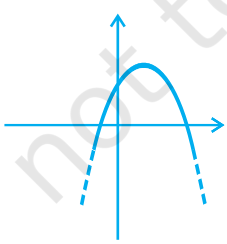
(C) can have a linear term but the constant term is negative.

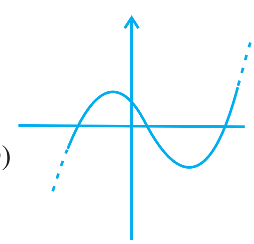
(D) can have a linear term but the constant term is positive.

11. Which of the following is not the graph of a quadratic polynomial?

(A) 

(B) 

(C) 

(D) 

(C) Short Answer Questions with Reasoning

Sample Question 1: Can $x - 1$ be the remainder on division of a polynomial $p(x)$ by $2x + 3$? Justify your answer.

Solution : No, since degree $(x - 1) = 1 =$ degree $(2x + 3)$.

Sample Question 2: Is the following statement True or False? Justify your answer.

If the zeroes of a quadratic polynomial $ax^2 + bx + c$ are both negative, then a, b and c all have the same sign.

Solution : True, because $-\frac{b}{a}$ = sum of the zeroes < 0, so that $\frac{b}{a} > 0$. Also the product of the zeroes $= \frac{c}{a} > 0$.

EXERCISE 2.2

1. Answer the following and justify:
   (i) Can $x^2 - 1$ be the quotient on division of $x^6 + 2x^3 + x - 1$ by a polynomial in x of degree 5?
   (ii) What will the quotient and remainder be on division of $ax^2 + bx + c$ by $px^3 + qx^2 + rx + s$, $p \neq 0$?
   (iii) If on division of a polynomial $p(x)$ by a polynomial $g(x)$, the quotient is zero, what is the relation between the degrees of $p(x)$ and $g(x)$?
   (iv) If on division of a non-zero polynomial $p(x)$ by a polynomial $g(x)$, the remainder is zero, what is the relation between the degrees of $p(x)$ and $g(x)$?
   (v) Can the quadratic polynomial $x^2 + kx + k$ have equal zeroes for some odd integer $k > 1$?
2. Are the following statements 'True' or 'False'? Justify your answers.
   (i) If the zeroes of a quadratic polynomial $ax^2 + bx + c$ are both positive, then a, b and c all have the same sign.
   (ii) If the graph of a polynomial intersects the x-axis at only one point, it cannot be a quadratic polynomial.
   (iii) If the graph of a polynomial intersects the x-axis at exactly two points, it need not be a quadratic polynomial.
   (iv) If two of the zeroes of a cubic polynomial are zero, then it does not have linear and constant terms.

(v) If all the zeroes of a cubic polynomial are negative, then all the coefficients and the constant term of the polynomial have the same sign.

(vi) If all three zeroes of a cubic polynomial $x^3 + ax^2 - bx + c$ are positive, then at least one of a, b and c is non-negative.

(vii) The only value of k for which the quadratic polynomial $kx^2 + x + k$ has equal zeros is $\frac{1}{2}$

(D) Short Answer Questions

Sample Question 1: Find the zeroes of the polynomial $x^2 + \frac{1}{6}x - 2$, and verify the relation between the coefficients and the zeroes of the polynomial.

Solution : $x^2 + \frac{1}{6}x - 2 = \frac{1}{6}(6x^2 + x - 12) = \frac{1}{6}[6x^2 + 9x - 8x - 12]$

$$= \frac{1}{6}[3x(2x + 3) - 4(2x + 3)] = \frac{1}{6}(3x - 4)(2x + 3)$$

Hence, $\frac{4}{3}$ and $-\frac{3}{2}$ are the zeroes of the given polynomial.

The given polynomial is $x^2 + \frac{1}{6}x - 2$.

The sum of zeroes $= \frac{4}{3} + \left(-\frac{3}{2}\right) = \frac{-1}{6} = -\frac{\text{Coefficient of } x}{\text{Coefficient of } x^2}$ and

the product of zeroes $= \frac{4}{3} \times \frac{-3}{2} = -2 = \frac{\text{Constant term}}{\text{Coefficient of } x^2}$

EXERCISE 2.3

Find the zeroes of the following polynomials by factorisation method and verify the relations between the zeroes and the coefficients of the polynomials:

1. $4x^2 - 3x - 1$ **2.** $3x^2 + 4x - 4$

3. $5t^2 + 12t + 7$

4. $t^3 - 2t^2 - 15t$

5. $2x^2 + \frac{7}{2}x + \frac{3}{4}$

6. $4x^2 + 5\sqrt{2}\,x - 3$

7. $2s^2 - (1 + 2\sqrt{2})s + \sqrt{2}$

8. $v^2 + 4\sqrt{3}\,v - 15$

9. $y^2 + \frac{3}{2}\sqrt{5}\,y - 5$

10. $7y^2 - \frac{11}{3}y - \frac{2}{3}$

(E) Long Answer Questions

Sample Question 1: Find a quadratic polynomial, the sum and product of whose zeroes are $\sqrt{2}$ and $-\frac{3}{2}$, respectively. Also find its zeroes.

Solution : A quadratic polynomial, the sum and product of whose zeroes are $\sqrt{2}$ and $-\frac{3}{2}$ is $x^2 - \sqrt{2}\,x - \frac{3}{2}$

$$x^2 - \sqrt{2}\,x - \frac{3}{2} = \frac{1}{2}[2x^2 - 2\sqrt{2}\,x - 3]$$

$$= \frac{1}{2}[2x^2 + \sqrt{2}\,x - 3\sqrt{2x} - 3]$$

$$= \frac{1}{2}[\sqrt{2}\,x(\sqrt{2}\,x + 1) - 3(\sqrt{2}\,x + 1)]$$

$$= \frac{1}{2}[\sqrt{2}\,x + 1][\sqrt{2}\,x - 3]$$

Hence, the zeroes are $-\frac{1}{\sqrt{2}}$ and $\frac{3}{\sqrt{2}}$.

Sample Question 2: If the remainder on division of $x^3 + 2x^2 + kx + 3$ by $x - 3$ is 21, find the quotient and the value of k. Hence, find the zeroes of the cubic polynomial $x^3 + 2x^2 + kx - 18$.

Solution : Let $p(x) = x^3 + 2x^2 + kx + 3$

Then, $p(3) = 3^3 + 2 \times 3^2 + 3k + 3 = 21$

i.e., $3k = -27$

i.e., $k = -9$

Hence, the given polynomial will become $x^3 + 2x^2 - 9x + 3$.

Now,

$$
\begin{array}{r|l}
x-3 & x^3 + 2x^2 - 9x + 3 \;\big(\; x^2 + 5x + 6 \\
 & \underline{x^3 - 3x^2} \\
 & \quad 5x^2 - 9x + 3 \\
 & \quad \underline{5x^2 - 15x} \\
 & \qquad 6x + 3 \\
 & \qquad \underline{6x - 18} \\
 & \qquad\quad 21
\end{array}
$$

So, $x^3 + 2x^2 - 9x + 3 = (x^2 + 5x + 6)(x - 3) + 21$

i.e., $x^3 + 2x^2 - 9x - 18 = (x - 3)(x^2 + 5x + 6)$

$= (x - 3)(x + 2)(x + 3)$

So, the zeroes of $x^3 + 2x^2 + kx - 18$ are 3, – 2, – 3.

EXERCISE 2.4

1. For each of the following, find a quadratic polynomial whose sum and product respectively of the zeroes are as given. Also find the zeroes of these polynomials by factorisation.

(i) $\frac{-8}{3}, \frac{4}{3}$

(ii) $\frac{21}{8}, \frac{5}{16}$

(iii) $-2\sqrt{3}, -9$

(iv) $\frac{-3}{2\sqrt{5}}, -\frac{1}{2}$

2. Given that the zeroes of the cubic polynomial $x^3 - 6x^2 + 3x + 10$ are of the form a, $a + b$, $a + 2b$ for some real numbers a and b, find the values of a and b as well as the zeroes of the given polynomial.

3. Given that $\sqrt{2}$ is a zero of the cubic polynomial $6x^3 + \sqrt{2}\,x^2 - 10x - 4\sqrt{2}$, find its other two zeroes.

4. Find k so that $x^2 + 2x + k$ is a factor of $2x^4 + x^3 - 14x^2 + 5x + 6$. Also find all the zeroes of the two polynomials.

5. Given that $x - \sqrt{5}$ is a factor of the cubic polynomial $x^3 - 3\sqrt{5}x^2 + 13x - 3\sqrt{5}$, find all the zeroes of the polynomial.

6. For which values of a and b, are the zeroes of $q(x) = x^3 + 2x^2 + a$ also the zeroes of the polynomial $p(x) = x^5 - x^4 - 4x^3 + 3x^2 + 3x + b$? Which zeroes of $p(x)$ are not the zeroes of $q(x)$?

CHAPTER 3

PAIR OF LINEAR EQUATIONS IN TWO VARIABLES

(A) Main Concepts and Results

- Two linear equations in the same two variables are said to form a pair of linear equations in two variables.
- The most general form of a pair of linear equations is

  $a_1x + b_1y + c_1 = 0$

  $a_2x + b_2y + c_2 = 0$,

  where $a_1, a_2, b_1, b_2, c_1, c_2$ are real numbers, such that $a_1^2 + b_1^2 \neq 0, a_2^2 + b_2^2 \neq 0$.
- A pair of linear equations is consistent if it has a solution – either a unique or infinitely many.

  In case of infinitely many solutions, the pair of linear equations is also said to be dependent. Thus, in this case, the pair of linear equations is dependent and consistent.
- A pair of linear equations is inconsistent, if it has no solution.
- Let a pair of linear equations in two variables be $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$.

(I) If $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$, then

(i) the pair of linear equations is consistent,

(ii) the graph will be a pair of lines intersecting at a unique point, which is the solution of the pair of equations.

(II) If $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$, then

(i) the pair of linear equations is inconsistent,

(ii) the graph will be a pair of parallel lines and so the pair of equations will have no solution.

(III) If $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$, then

(i) the pair of linear equations is dependent, and consistent,

(ii) the graph will be a pair of coincident lines. Each point on the lines will be a solution, and so the pair of equations will have infinitely many solutions.

- A pair of linear equations can be solved algebraically by any of the following methods:

  (i) Substitution Method

  (ii) Elimination Method

  (iii) Cross- multiplication Method

- The pair of linear equations can also be solved geometrically/graphically.

(B) Multiple Choice Questions

Choose the correct answer from the given four options:

Sample Question 1 : The pair of equations $5x - 15y = 8$ and $3x - 9y = \frac{24}{5}$ has

(A) one solution (B) two solutions (C) infinitely many solutions
(D) no solution

Solution : Answer (C)

Sample Question 2 : The sum of the digits of a two-digit number is 9. If 27 is added to it, the digits of the number get reversed. The number is

(A) 25 (B) 72 (C) 63 (D) 36

Solution : Answer (D)

EXERCISE 3.1

Choose the correct answer from the given four options:

1. Graphically, the pair of equations

$$6x - 3y + 10 = 0$$

$$2x - y + 9 = 0$$

represents two lines which are

(A) intersecting at exactly one point. (B) intersecting at exactly two points.

(C) coincident. (D) parallel.

2. The pair of equations $x + 2y + 5 = 0$ and $-3x - 6y + 1 = 0$ have

(A) a unique solution (B) exactly two solutions

(C) infinitely many solutions (D) no solution

3. If a pair of linear equations is consistent, then the lines will be

(A) parallel (B) always coincident

(C) intersecting or coincident (D) always intersecting

4. The pair of equations $y = 0$ and $y = -7$ has

(A) one solution (B) two solutions

(C) infinitely many solutions (D) no solution

5. The pair of equations $x = a$ and $y = b$ graphically represents lines which are

(A) parallel (B) intersecting at (b, a)

(C) coincident (D) intersecting at (a, b)

6. For what value of k, do the equations $3x - y + 8 = 0$ and $6x - ky = -16$ represent coincident lines?

(A) $\frac{1}{2}$ (B) $-\frac{1}{2}$ (C) 2 (D) -2

7. If the lines given by $3x + 2ky = 2$ and $2x + 5y + 1 = 0$ are parallel, then the value of k is

(A) $\frac{-5}{4}$ (B) $\frac{2}{5}$ (C) $\frac{15}{4}$ (D) $\frac{3}{2}$

8. The value of c for which the pair of equations $cx - y = 2$ and $6x - 2y = 3$ will have infinitely many solutions is

(A) 3 (B) – 3 (C) –12 (D) no value

9. One equation of a pair of dependent linear equations is $-5x + 7y = 2$. The second equation can be

(A) $10x + 14y + 4 = 0$
(B) $-10x - 14y + 4 = 0$
(C) $-10x + 14y + 4 = 0$
(D) $10x - 14y = -4$

10. A pair of linear equations which has a unique solution $x = 2, y = -3$ is

(A) $x + y = -1$
$2x - 3y = -5$

(B) $2x + 5y = -11$
$4x + 10y = -22$

(C) $2x - y = 1$
$3x + 2y = 0$

(D) $x - 4y - 14 = 0$
$5x - y - 13 = 0$

11. If $x = a, y = b$ is the solution of the equations $x - y = 2$ and $x + y = 4$, then the values of a and b are, respectively

(A) 3 and 5
(B) 5 and 3
(C) 3 and 1
(D) –1 and –3

12. Aruna has only Re 1 and Rs 2 coins with her. If the total number of coins that she has is 50 and the amount of money with her is Rs 75, then the number of Re 1 and Rs 2 coins are, respectively

(A) 35 and 15
(B) 35 and 20
(C) 15 and 35
(D) 25 and 25

13. The father's age is six times his son's age. Four years hence, the age of the father will be four times his son's age. The present ages, in years, of the son and the father are, respectively

(A) 4 and 24
(B) 5 and 30
(C) 6 and 36
(D) 3 and 24

(C) Short Answer Questions with Reasoning

Sample Question 1: Is it true to say that the pair of equations

$$-x + 2y + 2 = 0 \text{ and } \frac{1}{2}x - \frac{1}{4}y - 1 = 0$$

has a unique solution? Justify your answer.

Solution : Yes.

Here, $\frac{a_1}{a_2} = \frac{-1}{\frac{1}{2}} = -2$, $\frac{b_1}{b_2} = \frac{2}{-\frac{1}{4}} = -8$

As $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$, the pair of equations has a unique solution.

Sample Question 2 : Do the equations $4x + 3y - 1 = 5$ and $12x + 9y = 15$ represent a pair of coincident lines? Justify your answer.

Solution : No.

We may rewrite the equations as

$4x + 3y = 6$

$12x + 9y = 15$

Here, $\frac{a_1}{a_2} = \frac{1}{3}$, $\frac{b_1}{b_2} = \frac{1}{3}$ and $\frac{c_1}{c_2} = \frac{2}{5}$

As $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$, the given equations do not represent a pair of coincident lines.

Sample Question 3 : Is the pair of equations $x + 2y - 3 = 0$ and $6y + 3x - 9 = 0$ consistent? Justify your answer.

Solution : Yes.

Rearranging the terms in the equations, we get

$$x + 2y - 3 = 0$$

$$3x + 6y - 9 = 0$$

Here, $\frac{a_1}{a_2} = \frac{1}{3}, \frac{b_1}{b_2} = \frac{1}{3}, \frac{c_1}{c_2} = \frac{1}{3}$. As $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$, the pair of equations is consistent.

EXERCISE 3.2

1. Do the following pair of linear equations have no solution? Justify your answer.

(i) $2x + 4y = 3$
$12y + 6x = 6$

(ii) $x = 2y$
$y = 2x$

(iii) $3x + y - 3 = 0$
$2x + \frac{2}{3}y = 2$

2. Do the following equations represent a pair of coincident lines? Justify your answer.

(i) $3x + \frac{1}{7}y = 3$
$7x + 3y = 7$

(ii) $-2x - 3y = 1$
$6y + 4x = -2$

(iii) $\frac{x}{2} + y + \frac{2}{5} = 0$
$4x + 8y + \frac{5}{16} = 0$

3. Are the following pair of linear equations consistent? Justify your answer.

(i) $-3x - 4y = 12$
$4y + 3x = 12$

(ii) $\frac{3}{5}x - y = \frac{1}{2}$
$\frac{1}{5}x - 3y = \frac{1}{6}$

(iii) $2ax + by = a$
$4ax + 2by - 2a = 0; \quad a, b \neq 0$

(iv) $x + 3y = 11$
$2(2x + 6y) = 22$

4. For the pair of equations

$\lambda x + 3y = -7$

$2x + 6y = 14$

to have infinitely many solutions, the value of λ should be 1. Is the statement true? Give reasons.

5. For all real values of c, the pair of equations

$x - 2y = 8$

$5x - 10y = c$

have a unique solution. Justify whether it is true or false.

6. The line represented by $x = 7$ is parallel to the x–axis. Justify whether the statement is true or not.

(D) Short Answer Questions

Sample Question 1 : For which values of p and q, will the following pair of linear equations have infinitely many solutions?

$$4x + 5y = 2$$

$$(2p + 7q)\, x + (p + 8q)\, y = 2q - p + 1.$$

Solution :

Here, $\dfrac{a_1}{a_2} = \dfrac{4}{2p+7q}$

$$\frac{b_1}{b_2} = \frac{5}{p+8q}$$

$$\frac{c_1}{c_2} = \frac{2}{2q-p+1}$$

For a pair of linear equations to have infinitely many solutions

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

So, $\dfrac{4}{2p+7q} = \dfrac{5}{p+8q} = \dfrac{2}{2q-p+1}$

So, $\dfrac{4}{2p+7q} = \dfrac{5}{p+8q}$ and $\dfrac{4}{2p+7q} = \dfrac{2}{2q-p+1}$

i.e., $4p + 32q = 10p + 35q$ and $8q - 4p + 4 = 4p + 14q$

i.e., $6p + 3q = 0$ and $8p + 6q = 4$

i.e., $q = -2p$ (1) and $4p + 3q = 2$ (2)

Substituting the value of q obtained from Equation(1) in Equation(2), we get

$$4p - 6p = 2$$

or $p = -1$

Substituting the value of p in Equation (1), we get

$$q = 2$$

So, for $p = -1$, $q = 2$, the given pair of linear equations will have infinitely many solutions.

Sample Question 2: Solve the following pair of linear equations:

$$21x + 47y = 110$$

$$47x + 21y = 162$$

Solution: We have

$21x + 47y = 110$ (1)
$47x + 21y = 162$ (2)

Multiplying Equation (1) by 47 and Equation (2) by 21, we get

$987x + 2209\,y = 5170$ (3)

$987x + 441y = 3402$ (4)

Subtracting Equation (4) from Equation (3), we get

$$1768y = 1768$$

or $y = 1$

Substituting the value of y in Equation (1), we get

$$21x + 47 = 110$$

or $21x = 63$
or $x = 3$
So, $x = 3$, $y = 1$

Alternative Solution: We have

$21x + 47y = 110$ (1)

$47x + 21y = 162$ (2)

Adding Equations (1) and (2), we have

$$68x + 68y = 272$$

or $x + y = 4$ (5)

Subtracting Equation (1) from Equation (2), we have

$$26x - 26y = 52$$

or $x - y = 2$ (6)

On adding and subtracting Equations (5) and (6), we get

$$x = 3, \quad y = 1$$

Sample Question 3 : Draw the graphs of the pair of linear equations $x - y + 2 = 0$ and $4x - y - 4 = 0$. Calculate the area of the triangle formed by the lines so drawn and the x-axis.

Solution :

For drawing the graphs of the given equations, we find two solutions of each of the equations, which are given in Table 3.1

Table 3.1

x	0	–2
$y = x + 2$	2	0

x	0	1
$y = 4x - 4$	– 4	0

Plot the points A (0, 2), B (–2, 0), P (0, –4) and Q (1, 0) on the graph paper, and join the points to form the lines AB and PQ as shown in Fig 3.1

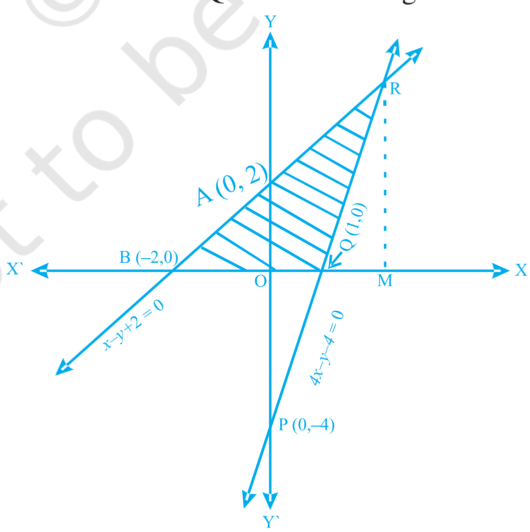


Fig. 3.1

We observe that there is a point R (2, 4) common to both the lines AB and PQ. The triangle formed by these lines and the x- axis is BQR.

The vertices of this triangle are B (–2, 0), Q (1, 0) and R (2, 4).

We know that;

Area of triangle = $\frac{1}{2}$ Base $\times$ Altitude

Here, Base = BQ = BO + OQ = 2 + 1 = 3 units.

Altitude = RM = Ordinate of R = 4 units.

So, area of Δ BQR = $\frac{1}{2} \times 3 \times 4 = 6$ sq. units.

EXERCISE 3.3

1. For which value(s) of λ, do the pair of linear equations

$\lambda x + y = \lambda^2$ and $x + \lambda y = 1$ have

(i) no solution?

(ii) infinitely many solutions?

(iii) a unique solution?

2. For which value(s) of k will the pair of equations

$kx + 3y = k - 3$

$12x + ky = k$

have no solution?

3. For which values of a and b, will the following pair of linear equations have infinitely many solutions?

$x + 2y = 1$

$(a - b)x + (a + b)y = a + b - 2$

4. Find the value(s) of p in (i) to (iv) and p and q in (v) for the following pair of equations:

(i) $3x - y - 5 = 0$ and $6x - 2y - p = 0$,
if the lines represented by these equations are parallel.

(ii) $-x + py = 1$ and $px - y = 1$,
if the pair of equations has no solution.

(iii) $-3x + 5y = 7$ and $2px - 3y = 1$,

if the lines represented by these equations are intersecting at a unique point.

(iv) $2x + 3y - 5 = 0$ and $px - 6y - 8 = 0$,
if the pair of equations has a unique solution.

(v) $2x + 3y = 7$ and $2px + py = 28 - qy$,
if the pair of equations have infinitely many solutions.

5. Two straight paths are represented by the equations $x - 3y = 2$ and $-2x + 6y = 5$. Check whether the paths cross each other or not.

6. Write a pair of linear equations which has the unique solution $x = -1$, $y = 3$. How many such pairs can you write?

7. If $2x + y = 23$ and $4x - y = 19$, find the values of $5y - 2x$ and $\frac{y}{x} - 2$.

8. Find the values of x and y in the following rectangle [see Fig. 3.2].

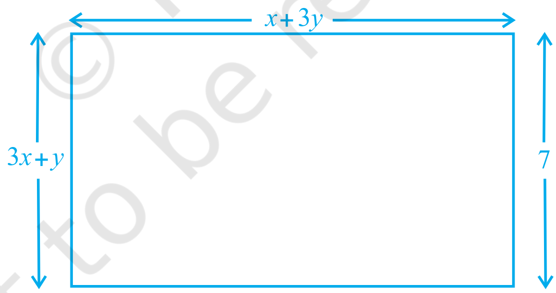


Fig. 3.2

9. Solve the following pairs of equations:

(i) $x + y = 3.3$

$\frac{0.6}{3x-2y} = -1, \quad 3x - 2y \neq 0$

(ii) $\frac{x}{3} + \frac{y}{4} = 4$

$\frac{5x}{6} - \frac{y}{8} = 4$

(iii) $4x + \frac{6}{y} = 15$

$6x - \frac{8}{y} = 14,\ y \neq 0$

(iv) $\frac{1}{2x} - \frac{1}{y} = -1$

$\frac{1}{x} + \frac{1}{2y} = 8, \qquad x, y \neq 0$

(v) $43x + 67y = -24$

$67x + 43y = 24$

(vi) $\frac{x}{a} + \frac{y}{b} = a + b$

$\frac{x}{a^2} + \frac{y}{b^2} = 2, \qquad a, b \neq 0$

(vii) $\frac{2xy}{x + y} = \frac{3}{2}$

$\frac{xy}{2x - y} = \frac{-3}{10}, \qquad x + y \neq 0,\ 2x - y \neq 0$

10. Find the solution of the pair of equations $\frac{x}{10} + \frac{y}{5} - 1 = 0$ and $\frac{x}{8} + \frac{y}{6} = 15$.

Hence, find λ, if $y = \lambda x + 5$.

11. By the graphical method, find whether the following pair of equations are consistent or not. If consistent, solve them.

(i) $3x + y + 4 = 0$

$6x - 2y + 4 = 0$

(ii) $x - 2y = 6$

$3x - 6y = 0$

(iii) $x + y = 3$

$3x + 3y = 9$

12. Draw the graph of the pair of equations $2x + y = 4$ and $2x - y = 4$. Write the vertices of the triangle formed by these lines and the y-axis. Also find the area of this triangle.

13. Write an equation of a line passing through the point representing solution of the pair of linear equations $x+y = 2$ and $2x-y = 1$. How many such lines can we find?

14. If $x+1$ is a factor of $2x^3 + ax^2 + 2bx + 1$, then find the values of a and b given that $2a-3b = 4$.

15. The angles of a triangle are x, y and 40°. The difference between the two angles x and y is 30°. Find x and y.

16. Two years ago, Salim was thrice as old as his daughter and six years later, he will be four years older than twice her age. How old are they now?

17. The age of the father is twice the sum of the ages of his two children. After 20 years, his age will be equal to the sum of the ages of his children. Find the age of the father.

18. Two numbers are in the ratio 5 : 6. If 8 is subtracted from each of the numbers, the ratio becomes 4 : 5. Find the numbers.

19. There are some students in the two examination halls A and B. To make the number of students equal in each hall, 10 students are sent from A to B. But if 20 students are sent from B to A, the number of students in A becomes double the number of students in B. Find the number of students in the two halls.

20. A shopkeeper gives books on rent for reading. She takes a fixed charge for the first two days, and an additional charge for each day thereafter. Latika paid Rs 22 for a book kept for six days, while Anand paid Rs 16 for the book kept for four days. Find the fixed charges and the charge for each extra day.

21. In a competitive examination, one mark is awarded for each correct answer while $\frac{1}{2}$ mark is deducted for every wrong answer. Jayanti answered 120 questions and got 90 marks. How many questions did she answer correctly?

22. The angles of a cyclic quadrilateral ABCD are

$\angle A = (6x + 10)^\circ$, $\angle B = (5x)^\circ$

$\angle C = (x + y)^\circ$, $\angle D = (3y - 10)^\circ$

Find x and y, and hence the values of the four angles.

(E) Long Answer Questions

Sample Question 1 : Draw the graphs of the lines $x = -2$ and $y = 3$. Write the vertices of the figure formed by these lines, the x-axis and the y-axis. Also, find the area of the figure.

Solution :

We know that the graph of $x = -2$ is a line parallel to y-axis at a distance of 2 units to the left of it.

So, the line l is the graph of $x = -2$ [see Fig. 3.3]

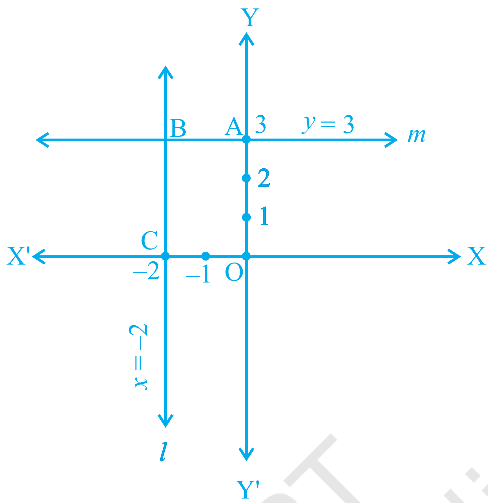


Fig. 3.3

The graph of $y = 3$ is a line parallel to the x-axis at a distance of 3 units above it.

So, the line m is the graph of $y = 3$.

The figure enclosed by the lines $x = –2$, $y = 3$, the x-axis and the y-axis is OABC, which is a rectangle. (Why?)

A is a point on the y-axis at a distance of 3 units above the x-axis. So, the coordinates of A are (0, 3);

C is a point on the x-axis at a distance of 2 units to the left of y-axis. So, the coordinates of C are (–2, 0)

B is the solution of the pair of equations $x = –2$ and $y = 3$. So, the coordinates of B are (–2, 3)

So, the vertices of the rectangle OABC are O (0, 0), A (0, 3), B (–2, 3), C (–2, 0)

The length and breadth of this rectangle are 2 units and 3 units, respectively.

As the area of a rectangle = length $\times$ breadth,

the area of rectangle OABC = $2 \times 3 = 6$ sq. units.

Sample Question 2: Determine, algebraically, the vertices of the triangle formed by the lines

$$5x - y = 5, \quad x + 2y = 1 \text{ and } 6x + y = 17.$$

Solution:

The vertex of a triangle is the common solution of the two equations forming its two sides. So, solving the given equations pairwise will give the vertices of the triangle.

From the given equations, we will have the following three pairs of equations:

$5x - y = 5$ and $x + 2y = 1$

$x + 2y = 1$ and $6x + y = 17$

$5x - y = 5$ and $6x + y = 17$

Solving the pair of equations

$$5x - y = 5$$

$$x + 2y = 1$$

we get, $x = 1$, $y = 0$

So, one vertex of the triangle is (1, 0)
Solving the second pair of equations

$$x + 2y = 1$$

$$6x + y = 17$$

we get $x = 3$, $y = -1$

So, another vertex of the triangle is (3, –1)

Solving the third pair of equations

$$5x - y = 5$$

$$6x + y = 17,$$

we get $x = 2$, $y = 5$.

So, the third vertex of the triangle is (2, 5). So, the three vertices of the triangle are (1, 0), (3, –1) and (2, 5).

Sample Question 3 : Jamila sold a table and a chair for Rs 1050, thereby making a profit of 10% on the table and 25% on the chair. If she had taken a profit of 25% on the table and 10% on the chair she would have got Rs 1065. Find the cost price of each.

Solution : Let the cost price of the table be Rs x and the cost price of the chair be Rs y.

The selling price of the table, when it is sold at a profit of 10%

$$= \text{Rs}\ \ x + \frac{10}{100}x\ \ = \text{Rs}\frac{110}{100}x$$

The selling price of the chair when it is sold at a profit of 25%

$$= \text{Rs}\ \ y + \frac{25}{100}y\ \ = \text{Rs}\frac{125}{100}y$$

So, $\qquad \frac{110}{100}x + \frac{125}{100}y = 1050 \qquad\qquad (1)$

When the table is sold at a profit of 25%, its selling price $= \text{Rs}\left(x + \frac{25}{100}x\right) = \text{Rs}\frac{125}{100}x$

When the chair is sold at a profit of 10%, its selling price $= \text{Rs}\left(y + \frac{10}{100}y\right) = \text{Rs}\frac{110}{100}y$

So, $\qquad \frac{125}{100}x + \frac{110}{100}y = 1065 \qquad\qquad (2)$

From Equations (1) and (2), we get

$$110x + 125y = 105000$$

and $\ 125x + 110y = 106500$

On adding and subtracting these equations, we get

$$235x + 235y = 211500$$

and $\qquad 15x - 15y = 1500$

i.e., $\qquad x+y = 900 \qquad\qquad (3)$

and $\qquad x - y = 100 \qquad\qquad (4)$

Solving Equations (3) and (4), we get

$$x = 500,\ y = 400$$

So, the cost price of the table is Rs 500 and the cost price of the chair is Rs 400.

Sample Question 4: It can take 12 hours to fill a swimming pool using two pipes. If the pipe of larger diameter is used for 4 hours and the pipe of smaller diameter for 9 hours, only half the pool can be filled.How long would it take for each pipe to fill the pool separately?

Solution:

Let the time taken by the pipe of larger diameter to fill the pool be x hours and that taken by the pipe of smaller diameter pipe alone be y hours.

In x hours, the pipe of larger diameter fills the pool.

So, in 1 hour the pipe of larger diameter fills $\frac{1}{x}$ part of the pool, and so, in 4 hours, the pipe of larger diameter fills $\frac{4}{x}$ parts of the pool.

Similarly, in 9 hours, the pipe of smaller diameter fills $\frac{9}{y}$ parts of the pool.

According to the question,

$$\frac{4}{x}+\frac{9}{y}=\frac{1}{2} \qquad (1)$$

Also, using both the pipes, the pool is filled in 12 hours.

So, $$\frac{12}{x}+\frac{12}{y}=1 \qquad (2)$$

Let $\frac{1}{x}=u$ and $\frac{1}{y}=v$. Then Equations (1) and (2) become

$$4u+9v=\frac{1}{2} \qquad (3)$$

$$12u+12v=1 \qquad (4)$$

Multiplying Equation (3) by 3 and subtracting Equation (4) from it, we get

$$15v=\frac{1}{2} \text{ or } v=\frac{1}{30}$$

Substituting the value of v in Equation (4), we get $u = \frac{1}{20}$

So, $u = \frac{1}{20}, \; v = \frac{1}{30}$

So, $\frac{1}{x} = \frac{1}{20}, \; \frac{1}{y} = \frac{1}{30}$

or, $x = 20, \; y = 30.$

So, the pipe of larger diameter alone can fill the pool in 20 hours and the pipe of smaller diameter alone can fill the pool in 30 hours.

EXERCISE 3.4

1. Graphically, solve the following pair of equations:

$2x + y = 6$

$2x - y + 2 = 0$

Find the ratio of the areas of the two triangles formed by the lines representing these equations with the x-axis and the lines with the y-axis.

2. Determine, graphically, the vertices of the triangle formed by the lines

$y = x, \qquad 3y = x, \qquad x + y = 8$

3. Draw the graphs of the equations $x = 3$, $x = 5$ and $2x - y - 4 = 0$. Also find the area of the quadrilateral formed by the lines and the x–axis.

4. The cost of 4 pens and 4 pencil boxes is Rs 100. Three times the cost of a pen is Rs 15 more than the cost of a pencil box. Form the pair of linear equations for the above situation. Find the cost of a pen and a pencil box.

5. Determine, algebraically, the vertices of the triangle formed by the lines

$3x - y = 3$

$2x - 3y = 2$

$x + 2y = 8$

6. Ankita travels 14 km to her home partly by rickshaw and partly by bus. She takes half an hour if she travels 2 km by rickshaw, and the remaining distance by bus.

On the other hand, if she travels 4 km by rickshaw and the remaining distance by bus, she takes 9 minutes longer. Find the speed of the rickshaw and of the bus.

7. A person, rowing at the rate of 5 km/h in still water, takes thrice as much time in going 40 km upstream as in going 40 km downstream. Find the speed of the stream.

8. A motor boat can travel 30 km upstream and 28 km downstream in 7 hours. It can travel 21 km upstream and return in 5 hours. Find the speed of the boat in still water and the speed of the stream.

9. A two-digit number is obtained by either multiplying the sum of the digits by 8 and then subtracting 5 or by multiplying the difference of the digits by 16 and then adding 3. Find the number.

10. A railway half ticket costs half the full fare, but the reservation charges are the same on a half ticket as on a full ticket. One reserved first class ticket from the station A to B costs Rs 2530. Also, one reserved first class ticket and one reserved first class half ticket from A to B costs Rs 3810. Find the full first class fare from station A to B, and also the reservation charges for a ticket.

11. A shopkeeper sells a saree at 8% profit and a sweater at 10% discount, thereby, getting a sum Rs 1008. If she had sold the saree at 10% profit and the sweater at 8% discount, she would have got Rs 1028. Find the cost price of the saree and the list price (price before discount) of the sweater.

12. Susan invested certain amount of money in two schemes A and B, which offer interest at the rate of 8% per annum and 9% per annum, respectively. She received Rs 1860 as annual interest. However, had she interchanged the amount of investments in the two schemes, she would have received Rs 20 more as annual interest. How much money did she invest in each scheme?

13. Vijay had some bananas, and he divided them into two lots A and B. He sold the first lot at the rate of Rs 2 for 3 bananas and the second lot at the rate of Re 1 per banana, and got a total of Rs 400. If he had sold the first lot at the rate of Re 1 per banana, and the second lot at the rate of Rs 4 for 5 bananas, his total collection would have been Rs 460. Find the total number of bananas he had.

CHAPTER 4

QUADRATIC EQUATIONS

(A) Main Concepts and Results

- Quadratic equation : A quadratic equation in the variable x is of the form $ax^2 + bx + c = 0$, where a, b, c are real numbers and $a \neq 0$.
- Roots of a quadratic equation : A real number α is said to be a root of the quadratic equation $ax^2 + bx + c = 0$, if $a\alpha^2 + b\alpha + c = 0$.
- The roots of the quadratic equation $ax^2 + bx + c = 0$ are the same as the zeroes of the quadratic polynomial $ax^2 + bx + c$.
- Finding the roots of a quadratic equation by the method of factorisation : If we can factorise the quadratic polynomial $ax^2 + bx + c$, then the roots of the quadratic equation $ax^2 + bx + c = 0$ can be found by equating to zero the linear factors of $ax^2 + bx + c$.
- Finding the roots of a quadratic equation by the method of completing the square : By adding and subtracting a suitable constant, we club the x^2 and x terms in the quadratic equation so that they become a complete square, and solve for x.
- Quadratic Formula : If $b^2 - 4ac \geq 0$, then the real roots of the quadratic equation $ax^2 + bx + c = 0$ are given by $\dfrac{-b}{2a} \pm \dfrac{\sqrt{b^2 - 4ac}}{2a}$.
- The expression $b^2 - 4ac$ is called the discriminant of the quadratic equation.
- Existence of roots of a quadratic equation: A quadratic equation $ax^2+bx+c=0$ has

(i) two distinct real roots if $b^2 - 4ac > 0$
(ii) two equal real roots if $b^2 - 4ac = 0$
(iii) no real roots if $b^2 - 4ac < 0$.

(B) Multiple Choice Questions

Choose the correct answer from the given four options:

Sample Question 1 : Which one of the following is not a quadratic equation?

(A) $(x + 2)^2 = 2(x + 3)$ (B) $x^2 + 3x = (-1)(1 - 3x)^2$

(C) $(x + 2)(x - 1) = x^2 - 2x - 3$ (D) $x^3 - x^2 + 2x + 1 = (x + 1)^3$

Solution : Answer (C)

Sample Question 2 : Which constant should be added and subtracted to solve the quadratic equation $4x^2 - \sqrt{3}x - 5 = 0$ by the method of completing the square?

(A) $\frac{9}{16}$ (B) $\frac{3}{16}$ (C) $\frac{3}{4}$ (D) $\frac{\sqrt{3}}{4}$

Solution : Answer (B)

EXERCISE 4.1

Choose the correct answer from the given four options in the following questions:

1. Which of the following is a quadratic equation?

(A) $x^2 + 2x + 1 = (4 - x)^2 + 3$ (B) $-2x^2 = (5 - x)\left(2x - \frac{2}{5}\right)$

(C) $(k + 1)x^2 + \frac{3}{2}x = 7$, where $k = -1$ (D) $x^3 - x^2 = (x - 1)^3$

2. Which of the following is not a quadratic equation?

(A) $2(x - 1)^2 = 4x^2 - 2x + 1$ (B) $2x - x^2 = x^2 + 5$

(C) $(\sqrt{2}x + \sqrt{3})^2 + x^2 = 3x^2 - 5x$ (D) $(x^2 + 2x)^2 = x^4 + 3 + 4x^3$

3. Which of the following equations has 2 as a root?

(A) $x^2 - 4x + 5 = 0$ (B) $x^2 + 3x - 12 = 0$

(C) $2x^2 - 7x + 6 = 0$ (D) $3x^2 - 6x - 2 = 0$

4. If $\frac{1}{2}$ is a root of the equation $x^2 + kx - \frac{5}{4} = 0$, then the value of k is

(A) 2 (B) – 2 (C) $\frac{1}{4}$ (D) $\frac{1}{2}$

5. Which of the following equations has the sum of its roots as 3?

(A) $2x^2 - 3x + 6 = 0$ (B) $-x^2 + 3x - 3 = 0$

(C) $\sqrt{2}x^2 - \frac{3}{\sqrt{2}}x + 1 = 0$ (D) $3x^2 - 3x + 3 = 0$

6. Values of k for which the quadratic equation $2x^2 - kx + k = 0$ has equal roots is

(A) 0 only (B) 4 (C) 8 only (D) 0, 8

7. Which constant must be added and subtracted to solve the quadratic equation $9x^2 + \frac{3}{4}x - \sqrt{2} = 0$ by the method of completing the square?

(A) $\frac{1}{8}$ (B) $\frac{1}{64}$ (C) $\frac{1}{4}$ (D) $\frac{9}{64}$

8. The quadratic equation $2x^2 - \sqrt{5}x + 1 = 0$ has

(A) two distinct real roots (B) two equal real roots

(C) no real roots (D) more than 2 real roots

9. Which of the following equations has two distinct real roots?

(A) $2x^2 - 3\sqrt{2}x + \frac{9}{4} = 0$ (B) $x^2 + x - 5 = 0$

(C) $x^2 + 3x + 2\sqrt{2} = 0$ (D) $5x^2 - 3x + 1 = 0$

10. Which of the following equations has no real roots?

(A) $x^2 - 4x + 3\sqrt{2} = 0$ (B) $x^2 + 4x - 3\sqrt{2} = 0$

(C) $x^2 - 4x - 3\sqrt{2} = 0$ (D) $3x^2 + 4\sqrt{3}x + 4 = 0$

11. $(x^2 + 1)^2 - x^2 = 0$ has

(A) four real roots (B) two real roots

(C) no real roots (D) one real root.

(C) Short Answer Questions with Reasoning

Sample Question 1 : Does $(x - 1)^2 + 2(x + 1) = 0$ have a real root? Justify your answer.

Solution : No, since the equation is simplified to $x^2 + 3 = 0$ whose discriminant is -12.

Sample Question 2 : Is the following statement 'True' or 'False'?Justify your answer. If in a quadratic equation the coefficient of x is zero, then the quadratic equation has no real roots.

Solution : False, since the discriminant in this case is $-4ac$ which can still be non-negative if a and c are of opposite signs or if one of a or c is zero.

EXERCISE 4.2

1. State whether the following quadratic equations have two distinct real roots. Justify your answer.

(i) $x^2 - 3x + 4 = 0$ (ii) $2x^2 + x - 1 = 0$

(iii) $2x^2 - 6x + \frac{9}{2} = 0$ (iv) $3x^2 - 4x + 1 = 0$

(v) $(x + 4)^2 - 8x = 0$ (vi) $(x - \sqrt{2})^2 - 2(x + 1) = 0$

(vii) $\sqrt{2}x^2 - \frac{3}{\sqrt{2}}x + \frac{1}{\sqrt{2}} = 0$ (viii) $x(1 - x) - 2 = 0$

(ix) $(x - 1)(x + 2) + 2 = 0$ (x) $(x + 1)(x - 2) + x = 0$

2. Write whether the following statements are true or false. Justify your answers.

(i) Every quadratic equation has exactly one root.

(ii) Every quadratic equation has at least one real root.

(iii) Every quadratic equation has at least two roots.

(iv) Every quadratic equations has at most two roots.

(v) If the coefficient of x^2 and the constant term of a quadratic equation have opposite signs, then the quadratic equation has real roots.

(vi) If the coefficient of x^2 and the constant term have the same sign and if the coefficient of x term is zero, then the quadratic equation has no real roots.

3. A quadratic equation with integral coefficient has integral roots. Justify your answer.

4. Does there exist a quadratic equation whose coefficients are rational but both of its roots are irrational? Justify your answer.

5. Does there exist a quadratic equation whose coefficients are all distinct irrationals but both the roots are rationals? Why?

6. Is 0.2 a root of the equation $x^2 - 0.4 = 0$? Justify.

7. If $b = 0$, $c < 0$, is it true that the roots of $x^2 + bx + c = 0$ are numerically equal and opposite in sign? Justify.

(D) Short Answer Questions

Sample Question 1 : Find the roots of the quadratic equation $2x^2 - \sqrt{5}x - 2 = 0$ using the quadratic formula.

Solution : $b^2 - 4ac = 5 - 4 \times 2 \times (-2) = 21$

Therefore, the roots are $\dfrac{\sqrt{5} \pm \sqrt{21}}{4}$, i.e., $\dfrac{\sqrt{5} + \sqrt{21}}{4}$ and $\dfrac{\sqrt{5} - \sqrt{21}}{4}$

Sample Question 2 : Find the roots of $6x^2 - \sqrt{2}x - 2 = 0$ by the factorisation of the corresponding quadratic polynomial.

Solution :

$$\begin{aligned} 6x^2 - \sqrt{2}x - 2 &= 6x^2 - 3\sqrt{2}\,x + 2\sqrt{2}\,x - 2 \\ &= 3x\,(2x - \sqrt{2}) + \sqrt{2}\,(2x - \sqrt{2}) \\ &= (3x + \sqrt{2})\,(2x - \sqrt{2}) \end{aligned}$$

Now, $6x^2 - \sqrt{2}x - 2 = 0$ gives $(3x + \sqrt{2})(2x - \sqrt{2}) = 0$, i.e., $3x + \sqrt{2} = 0$ or $2x - \sqrt{2} = 0$

So, the roots are $-\dfrac{\sqrt{2}}{3}$ and $\dfrac{\sqrt{2}}{2}$.

EXERCISE 4.3

1. Find the roots of the quadratic equations by using the quadratic formula in each of the following:

(i) $2x^2 - 3x - 5 = 0$

(ii) $5x^2 + 13x + 8 = 0$

(iii) $-3x^2 + 5x + 12 = 0$

(iv) $-x^2 + 7x - 10 = 0$

(v) $x^2 + 2\sqrt{2}\,x - 6 = 0$

(vi) $x^2 - 3\sqrt{5}\,x + 10 = 0$

(vii) $\frac{1}{2}x^2 - \sqrt{11}\,x + 1 = 0$

2. Find the roots of the following quadratic equations by the factorisation method:

(i) $2x^2 + \frac{5}{3}x - 2 = 0$

(ii) $\frac{2}{5}x^2 - x - \frac{3}{5} = 0$

(iii) $3\sqrt{2}x^2 - 5x - \sqrt{2} = 0$

(iv) $3x^2 + 5\sqrt{5}x - 10 = 0$

(v) $21x^2 - 2x + \frac{1}{21} = 0$

(E) Long Answer Questions

Sample Question 1 : Check whether the equation $6x^2 - 7x + 2 = 0$ has real roots, and if it has, find them by the method of completing the squares.

Solution : The discriminant = $b^2 - 4ac = 49 - 4 \times 6 \times 2 = 1 > 0$

So, the given equation has two distinct real roots.

Now, $6x^2 - 7x + 2 = 0$

i.e., $36x^2 - 42x + 12 = 0$

i.e., $6x - \frac{7}{2}^{\,2} + 12 - \frac{49}{4} = 0$

i.e., $6x - \frac{7}{2}^{\,2} - \frac{1}{2}^{\,2} = 0$ or $\left(6x - \frac{7}{2}\right)^2 = \left(\frac{1}{2}\right)^2$

The roots are given by $6x - \frac{7}{2} = \pm\frac{1}{2}$

i.e., $6x = 4, 3$

i.e., $x = \frac{2}{3}, \frac{1}{2}$.

Sample Question 2 : Had Ajita scored 10 more marks in her mathematics test out of 30 marks, 9 times these marks would have been the square of her actual marks. How many marks did she get in the test?

Solution : Let her actual marks be x

Therefore, $9(x+10) = x^2$

i.e., $x^2 - 9x - 90 = 0$

i.e., $x^2 - 15x + 6x - 90 = 0$

i.e., $x(x - 15) + 6(x - 15) = 0$

i.e., $(x + 6)(x - 15) = 0$

Therefore, $x = -6$ or $x = 15$

Since x is the marks obtained, $x \neq -6$. Therefore, $x = 15$.

So, Ajita got 15 marks in her mathematics test.

Sample Question 3 : A train travels at a certain average speed for a distance of 63 km and then travels a distance of 72 km at an average speed of 6 km/h more than its original speed. If it takes 3 hours to complete the total journey, what is its original average speed?

Solution : Let its original average speed be x km/h. Therefore,

$$\frac{63}{x} + \frac{72}{x+6} = 3$$

i.e., $$\frac{7}{x} + \frac{8}{x+6} = \frac{3}{9} = \frac{1}{3}$$

i.e., $$\frac{7(x+6)+8x}{x(x+6)} = \frac{1}{3}$$

i.e., $21(x + 6) + 24x = x(x + 6)$

i.e., $21x + 126 + 24x = x^2 + 6x$

i.e., $x^2 - 39x - 126 = 0$

i.e., $(x + 3)(x - 42) = 0$

i.e., $x = -3$ or $x = 42$

Since x is the average speed of the train, x cannot be negative.

Therefore, $x = 42$.

So, the original average speed of the train is 42 km/h.

EXERCISE 4.4

1. Find whether the following equations have real roots. If real roots exist, find them.

(i) $8x^2 + 2x - 3 = 0$

(ii) $-2x^2 + 3x + 2 = 0$

(iii) $5x^2 - 2x - 10 = 0$

(iv) $\frac{1}{2x-3} + \frac{1}{x-5} = 1, x \neq \frac{3}{2}, 5$

(v) $x^2 + 5\sqrt{5}x - 70 = 0$

2. Find a natural number whose square diminished by 84 is equal to thrice of 8 more than the given number.

3. A natural number, when increased by 12, equals 160 times its reciprocal. Find the number.

4. A train, travelling at a uniform speed for 360 km, would have taken 48 minutes less to travel the same distance if its speed were 5 km/h more. Find the original speed of the train.

5. If Zeba were younger by 5 years than what she really is, then the square of her age (in years) would have been 11 more than five times her actual age. What is her age now?

6. At present Asha's age (in years) is 2 more than the square of her daughter Nisha's age. When Nisha grows to her mother's present age, Asha's age would be one year less than 10 times the present age of Nisha. Find the present ages of both Asha and Nisha.

7. In the centre of a rectangular lawn of dimensions 50 m × 40 m, a rectangular pond has to be constructed so that the area of the grass surrounding the pond would be 1184 m^2 [see Fig. 4.1]. Find the length and breadth of the pond.

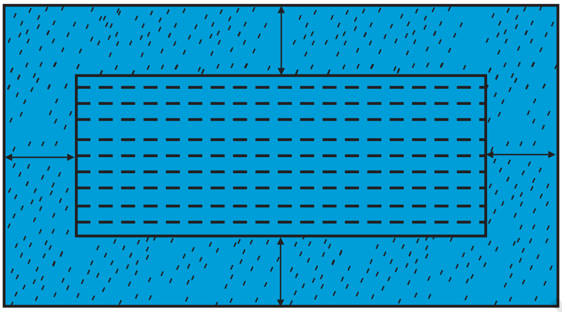


Fig. 4.1

8. At t minutes past 2 pm, the time needed by the minutes hand of a clock to show 3 pm was found to be 3 minutes less than $\frac{t^2}{4}$ minutes. Find t.

CHAPTER 5

ARITHMETIC PROGRESSIONS

(A) Main Concepts and Results

- An arithmetic progression (AP) is a list of numbers in which each term is obtained by adding a fixed number d to the preceding term, except the first term a. The fixed number d is called its common difference.

  The general form of an AP is $a, a + d, a + 2d, a + 3d,...$

- In the list of numbers $a_1, a_2, a_3,...$ if the differences $a_2 - a_1, a_3 - a_2, a_4 - a_3,...$ give the same value, i.e., if $a_{k+1} - a_k$ is the same for different values of k, then the given list of numbers is an AP.

- The n^{th} term a_n (or the general term) of an AP is $a_n = a + (n - 1) d$, where a is the first term and d is the common difference. Note that $a_1 = a$.

- The sum S_n of the first n terms of an AP is given by

$$S_n = \frac{n}{2} [2a + (n - 1) d]$$

  If l is the last term of an AP of n terms, then the sum of all the terms can also be given by

$$S_n = \frac{n}{2} [a + l]$$

  Sometimes S_n is also denoted by S.

- If S_n is the sum of the first n terms of an AP, then its n^{th} term a_n is given by

$$a_n = S_n - S_{n-1}$$

(B) Multiple Choice Questions

Choose the correct answer from the given four options:

Sample Question 1 : The 10th term of the AP: 5, 8, 11, 14, ... is

(A) 32 (B) 35 (C) 38 (D) 185

Solution : Answer (A)

Sample Question 2 : In an AP if $a = -7.2$, $d = 3.6$, $a_n = 7.2$, then n is

(A) 1 (B) 3 (C) 4 (D) 5

Solution : Answer (D)

EXERCISE 5.1

Choose the correct answer from the given four options:

1. In an AP, if $d = -4$, $n = 7$, $a_n = 4$, then a is

(A) 6 (B) 7 (C) 20 (D) 28

2. In an AP, if $a = 3.5$, $d = 0$, $n = 101$, then a_n will be

(A) 0 (B) 3.5 (C) 103.5 (D) 104.5

3. The list of numbers – 10, – 6, – 2, 2,... is

(A) an AP with $d = -16$

(B) an AP with $d = 4$

(C) an AP with $d = -4$

(D) not an AP

4. The 11th term of the AP: $-5, \frac{-5}{2}, 0, \frac{5}{2}, \ldots$ is

(A) –20 (B) 20 (C) –30 (D) 30

5. The first four terms of an AP, whose first term is –2 and the common difference is –2, are

(A) – 2, 0, 2, 4

(B) – 2, 4, – 8, 16

(C) – 2, – 4, – 6, – 8

(D) – 2, – 4, – 8, –16

6. The 21st term of the AP whose first two terms are –3 and 4 is

(A) 17 (B) 137 (C) 143 (D) –143

7. If the 2nd term of an AP is 13 and the 5th term is 25, what is its 7th term?

(A) 30 (B) 33 (C) 37 (D) 38

8. Which term of the AP: 21, 42, 63, 84,... is 210?

(A) 9th (B) 10th (C) 11th (D) 12th

9. If the common difference of an AP is 5, then what is $a_{18} - a_{13}$?

(A) 5 (B) 20 (C) 25 (D) 30

10. What is the common difference of an AP in which $a_{18} - a_{14} = 32$?

(A) 8 (B) – 8 (C) – 4 (D) 4

11. Two APs have the same common difference. The first term of one of these is –1 and that of the other is – 8. Then the difference between their 4th terms is

(A) –1 (B) – 8 (C) 7 (D) –9

12. If 7 times the 7th term of an AP is equal to 11 times its 11th term, then its 18th term will be

(A) 7 (B) 11 (C) 18 (D) 0

13. The 4th term from the end of the AP: –11, –8, –5, ..., 49 is

(A) 37 (B) 40 (C) 43 (D) 58

14. The famous mathematician associated with finding the sum of the first 100 natural numbers is

(A) Pythagoras (B) Newton

(C) Gauss (D) Euclid

15. If the first term of an AP is –5 and the common difference is 2, then the sum of the first 6 terms is

(A) 0 (B) 5 (C) 6 (D) 15

16. The sum of first 16 terms of the AP: 10, 6, 2,... is

(A) –320 (B) 320 (C) –352 (D) –400

17. In an AP if $a = 1$, $a_n = 20$ and $S_n = 399$, then n is

(A) 19 (B) 21 (C) 38 (D) 42

18. The sum of first five multiples of 3 is

(A) 45 (B) 55 (C) 65 (D) 75

(C) Short Answer Questions with Reasoning

Sample Question 1: In the AP: 10, 5, 0, –5, ... the common difference d is equal to 5. Justify whether the above statement is true or false.

Solution :

$$a_2 - a_1 = 5 - 10 = -5$$
$$a_3 - a_2 = 0 - 5 = -5$$
$$a_4 - a_3 = -5 - 0 = -5$$

Although the given list of numbers forms an AP, it is with $d = -5$ and not with $d = 5$

So, the given statement is false.

Sample Question 2 : Divya deposited Rs 1000 at compound interest at the rate of 10% per annum. The amounts at the end of first year, second year, third year, ..., form an AP. Justify your answer.

Solution : Amount at the end of the 1st year = Rs 1100

Amount at the end of the 2nd year = Rs 1210

Amount at the end of 3rd year = Rs 1331 and so on.

So, the amount (in Rs) at the end of 1st year, 2nd year, 3rd year, ... are

1100, 1210, 1331, ...

Here, $a_2 - a_1 = 110$

$a_3 - a_2 = 121$

As, $a_2 - a_1 \neq a_3 - a_2$, it does not form an AP.

Sample Question 3: The n^{th} term of an AP cannot be $n^2 + 1$. Justify your answer.

Solution :

Here, $a_n = n^2 + 1$

So, $a_1 = 1^2 + 1 = 2$

$a_2 = 2^2 + 1 = 5$

$a_3 = 3^2 + 1 = 10$

$\vdots$

List of numbers becomes 2, 5, 10, ...

Here, $5 - 2 \neq 10 - 5$, so it does not form an AP.

Alternative Solution 1:

We know that in an AP, $d = a_n - a_{n-1}$

Here, $a_n = n^2 + 1$

So, $a_n - a_{n-1} = (n^2 + 1) - \ (n-1)^2 + 1$

$= 2n - 1$

As $a_n - a_{n-1}$ depends upon n, d is not a fixed number.

So, $a_n = n^2 + 1$ cannot be the n^{th} term of an AP.

Alternative Solution 2:

We know that in an AP

$a_n = a + (n-1)d$. We observe that a_n is a linear polynomial in n.

Here, $a_n = n^2 + 1$ is not a linear polynomial in n. So, it cannot be the n^{th} term of an AP.

EXERCISE 5.2

1. Which of the following form an AP? Justify your answer.

(i) $-1, -1, -1, -1, \ldots$

(ii) $0, 2, 0, 2, \ldots$

(iii) $1, 1, 2, 2, 3, 3, \ldots$

(iv) $11, 22, 33, \ldots$

(v) $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots$

(vi) $2, 2^2, 2^3, 2^4, \ldots$

(vii) $\sqrt{3}, \sqrt{12}, \sqrt{27}, \sqrt{48}, \ldots$

2. Justify whether it is true to say that $-1, -\frac{3}{2}, -2, \frac{5}{2}, \ldots$ forms an AP as $a_2 - a_1 = a_3 - a_2$.

3. For the AP: $-3, -7, -11, \ldots$, can we find directly $a_{30} - a_{20}$ without actually finding a_{30} and a_{20}? Give reasons for your answer.

4. Two APs have the same common difference. The first term of one AP is 2 and that of the other is 7. The difference between their 10^{th} terms is the same as the difference between their 21^{st} terms, which is the same as the difference between any two corresponding terms. Why?

5. Is 0 a term of the AP: 31, 28, 25, ...? Justify your answer.

6. The taxi fare after each km, when the fare is Rs 15 for the first km and Rs 8 for each additional km, does not form an AP as the total fare (in Rs) after each km is

15, 8, 8, 8, ...

Is the statement true? Give reasons.

7. In which of the following situations, do the lists of numbers involved form an AP? Give reasons for your answers.

(i) The fee charged from a student every month by a school for the whole session, when the monthly fee is Rs 400.

(ii) The fee charged every month by a school from Classes I to XII, when the monthly fee for Class I is Rs 250, and it increases by Rs 50 for the next higher class.

(iii) The amount of money in the account of Varun at the end of every year when Rs 1000 is deposited at simple interest of 10% per annum.

(iv) The number of bacteria in a certain food item after each second, when they double in every second.

8. Justify whether it is true to say that the following are the n^{th} terms of an AP.

(i) $2n–3$ (ii) $3n^2+5$ (iii) $1+n+n^2$

(D) Short Answer Questions

Sample Question 1 : If the numbers $n – 2$, $4n – 1$ and $5n + 2$ are in AP, find the value of n.

Solution :

As $n – 2$, $4n – 1$, $5n + 2$ are in AP,

so $(4n – 1) – (n – 2) = (5n + 2) – (4n – 1)$

i.e, $3n + 1 = n + 3$

i.e, $n = 1$

Sample Question 2 : Find the value of the middle most term (s) of the AP : –11, –7, –3,..., 49.

Solution :

Here, $a = –11$, $d = –7 – (–11) = 4$, $a_n = 49$

We have $a_n = a + (n – 1)\, d$

So, $49 = –11 + (n – 1) \times 4$

i.e., $60 = (n – 1) \times 4$

i.e., $n = 16$

As n is an even number, there will be two middle terms which are $\frac{16}{2}$th and $\left(\frac{16}{2}+1\right)$th, i.e., the 8^{th} term and the 9^{th} term.

$$a_8 = a + 7d = –11 + 7 \times 4 = 17$$
$$a_9 = a + 8d = –11 + 8 \times 4 = 21$$

So, the values of the two middle most terms are 17 and 21, respectively.

Sample Question 3: The sum of the first three terms of an AP is 33. If the product of the first and the third term exceeds the second term by 29, find the AP.

Solution : Let the three terms in AP be

$a – d, a, a + d.$

So, $a – d + a + a + d = 33$

or $a = 11$

Also, $(a – d)(a + d) = a + 29$

i.e., $a^2 – d^2 = a + 29$

i.e., $121 – d^2 = 11 + 29$

i.e., $d^2 = 81$

i.e., $d = \pm 9$

So there will be two APs and they are : 2, 11, 20, ...

and 20, 11, 2, ...

EXERCISE 5.3

1. Match the APs given in column A with suitable common differences given in column B.

Column A		**Column B**	
(A_1)	$2, –2, –6, –10,...$	(B_1)	$\frac{2}{3}$
(A_2)	$a = –18, n = 10, a_n = 0$	(B_2)	$–5$
(A_3)	$a = 0, a_{10} = 6$	(B_3)	4
(A_4)	$a_2 = 13, a_4 = 3$	(B_4)	$–4$
		(B_5)	2
		(B_6)	$\frac{1}{2}$
		(B_7)	5

2. Verify that each of the following is an AP, and then write its next three terms.

(i) $0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, \ldots$

(ii) $5, \frac{14}{3}, \frac{13}{3}, 4, \ldots$

(iii) $\sqrt{3}, 2\sqrt{3}, 3\sqrt{3}, \ldots$

(iv) $a + b, (a + 1) + b, (a + 1) + (b + 1), \ldots$

(v) $a, 2a + 1, 3a + 2, 4a + 3, \ldots$

3. Write the first three terms of the APs when a and d are as given below:

(i) $a = \frac{1}{2}, d = -\frac{1}{6}$

(ii) $a = -5, d = -3$

(iii) $a = \sqrt{2}, \quad d = \frac{1}{\sqrt{2}}$

4. Find a, b and c such that the following numbers are in AP: $a, 7, b, 23, c$.

5. Determine the AP whose fifth term is 19 and the difference of the eighth term from the thirteenth term is 20.

6. The 26^{th}, 11^{th} and the last term of an AP are 0, 3 and $-\frac{1}{5}$, respectively. Find the common difference and the number of terms.

7. The sum of the 5^{th} and the 7^{th} terms of an AP is 52 and the 10^{th} term is 46. Find the AP.

8. Find the 20^{th} term of the AP whose 7^{th} term is 24 less than the 11^{th} term, first term being 12.

9. If the 9^{th} term of an AP is zero, prove that its 29^{th} term is twice its 19^{th} term.

10. Find whether 55 is a term of the AP: 7, 10, 13,--- or not. If yes, find which term it is.

11. Determine k so that $k^2 + 4k + 8$, $2k^2 + 3k + 6$, $3k^2 + 4k + 4$ are three consecutive terms of an AP.

12. Split 207 into three parts such that these are in AP and the product of the two smaller parts is 4623.

13. The angles of a triangle are in AP. The greatest angle is twice the least. Find all the angles of the triangle.

14. If the nth terms of the two APs: 9, 7, 5, ... and 24, 21, 18,... are the same, find the value of n. Also find that term.

15. If sum of the 3rd and the 8th terms of an AP is 7 and the sum of the 7th and the 14th terms is –3, find the 10th term.

16. Find the 12th term from the end of the AP: –2, –4, –6,..., –100.

17. Which term of the AP: 53, 48, 43,... is the first negative term?

18. How many numbers lie between 10 and 300, which when divided by 4 leave a remainder 3?

19. Find the sum of the two middle most terms of the AP: $-\frac{4}{3}, -1, -\frac{2}{3}, ..., 4\frac{1}{3}$.

20. The first term of an AP is –5 and the last term is 45. If the sum of the terms of the AP is 120, then find the number of terms and the common difference.

21. Find the sum:

(i) $1 + (-2) + (-5) + (-8) + ... + (-236)$

(ii) $4 - \frac{1}{n} + 4 - \frac{2}{n} + 4 - \frac{3}{n} + ...$ upto n terms

(iii) $\frac{a-b}{a+b} + \frac{3a-2b}{a+b} + \frac{5a-3b}{a+b} + \cdots$ to 11 terms.

22. Which term of the AP: –2, –7, –12,... will be –77? Find the sum of this AP upto the term –77.

23. If $a_n = 3 - 4n$, show that $a_1, a_2, a_3, ...$ form an AP. Also find S_{20}.

24. In an AP, if $S_n = n(4n + 1)$, find the AP.

25. In an AP, if $S_n = 3n^2 + 5n$ and $a_k = 164$, find the value of k.

26. If S_n denotes the sum of first n terms of an AP, prove that

$$S_{12} = 3(S_8 - S_4)$$

27. Find the sum of first 17 terms of an AP whose 4^{th} and 9^{th} terms are –15 and –30 respectively.

28. If sum of first 6 terms of an AP is 36 and that of the first 16 terms is 256, find the sum of first 10 terms.

29. Find the sum of all the 11 terms of an AP whose middle most term is 30.

30. Find the sum of last ten terms of the AP: 8, 10, 12,---, 126.

31. Find the sum of first seven numbers which are multiples of 2 as well as of 9.

[**Hint:** Take the LCM of 2 and 9]

32. How many terms of the AP: –15, –13, –11,--- are needed to make the sum –55? Explain the reason for double answer.

33. The sum of the first n terms of an AP whose first term is 8 and the common difference is 20 is equal to the sum of first $2n$ terms of another AP whose first term is – 30 and the common difference is 8. Find n.

34. Kanika was given her pocket money on Jan 1^{st}, 2008. She puts Re 1 on Day 1, Rs 2 on Day 2, Rs 3 on Day 3, and continued doing so till the end of the month, from this money into her piggy bank. She also spent Rs 204 of her pocket money, and found that at the end of the month she still had Rs 100 with her. How much was her pocket money for the month?

35. Yasmeen saves Rs 32 during the first month, Rs 36 in the second month and Rs 40 in the third month. If she continues to save in this manner, in how many months will she save Rs 2000?

(E) Long Answer Questions

Sample Question 1: The sum of four consecutive numbers in an AP is 32 and the ratio of the product of the first and the last terms to the product of the two middle terms is 7 : 15. Find the numbers.

Solution: Let the four consecutive numbers in AP be

$$a - 3d,\ a - d,\ a + d,\ a + 3d.$$

So, $a - 3d + a - d + a + d + a + 3d = 32$

or $4a = 32$

or $a = 8$

Also, $\dfrac{(a-3d)(a+3d)}{(a-d)(a+d)} = \dfrac{7}{15}$

or, $\dfrac{a^2 - 9d^2}{a^2 - d^2} = \dfrac{7}{15}$

or, $15a^2 - 135d^2 = 7a^2 - 7d^2$

or, $8a^2 - 128d^2 = 0$

or, $d^2 = \dfrac{8 \times 8 \times 8}{128} = 4$

or, $d = \pm 2$

So, when $a = 8$, $d = 2$, the numbers are 2, 6, 10, 14.

Sample Question 2: Solve the equation :

$1 + 4 + 7 + 10 + ... + x = 287$

Solution :

Here, 1, 4, 7, 10, ..., x form an AP with $a = 1$, $d = 3$, $a_n = x$

We have, $a_n = a + (n - 1)d$

So, $x = 1 + (n - 1) \times 3 = 3n - 2$

Also, $S = \dfrac{n}{2}(a + l)$

So, $287 = \dfrac{n}{2}(1 + x)$

$$= \frac{n}{2}(1+3n-2)$$

or, $574 = n\,(3n - 1)$

or, $3n^2 - n - 574 = 0$

Therefore, $n = \dfrac{1 \pm \sqrt{1+6888}}{6}$

$$= \frac{1 \pm 83}{6} = \frac{84}{6}, \frac{-82}{6}$$

$$= 14, \frac{-41}{3}$$

As n cannot be negative, so $n = 14$

Therefore, $x = 3n - 2 = 3 \times 14 - 2 = 40$.

Alternative solution:

Here, 1, 4, 7, 10, ... x form an AP with $a = 1$, $d = 3$, S = 287

We have, $\text{S} = \dfrac{n}{2}\ 2a + (n-1)d$

So, $287 = \dfrac{n}{2}\ 2 + (n-1) \times 3$

or, $574 = n\,(3n - 1)$

or, $3n^2 - n - 574 = 0$

Now proceed as above.

EXERCISE 5.4

1. The sum of the first five terms of an AP and the sum of the first seven terms of the same AP is 167. If the sum of the first ten terms of this AP is 235, find the sum of its first twenty terms.

2. Find the

(i) sum of those integers between 1 and 500 which are multiples of 2 as well as of 5.

(ii) sum of those integers from 1 to 500 which are multiples of 2 as well as of 5 .

(iii) sum of those integers from 1 to 500 which are multiples of 2 or 5.

[**Hint** (iii) **:** These numbers will be : multiples of 2 + multiples of 5 – multiples of 2 as well as of 5]

3. The eighth term of an AP is half its second term and the eleventh term exceeds one third of its fourth term by 1. Find the 15^{th} term.

4. An AP consists of 37 terms. The sum of the three middle most terms is 225 and the sum of the last three is 429. Find the AP.

5. Find the sum of the integers between 100 and 200 that are

(i) divisible by 9

(ii) not divisible by 9

[**Hint** (ii) **:** These numbers will be : Total numbers – Total numbers divisible by 9]

6. The ratio of the 11^{th} term to the 18^{th} term of an AP is 2 : 3. Find the ratio of the 5^{th} term to the 21^{st} term, and also the ratio of the sum of the first five terms to the sum of the first 21 terms.

7. Show that the sum of an AP whose first term is *a*, the second term *b* and the last term *c*, is equal to

$$\frac{(a+c)(b+c-2a)}{2(b-a)}$$

8. Solve the equation

$$-4 + (-1) + 2 + ... + x = 437$$

9. Jaspal Singh repays his total loan of Rs 118000 by paying every month starting with the first instalment of Rs 1000. If he increases the instalment by Rs 100 every month, what amount will be paid by him in the 30^{th} instalment? What amount of loan does he still have to pay after the 30^{th} instalment?

10. The students of a school decided to beautify the school on the Annual Day by fixing colourful flags on the straight passage of the school. They have 27 flags to be fixed at intervals of every 2 m. The flags are stored at the position of the middle most flag. Ruchi was given the responsibility of placing the flags. Ruchi kept her books where the flags were stored. She could carry only one flag at a time. How much distance did she cover in completing this job and returning back to collect her books? What is the maximum distance she travelled carrying a flag?

CHAPTER 6

TRIANGLES

(A) Main Concepts and Results

Congruence and similarity, Conditions for similarity of two polygons, Similarity of Triangles, Similarity and correspondence of vertices, Criteria for similarity of triangles; (i) AAA or AA (ii) SSS (iii) SAS

- If a line is drawn parallel to one side of a triangle to intersect the other two sides, then these two sides are divided in the same ratio (Basic Proportionality Theorem) and its converse.
- Ratio of the areas of two similar triangles is equal to the ratio of the squares of their corresponding sides.
- Perpendicular drawn from the vertex of the right angle of a right triangle to its hypotenuse divides the triangle into two triangles which are similar to the whole triangle and to each other.
- In a right triangle, the square on the hypotenuse is equal to the sum of the squares on the other two sides (Pythagoras Theorem) and its converse.

(B) Multiple Choice Questions

Choose the correct answer from the given four options:

Sample Question 1: If in Fig 6.1, O is the point of intersection of two chords AB and CD such that OB = OD, then triangles OAC and ODB are

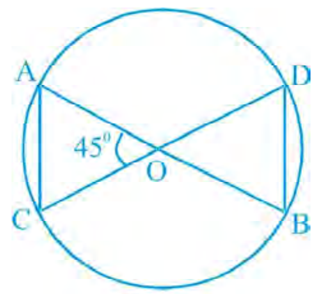


Fig. 6.1

(A) equilateral but not similar

(B) isosceles but not similar

(C) equilateral and similar

(D) isosceles and similar

Solution : Answer (D)

Sample Question 2: D and E are respectively the points on the sides AB and AC of a triangle ABC such that AD = 2 cm, BD = 3 cm, BC = 7.5 cm and DE || BC. Then, length of DE (in cm) is

(A) 2.5 (B) 3 (C) 5 (D) 6

Solution : Answer (B)

EXERCISE 6.1

Choose the correct answer from the given four options:

1. In Fig. 6.2, $\angle$ BAC = 90° and AD $\perp$ BC. Then,

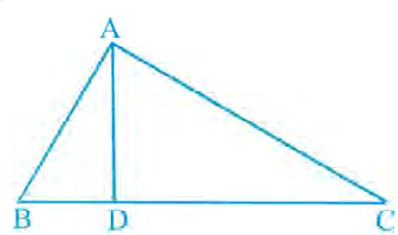


Fig. 6.2

(A) BD . CD = BC^2 (B) AB . AC = BC^2

(C) BD . CD = AD^2 (D) AB . AC = AD^2

2. The lengths of the diagonals of a rhombus are 16 cm and 12 cm. Then, the length of the side of the rhombus is

(A) 9 cm (B) 10 cm (C) 8 cm (D) 20 cm

3. If Δ A B C ~ Δ E D F and Δ A B C is not similar to Δ D E F, then which of the following is not true?

(A) BC . EF = A C. FD (B) AB . EF = AC . DE

(C) BC . DE = AB . EF (D) BC . DE = AB . FD

4. If in two triangles ABC and PQR, $\frac{AB}{QR} = \frac{BC}{PR} = \frac{CA}{PQ}$, then

(A) Δ PQR ~ Δ CAB (B) Δ PQR ~ Δ ABC

(C) Δ CBA ~ Δ PQR (D) Δ BCA ~ Δ PQR

5. In Fig.6.3, two line segments AC and BD intersect each other at the point P such that PA = 6 cm, PB = 3 cm, PC = 2.5 cm, PD = 5 cm, ∠ APB = 50° and ∠ CDP = 30°. Then, ∠ PBA is equal to

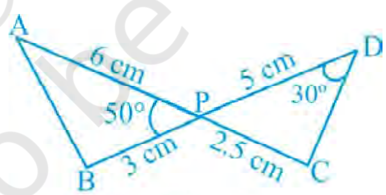


Fig. 6.3

(A) 50° (B) 30° (C) 60° (D) 100°

6. If in two triangles DEF and PQR, ∠D = ∠Q and ∠R = ∠E, then which of the following is not true?

(A) $\frac{EF}{PR} = \frac{DF}{PQ}$ (B) $\frac{DE}{PQ} = \frac{EF}{RP}$

(C) $\frac{DE}{QR} = \frac{DF}{PQ}$ (D) $\frac{EF}{RP} = \frac{DE}{QR}$

7. In triangles ABC and DEF, $\angle B = \angle E$, $\angle F = \angle C$ and AB = 3 DE. Then, the two triangles are

(A) congruent but not similar (B) similar but not congruent

(C) neither congruent nor similar (D) congruent as well as similar

8. It is given that Δ ABC ~ Δ PQR, with $\frac{BC}{QR} = \frac{1}{3}$. Then, $\frac{\text{ar (PRQ)}}{\text{ar (BCA)}}$ is equal to

(A) 9 (B) 3 (C) $\frac{1}{3}$ (D) $\frac{1}{9}$

9. It is given that Δ ABC ~ Δ DFE, $\angle A = 30°$, $\angle C = 50°$, AB = 5 cm, AC = 8 cm and DF= 7.5 cm. Then, the following is true:

(A) DE = 12 cm, $\angle F = 50°$ (B) DE = 12 cm, $\angle F = 100°$

(C) EF = 12 cm, $\angle D = 100°$ (D) EF = 12 cm, $\angle D = 30°$

10. If in triangles ABC and DEF, $\frac{AB}{DE} = \frac{BC}{FD}$, then they will be similar, when

(A) $\angle B = \angle E$ (B) $\angle A = \angle D$

(C) $\angle B = \angle D$ (D) $\angle A = \angle F$

11. If Δ ABC ~ Δ QRP, $\frac{\text{ar (ABC)}}{\text{ar (PQR)}} = \frac{9}{4}$, AB = 18 cm and BC = 15 cm, then PR is equal to

(A) 10 cm (B) 12 cm (C) $\frac{20}{3}$ cm (D) 8 cm

12. If S is a point on side PQ of a Δ PQR such that PS = QS = RS, then

(A) PR . QR = RS^2 (B) $QS^2 + RS^2 = QR^2$

(C) $PR^2 + QR^2 = PQ^2$ (D) $PS^2 + RS^2 = PR^2$

(C) Short Answer Questions with Reasoning

Sample Question 1: In Δ ABC, AB = 24 cm, BC = 10 cm and AC = 26 cm. Is this triangle a right triangle? Give reasons for your answer.

Solution : Here $AB^2 = 576$, $BC^2 = 100$ and $AC^2 = 676$. So, $AC^2 = AB^2 + BC^2$

Hence, the given triangle is a right triangle.

Sample Question 2: P and Q are the points on the sides DE and DF of a triangle DEF such that DP = 5 cm, DE = 15 cm, DQ= 6 cm and QF = 18 cm. Is PQ || EF? Give reasons for your answer.

Solution : Here, $\frac{DP}{PE} = \frac{5}{15-5} = \frac{1}{2}$ and $\frac{DQ}{QF} = \frac{6}{18} = \frac{1}{3}$

As $\frac{DP}{PE} \neq \frac{DQ}{QF}$, therefore PQ is not parallel to EF.

Sample Question 3: It is given that $\Delta FED \sim \Delta STU$. Is it true to say that $\frac{DE}{ST} = \frac{EF}{TU}$? Why?

Solution : No, because the correct correspondence is $F \leftrightarrow S$, $E \leftrightarrow T$, $D \leftrightarrow U$.

With this correspondence, $\frac{EF}{ST} = \frac{DE}{TU}$.

EXERCISE 6.2

1. Is the triangle with sides 25 cm, 5 cm and 24 cm a right triangle? Give reasons for your answer.
2. It is given that Δ DEF ~ Δ RPQ. Is it true to say that $\angle D = \angle R$ and $\angle F = \angle P$? Why?
3. A and B are respectively the points on the sides PQ and PR of a triangle PQR

such that PQ = 12.5 cm, PA = 5 cm, BR= 6 cm and PB = 4 cm. Is AB || QR? Give reasons for your answer.

4. In Fig 6.4, BD and CE intersect each other at the point P. Is Δ PBC ~ ΔPDE? Why?

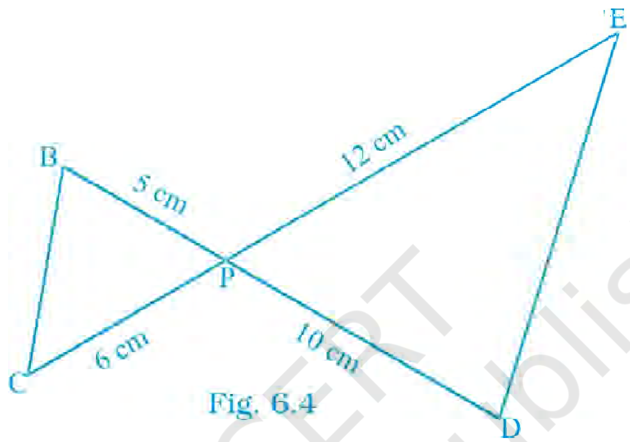


Fig. 6.4

5. In triangles PQR and MST, $\angle P = 55°$, $\angle Q = 25°$, $\angle M = 100°$ and $\angle S = 25°$. Is Δ QPR ~ Δ TSM? Why?

6. Is the following statement true? Why?

"Two quadrilaterals are similar, if their corresponding angles are equal".

7. Two sides and the perimeter of one triangle are respectively three times the corresponding sides and the perimeter of the other triangle. Are the two triangles similar? Why?

8. If in two right triangles, one of the acute angles of one triangle is equal to an acute angle of the other triangle, can you say that the two triangles will be similar? Why?

9. The ratio of the corresponding altitudes of two similar triangles is $\frac{3}{5}$. Is it correct to say that ratio of their areas is $\frac{6}{5}$? Why?

10. D is a point on side QR of ΔPQR such that PD ⊥ QR. Will it be correct to say that ΔPQD ~ Δ RPD? Why?

11. In Fig. 6.5, if ∠D = ∠C, then is it true that ΔADE ~ ΔACB? Why?

12. Is it true to say that if in two triangles, an angle of one triangle is equal to an angle of another triangle and two sides of one triangle are proportional to the two sides of the other triangle, then the triangles are similar? Give reasons for your answer.

(D) Short Answer Questions

Sample Question 1: Legs (sides other than the hypotenuse) of a right triangle are of lengths 16cm and 8 cm. Find the length of the side of the largest square that can be inscribed in the triangle.

Solution: Let ABC be a right triangle right angled at B with AB = 16 cm and BC = 8 cm. Then, the largest square BRSP which can be inscribed in this triangle will be as shown in Fig.6.6.

Let PB = x cm. So., AP = $(16-x)$ cm. In ΔAPS and ΔABC, ∠A = ∠A and ∠APS = ∠ABC (Each 90°)

So, ΔAPS ~ ΔABC (AA similarity)

Therefore, $\frac{AP}{AB} = \frac{PS}{BC}$

or $\frac{16-x}{16} = \frac{x}{8}$

or $128 - 8x = 16x$

or $x = \frac{128}{24} = \frac{16}{3}$

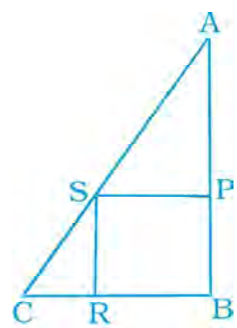


Fig. 6.6

Thus, the side of the required square is of length $\frac{16}{3}$ cm.

Sample Question 2: Hypotenuse of a right triangle is 25 cm and out of the remaining two sides, one is longer than the other by 5 cm. Find the lengths of the other two sides.

Solution : Let one side be x cm. Then the other side will be $(x + 5)$ cm.

Therefore, from Pythagoras Theorem

$$x^2 + (x + 5)^2 = (25)^2$$

or $\quad x^2 + x^2 + 10x + 25 = 625$

or $\quad x^2 + 5x - 300 = 0$

or $\quad x^2 + 20x - 15x - 300 = 0$

or $\quad x(x+20) - 15(x+20) = 0$

or $\quad (x-15)(x+20) = 0$

So, $\quad x = 15$ or $x = -20$

Rejecting $x = -20$, we have length of one side = 15 cm and that of the other side = (15 + 5) cm = 20 cm

Sample Question 3: In Fig 6.7,

$\angle D = \angle E$ and $\dfrac{AD}{DB} = \dfrac{AE}{EC}$. Prove that BAC is an isosceles triangle.

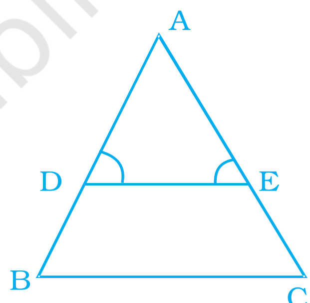


Fig. 6.7

Solution : $\dfrac{AD}{DB} = \dfrac{AE}{EC}$ (Given)

Therefore, DE || BC (Converse of Basic Proportionality Theorem)

So, $\angle D = \angle B$ and $\angle E = \angle C$ (Corresponding angles) (1)

But $\angle D = \angle E$ (Given)

Therefore, $\angle B = \angle C$ [From (1)]

So, AB = AC (Sides opposite to equal angles)

i.e., BAC is an isosceles triangle.

EXERCISE 6.3

1. In a Δ PQR, $PR^2 - PQ^2 = QR^2$ and M is a point on side PR such that QM $\perp$ PR. Prove that

$QM^2 = PM \times MR$.

2. Find the value of x for which DE || AB in Fig. 6.8.

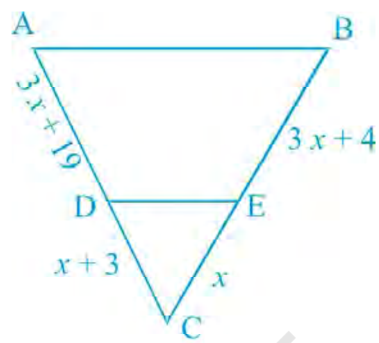


Fig. 6.8

3. In Fig. 6.9, if $\angle 1 = \angle 2$ and Δ NSQ $\cong$ Δ MTR, then prove that Δ PTS ~ Δ PRQ.

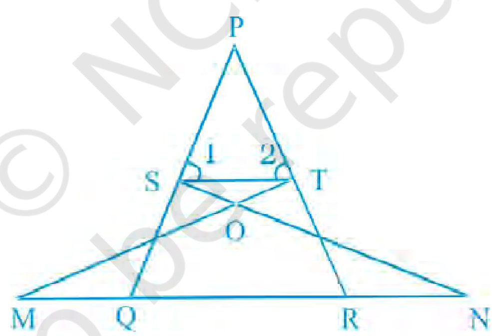


Fig. 6.9

4. Diagonals of a trapezium PQRS intersect each other at the point O, PQ || RS and PQ = 3 RS. Find the ratio of the areas of triangles POQ and ROS.

5. In Fig. 6.10, if AB || DC and AC and PQ intersect each other at the point O, prove that OA . CQ = OC . AP.

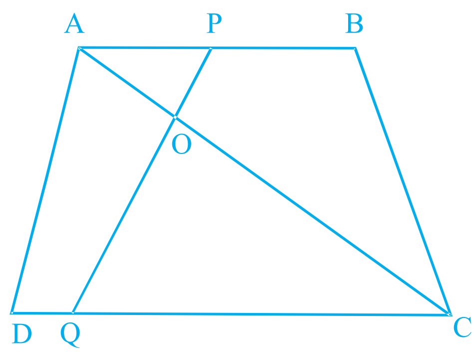


Fig. 6.10

6. Find the altitude of an equilateral triangle of side 8 cm.

7. If Δ ABC ~ Δ DEF, AB = 4 cm, DE = 6 cm, EF = 9 cm and FD = 12 cm, find the perimeter of Δ ABC.

8. In Fig. 6.11, if DE || BC, find the ratio of ar (ADE) and ar (DECB).

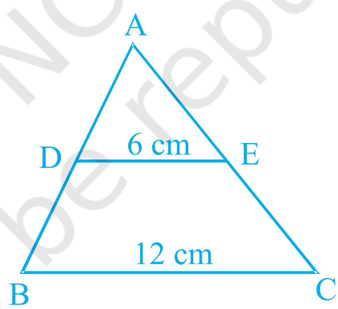


Fig. 6.11

9. ABCD is a trapezium in which AB || DC and P and Q are points on AD and BC, respectively such that PQ || DC. If PD = 18 cm, BQ = 35 cm and QC = 15 cm, find AD.

10. Corresponding sides of two similar triangles are in the ratio of 2 : 3. If the area of the smaller triangle is 48 cm^2, find the area of the larger triangle.

11. In a triangle PQR, N is a point on PR such that $QN \perp PR$. If PN. NR = QN^2, prove that $\angle PQR = 90°$.

12. Areas of two similar triangles are 36 cm^2 and 100 cm^2. If the length of a side of the larger triangle is 20 cm, find the length of the corresponding side of the smaller triangle.

13. In Fig. 6.12, if $\angle ACB = \angle CDA$, AC = 8 cm and AD = 3 cm, find BD.

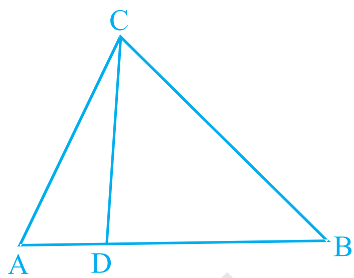


Fig. 6.12

14. A 15 metres high tower casts a shadow 24 metres long at a certain time and at the same time, a telephone pole casts a shadow 16 metres long. Find the height of the telephone pole.

15. Foot of a 10 m long ladder leaning against a vertical wall is 6 m away from the base of the wall. Find the height of the point on the wall where the top of the ladder reaches.

(E) Long Answer Questions

Sample Question 1: In Fig 6.13, OB is the perpendicular bisector of the line segment DE, FA ⊥ OB and F E intersects OB at the point C. Prove that $\frac{1}{OA}+\frac{1}{OB}=\frac{2}{OC}$.

Solution: In Δ AOF and Δ BOD.

$\angle O = \angle O$ (Same angle) and $\angle A = \angle B$ (each 90°)

Therefore, Δ AOF ~ Δ BOD (AA similarity)

So, $\frac{OA}{OB}=\frac{FA}{DB}$ (1)

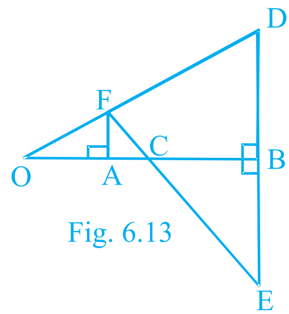


Fig. 6.13

Also, in Δ FAC and Δ EBC, ∠A = ∠B (Each 90°)

and ∠FCA = ∠ECB (Vertically opposite angles).

Therefore, Δ FAC ~ Δ EBC (AA similarity).

So, $$\frac{FA}{EB} = \frac{AC}{BC}$$

But EB = DB (B is mid-point of DE)

So, $$\frac{FA}{DB} = \frac{AC}{BC} \qquad (2)$$

Therefore, from (1) and (2), we have:

$$\frac{AC}{BC} = \frac{OA}{OB}$$

i.e., $$\frac{OC - OA}{OB - OC} = \frac{OA}{OB}$$

or $\quad OB \,.\, OC - OA \,.\, OB = OA \,.\, OB - OA \,.\, OC$

or $\quad OB \,.\, OC + OA \,.\, OC = 2\, OA \,.\, OB$

or $\quad (OB + OA).\, OC = 2\, OA \,.\, OB$

or $$\frac{1}{OA} + \frac{1}{OB} = \frac{2}{OC} \quad \text{[Dividing both the sides by OA . OB . OC]}$$

Sample Question 2: Prove that if in a triangle square on one side is equal to the sum of the squares on the other two sides, then the angle opposite the first side is a right angle.

Solution: See proof of Theorem 6.9 of Mathematics Textbook for Class X.

Sample Question 3: An aeroplane leaves an Airport and flies due North at 300 km/h. At the same time, another aeroplane leaves the same Airport and flies due West at 400 km/h. How far apart the two aeroplanes would be after $1\frac{1}{2}$ hours?

Solution: Distance travelled by first aeroplane in $1\frac{1}{2}$ hours $= 300 \times \frac{3}{2}$ km $= 450$ km

and that by second aeroplane $= \frac{400 \times 3}{2}\text{ km} = 600\text{ km}$

Position of the two aeroplanes after $1\frac{1}{2}$ hours would be A and B as shown in Fig. 6.14.

That is, OA = 450 km and OB = 600 km.

From Δ AOB, we have

$$AB^2 = OA^2 + OB^2$$

or $\quad AB^2 = (450)^2 + (600)^2$

$$= (150)^2 \times 3^2 + (150)^2 \times 4^2$$

$$= 150^2 \ (3^2 + 4^2)$$

$$= 150^2 \times 5^2$$

or $\quad AB = 150 \times 5 = 750$

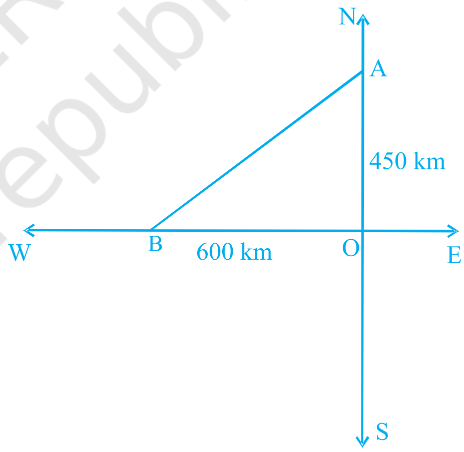


Fig. 6.14

Thus, the two aeroplanes will be 750 km apart after $1\frac{1}{2}$ hours.

Sample Question 4: In Fig. 6.15, if $\Delta ABC \sim \Delta DEF$ and their sides are of lengths (in cm) as marked along them, then find the lengths of the sides of each triangle.

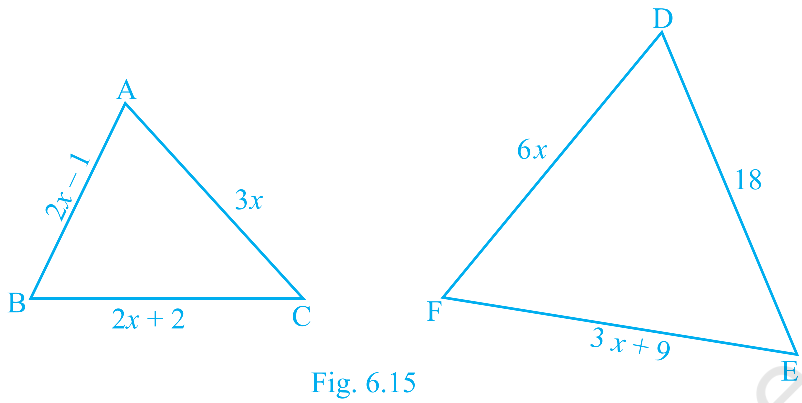


Fig. 6.15

Solution: Δ ABC ~ Δ DEF (Given)

Therefore, $\frac{AB}{DE}=\frac{BC}{EF}=\frac{CA}{FD}$

So, $\frac{2x-1}{18}=\frac{2x+2}{3x+9}=\frac{3x}{6x}$

Now, taking $\frac{2x-1}{18}=\frac{3x}{6x}$, we have

$$\frac{2x-1}{18}=\frac{1}{2}$$

or $4x - 2 = 18$

or $x = 5$

Therefore, AB = 2 × 5 –1= 9, BC = 2 × 5 + 2 = 12,

CA = 3 × 5 = 15, DE = 18, EF = 3× 5 + 9 = 24 and FD = 6 × 5 = 30

Hence, AB = 9 cm, BC = 12 cm, CA = 15 cm,

DE = 18 cm, EF = 24 cm and FD = 30 cm.

EXERCISE 6.4

1. In Fig. 6.16, if $\angle A = \angle C$, AB = 6 cm, BP = 15 cm, AP = 12 cm and CP = 4 cm, then find the lengths of PD and CD.

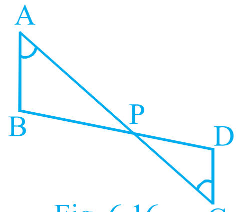


Fig. 6.16

2. It is given that Δ ABC ~ Δ EDF such that AB = 5 cm, AC = 7 cm, DF= 15 cm and DE = 12 cm. Find the lengths of the remaining sides of the triangles.

3. Prove that if a line is drawn parallel to one side of a triangle to intersect the other two sides, then the two sides are divided in the same ratio.

4. In Fig 6.17, if PQRS is a parallelogram and AB || PS, then prove that OC || SR.

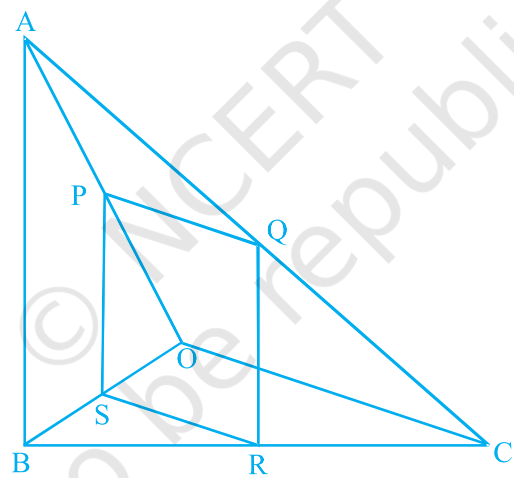


Fig. 6.17

5. A 5 m long ladder is placed leaning towards a vertical wall such that it reaches the wall at a point 4 m high. If the foot of the ladder is moved 1.6 m towards the wall, then find the distance by which the top of the ladder would slide upwards on the wall.

6. For going to a city B from city A, there is a route via city C such that AC$\perp$CB, AC = 2 x km and CB = 2 $(x + 7)$ km. It is proposed to construct a 26 km highway which directly connects the two cities A and B. Find how much distance will be saved in reaching city B from city A after the construction of the highway.

7. A flag pole 18 m high casts a shadow 9.6 m long. Find the distance of the top of the pole from the far end of the shadow.

8. A street light bulb is fixed on a pole 6 m above the level of the street. If a woman of height 1.5 m casts a shadow of 3m, find how far she is away from the base of the pole.

9. In Fig. 6.18, ABC is a triangle right angled at B and BD $\perp$ AC. If AD = 4 cm, and CD = 5 cm, find BD and AB.

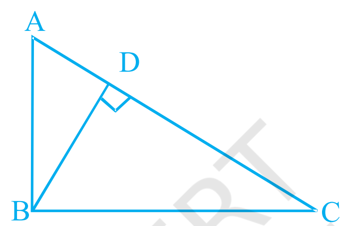


Fig. 6.18

10. In Fig. 6.19, PQR is a right triangle right angled at Q and QS $\perp$ PR . If PQ = 6 cm and PS = 4 cm, find QS, RS and QR.

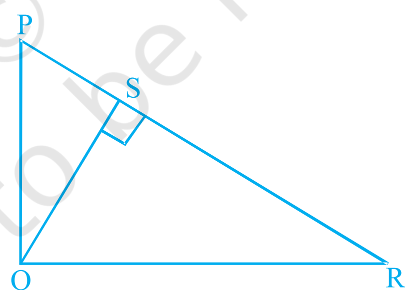


Fig. 6.19

11. In Δ PQR, PD $\perp$ QR such that D lies on QR . If PQ = a, PR = b, QD = c and DR = d, prove that $(a + b)(a – b) = (c + d)(c – d)$.

12. In a quadrilateral ABCD, $\angle$A + $\angle$D = 90°. Prove that $AC^2 + BD^2 = AD^2 + BC^2$

[**Hint:** Produce AB and DC to meet at E.]

13. In fig. 6.20, $l \parallel m$ and line segments AB, CD and EF are concurrent at point P.

Prove that $\frac{AE}{BF} = \frac{AC}{BD} = \frac{CE}{FD}$.

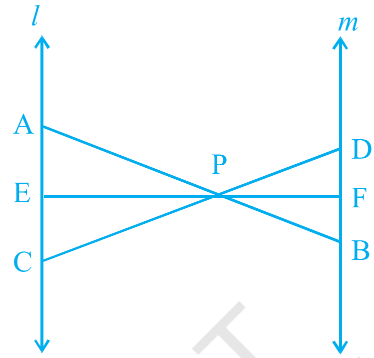


Fig. 6.20

14. In Fig. 6.21, PA, QB, RC and SD are all perpendiculars to a line l, AB = 6 cm, BC = 9 cm, CD = 12 cm and SP = 36 cm. Find PQ, QR and RS.

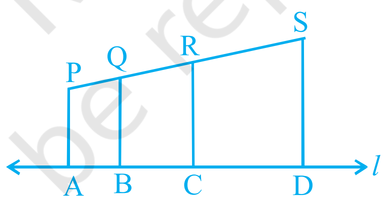


Fig. 6.21

15. O is the point of intersection of the diagonals AC and BD of a trapezium ABCD with $AB \parallel DC$. Through O, a line segment PQ is drawn parallel to AB meeting AD in P and BC in Q. Prove that PO = QO.

16. In Fig. 6.22, line segment DF intersect the side AC of a triangle ABC at the point E such that E is the mid-point of CA and $\angle AEF = \angle AFE$. Prove that

$\frac{BD}{CD} = \frac{BF}{CE}$.

[**Hint:** Take point G on AB such that $CG \parallel DF$.]

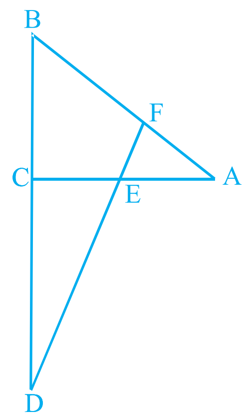


Fig. 6.22

17. Prove that the area of the semicircle drawn on the hypotenuse of a right angled triangle is equal to the sum of the areas of the semicircles drawn on the other two sides of the triangle.

18. Prove that the area of the equilateral triangle drawn on the hypotenuse of a right angled triangle is equal to the sum of the areas of the equilateral triangles drawn on the other two sides of the triangle.

CHAPTER 7

COORDINATE GEOMETRY

(A) Main Concepts and Results

Distance Formula, Section Formula, Area of a Triangle.

- The distance between two points P (x_1, y_1) and Q (x_2, y_2) is $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
- The distance of a point P (x,y) from the origin is $\sqrt{x^2 + y^2}$
- The coordinates of the point P which divides the line segment joining the points A (x_1, y_1) and B (x_2, y_2) internally in the ratio $m_1 : m_2$ are $\left(\frac{m_1 x_2 + m_2 x_1}{m_1 + m_2}, \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2}\right)$
- The coordinates of the mid-point of the line segment joining the points P (x_1, y_1) and Q (x_2, y_2) are $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$
- The area of a triangle with vertices A (x_1, y_1), B (x_2, y_2) and C (x_3, y_3) is

  $$\frac{1}{2}\,[x_1 (y_2 - y_3) + x_2 (y_3 - y_1) + x_3 (y_1 - y_2)]$$

  which is non–zero unless the points A, B and C are collinear.

(B) Multiple Choice Questions

Choose the correct answer from the given four options:

Sample Question 1: If the distance between the points (2, –2) and (–1, x) is 5, one of the values of x is

(A) –2 (B) 2 (C) –1 (D) 1

Solution : Answer (B)

Sample Question 2: The mid-point of the line segment joining the points A (–2, 8) and B (– 6, – 4) is

(A) (– 4, – 6) (B) (2, 6) (C) (– 4, 2) (D) (4, 2)

Solution : Answer (C)

Sample Question 3: The points A (9, 0), B (9, 6), C (–9, 6) and D (–9, 0) are the vertices of a

(A) square (B) rectangle (C) rhombus (D) trapezium

Solution : Answer (B)

EXERCISE 7.1

Choose the correct answer from the given four options:

1. The distance of the point P (2, 3) from the x-axis is

(A) 2 (B) 3 (C) 1 (D) 5

2. The distance between the points A (0, 6) and B (0, –2) is

(A) 6 (B) 8 (C) 4 (D) 2

3. The distance of the point P (–6, 8) from the origin is

(A) 8 (B) $2\sqrt{7}$ (C) 10 (D) 6

4. The distance between the points (0, 5) and (–5, 0) is

(A) 5 (B) $5\sqrt{2}$ (C) $2\sqrt{5}$ (D) 10

5. AOBC is a rectangle whose three vertices are vertices A (0, 3), O (0, 0) and B (5, 0). The length of its diagonal is

(A) 5 (B) 3 (C) $\sqrt{34}$ (D) 4

6. The perimeter of a triangle with vertices (0, 4), (0, 0) and (3, 0) is

(A) 5 (B) 12 (C) 11 (D) $7+\sqrt{5}$

7. The area of a triangle with vertices A (3, 0), B (7, 0) and C (8, 4) is

(A) 14 (B) 28 (C) 8 (D) 6

8. The points (–4, 0), (4, 0), (0, 3) are the vertices of a

(A) right triangle (B) isosceles triangle

(C) equilateral triangle (D) scalene triangle

9. The point which divides the line segment joining the points (7, –6) and (3, 4) in ratio 1 : 2 internally lies in the
(A) I quadrant (B) II quadrant
(C) III quadrant (D) IV quadrant

10. The point which lies on the perpendicular bisector of the line segment joining the points A (–2, –5) and B (2, 5) is
(A) (0, 0) (B) (0, 2) (C) (2, 0) (D) (–2, 0)

11. The fourth vertex D of a parallelogram ABCD whose three vertices are A (–2, 3), B (6, 7) and C (8, 3) is
(A) (0, 1) (B) (0, –1) (C) (–1, 0) (D) (1, 0)

12. If the point P (2, 1) lies on the line segment joining points A (4, 2) and B (8, 4), then

(A) $AP = \frac{1}{3} AB$ (B) AP = PB (C) $PB = \frac{1}{3} AB$ (D) $AP = \frac{1}{2} AB$

13. If P $\frac{a}{3}, 4$ is the mid-point of the line segment joining the points Q (– 6, 5) and R (– 2, 3), then the value of a is
(A) – 4 (B) – 12 (C) 12 (D) – 6

14. The perpendicular bisector of the line segment joining the points A (1, 5) and B (4, 6) cuts the y-axis at
(A) (0, 13) (B) (0, –13)
(C) (0, 12) (D) (13, 0)

15. The coordinates of the point which is equidistant from the three vertices of the Δ AOB as shown in the Fig. 7.1 is

(A) (x, y) (B) (y, x)

(C) $\frac{x}{2}, \frac{y}{2}$ (D) $\frac{y}{2}, \frac{x}{2}$

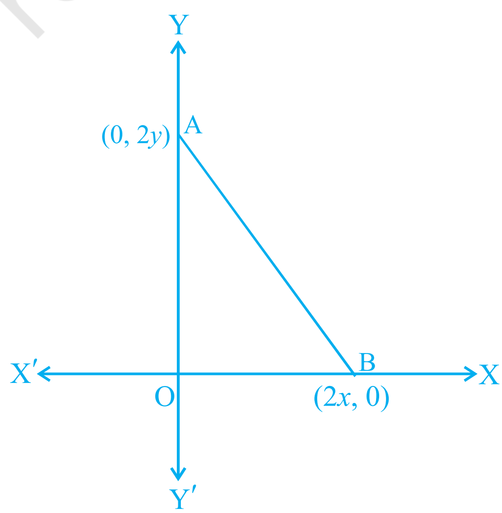


Fig. 7.1

16. A circle drawn with origin as the centre passes through $(\frac{13}{2}, 0)$. The point which does not lie in the interior of the circle is

(A) $\frac{-3}{4},1$ (B) $2,\frac{7}{3}$ (C) $5,\frac{-1}{2}$ (D) $\left(-6,\frac{5}{2}\right)$

17. A line intersects the *y*-axis and *x*-axis at the points P and Q, respectively. If (2, –5) is the mid-point of PQ, then the coordinates of P and Q are, respectively

(A) (0, – 5) and (2, 0) (B) (0, 10) and (– 4, 0)

(C) (0, 4) and (– 10, 0) (D) (0, – 10) and (4, 0)

18. The area of a triangle with vertices $(a, b + c)$, $(b, c + a)$ and $(c, a + b)$ is

(A) $(a + b + c)^2$ (B) 0 (C) $a + b + c$ (D) abc

19. If the distance between the points (4, *p*) and (1, 0) is 5, then the value of *p* is

(A) 4 only (B) ± 4 (C) – 4 only (D) 0

20. If the points A (1, 2), O (0, 0) and C (a, b) are collinear, then

(A) $a = b$ (B) $a = 2b$ (C) $2a = b$ (D) $a = -b$

(C) Short Answer Questions with Reasoning

State whether the following statements are true or false. Justify your answer.

Sample Question 1 : The points A (–1, 0), B (3, 1), C (2, 2) and D (–2, 1) are the vertices of a parallelogram.

Solution : True. The coordinates of the mid-points of both the diagonals AC and BD are $\frac{1}{2},1$, i.e., the diagonals bisect each other.

Sample Question 2 : The points (4, 5), (7, 6) and (6, 3) are collinear.

Solution : False. Since the area of the triangle formed by the points is 4 sq. units, the points are not collinear.

Sample Question 3 : Point P (0, –7) is the point of intersection of *y*-axis and perpendicular bisector of line segment joining the points A (–1, 0) and B (7, –6).

Solution : True. P (0, –7) lies on the *y* -axis. It is at a distance of $\sqrt{50}$ units from both the points (–1, 0) and (7, –6).

EXERCISE 7.2

State whether the following statements are true or false. Justify your answer.

1. Δ ABC with vertices A (–2, 0), B (2, 0) and C (0, 2) is similar to Δ DEF with vertices D (–4, 0) E (4, 0) and F (0, 4).

2. Point P (– 4, 2) lies on the line segment joining the points A (– 4, 6) and B (– 4, – 6).

3. The points (0, 5), (0, –9) and (3, 6) are collinear.

4. Point P (0, 2) is the point of intersection of y–axis and perpendicular bisector of line segment joining the points A (–1, 1) and B (3, 3).

5. Points A (3, 1), B (12, –2) and C (0, 2) cannot be the vertices of a triangle.

6. Points A (4, 3), B (6, 4), C (5, –6) and D (–3, 5) are the vertices of a parallelogram.

7. A circle has its centre at the origin and a point P (5, 0) lies on it. The point Q (6, 8) lies outside the circle.

8. The point A (2, 7) lies on the perpendicular bisector of line segment joining the points P (6, 5) and Q (0, – 4).

9. Point P (5, –3) is one of the two points of trisection of the line segment joining the points A (7, – 2) and B (1, – 5).

10. Points A (–6, 10), B (–4, 6) and C (3, –8) are collinear such that AB = $\frac{2}{9}$AC.

11. The point P (–2, 4) lies on a circle of radius 6 and centre C (3, 5).

12. The points A (–1, –2), B (4, 3), C (2, 5) and D (–3, 0) in that order form a rectangle.

(D) Short Answer Questions

Sample Question 1 : If the mid-point of the line segment joining the points A (3, 4) and B (k, 6) is P (x, y) and $x + y - 10 = 0$, find the value of k.

Solution : Mid-point of the line segment joining A (3, 4) and B (k, 6) = $\frac{3+k}{2}, \frac{4+6}{2}$

$$= \frac{3+k}{2}, 5$$

Then, $\frac{3+k}{2}, 5 = (x, y)$

Therefore, $\frac{3+k}{2} = x$ and $5 = y$.

Since $x + y - 10 = 0$, we have

$$\frac{3+k}{2} + 5 - 10 = 0$$

i.e., $3 + k = 10$

Therefore, $k = 7$.

Sample Question 2 : Find the area of the triangle ABC with A (1, –4) and the mid-points of sides through A being (2, – 1) and (0, – 1).

Solution: Let the coordinates of B and C be (a, b) and (x, y), respectively.

Then , $\left(\dfrac{1+a}{2}, \dfrac{-4+b}{2}\right) = (2, -1)$

Therefore, $1 + a = 4, \quad -4 + b = -2$

$a = 3 \qquad b = 2$

Also, $\left(\dfrac{1+x}{2}, \dfrac{-4+y}{2}\right) = (0, -1)$

Therefore, $1 + x = 0, \qquad -4 + y = -2$

i.e., $x = -1$ i.e., $y = 2$

The coordinates of the vertices of Δ ABC are A (1, –4), B (3, 2) and C (–1, 2).

$$\text{Area of } \Delta\text{ ABC} = \frac{1}{2}[1(2-2)+3(2+4)-1(-4-2)]$$

$$= \frac{1}{2}[18+6]$$

$$= 12 \text{ sq. units.}$$

Sample Question 3 : Name the type of triangle PQR formed by the points P $\left(\sqrt{2},\sqrt{2}\right)$, Q $\left(-\sqrt{2},-\sqrt{2}\right)$ and R $\left(-\sqrt{6},\sqrt{6}\right)$.

Solution : Using distance formula

$$PQ = \sqrt{\left(\sqrt{2}+\sqrt{2}\right)^2+\left(\sqrt{2}+\sqrt{2}\right)^2} = \sqrt{\left(2\sqrt{2}\right)^2+\left(2\sqrt{2}\right)^2} = \sqrt{16} = 4$$

$$PR = \sqrt{\left(\sqrt{2}+\sqrt{6}\right)^2+\left(\sqrt{2}-\sqrt{6}\right)^2} = \sqrt{2+6+2\sqrt{12}+2+6-2\sqrt{12}} = \sqrt{16} = 4$$

$$RQ = \sqrt{\left(-\sqrt{2}+\sqrt{6}\right)^2+\left(-\sqrt{2}-\sqrt{6}\right)^2} = \sqrt{2+6-2\sqrt{12}+2+6+2\sqrt{12}} = \sqrt{16} = 4$$

Since PQ = PR = RQ = 4, points P, Q, R form an equilateral triangle.

Sample Question 4 : ABCD is a parallelogram with vertices A (x_1, y_1), B (x_2, y_2) and C (x_3, y_3). Find the coordinates of the fourth vertex D in terms of x_1, x_2, x_3, y_1, y_2 and y_3.

Solution: Let the coordinates of D be (x, y). We know that diagonals of a parallelogram bisect each other.

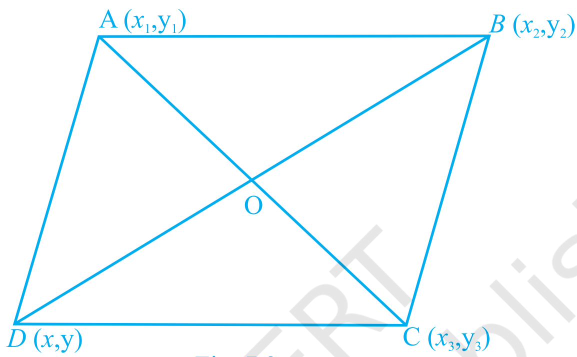


Fig. 7.2

Therefore, mid-point of AC = mid-point of BD $\quad \frac{x_1+x_3}{2}, \frac{y_1+y_3}{2} = \frac{x_2+x}{2}, \frac{y_2+y}{2}$

i.e., $x_1 + x_3 = x_2 + x$ and $y_1 + y_3 = y_2 + y$
i.e., $x_1 + x_3 - x_2 = x$ and $y_1 + y_3 - y_2 = y$
Thus, the coordinates of D are
$(x_1 + x_3 - x_2, y_1 + y_3 - y_2)$

EXERCISE 7.3

1. Name the type of triangle formed by the points A (–5, 6), B (–4, –2) and C (7, 5).
2. Find the points on the x–axis which are at a distance of $2\sqrt{5}$ from the point (7, –4). How many such points are there?
3. What type of a quadrilateral do the points A (2, –2), B (7, 3), C (11, –1) and D (6, –6) taken in that order, form?
4. Find the value of a, if the distance between the points A (–3, –14) and B (a, –5) is 9 units.
5. Find a point which is equidistant from the points A (–5, 4) and B (–1, 6)? How many such points are there?

6. Find the coordinates of the point Q on the x–axis which lies on the perpendicular bisector of the line segment joining the points A (–5, –2) and B(4, –2). Name the type of triangle formed by the points Q, A and B.

7. Find the value of m if the points (5, 1), (–2, –3) and (8, 2m) are collinear.

8. If the point A (2, – 4) is equidistant from P (3, 8) and Q (–10, y), find the values of y. Also find distance PQ.

9. Find the area of the triangle whose vertices are (–8, 4), (–6, 6) and (–3, 9).

10. In what ratio does the x–axis divide the line segment joining the points (– 4, – 6) and (–1, 7)? Find the coordinates of the point of division.

11. Find the ratio in which the point $P\left(\frac{3}{4},\frac{5}{12}\right)$ divides the line segment joining the points A $\frac{1}{2},\frac{3}{2}$ and B (2, –5).

12. If P (9a – 2, –b) divides line segment joining A (3a + 1, –3) and B (8a, 5) in the ratio 3 : 1, find the values of a and b.

13. If (a, b) is the mid-point of the line segment joining the points A (10, –6) and B (k, 4) and a – 2b = 18, find the value of k and the distance AB.

14. The centre of a circle is (2a, a – 7). Find the values of a if the circle passes through the point (11, –9) and has diameter $10\sqrt{2}$ units.

15. The line segment joining the points A (3, 2) and B (5,1) is divided at the point P in the ratio 1:2 and it lies on the line $3x - 18y + k = 0$. Find the value of k.

16. If $D\left(\frac{-1}{2},\frac{5}{2}\right)$, E (7, 3) and $F\left(\frac{7}{2},\frac{7}{2}\right)$ are the midpoints of sides of Δ ABC, find the area of the Δ ABC.

17. The points A (2, 9), B (a, 5) and C (5, 5) are the vertices of a triangle ABC right angled at B. Find the values of a and hence the area of ΔABC.

18. Find the coordinates of the point R on the line segment joining the points P (–1, 3) and Q (2, 5) such that $PR = \frac{3}{5}PQ$.

19. Find the values of k if the points A (k + 1, 2k), B (3k, 2k + 3) and C (5k – 1, 5k) are collinear.

20. Find the ratio in which the line $2x + 3y - 5 = 0$ divides the line segment joining the points (8, –9) and (2, 1). Also find the coordinates of the point of division.

(E) Long Answer Questions

Sample Question 1 : The mid-points D, E, F of the sides of a triangle ABC are (3, 4), (8, 9) and (6, 7). Find the coordinates of the vertices of the triangle.

Solution : Since D and F are the mid-points of AB and AC, respectively, by mid-point theorem, we can prove that DFEB is a parallelogram. Let the coordinates of B be (x, y).

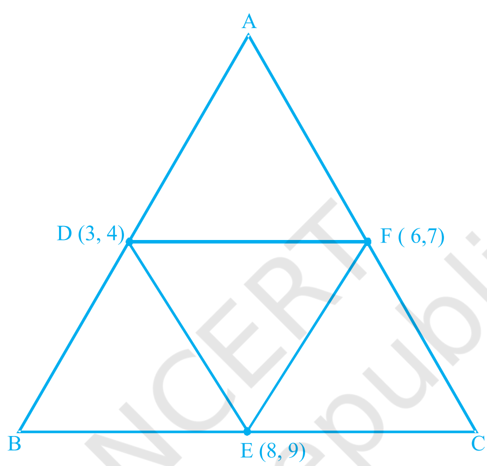


Fig. 7.3

Refer to Sample Question 4 of Section (D) to get

$$x = 3 + 8 - 6 = 5$$

$$y = 4 + 9 - 7 = 6$$

Therefore, B (5, 6) is one of the vertices of the triangle.

Similarly DFCE and DAFE are also parallelograms, and the coordinates of A are (3 + 6 – 8, 4 + 7 – 9) = (1, 2). Coordinates of C are (8 + 6 – 3, 9 + 7 – 4) = (11, 12). Thus, the coordinates of the vertices of the triangle are A (1, 2), B (5,6) and C (11, 12).

EXERCISE 7.4

1. If (– 4, 3) and (4, 3) are two vertices of an equilateral triangle, find the coordinates of the third vertex, given that the origin lies in the interior of the triangle.
2. A (6, 1), B (8, 2) and C (9, 4) are three vertices of a parallelogram ABCD. If E is the midpoint of DC, find the area of Δ ADE.

3. The points A (x_1, y_1), B (x_2, y_2) and C $(x_3\ y_3)$ are the vertices of Δ ABC.

(i) The median from A meets BC at D. Find the coordinates of the point D.

(ii) Find the coordinates of the point P on AD such that AP : PD = 2 : 1

(iii) Find the coordinates of points Q and R on medians BE and CF, respectively such that BQ : QE = 2 : 1 and CR : RF = 2 : 1

(iv) What are the coordinates of the centroid of the triangle ABC?

4. If the points A (1, –2), B (2, 3) C (a, 2) and D (– 4, –3) form a parallelogram, find the value of a and height of the parallelogram taking AB as base.

5. Students of a school are standing in rows and columns in their playground for a drill practice. A, B, C and D are the positions of four students as shown in figure 7.4. Is it possible to place Jaspal in the drill in such a way that he is equidistant from each of the four students A, B, C and D? If so, what should be his position?

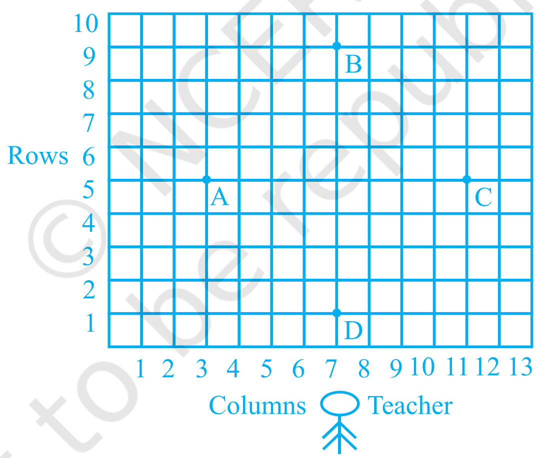


Fig. 7.4

6. Ayush starts walking from his house to office. Instead of going to the office directly, he goes to a bank first, from there to his daughter's school and then reaches the office. What is the extra distance travelled by Ayush in reaching his office? (Assume that all distances covered are in straight lines).

If the house is situated at (2, 4), bank at (5, 8), school at (13, 14) and office at (13, 26) and coordinates are in km.