Mathematics

(Chapter – 1) (Real Numbers)

(Class X)

Exercise 1.1

Question 1:

Use Euclid's division algorithm to find the HCF of:

(i) 135 and 225 **(ii)** 196 and 38220 **(iii)** 867 and 255

Answer 1:

(i) 135 and 225

Since 225 > 135, we apply the division lemma to 225 and 135 to obtain

$225 = 135 \times 1 + 90$

Since remainder $90 \neq 0$, we apply the division lemma to 135 and 90 to obtain

$135 = 90 \times 1 + 45$

We consider the new divisor 90 and new remainder 45, and apply the division lemma to obtain

$90 = 2 \times 45 + 0$

Since the remainder is zero, the process stops.

Since the divisor at this stage is 45,

Therefore, the HCF of 135 and 225 is 45.

(ii) 196 and 38220

Since 38220 > 196, we apply the division lemma to 38220 and 196 to obtain

$38220 = 196 \times 195 + 0$

Since the remainder is zero, the process stops.

Since the divisor at this stage is 196,

Therefore, HCF of 196 and 38220 is 196.

(iii) 867 and 255

Since 867 > 255, we apply the division lemma to 867 and 255 to obtain

$867 = 255 \times 3 + 102$

Since remainder $102 \neq 0$, we apply the division lemma to 255 and 102 to obtain

$255 = 102 \times 2 + 51$

We consider the new divisor 102 and new remainder 51, and apply the division lemma to obtain

$102 = 51 \times 2 + 0$

Since the remainder is zero, the process stops.

Since the divisor at this stage is 51, Therefore,

HCF of 867 and 255 is 51.

Question 2:

Show that any positive odd integer is of the form $6q + 1$, or $6q + 3$, or $6q + 5$, where q is some integer.

Answer 2:

Let a be any positive integer and $b = 6$.

Then, by Euclid's algorithm, $a = 6q + r$ for some integer $q \geq 0$, and

$r = 0, 1, 2, 3, 4, 5$ because $0 \leq r < 6$.

Therefore, $a = 6q$ or $6q + 1$ or $6q + 2$ or $6q + 3$ or $6q + 4$ or $6q + 5$

Also, $6q + 1 = 2 \times 3q + 1 = 2k_1 + 1$, where k_1 is a positive integer

$6q + 3 = (6q + 2) + 1 = 2\ (3q + 1) + 1 = 2k_2 + 1$, where k_2 is an integer

$6q + 5 = (6q + 4) + 1 = 2\ (3q + 2) + 1 = 2k_3 + 1$, where k_3 is an integer

Clearly,

$6q + 1$, $6q + 3$, $6q + 5$ are of the form $2k + 1$, where k is an integer.

Therefore, $6q + 1$, $6q + 3$, $6q + 5$ are not exactly divisible by 2.

Hence, these expressions of numbers are odd numbers.

And therefore, any odd integer can be expressed in the form $6q + 1$, or $6q + 3$, or $6q + 5$

Question 3:

An army contingent of 616 members is to march behind an army band of 32 members in a parade. The two groups are to march in the same number of columns. What is the maximum number of columns in which they can march?

Answer 3:

HCF (616, 32) will give the maximum number of columns in which they can march.

We can use Euclid's algorithm to find the HCF.

$616 = 32 \times 19 + 8$

$32 = 8 \times 4 + 0$

The HCF (616, 32) is 8.

Therefore, they can march in 8 columns each.

Question 4:

Use Euclid's division lemma to show that the square of any positive integer is either of form $3m$ or $3m + 1$ for some integer m.

[**Hint:** Let x be any positive integer then it is of the form $3q$, $3q + 1$ or $3q + 2$. Now square each of these and show that they can be rewritten in the form $3m$ or $3m + 1$.]

Answer 4:

Let a be any positive integer and $b = 3$.

Then $a = 3q + r$ for some integer $q \geq 0$

And $r = 0, 1, 2$ because $0 \leq r < 3$

Therefore, $a = 3q$ or $3q + 1$ or $3q + 2$ Or,

$$\begin{aligned} a^2 &= (3q)^2 \ or \ (3q+1)^2 \ or \ (3q+2)^2 \\ &= (3q)^2 \ or \ 9q^2+6q+1 \ or \ 9q^2+12q+4 \\ &= 3\times(3q^2) \ or \ 3\times(3q^2+2q)+1 \ or \ 3\times(3q^2+4q+1)+1 \\ &= 3k_1 \ or \ 3k_2+1 \ or \ 3k_3+1 \end{aligned}$$

Where k_1, k_2, and k_3 are some positive integers

Hence, it can be said that the square of any positive integer is either of the form $3m$ or $3m + 1$.

Question 5:

Use Euclid's division lemma to show that the cube of any positive integer is of the form $9m$, $9m + 1$ or $9m + 8$.

Answer 5:

Let a be any positive integer and $b = 3$

$a = 3q + r$, where $q \geq 0$ and $0 \leq r < 3$

$a = 3q$ or $3q + 1$ or $3q + 2$

Therefore, every number can be represented as these three forms.

There are three cases.

Case 1: When $a = 3q$,

$a^3 = (3q)^3 = 27q^3 = 9(3q^3) = 9m$

Where m is an integer such that $m = 3q^3$

Case 2: When $a = 3q + 1$,

$a^3 = (3q + 1)^3$

$a^3 = 27q^3 + 27q^2 + 9q + 1$

$a^3 = 9(3q^3 + 3q^2 + q) + 1$

$a^3 = 9m + 1$

Where m is an integer such that $m = (3q^3 + 3q^2 + q)$

Case 3: When $a = 3q + 2$,

$a^3 = (3q + 2)^3$

$a^3 = 27q^3 + 54q^2 + 36q + 8$

$a^3 = 9(3q^3 + 6q^2 + 4q) + 8$

$a^3 = 9m + 8$

Where m is an integer such that $m = (3q^3 + 6q^2 + 4q)$

Therefore, the cube of any positive integer is of the form $9m$, $9m + 1$, or $9m + 8$.

Mathematics

(Chapter – 1) (Real Numbers)

(Class X)

Exercise 1.2

Question 1:

Express each number as product of its prime factors:

(i) 140 (ii) 156 (iii) 3825 (iv) 5005 (v) 7429

Answer 1:

(i) $140 = 2\times2\times5\times7 = 2^2\times5\times7$

(ii) $156 = 2\times2\times3\times13 = 2^2\times3\times13$

(iii) $3825 = 3\times3\times5\times5\times17 = 3^2\times5^2\times17$

(iv) $5005 = 5\times7\times11\times13$

(v) $7429 = 17\times19\times23$

Question 2:

Find the LCM and HCF of the following pairs of integers and verify that LCM × HCF = product of the two numbers.

(i) 26 and 91 (ii) 510 and 92 (iii) 336 and 54

Answer 2:

(i) 26 and 91

$26 = 2\times13$

$91 = 7\times13$

$\text{HCF} = 13$

$\text{LCM} = 2\times7\times13 = 182$

Product of the two numbers $= 26\times91 = 2366$

$\text{HCF}\times\text{LCM} = 13\times182 = 2366$

Hence, product of two numbers = HCF × LCM

(ii) 510 and 92

$510 = 2\times3\times5\times17$

$92 = 2\times2\times23$

$\text{HCF} = 2$

$\text{LCM} = 2\times2\times3\times5\times17\times23 = 23460$

Product of the two numbers $= 510\times92 = 46920$

$\text{HCF}\times\text{LCM} = 2\times23460$

$= 46920$

Hence, product of two numbers = HCF × LCM

(iii) 336 and 54

$336 = 2\times2\times2\times2\times3\times7$

$336 = 2^4\times3\times7$

$54 = 2\times3\times3\times3$

$54 = 2\times3^3$

$\text{HCF} = 2\times3 = 6$

$\text{LCM} = 2^4\times3^3\times7 = 3024$

Product of the two numbers $= 336\times54 = 18144$

$\text{HCF}\times\text{LCM} = 6\times3024 = 18144$

Hence, product of two numbers = HCF × LCM

Question 3:

Find the LCM and HCF of the following integers by applying the prime factorisation method.

(i) 12, 15 and 21 (ii) 17, 23 and 29 (iii) 8, 9 and 25

Answer 3:

(i) 12,15 and 21

$12 = 2^2\times3$

$15 = 3\times5$

$21 = 3\times7$

$\text{HCF} = 3$

$\text{LCM} = 2^2\times3\times5\times7 = 420$

(ii) 17,23 and 29

$17 = 1\times17$

$23 = 1\times23$

$29 = 1\times29$

$\text{HCF} = 1$

$\text{LCM} = 17\times23\times29 = 11339$

(iii) 8,9 and 25

$8 = 2 \times 2 \times 2$

$9 = 3 \times 3$

$25 = 5 \times 5$

$\text{HCF} = 1$

$\text{LCM} = 2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 5 = 1800$

Question 4:

Given that HCF (306, 657) = 9, find LCM (306, 657).

Answer 4:

$\text{HCF}(306, 657) = 9$

We know that, $\text{LCM} \times \text{HCF} = $ Product of two numbers

$\therefore \text{LCM} \times \text{HCF} = 306 \times 657$

$$\text{LCM} = \frac{306 \times 657}{\text{HCF}} = \frac{306 \times 657}{9}$$

$\text{LCM} = 22338$

Question 5:

Check whether 6^n can end with the digit 0 for any natural number n.

Answer 5:

If any number ends with the digit 0, it should be divisible by 10 or in other words, it will also be divisible by 2 and 5 as $10 = 2 \times 5$

Prime factorisation of $6^n = (2 \times 3)^n$

It can be observed that 5 is not in the prime factorisation of 6^n.

Hence, for any value of n, 6^n will not be divisible by 5.

Therefore, 6^n cannot end with the digit 0 for any natural number n.

Question 6:

Explain why $7 \times 11 \times 13 + 13$ and $7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 + 5$ are composite numbers.

Answer 6:

Numbers are of two types - prime and composite. Prime numbers can be divided by 1 and only itself, whereas composite numbers have factors other than 1 and itself.

It can be observed that

$7 \times 11 \times 13 + 13 = 13 \times (7 \times 11 + 1) = 13 \times (77 + 1)$

$= 13 \times 78$

$= 13 \times 13 \times 6$

The given expression has 6 and 13 as its factors. Therefore, it is a composite number.

$7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 + 5 = 5 \times (7 \times 6 \times 4 \times 3 \times 2 \times 1 + 1)$

$= 5 \times (1008 + 1)$

$= 5 \times 1009$

1009 cannot be factorised further. Therefore, the given expression has 5 and 1009 as its factors. Hence, it is a composite number.

Question 7:

There is a circular path around a sports field. Sonia takes 18 minutes to drive one round of the field, while Ravi takes 12 minutes for the same. Suppose they both start at the same point and at the same time, and go in the same direction. After how many minutes will they meet again at the starting point?

Answer 7:

It can be observed that Ravi takes lesser time than Sonia for completing 1 round of the circular path. As they are going in the same direction, they will meet again at the same time when Ravi will have completed 1 round of that circular path with respect to Sonia. And the total time taken for completing this 1 round of circular path will be the LCM of time taken by Sonia and Ravi for completing 1 round of circular path respectively i.e., LCM of 18 minutes and 12 minutes.

$18 = 2 \times 3 \times 3$

And, $12 = 2 \times 2 \times 3$

LCM of 12 and 18 $= 2 \times 2 \times 3 \times 3 = 36$

Therefore, Ravi and Sonia will meet together at the starting point after 36 minutes.

Mathematics

(Chapter – 1) (Real Numbers)

(Class X)

Exercise 1.3

Question 1:

Prove that $\sqrt{5}$ is irrational.

Answer 1:

Let $\sqrt{5}$ is a rational number.

Therefore, we can find two integers *a*, *b* ($b \neq 0$) such that $\sqrt{5} = \frac{a}{b}$ Let *a* and *b* have a common factor other than 1. Then we can divide them by the common factor, and assume that *a* and *b* are co-prime.

$$a = \sqrt{5}b$$

$$\Rightarrow a^2 = 5b^2$$

Therefore, a^2 is divisible by 5 and it can be said that *a* is divisible by 5.

Let $a = 5k$, where *k* is an integer

$$(5k)^2 = 5b^2$$

$$\Rightarrow 5k^2 = b^2$$

This means that b^2 is divisible by 5 and hence, *b* is divisible by 5.

This implies that *a* and *b* have 5 as a common factor.

And this is a contradiction to the fact that *a* and *b* are co-prime.

Hence, $\sqrt{5}$ *cannot* be expressed as $\frac{p}{q}$ or it can be said that $\sqrt{5}$ is irrational.

Question 2:

Prove that $3 + 2\sqrt{5}$ is irrational.

Answer 2:

Let $3 + 2\sqrt{5}$ is rational.

Therefore, we can find two co-prime integers *a*, *b* ($b \neq 0$) such that

$$3 + 2\sqrt{5} = \frac{a}{b}$$

$$\Rightarrow 2\sqrt{5} = \frac{a}{b} - 3$$

$$\Rightarrow \sqrt{5} = \frac{1}{2}\left(\frac{a}{b} - 3\right)$$

Since *a* and *b* are integers, $\frac{1}{2}\left(\frac{a}{b}-3\right)$ will also be rational and therefore, $\sqrt{5}$ is rational.

This contradicts the fact that $\sqrt{5}$ is irrational. Hence, our assumption that $3+2\sqrt{5}$ is rational is false. Therefore, $3+2\sqrt{5}$ is irrational.

Question 3:

Prove that the following are irrationals:

(i) $\frac{1}{\sqrt{2}}$ **(ii)** $7\sqrt{5}$ **(iii)** $6+\sqrt{2}$

Answer 3:

(i) $\frac{1}{\sqrt{2}}$

Let $\frac{1}{\sqrt{2}}$ is rational.

Therefore, we can find two co-prime integers *a*, *b* (*b* ≠ 0) such that

$$\frac{1}{\sqrt{2}}=\frac{a}{b}$$

Or

$$\sqrt{2}=\frac{b}{a}$$

$\frac{b}{a}$ is rational as *a* and *b* are integers.

Therefore, $\sqrt{2}$ is rational which contradicts to the fact that $\sqrt{2}$ is irrational.

Hence, our assumption is false and $\frac{1}{\sqrt{2}}$ is irrational.

(ii) $7\sqrt{5}$

Let $7\sqrt{5}$ is rational.

Therefore, we can find two co-prime integers *a*, *b* (*b* ≠ 0) such that

$$7\sqrt{5}=\frac{a}{b}$$

$$\Rightarrow \sqrt{5}=\frac{a}{7b}$$

$\frac{a}{7b}$ is rational as *a* and *b* are integers.

Therefore, $\sqrt{5}$ should be rational.

This contradicts the fact that $\sqrt{5}$ is irrational. Therefore, our assumption that $7\sqrt{5}$ is rational is false. Hence, $7\sqrt{5}$ is irrational.

(iii) $6+\sqrt{2}$

Let $6+\sqrt{2}$ be rational.

Therefore, we can find two co-prime integers *a*, *b* (*b* ≠ 0) such that

$$6+\sqrt{2}=\frac{a}{b}$$

$$\Rightarrow \sqrt{2}=\frac{a}{b}-6$$

Since *a* and *b* are integers, $\frac{a}{b}-6$ is also rational and hence, $\sqrt{2}$ should be rational. This contradicts the fact that $\sqrt{2}$ is irrational. Therefore, our assumption is false and hence, $6+\sqrt{2}$ is irrational.

Mathematics

(Chapter – 1) (Real Numbers)

(Class X)

Exercise 1.4

Question 1:

Without actually performing the long division, state whether the following rational numbers will have a terminating decimal expansion or a non-terminating repeating decimal expansion:

(i) $\frac{13}{3125}$ (ii) $\frac{17}{8}$ (iii) $\frac{64}{455}$ (iv) $\frac{15}{1600}$

(v) $\frac{29}{343}$ (vi) $\frac{23}{2^3 5^2}$ (vii) $\frac{129}{2^2 5^7 7^5}$ (viii) $\frac{6}{15}$

(ix) $\frac{35}{50}$ (x) $\frac{77}{210}$

Answer 1:

(i) $\frac{13}{3125}$

$3125 = 5^5$

The denominator is of the form 5^m.

Hence, the decimal expansion of $\frac{13}{3125}$ is terminating.

(ii) $\frac{17}{8}$

$8 = 2^3$

The denominator is of the form 2^m.

Hence, the decimal expansion of $\frac{17}{8}$ is terminating.

(iii) $\frac{64}{455}$

$455 = 5 \times 7 \times 13$

Since the denominator is not in the form $2^m \times 5^n$, and it also contains 7 and 13 as its factors, its decimal expansion will be non-terminating repeating.

(iv) $\frac{15}{1600}$

$1600 = 2^6 \times 5^2$

The denominator is of the form $2^m \times 5^n$.

Hence, the decimal expansion of $\frac{15}{1600}$ is terminating.

(v) $\frac{29}{343}$

$343 = 7^3$

Since the denominator is not in the form $2^m \times 5^n$, and it has 7 as its factor, the decimal expansion of $\frac{29}{343}$ is non-terminating repeating.

(vi) $\frac{23}{2^3 \times 5^2}$

The denominator is of the form $2^m \times 5^n$.

Hence, the decimal expansion of $\frac{23}{2^3 \times 5^2}$ is terminating.

(vii) $\frac{129}{2^2 \times 5^7 \times 7^5}$

Since the denominator is not of the form $2^m \times 5^n$, and it also has 7 as its factor, the decimal expansion of $\frac{129}{2^2 \times 5^7 \times 7^5}$ is non-terminating repeating.

(viii) $\frac{6}{15} = \frac{2 \times 3}{3 \times 5} = \frac{2}{5}$

The denominator is of the form 5^n.

Hence, the decimal expansion of $\frac{6}{15}$ is terminating.

(ix) $\frac{35}{50} = \frac{7 \times 5}{10 \times 5} = \frac{7}{10}$

$10 = 2 \times 5$

The denominator is of the form $2^m \times 5^n$.

Hence, the decimal expansion of $\frac{35}{50}$ is terminating.

(x) $\frac{77}{210} = \frac{11 \times 7}{30 \times 7} = \frac{11}{30}$

$30 = 2 \times 3 \times 5$

Since the denominator is not of the form $2^m \times 5^n$, and it also has 3 as its factors, the decimal expansion of $\frac{77}{210}$ is non-terminating repeating.

Question 2:

Write down the decimal expansions of those rational numbers in Question 1 above which have terminating decimal expansions.

Answer 2:

(i) $\frac{13}{3125} = 0.00416$

$$\begin{array}{r} 0.00416 \\ 3125\overline{)13.00000} \\ \underline{0} \\ 130 \\ \underline{0} \\ 1300 \\ \underline{0} \\ 13000 \\ \underline{12500} \\ 5000 \\ \underline{3125} \\ 18750 \\ \underline{18750} \\ \times \end{array}$$

(ii) $\frac{17}{8} = 2.125$

$$
\begin{array}{r}
2.125 \\
8\overline{)\;17} \\
16 \\ \hline
10 \\
8 \\ \hline
20 \\
16 \\ \hline
40 \\
40 \\ \hline
\times \\ \hline
\end{array}
$$

(iv) $\frac{15}{1600} = 0.009375$

$$
\begin{array}{r}
0.009375 \\
1600\overline{)15.000000} \\
0 \\ \hline
150 \\
0 \\ \hline
1500 \\
0 \\ \hline
15000 \\
14400 \\ \hline
6000 \\
4800 \\ \hline
12000 \\
11200 \\ \hline
8000 \\
8000 \\ \hline
\times \\ \hline
\end{array}
$$

(vi) $\frac{23}{2^3 \times 5^2} = \frac{23}{200} = 0.115$

$$\begin{array}{r} 0.115 \\ 200\overline{)23.000} \\ 0 \\ \hline 230 \\ 200 \\ \hline 300 \\ 200 \\ \hline 1000 \\ 1000 \\ \hline \times \\ \hline \end{array}$$

(viii) $\frac{6}{15} = \frac{2 \times 3}{3 \times 5} = \frac{2}{5} = 0.4$

$$\begin{array}{r} 0.4 \\ 5\overline{)2.0} \\ 0 \\ \hline 20 \\ 20 \\ \hline \times \\ \hline \end{array}$$

(ix) $\frac{35}{50} = 0.7$

$$\begin{array}{r} 0.7 \\ 50\overline{)35.0} \\ 0 \\ \hline 350 \\ 350 \\ \hline \times \\ \hline \end{array}$$

Question 3:

The following real numbers have decimal expansions as given below. In each case, decide whether they are rational or not. If they are rational, and of the form $\frac{p}{q}$, what can you say about the prime factor of q?

(i) 43.123456789 (ii) 0.120120012000120000… (iii) $43.\overline{123456789}$

Answer 3:

(i) 43.123456789

Since this number has a terminating decimal expansion, it is a rational number of the form $\frac{p}{q}$ and q is of the form $2^m \times 5^n$

i.e., the prime factors of q will be either 2 or 5 or both.

(ii) 0.120120012000120000 …

The decimal expansion is neither terminating nor recurring. Therefore, the given number is an irrational number.

(iii) $43.\overline{123456789}$

Since the decimal expansion is non-terminating recurring, the given number is a rational number of the form $\frac{p}{q}$ and q is not of the form $2^m \times 5^n$ i.e., the prime factors of q will also have a factor other than 2 or 5.

Mathematics

(Chapter – 2) (Polynomials)

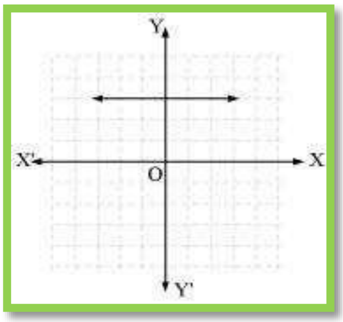
(Class – X)

Exercise 2.1

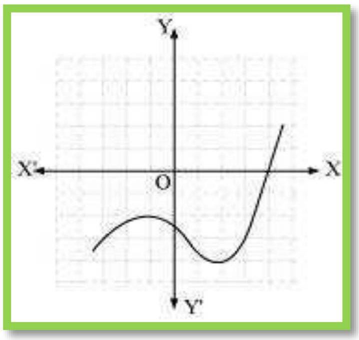
Question 1:

The graphs of $y = p(x)$ are given in following figure, for some polynomials $p(x)$. Find the number of zeroes of $p(x)$, in each case.

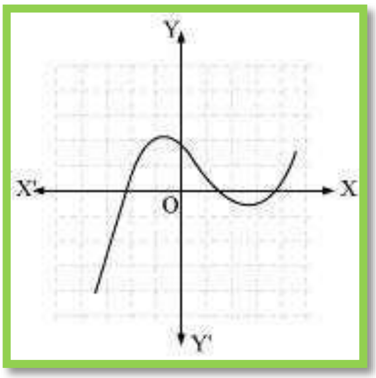
(i)



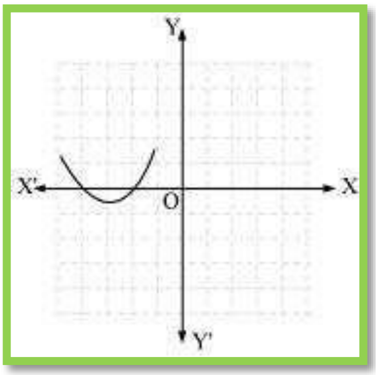
(ii)



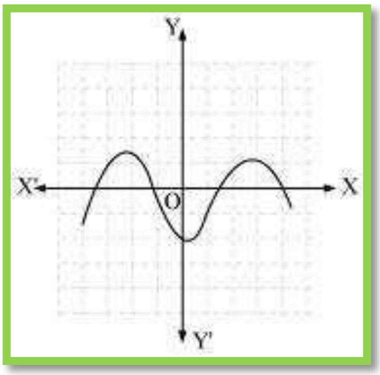
(iii)



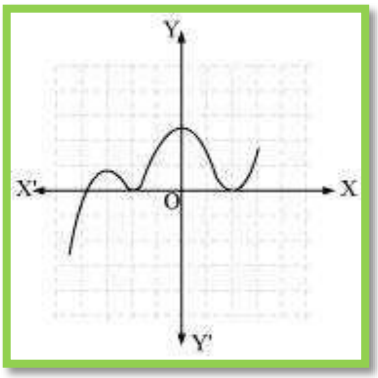
(iv)



(v)



(v)



Answer 1:

(i) The number of zeroes is 0 as the graph does not cut the x-axis at any point.

(ii) The number of zeroes is 1 as the graph intersects the x-axis at only 1 point.

(iii) The number of zeroes is 3 as the graph intersects the x-axis at 3 points.

(iv) The number of zeroes is 2 as the graph intersects the x-axis at 2 points.

(v) The number of zeroes is 4 as the graph intersects the x-axis at 4 points.

(vi) The number of zeroes is 3 as the graph intersects the x-axis at 3 points.

Mathematics

(Chapter – 2) (Polynomials)
(Class X)

Exercise 2.2

Question 1:

Find the zeroes of the following quadratic polynomials and verify the relationship between the zeroes and the coefficients.

$(\text{i})\, x^2-2x-8$ $(\text{ii})\, 4s^2-4s+1$ $(\text{iii})\, 6x^2-3-7x$

$(\text{iv})\, 4u^2+8u$ $(\text{v})\, t^2-15$ $(\text{vi})\, 3x^2-x-4$

Answer 1:

(i) $x^2-2x-8=(x-4)(x+2)$

The value of x^2-2x-8 is zero when $x-4=0$ or $x+2=0$, i.e., when $x=4$ or $x=-2$

Therefore, the zeroes of x^2-2x-8 are 4 and −2.

Sum of zeroes = $4-2=2=\frac{-(-2)}{1}=\frac{-(\text{Coefficient of } x)}{\text{Coefficient of } x^2}$

Product of zeroes $=4\times(-2)=-8=\frac{(-8)}{1}=\frac{\text{Constant term}}{\text{Coefficient of } x^2}$

(ii) $4s^2-4s+1=(2s-1)^2$

The value of $4s^2-4s+1$ is zero when $2s-1=0$, i.e., $s=\frac{1}{2}$ Therefore,

the zeroes of $4s^2-4s+1$ are ½ and ½ .

Sum of zeroes = $\frac{1}{2}+\frac{1}{2}=1=\frac{-(-4)}{4}=\frac{-(\text{Coefficient of } s)}{(\text{Coefficient of } s^2)}$

Product of zeroes $=\frac{1}{2}\times\frac{1}{2}=\frac{1}{4}=\frac{\text{Constant term}}{\text{Coefficient of } s^2}$

(iii) $6x^2-3-7x=6x^2-7x-3=(3x+1)(2x-3)$

The value of $6x^2-3-7x$ is zero when $3x+1=0$ or $2x-3=0$, i.e.,

$x = \frac{-1}{3}$ or $x = \frac{3}{2}$

Therefore, the zeroes of $6x^2 - 3 - 7x$ are $\frac{-1}{3}$ and $\frac{3}{2}$.

Sum of zeroes = $\frac{-1}{3} + \frac{3}{2} = \frac{7}{6} = \frac{-(-7)}{6} = \frac{-(\text{Coefficient of } x)}{\text{Coefficient of } x^2}$

Product of zeroes = $\frac{-1}{3} \times \frac{3}{2} = \frac{-1}{2} = \frac{-3}{6} = \frac{\text{Constant term}}{\text{Coefficient of } x^2}$

(iv) $4u^2 + 8u = 4u^2 + 8u + 0$

$= 4u(u+2)$

The value of $4u^2 + 8u$ is zero when $4u = 0$ or $u + 2 = 0$, i.e.,
$u = 0$ or $u = -2$

Therefore, the zeroes of $4u^2 + 8u$ are 0 and -2.

Sum of zeroes = $0 + (-2) = -2 = \frac{-(8)}{4} = \frac{-(\text{Coefficient of } u)}{\text{Coefficient of } u^2}$

Product of zeroes = $0 \times (-2) = 0 = \frac{0}{4} = \frac{\text{Constant term}}{\text{Coefficient of } u^2}$

(v) $t^2 - 15$

$= t^2 - 0t - 15$

$= (t - \sqrt{15})(t + \sqrt{15})$

The value of $t^2 - 15$ is zero when $t - \sqrt{15} = 0$ or $t + \sqrt{15} = 0$, i.e., when
$t = \sqrt{15}$ or $t = -\sqrt{15}$

Therefore, the zeroes of $t^2 - 15$ are $\sqrt{15}$ and $-\sqrt{15}$.

Sum of zeroes = $\sqrt{15}+\left(-\sqrt{15}\right)=0=\frac{-0}{1}=\frac{-(\text{Coefficient of } t)}{\left(\text{Coefficient of } t^2\right)}$

Product of zeroes = $\left(\sqrt{15}\right)\left(-\sqrt{15}\right)=-15=\frac{-15}{1}=\frac{\text{Constant term}}{\text{Coefficient of } x^2}$

(vi) $3x^2-x-4$

$=(3x-4)(x+1)$

The value of $3x^2 - x - 4$ is zero when $3x - 4 = 0$ or $x + 1 = 0$, i.e.,

when $x=\frac{4}{3}$ or $x = -1$

Therefore, the zeroes of $3x^2 - x - 4$ are 4/3 and −1.

Sum of zeroes = $\frac{4}{3}+(-1)=\frac{1}{3}=\frac{-(-1)}{3}=\frac{-(\text{Coefficient of } x)}{\text{Coefficient of } x^2}$

Product of zeroes $=\frac{4}{3}(-1)=\frac{-4}{3}=\frac{\text{Constant term}}{\text{Coefficient of } x^2}$

Question 2:

Find a quadratic polynomial each with the given numbers as the sum and product of its zeroes respectively.

(i) $\frac{1}{4},-1$ (ii) $\sqrt{2},\frac{1}{3}$ (iii) $0,\sqrt{5}$

(iv) $1,1$ (v) $-\frac{1}{4},\frac{1}{4}$ (vi) $4,1$

Answer 2:

(i) $\frac{1}{4},-1$

Let the polynomial be $ax^2 + bx + c$ and its zeroes be α and β.

$$\alpha + \beta = \frac{1}{4} = \frac{-b}{a}$$
$$\alpha\beta = -1 = \frac{-4}{4} = \frac{c}{a}$$
$$\text{If } a = 4, \text{ then } b = -1, \ c = -4$$

Therefore, the quadratic polynomial is $4x^2 - x - 4$.

(ii) $\sqrt{2}, \frac{1}{3}$

Let the polynomial be $ax^2 + bx + c$ and its zeroes be α and β.

$$\alpha + \beta = \sqrt{2} = \frac{3\sqrt{2}}{3} = \frac{-b}{a}$$
$$\alpha\beta = \frac{1}{3} = \frac{c}{a}$$
$$\text{If } a = 3, \text{ then } b = -3\sqrt{2}, \ c = 1$$

Therefore, the quadratic polynomial is $3x^2 - 3\sqrt{2}x + 1$.

(iii) $0, \sqrt{5}$

Let the polynomial be $ax^2 + bx + c$ and its zeroes be α and β.

$$\alpha + \beta = 0 = \frac{0}{1} = \frac{-b}{a}$$
$$\alpha \times \beta = \sqrt{5} = \frac{\sqrt{5}}{1} = \frac{c}{a}$$
$$\text{If } a = 1, \text{ then } b = 0, \ c = \sqrt{5}$$

Therefore, the quadratic polynomial is $x^2 + \sqrt{5}$.

(iv) 1, 1

Let the polynomial be $ax^2 + bx + c$ and its zeroes be α and β.

$$\alpha+\beta=1=\frac{1}{1}=\frac{-b}{a}$$
$$\alpha\times\beta=1=\frac{1}{1}=\frac{c}{a}$$
$$\text{If } a=1, \text{ then } b=-1,\ c=1$$

Therefore, the quadratic polynomial is x^2-x+1.

$(\text{v})\quad -\frac{1}{4},\frac{1}{4}$

Let the polynomial be $ax^2 + bx + c$ and its zeroes be α and β.

$$\alpha+\beta=\frac{-1}{4}=\frac{-b}{a}$$
$$\alpha\times\beta=\frac{1}{4}=\frac{c}{a}$$
$$\text{If } a=4, \text{ then } b=1,\ c=1$$

Therefore, the quadratic polynomial is $4x^2+x+1$.

$(\text{vi})\quad 4,\ 1$

Let the polynomial be $ax^2 + bx + c$ and its zeroes be α and β.

$$\alpha+\beta=4=\frac{4}{1}=\frac{-b}{a}$$
$$\alpha\times\beta=1=\frac{1}{1}=\frac{c}{a}$$
$$\text{If } a=1, \text{ then } b=-4,\ c=1$$

Therefore, the quadratic polynomial is x^2-4x+1.

Mathematics

(Chapter – 2) (Polynomials)

(Class – X)

Exercise 2.3

Question 1:

Divide the polynomial $p(x)$ by the polynomial $g(x)$ and find the quotient and remainder in each of the following:

(i) $p(x) = x^3 - 3x^2 + 5x - 3, \quad g(x) = x^2 - 2$

(ii) $p(x) = x^4 - 3x^2 + 4x + 5, \quad g(x) = x^2 + 1 - x$

(iii) $p(x) = x^4 - 5x + 6, \quad g(x) = 2 - x^2$

Answer 1:

(i) $p(x) = x^3 - 3x^2 + 5x - 3$

$q(x) = x^2 - 2$

$$
\begin{array}{r|l}
 & \quad x - 3 \\
\hline
x^2 - 2 & x^3 - 3x^2 + 5x - 3 \\
 & x^3 \qquad\quad - 2x \\
 & - \qquad\quad + \\
\hline
 & \quad -3x^2 + 7x - 3 \\
 & \quad -3x^2 \qquad + 6 \\
 & \quad + \qquad\quad - \\
\hline
 & \qquad\qquad 7x - 9 \\
\hline
\end{array}
$$

Quotient = $x - 3$

Remainder = $7x - 9$

(ii) $p(x) = x^4 - 3x^2 + 4x + 5 = x^4 + 0.x^3 - 3x^2 + 4x + 5$

$q(x) = x^2 + 1 - x = x^2 - x + 1$

$$
\begin{array}{r|l}
 & x^2 + x - 3 \\
\hline
x^2 - x + 1 & x^4 + 0.x^3 - 3x^2 + 4x + 5 \\
 & x^4 - x^3 + x^2 \\
 & - \quad + \quad - \\
\cline{2-2}
 & x^3 - 4x^2 + 4x + 5 \\
 & x^3 - x^2 + x \\
 & - \quad + \quad - \\
\cline{2-2}
 & -3x^2 + 3x + 5 \\
 & -3x^2 + 3x - 3 \\
 & + \quad - \quad + \\
\cline{2-2}
 & 8 \\
\cline{2-2}
\end{array}
$$

Quotient = $x^2 + x - 3$

Remainder = 8

(iii) $p(x) = x^4 - 5x + 6 = x^4 + 0.x^2 - 5x + 6$

$q(x) = 2 - x^2 = -x^2 + 2$

$$\begin{array}{r|l} & \quad -x^2 - 2 \\ \hline -x^2 + 2 & x^4 + 0.x^2 - 5x + 6 \\ & x^4 - 2x^2 \\ & - \quad + \\ \hline & \quad 2x^2 - 5x + 6 \\ & \quad 2x^2 \qquad - 4 \\ & \quad - \qquad\quad + \\ \hline & \qquad -5x + 10 \\ \hline \end{array}$$

Quotient = $-x^2 - 2$

Remainder = $-5x + 10$

Question 2:

Check whether the first polynomial is a factor of the second polynomial by dividing the second polynomial by the first polynomial:

(i) $t^2 - 3, 2t^4 + 3t^3 - 2t^2 - 9t - 12$

(ii) $x^2 + 3x + 1, 3x^4 + 5x^3 - 7x^2 + 2x + 2$

(iii) $x^3 - 3x + 1, x^5 - 4x^3 + x^2 + 3x + 1$

Answer 2:

(i) $t^2 - 3,\ 2t^4 + 3t^3 - 2t^2 - 9t - 12$

t^2-3 = $t^2+0.t-3$

$$\begin{array}{r|l} & 2t^2+3t+4 \\ \hline t^2+0.t-3 & 2t^4+3t^3-2t^2-9t-12 \\ & 2t^4+0.t^3-6t^2 \\ & - \quad - \quad + \\ \hline & \quad 3t^3+4t^2-9t-12 \\ & \quad 3t^3+0.t^2-9t \\ & \quad - \quad - \quad + \\ \hline & \quad\quad 4t^2+0.t-12 \\ & \quad\quad 4t^2+0.t-12 \\ & \quad\quad - \quad - \quad + \\ \hline & \quad\quad\quad 0 \\ \hline \end{array}$$

Since the remainder is 0,

Hence, t^2-3 is a factor of $2t^4+3t^3-2t^2-9t-12$.

(ii) $x^2+3x+1,\ 3x^4+5x^3-7x^2+2x+2$

$$\begin{array}{r|l} & 3x^2-4x+2 \\ \hline x^2+3x+1 & 3x^4+5x^3-7x^2+2x+2 \\ & 3x^4+9x^3+3x^2 \\ & - \quad - \quad - \\ \hline & \quad -4x^3-10x^2+2x+2 \\ & \quad -4x^3-12x^2-4x \\ & \quad + \quad + \quad + \\ \hline & \quad\quad 2x^2+6x+2 \\ & \quad\quad 2x^2+6x+2 \\ \hline & \quad\quad\quad 0 \\ \hline \end{array}$$

Since the remainder is 0,

Hence, x^2+3x+1 is a factor of $3x^4+5x^3-7x^2+2x+2$.

(iii) $x^3-3x+1,\ x^5-4x^3+x^2+3x+1$

$$\begin{array}{r|l} & x^2-1 \\ \hline x^3-3x+1 & x^5-4x^3+x^2+3x+1 \\ & x^5-3x^3+x^2 \\ & -\quad+\quad- \\ \hline & -x^3 \qquad +3x+1 \\ & -x^3 \qquad +3x-1 \\ & + \qquad\quad -\quad + \\ \hline & \qquad\qquad 2 \\ \hline \end{array}$$

Since the remainder $\neq 0$,

Hence, x^3-3x+1 is not a factor of $x^5-4x^3+x^2+3x+1$.

Question 3:

Obtain all other zeroes of $3x^4+6x^3-2x^2-10x-5$, if two of its zeroes are $\sqrt{\frac{5}{3}}$ and $-\sqrt{\frac{5}{3}}$.

Answer 3:

$$p(x)=3x^4+6x^3-2x^2-10x-5$$

Since the two zeroes are $\sqrt{\frac{5}{3}}$ and $-\sqrt{\frac{5}{3}}$

$\therefore \left(x-\sqrt{\frac{5}{3}}\right)\left(x+\sqrt{\frac{5}{3}}\right)=\left(x^2-\frac{5}{3}\right)$ is a factor of $3x^4+6x^3-2x^2-10x-5$

Therefore, we divide the given polynomial by $x^2-\frac{5}{3}$

$$
\begin{array}{r|l}
\multicolumn{1}{r}{} & 3x^2+6x+3 \\
\cline{2-2}
x^2+0.x-\frac{5}{3} & 3x^4+6x^3-2x^2-10x-5 \\
 & 3x^4+0x^3-5x^2 \\
 & -\quad -\quad + \\
\cline{2-2}
 & \qquad 6x^3+3x^2-10x-5 \\
 & \qquad 6x^3+0x^2-10x \\
 & \qquad -\quad -\quad + \\
\cline{2-2}
 & \qquad\qquad 3x^2+0x-5 \\
 & \qquad\qquad 3x^2+0x-5 \\
 & \qquad\qquad -\quad -\quad + \\
\cline{2-2}
 & \qquad\qquad\qquad 0 \\
\cline{2-2}
\end{array}
$$

$$
\begin{aligned}
3x^4+6x^3-2x^2-10x-5 &= \left(x^2-\frac{5}{3}\right)\left(3x^2+6x+3\right) \\
&= 3\left(x^2-\frac{5}{3}\right)\left(x^2+2x+1\right)
\end{aligned}
$$

We factorize x^2+2x+1

$=(x+1)^2$

Therefore, its zero is given by $x + 1 = 0$ or $x = -1$

As it has the term $(x+1)^2$, therefore, there will be 2 zeroes at $x = -1$.

Hence, the zeroes of the given polynomial are $\sqrt{\frac{5}{3}}, -\sqrt{\frac{5}{3}}$ -1 and -1.

Question 4:

On dividing x^3-3x^2+x+2 by a polynomial $g(x)$, the quotient and remainder were $x - 2$ and $- 2x + 4$, respectively. Find $g(x)$.

Answer 4:

$p(x)=x^3-3x^2+x+2 \qquad \text{(Dividend)}$

$g(x)$ = ? (Divisor)

Quotient = $(x - 2)$

Remainder = $(-2x + 4)$

Dividend = Divisor × Quotient + Remainder

$$x^3 - 3x^2 + x + 2 = g(x) \times (x-2) + (-2x+4)$$

$$x^3 - 3x^2 + x + 2 + 2x - 4 = g(x)(x-2)$$

$$x^3 - 3x^2 + 3x - 2 = g(x)(x-2)$$

$g(x)$ is the quotient when we divide $(x^3 - 3x^2 + 3x - 2)$ by $(x-2)$

$$\begin{array}{r|l} & x^2 - x + 1 \\ \hline x-2 & x^3 - 3x^2 + 3x - 2 \\ & x^3 - 2x^2 \\ & - \quad + \\ \hline & -x^2 + 3x - 2 \\ & -x^2 + 2x \\ & + \quad - \\ \hline & x - 2 \\ & x - 2 \\ & - \quad + \\ \hline & 0 \\ \hline \end{array}$$

$$\therefore g(x) = (x^2 - x + 1)$$

Question 5:

Give examples of polynomial $p(x)$, $g(x)$, $q(x)$ and $r(x)$, which satisfy the division algorithm and

(i) deg $p(x)$ = deg $q(x)$

(ii) deg $q(x)$ = deg $r(x)$

(iii) deg $r(x) = 0$

Answer 5:

According to the division algorithm, if $p(x)$ and $g(x)$ are two polynomials with $g(x) \neq 0$, then we can find polynomials $q(x)$ and $r(x)$ such that $p(x) = g(x) \times q(x) + r(x)$,

where $r(x) = 0$ or degree of $r(x) <$ degree of $g(x)$

Degree of a polynomial is the highest power of the variable in the polynomial.

(i) deg $p(x)$ = deg $q(x)$

Degree of quotient will be equal to degree of dividend when divisor is constant (i.e., when any polynomial is divided by a constant).

Let us assume the division of $6x^2 + 2x + 2$ by 2.

Here, $p(x) = 6x^2 + 2x + 2$

$g(x) = 2$

$q(x) = 3x^2 + x + 1$and $r(x) = 0$

Degree of $p(x)$ and $q(x)$ is the same i.e., 2.

Checking for division algorithm, $p(x) = g(x) \times q(x) + r(x)$

$6x^2 + 2x + 2 = (2)\ (3x^2 + x + 1) + 0$

Thus, the division algorithm is satisfied.

(ii) deg $q(x)$ = deg $r(x)$

Let us assume the division of $x^3 + x$ by x^2,

Here, $p(x) = x^3 + x$ $g(x) = x^2$ $q(x) = x$ and $r(x) = x$

Clearly, the degree of $q(x)$ and $r(x)$ is the same i.e., 1. Checking for division algorithm, $p(x) = g(x) \times q(x) + r(x)$

$x^3 + x = (x^2) \times x + x$ $x^3 + x = x^3 + x$

Thus, the division algorithm is satisfied.

(iii)deg $r(x) = 0$

Degree of remainder will be 0 when remainder comes to a constant.

Let us assume the division of $x^3 + 1$by x^2.

Here, $p(x) = x^3 + 1$ $g(x) = x^2$ $q(x) = x$ and $r(x) = 1$

Clearly, the degree of $r(x)$ is 0. Checking for division algorithm,

$p(x) = g(x) \times q(x) + r(x)$ $x^3 + 1 = (x^2) \times x + 1$ $x^3 + 1 = x^3 + 1$

Thus, the division algorithm is satisfied.

Mathematics

(Chapter – 2) (Polynomials)

(Class – X)

Exercise 2.4

Question 1:

Verify that the numbers given alongside of the cubic polynomials below are their zeroes. Also verify the relationship between the zeroes and the coefficients in each case:

(i) $2x^3 + x^2 - 5x + 2; \quad \frac{1}{2}, 1, -2$

(ii) $x^3 - 4x^2 + 5x - 2; \quad 2, 1, 1$

Answer 1:

(i) $p(x) = 2x^3 + x^2 - 5x + 2.$

Zeroes for this polynomial are $\frac{1}{2}, 1, -2$

$$p\left(\frac{1}{2}\right) = 2\left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^2 - 5\left(\frac{1}{2}\right) + 2$$
$$= \frac{1}{4} + \frac{1}{4} - \frac{5}{2} + 2$$
$$= 0$$
$$p(1) = 2 \times 1^3 + 1^2 - 5 \times 1 + 2$$
$$= 0$$
$$p(-2) = 2(-2)^3 + (-2)^2 - 5(-2) + 2$$
$$= -16 + 4 + 10 + 2 = 0$$

Therefore, ½ , 1, and −2 are the zeroes of the given polynomial.
Comparing the given polynomial with $ax^3 + bx^2 + cx + d$,

we obtain $a = 2$, $b = 1$, $c = -5$, $d = 2$

We can take $\alpha = \frac{1}{2}, \beta = 1, \gamma = -2$

$\alpha + \beta + \gamma = \frac{1}{2} + 1 + (-2) = -\frac{1}{2} = \frac{-b}{a}$

$\alpha\beta + \beta\gamma + \alpha\gamma = \frac{1}{2} \times 1 + 1(-2) + \frac{1}{2}(-2) = \frac{-5}{2} = \frac{c}{a}$

$\alpha\beta\gamma = \frac{1}{2} \times 1 \times (-2) = \frac{-1}{1} = \frac{-(2)}{2} = \frac{-d}{a}$

Therefore, the relationship between the zeroes and the coefficients is verified.

(ii) $p(x) = x^3 - 4x^2 + 5x - 2$

Zeroes for this polynomial are 2, 1, 1.

$p(2) = 2^3 - 4(2^2) + 5(2) - 2$

$= 8 - 16 + 10 - 2 = 0$

$p(1) = 1^3 - 4(1)^2 + 5(1) - 2$

$= 1 - 4 + 5 - 2 = 0$

Therefore, 2, 1, 1 are the zeroes of the given polynomial.

Comparing the given polynomial with $ax^3 + bx^2 + cx + d$,

we obtain *a* = 1, *b* = −4, *c* = 5, *d* = −2.

Verification of the relationship between zeroes and coefficient of the given polynomial

$\text{Sum of zeroes} = 2 + 1 + 1 = 4 = \frac{-(-4)}{1} = \frac{-b}{a}$

Multiplication of zeroes taking two at a time

= (2)(1) + (1)(1) + (2)(1) =2 + 1 + 2 = 5 $= \frac{(5)}{1} = \frac{c}{a}$

Multiplication of zeroes = 2 × 1 × 1 = 2 $= \frac{-(-2)}{1} = \frac{-d}{a}$

Hence, the relationship between the zeroes and the coefficients is verified.

Question 2:

Find a cubic polynomial with the sum, sum of the product of its zeroes taken two at a time, and the product of its zeroes as 2, − 7, − 14 respectively.

Answer 2:

Let the polynomial be ax^3+bx^2+cx+d and the zeroes be $\alpha, \beta, \text{ and } \gamma$.

It is given that

$$\alpha+\beta+\gamma=\frac{2}{1}=\frac{-b}{a}$$

$$\alpha\beta+\beta\gamma+\alpha\gamma=\frac{-7}{1}=\frac{c}{a}$$

$$\alpha\beta\gamma=\frac{-14}{1}=\frac{-d}{a}$$

If $a = 1$, then $b = -2$, $c = -7$, $d = 14$

Hence, the polynomial is $x^3-2x^2-7x+14$.

Question 3:

If the zeroes of polynomial , x^3-3x^2+x+1 are $a-b, a, a+b$ find a and b.

Answer 3:

$$p(x)=x^3-3x^2+x+1$$

Zeroes are $a - b$, $a + a + b$

Comparing the given polynomial with px^3+qx^2+rx+t , we obtain

$p = 1$, $q = -3$, $r = 1$, $t = 1$

$$\text{Sum of zeroes} = a - b + a + a + b$$
$$\frac{-q}{p} = 3a$$
$$\frac{-(-3)}{1} = 3a$$
$$3 = 3a$$
$$a = 1$$

The zeroes are $1-b, 1, 1+b$

$$\text{Multiplication of zeroes} = 1(1-b)(1+b)$$
$$\frac{-t}{p} = 1 - b^2$$
$$\frac{-1}{1} = 1 - b^2$$
$$1 - b^2 = -1$$
$$1 + 1 = b^2$$
$$b = \pm\sqrt{2}$$

Hence, $a = 1$ and $b = \sqrt{2}$ or $-\sqrt{2}$

Question 4:

]It two zeroes of the polynomial , $x^4 - 6x^3 - 26x^2 + 138x - 35$ are $2 \pm \sqrt{3}$ find other zeroes.

Answer 4:

Given $2 + \sqrt{3}$ and $2 - \sqrt{3}$ are zeroes of the given polynomial.

So, $(2 + \sqrt{3})(2 - \sqrt{3})$ is a factor of polynomial.

Therefore, $[x - (2 + \sqrt{3})][x - (2 - \sqrt{3})] = x^2 + 4 - 4x - 3$

$= x^2 - 4x + 1$ is a factor of the given polynomial

For finding the remaining zeroes of the given polynomial, we will find

$$
\begin{array}{r|l}
 & x^2-2x-35 \\
\hline
x^2-4x+1 & x^4-6x^3-26x^2+138x-35 \\
 & x^4-4x^3+\ \ x^2 \\
 & -\quad +\quad - \\
\hline
 & \quad -2x^3-27x^2+138x-35 \\
 & \quad -2x^3+\ 8x^2-\ \ 2x \\
 & \quad +\quad -\quad\quad + \\
\hline
 & \quad\quad -35x^2+140x-35 \\
 & \quad\quad -35x^2+140x-35 \\
 & \quad\quad +\quad\quad -\quad\quad + \\
\hline
 & \quad\quad\quad\quad 0
\end{array}
$$

Clearly, $x^4-6x^3-26x^2+138x-35 = \left(x^2-4x+1\right)\left(x^2-2x-35\right)$

$\left(x^2-2x-35\right)$ is also a factor of the given

It can be observed that polynomial $\left(x^2-2x-35\right) = (x-7)(x+5)$

Therefore, the value of the polynomial is also zero when $x-7=0$ or $x+5=0$

Or x = 7 or −5

Hence, 7 and −5 are also zeroes of this polynomial.

Question 5:

If the polynomial $x^4-6x^3+16x^2-25x+10$ is divided by another polynomial, x^2-2x+k the remainder comes out to be $x + a$, find k and a.

Answer 5:

By division algorithm,

Dividend = Divisor × Quotient + Remainder

Dividend − Remainder = Divisor × Quotient

$x^4 - 6x^3 + 16x^2 - 25x + 10 - x - a = x^4 - 6x^3 + 16x^2 - 26x + 10 - a$ will be divisible by $x^2 - 2x + k$.

Let us divide $x^4 - 6x^3 + 16x^2 - 26x + 10 - a$ by $x^2 - 2x + k$

$$
\begin{array}{r|l}
 & x^2 - 4x + (8-k) \\
\hline
x^2 - 2x + k & x^4 - 6x^3 + 16x^2 - 26x + 10 - a \\
 & x^4 - 2x^3 + kx^2 \\
 & - \quad + \quad - \\
\hline
 & -4x^3 + (16-k)x^2 - 26x \\
 & -4x^3 + 8x^2 - 4kx \\
 & + \quad - \quad + \\
\hline
 & (8-k)x^2 - (26-4k)x + 10 - a \\
 & (8-k)x^2 - (16-2k)x + (8k - k^2) \\
 & - \quad + \quad - \\
\hline
 & (-10+2k)x + (10 - a - 8k + k^2)
\end{array}
$$

It can be observed that $(-10+2k)x + (10 - a - 8k + k^2)$ Will be 0.

Therefore, $(-10+2k)$ = 0 and $(10 - a - 8k + k^2)$ = 0

For $(-10+2k)$ = 0, 2 k =10 And thus, k = 5

For $(10 - a - 8k + k^2)$ = 0

$10 - a - 8 \times 5 + 25 = 0$

$10 - a - 40 + 25 = 0$

$-5 - a = 0$

Therefore, $a = -5$

Hence, $k = 5$ and $a = -5$

Mathematics

(Chapter – 3) (Linear equations in two variables)

(Class – X)

Exercise 3.1

Question 1:

Aftab tells his daughter, "Seven years ago, I was seven times as old as you were then. Also, three years from now, I shall be three times as old as you will be." (Isn't this interesting?) Represent this situation algebraically and graphically.

Answer 1:

Let the present age of Aftab be x.

And, present age of his daughter = y

Seven years ago,

Age of Aftab = $x - 7$

Age of his daughter = $y - 7$

According to the question,

$$(x - 7) = 7(y - 7)$$
$$\Rightarrow \quad x - 7 = 7y - 49$$
$$\Rightarrow \quad x - 7y = -42 \quad \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

Three years hence,

Age of Aftab = $x + 3$ Age of

his daughter = $y + 3$

According to the question,

$$(x+3) = 3(y+3)$$
$$x+3 = 3y+9$$
$$x-3y = 6 \qquad (2)$$

Therefore, the algebraic representation is

$$x-7y=-42$$
$$x-3y=6$$

For $x-7y=-42$

$$x=-42+7y$$

The solution table is

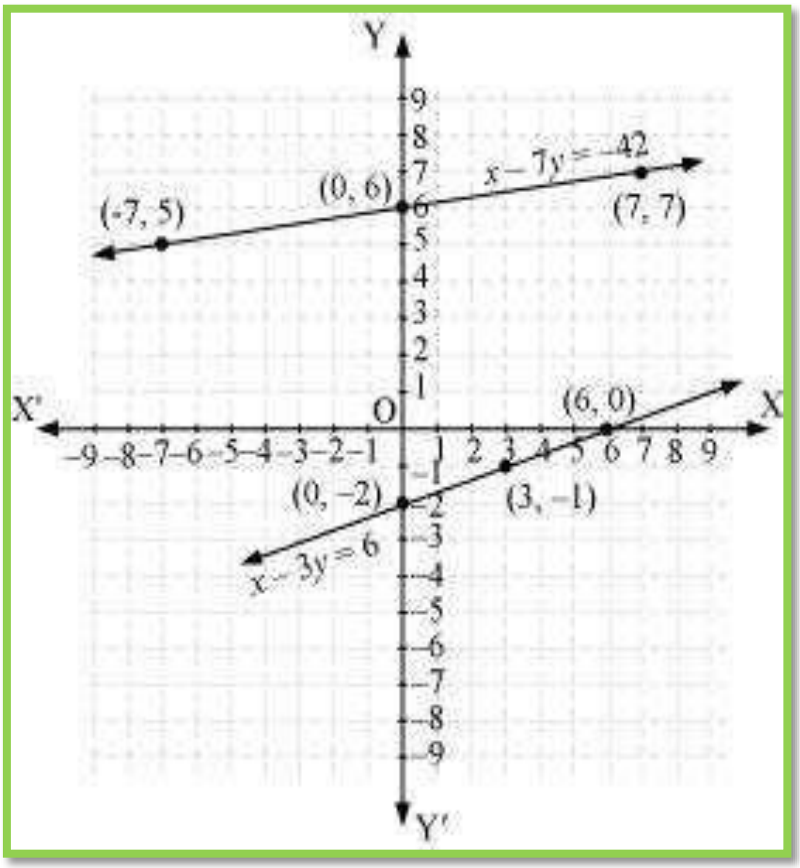
x	− 7	0	7
y	5	6	7

For x − 3 y = 6 or $x = 6 + 3y$

The solution table is

x	6	3	0
y	0	− 1	− 2

The graphical representation is as follows.



Question 2:

The coach of a cricket team buys 3 bats and 6 balls for Rs 3900. Later, she buys another bat and 2 more balls of the same kind for Rs 1300. Represent this situation algebraically and geometrically.

Answer 1:

Let the cost of a bat be Rs *x*.

And, cost of a ball = Rs *y*

According to the question, the algebraic representation is

$$3x + 6y = 3900$$
$$x + 2y = 1300$$

For $3x + 6y = 3900$

$$x = \frac{3900 - 6y}{3}$$

,

The solution table is

x	300	100	− 100
y	500	600	700

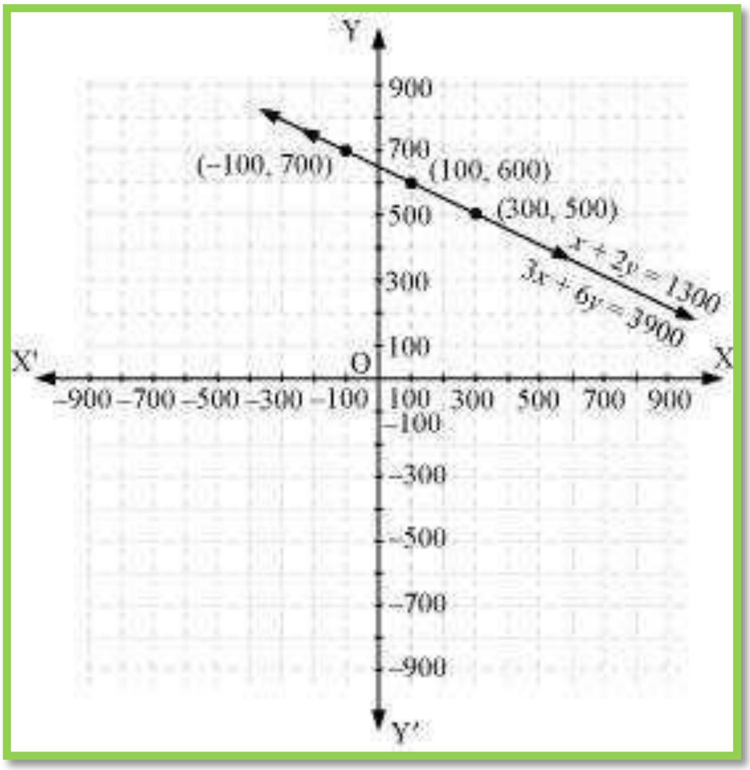
For, $x + 2y = 1300$

$$x = 1300 - 2y$$

The solution table is

x	300	100	− 100
y	500	600	700

The graphical representation is as follows.



Question 3:

The cost of 2 kg of apples and 1 kg of grapes on a day was found to be Rs 160. After a month, the cost of 4 kg of apples and 2 kg of grapes is Rs 300. Represent the situation algebraically and geometrically.

Answer 3:

Let the cost of 1 kg of apples be Rs *x*.

And, cost of 1 kg of grapes = Rs *y*

According to the question, the algebraic representation is

$$2x + y = 160$$
$$4x + 2y = 300$$

For $2x + y = 160$

$$y = 160 - 2x$$

The solution table is

x	50	60	70
y	60	40	20

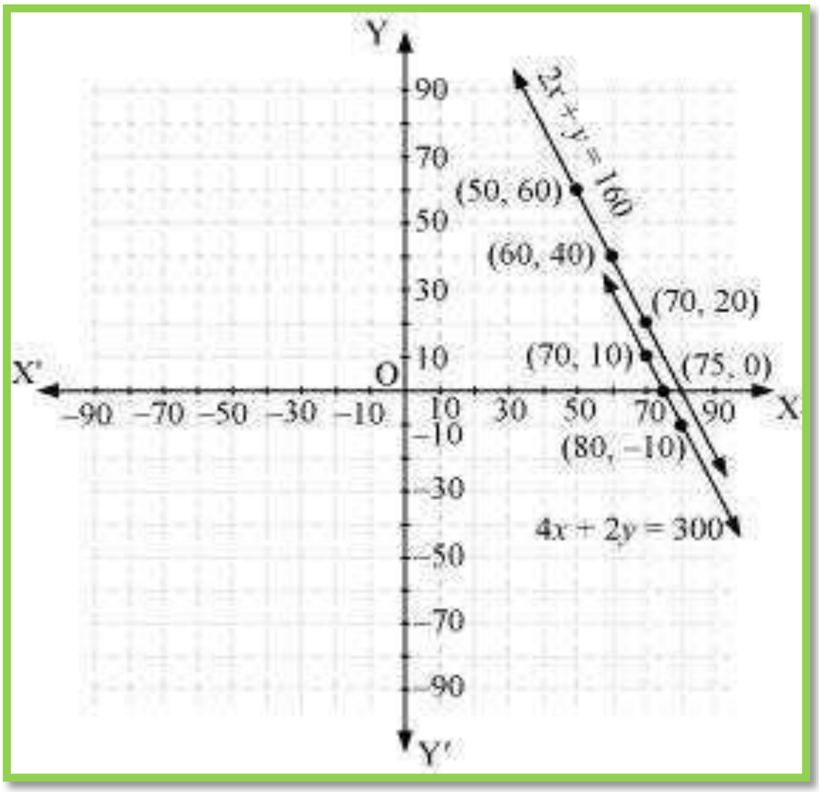
For $4x + 2y = 300$,

$$y = \frac{300 - 4x}{2}$$

The solution table is

x	70	80	75
y	10	−10	0

The graphical representation is as follows.



Mathematics

(Chapter – 3) (Linear equations in two variables)

(Class – X)

Exercise 3.2

Question 1:

Form the pair of linear equations in the following problems, and find their solutions graphically.

(i). 10 students of Class X took part in a Mathematics quiz. If the number of girls is 4 more than the number of boys, find the number of boys and girls who took part in the quiz.

(ii). 5 pencils and 7 pens together cost Rs 50, whereas 7 pencils and 5 pens together cost Rs 46. Find the cost of one pencil and that of one pen.

Answer 1:

(i) Let the number of girls be x and the number of boys be y.

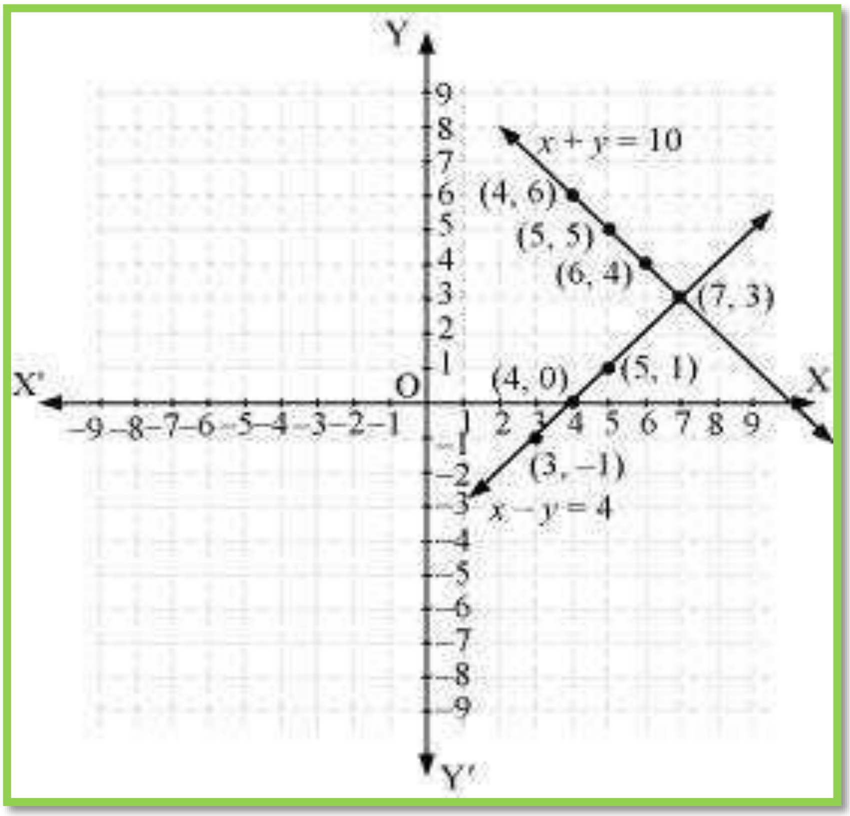
According to the question, the algebraic representation is $x + y = 10$ $x - y = 4$
For $x + y = 10$, $x = 10 - y$

x	5	4	6
y	5	6	4

For $x - y = 4$, $x = 4 + y$

x	5	4	3
y	1	0	−1

Hence, the graphic representation is as follows.



From the figure, it can be observed that these lines intersect each other at point (7, 3).

Therefore, the number of girls and boys in the class are 7 and 3 respectively.

(ii) Let the cost of 1 pencil be Rs x and the cost of 1 pen be Rs y.

According to the question, the algebraic representation is

$5x + 7y = 50$

$7x + 5y = 46$

For $5x + 7y = 50$,

$$x = \frac{50 - 7y}{5}$$

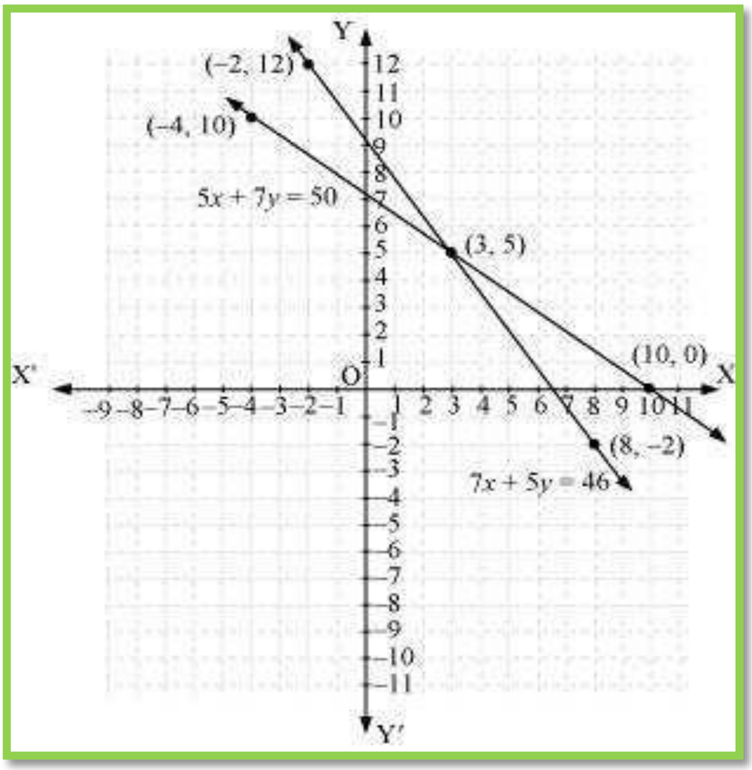
x	3	10	− 4
y	5	0	10

$7x + 5y = 46$

$$x = \frac{46 - 5y}{7}$$

x	**8**	**3**	**− 2**
y	**− 2**	**5**	**12**

Hence, the graphic representation is as follows.



From the figure, it can be observed that these lines intersect each other at point (3, 5).

Therefore, the cost of a pencil and a pen are Rs 3 and Rs 5 respectively.

Question 2:

On comparing the ratios, $\frac{a_1}{a_2}, \frac{b_1}{b_2}$ and $\frac{c_1}{c_2}$ find out whether the lines representing the following pairs of linear equations at a point, are parallel or coincident:

(i) $5x-4y+8=0$
$7x+6y-9=0$

(ii) $9x+3y+12=0$
$18x+6y+24=0$

(iii) $6x-3y+10=0$
$2x-y+9=0$

Answer 2:

(i) $5x - 4y + 8 = 0$

$7x + 6y - 9 = 0$

Comparing these equations with $a_1x+b_1y+c_1=0$ and $a_2x+b_2y+c_2=0$, we obtain

$a_1=5, \quad b_1=-4, \quad c_1=8$

$a_2=7, \quad b_2=6, \quad c_2=-9$

$\frac{a_1}{a_2}=\frac{5}{7}$

$\frac{b_1}{b_2}=\frac{-4}{6}=\frac{-2}{3}$

Since $\frac{a_1}{a_2}\neq\frac{b_1}{b_2}$,

Hence, the lines representing the given pair of equations have a unique solution and the pair of lines intersects at exactly one point.

(ii) $9x + 3y + 12 = 0$

$18x + 6y + 24 = 0$

Comparing these equations with $a_1x+b_1y+c_1=0$

and $a_2x+b_2y+c_2=0$

We obtain, $a_1=9, \quad b_1=3, \quad c_1=12$

$a_2=18, \quad b_2=6, \quad c_2=24$

$\frac{a_1}{a_2}=\frac{9}{18}=\frac{1}{2}$

$\frac{b_1}{b_2}=\frac{3}{6}=\frac{1}{2}$

$\frac{c_1}{c_2}=\frac{12}{24}=\frac{1}{2}$

Since $\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$

Hence, the lines representing the given pair of equations are coincident and there are infinite possible solutions for the given pair of equations.

(iii)$6x - 3y + 10 = 0$

$2x - y + 9 = 0$

Comparing these equations with $a_1x+b_1y+c_1=0$

and $a_2x+b_2y+c_2=0$, we obtain

$a_1=6,\quad b_1=-3,\quad c_1=10$

$a_2=2,\quad b_2=-1,\quad c_2=9$

$\frac{a_1}{a_2}=\frac{6}{2}=\frac{3}{1}$

$\frac{b_1}{b_2}=\frac{-3}{-1}=\frac{3}{1}$

$\frac{c_1}{c_2}=\frac{10}{9}$

Since $\frac{a_1}{a_2}=\frac{b_1}{b_2}\neq\frac{c_1}{c_2}$,

Hence, the lines representing the given pair of equations are parallel to each other and hence, these lines will never intersect each other at any point or there is no possible solution for the given pair of equations.

Question 3:

On comparing the ratios, $\frac{a_1}{a_2},\frac{b_1}{b_2}$ and $\frac{c_1}{c_2}$ find out whether the following pair of linear equations are consistent, or inconsistent.

(i) $3x+2y=5;\quad 2x-3y=7$ (ii) $2x-3y=8;\quad 4x-6y=9$

(iii) $\frac{3}{2}x+\frac{5}{3}y=7;\quad 9x-10y=14$ (iv) $5x-3y=11;\,-10x+6y=-22$

(v) $\frac{4}{3}x+2y=8;\quad 2x+3y=12$

Answer 3:

(i) $3x + 2y = 5$, $2x - 3y = 7$

$$\frac{a_1}{a_2} = \frac{3}{2}, \quad \frac{b_1}{b_2} = \frac{-2}{3}, \quad \frac{c_1}{c_2} = \frac{5}{7}$$

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

These linear equations are intersecting each other at one point and thus have only one possible solution. Hence, the pair of linear equations is consistent.

(ii) $2x - 3y = 8$

$4x - 6y = 9$

$$\frac{a_1}{a_2} = \frac{2}{4} = \frac{1}{2}, \quad \frac{b_1}{b_2} = \frac{-3}{-6} = \frac{1}{2}, \quad \frac{c_1}{c_2} = \frac{8}{9}$$

Since $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$,

Therefore, these linear equations are parallel to each other and thus have no possible solution. Hence, the pair of linear equations is inconsistent.

(iii) $\frac{3}{2}x + \frac{5}{3}y = 7$

$9x - 10y = 14$

$$\frac{a_1}{a_2} = \frac{\frac{3}{2}}{9} = \frac{1}{6}, \quad \frac{b_1}{b_2} = \frac{\frac{5}{3}}{-10} = \frac{-1}{6}, \quad \frac{c_1}{c_2} = \frac{7}{14} = \frac{1}{2}$$

Since $\frac{a_1}{a_2} \neq \frac{b_1}{b_2'}$

Therefore, these linear equations are intersecting each other at one point and thus have only one possible solution. Hence, the pair of linear equations is consistent.

(iv) $5x - 3y = 11$

$-10x + 6y = -22$

$$\frac{a_1}{a_2} = \frac{5}{-10} = \frac{-1}{2}, \quad \frac{b_1}{b_2} = \frac{-3}{6} = \frac{-1}{2}, \quad \frac{c_1}{c_2} = \frac{11}{-22} = \frac{-1}{2}$$

Since $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

Therefore, these linear equations are coincident pair of lines and thus have infinite number of possible solutions. Hence, the pair of linear equations is consistent.

(v) $\frac{4}{3}x + 2y = 8$

$2x + 3y = 12$

$$\frac{a_1}{a_2} = \frac{\frac{4}{3}}{2} = \frac{2}{3}, \quad \frac{b_1}{b_2} = \frac{2}{3}, \quad \frac{c_1}{c_2} = \frac{8}{12} = \frac{2}{3}$$

Since $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$,

Therefore, these linear equations are coincident pair of lines and thus have infinite number of possible solutions. Hence, the pair of linear equations is consistent.

Question 4:

Which of the following pairs of linear equations are consistent/ inconsistent? If consistent, obtain the solution graphically:

(i) $x + y = 5, \quad 2x + 2y = 10$

(ii) $x - y = 8, \quad 3x - 3y = 16$

(iii) $2x + y - 6 = 0, \quad 4x - 2y - 4 = 0$

(iv) $2x - 2y - 2 = 0, 4x - 4y - 5 = 0$

Answer 4:

(i) $x + y = 5$

$2x + 2y = 10$

$$\frac{a_1}{a_2} = \frac{1}{2}, \quad \frac{b_1}{b_2} = \frac{1}{2}, \quad \frac{c_1}{c_2} = \frac{5}{10} = \frac{1}{2}$$

Since, $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

Therefore, these linear equations are coincident pair of lines and thus have infinite number of possible solutions. Hence, the pair of linear equations is consistent.

$x + y = 5$, $x = 5 - y$

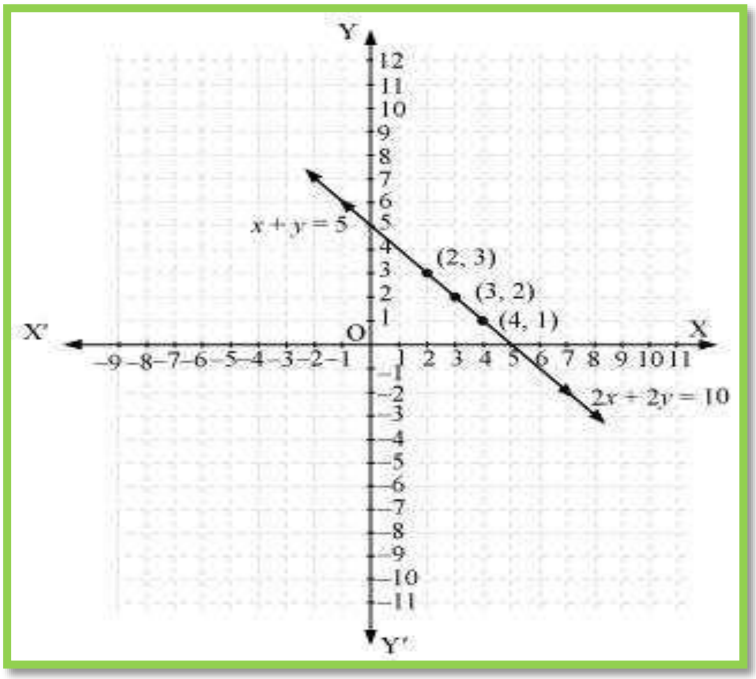
x	**4**	**3**	**2**
y	**1**	**2**	**3**

And, $2x + 2y = 10$

$x = \frac{10-2y}{2}$

x	**4**	**3**	**2**
y	**1**	**2**	**3**

Hence, the graphic representation is as follows.



From the figure, it can be observed that these lines are overlapping each other.

Therefore, infinite solutions are possible for the given pair of equations.

(ii)$x - y = 8$

$3x - 3y = 16$

$$\frac{a_1}{a_2} = \frac{1}{3}, \quad \frac{b_1}{b_2} = \frac{-1}{-3} = \frac{1}{3}, \quad \frac{c_1}{c_2} = \frac{8}{16} = \frac{1}{2}$$

Since $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

Therefore, these linear equations are parallel to each other and thus have no possible solution. Hence, the pair of linear equations is inconsistent.

(iii)$2x + y - 6 = 0$

$4x - 2y - 4 = 0$

$$\frac{a_1}{a_2} = \frac{2}{4} = \frac{1}{2}, \quad \frac{b_1}{b_2} = \frac{-1}{2}, \quad \frac{c_1}{c_2} = \frac{-6}{-4} = \frac{3}{2}$$

Since $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$,

Therefore, these linear equations are intersecting each other at one point and thus have only one possible solution. Hence, the pair of linear equations is consistent.

$2x + y - 6 = 0$, $y = 6 - 2x$

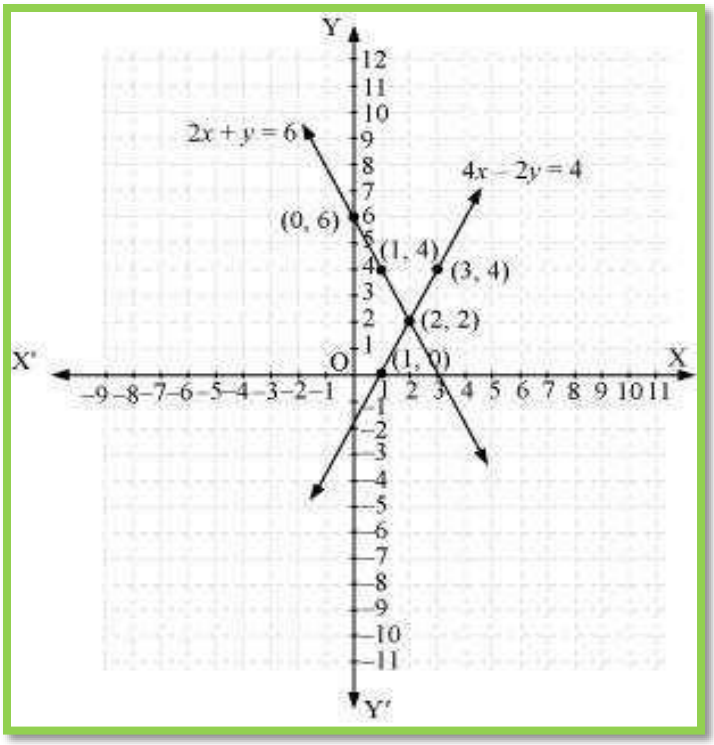
x	**0**	**1**	**2**
y	**6**	**4**	**2**

And $4x - 2y - 4 = 0$

$$y = \frac{4x - 4}{2}$$

x	**1**	**2**	**3**
y	**0**	**2**	**4**

Hence, the graphic representation is as follows.



From the figure, it can be observed that these lines are intersecting each other at the only point i.e., (2, 2) and it is the solution for the given pair of equations.

(iv) $2x - 2y - 2 = 0$

$4x - 4y - 5 = 0$

$$\frac{a_1}{a_2} = \frac{2}{4} = \frac{1}{2}, \quad \frac{b_1}{b_2} = \frac{-2}{-4} = \frac{1}{2}, \quad \frac{c_1}{c_2} = \frac{2}{5}$$

Since $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

Therefore, these linear equations are parallel to each other and thus have no possible solution. Hence, the pair of linear equations is inconsistent.

Question 5:

Half the perimeter of a rectangular garden, whose length is 4 m more than its width, is 36 m. Find the dimensions of the garden.

Answer 5:

Let the width of the garden be *x* and length be *y*.

According to the question,

$y - x = 4$ ………………. (1)
$y + x = 36$ ……………..(2)

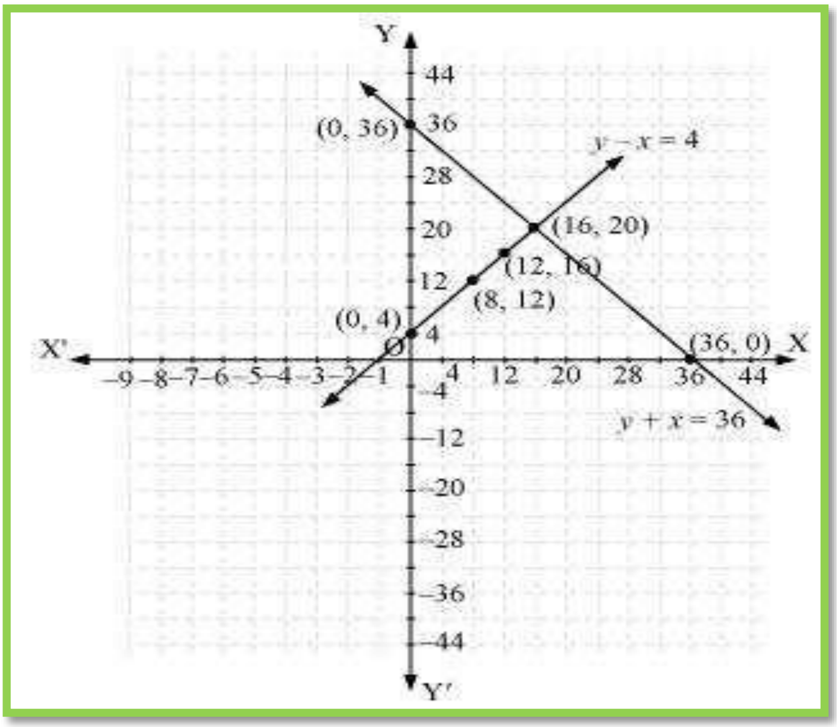
$y - x = 4$

x	0	8	12
y	4	12	16

$y + x = 36$

x	0	36	16
y	36	0	20

Hence, the graphic representation is as follows.



From the figure, it can be observed that these lines are intersecting each other at only point i.e., (16, 20). Therefore, the length and width of the given garden is 20 m and 16 m respectively.

Question 6:

Given the linear equation $2x + 3y - 8 = 0$, write another linear equations in two variables such that the geometrical representation of the pair so formed is:

(i) Intersecting lines (ii) Parallel lines

(iii) Coincident lines

Answer 6:

(i)**Intersecting lines:**

For this condition,

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

The second line such that it is intersecting the given line is

$$2x+4y-6=0 \quad \text{as} \quad \frac{a_1}{a_2}=\frac{2}{2}=1, \quad \frac{b_1}{b_2}=\frac{3}{4} \text{ and } \frac{a_1}{a_2} \neq \frac{b_1}{b_2}.$$

(ii) **Parallel lines:**

For this condition,

$$\frac{a_1}{a_2}=\frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

Hence, the second line can be

$4x + 6y - 8 = 0$

$$\text{as} \frac{a_1}{a_2}=\frac{2}{4}=\frac{1}{2}, \quad \frac{b_1}{b_2}=\frac{3}{6}=\frac{1}{2}, \frac{c_1}{c_2}=\frac{-8}{-8}=1$$

$$\text{And clearly, } \frac{a_1}{a_2}=\frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

(iii)**Coincident lines**:

For coincident lines,

$$\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$$

Hence, the second line can be

$6x + 9y - 24 = 0$

as $\frac{a_1}{a_2} = \frac{2}{6} = \frac{1}{3}$, $\frac{b_1}{b_2} = \frac{3}{9} = \frac{1}{3}$, $\frac{c_1}{c_2} = \frac{-8}{-24} = \frac{1}{3}$

And clearly, $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

Question 7:

Draw the graphs of the equations $x - y + 1 = 0$ and $3x + 2y - 12 = 0$. Determine the coordinates of the vertices of the triangle formed by these lines and the x-axis, and shade the triangular region.

Answer 7:

$x - y + 1 = 0$ or $x = y - 1$

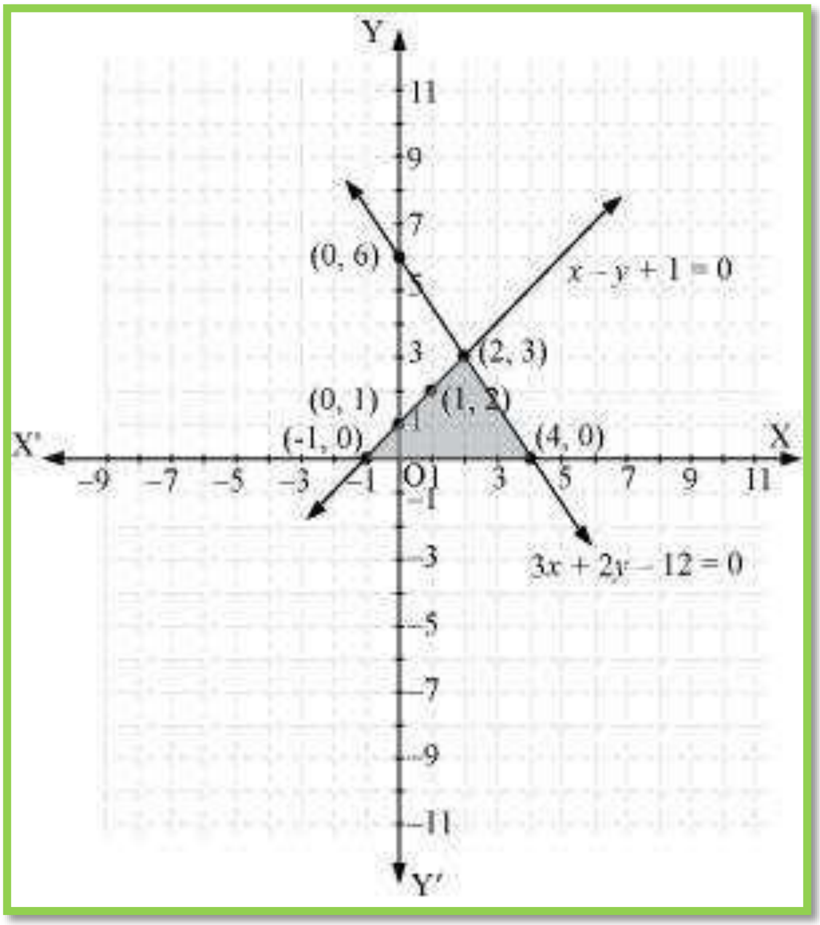
x	**0**	**1**	**2**
y	**1**	**2**	**3**

$3x + 2y - 12 = 0$

$x = \frac{12 - 2y}{3}$

x	**4**	**2**	**0**
y	**0**	**3**	**6**

Hence, the graphic representation is as follows.



From the figure, it can be observed that these lines are intersecting each other at point (2, 3) and *x*-axis at (−1, 0) and (4, 0). Therefore, the vertices of the triangle are (2, 3), (−1, 0), and (4, 0).

Mathematics

(Chapter – 3) (Linear equations in two variables)

(Class – X)

Exercise 3.3

Question 1:

Solve the following pair of linear equations by the substitution method.

(i) $x+y=14$

$x-y=4$

(ii) $s-t=3$

$\frac{s}{3}+\frac{t}{2}=6$

(iii) $3x-y=3$

$9x-3y=9$

(iv) $0.2x+0.3y=1.3$

$0.4x+0.5y=2.3$

(v) $\sqrt{2}x+\sqrt{3}y=0$

$\sqrt{3}x-\sqrt{8}y=0$

(vi) $\frac{3x}{2}-\frac{5y}{3}=-2$

$\frac{x}{3}+\frac{y}{2}=\frac{13}{6}$

Answer 1:

(i) $x + y = 14$ ……………………..(1)

$x - y = 4$ ………………………..(2)

From (1), we obtain $x = 14 - y$ (3)

Substituting this value in equation (2), we obtain

$$(14-y)-y=4$$
$$14-2y=4$$
$$10=2y$$
$$y=5 \qquad (4)$$

Substituting this in equation (3), we obtain

$$x=9$$
$$\therefore x=9, y=5$$

(ii) $s-t=3 \qquad (1)$

$$\frac{s}{3}+\frac{t}{2}=6 \qquad (2)$$

From (1), we obtain

$$s=t+3 \qquad (3)$$

Substituting this value in equation (2), we obtain

$$\frac{t+3}{3}+\frac{t}{2}=6$$
$$2t+6+3t=36$$
$$5t=30$$
$$t=6 \qquad (4)$$

Substituting in equation (3), we obtain s
= 9

$\therefore s = 9, t = 6$

(iii)$3x - y = 3$ (1)

$9x - 3y = 9$ (2) From (1), we obtain $y = 3x - 3$ (3)

Substituting this value in equation (2), we obtain

$$9x-3(3x-3)=9$$
$$9x-9x+9=9$$

9 = 9

This is always true.

Hence, the given pair of equations has infinite possible solutions and the relation between these variables can be given by $y = 3x - 3$

Therefore, one of its possible solutions is $x = 1$, $y = 0$.

(iv) $0.2x+0.3y=1.3 \qquad (1)$

$$0.4x+0.5y=2.3 \qquad (2)$$

From equation (1), we obtain

$$x=\frac{1.3-0.3y}{0.2} \qquad (3)$$

Substituting this value in equation (2), we obtain

$$0.4\left(\frac{1.3-0.3y}{0.2}\right)+0.5y=2.3$$
$$2.6-0.6y+0.5y=2.3$$
$$2.6-2.3=0.1y$$
$$0.3=0.1y$$
$$y=3 \qquad (4)$$

Substituting this value in equation (3), we obtain

$$x = \frac{1.3 - 0.3 \times 3}{0.2}$$
$$= \frac{1.3 - 0.9}{0.2} = \frac{0.4}{0.2} = 2$$
$$\therefore x = 2,\ y = 3$$

(v) $\sqrt{2}x + \sqrt{3}y = 0 \qquad (1)$

$\sqrt{3}x - \sqrt{8}y = 0 \qquad (2)$

From equation (1), we obtain

$$x = \frac{-\sqrt{3}y}{\sqrt{2}} \qquad (3)$$

Substituting this value in equation (2), we obtain

$$\sqrt{3}\left(-\frac{\sqrt{3}y}{\sqrt{2}}\right) - \sqrt{8}y = 0$$
$$-\frac{3y}{\sqrt{2}} - 2\sqrt{2}y = 0$$
$$y\left(-\frac{3}{\sqrt{2}} - 2\sqrt{2}\right) = 0$$
$$y = 0 \qquad (4)$$

Substituting this value in equation (3), we obtain

$x = 0$

$\therefore x = 0,\ y = 0$

(vi) $\frac{3}{2}x - \frac{5}{3}y = -2 \qquad (1)$

$\frac{x}{3} + \frac{y}{2} = \frac{13}{6} \qquad (2)$

From equation (1), we obtain

$$9x - 10y = -12$$
$$x = \frac{-12 + 10y}{9} \qquad (3)$$

Substituting this value in equation (2), we obtain

$$\frac{\frac{-12+10y}{9}}{3}+\frac{y}{2}=\frac{13}{6}$$

$$\frac{-12+10y}{27}+\frac{y}{2}=\frac{13}{6}$$

$$\frac{-24+20y+27y}{54}=\frac{13}{6}$$

$$47y=117+24$$

$$47y=141$$

$$y=3 \qquad (4)$$

Substituting this value in equation (3), we obtain

$$x=\frac{-12+10\times3}{9}=\frac{18}{9}=2$$

Hence, $x = 2$, $y = 3$

Question 2:

Solve $2x + 3y = 11$ and $2x - 4y = -24$ and hence find the value of 'm' for which $y = mx + 3$.

Answer 2:

$$2x+3y=11 \qquad (1)$$

$$2x-4y=-24 \qquad (2)$$

From equation (1), we obtain

$$x=\frac{11-3y}{2} \qquad (3)$$

Substituting this value in equation (2), we obtain

$$2\left(\frac{11-3y}{2}\right)-4y=-24$$

$$11-3y-4y=-24$$

$$-7y=-35$$

$$y=5 \qquad (4)$$

Putting this value in equation (3), we obtain

$$x=\frac{11-3\times5}{2}=-\frac{4}{2}=-2$$

Hence, $x = -2$, $y = 5$

Also,

$$y = mx + 3$$
$$5 = -2m + 3$$
$$-2m = 2$$
$$m = -1$$

Question 3:

Form the pair of linear equations for the following problems and find their solution by substitution method.

(i) The difference between two numbers is 26 and one number is three times the other. Find them.

(ii) The larger of two supplementary angles exceeds the smaller by 18 degrees. Find them.

(iii) The coach of a cricket team buys 7 bats and 6 balls for Rs 3800. Later, she buys 3 bats and 5 balls for Rs 1750. Find the cost of each bat and each ball.

(iv) The taxi charges in a city consist of a fixed charge together with the charge for the distance covered. For a distance of 10 km, the charge paid is Rs 105 and for a journey of 15 km, the charge paid is Rs 155. What are the fixed charges and the charge per km? How much does a person have to pay for travelling a distance of 25 km.

(v) A fraction becomes 8/11, if 2 is added to both the numerator and the denominator. If, 3 is added to both the numerator and the denominator it becomes 5/6. Find the fraction.

(vi) Five years hence, the age of Jacob will be three times that of his son. Five years ago, Jacob's age was seven times that of his son. What are their present ages?

Answer 3:

(i) Let the first number be x and the other number be y such that $y > x$.

According to the given information,

$$y = 3x \qquad (1)$$
$$y - x = 26 \qquad (2)$$

On substituting the value of y from equation (1) into equation (2), we obtain

$$3x - x = 26$$
$$x = 13 \qquad (3)$$

Substituting this in equation (1), we obtain y = 39

Hence, the numbers are 13 and 39.

(ii) Let the larger angle be *x* and smaller angle be *y*.

We know that the sum of the measures of angles of a supplementary pair is always 180º.

According to the given information,

$$x + y = 180^\circ \qquad (1)$$
$$x - y = 18^\circ \qquad (2)$$

From (1), we obtain $x = 180^\circ - y$ (3)

Substituting this in equation (2), we obtain

$$180^\circ - y - y = 18^\circ$$
$$162^\circ = 2y$$
$$81^\circ = y \qquad (4)$$

Putting this in equation (3), we obtain *x*

= 180º − 81º

= 99º

Hence, the angles are 99º and 81º.

(iii) Let the cost of a bat and a ball be *x* and *y* respectively.

According to the given information,

$$7x + 6y = 3800 \qquad (1)$$
$$3x + 5y = 1750 \qquad (2)$$

From (1), we obtain

$$y = \frac{3800 - 7x}{6} \qquad (3)$$

Substituting this value in equation (2), we obtain

$$3x + 5\left(\frac{3800 - 7x}{6}\right) = 1750$$
$$3x + \frac{9500}{3} - \frac{35x}{6} = 1750$$
$$3x - \frac{35x}{6} = 1750 - \frac{9500}{3}$$

$$\frac{18x-35x}{6}=\frac{5250-9500}{3}$$
$$-\frac{17x}{6}=\frac{-4250}{3}$$
$$-17x=-8500$$
$$x=500 \qquad (4)$$

Substituting this in equation (3), we obtain

$$y=\frac{3800-7\times 500}{6}$$
$$=\frac{300}{6}=50$$

Hence, the cost of a bat is Rs 500 and that of a ball is Rs 50.

(iv)Let the fixed charge be Rs *x* and per km charge be Rs *y*.

According to the given information,

$$x+10y=105 \qquad (1)$$
$$x+15y=155 \qquad (2)$$

From (3), we obtain

$$x=105-10y \qquad (3)$$

Substituting this in equation (2), we obtain

$$105-10y+15y=155$$
$$5y=50$$
$$y=10 \qquad (4)$$

Putting this in equation (3), we obtain

$$x=105-10\times 10$$
$$x=5$$

Hence, fixed charge = Rs 5

And per km charge = Rs 10

Charge for 25 km = *x* + 25*y*

= 5 + 250 = Rs 255

(v) Let the fraction be x/y.

According to the given information,

$$\frac{x+2}{y+2}=\frac{9}{11}$$
$$11x+22=9y+18$$
$$11x-9y=-4 \qquad (1)$$
$$\frac{x+3}{y+3}=\frac{5}{6}$$
$$6x+18=5y+15$$
$$6x-5y=-3 \qquad (2)$$

From equation (1), we obtain $x=\frac{-4+9y}{11} \qquad (3)$

Substituting this in equation (2), we obtain

$$6\left(\frac{-4+9y}{11}\right)-5y=-3$$
$$-24+54y-55y=-33$$
$$-y=-9$$
$$y=9 \qquad (4)$$

Substituting this in equation (3), we obtain

$$x=\frac{-4+81}{11}=7$$

Hence, the fraction is 7/9.

(vi) Let the age of Jacob be *x* and the age of his son be *y*.

According to the given information,

$$(x+5)=3(y+5)$$
$$x-3y=10 \qquad (1)$$
$$(x-5)=7(y-5)$$
$$x-7y=-30 \qquad (2)$$

From (1), we obtain

$$x=3y+10 \qquad (3)$$

Substituting this value in equation (2), we obtain

$$3y+10-7y=-30$$
$$-4y=-40$$
$$y=10 \qquad (4)$$

Substituting this value in equation (3), we obtain

$$x=3\times10+10$$
$$=40$$

Hence, the present age of Jacob is 40 years whereas the present age of his son is 10 years.

Mathematics

(Chapter – 3) (Linear equations in two variables)

(Class – X)

Exercise 3.4

Question 1:

Solve the following pair of linear equations by the elimination method and the substitution method:

(i) $x+y=5$ and $2x-3y=4$ (ii) $3x+4y=10$ and $2x-2y=2$

(iii) $3x-5y-4=0$ and $9x=2y+7$ (iv) $\frac{x}{2}+\frac{2y}{3}=-1$ and $x-\frac{y}{3}=3$

Answer 1:

(i) ***By elimination method***

X + y = 5 ....(1)

2x – 3y = 4 ....(2)

Multiplying equation (1) by 2, we obtain

$2x+2y=10 \quad (3)$

Subtracting equation (2) from equation (3), we obtain

$5y=6$

$y=\frac{6}{5} \qquad (4)$

Substituting the value in equation (1), we obtain

$x=5-\frac{6}{5}=\frac{19}{5}$

$\therefore x=\frac{19}{5}, y=\frac{6}{5}$

By substitution method

From equation (1), we obtain

$x=5-y$ (5)

Putting this value in equation (2), we obtain

$2(5-y)-3y=4$

$-5y = -6$

$y=\frac{6}{5}$

Substituting the value in equation (5), we obtain

$x = 5 - \frac{6}{5} = \frac{19}{5}$

$\therefore x = \frac{19}{5}, y = \frac{6}{5}$

(ii) *By elimination method*

$3x + 4y = 10 \quad (1)$

$2x - 2y = 2 \quad (2)$

Multiplying equation (2) by 2, we obtain

$4x - 4y = 4 \quad (3)$

Adding equation (1) and (3), we obtain

$7x = 14$

$x = 2 \quad (4)$

Substituting in equation (1), we obtain

$6 + 4y = 10$

$4y = 4$

$y = 1$

Hence, $x = 2$, $y = 1$

By substitution method

From equation (2), we obtain

$x = 1 + y$ (5)

Putting this value in equation (1), we obtain

$3(1 + y) + 4y = 10$

$7y = 7$

$y = 1$

Substituting the value in equation (5), we obtain

$x = 1 + 1 = 2$

$\therefore x = 2, y = 1$

(iii)*By elimination method*

$3x - 5y - 4 = 0 \quad (1)$

$9x = 2y + 7$

$9x - 2y - 7 = 0 \quad (2)$

Multiplying equation (1) by 3, we obtain

$$9x - 15y - 12 = 0 \qquad (3)$$

Subtracting equation (3) from equation (2), we obtain

$$13y = -5$$

$$y = \frac{-5}{13} \qquad (4)$$

Substituting in equation (1), we obtain

$$3x + \frac{25}{13} - 4 = 0$$

$$3x = \frac{27}{13}$$

$$x = \frac{9}{13}$$

$$\therefore \quad x = \frac{9}{13}, y = \frac{-5}{13}$$

By substitution method

From equation (1), we obtain

$$x = \frac{5y+4}{3} \qquad (5)$$

Putting this value in equation (2), we obtain

$$9\left(\frac{5y+4}{3}\right) - 2y - 7 = 0$$

$$13y = -5$$

$$y = -\frac{5}{13}$$

Substituting the value in equation (5), we obtain

$$x = \frac{5\left(\frac{-5}{13}\right)+4}{3}$$

$$x = \frac{9}{13}$$

$$\therefore \quad x = \frac{9}{13}, y = \frac{-5}{13}$$

(iv) ***By elimination method***

$$\frac{x}{2}+\frac{2y}{3}=-1$$

$$3x+4y=-6 \quad (1)$$

$$x-\frac{y}{3}=3$$

$$3x-y=9 \qquad (2)$$

Subtracting equation (2) from equation (1), we obtain

$$5y=-15$$

$$y=-3 \qquad (3)$$

Substituting this value in equation (1), we obtain

$$3x-12=-6$$

$$3x=6$$

$$x=2$$

Hence, x = 2, y = −3

By substitution method

From equation (2), we obtain

$$x=\frac{y+9}{3} \quad (5)$$

Putting this value in equation (1), we obtain

$$3\left(\frac{y+9}{3}\right)+4y=-6$$

$5y$ = −15

$$y=-3$$

Substituting the value in equation (5), we obtain

$$x=\frac{-3+9}{3}=2$$

∴ x = 2, y = −3

Question 2:

Form the pair of linear equations in the following problems, and find their solutions (if they exist) by the elimination method:

(i) If we add 1 to the numerator and subtract 1 from the denominator, a fraction reduces to 1. It becomes ½ if we only add 1 to the denominator. What is the fraction?

(ii) Five years ago, Nuri was thrice as old as Sonu. Ten years later, Nuri will be twice as old as Sonu. How old are Nuri and Sonu?

(iii) The sum of the digits of a two-digit number is 9. Also, nine times this number is twice the number obtained by reversing the order of the digits. Find the number.

(iv) Meena went to bank to withdraw Rs 2000. She asked the cashier to give her Rs 50 and Rs 100 notes only. Meena got 25 notes in all. Find how many notes of Rs 50 and Rs 100 she received.

(v) A lending library has a fixed charge for the first three days and an additional charge for each day thereafter. Saritha paid Rs 27 for a book kept for seven days, while Susy paid Rs 21 for the book she kept for five days. Find the fixed charge and the charge for each extra day.

Answer 2:

(i) Let the fraction be x/y.

According to the given information,

$$\frac{x+1}{y-1}=1 \qquad \Rightarrow x-y=-2 \qquad (1)$$

$$\frac{x}{y+1}=\frac{1}{2} \qquad \Rightarrow 2x-y=1 \qquad (2)$$

Subtracting equation (1) from equation (2), we obtain $x = 3$ …………(3)

Substituting this value in equation (1), we obtain

$3-y=-2$

$-y=-5$

$y=5$

Hence, the fraction is 3/5.

(ii) Let present age of Nuri = x

and present age of Sonu = y

According to the given information,

$$(x-5)=3(y-5)$$
$$x-3y=-10 \qquad (1)$$
$$(x+10)=2(y+10)$$
$$x-2y=10 \qquad (2)$$

Subtracting equation (1) from equation (2), we obtain y = 20 (3)

Substituting it in equation (1), we obtain

$$x-60=-10$$
$$x=50$$

Hence, age of Nuri = 50 years

And, age of Sonu = 20 years

(iii) Let the unit digit and tens digits of the number be x and y respectively.

Then, number = 10y + x

Number after reversing the digits = 10x + y

According to the given information, x + y = 9 …………………(1)

9(10y + x) = 2(10x + y)

88y − 11x = 0

− x + 8y =0 …………………(2)

Adding equation (1) and (2), we obtain

9y = 9 y

= 1 (3)

Substituting the value in equation (1), we obtain x = 8

Hence, the number is 10y + x = 10 × 1 + 8 = 18

(iv) Let the number of Rs 50 notes and Rs 100 notes be x and y respectively.

According to the given information,

$$x+y=25 \qquad (1)$$
$$50x+100y=2000 \qquad (2)$$

Multiplying equation (1) by 50, we obtain

$$50x+50y=1250 \qquad (3)$$

Subtracting equation (3) from equation (2), we obtain

$$50y=750$$
$$y=15$$

Substituting in equation (1), we have x = 10

Hence, Meena has 10 notes of Rs 50 and 15 notes of Rs 100.

(v) Let the fixed charge for first three days and each day charge thereafter be Rs *x* and Rs *y* respectively.

According to the given information,

$$x + 4y = 27 \qquad (1)$$

$$x + 2y = 21 \qquad (2)$$

Subtracting equation (2) from equation (1), we obtain

$$2y = 6$$

$$y = 3 \qquad (3)$$

Substituting in equation (1), we obtain

$$x + 12 = 27$$

$$x = 15$$

Hence,

fixed charge = Rs 15

And

Charge per day = Rs 3

Mathematics

(Chapter – 3) (Linear equations in two variables)

(Class – X)

Exercise 3.5

Question 1:

Which of the following pairs of linear equations has unique solution, no solution or infinitely many solutions? In case there is a unique solution, find it by using cross multiplication method.

(i) $x-3y-3=0$
$3x-9y-2=0$

(ii) $2x+y=5$
$3x+2y=8$

(iii) $3x-5y=20$
$6x-10y=40$

(iv) $x-3y-7=0$
$3x-3y-15=0$

Answer 1:

(i) $x-3y-3=0$

$3x-9y-2=0$

$$\frac{a_1}{a_2}=\frac{1}{3},\quad \frac{b_1}{b_2}=\frac{-3}{-9}=\frac{1}{3},\quad \frac{c_1}{c_2}=\frac{-3}{-2}=\frac{3}{2}$$

$$\frac{a_1}{a_2}=\frac{b_1}{b_2}\neq\frac{c_1}{c_2}$$

Therefore, the given sets of lines are parallel to each other. Therefore, they will not intersect each other and thus, there will not be any solution for these equations.

(ii) $2x+y=5$

$3x+2y=8$

$$\frac{a_1}{a_2}=\frac{2}{3},\quad \frac{b_1}{b_2}=\frac{1}{2},\quad \frac{c_1}{c_2}=\frac{-5}{-8}$$

$$\frac{a_1}{a_2}\neq\frac{b_1}{b_2}$$

Therefore, they will intersect each other at a unique point and thus, there will be a unique solution for these equations. By cross-multiplication method,

$$\frac{x}{b_1c_2 - b_2c_1} = \frac{y}{c_1a_2 - c_2a_1} = \frac{1}{a_1b_2 - a_2b_1}$$
$$\frac{x}{-8-(-10)} = \frac{y}{-15+16} = \frac{1}{4-3}$$
$$\frac{x}{2} = \frac{y}{1} = 1$$
$$\frac{x}{2} = 1, \quad \frac{y}{1} = 1$$
$$x = 2, \quad y = 1$$

$\therefore$ $x = 2$, $y = 1$

(iii) $3x - 5y = 20$

$$6x - 10y = 40$$
$$\frac{a_1}{a_2} = \frac{3}{6} = \frac{1}{2}, \quad \frac{b_1}{b_2} = \frac{-5}{-10} = \frac{1}{2}, \quad \frac{c_1}{c_2} = \frac{-20}{-40} = \frac{1}{2}$$
$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

Therefore, the given sets of lines will be overlapping each other i.e., the lines will be coincident to each other and thus, there are infinite solutions possible for these equations.

(iv) $x - 3y - 7 = 0$

$$3x - 3y - 15 = 0$$
$$\frac{a_1}{a_2} = \frac{1}{3}, \quad \frac{b_1}{b_2} = \frac{-3}{-3} = 1, \quad \frac{c_1}{c_2} = \frac{-7}{-15} = \frac{7}{15}$$
$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

Therefore, they will intersect each other at a unique point and thus, there will be a unique solution for these equations.

By cross-multiplication,

$$\frac{x}{45-(21)}=\frac{y}{-21-(-15)}=\frac{1}{-3-(-9)}$$

$$\frac{x}{24}=\frac{y}{-6}=\frac{1}{6}$$

$$\frac{x}{24}=\frac{1}{6} \text{ and } \frac{y}{-6}=\frac{1}{6}$$

$$x=4 \text{ and } y=-1$$

$$\therefore \quad x=4, y=-1$$

Question 2:

(i) For which values of *a* and *b* will the following pair of linear equations have an infinite number of solutions?

$$2x+3y=7$$

$$(a-b)x+(a+b)y=3a+b-2$$

(ii) For which value of *k* will the following pair of linear equations have no solution?

$$3x+y=1$$

$$(2k-1)x+(k-1)y=2k+1$$

Answer 2:

(i) $\quad 2x+3y-7=0$

$$(a-b)x+(a+b)y-(3a+b-2)=0$$

$$\frac{a_1}{a_2}=\frac{2}{a-b}, \quad \frac{b_1}{b_2}=\frac{3}{a+b}, \quad \frac{c_1}{c_2}=\frac{-7}{-(3a+b-2)}=\frac{7}{(3a+b-2)}$$

For infinitely many solutions,

$$\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$$

$$\frac{2}{a-b}=\frac{7}{3a+b-2}$$

$$6a+2b-4=7a-7b$$

$$a-9b=-4 \qquad (1)$$

$$\frac{2}{a-b}=\frac{3}{a+b}$$

$$2a+2b=3a-3b$$

$$a-5b=0 \qquad (2)$$

Subtracting (1) from (2), we obtain

$4b = 4$

$b = 1$

Substituting this in equation (2), we obtain

$a - 5 \times 1 = 0$

$a = 5$

Hence, *a* = 5 and *b* = 1 are the values for which the given equations give infinitely many solutions.

(ii) $3x + y - 1 = 0$

$(2k-1)x + (k-1)y - 2k - 1 = 0$

$$\frac{a_1}{a_2} = \frac{3}{2k-1}, \quad \frac{b_1}{b_2} = \frac{1}{k-1}, \quad \frac{c_1}{c_2} = \frac{-1}{-2k-1} = \frac{1}{2k+1}$$

For no solution,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

$$\frac{3}{2k-1} = \frac{1}{k-1} \neq \frac{1}{2k+1}$$

$$\frac{3}{2k-1} = \frac{1}{k-1}$$

$3k - 3 = 2k - 1$

$k = 2$

Hence, for *k* = 2, the given equation has no solution.

Question 3:

Solve the following pair of linear equations by the substitution and cross multiplication methods:

$8x + 5y = 9$

$3x + 2y = 4$

Answer 3:

$8x + 5y = 9 \qquad (i)$

$3x + 2y = 4 \qquad (ii)$

From equation (*ii*), we obtain

$$x = \frac{4-2y}{3} \qquad (iii)$$

Substituting this value in equation (*i*), we obtain

$$8\left(\frac{4-2y}{3}\right)+5y=9$$
$$32-16y+15y=27$$
$$-y=-5$$
$$y=5 \qquad (iv)$$

Substituting this value in equation (*ii*), we obtain

$$3x+10=4$$
$$x=-2$$

Hence, $x=-2, y=5$

Again, by cross-multiplication method, we obtain

$$8x+5y-9=0$$
$$3x+2y-4=0$$
$$\frac{x}{-20-(-18)}=\frac{y}{-27-(-32)}=\frac{1}{16-15}$$
$$\frac{x}{-2}=\frac{y}{5}=\frac{1}{1}$$
$$\frac{x}{-2}=1 \text{ and } \frac{y}{5}=1$$
$$x=-2 \text{ and } y=5$$

Question 4:

Form the pair of linear equations in the following problems and find their solutions (if they exist) by any algebraic method:

(i). A part of monthly hostel charges is fixed and the remaining depends on the number of days one has taken food in the mess. When a student A takes food for 20 days she has to pay Rs 1000 as hostel charges whereas a student B, who takes food for 26 days, pays Rs 1180 as hostel charges. Find the fixed charges and the cost of food per day.

(ii). A fraction becomes 1/3 when 1 is subtracted from the numerator and it becomes ¼ when 8 is added to its denominator. Find the fraction.

(iii). Yash scored 40 marks in a test, getting 3 marks for each right answer and losing 1 mark for each wrong answer. Had 4 marks been awarded for each correct answer and 2 marks been deducted for each incorrect answer, then Yash would have scored 50 marks. How many questions were there in the test?

(iv). Places A and B are 100 km apart on a highway. One car starts from A and another from B at the same time. If the cars travel in the same direction at different speeds, they meet in 5 hours. If they travel towards each other, they meet in 1 hour. What are the speeds of the two cars?

(v). The area of a rectangle gets reduced by 9 square units, if its length is reduced by 5 units and breadth is increased by 3 units. If we increase the length by 3 units and the breadth by 2 units, the area increases by 67 square units. Find the dimensions of the rectangle.

Answer 4:

(i) Let *x* be the fixed charge of the food and *y* be the charge for food per day.

According to the given information,

$$x+20y=1000 \qquad (1)$$
$$x+26y=1180 \qquad (2)$$

Subtracting equation (1) from equation (2), we obtain

$$6y=180$$
$$y=30$$

Substituting this value in equation (1), we obtain

$$x+20\times 30=1000$$
$$x=1000-600$$
$$x=400$$

Hence, fixed charge = Rs 400

And charge per day = Rs 30

(ii) Let the fraction be x/y.

According to the given information,

$$\frac{x-1}{y}=\frac{1}{3} \quad \Rightarrow \quad 3x-y=3 \qquad (1)$$
$$\frac{x}{y+8}=\frac{1}{4} \quad \Rightarrow \quad 4x-y=8 \qquad (2)$$

Subtracting equation (1) from equation (2), we obtain

$$x=5 \qquad (3)$$

Putting this value in equation (1), we obtain

$$15-y=3$$
$$y=12$$

Hence, the fraction is 5/12.

(iii) Let the number of right answers and wrong answers be *x* and *y* respectively.

According to the given information,

$$3x - y = 40 \qquad (1)$$

$$4x - 2y = 50$$

$$\Rightarrow 2x - y = 25 \qquad (2)$$

Subtracting equation (2) from equation (1), we obtain *x*

= 15 (3)

Substituting this in equation (2), we obtain

$$30 - y = 25$$

$$y = 5$$

Therefore, number of right answers = 15

And number of wrong answers = 5

Total number of questions = 20

(iv) Let the speed of 1st car and 2nd car be *u* km/h and *v* km/h.

Respective speed of both cars while they are travelling in same direction = (u - v) km/h

Respective speed of both cars while they are travelling in opposite directions i.e., travelling towards each other = (u + v) km/h

According to the given information,

$$5(u - v) = 100$$

$$\Rightarrow u - v = 20 \quad ...(1)$$

$$1(u + v) = 100 \quad ...(2)$$

Adding both the equations, we obtain

$$2u = 120$$

$$u = 60 \text{ km/h} \qquad (3)$$

Substituting this value in equation (2), we obtain *v*

= 40 km/h

Hence, speed of one car = 60 km/h and speed of other car = 40 km/h

(v) Let length and breadth of rectangle be *x* unit and *y* unit respectively.

Area = *xy*

According to the question,

$$(x-5)(y+3) = xy - 9$$

$$\Rightarrow 3x - 5y - 6 = 0 \qquad (1)$$

$$(x+3)(y+2) = xy + 67$$

$$\Rightarrow 2x + 3y - 61 = 0 \qquad (2)$$

By cross-multiplication method, we obtain

$$\frac{x}{305-(-18)} = \frac{y}{-12-(-183)} = \frac{1}{9-(-10)}$$

$$\frac{x}{323} = \frac{y}{171} = \frac{1}{19}$$

$$x = 17, y = 9$$

Hence, the length and breadth of the rectangle are 17 units and 9 units respectively.

Mathematics

(Chapter – 3) (Linear equations in two variables)

(Class – X)

Exercise 3.6

Question 1:

Solve the following pairs of equations by reducing them to a pair of linear equations:

(i) $\frac{1}{2x}+\frac{1}{3y}=2$

$\frac{1}{3x}+\frac{1}{2y}=\frac{13}{6}$

(ii) $\frac{2}{\sqrt{x}}+\frac{3}{\sqrt{y}}=2$

$\frac{4}{\sqrt{x}}-\frac{9}{\sqrt{y}}=-1$

(iii) $\frac{4}{x}+3y=14$

$\frac{3}{x}-4y=23$

(iv) $\frac{5}{x-1}+\frac{1}{y-2}=2$

$\frac{6}{x-1}-\frac{3}{y-2}=1$

(v) $\frac{7x-2y}{xy}=5$

$\frac{8x+7y}{xy}=15$

(vi) $6x+3y=6xy$

$2x+4y=5xy$

(vii) $\frac{10}{x+y}+\frac{2}{x-y}=4$

$\frac{15}{x+y}-\frac{5}{x-y}=-2$

(viii) $\frac{1}{3x+y}+\frac{1}{3x-y}=\frac{3}{4}$

$\frac{1}{2(3x+y)}-\frac{1}{2(3x-y)}=\frac{-1}{8}$

Answer 1:

(i) $\frac{1}{2x}+\frac{1}{3y}=2$

$\frac{1}{3x}+\frac{1}{2y}=\frac{13}{6}$

Let $\frac{1}{x}=p$ and $\frac{1}{y}=q$, then the equations change as follows.

$$\frac{p}{2}+\frac{q}{3}=2 \quad \Rightarrow \quad 3p+2q-12=0 \qquad (1)$$

$$\frac{p}{3}+\frac{q}{2}=\frac{13}{6} \quad \Rightarrow \quad 2p+3q-13=0 \qquad (2)$$

Using cross-multiplication method, we obtain

$$\frac{p}{-26-(-36)}=\frac{q}{-24-(-39)}=\frac{1}{9-4}$$

$$\frac{p}{10}=\frac{q}{15}=\frac{1}{5}$$

$$\frac{p}{10}=\frac{1}{5} \text{ and } \frac{q}{15}=\frac{1}{5}$$

$$p=2 \text{ and } q=3$$

$$\frac{1}{x}=2 \text{ and } \frac{1}{y}=3$$

$$x=\frac{1}{2} \text{ and } y=\frac{1}{3}$$

(ii) $$\frac{2}{\sqrt{x}}+\frac{3}{\sqrt{y}}=2$$

$$\frac{4}{\sqrt{x}}-\frac{9}{\sqrt{y}}=-1$$

Putting $\frac{1}{\sqrt{x}}=p$ and $\frac{1}{\sqrt{y}}=q$ in the given equations, we obtain

$$2p+3q=2 \qquad (1)$$

$$4p-9q=-1 \qquad (2)$$

Multiplying equation (1) by 3, we obtain

$6p + 9q = 6$ (3)

Adding equation (2) and (3), we obtain

$$10p=5$$

$$p=\frac{1}{2} \qquad (4)$$

Putting in equation (1), we obtain

$$2\times\frac{1}{2}+3q=2$$

$$3q=1$$

$$q=\frac{1}{3}$$

$$p = \frac{1}{\sqrt{x}} = \frac{1}{2}$$
$$\sqrt{x} = 2$$
$$x = 4$$
$$\text{and } q = \frac{1}{\sqrt{y}} = \frac{1}{3}$$
$$\sqrt{y} = 3$$
$$y = 9$$

Hence, $x = 4, y = 9$

$$\text{(iii)} \quad \frac{4}{x} + 3y = 14$$
$$\frac{3}{x} - 4y = 23$$

Substituting $\frac{1}{x} = p$ in the given equations, we obtain

$$4p + 3y = 14 \quad \Rightarrow \quad 4p + 3y - 14 = 0 \quad (1)$$
$$3p - 4y = 23 \quad \Rightarrow \quad 3p - 4y - 23 = 0 \quad (2)$$

By cross-multiplication, we obtain

$$\frac{p}{-69-56} = \frac{y}{-42-(-92)} = \frac{1}{-16-9}$$
$$\frac{p}{-125} = \frac{y}{50} = \frac{-1}{25}$$
$$\frac{p}{-125} = \frac{-1}{25} \text{ and } \frac{y}{50} = \frac{-1}{25}$$
$$p = 5 \text{ and } y = -2$$
$$p = \frac{1}{x} = 5$$
$$x = \frac{1}{5}$$
$$y = -2$$

(iv) $\frac{5}{x-1}+\frac{1}{y-2}=2$

$\frac{6}{x-1}-\frac{3}{y-2}=1$

Putting $\frac{1}{x-1}=p$ and $\frac{1}{y-2}=q$ in the given equation, we obtain

$5p+q=2 \quad (1)$

$6p-3q=1 \quad (2)$

Multiplying equation (1) by 3, we obtain

$15p+3q=6 \quad (3)$

Adding (2) an (3), we obtain

$21p=7$

$p=\frac{1}{3}$

Putting this value in equation (1), we obtain

$$5\times\frac{1}{3}+q=2$$

$$q=2-\frac{5}{3}=\frac{1}{3}$$

$$p=\frac{1}{x-1}=\frac{1}{3}$$

$$\Rightarrow x-1=3$$

$$\Rightarrow x=4$$

$$q=\frac{1}{y-2}=\frac{1}{3}$$

$$y-2=3$$

$$y=5$$

$$\therefore x=4, y=5$$

(v) $\dfrac{7x-2y}{xy}=5$

$\dfrac{7}{y}-\dfrac{2}{x}=5 \qquad (1)$

$\dfrac{8x+7y}{xy}=15$

$\dfrac{8}{y}+\dfrac{7}{x}=15 \qquad (2)$

Putting $\dfrac{1}{x}=p$ and $\dfrac{1}{y}=q$ in the given equation, we obtain

$-2p+7q=5 \quad \Rightarrow \quad -2p+7q-5=0 \qquad (3)$

$7p+8q=15 \quad \Rightarrow \quad 7p+8q-15=0 \qquad (4)$

By cross-multiplication method, we obtain

$$\frac{p}{-105-(-40)}=\frac{q}{-35-30}=\frac{1}{-16-49}$$

$$\frac{p}{-65}=\frac{q}{-65}=\frac{1}{-65}$$

$$\frac{p}{-65}=\frac{1}{-65} \text{ and } \frac{q}{-65}=\frac{1}{-65}$$

$p=1$ and $q=1$

$p=\dfrac{1}{x}=1 \qquad q=\dfrac{1}{y}=1$

$x=1 \qquad y=1$

(vi) $6x+3y=6xy$

$\Rightarrow \dfrac{6}{y}+\dfrac{3}{x}=6 \qquad (1)$

$2x+4y=5xy$

$\dfrac{2}{y}+\dfrac{4}{x}=5 \qquad (2)$

Putting $\frac{1}{x} = p$ and $\frac{1}{y} = q$

$3p + 6q - 6 = 0$

$4p + 2q - 5 = 0$

By cross-multiplication method, we obtain

$$\frac{p}{-30-(-12)} = \frac{q}{-24-(-15)} = \frac{1}{6-24}$$

$$\frac{p}{-18} = \frac{q}{-9} = \frac{1}{-18}$$

$$\frac{p}{-18} = \frac{1}{-18} \text{ and } \frac{q}{-9} = \frac{1}{-18}$$

$$p = 1 \quad \text{and} \quad q = \frac{1}{2}$$

$$p = \frac{1}{x} = 1 \qquad q = \frac{1}{y} = \frac{1}{2}$$

$$x = 1 \qquad y = 2$$

Hence, $x = 1$, $y = 2$

$$\text{(vii)} \quad \frac{10}{x+y} + \frac{2}{x-y} = 4$$

$$\frac{15}{x+y} - \frac{5}{x-y} = -2$$

Putting $\frac{1}{x+y} = p$ and $\frac{1}{x-y} = q$ in the given equations, we obtain

$$10p + 2q = 4 \quad \Rightarrow \quad 10p + 2q - 4 = 0 \quad (1)$$

$$15p - 5q = -2 \quad \Rightarrow \quad 15p - 5q + 2 = 0 \quad (2)$$

Using cross-multiplication method, we obtain

$$\frac{p}{4-20}=\frac{q}{-60-(20)}=\frac{1}{-50-30}$$

$$\frac{p}{-16}=\frac{q}{-80}=\frac{1}{-80}$$

$$\frac{p}{-16}=\frac{1}{-80} \text{ and } \frac{q}{-80}=\frac{1}{-80}$$

$$p=\frac{1}{5} \text{ and } q=1$$

$$p=\frac{1}{x+y}=\frac{1}{5} \text{ and } q=\frac{1}{x-y}=1$$

$$x+y=5 \qquad (3)$$

$$\text{and } x-y=1 \qquad (4)$$

Adding equation (3) and (4), we obtain

$$2x=6$$

$$x=3 \qquad (5)$$

Substituting in equation (3), we obtain y

= 2

Hence, x = 3, y = 2

$$(\text{viii}) \quad \frac{1}{3x+y}+\frac{1}{3x-y}=\frac{3}{4}$$

$$\frac{1}{2(3x+y)}-\frac{1}{2(3x-y)}=\frac{-1}{8}$$

Putting $\frac{1}{3x+y}=p$ and $\frac{1}{3x-y}=q$ in these equations,

$$p+q=\frac{3}{4} \qquad (1)$$

$$\frac{p}{2}-\frac{q}{2}=\frac{-1}{8}$$

$$p-q=\frac{-1}{4} \qquad (2)$$

Adding (1) and (2), we obtain

$$2p = \frac{3}{4} - \frac{1}{4}$$

$$2p = \frac{1}{2}$$

$$p = \frac{1}{4}$$

Substituting in (2), we obtain

$$\frac{1}{4} - q = \frac{-1}{4}$$

$$q = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$$

$$p = \frac{1}{3x+y} = \frac{1}{4}$$

$$3x + y = 4 \qquad (3)$$

$$q = \frac{1}{3x-y} = \frac{1}{2}$$

$$3x - y = 2 \qquad (4)$$

Adding equations (3) and (4), we obtain

$$6x = 6$$

$$x = 1 \qquad (5)$$

Substituting in (3), we obtain

$$3(1) + y = 4$$

$$y = 1$$

Hence, $x = 1$, $y = 1$

Question 2:

Formulate the following problems as a pair of equations, and hence find their solutions:

(i) Ritu can row downstream 20 km in 2 hours, and upstream 4 km in 2 hours. Find her speed of rowing in still water and the speed of the current.

(ii) 2 women and 5 men can together finish an embroidery work in 4 days, while 3 women and 6 men can finish it in 3 days. Find the time taken by 1 woman alone to finish the work, and also that taken by 1 man alone.

(iii) Roohi travels 300 km to her home partly by train and partly by bus. She takes 4 hours if she travels 60 km by train and remaining by bus. If she travels 100 km by train and the remaining by bus, she takes 10 minutes longer. Find the speed of the train and the bus separately.

Answer 2:

(i) Let the speed of Ritu in still water and the speed of stream be *x* km/h and *y* km/h respectively.

Speed of Ritu while rowing

Upstream = x − y km/h

Downstream = x + y km/h

According to question,

$$2(x+y)=20$$

$$\Rightarrow x+y=10 \qquad (1)$$

$$2(x-y)=4$$

$$\Rightarrow x-y=2 \qquad (2)$$

Adding equation (1) and (2), we obtain

$$2x=12 \quad \Rightarrow \quad x=6$$

Putting this in equation (1), we obtain $y = 4$

Hence, Ritu's speed in still water is 6 km/h and the speed of the current is 4 km/h.

(ii) Let the number of days taken by a woman and a man be *x* and *y* respectively.

Therefore, work done by a woman in 1 day = 1/x

Work done by a man in 1 day = 1/y

According to the question,

$$4\left(\frac{2}{x}+\frac{5}{y}\right)=1$$

$$\frac{2}{x}+\frac{5}{y}=\frac{1}{4}$$

$$3\left(\frac{3}{x}+\frac{6}{y}\right)=1$$

$$\frac{3}{x}+\frac{6}{y}=\frac{1}{3}$$

Putting $\frac{1}{x}=p$ and $\frac{1}{y}=q$

$$2p+5q=\frac{1}{4}$$

$$\Rightarrow \quad 8p+20q=1$$

$$3p+6q=\frac{1}{3}$$

$$\Rightarrow 9p+18q=1$$

By cross-multiplication, we obtain

$$\frac{p}{-20-(-18)}=\frac{q}{-9-(-8)}=\frac{1}{144-180}$$

$$\frac{p}{-2}=\frac{q}{-1}=\frac{1}{-36}$$

$$\frac{p}{-2}=\frac{-1}{36} \text{ and } \frac{q}{-1}=\frac{1}{-36}$$

$$p=\frac{1}{18} \text{ and } q=\frac{1}{36}$$

$$p=\frac{1}{x}=\frac{1}{18} \text{ and } q=\frac{1}{y}=\frac{1}{36}$$

$$x=18 \qquad y=36$$

Hence, number of days taken by a woman = 18

Number of days taken by a man = 36

(iii) Let the speed of train and bus be *u* km/h and *v* km/h respectively.

According to the given information,

$$\frac{60}{u}+\frac{240}{v}=4 \qquad (1)$$

$$\frac{100}{u}+\frac{200}{v}=\frac{25}{6} \qquad (2)$$

Putting $\frac{1}{u}=p$ and $\frac{1}{v}=q$ in these equations, we obtain

$$60p+240q=4 \qquad (3)$$

$$100p+200q=\frac{25}{6}$$

$$600p+1200q=25 \qquad (4)$$

Multiplying equation (3) by 10, we obtain

$$600p+2400q=40 \qquad (5)$$

Subtracting equation (4) from (5), we obtain

$$1200q=15$$

$$q=\frac{15}{1200}=\frac{1}{80} \qquad (6)$$

Substituting in equation (3), we obtain

$$60p + 3 = 4$$
$$60p = 1$$
$$p = \frac{1}{60}$$
$$p = \frac{1}{u} = \frac{1}{60} \quad \text{and} \quad q = \frac{1}{v} = \frac{1}{80}$$
$$u = 60 \text{ km/h} \quad \text{and} \quad v = 80 \text{ km/h}$$

Hence, speed of train = 60 km/h

Speed of bus = 80 km/h

Mathematics

(Chapter – 3) (Linear equations in two variables)

(Class – X)

Exercise 3.7

Question 1:

The ages of two friends Ani and Biju differ by 3 years. Ani's father Dharam is twice as old as Ani and Biju is twice as old as his sister Cathy. The ages of Cathy and Dharam differs by 30 years. Find the ages of Ani and Biju.

Answer 1:

The difference between the ages of Biju and Ani is 3 years. Either Biju is 3 years older than Ani or Ani is 3 years older than Biju. However, it is obvious that in both cases, Ani's father's age will be 30 years more than that of Cathy's age.

Let the age of Ani and Biju be x and y years respectively.

Therefore, age of Ani's father, Dharam = $2 \times x = 2x$ years

And age of Biju's sister Cathy y/2 years

By using the information given in the question,

Case (I)

When Ani is older than Biju by 3 years, $x - y = 3$ ....(i)

$$2x - \frac{y}{2} = 30$$

$4x - y = 60$ ....(ii)

Subtracting (i) from (ii), we obtain

$3x = 60 - 3 = 57$

$$x = \frac{57}{3} = 19$$

Therefore, age of Ani = 19 years

And age of Biju = 19 – 3 = 16 years

Case (II)

When Biju is older than Ani, $y - x = 3$ ....(i)

$$2x - \frac{y}{2} = 30$$

$4x - y = 60$ ....(ii)

Adding (i) and (ii), we obtain

$3x = 63$

$x = 21$

Therefore, age of Ani = 21 years And

age of Biju = 21 + 3 = 24 years

Question 2:

One says, "Give me a hundred, friend! I shall then become twice as rich as you". The other replies, "If you give me ten, I shall be six times as rich as you". Tell me what is the amount of their (respective) capital? [From the Bijaganita of Bhaskara II)

[**Hint:** $x + 100 = 2(y - 100)$, $y + 10 = 6(x - 10)$]

Answer 2:

Let those friends were having Rs x and y with them.

Using the information given in the question, we obtain

$x + 100 = 2(y - 100)$ $x + 100 = 2y - 200$

or $x - 2y = -300$ ....(i)

And,

$6(x - 10) = (y + 10)$

Or $6x - 60 = y + 10$

Or $6x - y = 70$ ....(ii)

Multiplying equation (ii) by 2, we obtain

$12x - 2y = 140$ (iii)

Subtracting equation (i) from equation (iii), we obtain

$11x = 140 + 300$

$11x = 440$ $x =$

40

Using this in equation (i), we obtain

$40 - 2y = -300$

$40 + 300 = 2y$

$2y = 340$

$y = 170$

Therefore, those friends had Rs 40 and Rs 170 with them respectively.

Question 3:

A train covered a certain distance at a uniform speed. If the train would have been 10 km/h faster, it would have taken 2 hours less than the scheduled time. And if the train were slower by 10 km/h; it would have taken 3 hours more than the scheduled time. Find the distance covered by the train.

Answer 3:

Let the speed of the train be x km/h and the time taken by train to travel the given distance be t hours and the distance to travel was d km. We know that,

$$\text{Speed} = \frac{\text{Distance travelled}}{\text{Time taken to travel that distance}}$$

$$x = \frac{d}{t}$$

Or, $d = xt$ .... (i)

Using the information given in the question, we obtain

$$(x+10) = \frac{d}{(t-2)}$$

$$(x+10)(t-2) = d$$

$$xt + 10t - 2x - 20 = d$$

By using equation (i), we obtain

$-2x + 10t = 20$ .... (ii)

$$(x-10) = \frac{d}{(t+3)}$$

$$(x-10)(t+3) = d$$

$$xt - 10t + 3x - 30 = d$$

By using equation (i), we obtain

$3x - 10t = 30$ (iii)

Adding equations (ii) and (iii), we obtain

$x = 50$

Using equation (ii), we obtain

$(-2) \times (50) + 10t = 20$

$-100 + 10t = 20$ $10t = 120$ $t = 12$ hours

From equation (i), we obtain

Distance to travel = $d = xt$

$= 50 \times 12$

$= 600$ km

Hence, the distance covered by the train is 600 km.

Question 4:

The students of a class are made to stand in rows. If 3 students are extra in a row, there would be 1 row less. If 3 students are less in a row, there would be 2 rows more. Find the number of students in the class.

Answer 4:

Let the number of rows be x and number of students in a row be y.

Total students of the class

= Number of rows × Number of students in a row

$= xy$

Using the information given in the question,

Condition 1

Total number of students $= (x - 1)(y + 3)$

Or $xy = (x - 1)(y + 3) = xy - y + 3x - 3$

Or $3x - y - 3 = 0$

Or $3x - y = 3$ ……………………………………. (i)

Condition 2

Total number of students $= (x + 2)(y - 3)$

Or $xy = xy + 2y - 3x - 6$

Or $3x - 2y = -6$ ……………………………….(ii)

Subtracting equation (ii) from (i),

$(3x - y) - (3x - 2y) = 3 - (-6)$

$-y + 2y = 3 + 6$ $y = 9$

By using equation (i), we obtain

$3x - 9 = 3$

$3x = 9 + 3 = 12$

$x = 4$

Number of rows $= x = 4$

Number of students in a row $= y = 9$

Number of total students in a class $= xy = 4 \times 9 = 36$

Question 5:

In a ΔABC, $\angle C = 3\angle B = 2(\angle A + \angle B)$. Find the three angles.

Answer 5:

Given that,

$\angle C = 3\angle B = 2(\angle A + \angle B)$

$3\angle B = 2(\angle A + \angle B)$

$3\angle B = 2\angle A + 2\angle B$

$\angle B = 2\angle A$

$2\angle A - \angle B = 0$ ………………………………… (*i*)

We know that the sum of the measures of all angles of a triangle is 180°. Therefore,

$\angle A + \angle B + \angle C = 180°$

$\angle A + \angle B + 3\angle B = 180°$

$\angle A + 4\angle B = 180°$ …………………………….. (*ii*)

Multiplying equation (*i*) by 4, we obtain

$8\angle A - 4\angle B = 0$ …………………………….… (*iii*)

Adding equations (*ii*) and (*iii*), we obtain

$9\angle A = 180°$

$\angle A = 20°$

From equation (*ii*), we obtain

$20° + 4\angle B = 180°$

$4\angle B = 160°$

$\angle B = 40°$

$\angle C = 3\angle B$

$= 3 \times 40° = 120°$

Therefore, $\angle A$, $\angle B$, $\angle C$ are 20°, 40°, and 120° respectively.

Question 6:

Draw the graphs of the equations $5x - y = 5$ and $3x - y = 3$. Determine the coordinates of the vertices of the triangle formed by these lines and the y axis.

Answer 6:

$5x - y = 5$

Or, $y = 5x - 5$

The solution table will be as follows.

x	0	1	2
y	−5	0	5

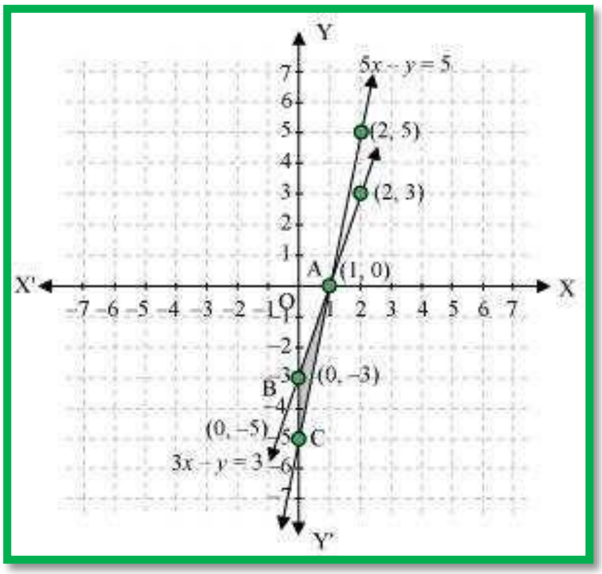
$3x - y = 3$

Or, $y = 3x - 3$

The solution table will be as follows.

x	0	1	2
y	− 3	0	3

The graphical representation of these lines will be as follows.



It can be observed that the required triangle is ΔABC formed by these lines and y-axis.

The coordinates of vertices are A (1, 0), B (0, − 3), C (0, − 5).

Question 7:

Solve the following pair of linear equations.

(i). $px + qy = p - q$, $qx - py = p + q$ (ii). $ax + by = c$, $bx + ay = 1 + c$

(iii). $x/a - y/b = 0$, $ax + by = a^2 + b^2$

(iv). $(a - b)x + (a + b)y = a^2 - 2ab - b^2$

$(a + b)(x + y) = a^2 + b^2$

(v) $152x - 378y = -74$

$-378x + 152y = -604$

Answer 7:

(i) $px + qy = p - q$ ………………….… (1)

$qx - py = p + q$ ………………….… (2)

Multiplying equation (1) by p and equation (2) by q, we obtain

$p^2x + pqy = p^2 - pq$ ………………………… (3)

$q^2x - pqy = pq + q^2$ ……………………..… (4)

Adding equations (3) and (4), we obtain

$p^2x + q^2x = p^2 + q^2$

$(p^2 + q^2)x = p^2 + q^2$

$$x = \frac{p^2 + q^2}{p^2 + q^2} = 1$$

From equation (1), we obtain

$p(1) + qy = p - q$ $qy = -q$ $y = -1$

(ii)$ax + by = c$ …………….. (1)

$bx + ay = 1 + c$ ………. (2)

Multiplying equation (1) by a and equation (2) by b, we obtain

$a^2x + aby = ac$ ………………… (3)

$b^2x + aby = b + bc$ ………….. (4)

Subtracting equation (4) from equation (3),

$(a^2 - b^2)x = ac - bc - b$

$$x = \frac{c(a-b)-b}{a^2-b^2}$$

From equation (1), we obtain

$ax + by = c$

$$a\left\{\frac{c(a-b)-b}{a^2-b^2}\right\}+by=c$$

$$\frac{ac(a-b)-ab}{a^2-b^2}+by=c$$

$$by=c-\frac{ac(a-b)-ab}{a^2-b^2}$$

$$by=\frac{a^2c-b^2c-a^2c+abc+ab}{a^2-b^2}$$

$$by=\frac{abc-b^2c+ab}{a^2-b^2}$$

$$by=\frac{bc(a-b)+ab}{a^2-b^2}$$

$$y=\frac{c(a-b)+a}{a^2-b^2}$$

(iii) $\frac{x}{a}-\frac{y}{b}=0$

Or, $bx - ay = 0$ …………… (1)

$ax + by = a^2 + b^2$ ………….. (2)

Multiplying equation (1) and (2) by b and a respectively, we obtain

$b^2x - aby = 0$ ……………..…… (3)

$a^2x + aby = a^3 + ab^2$ ……………… (4)

Adding equations (3) and (4), we obtain

$b^2x + a^2x = a^3 + ab^2$

$x(b^2 + a^2) = a(a^2 + b^2)$ $x = a$

By using (1), we obtain

$b(a) - ay = 0$

$ab - ay = 0$

$ay = ab$

$y = b$

(iv) $(a - b)x + (a + b)y = a^2 - 2ab - b^2$ ………………… (1)

$(a + b)(x + y) = a^2 + b^2$

$(a + b)x + (a + b)y = a^2 + b^2$ ………………….… (2)

Subtracting equation (2) from (1), we obtain

$(a - b)x - (a + b)x = (a^2 - 2ab - b^2) - (a^2 + b^2)$

$(a - b - a - b)x = -2ab - 2b^2$

$-2bx = -2b(a + b)$

$x = a + b$

Using equation (1), we obtain

$(a - b)(a + b) + (a + b)y = a^2 - 2ab - b^2$

$a^2 - b^2 + (a + b)y = a^2 - 2ab - b^2$

$(a + b)y = -2ab$

$$y = \frac{-2ab}{a+b}$$

(v) $152x - 378y = -74$

$76x - 189y = -37$

$$x = \frac{189y - 37}{76} \quad \text{……………..… (1)}$$

$-378x + 152y = -604$

$-189x + 76y = -302$ … (2)

Substituting the value of x in equation (2), we obtain

$$-189\left(\frac{189y-37}{76}\right) + 76y = -302$$

$-(189)^2 y + 189 \times 37 + (76)^2 y = -302 \times 76$

$189 \times 37 + 302 \times 76 = (189)^2 y - (76)^2 y$

$6993 + 22952 = (189 - 76)(189 + 76)y$

$29945 = (113)(265)y$

$y = 1$

From equation (1), we obtain

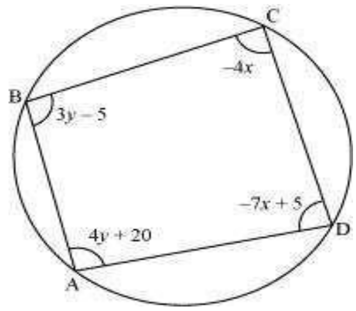
$$x = \frac{189(1) - 37}{76}$$

$$x = \frac{189 - 37}{76} = \frac{152}{76}$$

$$x = 2$$

Question 8:

ABCD is a cyclic quadrilateral finds the angles of the cyclic quadrilateral.



Answer 8:

We know that the sum of the measures of opposite angles in a cyclic quadrilateral is 180°.

Therefore, $\angle A + \angle C = 180$

$4y + 20 - 4x = 180$

$-4x + 4y = 160$

$x - y = -40$ ……………………..(*i*)

Also, $\angle B + \angle D = 180$

$3y - 5 - 7x + 5 = 180$

$-7x + 3y = 180$ …………………..(*ii*)

Multiplying equation (*i*) by 3, we obtain

$3x - 3y = -120$ …………………..(*iii*)

Adding equations (*ii*) and (*iii*), we obtain

$-7x + 3x = 180 - 120$

$-4x = 60$

$x = -15$

By using equation (*i*), we obtain

$x - y = -40$

$-15 - y = -40$

$y = -15 + 40 = 25$

$\angle A = 4y + 20 = 4(25) + 20 = 120°$

$\angle B = 3y - 5 = 3(25) - 5 = 70°$

$\angle C = -4x = -4(-15) = 60°$

$\angle D = -7x + 5 = -7(-15) + 5 = 110°$

Mathematic

(Chapter – 4) (Quadratic Equations)

(Class X)

Exercise 4.1

Question 1:

Check whether the following are quadratic equations:

(i) $(x+1)^2 = 2(x-3)$ (ii) $x^2 - 2x = (-2)(3-x)$

(iii) $(x-2)(x+1) = (x-1)(x+3)$ (iv) $(x-3)(2x+1) = x(x+5)$

(v) $(2x-1)(x-3) = (x+5)(x-1)$ (vi) $x^2 + 3x + 1 = (x-2)^2$

(vii) $(x+2)^3 = 2x(x^2-1)$ (viii) $x^3 - 4x^2 - x + 1 = (x-2)^3$

Answer 1:

(i) $(x+1)^2 = 2(x-3) \Rightarrow x^2 + 2x + 1 = 2x - 6 \Rightarrow x^2 + 7 = 0$

It is of the form $ax^2 + bx + c = 0$.

Hence, the given equation is a quadratic equation.

(ii) $x^2 - 2x = (-2)(3-x) \Rightarrow x^2 - 2x = -6 + 2x \Rightarrow x^2 - 4x + 6 = 0$

It is of the form $ax^2 + bx + c = 0$.

Hence, the given equation is a quadratic equation.

(iii) $(x-2)(x+1) = (x-1)(x+3) \Rightarrow x^2 - x - 2 = x^2 + 2x - 3 \Rightarrow 3x - 1 = 0$

It is not of the form $ax^2 + bx + c = 0$.

Hence, the given equation is not a quadratic equation.

(iv) $(x-3)(2x+1) = x(x+5) \Rightarrow 2x^2 - 5x - 3 = x^2 + 5x \Rightarrow x^2 - 10x - 3 = 0$

It is of the form $ax^2 + bx + c = 0$.

Hence, the given equation is a quadratic equation.

(v) $(2x-1)(x-3) = (x+5)(x-1) \Rightarrow 2x^2 - 7x + 3 = x^2 + 4x - 5 \Rightarrow x^2 - 11x + 8 = 0$

It is of the form $ax^2 + bx + c = 0$.

Hence, the given equation is a quadratic equation.

(vi) $x^2 + 3x + 1 = (x-2)^2 \Rightarrow x^2 + 3x + 1 = x^2 + 4 - 4x \Rightarrow 7x - 3 = 0$

It is not of the form $ax^2 + bx + c = 0$.

Hence, the given equation is not a quadratic equation.

(vii) $(x+2)^3 = 2x(x^2-1) \Rightarrow x^3+8+6x^2+12x = 2x^3-2x \Rightarrow x^3-14x-6x^2-8=0$

It is not of the form $ax^2+bx+c=0$.

Hence, the given equation is not a quadratic equation.

(viii) $x^3-4x^3-x+1=(x-2)^3 \Rightarrow x^3-4x^2-x+1=x^3-8-6x^2+12x \Rightarrow 2x^2-13x+9=0$

It is of the form $ax^2+bx+c=0$.

Hence, the given equation is a quadratic equation.

Question 2:

Represent the following situations in the form of quadratic equations.

(i) The area of a rectangular plot is 528 m^2. The length of the plot (in metres) is one more than twice its breadth. We need to find the length and breadth of the plot.

(ii) The product of two consecutive positive integers is 306. We need to find the integers.

(iii) Rohan's mother is 26 years older than him. The product of their ages (in years) 3 years from now will be 360. We would like to find Rohan's present age.

(iv) A train travels a distance of 480 km at a uniform speed. If the speed had been 8 km/h less, then it would have taken 3 hours more to cover the same distance. We need to find the speed of the train.

Answer 2:

(i) Let the breadth of the plot be x m.

Hence, the length of the plot is $(2x + 1)$ m.

Area of a rectangle = Length × Breadth

∴ $528 = x(2x + 1)$

$\Rightarrow 2x^2+x-528=0$

(ii) Let the consecutive integers be x and $x + 1$.

It is given that their product is 306.

$\therefore\ x(x+1)=306 \Rightarrow x^2+x-306=0$

(iii) Let Rohan's age be *x*.

Hence, his mother's age = $x + 26$

3 years hence,

Rohan's age = $x + 3$

Mother's age = $x + 26 + 3 = x + 29$

It is given that the product of their ages after 3 years is 360.

$\therefore (x+3)(x+29)=360$

$\Rightarrow x^2+32x-273=0$

(iv) Let the speed of train be *x* km/h.

Time taken to travel 480 km = $\frac{480}{x}$ hrs

In second condition, let the speed of train = $(x-8)$ km/h

It is also given that the train will take 3 hours to cover the same distance.

Therefore, time taken to travel 480 km = $\left(\frac{480}{x}+3\right)$ hrs

Speed × Time = Distance

$(x-8)\left(\frac{480}{x}+3\right)=480$

$\Rightarrow 480+3x-\frac{3840}{x}-24=480$

$\Rightarrow 3x-\frac{3840}{x}=24$

$\Rightarrow 3x^2-24x+3840=0$

$\Rightarrow x^2-8x+1280=0$

Mathematic

(Chapter – 4) (Quadratic Equations)
(Class X)

Exercise 4.2

Question 1:

Find the roots of the following quadratic equations by factorisation:

(i) $x^2-3x-10=0$ (ii) $2x^2+x-6=0$

(iii) $\sqrt{2}x^2+7x+5\sqrt{2}=0$ (iv) $2x^2-x+\frac{1}{8}=0$

(v) $100x^2-20x+1=0$

Answer 1:

(i)
$$\begin{aligned}&x^2-3x-10\\&=x^2-5x+2x-10\\&=x(x-5)+2(x-5)\\&=(x-5)(x+2)\end{aligned}$$

Roots of this equation are the values for which $(x-5)(x+2)=0$

∴ $x-5=0$ or $x+2=0$

i.e., $x=5$ or $x=-2$

(ii)
$$\begin{aligned}&2x^2+x-6\\&=2x^2+4x-3x-6\\&=2x(x+2)-3(x+2)\\&=(x+2)(2x-3)\end{aligned}$$

Roots of this equation are the values for which $(x+2)(2x-3)=0$

∴ $x+2=0$ or $2x-3=0$

i.e., $x=-2$ or $x=3/2$

(iii)
$$\begin{aligned}&\sqrt{2}x^2+7x+5\sqrt{2}\\&=\sqrt{2}x^2+5x+2x+5\sqrt{2}\\&=x\left(\sqrt{2}x+5\right)+\sqrt{2}\left(\sqrt{2}x+5\right)\\&=\left(\sqrt{2}x+5\right)\left(x+\sqrt{2}\right)\end{aligned}$$

Roots of this equation are the values for which $\left(\sqrt{2}x+5\right)\left(x+\sqrt{2}\right)$ = 0

$\sqrt{2}x+5$ = 0 or $x+\sqrt{2}$ = 0

i.e., x = $\frac{-5}{\sqrt{2}}$ or x = $-\sqrt{2}$

(iv) $2x^2 - x + \frac{1}{8}$

$$= \frac{1}{8}\left(16x^2 - 8x + 1\right)$$
$$= \frac{1}{8}\left(16x^2 - 4x - 4x + 1\right)$$
$$= \frac{1}{8}\left(4x\left(4x-1\right) - 1\left(4x-1\right)\right)$$
$$= \frac{1}{8}\left(4x-1\right)^2$$

Roots of this equation are the values for which $\left(4x-1\right)^2$ = 0

Therefore, $\left(4x-1\right)=0$ or $\left(4x-1\right)=0$

$$x = \frac{1}{4} \text{ or } x = \frac{1}{4}$$

(v) $100x^2 - 20x + 1$

$$= 100x^2 - 10x - 10x + 1$$
$$= 10x\left(10x-1\right) - 1\left(10x-1\right)$$
$$= \left(10x-1\right)^2$$

Roots of this equation are the values for which $\left(10x-1\right)^2$ = 0

Therefore, $\left(10x-1\right)=0$ or $\left(10x-1\right)=0$

$$x = \frac{1}{10} \text{ or } x = \frac{1}{10}$$

Question 2:

(i) John and Jivanti together have 45 marbles. Both of them lost 5 marbles each, and the product of the number of marbles they now have is 124. Find out how many marbles they had to start with. **(ii)** A cottage industry produces a certain number of toys in a day. The cost of production of each toy (in rupees) was found to be 55 minus the number of toys produced in a day. On a particular day, the total cost of production was Rs 750. Find out the number of toys produced on that day.

Answer 2:

(i) Let the number of John's marbles be x.

Therefore, number of Jivanti's marble = $45 - x$

After losing 5 marbles,

Number of John's marbles = $x - 5$

Number of Jivanti's marbles = $45 - x - 5 = 40 - x$

It is given that the product of their marbles is 124.

$$\therefore (x-5)(40-x)=124$$
$$\Rightarrow x^2-45x+324=0$$
$$\Rightarrow x^2-36x-9x+324=0$$
$$\Rightarrow x(x-36)-9(x-36)=0$$
$$\Rightarrow (x-36)(x-9)=0$$

X – 36 = 0 or $x - 9 = 0$

i.e., $x = 36$ or $x = 9$

If the number of John's marbles = 36,

Then, number of Jivanti's marbles = 45 − 36 = 9

If number of John's marbles = 9,

Then, number of Jivanti's marbles = 45 − 9 = 36

(ii) Let the number of toys produced be x.

∴ Cost of production of each toy = Rs $(55 - x)$

It is given that, total production of the toys = Rs 750

$\therefore x(55-x)=750$
$\Rightarrow x^2-55x+750=0$
$\Rightarrow x^2-25x-30x+750=0$
$\Rightarrow x(x-25)-30(x-25)=0$
$\Rightarrow (x-25)(x-30)=0$

x − 25 = 0 or x − 30 = 0

i.e., x = 25 or x = 30

Hence, the number of toys will be either 25 or 30.

Question 3:

Find two numbers whose sum is 27 and product is 182.

Answer 3:

Let the first number be x and the second number is 27 − x.

Therefore, their product = x (27 − x)

It is given that the product of these numbers is 182.

Therefore, $x(27-x)=182$
$\Rightarrow x^2-27x+182=0$
$\Rightarrow x^2-13x-14x+182=0$
$\Rightarrow x(x-13)-14(x-13)=0$
$\Rightarrow (x-13)(x-14)=0$

x − 13 = 0 or x − 14 = 0

i.e., x = 13 or x = 14

If first number = 13, then

Other number = 27 − 13 = 14

If first number = 14, then

Other number = 27 − 14 = 13 Therefore,

the numbers are 13 and 14.

Question 4:

Find two consecutive positive integers, sum of whose squares is 365.

Answer 4:

Let the consecutive positive integers be x and $x + 1$.

Given that $x^2 + (x+1)^2 = 365$

$\Rightarrow x^2 + x^2 + 1 + 2x = 365$

$\Rightarrow 2x^2 + 2x - 364 = 0$

$\Rightarrow x^2 + x - 182 = 0$

$\Rightarrow x^2 + 14x - 13x - 182 = 0$

$\Rightarrow x(x+14) - 13(x+14) = 0$

$\Rightarrow (x+14)(x-13) = 0$

Either $x + 14 = 0$ or $x - 13 = 0$, i.e., $x = -14$ or $x = 13$

Since the integers are positive, x can only be 13.

$\therefore x + 1 = 13 + 1 = 14$

Therefore, two consecutive positive integers will be 13 and 14.

Question 5:

The altitude of a right triangle is 7 cm less than its base. If the hypotenuse is 13 cm, find the other two sides.

Answer 5:

Let the base of the right triangle be x cm.

Its altitude = $(x - 7)$ cm

From pythagoras theorem,

$\text{Base}^2 + \text{Altitude}^2 = \text{Hypotenuse}^2$

$\therefore x^2 + (x-7)^2 = 13^2$

$\Rightarrow x^2 + x^2 + 49 - 14x = 169$

$\Rightarrow 2x^2 - 14x - 120 = 0$

$\Rightarrow x^2 - 7x - 60 = 0$

$\Rightarrow x^2 - 12x + 5x - 60 = 0$

$\Rightarrow x(x-12) + 5(x-12) = 0$

$\Rightarrow (x-12)(x+5) = 0$

Either $x - 12 = 0$ or $x + 5 = 0$, i.e., $x = 12$ or $x = -5$

Since sides are positive, x can only be 12.

Therefore, the base of the given triangle is 12 cm and the altitude of this triangle will be $(12 - 7)$ cm = 5 cm.

Question 6:

A cottage industry produces a certain number of pottery articles in a day. It was observed on a particular day that the cost of production of each article (in rupees) was 3 more than twice the number of articles produced on that day. If the total cost of production on that day was Rs 90, find the number of articles produced and the cost of each article.

Answer 6:

Let the number of articles produced be x.

Therefore, cost of production of each article = Rs $(2x + 3)$

It is given that the total production is Rs 90.

$$\therefore x(2x+3)=90$$
$$\Rightarrow 2x^2+3x-90=0$$
$$\Rightarrow 2x^2+15x-12x-90=0$$
$$\Rightarrow x(2x+15)-6(2x+15)=0$$
$$\Rightarrow (2x+15)(x-6)=0$$

Either $2x + 15 = 0$ or $x - 6 = 0$, i.e., $x = \frac{-15}{2}$ or $x = 6$

As the number of articles produced can only be a positive integer, therefore, x can only be 6.

Hence, number of articles produced = 6

Cost of each article = $2 \times 6 + 3$ = Rs 15

Mathematic

(Chapter – 4) (Quadratic Equations)
(Class X)

Exercise 4.3

Question 1:

Find the roots of the following quadratic equations, if they exist, by the method of completing the square:

(i) $2x^2-7x+3=0$ (ii) $2x^2+x-4=0$

(iii) $4x^2+4\sqrt{3x}+3=0$ (iv) $2x^2+x+4=0$

Answer 1:

(i) $2x^2-7x+3=0$

$\Rightarrow 2x^2-7x=-3$

On dividing both sides of the equation by 2, we obtain

$\Rightarrow x^2-\frac{7}{2}x=-\frac{3}{2}$

$\Rightarrow x^2-2\times x\times\frac{7}{4}=-\frac{3}{2}$

On adding $\left(\frac{7}{4}\right)^2$ to both sides of equation, we obtain

$\Rightarrow (x)^2-2\times x\times\frac{7}{4}+\left(\frac{7}{4}\right)^2=\left(\frac{7}{4}\right)^2-\frac{3}{2}$

$\Rightarrow \left(x-\frac{7}{4}\right)^2=\frac{49}{16}-\frac{3}{2}$

$\Rightarrow \left(x-\frac{7}{4}\right)^2=\frac{25}{16}$

$\Rightarrow \left(x-\frac{7}{4}\right)=\pm\frac{5}{4}$

$\Rightarrow x=\frac{7}{4}\pm\frac{5}{4}$

$\Rightarrow x=\frac{7}{4}+\frac{5}{4}$ or $x=\frac{7}{4}-\frac{5}{4}$

$\Rightarrow x=\frac{12}{4}$ or $x=\frac{2}{4}$

$\Rightarrow x=3$ or $\frac{1}{2}$

(ii) $2x^2+x-4=0$

$\Rightarrow 2x^2+x=4$

On dividing both sides of the equation by 2, we obtain

$\Rightarrow x^2+\frac{1}{2}x=2$

On adding $\left(\frac{1}{4}\right)^2$ to both sides of the equation, we obtain

$\Rightarrow (x)^2+2\times x\times\frac{1}{4}+\left(\frac{1}{4}\right)^2=2+\left(\frac{1}{4}\right)^2$

$\Rightarrow \left(x+\frac{1}{4}\right)^2=\frac{33}{16}$

$\Rightarrow x+\frac{1}{4}=\pm\frac{\sqrt{33}}{4}$

$\Rightarrow x=\pm\frac{\sqrt{33}}{4}-\frac{1}{4}$

$\Rightarrow x=\frac{\pm\sqrt{33}-1}{4}$

$\Rightarrow x=\frac{\sqrt{33}-1}{4}$ or $\frac{-\sqrt{33}-1}{4}$

(iii) $4x^2+4\sqrt{3}x+3=0$

$\Rightarrow (2x)^2+2\times 2x\times\sqrt{3}+\left(\sqrt{3}\right)^2=0$

$\Rightarrow \left(2x+\sqrt{3}\right)^2=0$

$\Rightarrow \left(2x+\sqrt{3}\right)=0$ and $\left(2x+\sqrt{3}\right)=0$

$\Rightarrow x=\frac{-\sqrt{3}}{2}$ and $x=\frac{-\sqrt{3}}{2}$

(iv) $2x^2 + x + 4 = 0$

$\Rightarrow 2x^2 + x = -4$

On dividing both sides of the equation by 2, we obtain

$\Rightarrow x^2 + \frac{1}{2}x = -2$

$\Rightarrow x^2 + 2 \times x \times \frac{1}{4} = -2$

On adding $\left(\frac{1}{4}\right)^2$ to both sides of the equation, we obtain

$\Rightarrow (x)^2 + 2 \times x \times \frac{1}{4} + \left(\frac{1}{4}\right)^2 = \left(\frac{1}{4}\right)^2 - 2$

$\Rightarrow \left(x + \frac{1}{4}\right)^2 = \frac{1}{16} - 2$

$\Rightarrow \left(x + \frac{1}{4}\right)^2 = -\frac{31}{16}$

However, the square of a number cannot be negative.

Therefore, there is no real root for the given equation.

Question 2:

Find the roots of the quadratic equations given in Q.1 above by applying the quadratic formula.

Answer 2: (i) $2x^2 - 7x + 3 = 0$

On comparing this equation with $ax^2 + bx + c = 0$, we obtain

$a = 2, b = -7, c = 3$

By using quadratic formula, we obtain

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\Rightarrow x = \frac{7 \pm \sqrt{49 - 24}}{4}$$

$$\Rightarrow x = \frac{7 \pm \sqrt{25}}{4}$$

$$\Rightarrow x = \frac{7 \pm 5}{4}$$

$$\Rightarrow x = \frac{7+5}{4} \text{ or } \frac{7-5}{4}$$

$$\Rightarrow x = \frac{12}{4} \text{ or } \frac{2}{4}$$

$$\therefore x = 3 \text{ or } \frac{1}{2}$$

(ii) $2x^2 + x - 4 = 0$

On comparing this equation with $ax^2 + bx + c = 0$, we obtain

$a = 2, b = 1, c = -4$

By using quadratic formula, we obtain

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\Rightarrow x = \frac{-1 \pm \sqrt{1+32}}{4}$$

$$\Rightarrow x = \frac{-1 \pm \sqrt{33}}{4}$$

$$\therefore\ x = \frac{-1+\sqrt{33}}{4} \text{ or } \frac{-1-\sqrt{33}}{4}$$

(iii) $4x^2 + 4\sqrt{3}x + 3 = 0$

On comparing this equation with $ax^2 + bx + c = 0$, we obtain

$a = 4, b = 4\sqrt{3}, c = 3$

By using quadratic formula, we obtain

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\Rightarrow x = \frac{-4\sqrt{3} \pm \sqrt{48-48}}{8}$$

$$\Rightarrow x = \frac{-4\sqrt{3} \pm 0}{8}$$

$$\therefore\ x = \frac{-\sqrt{3}}{2} \text{ or } \frac{-\sqrt{3}}{2}$$

(iv) $2x^2 + x + 4 = 0$

On comparing this equation with $ax^2 + bx + c = 0$, we obtain

$a = 2, b = 1, c = 4$

By using quadratic formula, we obtain

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\Rightarrow x = \frac{-1 \pm \sqrt{1-32}}{4}$$

$$\Rightarrow x = \frac{-1 \pm \sqrt{-31}}{4}$$

However, the square of a number cannot be negative.

Therefore, there is no real root for the given equation.

Question 3:

Find the roots of the following equations:

(i) $x - \frac{1}{x} = 3, x \neq 0$ (ii) $\frac{1}{x+4} - \frac{1}{x-7} = \frac{11}{30}, x \neq -4, 7$

Answer 3:

(i) $x - \frac{1}{x} = 3 \Rightarrow x^2 - 3x - 1 = 0$

On comparing this equation with $ax^2 + bx + c = 0$, we obtain

$a = 1, b = -3, c = -1$

By using quadratic formula, we obtain

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\Rightarrow x = \frac{3 \pm \sqrt{9+4}}{2}$$

$$\Rightarrow x = \frac{3 \pm \sqrt{13}}{2}$$

Therefore, $x = \frac{3+\sqrt{13}}{2}$ or $\frac{3-\sqrt{13}}{2}$

(ii) $\frac{1}{x+4} - \frac{1}{x-7} = \frac{11}{30}$

$$\Rightarrow \frac{x-7-x-4}{(x+4)(x-7)} = \frac{11}{30}$$

$$\Rightarrow \frac{-11}{(x+4)(x-7)} = \frac{11}{30}$$

$$\Rightarrow (x+4)(x-7) = -30$$

$$\Rightarrow x^2 - 3x - 28 = -30$$

$$\Rightarrow x^2 - 3x + 2 = 0$$

$$\Rightarrow x^2 - 2x - x + 2 = 0$$

$$\Rightarrow x(x-2) - 1(x-2) = 0$$

$$\Rightarrow (x-2)(x-1) = 0$$

$$\Rightarrow x = 1 \text{ or } 2$$

Question 4:

The sum of the reciprocals of Rehman's ages, (in years) 3 years ago and 5 years from now is 1/3. Find his present age.

Answer 4:

Let the present age of Rehman be *x* years.

Three years ago, his age was (x − 3) years.

Five years hence, his age will be (x + 5) years.

It is given that the sum of the reciprocals of Rehman's ages 3 years ago and 5 years from now is 1/3.

$$\therefore \frac{1}{x-3}+\frac{1}{x+5}=\frac{1}{3}$$
$$\frac{x+5+x-3}{(x-3)(x+5)}=\frac{1}{3}$$
$$\frac{2x+2}{(x-3)(x+5)}=\frac{1}{3}$$
$$\Rightarrow 3(2x+2)=(x-3)(x+5)$$
$$\Rightarrow 6x+6=x^2+2x-15$$
$$\Rightarrow x^2-4x-21=0$$
$$\Rightarrow x^2-7x+3x-21=0$$
$$\Rightarrow x(x-7)+3(x-7)=0$$
$$\Rightarrow (x-7)(x+3)=0$$
$$\Rightarrow x=7,-3$$

However, age cannot be negative.

Therefore, Rehman's present age is 7 years.

Question 5:

In a class test, the sum of Shefali's marks in Mathematics and English is 30. Had she got 2 marks more in Mathematics and 3 marks less in English, the product of their marks would have been 210. Find her marks in the two subjects.

Answer 5:
Let the marks in Maths be *x*.
Then, the marks in English will be 30 − *x*.
According to the given question,

$$(x+2)(30-x-3)=210$$
$$(x+2)(27-x)=210$$
$$\Rightarrow -x^2+25x+54=210$$
$$\Rightarrow x^2-25x+156=0$$
$$\Rightarrow x^2-12x-13x+156=0$$
$$\Rightarrow x(x-12)-13(x-12)=0$$
$$\Rightarrow (x-12)(x-13)=0$$
$$\Rightarrow x=12,13$$

If the marks in Maths are 12,
then marks in English will be 30 − 12 =18
If the marks in Maths are 13,
then marks in English will be 30 − 13 =17

Question 6:
The diagonal of a rectangular field is 60 metres more than the shorter side. If the longer side is 30 metres more than the shorter side, find the sides of the field.

Answer 6:
Let the shorter side of the rectangle be *x* m.
Then, larger side of the rectangle = (*x* + 30) m

Diagonal of the rectangle $=\sqrt{x^2+(x+30)^2}$

It is given that the diagonal of the rectangle is 60 m more than the shorter side.

$$\therefore \sqrt{x^2+(x+30)^2}=x+60$$
$$\Rightarrow x^2+(x+30)^2=(x+60)^2$$
$$\Rightarrow x^2+x^2+900+60x=x^2+3600+120x$$
$$\Rightarrow x^2-60x-2700=0$$
$$\Rightarrow x^2-90x+30x-2700=0$$
$$\Rightarrow x(x-90)+30(x-90)$$
$$\Rightarrow (x-90)(x+30)=0$$
$$\Rightarrow x=90,-30$$

However, side cannot be negative. Therefore, the length of the shorter side will be 90 m.

Hence, length of the larger side will be (90 + 30) m = 120 m

Question 7:

The difference of squares of two numbers is 180. The square of the smaller number is 8 times the larger number. Find the two numbers.

Answer 7:

Let the larger and smaller number be *x* and *y* respectively.

According to the given question,

$$x^2 - y^2 = 180 \text{ and } y^2 = 8x$$
$$\Rightarrow x^2 - 8x = 180$$
$$\Rightarrow x^2 - 8x - 180 = 0$$
$$\Rightarrow x^2 - 18x + 10x - 180 = 0$$
$$\Rightarrow x(x-18) + 10(x-18) = 0$$
$$\Rightarrow (x-18)(x+10) = 0$$
$$\Rightarrow x = 18, -10$$

However, the larger number cannot be negative as 8 times of the larger number will be negative and hence, the square of the smaller number will be negative which is not possible.

Therefore, the larger number will be 18 only.

$$x = 18$$
$$\therefore\ y^2 = 8x = 8 \times 18 = 144$$
$$\Rightarrow\ y = \pm\sqrt{144} = \pm 12$$
$$\therefore\ \text{Smaller number} = \pm 12$$

Therefore, the numbers are 18 and 12 or 18 and −12.

Question 8:

A train travels 360 km at a uniform speed. If the speed had been 5 km/h more, it would have taken 1 hour less for the same journey. Find the speed of the train.

Answer 8:

Let the speed of the train be x km/hr.

Time taken to cover 360 km $= \frac{360}{x}$ hr

According to the given question,

$$(x+5)\left(\frac{360}{x}-1\right)=360$$

$$\Rightarrow (x+5)\left(\frac{360}{x}-1\right)=360$$

$$\Rightarrow 360-x+\frac{1800}{x}-5=360$$

$$\Rightarrow x^2+5x-1800=0$$

$$\Rightarrow x^2+45x-40x-1800=0$$

$$\Rightarrow x(x+45)-40(x+45)=0$$

$$\Rightarrow (x+45)(x-40)=0$$

$$\Rightarrow x=40,-45$$

However, speed cannot be negative. Therefore,

the speed of train is 40 km/h

Question 9:

Two water taps together can fill a tank in $9\frac{3}{8}$ hours. The tap of larger diameter takes 10 hours less than the smaller one to fill the tank separately. Find the time in which each tap can separately fill the tank.

Answer 9:

Let the time taken by the smaller pipe to fill the tank be x hr.

Time taken by the larger pipe = (x − 10) hr

Part of tank filled by smaller pipe in 1 hour = $\frac{1}{x}$

Part of tank filled by larger pipe in 1 hour = $\frac{1}{x-10}$

It is given that the tank can be filled in $9\frac{3}{8} = \frac{75}{8}$ hours by both the pipes together.

Therefore,

$$\frac{1}{x}+\frac{1}{x-10}=\frac{8}{75}$$
$$\frac{x-10+x}{x(x-10)}=\frac{8}{75}$$
$$\Rightarrow \frac{2x-10}{x(x-10)}=\frac{8}{75}$$
$$\Rightarrow 75(2x-10)=8x^2-80x$$
$$\Rightarrow 150x-750=8x^2-80x$$
$$\Rightarrow 8x^2-230x+750=0$$
$$\Rightarrow 8x^2-200x-30x+750=0$$
$$\Rightarrow 8x(x-25)-30(x-25)=0$$
$$\Rightarrow (x-25)(8x-30)=0$$
$$i.e., x=25, \frac{30}{8}$$

Time taken by the smaller pipe cannot be $\frac{30}{8}$ = 3.75 hours. As in this case, the time taken by the larger pipe will be negative, which is logically not possible.

Therefore, time taken individually by the smaller pipe and the larger pipe will be 25 and 25 − 10 =15 hours respectively.

Question 10:

An express train takes 1 hour less than a passenger train to travel 132 km between Mysore and Bangalore (without taking into consideration the time they stop at intermediate stations). If the average speeds of the express train is 11 km/h more than that of the passenger train, find the average speed of the two trains.

Answer 10:

Let the average speed of passenger train be x km/h.

Average speed of express train = $(x + 11)$ km/h

It is given that the time taken by the express train to cover 132 km is 1 hour less than the passenger train to cover the same distance.

$$\therefore \frac{132}{x} - \frac{132}{x+11} = 1$$

$$\Rightarrow 132\left[\frac{x+11-x}{x(x+11)}\right] = 1$$

$$\Rightarrow \frac{132\times 11}{x(x+11)} = 1$$

$$\Rightarrow 132\times 11 = x(x+11)$$

$$\Rightarrow x^2 + 11x - 1452 = 0$$

$$\Rightarrow x^2 + 44x - 33x - 1452 = 0$$

$$\Rightarrow x(x+44) - 33(x+44) = 0$$

$$\Rightarrow (x+44)(x-33) = 0$$

$$\Rightarrow x = -44, 33$$

Speed cannot be negative.

Therefore, the speed of the passenger train will be 33 km/h and thus, the speed of the express train will be 33 + 11 = 44 km/h.

Question 11:

Sum of the areas of two squares is 468 m^2. If the difference of their perimeters is 24 m, find the sides of the two squares.

Answer 11:

Let the sides of the two squares be x m and y m. Therefore, their perimeter will be $4x$ and $4y$ respectively and their areas will be x^2 and y^2 respectively. It is given that $4x - 4y = 24$ $x - y = 6$ $x = y + 6$

$$\text{Also, } x^2 + y^2 = 468$$
$$\Rightarrow (6+y)^2 + y^2 = 468$$
$$\Rightarrow 36 + y^2 + 12y + y^2 = 468$$
$$\Rightarrow 2y^2 + 12y - 432 = 0$$
$$\Rightarrow y^2 + 6y - 216 = 0$$
$$\Rightarrow y^2 + 18y - 12y - 216 = 0$$
$$\Rightarrow y(y+18) - 12(y+18) = 0$$
$$\Rightarrow (y+18)(y-12) = 0$$
$$\Rightarrow y = -18 \text{ or } 12.$$

However, side of a square cannot be negative.

Hence, the sides of the squares are 12 m and (12 + 6) m = 18 m

Mathematic

(Chapter – 4) (Quadratic Equations)

(Class X)

Exercise 4.4

Question 1:

Find the nature of the roots of the following quadratic equations.

If the real roots exist, find them;

(i) $2x^2 - 3x + 5 = 0$

(ii) $3x^2 - 4\sqrt{3}x + 4 = 0$

(iii) $2x^2 - 6x + 3 = 0$

Answer 1:

We know that for a quadratic equation $ax^2 + bx + c = 0$, discriminant is $b^2 - 4ac$.

- If $b^2 - 4ac > 0 \rightarrow$ two distinct real roots
- If $b^2 - 4ac = 0 \rightarrow$ two equal real roots
- If $b^2 - 4ac < 0 \rightarrow$ no real roots

(i) $2x^2 - 3x + 5 = 0$

Comparing this equation with $ax^2 + bx + c = 0$,

we obtain $a = 2$, $b = -3$, $c = 5$

Discriminant $= b^2 - 4ac = (-3)^2 - 4(2)(5) = 9 - 40 = -31$

As $b^2 - 4ac < 0$,

Therefore, no real root is possible for the given equation.

(ii) $3x^2 - 4\sqrt{3}x + 4 = 0$

Comparing this equation with $ax^2 + bx + c = 0$,

we obtain $a = 3,\ b = -4\sqrt{3}, c = 4$

Discriminant $= b^2 - 4ac = \left(-4\sqrt{3}\right)^2 - 4(3)(4) = 48 - 48 = 0$

As $b^2 - 4ac = 0$,

Therefore, real roots exist for the given equation and they are equal to each other.

And the roots will be $\frac{-b}{2a}$ and $\frac{-b}{2a}$.

$$\frac{-b}{2a}=\frac{-\left(-4\sqrt{3}\right)}{2\times 3}=\frac{4\sqrt{3}}{6}=\frac{2\sqrt{3}}{3}=\frac{2}{\sqrt{3}}$$

Therefore, the roots are $\frac{2}{\sqrt{3}}$ and $\frac{2}{\sqrt{3}}$.

(iii) $2x^2 - 6x + 3 = 0$

Comparing this equation with $ax^2 + bx + c = 0$, we obtain a

$= 2$, $b = -6$, $c = 3$

Discriminant $= b^2 - 4ac = (-6)^2 - 4\,(2)\,(3)$

$= 36 - 24 = 12$

As $b^2 - 4ac > 0$,

Therefore, distinct real roots exist for this equation as follows.

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$=\frac{-(-6)\pm\sqrt{(-6)^2-4(2)(3)}}{2(2)}$$

$$=\frac{6\pm\sqrt{12}}{4}=\frac{6\pm 2\sqrt{3}}{4}$$

$$=\frac{3\pm\sqrt{3}}{2}$$

Therefore, the roots are $\frac{3+\sqrt{3}}{2}$ or $\frac{3-\sqrt{3}}{2}$.

Question 2:

Find the values of k for each of the following quadratic equations, so that they have two equal roots.

(i) $2x^2 + kx + 3 = 0$

(ii) $kx\,(x - 2) + 6 = 0$

Answer 2:

We know that if an equation $ax^2 + bx + c = 0$ has two equal roots, its discriminant $(b^2 - 4ac)$ will be 0.

(i) $2x^2 + kx + 3 = 0$

Comparing equation with $ax^2 + bx + c = 0$,

we obtain $a = 2$, $b = k$, $c = 3$

Discriminant $= b^2 - 4ac = (k)^2 - 4(2)(3) = k^2 - 24$

For equal roots,

Discriminant = 0

$k^2 - 24 = 0$

$k^2 = 24$

$k = \pm\sqrt{24} = \pm 2\sqrt{6}$

(ii) $kx(x - 2) + 6 = 0$

or $kx^2 - 2kx + 6 = 0$

Comparing this equation with $ax^2 + bx + c = 0$,

we obtain $a = k$, $b = -2k$, $c = 6$

Discriminant $= b^2 - 4ac = (-2k)^2 - 4(k)(6) = 4k^2 - 24k$

For equal roots, $b^2 - 4ac = 0$

$4k^2 - 24k = 0$

$4k(k - 6) = 0$

Either $4k = 0$ or $k = 6 = 0$

$k = 0$ or $k = 6$

However, if $k = 0$, then the equation will not have the terms 'x^2' and 'x'.

Therefore, if this equation has two equal roots, k should be 6 only.

Question 3:

Is it possible to design a rectangular mango grove whose length is twice its breadth, and the area is 800 m^2?

If so, find its length and breadth.

Answer 3:

Let the breadth of mango grove be l.

Length of mango grove will be $2l$.

Area of mango grove = $(2l)(l)$

$= 2l^2$

$$2l^2 = 800$$

$$l^2 = \frac{800}{2} = 400$$

$$l^2 - 400 = 0$$

Comparing this equation with $al^2 + bl + c = 0$,

we obtain $a = 1$ $b = 0$, $c = 400$

Discriminant = $b^2 - 4ac = (0)^2 - 4 \times (1) \times (-400) = 1600$

Here, $b^2 - 4ac > 0$

Therefore, the equation will have real roots. And hence, the desired rectangular mango grove can be designed.

$$l = \pm 20$$

However, length cannot be negative.

Therefore, breadth of mango grove = 20 m

Length of mango grove = $2 \times 20 = 40$ m

Question 4:

Is the following situation possible? If so, determine their present ages. The sum of the ages of two friends is 20 years. Four years ago, the product of their ages in years was 48.

Answer 4:

Let the age of one friend be x years.

Age of the other friend will be $(20 - x)$ years.

4 years ago, age of 1st friend = $(x - 4)$ years

And, age of 2nd friend = $(20 - x - 4) = (16 - x)$ years

Given that,

$(x - 4)(16 - x) = 48$

$16x - 64 - x^2 + 4x = 48$

$-x^2 + 20x - 112 = 0$

$x^2 - 20x + 112 = 0$

Comparing this equation with $ax^2 + bx + c = 0$,

we obtain $a = 1$, $b = -20$, $c = 112$

Discriminant $= b^2 - 4ac = (-20)^2 - 4(1)(112) = 400 - 448 = -48$

As $b^2 - 4ac < 0$,

Therefore, no real root is possible for this equation and hence, this situation is not possible.

Question 5:

Is it possible to design a rectangular park of perimeter 80 and area 400 m^2? If so find its length and breadth.

Answer 5:

Let the length and breadth of the park be l and b.

Perimeter $= 2(l + b) = 80$ $l + b = 40$

Or, $b = 40 - l$

Area $= l \times b = l(40 - l) = 40l - l^2$

$40l - l^2 = 400$

$l^2 - 40l + 400 = 0$

Comparing this equation with $al^2 + bl + c = 0$,

we obtain $a = 1$, $b = -40$, $c = 400$

Discriminate = $b^2 - 4ac = (-40)^2 - 4(1)(400) = 1600 - 1600 = 0$

As $b^2 - 4ac = 0$,

Therefore, this equation has equal real roots. And hence, this situation is possible.

Root of this equation,

$$l = -\frac{b}{2a}$$

$$l = -\frac{(-40)}{2(1)} = \frac{40}{2} = 20$$

Therefore, length of park, $l = 20$ m

And breadth of park, $b = 40 - l = 40 - 20 = 20$ m

Mathematics

(Chapter – 5) (Arithmetic Progressions)

(Class – X)

Exercise 5.1

Question 1:

In which of the following situations, does the list of numbers involved make as arithmetic progression and why?

(i). The taxi fare after each km when the fare is Rs 15 for the first km and Rs 8 for each additional km.

(ii). The amount of air present in a cylinder when a vacuum pump removes ¼ of the air remaining in the cylinder at a time.

(iii). The cost of digging a well after every metre of digging, when it costs Rs 150 for the first metre and rises by Rs 50 for each subsequent metre.

(iv). The amount of money in the account every year, when Rs 10000 is deposited at compound interest at 8% per annum.

Answer 1:

(i). It can be observed that
Taxi fare for 1st km = 15
Taxi fare for first 2 km = 15 + 8 = 23
Taxi fare for first 3 km = 23 + 8 = 31
Taxi fare for first 4 km = 31 + 8 = 39
Clearly 15, 23, 31, 39 … forms an A.P. because every term is 8 more than the preceding term.

(ii). Let the initial volume of air in a cylinder be *V* lit. In each stroke, the vacuum pump removes ¼ of air remaining in the cylinder at a time. In other words, after every stroke, only $1-\frac{1}{4}=\frac{3}{4}th$ part of air will remain.

Therefore, volumes will be $V, \left(\frac{3V}{4}\right), \left(\frac{3V}{4}\right)^2, \left(\frac{3V}{4}\right)^3 \ldots$

Clearly, it can be observed that the adjacent terms of this series do not have the same difference between them. Therefore, this is not an A.P.

(iii). Cost of digging for first metre = 150
Cost of digging for first 2 metres = 150 + 50 = 200
Cost of digging for first 3 metres = 200 + 50 = 250

Cost of digging for first 4 metres = 250 + 50 = 300
Clearly, 150, 200, 250, 300 … forms an A.P. because every term is 50 more than the preceding term.

(iv). We know that if Rs P is deposited at *r*% compound interest per annum for *n* years, our money will be $P\left(1+\frac{r}{100}\right)^n$ after *n* years.
Therefore, after every year, our money will be

$$10000\left(1+\frac{8}{100}\right), 10000\left(1+\frac{8}{100}\right)^2, 10000\left(1+\frac{8}{100}\right)^3, 10000\left(1+\frac{8}{100}\right)^4, \ldots$$

Clearly, adjacent terms of this series do not have the same difference between them. Therefore, this is not an A.P.

Question 2:
Write first four terms of the A.P. when the first term *a* and the common difference *d* are given as follows

(i) $a = 10$, $d = 10$

(ii) $a = -2$, $d = 0$

(iii) $a = 4$, $d = -3$

(iv) $a = -1$ $d = \frac{1}{2}$

(v) $a = -1.25$, $d = -0.25$

Answer 2:

(i) $a = 10$, $d = 10$

Let the series be a_1, a_2, a_3, a_4, a_5 …
$a_1 = a = 10$
$a_2 = a_1 + d = 10 + 10 = 20$
$a_3 = a_2 + d = 20 + 10 = 30$
$a_4 = a_3 + d = 30 + 10 = 40$
$a_5 = a_4 + d = 40 + 10 = 50$

Therefore, the series will be 10, 20, 30, 40, 50 …
First four terms of this A.P. will be 10, 20, 30, and 40.

(ii) $a = -2, d = 0$

Let the series be a_1, a_2, a_3, a_4 …
$a_1 = a = -2$
$a_2 = a_1 + d = -2 + 0 = -2$
$a_3 = a_2 + d = -2 + 0 = -2$
$a_4 = a_3 + d = -2 + 0 = -2$

Therefore, the series will be −2, −2, −2, −2 …
First four terms of this A.P. will be −2, −2, −2 and −2.

(iii) $a = 4, d = -3$

Let the series be a_1, a_2, a_3, a_4 …
$a_1 = a = 4$
$a_2 = a_1 + d = 4 - 3 = 1$
$a_3 = a_2 + d = 1 - 3 = -2$
$a_4 = a_3 + d = -2 - 3 = -5$

Therefore, the series will be 4, 1, −2 −5 …
First four terms of this A.P. will be 4, 1, −2 and −5.

(iv) $a = -1, d = \frac{1}{2}$

Let the series be a_1, a_2, a_3, a_4 …

$$a_1 = a = -1$$

$$a_2 = a_1 + d = -1 + \frac{1}{2} = -\frac{1}{2}$$

$$a_3 = a_2 + d = -\frac{1}{2} + \frac{1}{2} = 0$$

$$a_4 = a_3 + d = 0 + \frac{1}{2} = \frac{1}{2}$$

Clearly, the series will be

$-1, -\frac{1}{2}, 0, \frac{1}{2}$ …………

First four terms of this A.P. will be $-1, -\frac{1}{2}, 0 \text{ and } \frac{1}{2}$.

(v) $a = -1.25$, $d = -0.25$

Let the series be a_1, a_2, a_3, a_4 …
$a_1 = a = -1.25$
$a_2 = a_1 + d = -1.25 - 0.25 = -1.50$
$a_3 = a_2 + d = -1.50 - 0.25 = -1.75$
$a_4 = a_3 + d = -1.75 - 0.25 = -2.00$

Clearly, the series will be 1.25, −1.50, −1.75, −2.00 ……..
First four terms of this A.P. will be −1.25, −1.50, −1.75 and −2.00.

Question 3:
For the following A.P.s, write the first term and the common difference.

(i) 3, 1, − 1, − 3 …

(ii) − 5, − 1, 3, 7 …

(iii) $\frac{1}{3}, \frac{5}{3}, \frac{9}{3}, \frac{13}{3}, \ldots$

(iv) 0.6, 1.7, 2.8, 3.9 …

Answer 3:

(i) 3, 1, −1, −3 …

Here, first term, $a = 3$
Common difference, d = Second term − First term
$= 1 - 3 = -2$

(ii) −5, −1, 3, 7 …

Here, first term, a = −5
Common difference, d = Second term − First term
= (−1) − (−5) = − 1 + 5 = 4

(iii) $\frac{1}{3}, \frac{5}{3}, \frac{9}{3}, \frac{13}{3}, \ldots$

Here, first term, a = 1/3
Common difference, d = Second term − First term
$= \frac{5}{3} - \frac{1}{3} = \frac{4}{3}$

(iv) 0.6, 1.7, 2.8, 3.9 …

Here, first term, a = 0.6
Common difference, d = Second term − First term
= 1.7 − 0.6 = 1.1

Mathematics

(Chapter – 5) (Arithmetic Progressions)

(Class – X)

Exercise 5.2

Question 1:

Fill in the blanks in the following table, given that a is the first term, d the common difference and a_n the n^{th} term of the A.P.

	a	d	n	a_n
I	7	3	8	…...
II	− 18	…..	10	0
III	…..	− 3	18	− 5
IV	− 18.9	2.5	…..	3.6
V	3.5	0	105	…..

Answer 1:

I. $a = 7$, $d = 3$, $n = 8$, $a_n = ?$
We know that,

For an A.P. $a_n = a + (n - 1)\,d$
$= 7 + (8 - 1)\,3$
$= 7 + (7)\,3$
$= 7 + 21 = 28$

Hence, $a_n = 28$

II. Given that $a = -18$, $n = 10$, $a_n = 0$, $d = ?$

We know that,
$a_n = a + (n - 1)\,d$
$0 = -18 + (10 - 1)\,d$
$18 = 9d$

$d = \frac{18}{9} = 2$

Hence, common difference, $d = 2$

III. Given that $d = -3$, $n = 18$, $a_n = -5$

We know that, $a_n = a + (n - 1) d$

$-5 = a + (18 - 1) (-3)$
$-5 = a + (17) (-3)$
$-5 = a - 51$
$a = 51 - 5 = 46$

Hence, $a = 46$

IV. $a = -18.9$, $d = 2.5$, $a_n = 3.6$, $n = ?$

We know that, $a_n = a + (n - 1) d$
$3.6 = -18.9 + (n - 1) 2.5$
$3.6 + 18.9 = (n - 1) 2.5$
$22.5 = (n - 1) 2.5$
$(n-1) = \frac{22.5}{2.5}$
$n - 1 = 9$
$n = 10$
Hence, $n = 10$

V. $a = 3.5$, $d = 0$, $n = 105$, $a_n = ?$

We know that, $a_n = a + (n - 1) d$
$a_n = 3.5 + (105 - 1) 0$
$a_n = 3.5 + 104 \times 0$
$a_n = 3.5$
Hence, $a_n = 3.5$

Question 2:

Choose the correct choice in the following and justify

(i). 30^{th} term of the A.P: 10, 7, 4, …, is

(A). 97 **(B).** 77 **(C).** − 77 **(D).** − 87

(ii). 11^{th} term of the A.P. $-3, -\frac{1}{2}, 2, \ldots$ is

(A). 28 **(B).** 22 **(C).** − 38 **(D).** $-48\frac{1}{2}$

Answer 2:

(i) Given that

A.P. 10, 7, 4, …

First term, $a = 10$

Common difference, $d = a_2 - a_1 = 7 - 10 = -3$

We know that, $a_n = a + (n - 1)\, d$

$a_{30} = 10 + (30 - 1)\,(-3)$
$a_{30} = 10 + (29)\,(-3)$
$a_{30} = 10 - 87 = -77$

Hence, the correct answer is **C.**

(ii) Given that, A.P. $-3, -\frac{1}{2}, 2, \ldots$

First term $a = -3$
Common difference, $d = a_2 - a_1$

$$= -\frac{1}{2} - (-3)$$
$$= -\frac{1}{2} + 3 = \frac{5}{2}$$

We know that,

$$a_n = a + (n-1)d$$
$$a_{11} = -3 + (11-1)\left(\frac{5}{2}\right)$$
$$a_{11} = -3 + (10)\left(\frac{5}{2}\right)$$
$$a_{11} = -3 + 25$$
$$a_{11} = 22$$

Hence, the answer is **B**.

Question 3:

In the following APs find the missing term in the boxes

(i) $2, \square, 26$

(ii) $\square, 13, \square, 3$

(iii) $5, \square, \square, 9\frac{1}{2}$

(iv) $-4, \square, \square, \square, \square, 6$

(v) $\square, 38, \square, \square, \square, -22$

Answer 3:

(i) $2, \square, 26$

For this A.P., a = 2 and a_3 = 26

We know that, $a_n = a + (n - 1)\,d$

$a_3 = 2 + (3 - 1)\,d$
$26 = 2 + 2d$
$24 = 2d$ $d = 12$

$a_2 = 2 + (2 - 1)\,12$
$= 14$

Therefore, 14 is the missing term.

(ii) $\square, 13, \square, 3$

For this A.P., $a_2 = 13$ and $a_4 = 3$

We know that, $a_n = a + (n - 1)\,d$
$a_2 = a + (2 - 1)\,d$
$13 = a + d$ ....(I)

$a_4 = a + (4 - 1)\,d$
$3 = a + 3d$ ....(II)

On subtracting (I) from (II), we obtain $-10 = 2d$
$d = -5$

From equation (I), we obtain
$13 = a + (-5)$
$a = 18$

$a_3 = 18 + (3 - 1)(-5)$
$= 18 + 2(-5) = 18 - 10 = 8$

Therefore, the missing terms are 18 and 8 respectively.

(iii) $5, \square, \square, 9\frac{1}{2}$

For this A.P.,

$a = 5$

$a_4 = 9\frac{1}{2} = \frac{19}{2}$

We know that,

$a_n = a + (n-1)d$

$a_4 = a + (4-1)d$

$\frac{19}{2} = 5 + 3d$

$\frac{19}{2} - 5 = 3d$

$\frac{9}{2} = 3d$

$d = \frac{3}{2}$

$a_2 = a + d = 5 + \frac{3}{2} = \frac{13}{2}$

$a_3 = a + 2d = 5 + 2\left(\frac{3}{2}\right) = 8$

Therefore, the missing terms are 13/2 and 8 respectively.

(iv) $-4, \square, \square, \square, \square, 6$

For this A.P., $a = -4$ and $a_6 = 6$
We know that, $a_n = a + (n - 1)\, d$
$a_6 = a + (6 - 1)\, d$
$6 = -4 + 5d$
$10 = 5d$
$d = 2$

$a_2 = a + d = -4 + 2 = -2$
$a_3 = a + 2d = -4 + 2\,(2) = 0$
$a_4 = a + 3d = -4 + 3\,(2) = 2$
$a_5 = a + 4d = -4 + 4\,(2) = 4$
Therefore, the missing terms are −2, 0, 2, and 4 respectively.

(v) $\square, 38, \square, \square, \square, -22$

For this A.P., $a_2 = 38$ and $a_6 = -22$
We know that $a_n = a + (n - 1)\,d$
$a_2 = a + (2 - 1)\,d$
$38 = a + d$ ………………………(1)

$a_6 = a + (6 - 1)\,d$
$-22 = a + 5d$ …………………….(2)

On subtracting equation (1) from (2), we obtain

$-22 - 38 = 4d$
$-60 = 4d$
$d = -15$

$a = a_2 - d = 38 - (-15) = 53$
$a_3 = a + 2d = 53 + 2\,(-15) = 23$
$a_4 = a + 3d = 53 + 3\,(-15) = 8$
$a_5 = a + 4d = 53 + 4\,(-15) = -7$
Therefore, the missing terms are 53, 23, 8, and −7 respectively.

Question 4:
Which term of the A.P. 3, 8, 13, 18, … is 78?

Answer 4:

3, 8, 13, 18, … For this A.P., $a = 3$ and $d = a_2 - a_1 = 8 - 3 = 5$
Let n^{th} term of this A.P. be 78.
$a_n = a + (n - 1)\,d$
$78 = 3 + (n - 1)\,5$
$75 = (n - 1)\,5$
$(n - 1) = 15$
$n = 16$

Hence, 16^{th} term of this A.P. is 78.

Question 5:

Find the number of terms in each of the following A.P.

(i). 7, 13, 19, …, 205 **(ii).** $18, 15\frac{1}{2}, 13, \ldots, -47$

Answer 5:

(i). 7, 13, 19, …, 205

For this A.P., $a = 7$ and $d = a_2 - a_1 = 13 - 7 = 6$
Let there are n terms in this A.P. $a_n = 205$
We know that $a_n = a + (n - 1)\, d$
Therefore, $205 = 7 + (n - 1)\, 6$
$198 = (n - 1)\, 6$
$33 = (n - 1)$
$n = 34$
Therefore, this given series has 34 terms in it.

(ii). $18, 15\frac{1}{2}, 13, \ldots, -47$

For this A.P.,

$$a = 18$$

$$d = a_2 - a_1 = 15\frac{1}{2} - 18$$

$$d = \frac{31 - 36}{2} = -\frac{5}{2}$$

Let there are n terms in this A.P.
Therefore, $a_n = -47$ and we know that,

$$a_n = a + (n-1)d$$

$$-47 = 18 + (n-1)\left(-\frac{5}{2}\right)$$

$$-47 - 18 = (n-1)\left(-\frac{5}{2}\right)$$

$$-65 = (n-1)\left(-\frac{5}{2}\right)$$

$$(n-1) = \frac{-130}{-5}$$

$$(n-1) = 26$$

$$n = 27$$

Therefore, this given A.P. has 27 terms in it.

Question 6:

Check whether − 150 is a term of the A.P. 11, 8, 5, 2, …

Answer 6:

For this A.P., a = 11 and $d = a_2 - a_1$ = 8 − 11 = −3

Let −150 be the n^{th} term of this A.P.

We know that,

$$a_n = a + (n-1)d$$
$$-150 = 11 + (n-1)(-3)$$
$$-150 = 11 - 3n + 3$$
$$-164 = -3n$$
$$n = \frac{164}{3}$$

Clearly, n is not an integer.

Therefore, −150 is not a term of this A.P.

Question 7:

Find the 31st term of an A.P. whose 11th term is 38 and the 16th term is 73.

Answer 7:

Given that, a_{11} = 38 and a_{16} = 73

We know that, $a_n = a + (n - 1)\, d$

$a_{11} = a + (11 - 1)\, d$

$38 = a + 10d$ ………………………..(1)

Similarly, $a_{16} = a + (16 - 1)\, d$

$73 = a + 15d$ …………………………(2)

On subtracting (1) from (2), we obtain

$35 = 5d$

$d = 7$

From equation (1),

$38 = a + 10 \times (7)$

$38 - 70 = a$
$a = -32$

$a_{31} = a + (31 - 1)\ d$
$= -32 + 30\ (7)$
$= -32 + 210$
$= 178$

Hence, 31^{st} term is 178.

Question 8:
An A.P. consists of 50 terms of which 3^{rd} term is 12 and the last term is 106. Find the 29^{th} term

Answer 8:

Given that, $a_3 = 12$ and $a_{50} = 106$
We know that, $a_n = a + (n - 1)\ d$
$a_3 = a + (3 - 1)\ d$
$12 = a + 2d$ …………………………..(I)

Similarly, $a_{50} = a + (50 - 1)\ d$
$106 = a + 49d$ ………………………(II)

On subtracting (I) from (II), we obtain
$94 = 47d$
$d = 2$

From equation (I), we obtain
$12 = a + 2\ (2)$
$a = 12 - 4 = 8$

$a_{29} = a + (29 - 1)\ d$
$a_{29} = 8 + (28)2$
$a_{29} = 8 + 56 = 64$
Therefore, 29^{th} term is 64.

Question 9:
If the 3^{rd} and the 9^{th} terms of an A.P. are 4 and − 8 respectively. Which term of this A.P. is zero?

Answer 9:

Given that, $a_3 = 4$ and $a_9 = -8$
We know that, $a_n = a + (n - 1)\,d$
$a_3 = a + (3 - 1)\,d$
$4 = a + 2d$ ……………………(I)
$a_9 = a + (9 - 1)\,d$
$-8 = a + 8d$ …………………(II)

On subtracting equation (I) from (II), we obtain
$-12 = 6d$
$d = -2$

From equation (I), we obtain
$4 = a + 2\,(-2)$
$4 = a - 4$
$a = 8$

Let n^{th} term of this A.P. be zero.
$a_n = a + (n - 1)\,d$
$0 = 8 + (n - 1)\,(-2)$
$0 = 8 - 2n + 2$
$2n = 10$

$n = 5$

Hence, 5^{th} term of this A.P. is 0.

Question 10:
If 17^{th} term of an A.P. exceeds its 10^{th} term by 7. Find the common difference.

Answer 10:
We know that,

For an A.P., $a_n = a + (n - 1)\,d$
$a_{17} = a + (17 - 1)\,d$
$a_{17} = a + 16d$

Similarly, $a_{10} = a + 9d$
It is given that $a_{17} - a_{10} = 7$
$(a + 16d) - (a + 9d) = 7$
$7d = 7$
$d = 1$

Therefore, the common difference is 1.

Question 11:
Which term of the A.P. 3, 15, 27, 39, … will be 132 more than its 54th term?

Answer 11:
Given A.P. is 3, 15, 27, 39, …
$a = 3\ d = a_2 - a_1 = 15 - 3 = 12$
$a_{54} = a + (54 - 1)\,d$
$= 3 + (53)\,(12)$
$= 3 + 636 = 639$
Now $a_{54} + 132 = 639 + 132 = 771$

We have to find the term of this A.P. which is 771.

Let n^{th} term be 771.
$a_n = a + (n - 1)\,d$
$771 = 3 + (n - 1)\,12$
$768 = (n - 1)\,12$
$(n - 1) = 64$
$n = 65$

Therefore, 65th term was 132 more than 54th term.

Alternatively,
Let n^{th} term be 132 more than 54th term.

$$n = 54 + \frac{132}{12}$$
$$= 54 + 11 = 65^{th} \text{ term}$$

Question 12:

Two APs have the same common difference. The difference between their 100^{th} term is 100, what is the difference between their 1000^{th} terms?

Answer 12:

Let the first term of these A.P.s be a_1 and a_2 respectively and the common difference of these A.P.s be d.

For first A.P.,

$a_{100} = a_1 + (100 - 1)\,d$
$= a_1 + 99d$

$a_{1000} = a_1 + (1000 - 1)\,d$
$a_{1000} = a_1 + 999d$

For second A.P.,

$a_{100} = a_2 + (100 - 1)\,d$
$= a_2 + 99d$

$a_{1000} = a_2 + (1000 - 1)\,d$
$= a_2 + 999d$

Given that, difference between

100^{th} term of these A.P.s = 100 Therefore,
$(a_1 + 99d) - (a_2 + 99d) = 100$ $a_1 - a_2 = 100$ ……………………(1)

Difference between 1000^{th} terms of these A.P.s

$(a_1 + 999d) - (a_2 + 999d) = a_1 - a_2$
From equation (1),
This difference, $a_1 - a_2 = 100$

Hence, the difference between 1000^{th} terms of these A.P. will be 100.

Question 13:
How many three digit numbers are divisible by 7?

Answer 13:
First three-digit number that is divisible by 7 = 105
Next number = 105 + 7 = 112
Therefore, 105, 112, 119, …
All are three digit numbers which are divisible by 7 and thus, all these are terms of an A.P. having first term as 105 and common difference as 7.

The maximum possible three-digit number is 999. When we divide it by 7, the remainder will be 5. Clearly, 999 − 5 = 994 is the maximum possible three-digit number that is divisible by 7.

The series is as follows.
105, 112, 119, …, 994

Let 994 be the nth term of this A.P.
$a = 105$, $d = 7$ and $a_n = 994$. $n = ?$

$a_n = a + (n - 1)\,d$
$994 = 105 + (n - 1)\,7$
$889 = (n - 1)\,7$
$(n - 1) = 127$
$n = 128$

Therefore, 128 three-digit numbers are divisible by 7.

Question 14:
How many multiples of 4 lie between 10 and 250?

Answer 14:
First multiple of 4 that is greater than 10 is 12. Next will be 16.

Therefore, 12, 16, 20, 24, …

All these are divisible by 4 and thus, all these are terms of an A.P. with first term as 12 and common difference as 4.

When we divide 250 by 4, the remainder will be 2.
Therefore, 250 − 2 = 248 is divisible by 4.

The series is as follows.

12, 16, 20, 24, …, 248

Let 248 be the n^{th} term of this A.P.

$$a = 12$$
$$d = 4$$
$$a_n = 248$$
$$a_n = a + (n-1)d$$
$$248 = 12 + (n-1)4$$
$$\frac{236}{4} = n - 1$$
$$59 = n - 1$$
$$n = 60$$

Therefore, there are 60 multiples of 4 between 10 and 250.

Question 15:
For what value of n, are the n^{th} terms of two APs 63, 65, 67, and 3, 10, 17, … equal

Answer 15:

For AP: 63, 65, 67, …
a = 63 and $d = a_2 - a_1$ = 65 − 63 = 2
n^{th} term of this A.P. = $a_n = a + (n - 1)\, d$
a_n= 63 + (n − 1) 2 = 63 + 2n − 2
a_n = 61 + 2n ……………(1)

For AP:3, 10, 17, …

$a = 3$ and $d = a_2 - a_1 = 10 - 3 = 7$
n^{th} term of this A.P. $= 3 + (n - 1)\ 7$
$a_n = 3 + 7n - 7$
$a_n = 7n - 4$ ……………………………….(2)

It is given that, n^{th} term of these A.P.s are equal to each other.
Equating both these equations, we obtain

$61 + 2n = 7n - 4$
$61 + 4 = 5n$
$5n = 65$
$n = 13$

Therefore, 13^{th} terms of both these A.P.s are equal to each other.

Question 16:
Determine the A.P. whose third term is 16 and the 7^{th} term exceeds the 5^{th} term by 12.

Answer 16:
$a_3 = 16$
$a + (3 - 1)\ d = 16$
$a + 2d = 16$ ………………………….(1)

$a_7 - a_5 = 12$
$[a + (7 - 1)\ d] - [a + (5 - 1)\ d] = 12$
$(a + 6d) - (a + 4d) = 12$
$2d = 12$
$d = 6$

From equation (1), we obtain

$a + 2\ (6) = 16$
$a + 12 = 16$
$a = 4$

Therefore, A.P. will be 4, 10, 16, 22, …

Question 17:
Find the 20th term from the last term of the A.P. 3, 8, 13, …, 253

Answer 17:
Given A.P. is 3, 8, 13, …, 253
Common difference for this A.P. is 5.

Therefore, this A.P. can be written in reverse order as
253, 248, 243, …, 13, 8, 5

For this A.P.,
$a = 253$
$d = 248 - 253 = -5$
$n = 20$
$a_{20} = a + (20 - 1)\ d$
$a_{20} = 253 + (19)\ (-5)$
$a_{20} = 253 - 95\ a = 158$

Therefore, 20th term from the last term is 158.

Question 18:
The sum of 4th and 8th terms of an A.P. is 24 and the sum of the 6th and 10th terms is 44. Find the first three terms of the A.P.

Answer 18:
We know that, $a_n = a + (n - 1)\ d$

$a_4 = a + (4 - 1)\ d$
$a_4 = a + 3d$

Similarly,
$a_8 = a + 7d$
$a_6 = a + 5d$
$a_{10} = a + 9d$

Given that, $a_4 + a_8 = 24$
$a + 3d + a + 7d = 24$
$2a + 10d = 24$

$a + 5d = 12$ ……………………………(1)

$a_6 + a_{10} = 44$
$a + 5d + a + 9d = 44$
$2a + 14d = 44$
$a + 7d = 22$ ……………………………(2)

On subtracting equation (1) from (2), we obtain

$2d = 22 - 12$
$2d = 10$
$d = 5$

From equation (1), we obtain

$a + 5d = 12$
$a + 5\ (5) = 12$
$a + 25 = 12$
$a = -13$

$a_2 = a + d = -\ 13 + 5 = -8$
$a_3 = a_2 + d = -\ 8 + 5 = -3$

Therefore, the first three terms of this A.P. are −13, −8, and −3.

Question 19:
Subba Rao started work in 1995 at an annual salary of *Rs* 5000 and received an increment of *Rs* 200 each year. In which year did his income reach *Rs* 7000?

Answer 19:
It can be observed that the incomes that Subba Rao obtained in various years are in A.P. as every year, his salary is increased by *Rs* 200.

Therefore, the salaries of each year after 1995 are

5000, 5200, 5400, …

Here, a = 5000 and d = 200

Let after n^{th} year, his salary be *Rs* 7000.
Therefore, $a_n = a + (n - 1)\,d$
$7000 = 5000 + (n - 1)\,200$
$200(n - 1) = 2000$
$(n - 1) = 10$
$n = 11$

Therefore, in 11th year, his salary will be *Rs* 7000.

Question 20:
Ramkali saved *Rs* 5 in the first week of a year and then increased her weekly saving by *Rs* 1.75. If in the n^{th} week, her week, her weekly savings become *Rs* 20.75, find *n.*

Answer 20:
Given that, $a = 5$, $d = 1.75$ and $a_n = 20.75$. $n = ?$

$a_n = a + (n - 1)\,d$

$$20.75 = 5 + (n-1)1.75$$

$$15.75 = (n-1)1.75$$

$$(n-1) = \frac{15.75}{1.75} = \frac{1575}{175}$$

$$= \frac{63}{7} = 9$$

$n - 1 = 9$
$n = 10$

Hence, n is 10.

Mathematics

(Chapter – 5) (Arithmetic Progressions)

(Class – X)

Exercise 5.3

Question 1:

Find the sum of the following APs.
(i) 2, 7, 12 ,…., to 10 terms.

(ii) − 37, − 33, − 29 ,…, to 12 terms

(iii) 0.6, 1.7, 2.8 ,…….., to 100 terms

(iv) $\frac{1}{15}, \frac{1}{12}, \frac{1}{10}$,………, to 11 terms

Answer 1:

(i) 2, 7, 12 ,…, to 10 terms

For this A.P., $a = 2$, $d = a_2 - a_1 = 7 - 2 = 5$ and $n = 10$

We know that,

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$S_{10} = \frac{10}{2}\left[2(2) + (10-1)5\right]$$

$$= 5\left[4 + (9)\times(5)\right]$$

$$= 5 \times 49 = 245$$

(ii) −37, −33, −29 ,…, to 12 terms

For this A.P., $a = -37$, $d = a_2 - a_1 = (-33) - (-37) = -33 + 37 = 4$
$n = 12$

We know that,

$$S_n = \frac{n}{2}\left[2a+(n-1)d\right]$$

$$S_{12} = \frac{12}{2}\left[2(-37)+(12-1)4\right]$$

$$= 6\left[-74+11\times 4\right]$$

$$= 6\left[-74+44\right]$$

$$= 6(-30) = -180$$

(iii) 0.6, 1.7, 2.8 ,…, to 100 terms

For this A.P., a = 0.6, $d = a_2 - a_1$ = 1.7 − 0.6 = 1.1 and n = 100

We know that,

$$S_n = \frac{n}{2}\left[2a+(n-1)d\right]$$

$$S_{100} = \frac{100}{2}\left[2(0.6)+(100-1)1.1\right]$$

$$= 50\left[1.2+(99)\times(1.1)\right]$$

$$= 50\left[1.2+108.9\right]$$

$$= 50\left[110.1\right]$$

$$= 5505$$

(iv) $\frac{1}{15}, \frac{1}{12}, \frac{1}{10}$,………, to 11 terms

For this A.P.,

$$a = \frac{1}{15}$$

n = 11

$$d = a_2 - a_1 = \frac{1}{12} - \frac{1}{15}$$

$$= \frac{5-4}{60} = \frac{1}{60}$$

We know that,

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$S_{11} = \frac{11}{2}\left[2\left(\frac{1}{15}\right) + (11-1)\frac{1}{60}\right]$$

$$= \frac{11}{2}\left[\frac{2}{15} + \frac{10}{60}\right]$$

$$= \frac{11}{2}\left[\frac{2}{15} + \frac{1}{6}\right] = \frac{11}{2}\left[\frac{4+5}{30}\right]$$

$$= \left(\frac{11}{2}\right)\left(\frac{9}{30}\right) = \frac{33}{20}$$

Question 2:

Find the sums given below

(i) 7 + $10\frac{1}{2}$ + 14 + ………… + 84

(ii) 34 + 32 + 30 + ……….. + 10

(iii) − 5 + (− 8) + (− 11) + ………… + (− 230)

Answer 2:

(i) 7 + $10\frac{1}{2}$ + 14 + …………+ 84

For this A.P.,

a = 7 and *l* = 84

$$d = a_2 - a_1 = 10\frac{1}{2} - 7 = \frac{21}{2} - 7 = \frac{7}{2}$$

Let 84 be the n^{th} term of this A.P.

$l = a + (n - 1)d$

$$84 = 7 + (n-1)\frac{7}{2}$$

$$77 = (n-1)\frac{7}{2}$$

$22 = n - 1$
$n = 23$

We know that,

$$S_n = \frac{n}{2}(a+l)$$

$$S_n = \frac{23}{2}[7+84]$$

$$= \frac{23 \times 91}{2} = \frac{2093}{2}$$

$$= 1046\frac{1}{2}$$

(ii) 34 + 32 + 30 + ……….. + 10

For this A.P., $a = 34$, $d = a_2 - a_1 = 32 - 34 = -2$ and $l = 10$

Let 10 be the n^{th} term of this A.P. $l = a + (n - 1)\, d$

$10 = 34 + (n - 1)(-2)$
$-24 = (n - 1)(-2)$
$12 = n - 1$
$n = 13$

$$S_n = \frac{n}{2}(a+l)$$

$$= \frac{13}{2}(34+10)$$

$$= \frac{13 \times 44}{2} = 13 \times 22$$

$$= 286$$

(iii)$(-5) + (-8) + (-11) + \dots\dots\dots + (-230)$

For this A.P.,
$a = -5$, $l = -230$ and $d = a_2 - a_1 = (-8) - (-5) = -8 + 5 = -3$
Let -230 be the n^{th} term of this A.P.

$l = a + (n - 1)d$
$-230 = -5 + (n - 1)(-3)$
$-225 = (n - 1)(-3)$
$(n - 1) = 75$
$n = 76$

And, $S_n = \frac{n}{2}(a + l)$

$= \frac{76}{2}[(-5) + (-230)]$
$= 38(-235)$
$= -8930$

Question 3:
In an AP

(i) Given $a = 5$, $d = 3$, $a_n = 50$, find n and S_n.

(ii) Given $a = 7$, $a_{13} = 35$, find d and S_{13}.

(iii) Given $a_{12} = 37$, $d = 3$, find a and S_{12}.

(iv) Given $a_3 = 15$, $S_{10} = 125$, find d and a_{10}.

(v) Given $d = 5$, $S_9 = 75$, find a and a_9.

(vi) Given $a = 2$, $d = 8$, $S_n = 90$, find n and a_n.

(vii) Given $a = 8$, $a_n = 62$, $S_n = 210$, find n and d.

(viii) Given $a_n = 4$, $d = 2$, $S_n = -14$, find n and a.

(ix) Given $a = 3$, $n = 8$, $S = 192$, find d.

(x) Given $l = 28$, $S = 144$ and there are total 9 terms. Find a.

Answer 3:

(i) Given that, a = 5, d = 3, a_n = 50

As $a_n = a + (n - 1)d$,
$\therefore$ 50 = 5 + (n − 1)3
45 = (n − 1)3
15 = n − 1 n = 16

$$S_n = \frac{n}{2}[a + a_n]$$

$$S_{16} = \frac{16}{2}[5 + 50]$$
$$= 8 \times 55$$
$$= 440$$

(ii) Given that, a = 7, a_{13} = 35

As $a_n = a + (n - 1)\ d$,
$\therefore a_{13} = a + (13 - 1)\ d$
35 = 7 + 12 d
35 − 7 = 12d
28 = 12d

$$d = \frac{7}{3}$$

$$S_n = \frac{n}{2}[a + a_n]$$

$$S_{13} = \frac{n}{2}[a + a_{13}]$$
$$= \frac{13}{2}[7 + 35]$$
$$= \frac{13 \times 42}{2} = 13 \times 21$$
$$= 273$$

(iii) Given that, a_{12} = 37, d = 3

As $a_n = a + (n - 1)d$,

$a_{12} = a + (12 - 1)3$

$37 = a + 33$

$a = 4$

$$S_n = \frac{n}{2}[a + a_n]$$

$$S_n = \frac{12}{2}[4 + 37]$$

$$S_n = 6(41)$$

$$S_n = 246$$

(iv) Given that, $a_3 = 15$, $S_{10} = 125$

As $a_n = a + (n - 1)d$,

$a_3 = a + (3 - 1)d$

$15 = a + 2d$ ……………………….(i)

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_{10} = \frac{10}{2}[2a + (10-1)d]$$

$$125 = 5(2a + 9d)$$

$$25 = 2a + 9d \qquad \text{(ii)}$$

On multiplying equation (i) by 2, we obtain

$30 = 2a + 4d$ …………………..(iii)

On subtracting equation (iii) from (ii), we obtain

$-5 = 5d$

$d = -1$

From equation (i),

$15 = a + 2(-1)$

$15 = a - 2$

$a = 17$

$a_{10} = a + (10 - 1)d$
$a_{10} = 17 + (9)(-1)$
$a_{10} = 17 - 9 = 8$

(v) Given that, $d = 5$, $S_9 = 75$

As $S_n = \frac{n}{2}[2a + (n-1)d]$

$S_9 = \frac{9}{2}[2a + (9-1)5]$

$75 = \frac{9}{2}(2a + 40)$

$25 = 3(a + 20)$
$25 = 3a + 60$
$3a = 25 - 60$

$a = \frac{-35}{3}$

$a_n = a + (n - 1)d$
$a_9 = a + (9 - 1)(5)$

$= \frac{-35}{3} + 8(5)$

$= \frac{-35}{3} + 40$

$= \frac{-35+120}{3} = \frac{85}{3}$

(vi) Given that, $a = 2$, $d = 8$, $S_n = 90$

As $S_n = \frac{n}{2}[2a + (n-1)d]$

$90 = \frac{n}{2}[4 + (n-1)8]$

$90 = n[2 + (n - 1)4]$
$90 = n[2 + 4n - 4]$

$90 = n(4n - 2) = 4n^2 - 2n$
$4n^2 - 2n - 90 = 0$
$4n^2 - 20n + 18n - 90 = 0$
$4n(n - 5) + 18(n - 5) = 0$
$(n - 5)(4n + 18) = 0$

Either $n - 5 = 0$ or $4n + 18 = 0$

$n = 5$ or $n = -\frac{18}{4} = \frac{-9}{2}$

However, n can neither be negative nor fractional.
Therefore, $n = 5$
$a_n = a + (n - 1)d$
$a_5 = 2 + (5 - 1)8$
$= 2 + (4)(8)$
$= 2 + 32 = 34$

(vii) Given that, $a = 8$, $a_n = 62$, $S_n = 210$

$$S_n = \frac{n}{2}[a + a_n]$$

$$210 = \frac{n}{2}[8 + 62]$$

$$210 = \frac{n}{2}(70)$$

$n = 6$
$a_n = a + (n - 1)d$
$62 = 8 + (6 - 1)d$
$62 - 8 = 5d$
$54 = 5d$

$$d = \frac{54}{5}$$

(viii) Given that, $a_n = 4$, $d = 2$, $S_n = -14$

$a_n = a + (n - 1)d$
$4 = a + (n - 1)2$
$4 = a + 2n - 2$
$a + 2n = 6$
$a = 6 - 2n$ ………………………….(i)

$$S_n = \frac{n}{2}[a + a_n]$$

$$-14 = \frac{n}{2}[a + 4]$$

$-28 = n\ (a + 4)$
$-28 = n\ (6 - 2n + 4)$ {From equation (i)}
$-28 = n\ (- 2n + 10)$
$-28 = - 2n^2 + 10n$
$2n^2 - 10n - 28 = 0$
$n^2 - 5n - 14 = 0$
$n^2 - 7n + 2n - 14 = 0$
$n\ (n - 7) + 2(n - 7) = 0$
$(n - 7)\ (n + 2) = 0$

Either $n - 7 = 0$ or $n + 2 = 0$
$n = 7$ or $n = -2$
However, n can neither be negative nor fractional.

Therefore, $n = 7$

From equation (i), we obtain
$a = 6 - 2n$
$a = 6 - 2(7)$
$= 6 - 14$
$= -8$

(ix) Given that, $a = 3$, $n = 8$, $S = 192$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$192 = \frac{8}{2}[2 \times 3 + (8-1)d]$$

$192 = 4\,[6 + 7d]$
$48 = 6 + 7d$
$42 = 7d$
$d = 6$

(x) Given that, $l = 28$, $S = 144$ and there are total of 9 terms.

$$S_n = \frac{n}{2}(a + l)$$

$$144 = \frac{9}{2}(a + 28)$$

$(16) \times (2) = a + 28$
$32 = a + 28$
$a = 4$

Question 4:
How many terms of the AP. 9, 17, 25 … must be taken to give a sum of 636?

Answer 4:

Let there be n terms of this A.P.
For this A.P.,
$a = 9$ and $d = a_2 - a_1 = 17 - 9 = 8$

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$636 = \frac{n}{2}\left[2 \times a + (n-1)8\right]$$

$$636 = \frac{n}{2}\left[18 + (n-1)8\right]$$

$636 = n\,[9 + 4n - 4]$
$636 = n\,(4n + 5)$

$4n^2 + 5n - 636 = 0$
$4n^2 + 53n - 48n - 636 = 0$
$n(4n + 53) - 12(4n + 53) = 0$
$(4n + 53)(n - 12) = 0$

Either $4n + 53 = 0$ or $n - 12 = 0$

$n = \frac{-53}{4}$ or $n = 12$

n cannot be $\frac{-53}{4}$. As the number of terms can neither be negative nor fractional, therefore, $n = 12$ only.

Question 5:
The first term of an AP is 5, the last term is 45 and the sum is 400. Find the number of terms and the common difference.

Answer 5:
Given that, $a = 5$, $l = 45$ and $S_n = 400$

$$S_n = \frac{n}{2}(a+l)$$

$$400 = \frac{n}{2}(5+45)$$

$$400 = \frac{n}{2}(50)$$

$n = 16$ $l = a + (n$

$- 1) d$

$45 = 5 + (16 - 1) d$

$40 = 15d$

$$d = \frac{40}{15} = \frac{8}{3}$$

Question 6:

The first and the last term of an AP are 17 and 350 respectively. If the common difference is 9, how many terms are there and what is their sum?

Answer 6:

Given that, a = 17, l = 350 and d = 9

Let there be n terms in the A.P.

l = a + (n − 1) d
350 = 17 + (n − 1)9
333 = (n − 1)9
(n − 1) = 37
n = 38

$$S_n = \frac{n}{2}(a+l)$$

$$\Rightarrow S_n = \frac{38}{2}(17+350) = 19(367) = 6973$$

Thus, this A.P. contains 38 terms and the sum of the terms of this A.P. is 6973.

Question 7:

Find the sum of first 22 terms of an AP in which d = 7 and 22nd term is 149.

Answer 7:

d = 7 and a_{22} = 149. S_{22} =?

$a_n = a + (n - 1)d$
$a_{22} = a + (22 - 1)d$
149 = a + 21 × 7
149 = a + 147
a = 2

$$S_n = \frac{n}{2}(a + a_n)$$
$$= \frac{22}{2}(2 + 149)$$
$$= 11(151) = 1661$$

Question 8:
Find the sum of first 51 terms of an AP whose second and third terms are 14 and 18 respectively.

Answer 8:
Given that, $a_2 = 14$ and $a_3 = 18$

$d = a_3 - a_2 = 18 - 14 = 4$
$a_2 = a + d$
$14 = a + 4$
$a = 10$

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$
$$S_{51} = \frac{51}{2}\left[2 \times 10 + (51-1)4\right]$$
$$= \frac{51}{2}\left[20 + (50)(4)\right]$$
$$= \frac{51(220)}{2} = 51(110)$$
$= 5610$

Question 9:
If the sum of first 7 terms of an AP is 49 and that of 17 terms is 289, find the sum of first n terms.

Answer 9:

Given that, $S_7 = 49$

$S_{17} = 289$

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$S_7 = \frac{7}{2}\left[2a + (7-1)d\right]$$

$$49 = \frac{7}{2}(2a + 6d)$$

$7 = (a + 3d)$

$a + 3d = 7$ …………………………..(i)

Similarly, $S_{17} = \frac{17}{2}\left[2a + (17-1)d\right]$

$$289 = \frac{17}{2}\left[2a + 16d\right]$$

$17 = (a + 8d)$

$a + 8d = 17$ ……………………….(ii)

Subtracting equation (i) from equation (ii),

$5d = 10$

$d = 2$

From equation (i),

$a + 3(2) = 7$

$a + 6 = 7$

$a = 1$

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$= \frac{n}{2}\left[2(1) + (n-1)(2)\right]$$

$$= \frac{n}{2}(2 + 2n - 2)$$

$$= \frac{n}{2}(2n)$$

$$= n^2$$

Question 10:
Show that $a_1, a_2 \ldots, a_n, \ldots$ form an AP where a_n is defined as below
(i) $a_n = 3 + 4n$

(ii) $a_n = 9 - 5n$

Also find the sum of the first 15 terms in each case.

Answer 10:
(i) $a_n = 3 + 4n$
$a_1 = 3 + 4(1) = 7$
$a_2 = 3 + 4(2) = 3 + 8 = 11$
$a_3 = 3 + 4(3) = 3 + 12 = 15$
$a_4 = 3 + 4(4) = 3 + 16 = 19$

It can be observed that

$a_2 - a_1 = 11 - 7 = 4$
$a_3 - a_2 = 15 - 11 = 4$
$a_4 - a_3 = 19 - 15 = 4$

i.e., $a_{k+1} - a_k$ is same every time. Therefore, this is an AP with common difference as 4 and first term as 7.

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$S_{15} = \frac{15}{2}\left[2(7) + (15-1)4\right]$$

$$= \frac{15}{2}\left[(14) + 56\right]$$

$$= \frac{15}{2}(70)$$

$= 15 \times 35$
$= 525$

(ii) $a_n = 9 - 5n$
$a_1 = 9 - 5 \times 1 = 9 - 5 = 4$
$a_2 = 9 - 5 \times 2 = 9 - 10 = -1$

$a_3 = 9 - 5 \times 3 = 9 - 15 = -6$
$a_4 = 9 - 5 \times 4 = 9 - 20 = -11$

It can be observed that

$a_2 - a_1 = -1 - 4 = -5$
$a_3 - a_2 = -6 - (-1) = -5$
$a_4 - a_3 = -11 - (-6) = -5$

i.e., $a_{k+1} - a_k$ is same every time. Therefore, this is an A.P. with common difference as -5 and first term as 4.

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$S_{15} = \frac{15}{2}\left[2(4) + (15-1)(-5)\right]$$

$$= \frac{15}{2}\left[8 + 14(-5)\right]$$

$$= \frac{15}{2}(8 - 70)$$

$$= \frac{15}{2}(-62) = 15(-31)$$

$= -465$

Question 11:
If the sum of the first n terms of an AP is $4n - n^2$, what is the first term (that is S_1)? What is the sum of first two terms? What is the second term? Similarly find the 3^{rd}, the10^{th} and the n^{th} terms.

Answer 11:
Given that,
$S_n = 4n - n^2$
First term, $a = S_1 = 4(1) - (1)^2 = 4 - 1 = 3$

Sum of first two terms $= S_2 = 4(2) - (2)^2 = 8 - 4 = 4$
Second term, $a_2 = S_2 - S_1 = 4 - 3 = 1$

$d = a_2 - a = 1 - 3 = -2$

$a_n = a + (n - 1)d$
$= 3 + (n - 1)(-2)$

$= 3 - 2n + 2$

$= 5 - 2n$

Therefore,
$a_3 = 5 - 2(3) = 5 - 6 = -1$
$a_{10} = 5 - 2(10) = 5 - 20 = -15$

Hence, the sum of first two terms is 4. The second term is 1. 3^{rd}, 10^{th}, and n^{th} terms are −1, −15, and $5 - 2n$ respectively.

Question 12:
Find the sum of first 40 positive integers divisible by 6.

Answer 12:
The positive integers that are divisible by 6 are

6, 12, 18, 24 …

It can be observed that these are making an A.P. whose first term is 6 and common difference is 6.

$a = 6$ and $d = 6$
$S_{40} = ?$

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$S_{40} = \frac{40}{2}\left[2(6) + (40-1)6\right]$$

$= 20[12 + (39)(6)]$
$= 20(12 + 234)$
$= 20 \times 246$
$= 4920$

Question 13:

Find the sum of first 15 multiples of 8.

Answer 13:

The multiples of 8 are

8, 16, 24, 32…

These are in an A.P., having first term as 8 and common difference as 8.

Therefore, a = 8 and d = 8

S_{15} =?

$$S_n = \frac{n}{2}\left[2a+(n-1)d\right]$$

$$= \frac{15}{2}\left[2(8)+(15-1)8\right]$$

$$= \frac{15}{2}\left[16+14(8)\right]$$

$$= \frac{15}{2}(16+112)$$

$$= \frac{15(128)}{2} = 15\times 64$$

= 960

Question 14:

Find the sum of the odd numbers between 0 and 50.

Answer 14:

The odd numbers between 0 and 50 are

1, 3, 5, 7, 9 … 49

Therefore, it can be observed that these odd numbers are in an A.P.

a = 1, d = 2 and l = 49

$l = a + (n - 1)\, d$

$49 = 1 + (n - 1)2$

$48 = 2(n - 1)$

$n - 1 = 24$
$n = 25$

$$S_n = \frac{n}{2}(a + l)$$

$$S_{25} = \frac{25}{2}(1 + 49)$$

$$= \frac{25(50)}{2} = (25)(25)$$

$= 625$

Question 15:
A contract on construction job specifies a penalty for delay of completion beyond a certain date as follows: Rs. 200 for the first day, Rs. 250 for the second day, Rs. 300 for the third day, etc., the penalty for each succeeding day being Rs. 50 more than for the preceding day. How much money the contractor has to pay as penalty, if he has delayed the work by 30 days.

Answer 15:
It can be observed that these penalties are in an A.P. having first term as 200 and common difference as 50.

$a = 200$ and $d = 50$

Penalty that has to be paid if he has delayed the work by 30 days
$= S_{30}$

$$= \frac{30}{2}\left[2(200) + (30 - 1)50\right]$$

$= 15 [400 + 1450]$
$= 15 (1850)$
$= 27750$

Therefore, the contractor has to pay Rs 27750 as penalty.

Question 16:
A sum of Rs 700 is to be used to give seven cash prizes to students of a school for their overall academic performance. If each prize is Rs 20 less than its preceding prize, find the value of each of the prizes.

Answer 16:
Let the cost of 1st prize be P.
Cost of 2nd prize = $P - 20$
And cost of 3rd prize = $P - 40$
It can be observed that the cost of these prizes are in an A.P. having common difference as -20 and first term as P.

$a = P$ and $d = -20$

Given that, $S_7 = 700$

$$\frac{7}{2}\left[2a+(7-1)d\right]=700$$

$$\frac{\left[2a+(6)(-20)\right]}{2}=100$$

$a + 3(-20) = 100$
$a - 60 = 100$
$a = 160$

Therefore, the value of each of the prizes was Rs 160, Rs 140, Rs 120, Rs 100, Rs 80, Rs 60, and Rs 40.

Question 17:
In a school, students thought of planting trees in and around the school to reduce air pollution. It was decided that the number of trees, that each section of each class will plant, will be the same as the class, in which they are studying, e.g., a section of class I will plant 1 tree, a section of class II will plant 2 trees and so on till class XII. There are three sections of each class. How many trees will be planted by the students?

Answer 17:

It can be observed that the number of trees planted by the students is in an AP. 1, 2, 3, 4, 5………………..12

First term, a = 1

Common difference, d = 2 − 1 = 1

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$S_{12} = \frac{12}{2}\left[2(1) + (12-1)(1)\right]$$

= 6 (2 + 11)

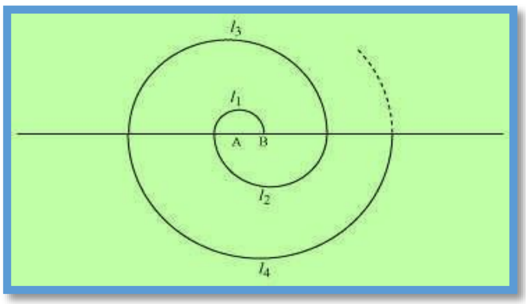
= 6 (13)

= 78

Therefore, number of trees planted by 1 section of the classes = 78
Number of trees planted by 3 sections of the classes = 3 × 78 = 234
Therefore, 234 trees will be planted by the students.

Question 18:

A spiral is made up of successive semicircles, with centres alternately at A and B, starting with centre at A of radii 0.5, 1.0 cm, 1.5 cm, 2.0 cm, ……… as shown in figure. What is the total length of such a spiral made up of thirteen consecutive semi-circles? $\left[Take\ \pi = \frac{22}{7}\right]$



Answer 18:

Semi-perimeter of circle = πr

$I_1 = \pi\ (0.5) = \frac{\pi}{2} cm$

$I_2 = \pi\ (1) = \pi\ cm$

$I_3 = \pi\ (1.5) = \frac{3\pi}{2} cm$

Therefore, I_1, I_2, I_3 ,i.e. the lengths of the semi-circles are in an A.P.,

$$\frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi, \ldots\ldots\ldots\ldots$$

$$a = \frac{\pi}{2}$$

$$d = \pi - \frac{\pi}{2} = \frac{\pi}{2}$$

S_{13} =?

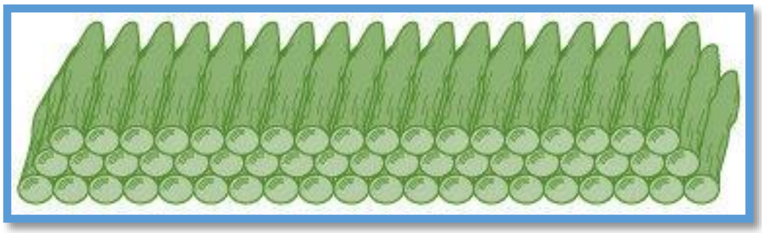
We know that the sum of *n* terms of an a A.P. is given by

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$
$$= \frac{13}{2}\left[2\left(\frac{\pi}{2}\right) + (13-1)\left(\frac{\pi}{2}\right)\right]$$
$$= \frac{13}{2}\left[\pi + \frac{12\pi}{2}\right]$$
$$= \left(\frac{13}{2}\right)(7\pi)$$
$$= \frac{91\pi}{2}$$
$$= \frac{91 \times 22}{2 \times 7} = 13 \times 11$$
$$= 143$$

Therefore, the length of such spiral of thirteen consecutive semi-circles will be 143 cm.

Question 19:

200 logs are stacked in the following manner: 20 logs in the bottom row, 19 in the next row, 18 in the row next to it and so on. In how many rows are the 200 logs placed and how many logs are in the top row?



Answer 19:

It can be observed that the numbers of logs in rows are in an A.P.
20, 19, 18…

For this A.P.,
$a = 20$ and $d = a_2 - a_1 = 19 - 20 = -1$

Let a total of 200 logs be placed in n rows.

$S_n = 200$

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$200 = \frac{n}{2}\left[2(20) + (n-1)(-1)\right]$$

$400 = n\,(40 - n + 1)$
$400 = n\,(41 - n)$
$400 = 41n - n^2$
$n^2 - 41n + 400 = 0$
$n^2 - 16n - 25n + 400 = 0$
$n\,(n - 16) - 25\,(n - 16) = 0$
$(n - 16)\,(n - 25) = 0$

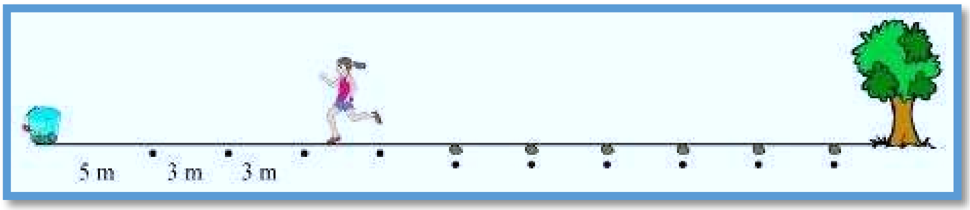
Either $(n - 16) = 0$ or $n - 25 = 0$
$n = 16$ or $n = 25$
$a_n = a + (n - 1)d$
$a_{16} = 20 + (16 - 1)\,(-1)$
$a_{16} = 20 - 15$

$a_{16} = 5$

Similarly, $a_{25} = 20 + (25 - 1)(-1)$
$a_{25} = 20 - 24$
$= -4$

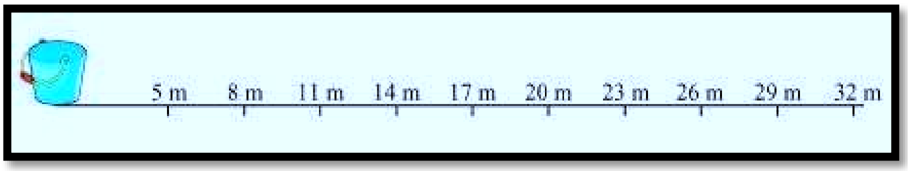
Clearly, the number of logs in 16th row is 5. However, the number of logs in 25th row is negative, which is not possible.
Therefore, 200 logs can be placed in 16 rows and the number of logs in the 16th row is 5.

Question 20:
In a potato race, a bucket is placed at the starting point, which is 5 m from the first potato and other potatoes are placed 3 m apart in a straight line. There are ten potatoes in the line.



A competitor starts from the bucket, picks up the nearest potato, runs back with it, drops it in the bucket, runs back to pick up the next potato, runs to the bucket to drop it in, and she continues in the same way until all the potatoes are in the bucket. What is the total distance the competitor has to run?
[Hint: to pick up the first potato and the second potato, the total distance (in metres) run by a competitor is $2 \times 5 + 2 \times (5 + 3)$]

Answer 20:



The distances of potatoes are as follows.
5, 8, 11, 14…

It can be observed that these distances are in A.P.
$a = 5$ $d = 8 - 5 = 3$

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$S_{10} = \frac{10}{2}\left[2(5) + (10-1)3\right]$$

= 5[10 + 9 × 3]

= 5(10 + 27) = 5(37)

= 185

As every time she has to run back to the bucket, therefore, the total distance that the competitor has to run will be two times of it.

Therefore, total distance that the competitor will run = 2 × 185 = 370 m

Alternatively,

The distances of potatoes from the bucket are 5, 8, 11, 14… Distance run by the competitor for collecting these potatoes are two times of the distance at which the potatoes have been kept.

Therefore, distances to be run are 10, 16, 22, 28, 34,……….
$a = 10$ and $d = 16 - 10 = 6$. $S_{10} = ?$

$$S_{10} = \frac{10}{2}\left[2 \times 10 + (10-1)6\right]$$

= 5[20 + 54]
= 5 (74)
= 370

Therefore, the competitor will run a total distance of 370 m.

Mathematics

(Chapter – 5) (Arithmetic Progressions)

(Class – X)

Exercise 5.4

Question 1:
Which term of the A.P. 121, 117, 113 … is its first negative term?
[Hint: Find n for $a_n < 0$]

Answer 1:
Given A.P. is 121, 117, 113 …

$a = 121$ $d = 117 - 121 = -4$
$a_n = a + (n - 1)\,d$
$= 121 + (n - 1)(-4)$
$= 121 - 4n + 4$
$= 125 - 4n$

We have to find the first negative term of this A.P.

$$\text{Therefore}, a_n < 0$$
$$125 - 4n < 0$$
$$125 < 4n$$
$$n > \frac{125}{4}$$
$$n > 31.25$$

Therefore, 32^{nd} term will be the first negative term of this A.P.

Question 2:
The sum of the third and the seventh terms of an A.P is 6 and their product is 8. Find the sum of first sixteen terms of the A.P.

Answer 2:
We know that,
$a_n = a + (n - 1)\,d$
$a_3 = a + (3 - 1)\,d$
$a_3 = a + 2d$
Similarly, $a_7 = a + 6d$

Given that,

$a_3 + a_7 = 6$
$(a + 2d) + (a + 6d) = 6$
$2a + 8d = 6$
$a + 4d = 3$
$a = 3 - 4d$ ……………………(i)

Also,

it is given that $(a_3) \times (a_7) = 8$

$(a + 2d) \times (a + 6d) = 8$

From equation (i),

$$(3-4d+2d)\times(3-4d+6d)=8$$
$$(3-2d)\times(3+2d)=8$$
$$9-4d^2=8$$
$$4d^2=9-8=1$$
$$d^2=\frac{1}{4}$$
$$d=\pm\frac{1}{2}$$
$$d=\frac{1}{2}\text{ or }-\frac{1}{2}$$

From equation (i),

$\left(\text{When } d \text{ is } \frac{1}{2}\right)$

$a=3-4d$

$$a = 3 - 4\left(\frac{1}{2}\right)$$

$$= 3 - 2 = 1$$

$\left(\text{When } d \text{ is } -\frac{1}{2}\right)$

$$a = 3 - 4\left(-\frac{1}{2}\right)$$

$$a = 3 + 2 = 5$$

$$S_n = \frac{n}{2}\left[2a(n-1)d\right]$$

$\left(\text{When } a \text{ is } 1 \text{ and } d \text{ is } \frac{1}{2}\right)$

$$S_{16} = \frac{16}{2}\left[2(1) + (16-1)\left(\frac{1}{2}\right)\right]$$

$$= 8\left[2 + \frac{15}{2}\right]$$

$$= 4(19) = 76$$

$\left(\text{When } a \text{ is } 5 \text{ and } d \text{ is } -\frac{1}{2}\right)$

$$S_{16} = \frac{16}{2}\left[2(5) + (16-1)\left(-\frac{1}{2}\right)\right]$$

$$= 8\left[10 + (15)\left(-\frac{1}{2}\right)\right]$$

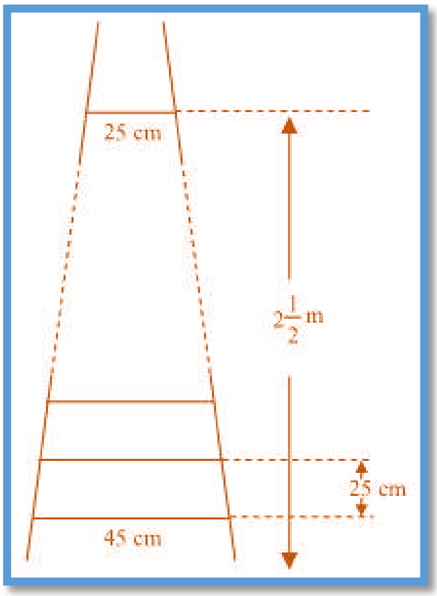
$$= 8\left(\frac{5}{2}\right)$$

$$= 20$$

Question 3:

A ladder has rungs 25 cm apart. (See figure). The rungs decrease uniformly in length from 45 cm at the bottom to 25 cm at the top. If the top and bottom rungs are $2\frac{1}{2}$ m apart, what is the length of the wood required for the rungs?

[**Hint**: number of rungs $=\frac{250}{25}$]



Answer 3:

It is given that the rungs are 25 cm apart and the top and bottom rungs are $2\frac{1}{2}$ m apart.

$\therefore$ Total number of rungs $= \dfrac{2\frac{1}{2}\times 100}{25} + 1 = \dfrac{250}{25} + 1 = 11$

Now, as the lengths of the rungs decrease uniformly, they will be in an A.P.

The length of the wood required for the rungs equals the sum of all the terms of this A.P.

First term, a = 45

Last term, l = 25 and n = 11

$$S_n = \frac{n}{2}(a+l)$$

$$\therefore S_{10} = \frac{11}{2}(45+25) = \frac{11}{2}(70) = 385 \text{ cm}$$

Therefore, the length of the wood required for the rungs is 385 cm.

Question 4:

The houses of a row are number consecutively from 1 to 49. Show that there is a value of x such that the sum of numbers of the houses preceding the house numbered x is equal to the sum of the number of houses following it. Find this value of x.

[**Hint**: $S_{x-1} = S_{49} - S_x$]

Answer 4:

The number of houses was

1, 2, 3 … 49

It can be observed that the number of houses are in an A.P. having a as 1 and d also as 1.

Let us assume that the number of x^{th} house was like this.

We know that,

Sum of n terms in an A.P. $= \frac{n}{2}[2a + (n-1)d]$

Sum of number of houses preceding x^{th} house = S_{x-1}

$$= \frac{(x-1)}{2}\left[2a + (x-1-1)d\right]$$

$$= \frac{x-1}{2}\left[2(1) + (x-2)(1)\right]$$

$$= \frac{x-1}{2}[2 + x - 2]$$

$$= \frac{(x)(x-1)}{2}$$

Sum of number of houses following x^{th} house = $S_{49} - S_x$

$$= \frac{49}{2}\left[2(1)+(49-1)(1)\right]-\frac{x}{2}\left[2(1)+(x-1)(1)\right]$$

$$= \frac{49}{2}(2+49-1)-\frac{x}{2}(2+x-1)$$

$$= \left(\frac{49}{2}\right)(50)-\frac{x}{2}(x+1)$$

$$= 25(49)-\frac{x(x+1)}{2}$$

It is given that these sums are equal to each other.

$$\frac{x(x-1)}{2} = 25(49)-x\left(\frac{x+1}{2}\right)$$

$$\frac{x^2}{2}-\frac{x}{2} = 1225-\frac{x^2}{2}-\frac{x}{2}$$

$$x^2 = 1225$$

$$x = \pm 35$$

However, the house numbers are positive integers.

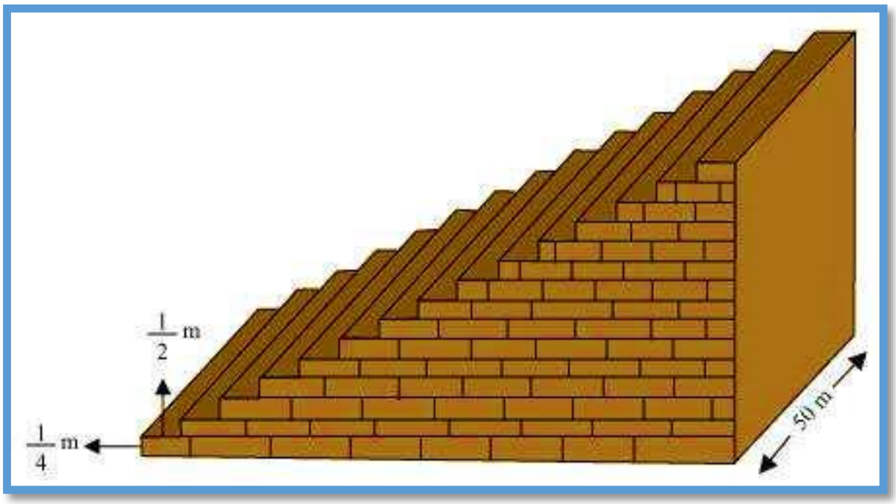
The value of x will be 35 only.

Therefore, house number 35 is such that the sum of the numbers of houses preceding the house numbered 35 is equal to the sum of the numbers of the houses following it.

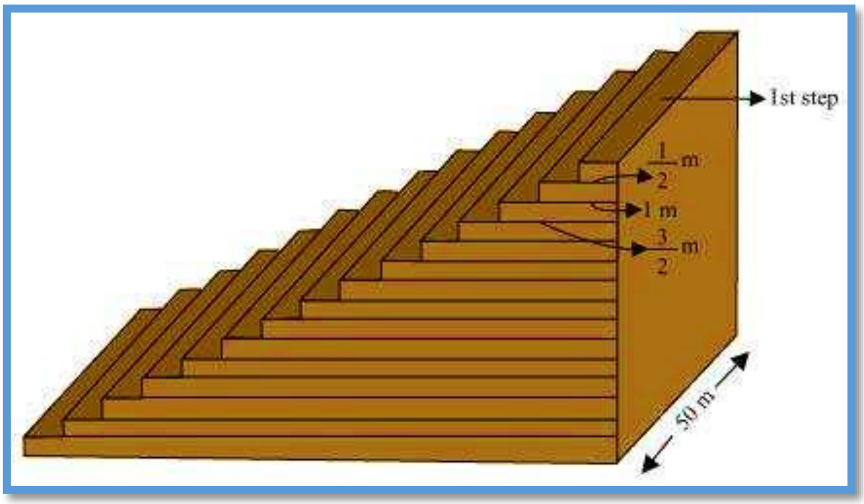
Question 5:

A small terrace at a football ground comprises of 15 steps each of which is 50 m long and built of solid concrete.

Each step has a rise of ¼ m and a tread of ½ m (See figure) calculate the total volume of concrete required to build the terrace.



Answer 5:



From the figure, it can be observed that

1st step is ½ m wide,
2nd step is 1 m wide, 3rd step is 3/2 m wide.
Therefore, the width of each step is increasing by ½ m each time
Whereas their height ¼ m and length 50 m remains the same.

Therefore, the widths of these steps are

$$\frac{1}{2}, 1, \frac{3}{2}, 2, \ldots$$

Volume of concrete in 1st step $= \frac{1}{4} \times \frac{1}{2} \times 50 = \frac{25}{4}$

Volume of concrete in 2nd step $= \frac{1}{4} \times 1 \times 50 = \frac{25}{2}$

Volume of concrete in 3rd step $= \frac{1}{4} \times \frac{3}{2} \times 50 = \frac{75}{4}$

It can be observed that the volumes of concrete in these steps are in an A.P.

$$\frac{25}{4}, \frac{25}{2}, \frac{75}{4}, \ldots$$

$$a = \frac{25}{4}$$

$$d = \frac{25}{2} - \frac{25}{4} = \frac{25}{4}$$

$$\text{and } S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$S_{15} = \frac{15}{2}\left[2\left(\frac{25}{4}\right) + (15-1)\frac{25}{4}\right]$$

$$= \frac{15}{2}\left[\frac{25}{2} + \frac{(14)25}{4}\right]$$

$$= \frac{15}{2}\left[\frac{25}{2} + \frac{175}{2}\right]$$

$$= \frac{15}{2}(100) = 750$$

Volume of concrete required to build the terrace is 750 m^3.

Mathematics

(Chapter – 6) (Triangles)

(Class – X)

Exercise 6.1

Question 1:

Fill in the blanks using correct word given in the brackets: –

(i) All circles are ___________. (congruent, similar)

(ii) All squares are ___________. (similar, congruent)

(iii) All ___________ triangles are similar. (isosceles, equilateral)

(iv) Two polygons of the same number of sides are similar, if (a) their corresponding angles are ___________ and (b) their corresponding sides are ___________. (equal, proportional)

Answer 1:

(i) Similar

(ii) Similar

(iii) Equilateral

(iv) (a) Equal

(b) Proportional

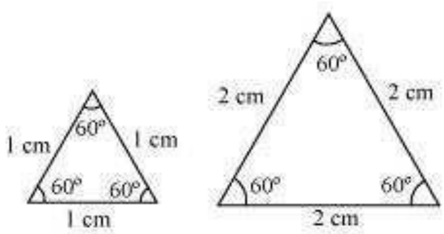
Question 2:

Give two different examples of pair of

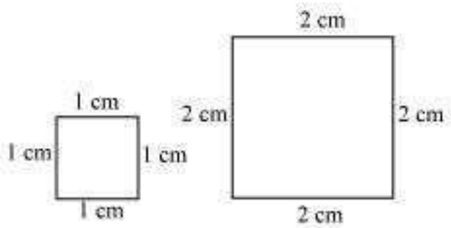
(i) Similar figures (ii)Non-similar figures

Answer 2:

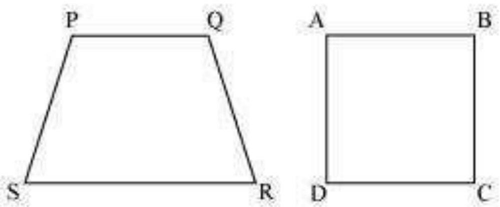
(i) Two equilateral triangles with sides 1 cm and 2 cm



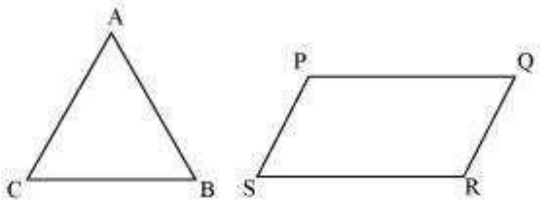
Two squares with sides 1 cm and 2 cm



(ii) Trapezium and square

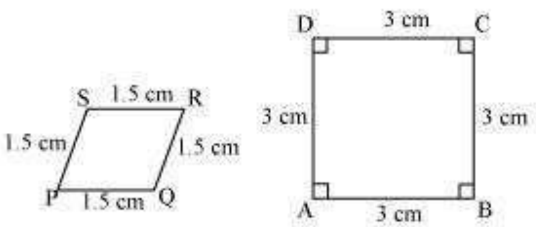


Triangle and parallelogram



Question 3:

State whether the following quadrilaterals are similar or not:



Answer 3:

Quadrilateral PQRS and ABCD are not similar as their corresponding sides are proportional, i.e. 1:2, but their corresponding angles are not equal.

Mathematics

(Chapter – 6) (Triangles)

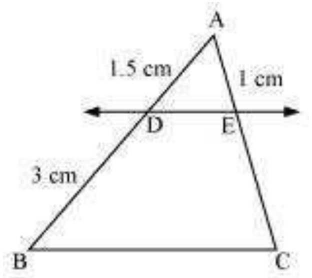
(Class – X)

Exercise 6.2

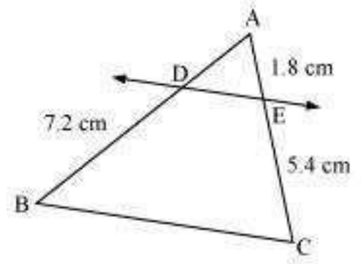
Question 1:

In figure.6.17. (i) and (ii), DE || BC. Find EC in (i) and AD in (ii).

(i)

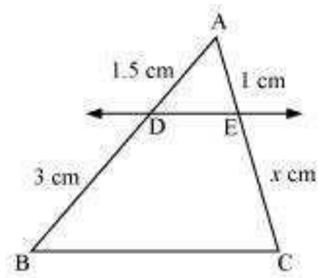


(ii)



Answer 1:

(i)



Let EC = x cm

It is given that DE || BC.

By using basic proportionality theorem, we obtain

$$\frac{AD}{DB} = \frac{AE}{EC}$$

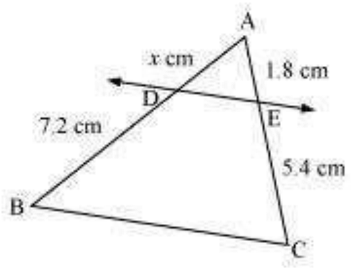
$$\frac{1.5}{3} = \frac{1}{x}$$

$$x = \frac{3 \times 1}{1.5}$$

$$x = 2$$

$$\therefore EC = 2 \text{ cm}$$

(ii)



Let AD = *x* cm

It is given that DE || BC.

By using basic proportionality theorem, we obtain

$$\frac{AD}{DB} = \frac{AE}{EC}$$

$$\frac{x}{7.2} = \frac{1.8}{5.4}$$

$$x = \frac{1.8 \times 7.2}{5.4}$$

$$x = 2.4$$

$$\therefore AD = 2.4 \text{ cm}$$

Question 2:

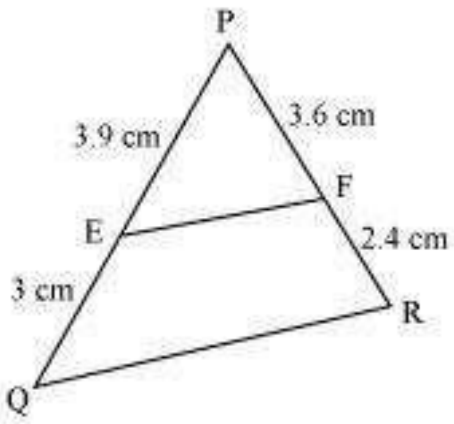
E and F are points on the sides PQ and PR respectively of a ΔPQR. For each of the following cases, state whether EF || QR.

(i) PE = 3.9 cm, EQ = 3 cm, PF = 3.6 cm and FR = 2.4 cm

(ii) PE = 4 cm, QE = 4.5 cm, PF = 8 cm and RF = 9 cm (iii)PQ = 1.28 cm, PR = 2.56 cm, PE = 0.18 cm and PF = 0.63 cm

Answer 2:

(i)



Given that, PE = 3.9 cm, EQ = 3 cm, PF = 3.6 cm, FR = 2.4 cm

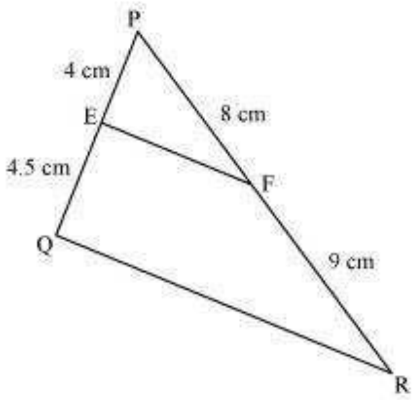
$$\frac{PE}{EQ}=\frac{3.9}{3}=1.3$$

$$\frac{PF}{FR}=\frac{3.6}{2.4}=1.5$$

Hence, $\frac{PE}{EQ}\neq\frac{PF}{FR}$

Therefore, EF is not parallel to QR.

(ii)



PE = 4 cm, QE = 4.5 cm, PF = 8 cm, RF = 9 cm

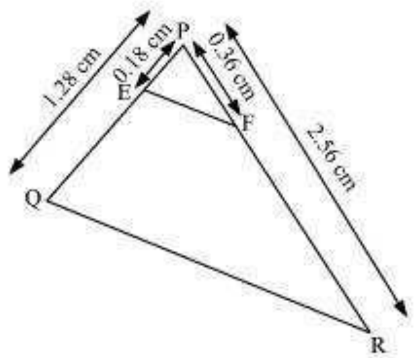
$$\frac{PE}{EQ} = \frac{4}{4.5} = \frac{8}{9}$$

$$\frac{PF}{FR} = \frac{8}{9}$$

Hence, $\frac{PE}{EQ} = \frac{PF}{FR}$

Therefore, EF is parallel to QR.

(iii)



PQ = 1.28 cm, PR = 2.56 cm, PE = 0.18 cm, PF = 0.36 cm

$$\frac{PE}{PQ} = \frac{0.18}{1.28} = \frac{18}{128} = \frac{9}{64}$$

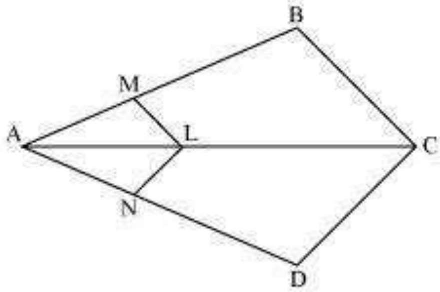
$$\frac{PF}{PR} = \frac{0.36}{2.56} = \frac{9}{64}$$

Hence, $\frac{PE}{PQ} = \frac{PF}{PR}$

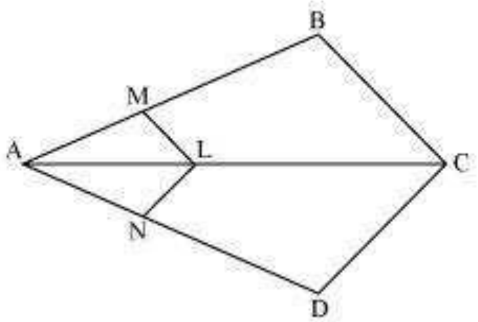
Therefore, EF is parallel to QR.

Question 3:

In the following figure, if LM || CB and LN || CD, prove that $\frac{AM}{AB} = \frac{AN}{AD}$.



Answer 3:



In the given figure, LM || CB

By using basic proportionality theorem, we obtain

$$\frac{AM}{AB} = \frac{AL}{AC} \qquad (i)$$

Similarly, LN || CD

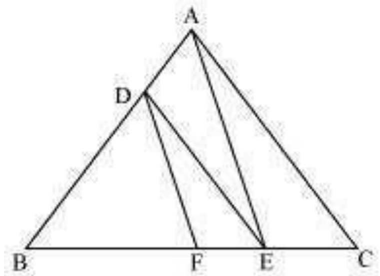
$$\therefore \frac{AN}{AD} = \frac{AL}{AC} \qquad (ii)$$

From (i) and (ii), we obtain

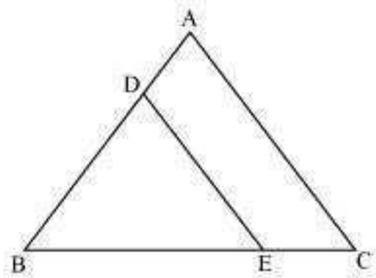
$$\frac{AM}{AB} = \frac{AN}{AD}$$

Question 4:

In the following figure, DE || AC and DF || AE. Prove that $\frac{BF}{FE} = \frac{BE}{EC}$.

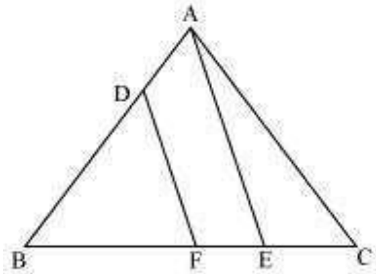


Answer 4:



In ΔABC, DE || AC

$$\therefore \frac{BD}{DA} = \frac{BE}{EC} \qquad \text{(Basic Proportionality Theorem)} \qquad (i)$$



In ΔBAE, $DF \parallel AE$

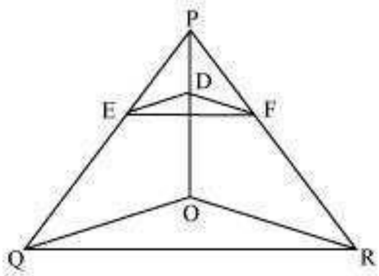
$$\therefore \frac{BD}{DA} = \frac{BF}{FE} \qquad \text{(Basic Proportionality Theorem)} \qquad (ii)$$

From (i) and (ii), we obtain

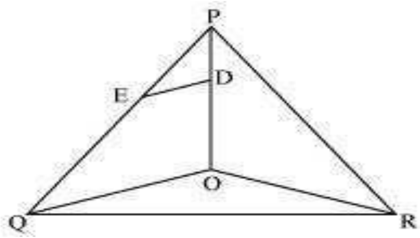
$$\frac{BE}{EC} = \frac{BF}{FE}$$

Question 5:

In the following figure, DE || OQ and DF || OR, show that EF || QR.

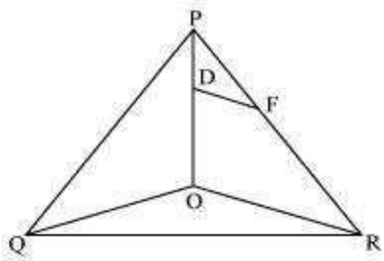


Answer 5:



In Δ POQ, DE || OQ

$$\therefore \frac{PE}{EQ}=\frac{PD}{DO} \qquad \text{(Basic proportionality theorem)} \qquad (i)$$



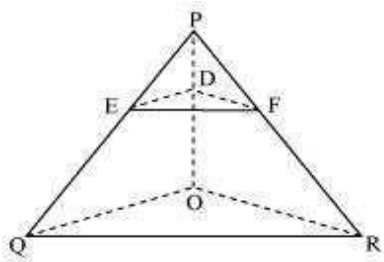
In ΔPOR, $DF \parallel OR$

$$\therefore \frac{PF}{FR}=\frac{PD}{DO} \qquad \text{(Basic proportionality theorem)} \qquad (ii)$$

From (i) and (ii), we obtain

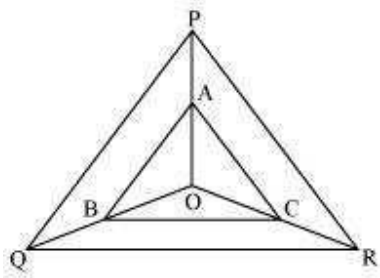
$$\frac{PE}{EQ}=\frac{PF}{FR}$$

$\therefore EF \parallel QR$ (Converse of basic proportionality theorem)

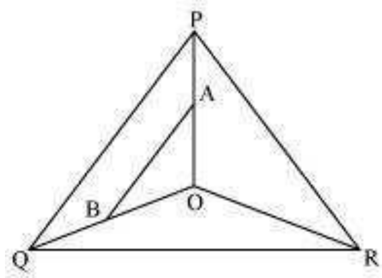


Question 6:

In the following figure, A, B and C are points on OP, OQ and OR respectively such that AB || PQ and AC || PR. Show that BC || QR.

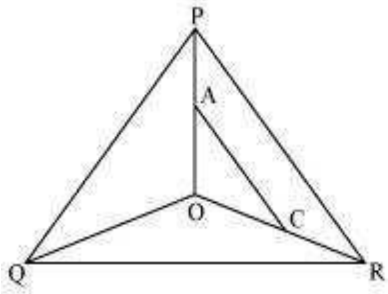


Answer 6:



In Δ POQ, AB || PQ

$$\therefore \frac{OA}{AP} = \frac{OB}{BQ} \qquad \text{(Basic proportionality theorem)} \qquad (i)$$



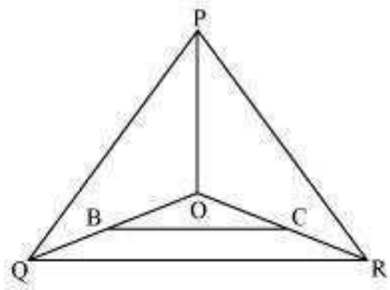
In ΔPOR, $AC \parallel PR$

$$\therefore \frac{OA}{AP} = \frac{OC}{CR} \qquad \text{(By basic proportionality theorem)} \quad (ii)$$

From (i) and (ii), we obtain

$$\frac{OB}{BQ} = \frac{OC}{CR}$$

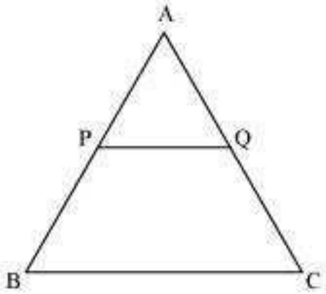
$$\therefore BC \parallel QR \qquad \text{(By the converse of basic proportionality theorem)}$$



Question 7:

Using Basic proportionality theorem, prove that a line drawn through the mid-points of one side of a triangle parallel to another side bisects the third side. (Recall that you have proved it in Class IX).

Answer 7:



Consider the given figure in which PQ is a line segment drawn through the mid-point P of line AB, such that $PQ \parallel BC$

By using basic proportionality theorem, we obtain

$$\frac{AQ}{QC} = \frac{AP}{PB}$$

$$\frac{AQ}{QC} = \frac{1}{1} \qquad (\text{P is the mid-point of AB. } \therefore\ AP = PB\ )$$

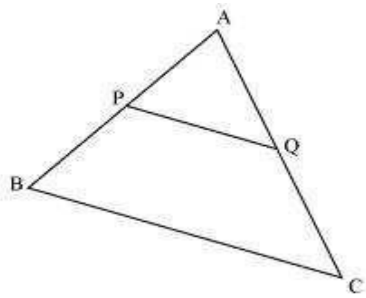
$$\Rightarrow AQ = QC$$

Or, Q is the mid-point of AC.

Question 8:

Using Converse of basic proportionality theorem, prove that the line joining the midpoints of any two sides of a triangle is parallel to the third side. (Recall that you have done it in Class IX).

Answer 8:



Consider the given figure in which PQ is a line segment joining the mid-points P and Q of line AB and AC respectively.

i.e., AP = PB and AQ = QC It can be observed that

$$\frac{AP}{PB} = \frac{1}{1}$$

and $\frac{AQ}{QC} = \frac{1}{1}$

$$\therefore \frac{AP}{PB} = \frac{AQ}{QC}$$

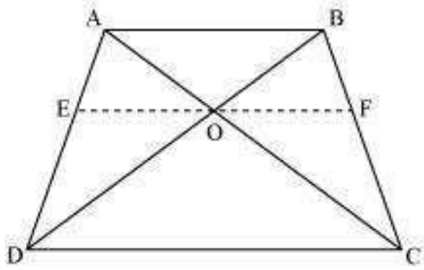
Hence, by using basic proportionality theorem, we obtain

$PQ \| BC$

Question 9:

ABCD is a trapezium in which AB || DC and its diagonals intersect each other at the point O. Show that $\frac{AO}{BO} = \frac{CO}{DO}$.

Answer 9:



Draw a line EF through point O, such that $EF \| CD$

In ΔADC, $EO \| CD$

By using basic proportionality theorem, we obtain

$$\frac{AE}{ED} = \frac{AO}{OC} \qquad (1)$$

In ΔABD, $OE \parallel AB$

So, by using basic proportionality theorem, we obtain

$$\frac{ED}{AE} = \frac{OD}{BO}$$

$$\Rightarrow \frac{AE}{ED} = \frac{BO}{OD} \qquad (2)$$

From equations (1) and (2), we obtain

$$\frac{AO}{OC} = \frac{BO}{OD}$$

$$\Rightarrow \frac{AO}{BO} = \frac{OC}{OD}$$

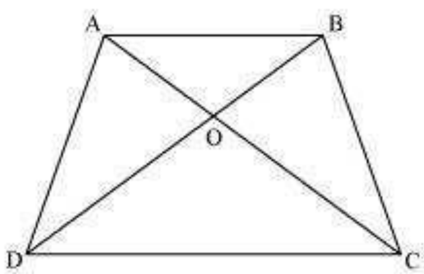
Question 10:

The diagonals of a quadrilateral ABCD intersect each other at the point O such that

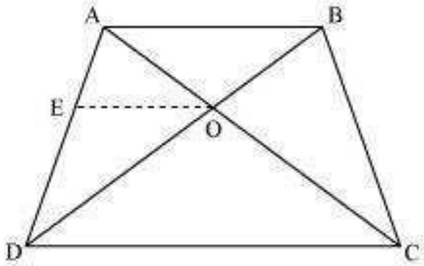
$\frac{AO}{BO} = \frac{CO}{DO}$. Show that ABCD is a trapezium.

Answer 10:

Let us consider the following figure for the given question.



Draw a line OE || AB



In ΔABD, OE || AB

By using basic proportionality theorem, we obtain

$$\frac{AE}{ED} = \frac{BO}{OD} \qquad (1)$$

However, it is given that

$$\frac{AO}{OC} = \frac{OB}{OD} \qquad (2)$$

From equations (1) and (2), we obtain

$$\frac{AE}{ED} = \frac{AO}{OC}$$

⇒ EO || DC [By the converse of basic proportionality theorem]

⇒ AB || OE || DC

⇒ AB || CD

∴ ABCD is a trapezium.

Mathematics

(Chapter – 6) (Triangles)

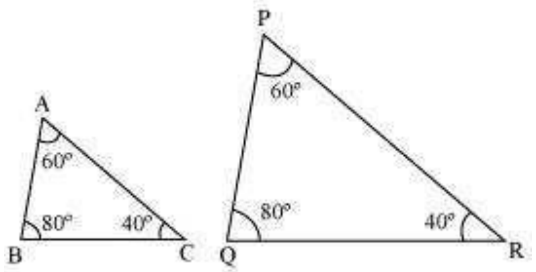
(Class – X)

Exercise 6.3

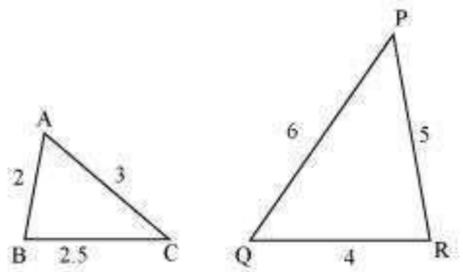
Question 1:

State which pairs of triangles in the following figure are similar? Write the similarity criterion used by you for answering the question and also write the pairs of similar triangles in the symbolic form:

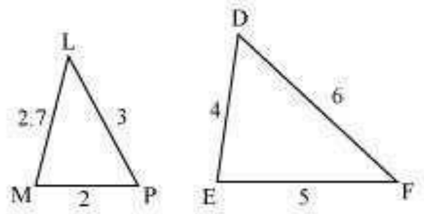
(i)



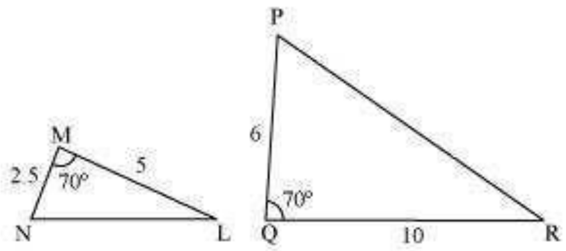
(ii)



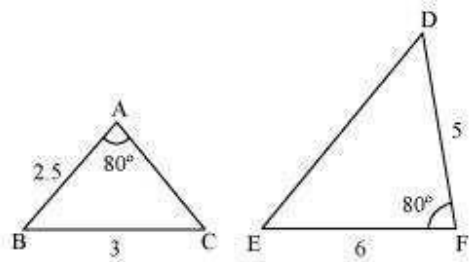
(iii)



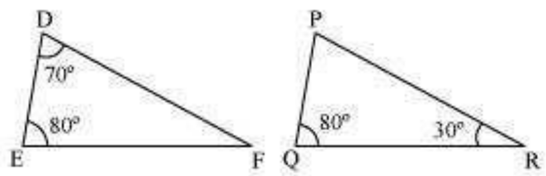
(iv)



(v)



(vi)



Answer 1:

(i) $\angle A = \angle P = 60°$

$\angle B = \angle Q = 80°$

$\angle C = \angle R = 40°$

Therefore, $\Delta ABC \sim \Delta PQR$ [By AAA similarity criterion]

$$\frac{AB}{QR} = \frac{BC}{RP} = \frac{CA}{PQ}$$

(ii)

$\therefore\ \Delta ABC \sim \Delta QRP$ [By SSS similarity criterion]

(iii)The given triangles are not similar as the corresponding sides are not proportional.

(iv)The given triangles are not similar as the corresponding sides are not proportional.

(v)The given triangles are not similar as the corresponding sides are not proportional.

(vi) In ΔDEF,

$\angle D + \angle E + \angle F = 180^{\circ}$ (Sum of the measures of the angles of a triangle is 180º.)

$70^{\circ} + 80^{\circ} + \angle F = 180^{\circ}$

$\angle F = 30^{\circ}$

Similarly, in ΔPQR,

$\angle P + \angle Q + \angle R = 180^{\circ}$

(Sum of the measures of the angles of a triangle is 180º.)

$\angle P + 80^{\circ} + 30^{\circ} = 180^{\circ}$

$\angle P = 70^{\circ}$

In ΔDEF and ΔPQR,

$\angle D = \angle P$ (Each 70°)

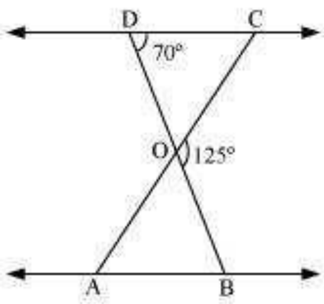
$\angle E = \angle Q$ (Each 80°)

$\angle F = \angle R$ (Each 30°)

∴ ΔDEF ~ ΔPQR [By AAA similarity criterion]

Question 2:

In the following figure, ΔODC ~ ΔOBA, $\angle BOC = 125^{\circ}$ and $\angle CDO = 70^{\circ}$. Find $\angle DOC$, $\angle DCO$ and $\angle OAB$



Answer 2:

DOB is a straight line.

∴ ∠DOC + ∠COB = 180°

⇒ ∠DOC = 180° − 125° = 55°

In ΔDOC,

∠DCO + ∠CDO + ∠DOC = 180°

(Sum of the measures of the angles of a triangle is 180º.)

⇒ ∠DCO + 70º + 55º = 180°

⇒ ∠DCO = 55°

It is given that ΔODC ~ ΔOBA.

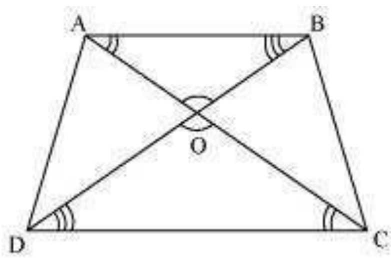
∴ ∠OAB = ∠ OCD [Corresponding angles are equal in similar triangles.]

⇒ ∠OAB = 55°

Question 3:

Diagonals AC and BD of a trapezium ABCD with AB || DC intersect each other at the point O. Using a similarity criterion for two triangles, show that $\frac{AO}{OC}=\frac{OB}{OD}$

Answer 3:



In ΔDOC and ΔBOA,

∠CDO = ∠ABO [Alternate interior angles as AB || CD]

∠DCO = ∠BAO [Alternate interior angles as AB || CD]

∠DOC = ∠BOA [Vertically opposite angles]

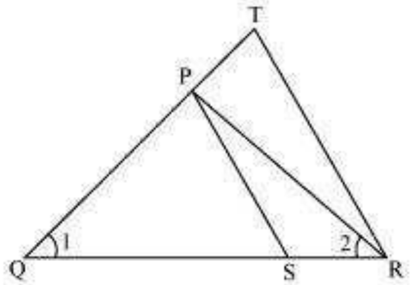
∴ ΔDOC ~ ΔBOA [AAA similarity criterion]

$$\therefore \frac{DO}{BO} = \frac{OC}{OA} \quad \left[\text{Corresponding sides are proportional}\right]$$

$$\Rightarrow \frac{OA}{OC} = \frac{OB}{OD}$$

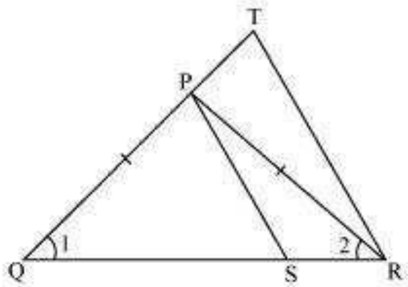
Question 4:

In the following figure, $\frac{QR}{QS} = \frac{QT}{PR}$ and $\angle 1 = \angle 2$.



Show that $\Delta PQS \sim \Delta TQR$

Answer 4:



In ΔPQR, ∠PQR = ∠PRQ

∴ PQ = PR ………………(i)

Given,

$$\frac{QR}{QS} = \frac{QT}{PR}$$

Using (i), we obtain

$$\frac{QR}{QS} = \frac{QT}{QP} \qquad (ii)$$

In ΔPQS and ΔTQR,

$$\frac{QR}{QS} = \frac{QT}{QP} \qquad [\text{Using}(ii)]$$

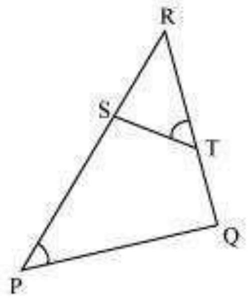
$$\angle Q = \angle Q$$

$$\therefore \ \Delta PQS \sim \Delta TQR \qquad [\text{SAS similarity criterion}]$$

Question 5:

S and T are point on sides PR and QR of ΔPQR such that ∠P = ∠RTS. Show that ΔRPQ ~ ΔRTS.

Answer 5:



In ΔRPQ and ΔRST,

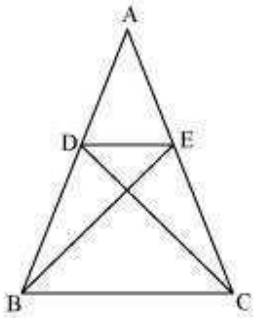
∠RTS = ∠QPS (Given)

∠R = ∠R (Common angle)

∴ ΔRPQ ~ ΔRTS (By AA similarity criterion)

Question 6:

In the following figure, if ΔABE ≅ ΔACD, show that ΔADE ~ ΔABC.



Answer 6:

It is given that ΔABE ≅ ΔACD.

∴ AB = AC [By CPCT] …………………(1)

And, AD = AE [By CPCT] ……………(2)

In ΔADE and ΔABC,

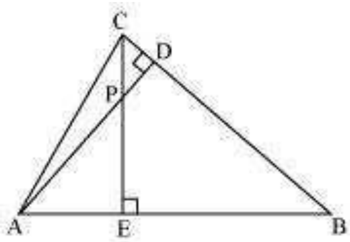
$\frac{AD}{AB} = \frac{AE}{AC}$ [Dividing equation (2) by (1)]

∠A = ∠A [Common angle]

∴ ΔADE ~ ΔABC [By SAS similarity criterion]

Question 7:

In the following figure, altitudes AD and CE of ΔABC intersect each other at the point P. Show that:



(i) ΔAEP ~ ΔCDP

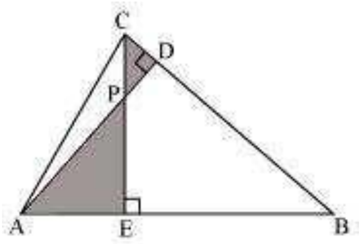
(ii) ΔABD ~ ΔCBE

(iii) ΔAEP ~ ΔADB

(v) ΔPDC ~ ΔBEC

Answer 7:

(i)



In ΔAEP and ΔCDP,

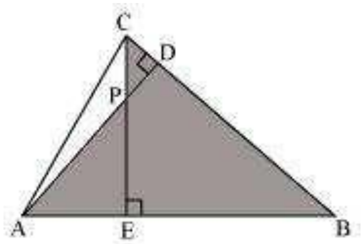
∠AEP = ∠CDP (Each 90°)

∠APE = ∠CPD (Vertically opposite angles)

Hence, by using AA similarity criterion,

ΔAEP ~ ΔCDP

(ii)



In ΔABD and ΔCBE,

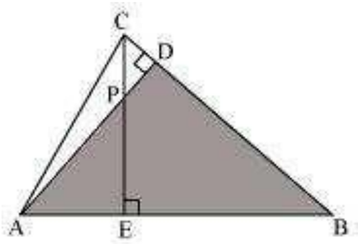
∠ADB = ∠CEB (Each 90°)

∠ABD = ∠CBE (Common)

Hence, by using AA similarity criterion,

ΔABD ~ ΔCBE

(iii)



In ΔAEP and ΔADB,

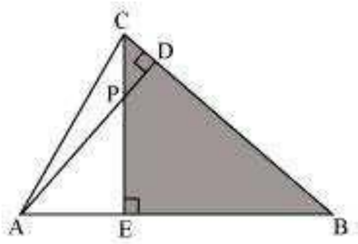
∠AEP = ∠ADB (Each 90°)

∠PAE = ∠DAB (Common)

Hence, by using AA similarity criterion,

ΔAEP ~ ΔADB

(iv)



In ΔPDC and ΔBEC,

∠PDC = ∠BEC (Each 90°)

∠PCD = ∠BCE (Common angle)

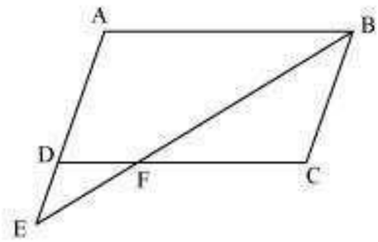
Hence, by using AA similarity criterion,

ΔPDC ~ ΔBEC

Question 8:

E is a point on the side AD produced of a parallelogram ABCD and BE intersects CD at F. Show that ΔABE ~ ΔCFB

Answer 8:



In ΔABE and ΔCFB,

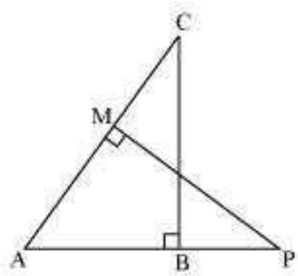
∠A = ∠C (Opposite angles of a parallelogram)

∠AEB = ∠CBF (Alternate interior angles as AE || BC)

∴ ΔABE ~ ΔCFB (By AA similarity criterion)

Question 9:

In the following figure, ABC and AMP are two right triangles, right angled at B and M respectively, prove that:



(i) ΔABC ~ ΔAMP

(ii) $\frac{CA}{PA} = \frac{BC}{MP}$

Answer 9:

In ΔABC and ΔAMP,

∠ABC = ∠AMP (Each 90°)

∠A = ∠A (Common)

∴ ΔABC ~ ΔAMP (By AA similarity criterion)

$\Rightarrow \frac{CA}{PA} = \frac{BC}{MP}$ (Corresponding sides of similar triangles are proportional)

Question 10:

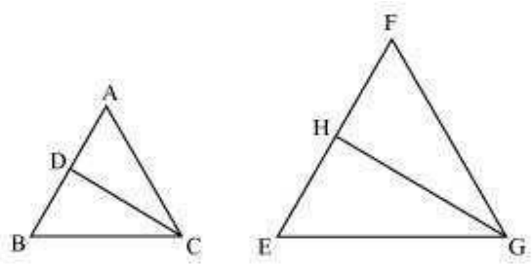
CD and GH are respectively the bisectors of ∠ACB and ∠EGF such that D and H lie on sides AB and FE of ΔABC and ΔEFG respectively. If ΔABC ~ ΔFEG, Show that:

(i) $\frac{CD}{GH} = \frac{AC}{FG}$

(ii) ΔDCB ~ ΔHGE

(iii) ΔDCA ~ ΔHGF

Answer 10:



It is given that ΔABC ~ ΔFEG.

∴ ∠A = ∠F, ∠B = ∠E, and ∠ACB = ∠FGE

∠ACB = ∠FGE

∴ ∠ACD = ∠FGH (Angle bisector)

And, ∠DCB = ∠HGE (Angle bisector)

In ΔACD and ΔFGH,

∠A = ∠F (Proved above)

∠ACD = ∠FGH (Proved above)

∴ ΔACD ~ ΔFGH (By AA similarity criterion)

$$\Rightarrow \frac{CD}{GH} = \frac{AC}{FG}$$

In ΔDCB and ΔHGE,

∠DCB = ∠HGE (Proved above)

∠B = ∠E (Proved above)

∴ ΔDCB ~ ΔHGE (By AA similarity criterion)

In ΔDCA and ΔHGF,

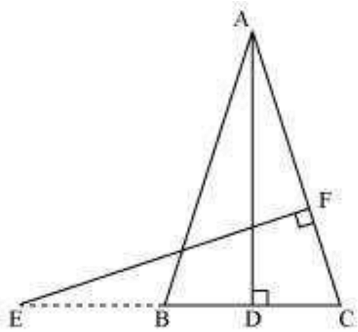
∠ACD = ∠FGH (Proved above)

∠A = ∠F (Proved above)

∴ ΔDCA ~ ΔHGF (By AA similarity criterion)

Question 11:

In the following figure, E is a point on side CB produced of an isosceles triangle ABC with AB = AC. If AD ⊥ BC and EF ⊥ AC, prove that ΔABD ~ ΔECF



Answer 11:

It is given that ABC is an isosceles triangle.

∴ AB = AC

⇒ ∠ABD = ∠ECF

In ΔABD and ΔECF,

∠ADB = ∠EFC (Each 90°)

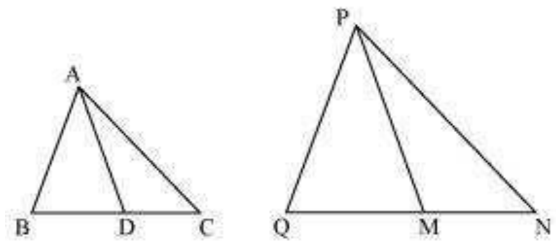
∠BAD = ∠CEF (Proved above)

∴ ΔABD ~ ΔECF (By using AA similarity criterion)

Question 12:

Sides AB and BC and median AD of a triangle ABC are respectively proportional to sides PQ and QR and median PM of ΔPQR (see the given figure). Show that ΔABC ~ ΔPQR.

Answer 12:



Median divides the opposite side.

∴ $BD = \frac{BC}{2}$ and $QM = \frac{QR}{2}$

Given that,

$$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AD}{PM}$$

$$\Rightarrow \frac{AB}{PQ} = \frac{\frac{1}{2}BC}{\frac{1}{2}QR} = \frac{AD}{PM}$$

$$\Rightarrow \frac{AB}{PQ} = \frac{BD}{QM} = \frac{AD}{PM}$$

In ΔABD and ΔPQM,

$\frac{AB}{PQ} = \frac{BD}{QM} = \frac{AD}{PM}$ (Proved above)

∴ ΔABD ~ ΔPQM (By SSS similarity criterion)

⇒ ∠ABD = ∠PQM (Corresponding angles of similar triangles)

In ΔABC and ΔPQR,

∠ABD = ∠PQM (Proved above)

$\frac{AB}{PQ} = \frac{BC}{QR}$

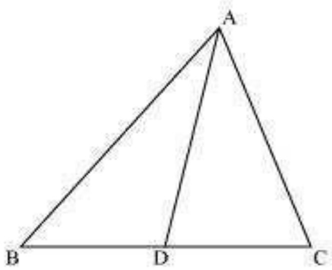
∴ ΔABC ~ ΔPQR (By SAS similarity criterion)

Question 13:

D is a point on the side BC of a triangle ABC such that ∠ADC = ∠BAC. Show that

$CA^2 = CB.CD.$

Answer 13:



In ΔADC and ΔBAC,

∠ADC = ∠BAC (Given)

∠ACD = ∠BCA (Common angle)

∴ ΔADC ~ ΔBAC (By AA similarity criterion)

We know that corresponding sides of similar triangles are in proportion.

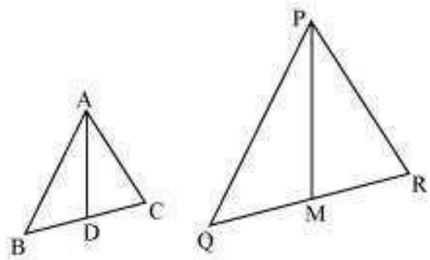
$\therefore \frac{CA}{CB} = \frac{CD}{CA}$

$\Rightarrow CA^2 = CB \times CD$

Question 14:

Sides AB and AC and median AD of a triangle ABC are respectively proportional to sides PQ and PR and median PM of another triangle PQR. Show that $\Delta ABC \sim \Delta PQR$

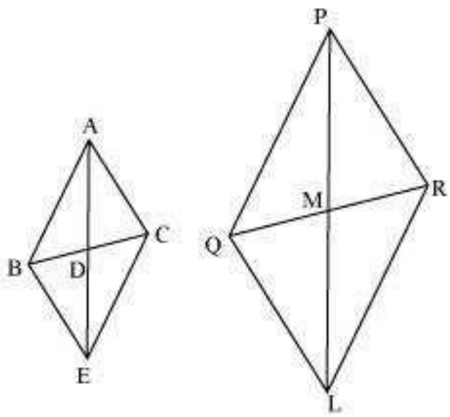
Answer 14:



Given that,

$$\frac{AB}{PQ} = \frac{AC}{PR} = \frac{AD}{PM}$$

Let us extend AD and PM up to point E and L respectively, such that AD = DE and PM = ML. Then, join B to E, C to E, Q to L, and R to L.



We know that medians divide opposite sides.

Therefore, BD = DC and QM = MR

Also, AD = DE (By construction)

And, PM = ML (By construction)

In quadrilateral ABEC, diagonals AE and BC bisect each other at point D.

Therefore, quadrilateral ABEC is a parallelogram.

∴ AC = BE and AB = EC (Opposite sides of a parallelogram are equal)

Similarly, we can prove that quadrilateral PQLR is a parallelogram and PR = QL, PQ = LR

It was given that

$$\frac{AB}{PQ}=\frac{AC}{PR}=\frac{AD}{PM}$$

$$\Rightarrow \frac{AB}{PQ}=\frac{BE}{QL}=\frac{2AD}{2PM}$$

$$\Rightarrow \frac{AB}{PQ}=\frac{BE}{QL}=\frac{AE}{PL}$$

∴ ΔABE ~ ΔPQL (By SSS similarity criterion)

We know that corresponding angles of similar triangles are equal.

∴ ∠BAE = ∠QPL … (1)

Similarly, it can be proved that ΔAEC ~ ΔPLR and

∠CAE = ∠RPL … (2)

Adding equation (1) and (2), we obtain

∠BAE + ∠CAE = ∠QPL + ∠RPL

⇒ ∠CAB = ∠RPQ … (3)

In ΔABC and ΔPQR,

$\frac{AB}{PQ}=\frac{AC}{PR}$ (Given)

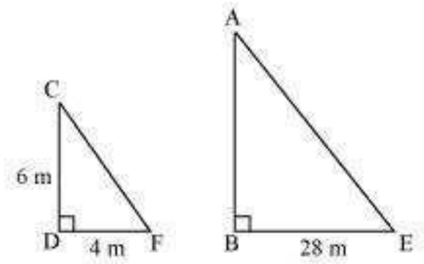
∠CAB = ∠RPQ [Using equation (3)]

∴ ΔABC ~ ΔPQR (By SAS similarity criterion)

Question 15:

A vertical pole of a length 6 m casts a shadow 4m long on the ground and at the same time a tower casts a shadow 28 m long. Find the height of the tower.

Answer 15:



Let AB and CD be a tower and a pole respectively.

Let the shadow of BE and DF be the shadow of AB and CD respectively.

At the same time, the light rays from the sun will fall on the tower and the pole at the same angle.

Therefore, ∠DCF = ∠BAE

And, ∠DFC = ∠BEA

∠CDF = ∠ABE (Tower and pole are vertical to the ground)

∴ ΔABE ~ ΔCDF (AAA similarity criterion)

$$\Rightarrow \frac{AB}{CD} = \frac{BE}{DF}$$

$$\Rightarrow \frac{AB}{6\text{ m}} = \frac{28}{4}$$

$$\Rightarrow AB = 42\text{ m}$$

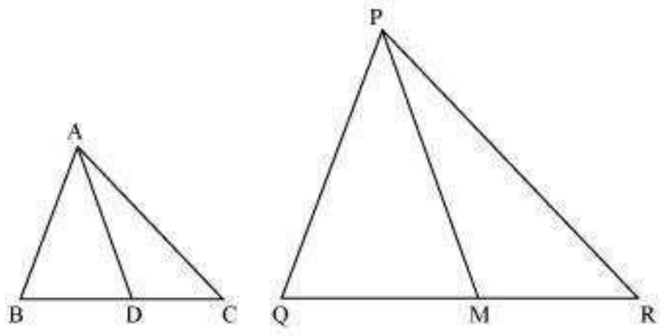
Therefore, the height of the tower will be 42 metres.

Question 16:

If AD and PM are medians of triangles ABC and PQR, respectively where

$\Delta ABC \sim \Delta PQR$ prove tha t $\frac{AB}{PQ} = \frac{AD}{PM}$

Answer 16:



It is given that ΔABC ~ ΔPQR

We know that the corresponding sides of similar triangles are in proportion.

$\therefore \frac{AB}{PQ}=\frac{AC}{PR}=\frac{BC}{QR}$ … (1)

Also, ∠A = ∠P, ∠B = ∠Q, ∠C = ∠R … (2)

Since AD and PM are medians, they will divide their opposite sides.

$\therefore BD=\frac{BC}{2}$ and $QM=\frac{QR}{2}$ … (3)

From equations (1) and (3), we obtain

$\frac{AB}{PQ}=\frac{BD}{QM}$ … (4)

In ΔABD and ΔPQM,

∠B = ∠Q [Using equation (2)]

$\frac{AB}{PQ}=\frac{BD}{QM}$ [Using equation (4)]

∴ ΔABD ~ ΔPQM (By SAS similarity criterion)

$\Rightarrow \frac{AB}{PQ}=\frac{BD}{QM}=\frac{AD}{PM}$

Mathematics

(Chapter – 6) (Triangles)

(Class – X)

Exercise 6.4

Question 1:

Let $\Delta ABC \sim \Delta DEF$ and their areas be, respectively, 64 cm^2 and 121 cm^2. If EF = 15.4 cm, find BC.

Answer 1:

It is given that $\Delta ABC \sim \Delta DEF$.

$$\therefore \frac{ar(\Delta ABC)}{ar(\Delta DEF)} = \left(\frac{AB}{DE}\right)^2 = \left(\frac{BC}{EF}\right)^2 = \left(\frac{AC}{DF}\right)^2$$

Given that,

$EF = 15.4$ cm,

$ar(\Delta ABC) = 64\ cm^2$,

$ar(\Delta DEF) = 121\ cm^2$

$$\therefore \frac{ar(ABC)}{ar(DEF)} = \left(\frac{BC}{EF}\right)^2$$

$$\Rightarrow \left(\frac{64\ cm^2}{121\ cm^2}\right) = \frac{BC^2}{(15.4\ cm)^2}$$

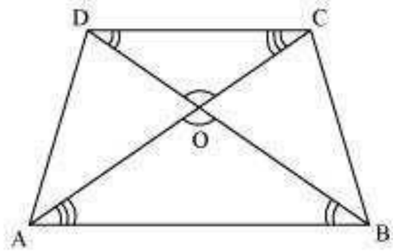
$$\Rightarrow \frac{BC}{15.4} = \left(\frac{8}{11}\right) cm$$

$$\Rightarrow BC = \left(\frac{8 \times 15.4}{11}\right) cm = (8 \times 1.4) cm = 11.2\, cm$$

Question 2:

Diagonals of a trapezium ABCD with AB || DC intersect each other at the point O. If AB = 2CD, find the ratio of the areas of triangles AOB and COD.

Answer 2:



Since AB || CD,

∴ ∠OAB = ∠OCD and ∠OBA = ∠ODC (Alternate interior angles)

In ΔAOB and ΔCOD,

∠AOB = ∠COD (Vertically opposite angles)

∠OAB = ∠OCD (Alternate interior angles)

∠OBA = ∠ODC (Alternate interior angles)

∴ ΔAOB ~ ΔCOD (By AAA similarity criterion)

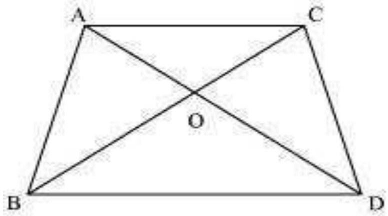
$$\therefore \frac{ar(\Delta AOB)}{ar(\Delta COD)} = \left(\frac{AB}{CD}\right)^2$$

Since $AB = 2\,CD$,

$$\therefore \frac{ar(\Delta AOB)}{ar(\Delta COD)} = \left(\frac{2\,CD}{CD}\right)^2 = \frac{4}{1} = 4:1$$

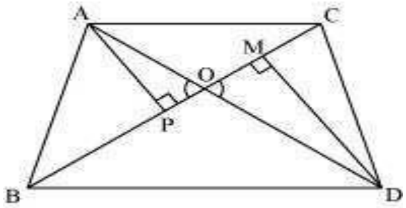
Question 3:

In the following figure, ABC and DBC are two triangles on the same base BC. If AD intersects BC at O, show that $\frac{\text{area}(\Delta ABC)}{\text{area}(\Delta DBC)} = \frac{AO}{DO}$



Answer 3:

Let us draw two perpendiculars AP and DM on line BC.



We know that area of a triangle = $\frac{1}{2} \times \text{Base} \times \text{Height}$

$$\therefore \frac{ar(\Delta ABC)}{ar(\Delta DBC)} = \frac{\frac{1}{2} BC \times AP}{\frac{1}{2} BC \times DM} = \frac{AP}{DM}$$

.

In ΔAPO and ΔDMO,

∠APO = ∠DMO (Each = 90°)

∠AOP = ∠DOM (Vertically opposite angles)

∴ ΔAPO ~ ΔDMO (By AA similarity criterion)

$$\therefore \frac{AP}{DM} = \frac{AO}{DO}$$

$$\Rightarrow \frac{ar(\Delta ABC)}{ar(\Delta DBC)} = \frac{AO}{DO}$$

Question 4:

If the areas of two similar triangles are equal, prove that they are congruent.

Answer 4:

Let us assume two similar triangles as ΔABC ~ ΔPQR.

$$\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \left(\frac{AB}{PQ}\right)^2 = \left(\frac{BC}{QR}\right)^2 = \left(\frac{AC}{PR}\right)^2 \qquad (1)$$

Given that, ar $(\Delta ABC) = $ ar (ΔPQR)

$$\Rightarrow \frac{ar(\Delta ABC)}{ar(\Delta PQR)} = 1$$

Putting this value in equation (1), we obtain

$$1 = \left(\frac{AB}{PQ}\right)^2 = \left(\frac{BC}{QR}\right)^2 = \left(\frac{AC}{PR}\right)^2$$

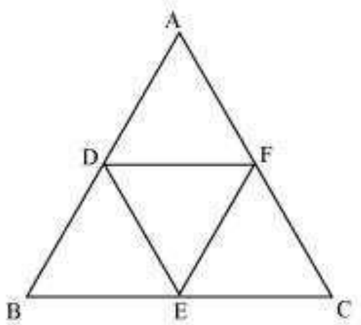
$\Rightarrow AB = PQ, BC = QR,$ and $AC = PR$

$\therefore \Delta ABC \cong \Delta PQR$ (By SSS congruence criterion)

Question 5:

D, E and F are respectively the mid-points of sides AB, BC and CA of ΔABC. Find the ratio of the area of ΔDEF and ΔABC.

Answer 5:



D and E are the mid-points of ΔABC.

$\therefore DE \parallel AC$ and $DE = \frac{1}{2}AC$

In ΔBED and ΔBCA,

$\angle BED = \angle BCA$ (Corresponding angles)

$\angle BDE = \angle BAC$ (Corresponding angles)

$\angle EBD = \angle CBA$ (Common angles)

$\therefore \Delta BED \sim \Delta BCA$ (AAA similarity criterion)

$$\frac{ar(\Delta BED)}{ar(\Delta BCA)} = \left(\frac{DE}{AC}\right)^2$$

$$\Rightarrow \frac{ar(\Delta BED)}{ar(\Delta BCA)} = \frac{1}{4}$$

$$\Rightarrow ar(\Delta BED) = \frac{1}{4}ar(\Delta BCA)$$

Similarly, $ar(\Delta CFE) = \frac{1}{4}ar(CBA)$ and $ar(\Delta ADF) = \frac{1}{4}ar(\Delta ABC)$

Also, $ar(\Delta DEF) = ar(\Delta ABC) - \left[ar(\Delta BED) + ar(\Delta CFE) + ar(\Delta ADF)\right]$

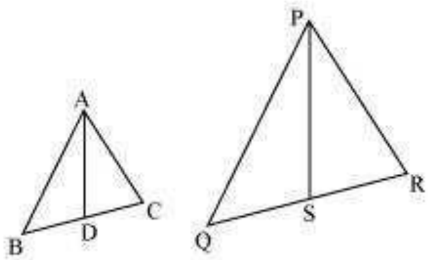
$$\Rightarrow ar(\Delta DEF) = ar(\Delta ABC) - \frac{3}{4}ar(\Delta ABC) = \frac{1}{4}ar(\Delta ABC)$$

$$\Rightarrow \frac{ar(\Delta DEF)}{ar(\Delta ABC)} = \frac{1}{4}$$

Question 6:

Prove that the ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding medians.

Answer 6:



Let us assume two similar triangles as ΔABC ~ ΔPQR. Let AD and PS be the medians of these triangles.

∵ ΔABC ~ ΔPQR

$\therefore \frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$ …………(1)

∠A = ∠P, ∠B = ∠Q, ∠C = ∠R ………………. (2)

Since AD and PS are medians,

∴ BD = DC = $\frac{BC}{2}$

And, QS = SR = $\frac{QR}{2}$

Equation (1) becomes

$\frac{AB}{PQ} = \frac{BD}{QS} = \frac{AC}{PR}$ ………….. (3)

In ΔABD and ΔPQS,

∠B = ∠Q [Using equation (2)]

And, $\frac{AB}{PQ} = \frac{BD}{QS}$ [Using equation (3)]

∴ ΔABD ~ ΔPQS (SAS similarity criterion)

Therefore, it can be said that

$$\frac{AB}{PQ}=\frac{BD}{QS}=\frac{AD}{PS}$$ ………………… (4)

$$\frac{ar(\Delta ABC)}{ar(\Delta PQR)}=\left(\frac{AB}{PQ}\right)^2=\left(\frac{BC}{QR}\right)^2=\left(\frac{AC}{PR}\right)^2$$

From equations (1) and (4), we may find that

$$\frac{AB}{PQ}=\frac{BC}{QR}=\frac{AC}{PR}=\frac{AD}{PS}$$

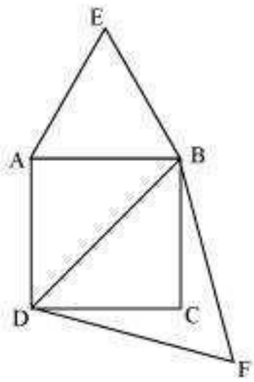
And hence,

$$\frac{ar(\Delta ABC)}{ar(\Delta PQR)}=\left(\frac{AD}{PS}\right)^2$$

Question 7:

Prove that the area of an equilateral triangle described on one side of a square is equal to half the area of the equilateral triangle described on one of its diagonals.

Answer 7:



Let ABCD be a square of side a.

Therefore, its diagonal $=\sqrt{2}a$

Two desired equilateral triangles are formed as ΔABE and ΔDBF.

Side of an equilateral triangle, ΔABE, described on one of its sides = *a*

Side of an equilateral triangle, ΔDBF, described on one of its diagonals $= \sqrt{2}a$

We know that equilateral triangles have all its angles as 60º and all its sides of the same length. Therefore, all equilateral triangles are similar to each other. Hence, the ratio between the areas of these triangles will be equal to the square of the ratio between the sides of these triangles.

$$\frac{\text{Area of } \Delta \text{ABE}}{\text{Area of } \Delta \text{DBF}} = \left(\frac{a}{\sqrt{2}a}\right)^2 = \frac{1}{2}$$

Question 8:

ABC and BDE are two equilateral triangles such that D is the mid-point of BC. Ratio of the area of triangles ABC and BDE is

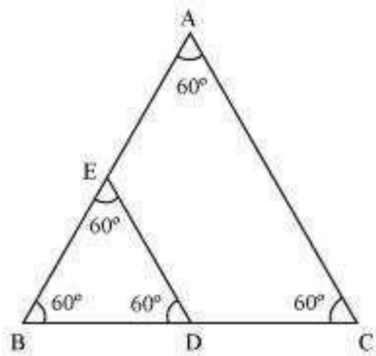
(A) 2 : 1 (B)

1 : 2

(C) 4 : 1

(D) 1 : 4

Answer 8:



We know that equilateral triangles have all its angles as 60º and all its sides of the same length. Therefore, all equilateral triangles are similar to each other.

Hence, the ratio between the areas of these triangles will be equal to the square of the ratio between the sides of these triangles.

Let side of ΔABC = x

Therefore, side of $\Delta BDE = \frac{x}{2}$

$$\therefore \frac{\text{area}(\Delta ABC)}{\text{area}(\Delta BDE)} = \left(\frac{x}{\frac{x}{2}}\right)^2 = \frac{4}{1}$$

Hence, the correct answer is (C).

Question 9:

Sides of two similar triangles are in the ratio 4 : 9. Areas of these triangles are in the ratio

(A) 2 : 3 (B) 4 : 9 (C) 81 : 16 (D) 16 : 81

Answer 9:

If two triangles are similar to each other, then the ratio of the areas of these triangles will be equal to the square of the ratio of the corresponding sides of these triangles.

It is given that the sides are in the ratio 4:9.

Therefore, ratio between areas of these triangles = $\left(\frac{4}{9}\right)^2 = \frac{16}{81}$

Hence, the correct answer is (D).

Mathematics

(Chapter – 6) (Triangles)
(Class 10)

Exercise 6.5

Question 1:

Sides of triangles are given below. Determine which of them are right triangles. In case of a right triangle, write the length of its hypotenuse.

(i) 7 cm, 24 cm, 25 cm
(ii) 3 cm, 8 cm, 6 cm
(iii) 50 cm, 80 cm, 100 cm
(iv) 13 cm, 12 cm, 5 cm

Answer 1:

(i) Sides of triangle: 7 cm, 24 cm and 25 cm.
Squaring these sides, we get 49, 576 and 625.
$49 + 576 = 625 \quad \Rightarrow \; 7^2 + 24^2 = 25^2$
These sides satisfy the Pythagoras triplet, hence these are sides of right angled triangle.
We know that the hypotenuses is the longest side in right angled triangle.
Hence, its length is 25 cm.

(ii) Sides of triangle: 3 cm, 6 cm and 8 cm.
Squaring these sides, we get 9, 36 and 64.
$9 + 36 \neq 64 \quad \Rightarrow \; 3^2 + 6^2 \neq 8^2$
These sides do not satisfy the Pythagoras triplet, hence these are not the sides of right angled triangle.

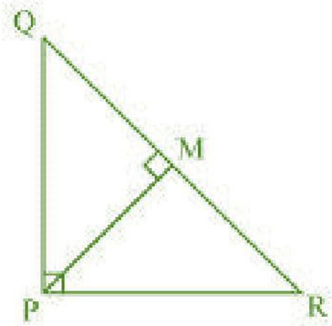
(iii) Sides of triangle: 50 cm, 80 cm and 100 cm.
Squaring these sides, we get 2500, 6400 and 10000.
$2500 + 6400 \neq 10000 \quad \Rightarrow \; 50^2 + 80^2 \neq 100^2$
These sides do not satisfy the Pythagoras triplet, hence these are not the sides of right angled triangle.

(iv) Sides of triangle: 5 cm, 12 cm and 13 cm.
Squaring these sides, we get 25, 144 and 169.
$25 + 144 = 169 \quad \Rightarrow \; 5^2 + 12^2 = 13^2$
These sides satisfy the Pythagoras triplet, hence these are sides of right angled triangle.
We know that the hypotenuses is the longest side in right angled triangle.
Hence, its length is 13 cm.

Question 2:

PQR is a triangle right angled at P and M is a point on QR such that PM ⊥ QR. Show that PM^2 = QM.MR.

Answer 2:



Let $\angle MPR = x$
In ΔMPR,
$\angle MRP = 180° - 90° - x$
Similarly,
In ΔMPR,
$\angle MPQ = \; 90° - \angle MPR \; = 90° - x$
$\angle MQP \; = 180° - 90° - (90° - x) = x$
In ΔQMP and ΔPMR,
∠MPQ = ∠MRP
∠PMQ = ∠RMP
∠MQP = ∠MPR
⇒ΔQMP ~ ΔPMR [AAA similarity]
We know that the corresponding sides of similar triangles are proportional.
Therefore,
$$\frac{QM}{PM} = \frac{MP}{MR}$$
$\Rightarrow PM^2 = MQ \times MR$

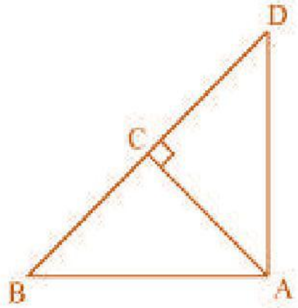
Question 3:

In Figure, ABD is a triangle right angled at A and AC ⊥ BD. Show that

(i) $AB^2 = BC \times BD$

(ii) $AC^2 = BC \times DC$

(iii) $AD^2 = BD \times CD$



Answer 3:

(i) In ΔADB and ΔCAB,

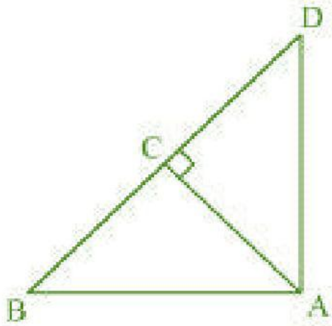
$\angle DAB = \angle ACB$ [Each 90°]

$\angle ABD = \angle CBA$ [Common]

$\therefore \Delta DCM \sim \Delta BDM$ [AA similarity]

$\Rightarrow \dfrac{AB}{CB} = \dfrac{BD}{AB}$

$\Rightarrow AB^2 = CB \times BD$



(ii) Let $\angle CAB = x$

In ΔCBA,

$\angle CBA = 180° - 90° - x$

$\Rightarrow \angle CBA = 90° - x$

Similarly, in ΔCAD,

$\angle CAD = 90° - \angle CAB$

$\Rightarrow \angle CAD = 90° - x$

$\angle CDA = 180° - 90° - (90° - x)$

$\Rightarrow \angle CDA = x$

In ΔCBA and ΔCAD,

$\angle CBA = \angle CAD$ [Proved above]

$\angle CAB = \angle CDA$ [Proved above]

$\angle ACB = \angle DCA$ [Each 90°]

$\therefore \Delta CBA \sim \Delta CAD$ [AAA similarity]

$\Rightarrow \dfrac{AC}{DC} = \dfrac{BC}{AC}$

$\Rightarrow AC^2 = BC \times DC$

(iii) In ΔDCA and ΔDAB,

$\angle DCA = \angle DAB$ [Each 90°]

$\angle CDA = \angle ADB$ [Common]

$\therefore \Delta DCA \sim \Delta DAB$ [AA similarity]

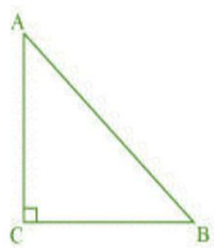
$\Rightarrow \dfrac{DC}{DA} = \dfrac{DA}{DB}$

$\Rightarrow AD^2 = BD \times CD$

Question 4:

ABC is an isosceles triangle right angled at C. Prove that $AB^2 = 2AC^2$.

Answer 4:



Given that the triangle ABC is an isosceles triangle such that AC = BC and ∠C = 90°,

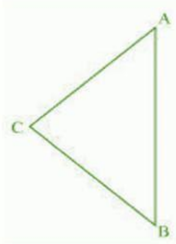
In ΔABC, by Pythagoras theorem

$AB^2 = AC^2 + BC^2$

$\Rightarrow AB^2 = AC^2 + AC^2$ [Because AC = BC]

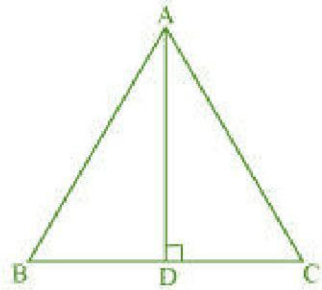
$\Rightarrow AB^2 = 2AC^2$

Question 5:
ABC is an isosceles triangle with AC = BC. If $AB^2 = 2AC^2$, prove that ABC is a right triangle.

Answer 5:


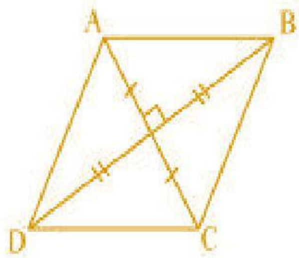
Given that: $AB^2 = 2AC^2$
$\Rightarrow AB^2 = AC^2 + AC^2$
$\Rightarrow AB^2 = AC^2 + BC^2$ [Because AC = BC]
These sides satisfy the Pythagoras theorem.
Hence, the triangle ABC is a right angled triangle.

Question 6:
ABC is an equilateral triangle of side $2a$. Find each of its altitudes.

Answer 6:


Let ABC be any equilateral triangle with each sides of length 2a. Perpendicular AD is drawn from A to BC.
We know that the altitude in equilateral triangle, bisects the opposite sides.
Therefore, ∴ BD = DC = a
In ΔADB, by Pythagoras theorem
$AB^2 = AD^2 + BD^2$
$\Rightarrow (2a)^2 = AD^2 + a^2$ [Because AB = 2a]
$\Rightarrow 4a^2 = AD^2 + a^2$
$\Rightarrow AD^2 = 3a^2 \Rightarrow AD = \sqrt{3}a$
Hence, the length of each altitude is $\sqrt{3}a$.

Question 7:
Prove that the sum of the squares of the sides of a rhombus is equal to the sum of the squares of its diagonals.

Answer 7:


In ΔAOB, by Pythagoras theorem
$AB^2 = AO^2 + OB^2$ ... (i)
In ΔBOC, by Pythagoras theorem
$BC^2 = BO^2 + OC^2$ ... (ii)
In ΔCOD, by Pythagoras theorem
$CD^2 = CO^2 + OD^2$ ... (iii)
In ΔAOD, by Pythagoras theorem
$AD^2 = AO^2 + OD^2$ ... (iv)

Adding the equations (i), (ii), (iii) and (iv), we have

$AB^2 + BC^2 + CD^2 + AD^2 = OA^2 + OB^2 + OB^2 + OC^2 + OC^2 + OD^2 + OD^2 + OA^2$

$= 2[OA^2 + OB^2 + OC^2 + OD^2]$

= 2[2OA2 + 2OB2] [Because OA = OC, OB = OD]

= 4[OA2 + OB2]

$= 4\left[\left(\frac{AC}{2}\right)^2 + \left(\frac{BD}{2}\right)^2\right]$ [Because OA = ½ AC, OB = ½ BD]

$= 4\left[\frac{AC^2}{4} + \frac{BD^2}{4}\right]$

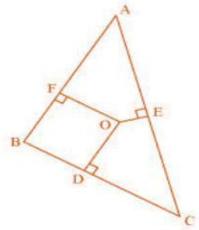
$= AC^2 + BD^2$

Question 8:

In Figure, O is a point in the interior of a triangle ABC, OD ⊥ BC, OE ⊥ AC and OF ⊥ AB. Show that

(i) $OA^2 + OB^2 + OC^2 - OD^2 - OE^2 - OF^2 = AF^2 + BD^2 + CE^2$

(ii) $AF^2 + BD^2 + CE^2 = AE^2 + CD^2 + BF^2$



Answer 8:

Join OA, OB and OC.

(i) In ΔAOF, by Pythagoras theorem

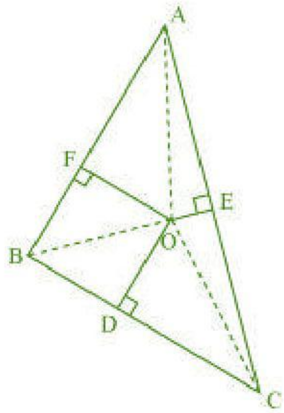
$OA^2 = OF^2 + AF^2$ ... (i)

In ΔBOD, by Pythagoras theorem

$OB^2 = OD^2 + BD^2$ ... (ii)

In ΔCOE, by Pythagoras theorem

$OC^2 = OE^2 + EC^2$ ... (iii)



Adding equations (i), (ii) and (iii), we have

$OA^2 + OB^2 + OC^2$

$= OF^2 + AF^2 + OD^2 + BD^2 + OE^2 + EC^2$

$\Rightarrow OA^2 + OB^2 + OC^2 - OD^2 - OE^2 - OF^2 = AF^2 + BD^2 + CE^2$ ... (iv)

(ii) From the equation (iv), we have

$AF^2 + BD^2 + CE^2$

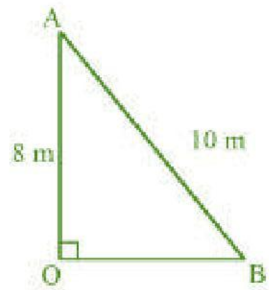
$= OA^2 + OB^2 + OC^2 - OD^2 - OE^2 - OF^2$

$= (OA^2 - OE^2) + (OC^2 - OD^2) + (OB^2 - OF^2)$

$= AE^2 + CD^2 + BF^2$

Question 9:

A ladder 10 m long reaches a window 8 m above the ground. Find the distance of the foot of the ladder from base of the wall.



Answer 9:

Let OA is wall and AB is ladder in the figure.

In ΔAOB, by Pythagoras theorem

$AB^2 = OA^2 + OB^2$

$\Rightarrow 10^2 = 8^2 + BO^2$

$\Rightarrow 100 = 64 + BO^2$

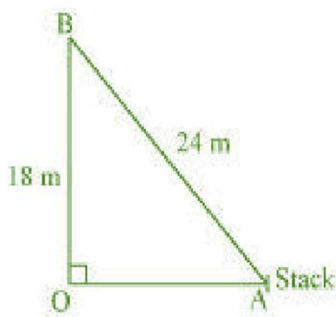
$\Rightarrow BO^2 = 36$

$\Rightarrow BO = 6\ m$

Hence, the distance of the foot of the ladder from the base of the wall is 6 m.

Question 10:

A guy wire attached to a vertical pole of height 18 m is 24 m long and has a stake attached to the other end. How far from the base of the pole should the stake be driven so that the wire will be taut?



Answer 10:

Let OB is vertical pole in the figure.

In ΔAOB, by Pythagoras theorem

$AB^2 = OB^2 + OA^2$

$\Rightarrow 24^2 = 18^2 + OA^2$

$\Rightarrow 576 = 324 + OA^2$

$\Rightarrow OA^2 = 252$

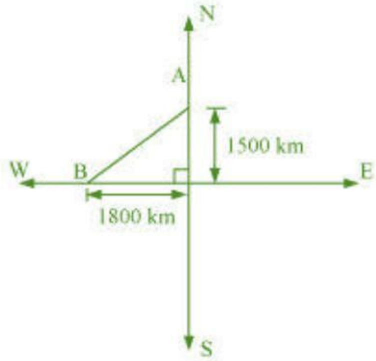
$\Rightarrow OA = 6\sqrt{7}\ m$

Hence, the distance of stake from the base of the pole is $6\sqrt{7}\ m$.

Question 11:

An aeroplane leaves an airport and flies due north at a speed of 1000 km per hour. At the same time, another aeroplane leaves the same airport and flies due west at a speed of 1200 km per hour. How far apart will be the two planes after $1\frac{1}{2}$ hours?

Answer 11:



Distance travelled by first aeroplane (due north) in $1\frac{1}{2}$ hours

$= 1000 \times \frac{3}{2} = 1500\ km$

Distance travelled by second aeroplane (due west) in $1\frac{1}{2}$ hours

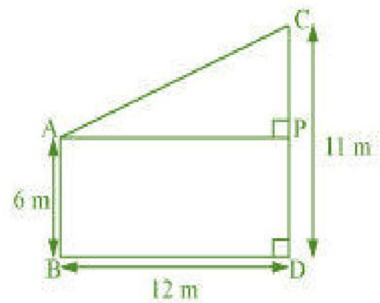
$= 1200 \times \frac{3}{2} = 1800\ km$

Now, OA and OB are the distance travelled.
Now by Pythagoras theorem, the distance between the two planes
$AB = \sqrt{OA^2 + OB^2}$
$= \sqrt{(1500)^2 + (1800)^2} = \sqrt{2250000 + 3240000}$
$= \sqrt{5490000} = 300\sqrt{61}\ km$
Hence, $1\frac{1}{2}$ hours, the distance between two planes is $300\sqrt{61}\ km$.

Question 12:

Two poles of heights 6 m and 11 m stand on a plane ground. If the distance between the feet of the poles is 12 m, find the distance between their tops.

Answer 12:

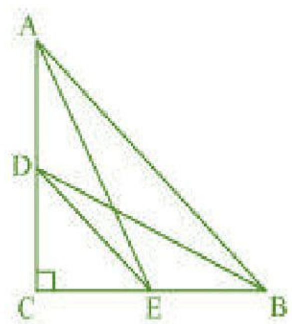


Let AB and CD are the two pole with height 6 m and 11 m respectively.
Therefore, CP = 11 − 6 = 5 m and AP = 12 m
In ΔAPC, by Pythagoras theorem
$AP^2 + PC^2 = AC^2$
$\Rightarrow 12^2 + 5^2 = AC^2$
$\Rightarrow AC^2 = 144 + 25 = 169$
$\Rightarrow AC = 13\ m$
Hence, the distance between the tops of two poles is 13 m.

Question 13:

D and E are points on the sides CA and CB respectively of a triangle ABC right angled at C. Prove that $AE^2 + BD^2 = AB^2 + DE^2$.

Answer 13:

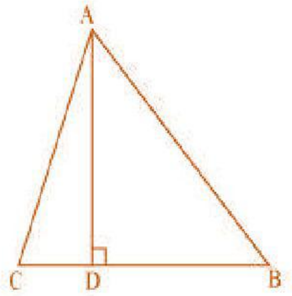


In ΔACE, by Pythagoras theorem
$AC^2 + CE^2 = AE^2$ ... (1)
In ΔBCD, by Pythagoras theorem
$BC^2 + CD^2 = DB^2$ ... (2)
From the equation (1) and (2), we have
$AC^2 + CE^2 + BC^2 + CD^2 = AE^2 + DB^2$ ... (3)
In ΔCDE, by Pythagoras theorem
$DE^2 = CD^2 + CE^2$ ... (4)
In ΔABC, by Pythagoras theorem
$AB^2 = AC^2 + CB^2$ ... (5)
From the equation (3), (4) and (5), we have
$DE^2 + AB^2 = AE^2 + DB^2$

Question 14:

The perpendicular from A on side BC of a Δ ABC intersects BC at D such that DB = 3CD (see Figure). Prove that $2AB^2 = 2AC^2 + BC^2$.

Answer 14:



In ΔACD, by Pythagoras theorem

$AC^2 = AD^2 + DC^2$

$\Rightarrow AC^2 - CD^2 = AD^2$ … (1)

In ΔABD, by Pythagoras theorem

$AB^2 = AD^2 + BD^2$

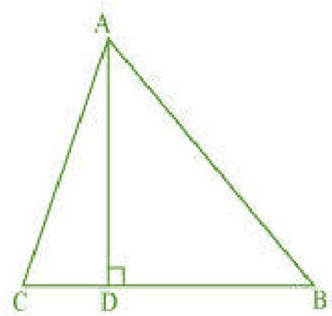
$\Rightarrow AB^2 - BD^2 = AD^2$ … (2)

From the equation (1) and (2), we have

$AC^2 - CD^2 = AB^2 - BD^2$ … (3)

Given that: 3DC = DB, therefore

$DC = \frac{BC}{4}$ and $BD = \frac{3BC}{4}$ … (4)



From the equation (3) and (4), we have

$$AC^2 - \left(\frac{BC}{4}\right)^2 = AB^2 - \left(\frac{3BC}{4}\right)^2$$

$$\Rightarrow AC^2 - \frac{BC^2}{16} = AB^2 - \frac{9BC^2}{16}$$

$\Rightarrow 16AC^2 - BC^2 = 16AB^2 - 9BC^2$

$\Rightarrow 16AC^2 = 16AB^2 - 8BC^2$

$\Rightarrow 2AC^2 = 2AB^2 - BC^2$

$\Rightarrow 2AB^2 = 2AC^2 + BC^2$

Question 15:

In an equilateral triangle ABC, D is a point on side BC such that BD =1/3 BC. Prove that $9AD^2 = 7AB^2$.

Answer 15:

Triangle ABC is an equilateral triangle with each side a. Draw an altitude AE from A to BC.

We know that the altitude in equilateral triangle, bisects the opposite sides.

Therefore, BE = EC = a/2

In ΔAEB, by Pythagoras theorem

$AB^2 = AE^2 + BE^2$

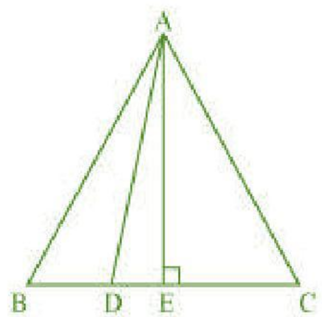
$\Rightarrow (a)^2 = AD^2 + (a/2)^2$ [Because AB = a]

$\Rightarrow a^2 = AD^2 + a^2/4$ $\Rightarrow AD^2 = 3a^2/4$ $\Rightarrow AD = \sqrt{3}a/2$

Given that: BD = 1/3 BC

∴ BD = a/3

DE = BE – BD = a/2 – a/3 = a/6



In ΔADE, by Pythagoras theorem,

$AD^2 = AE^2 + DE^2$

$$AD^2 = \left(\frac{a\sqrt{3}}{2}\right)^2 + \left(\frac{a}{6}\right)^2$$

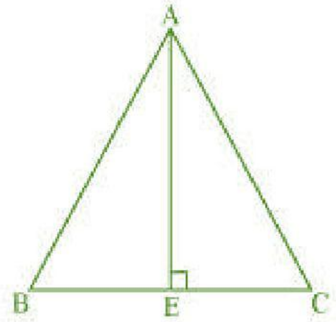
$$= \frac{3a^2}{4} + \frac{a^2}{36} = \frac{28a^2}{36} = \frac{7}{9}a^2$$

$$\Rightarrow AD^2 = \frac{7}{9}AB^2$$

$$\Rightarrow 9AD^2 = 7AB^2$$

Question 16:
In an equilateral triangle, prove that three times the square of one side is equal to four times the square of one of its altitudes.

Answer 16:
Let triangle ABC be an equilateral triangle with side a. Altitude AE is drown from A to BC.
We know that the altitude in equilateral triangle, bisects the opposite sides.
∴ BE = EC = BC/2 = a/2



In ΔABE, by Pythagoras theorem

$AB^2 = AE^2 + BE^2$

$$\Rightarrow a^2 = AE^2 + \left(\frac{a}{2}\right)^2 = AE^2 + \frac{a^2}{4}$$

$$\Rightarrow AE^2 = a^2 - \frac{a^2}{4} = \frac{3a^2}{4}$$

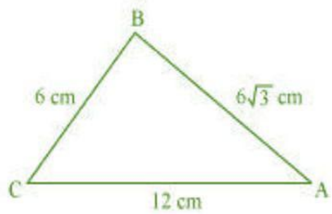
$4AE^2 = 3a^2$

⇒ 4 × (Altitude) = 3 × (Side)

Question 17:
Tick the correct answer and justify: In Δ ABC, AB = $6\sqrt{3}$ cm, AC = 12 cm and BC = 6 cm. The angle B is:

(A) 120°
(B) 60°
(C) 90°
(D) 45°

Answer 17:
Given that: AB = $6\sqrt{3}$ cm, AC = 12 cm and BC = 6 cm.
Therefore, $AB^2 = 108$, $AC^2 = 144$ and $BC^2 = 36$
Now,



$AB^2 + BC^2$
= 108 + 36
= 144
= AC^2

The sides are satisfying the Pythagoras triplet in ΔABC. Hence, these are the sides of a right angled triangle.
∴ ∠B = 90°
Hence, the option (C) is correct.

Mathematics

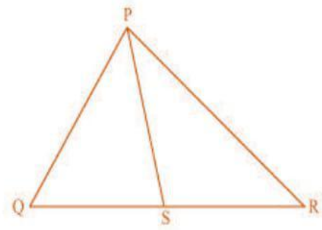
(Chapter – 6) (Triangles)

(Class 10)

Exercise 6.6 (Optional)

Question 1:

In Figure, PS is the bisector of ∠QPR of Δ PQR. Prove that $\frac{QS}{SR} = \frac{PQ}{QR}$.



Answer 1:

A line RT is drawn parallel to SP, which intersects QP produced at T.

Given that, SP bisects angle QPR, therefore

∠QPS = ∠SPR ... (1)

By construction,

∠SPR = ∠PRT (As PS || TR) ... (2)

∠QPS = ∠QTR (As PS || TR) ... (3)

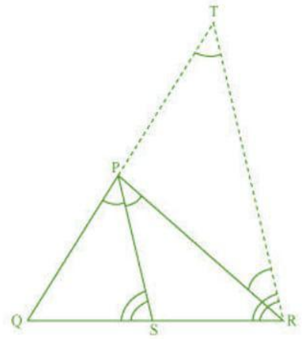
From the above equations, we have

∠PRT = ∠QTR

∴ PT = PR

By construction, PS || TR

In ΔQTR, by Thales theorem

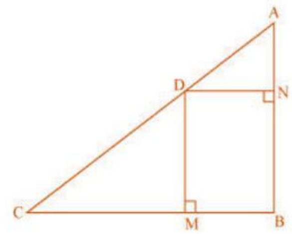


$\frac{QS}{SR} = \frac{QP}{PT} \Rightarrow \frac{QS}{SR} = \frac{PQ}{QR}$ [∵ PT = TR]

Question 2:

In Figure, D is a point on hypotenuse AC of Δ ABC, DM ⊥ BC and DN ⊥ AB. Prove that:

(i) $DM^2 = DN.MC$ **(ii)** $DN^2 = DM.AN$



Answer 2:

(i) Join B and D.

Given that, DN || CB, DM || AB and ∠B = 90°, ∴ DMBN is a rectangle.

∴ DN = MB and DM = NB

Given that, BD ⊥ AC, ∴ ∠CDB = 90°

⇒ ∠2 + ∠3 = 90° ... (1)

In ΔCDM, ∠1 + ∠2 + ∠DMC = 180°

⇒ ∠1 + ∠2 = 90° ... (2)

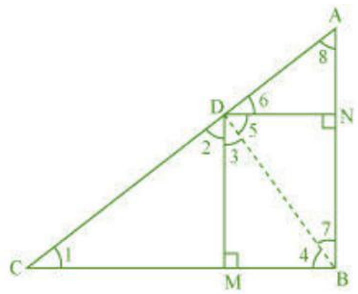
In ΔDMB, ∠3 + ∠DMB + ∠4 = 180°

⇒ ∠3 + ∠4 = 90° ... (3)

From the equations (1) and (2), we have, ∠1 = ∠3

From the equations (1) and (3), we have, ∠2 = ∠4

In ΔDCM and ΔBDM,



∠1 = ∠3 [Proved above]

∠2 = ∠4 [Proved above]

∴ ΔDCM ~ ΔBDM [AA similarity]

$\Rightarrow \frac{BM}{DM} = \frac{DM}{MC} \Rightarrow \frac{DN}{DM} = \frac{DM}{MC}$ $[\because BM = DN]$

$\Rightarrow DM^2 = DN \times MC$

(ii) In ΔDBN, ∠5 + ∠7 = 90° ... (4)

In ΔDAN, ∠6 + ∠8 = 90° ... (5)

BD ⊥ AC, ∴ ∠ADB = 90°

⇒ ∠5 + ∠6 = 90° ... (6)

From the equations (4) and (6), we have, ∠6 = ∠7

From the equations (5) and (6), we have, ∠8 = ∠5

In ΔDNA and ΔBND,

∠6 = ∠7 [Proved above]

∠8 = ∠5 [Proved above]

∴ ΔDNA ~ ΔBND [AA similarity]

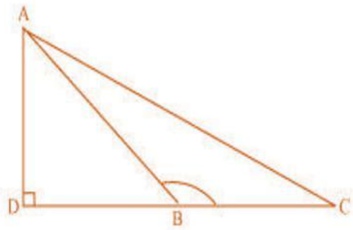
$\Rightarrow \frac{AN}{DN} = \frac{DN}{NB} \quad \Rightarrow DN^2 = AN \times NB$

$\Rightarrow DN^2 = AN \times DM$ $[\because NB = DM]$

Question 3:

In Figure, ABC is a triangle in which ∠ABC > 90° and AD ⊥ CB produced. Prove that $AC^2 = AB^2 + BC^2 + 2BC.BD$.

Answer 3:



In ΔADB, by Pythagoras theorem

$AB^2 = AD^2 + DB^2$ ... (1)

In ΔACD, by Pythagoras theorem

$AC^2 = AD^2 + DC^2$

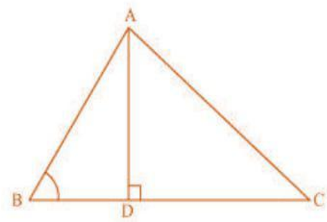
$\Rightarrow AC^2 = AD^2 + (DB + BC)^2$

$\Rightarrow AC^2 = AD^2 + DB^2 + BC^2 + 2DB \times BC$

$\Rightarrow AC^2 = AB^2 + BC^2 + 2DB \times BC$ [From the equation (1)]

Question 4:

In Figure, ABC is a triangle in which ∠ABC < 90° and AD ⊥ BC. Prove that
$AC^2 = AB^2 + BC^2 + 2BC.BD$.



Answer 4:

In ΔADB, by Pythagoras theorem

$AD^2 + DB^2 = AB^2$

$\Rightarrow AD2 = AB2 - DB2$ ... (1)

ΔADC में, by Pythagoras theorem, $AD^2 + DC^2 = AC^2$

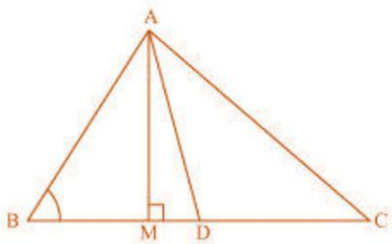
$\Rightarrow AB^2 - BD^2 + DC^2 = AC^2$ [From the equation (1)]

$\Rightarrow AB^2 - BD^2 + (BC - BD)^2 = AC^2$

$\Rightarrow AC^2 = AB^2 - BD^2 + BC^2 + BD^2 - 2BC \times BD = AB^2 + BC^2 - 2BC \times BD$

Question 5:

In Figure, AD is a median of a triangle ABC and AM ⊥ BC. Prove that:



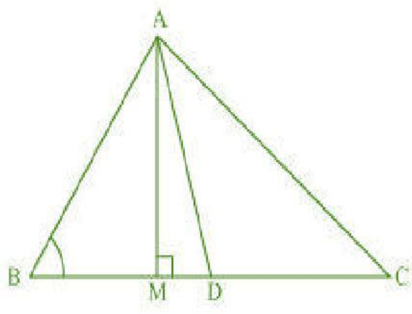
(i) $AC^2 = AD^2 + BC.DM + \left(\frac{BC}{2}\right)^2$

(ii) $AB^2 = AD^2 - BC.DM + \left(\frac{BC}{2}\right)^2$

(ii) $AC^2 + AB^2 = 2AD^2 + \frac{1}{2}BC^2$

Answer 5:

(i) In ΔAMD, by Pythagoras theorem



$AM^2 + MD^2 = AD^2$ ... (1)

In ΔAMC, by Pythagoras theorem, $AM^2 + MC^2 = AC^2$

$\Rightarrow AM^2 + (MD + DC)^2 = AC^2$

$\Rightarrow (AM^2 + MD^2) + DC^2 + 2MD.DC = AC^2$

$\Rightarrow AD^2 + DC^2 + 2MD.DC = AC^2$ [From equation (1)]

$\Rightarrow AD^2 + \left(\frac{BC}{2}\right)^2 + 2MD.\left(\frac{BC}{2}\right) = AC^2 \quad \left[\because DC = \frac{BC}{2}\right]$

$\Rightarrow AD^2 + \left(\frac{BC}{2}\right)^2 + MD.BC = AC^2$

(ii) In ΔABM, by Pythagoras theorem
$AB^2 = AM^2 + MB^2$
$= (AD^2 - DM^2) + MB^2$
$= (AD^2 - DM^2) + (BD - MD)^2$
$= AD^2 - DM^2 + BD^2 + MD^2 - 2BD \times MD$
$= AD^2 + BD^2 - 2BD \times MD$

$$= AD^2 + \left(\frac{BC}{2}\right)^2 - 2\left(\frac{BC}{2}\right)MD = AC^2 \qquad \left[\because BD = \frac{BC}{2}\right]$$

$$\Rightarrow AD^2 + \left(\frac{BC}{2}\right)^2 - BC.MD = AC^2$$

(iii) In ΔABM, by Pythagoras theorem, $AM^2 + MB^2 = AB^2$ ... (2)
In ΔAMC, by Pythagoras theorem, $AM^2 + MC^2 = AC^2$ ... (3)
Adding the equations (2) and (3), we have
$2AM^2 + MB^2 + MC^2 = AB^2 + AC^2$
$\Rightarrow 2AM^2 + (BD - DM)^2 + (MD + DC)^2 = AB^2 + AC^2$
$\Rightarrow 2AM^2 + BD^2 + DM^2 - 2BD.DM + MD^2 + DC^2 + 2MD.DC = AB^2 + AC^2$
$\Rightarrow 2AM^2 + 2MD^2 + BD^2 + DC^2 + 2MD\,(-BD + DC) = AB^2 + AC^2$

$$\Rightarrow 2(AM^2 + MD^2) + \left(\frac{BC}{2}\right)^2 + \left(\frac{BC}{2}\right)^2 + 2MD\left(-\frac{BC}{2} + \frac{BC}{2}\right) = AB^2 + AC^2$$

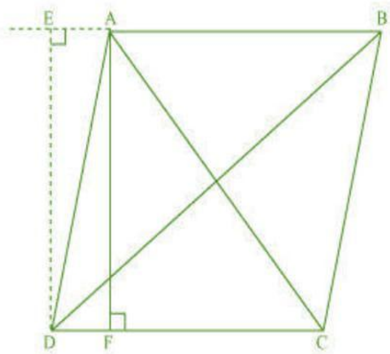
$$\Rightarrow 2AD^2 + \frac{1}{2}BC^2 = AB^2 + AC^2$$

Question 6:

Prove that the sum of the squares of the diagonals of parallelogram is equal to the sum of the squares of its sides.

Answer 6:

In parallelogram ABCD, altitudes AF and DE is drawn on DC and produced BA.



In ΔDEA, by Pythagoras theorem, $DE^2 + EA^2 = DA^2$ ... (i)
In ΔDEB, by Pythagoras theorem, $DE^2 + EB^2 = DB^2$
$\Rightarrow DE^2 + (EA + AB)^2 = DB^2$
$\Rightarrow (DE^2 + EA^2) + AB^2 + 2EA \times AB = DB^2$
$\Rightarrow DA^2 + AB^2 + 2EA \times AB = DB^2$ ... (ii)
In ΔADF, by Pythagoras theorem, $AD^2 = AF^2 + FD^2$
In ΔAFC, by Pythagoras theorem
$AC^2 = AF^2 + FC^2 = AF^2 + (DC - FD)^2 = AF^2 + DC^2 + FD^2 - 2DC \times FD$
$= (AF^2 + FD^2) + DC^2 - 2DC \times FD$
$\Rightarrow AC^2 = AD^2 + DC^2 - 2DC \times FD$ ... (iii)
ABCD is a parallelogram.
Therefore
AB = CD ... (iv)
and, BC = AD ... (v)
In ΔDEA and ΔADF,
∠DEA = ∠AFD [Each 90°]
∠EAD = ∠ADF [EA || DF]
AD = AD [Common]
∴ ΔEAD ≅ ΔFDA [AAS congruency rule]
⇒ EA = DF ... (vi)

Adding equations (ii) and (iii), we have

$DA^2 + AB^2 + 2EA \times AB + AD^2 + DC^2 - 2DC \times FD = DB^2 + AC^2$

$\Rightarrow DA^2 + AB^2 + AD^2 + DC^2 + 2EA \times AB - 2DC \times FD = DB^2 + AC^2$

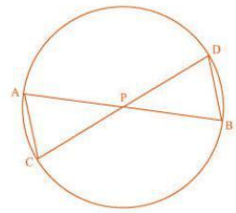
$\Rightarrow BC^2 + AB^2 + AD^2 + DC^2 + 2EA \times AB - 2AB \times EA = DB^2 + AC^2$ [From the equation (iv) and (vi)]

$\Rightarrow AB^2 + BC^2 + CD^2 + DA^2 = AC^2 + BD^2$

Question 7:

In Figure, two chords AB and CD intersect each other at the point P. Prove that:

(i) $\Delta APC \sim \Delta DPB$ **(ii)** AP.BP = CP.DP



Answer 7:

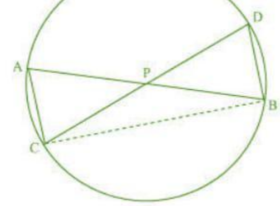
Join CB.

(i) In ΔAPC and ΔDPB,

$\angle APC = \angle DPB$ [Vertically Opposite Angles]

$\angle CAP = \angle BDP$ [Angles in the same segment]

$\Delta APC \sim \Delta DPB$ [AA similarity]



(ii) We have already proved that $\Delta APC \sim \Delta DPB$.

We know that the corresponding sides of similar triangles are proportional. So,

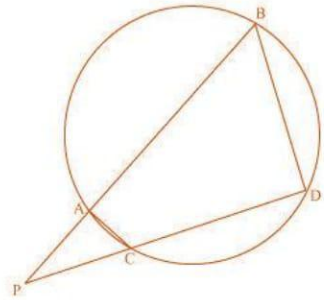
$$\frac{AP}{DP} = \frac{PC}{PB} = \frac{CA}{BD} \quad \Rightarrow \frac{AP}{DP} = \frac{PC}{PB} \quad \Rightarrow AP.PB = PC.DP$$

Question 8:

In Figure, two chords AB and CD of a circle intersect each other at the point P (when produced) outside the circle. Prove that

(i) $\Delta PAC \sim \Delta PDB$

(ii) PA.PB = PC.PD



Answer 8:

(i) In ΔPAC and ΔPDB,

$\angle P = \angle P$ [Common]

$\angle PAC = \angle PDB$

[The exterior angle of cyclic quadrilateral is equal to opposite interior angle]

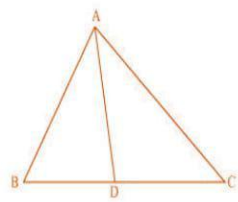
$\therefore \Delta PAC \sim \Delta PDB$ [AA similarity]

(ii) We know that the corresponding sides of similar triangles are proportional. Therefore,

$$\frac{PA}{PD} = \frac{AC}{BD} = \frac{PC}{PB} \quad \Rightarrow \frac{PA}{PD} = \frac{PC}{PB} \quad \Rightarrow PA.PB = PC.DP$$

Question 9:

In Figure, D is a point on side BC of Δ ABC such that $\frac{BD}{CD} = \frac{AB}{AC}$. Prove that AD is the bisector of ∠ BAC.

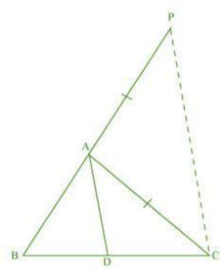


Answer 9:

Produce BA to P, such that AP = AC and join P to C.

Given that:

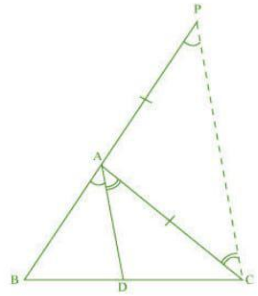
$$\frac{BD}{CD} = \frac{AB}{AC} \quad \Rightarrow \frac{BD}{CD} = \frac{AP}{AC}$$



By the converse of Thales theorem, we have

$AD \parallel PC \Rightarrow \angle BAD = \angle APC$ [Corresponding angle] ... (1)

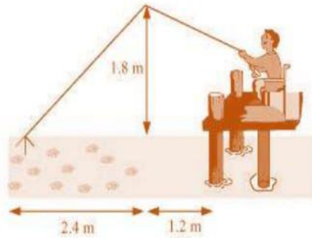
and, $\angle DAC = \angle ACP$ [Alternate angle] ... (2)



By construction,
AP = AC
⇒ ∠APC = ∠ACP ... (3)
From the equations (1), (2) and (3), we have
∠BAD = ∠APC
⇒ AD, bisects angle BAC.

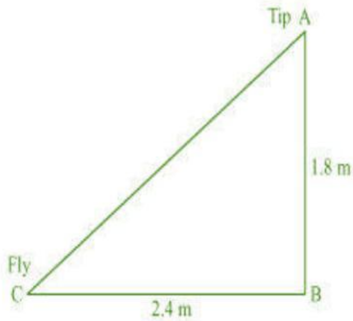
Question 10:

Nazima is fly fishing in a stream. The tip of her fishing rod is 1.8 m above the surface of the water and the fly at the end of the string rests on the water 3.6 m away and 2.4 m from a point directly under the tip of the rod. Assuming that her string (from the tip of her rod to the fly) is taut, how much string does she have out (see Figure)? If she pulls in the string at the rate of 5 cm per second, what will be the horizontal distance of the fly from her after 12 seconds?



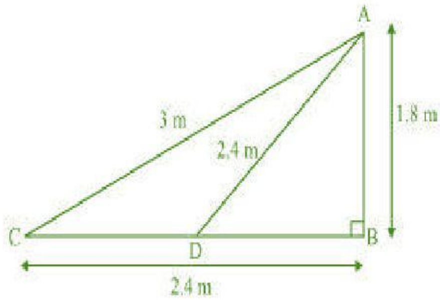
Answer 10:

Let AB be the height of rod tip from the surface of water and BC is the horizontal distance between fly to tip of the rod.
Then, the length of the string is AC.



In ΔABC, by Pythagoras theorem
$AC^2 = AB^2 + BC^2$
$\Rightarrow AB^2 = (1.8\text{ m})^2 + (2.4\text{ m})^2$
$\Rightarrow AB^2 = (3.24 + 5.76)\text{ m}^2$
$\Rightarrow AB^2 = 9.00\text{ m}^2$
$\Rightarrow AB = \sqrt{9} = 3\text{ m}$
Hence, the length of string, which is out, is 3 m.

If she pulls in the string at the rate of 5 cm/s, then the distance travelled by fly in 12 seconds
= 12 × 5 = 60 cm = 0.6 m
Let, D be the position of fly after 12 seconds.
Hence, AD is the length of string that is out after 12 seconds.
The length of the string pulls in by Nazima = AD = AC − 12
= (3.00 − 0.6) m
= 2.4 m



In ΔADB,
$AB^2 + BD^2 = AD^2$
$\Rightarrow (1.8\text{ m})^2 + BD^2 = (2.4\text{ m})^2$
$\Rightarrow BD^2 = (5.76 - 3.24)\text{ m}^2 = 2.52\text{ m}^2$
$\Rightarrow BD = 1.587\text{ m}$

Horizontal distance travelled by Fly
= BD + 1.2 m
= (1.587 + 1.2) m
= 2.787 m
= 2.79 m

Mathematics

(Chapter – 7) (Co-ordinate Geometry)

(Class – X)

Exercise 7.1

Question 1:

Find the distance between the following pairs of points:

(i) (2, 3), (4, 1) **(ii)** (−5, 7), (−1, 3) **(iii)** (*a*, *b*), (− *a*, − *b*)

Answer 1:

(i) Distance between the two points is given by

$$\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$$

Therefore, distance between $(2, 3)$ and $(4, 1)$ is given by

$$l=\sqrt{(2-4)^2+(3-1)^2}=\sqrt{(-2)^2+(2)^2}$$
$$=\sqrt{4+4}=\sqrt{8}=2\sqrt{2}$$

(ii) Distance between $(-5,7)$ and $(-1,3)$ is given by

$$l=\sqrt{(-5-(-1))^2+(7-3)^2}=\sqrt{(-4)^2+(4)^2}$$
$$=\sqrt{16+16}=\sqrt{32}=4\sqrt{2}$$

(iii) Distance between (a,b) and $(-a,-b)$ is given by

$$l=\sqrt{(a-(-a))^2+(b-(-b))^2}$$
$$=\sqrt{(2a)^2+(2b)^2}=\sqrt{4a^2+4b^2}=2\sqrt{a^2+b^2}$$

Question 2:

Find the distance between the points (0, 0) and (36, 15). Can you now find the distance between the two towns A and B discussed in Section 7.2.

Answer 2:

Distance between points (0, 0) and (36, 15).

$$= \sqrt{(36-0)^2 + (15-0)^2} = \sqrt{36^2 + 15^2}$$
$$= \sqrt{1296 + 225} = \sqrt{1521} = 39$$

Yes, we can find the distance between the given towns A and B.

Assume town A at origin point (0, 0).

Therefore, town B will be at point (36, 15) with respect to town A. And hence, as calculated above, the distance between town A and B will be 39 km.

Question 3:

Determine if the points (1, 5), (2, 3) and (− 2, − 11) are collinear.

Answer 3:

Let the points (1, 5), (2, 3), and (−2, −11) be representing the vertices A, B, and C of the given triangle respectively.

Let $A = (1,5), B = (2,3), C = (-2,-11)$

$$\therefore AB = \sqrt{(1-2)^2 + (5-3)^2} = \sqrt{5}$$
$$BC = \sqrt{(2-(-2))^2 + (3-(-11))^2} = \sqrt{4^2 + 14^2} = \sqrt{16+196} = \sqrt{212}$$
$$CA = \sqrt{(1-(-2))^2 + (5-(-11))^2} = \sqrt{3^2 + 16^2} = \sqrt{9+256} = \sqrt{265}$$

Since $AB + BC \neq CA$,

Therefore, the points (1, 5), (2, 3), and (−2, −11) are not collinear.

Question 4:

Check whether (5, − 2), (6, 4) and (7, − 2) are the vertices of an isosceles triangle.

Answer 4:

Let the points (5, −2), (6, 4), and (7, −2) are representing the vertices A, B, and C of the given triangle respectively.

$$AB = \sqrt{(5-6)^2 + (-2-4)^2} = \sqrt{(-1)^2 + (-6)^2} = \sqrt{1+36} = \sqrt{37}$$

$$BC = \sqrt{(6-7)^2 + (4-(-2))^2} = \sqrt{(-1)^2 + (6)^2} = \sqrt{1+36} = \sqrt{37}$$

$$CA = \sqrt{(5-7)^2 + (-2-(-2))^2} = \sqrt{(-2)^2 + 0^2} = 2$$

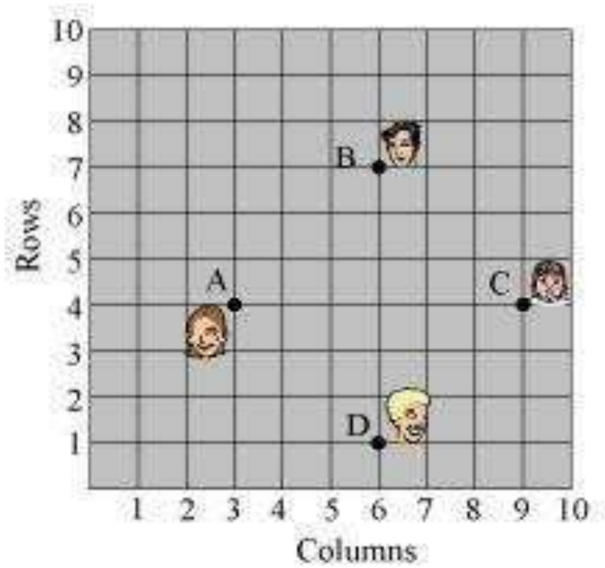
Therefore, $AB = BC$

As two sides are equal in length, therefore, ABC is an isosceles triangle.

Question 5:

In a classroom, 4 friends are seated at the points A, B, C and D as shown in the following figure. Champa and Chameli walk into the class and after observing for a few minutes Champa asks Chameli, "Don't you think ABCD is a square?" Chameli disagrees.

Using distance formula, find which of them is correct.



Answer 5:

It can be observed that A (3, 4), B (6, 7), C (9, 4), and D (6, 1) are the positions of these 4 friends.

$$AB = \sqrt{(3-6)^2 + (4-7)^2} = \sqrt{(-3)^2 + (-3)^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$$

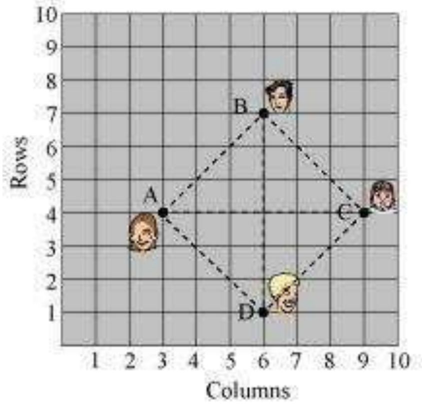
$$BC = \sqrt{(6-9)^2 + (7-4)^2} = \sqrt{(-3)^2 + (3)^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$$

$$CB = \sqrt{(9-6)^2 + (4-1)^2} = \sqrt{(3)^2 + (3)^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$$

$$AD = \sqrt{(3-6)^2 + (4-1)^2} = \sqrt{(-3)^2 + (3)^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$$

$$\text{Diagonal AC} = \sqrt{(3-9)^2 + (4-4)^2} = \sqrt{(-6)^2 + 0^2} = 6$$

$$\text{Diagonal BD} = \sqrt{(6-6)^2 + (7-1)^2} = \sqrt{0^2 + (6)^2} = 6$$



It can be observed that all sides of this quadrilateral ABCD are of the same length and also the diagonals are of the same length.

Therefore, ABCD is a square and hence, Champa was correct

Question 6:

Name the type of quadrilateral formed, if any, by the following points, and give reasons for your answer:

(i) (− 1, − 2), (1, 0), (− 1, 2), (− 3, 0)

(ii) (− 3, 5), (3, 1), (0, 3), (− 1, − 4)

(iii)(4, 5), (7, 6), (4, 3), (1, 2)

Answer 6:

(i) Let the points (−1, −2), (1, 0), (−1, 2), and (−3, 0) be representing the vertices A, B, C, and D of the given quadrilateral respectively.

$$\therefore \text{AB} = \sqrt{(-1-1)^2 + (-2-0)^2} = \sqrt{(-2)^2 + (-2)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$$

$$\text{BC} = \sqrt{(1-(-1))^2 + (0-2)^2} = \sqrt{(2)^2 + (-2)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$$

$$\text{CD} = \sqrt{(-1-(-3))^2 + (2-0)^2} = \sqrt{(2)^2 + (2)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$$

$$\text{AD} = \sqrt{(-1-(-3))^2 + (-2-0)^2} = \sqrt{(2)^2 + (-2)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$$

$$\text{Diagonal AC} = \sqrt{(-1-(-1))^2 + (-2-2)^2} = \sqrt{0^2 + (-4)^2} = \sqrt{16} = 4$$

$$\text{Diagonal BD} = \sqrt{(1-(-3))^2 + (0-0)^2} = \sqrt{(4)^2 + 0^2} = \sqrt{16} = 4$$

It can be observed that all sides of this quadrilateral are of the same length and also, the diagonals are of the same length. Therefore, the given points are the vertices of a square.

(ii) Let the points (− 3, 5), (3, 1), (0, 3), and (−1, −4) be representing the vertices A, B, C, and D of the given quadrilateral respectively.

$$\text{AB} = \sqrt{(-3-3)^2 + (5-1)^2} = \sqrt{(-6)^2 + (4)^2} = \sqrt{36+16} = \sqrt{52} = 2\sqrt{13}$$

$$\text{BC} = \sqrt{(3-0)^2 + (1-3)^2} = \sqrt{(3)^2 + (-2)^2} = \sqrt{9+4} = \sqrt{13}$$

$$\text{CD} = \sqrt{(0-(-1))^2 + (3-(-4))^2} = \sqrt{(1)^2 + (7)^2} = \sqrt{1+49} = \sqrt{50} = 5\sqrt{2}$$

$$\text{AD} = \sqrt{(-3-(-1))^2 + (5-(-4))^2} = \sqrt{(-2)^2 + (9)^2} = \sqrt{4+81} = \sqrt{85}$$

It can be observed that all sides of this quadrilateral are of different lengths. Therefore, it can be said that it is only a general quadrilateral, and not specific such as square, rectangle, etc.

(iii) Let the points (4, 5), (7, 6), (4, 3), and (1, 2) be representing the vertices A, B, C, and D of the given quadrilateral respectively.

$$\text{AB} = \sqrt{(4-7)^2 + (5-6)^2} = \sqrt{(-3)^2 + (-1)^2} = \sqrt{9+1} = \sqrt{10}$$
$$\text{BC} = \sqrt{(7-4)^2 + (6-3)^2} = \sqrt{(3)^2 + (3)^2} = \sqrt{9+9} = \sqrt{18}$$
$$\text{CD} = \sqrt{(4-1)^2 + (3-2)^2} = \sqrt{(3)^2 + (1)^2} = \sqrt{9+1} = \sqrt{10}$$
$$\text{AD} = \sqrt{(4-1)^2 + (5-2)^2} = \sqrt{(3)^2 + (3)^2} = \sqrt{9+9} = \sqrt{18}$$
$$\text{Diagonal AC} = \sqrt{(4-4)^2 + (5-3)^2} = \sqrt{(0)^2 + (2)^2} = \sqrt{0+4} = 2$$
$$\text{Diagonal CD} = \sqrt{(7-1)^2 + (6-2)^2} = \sqrt{(6)^2 + (4)^2} = \sqrt{36+16} = \sqrt{52} = 13\sqrt{2}$$

It can be observed that opposite sides of this quadrilateral are of the same length. However, the diagonals are of different lengths. Therefore, the given points are the vertices of a parallelogram.

Question 7:

Find the point on the *x*-axis which is equidistant from (2, − 5) and (− 2, 9).

Answer 7:

We have to find a point on *x*-axis. Therefore, its *y*-coordinate will be 0.

Let the point on *x*-axis be $(x, 0)$.

$$\text{Distance between } (x,0) \text{ and } (2,-5) = \sqrt{(x-2)^2 + (0-(-5))^2} = \sqrt{(x-2)^2 + (5)^2}$$
$$\text{Distance between } (x,0) \text{ and } (-2,9) = \sqrt{(x-(-2))^2 + (0-(-9))^2} = \sqrt{(x+2)^2 + (9)^2}$$

By the given condition, these distances are equal in measure.

$$\sqrt{(x-2)^2+(5)^2}=\sqrt{(x+2)^2+(9)^2}$$
$$(x-2)^2+25=(x+2)^2+81$$
$$x^2+4-4x+25=x^2+4+4x+81$$
$$8x=25-81$$
$$8x=-56$$
$$x=-7$$

Therefore, the point is (− 7, 0).

Question 8:

Find the values of *y* for which the distance between the points P (2, − 3) and Q (10, *y*) is 10 units.

Answer 8:

It is given that the distance between (2, −3) and (10, *y*) is 10.

Therefore, $\sqrt{(2-10)^2+(-3-y)^2}=10$
$$\sqrt{(-8)^2+(3+y)^2}=10$$
$$64+(y+3)^2=100$$
$$(y+3)^2=36$$
$$y+3=\pm 6$$
$$y+3=6 \text{ or } y+3=-6$$
Therefore, $y=3$ or -9

Question 9:

If Q (0, 1) is equidistant from P (5, − 3) and R (x, 6), find the values of x. Also find the distance QR and PR.

Answer 9:

$$\text{PQ} = \text{QR}$$

$$\sqrt{(5-0)^2+(-3-1)^2} = \sqrt{(0-x)^2+(1-6)^2}$$

$$\sqrt{(5)^2+(-4)^2} = \sqrt{(-x)^2+(-5)^2}$$

$$\sqrt{25+16} = \sqrt{x^2+25}$$

$$41 = x^2+25$$

$$16 = x^2$$

$$x = \pm 4$$

Therefore, point R is (4, 6) or (−4, 6).

When point R is (4, 6),

$$\text{PR} = \sqrt{(5-4)^2+(-3-6)^2} = \sqrt{1^2+(-9)^2} = \sqrt{1+81} = \sqrt{82}$$

$$\text{QR} = \sqrt{(0-4)^2+(1-6)^2} = \sqrt{(-4)^2+(-5)^2} = \sqrt{16+25} = \sqrt{41}$$

When point R is (−4, 6),

$$\text{PR} = \sqrt{(5-(-4))^2+(-3-6)^2} = \sqrt{(9)^2+(-9)^2} = \sqrt{81+81} = 9\sqrt{2}$$

$$\text{QR} = \sqrt{(0-(-4))^2+(1-6)^2} = \sqrt{(4)^2+(-5)^2} = \sqrt{16+25} = \sqrt{41}$$

Question 10:

Find a relation between x and y such that the point (x, y) is equidistant from the point (3, 6) and (− 3, 4).

Answer 10:

Point (x, y) is equidistant from (3, 6) and (−3, 4).

$$\therefore \sqrt{(x-3)^2+(y-6)^2}=\sqrt{(x-(-3))^2+(y-4)^2}$$

$$\sqrt{(x-3)^2+(y-6)^2}=\sqrt{(x+3)^2+(y-4)^2}$$

$$(x-3)^2+(y-6)^2=(x+3)^2+(y-4)^2$$

$$x^2+9-6x+y^2+36-12y=x^2+9+6x+y^2+16-8y$$

$$36-16=6x+6x+12y-8y$$

$$20=12x+4y$$

$$3x+y=5$$

$$3x+y-5=0$$

Mathematics

(Chapter – 7) (Co-ordinate Geometry)

(Class – X)

Exercise 7.2

Question 1:

Find the coordinates of the point which divides the join of (− 1, 7) and (4, − 3) in the ratio 2:3.

Answer 1:

Let P(x, y) be the required point. Using the section formula, we obtain

$$x = \frac{2\times 4 + 3\times(-1)}{2+3} = \frac{8-3}{5} = \frac{5}{5} = 1$$

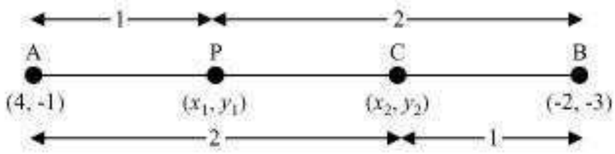
$$y = \frac{2\times(-3) + 3\times 7}{2+3} = \frac{-6+21}{5} = \frac{15}{5} = 3$$

Therefore, the point is (1, 3).

Question 2:

Find the coordinates of the points of trisection of the line segment joining (4, − 1) and (− 2, − 3).

Answer 2:



Let P (x_1, y_1) and Q (x_2, y_2) are the points of trisection of the line segment joining the given points i.e., AP = PQ = QB

Therefore, point P divides AB internally in the ratio 1:2.

$$x_1 = \frac{1\times(-2)+2\times4}{1+2}, \quad y_1 = \frac{1\times(-3)+2\times(-1)}{1+2}$$

$$x_1 = \frac{-2+8}{3} = \frac{6}{3} = 2, \quad y_1 = \frac{-3-2}{3} = \frac{-5}{3}$$

$$\text{Therefore, } P(x_1, y_1) = \left(2, -\frac{5}{3}\right)$$

Point Q divides AB internally in the ratio 2:1.

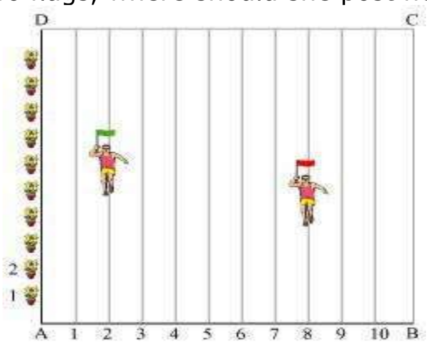
$$x_2 = \frac{2\times(-2)+1\times4}{2+1}, \quad y_2 = \frac{2\times(-3)+1\times(-1)}{2+1}$$

$$x_2 = \frac{-4+4}{3} = 0, \qquad y_2 = \frac{-6-1}{3} = \frac{-7}{3}$$

$$Q(x_2, y_2) = \left(0, -\frac{7}{3}\right)$$

Question 3:

To conduct Sports Day activities, in your rectangular shaped school ground ABCD, lines have been drawn with chalk powder at a distance of 1 m each. 100 flower pots have been placed at a distance of 1 m from each other along AD, as shown in the following figure. Niharika runs $\frac{1}{4}th$ the distance AD on the 2nd line and posts a green flag. Preet runs $\frac{1}{5}th$ the distance AD on the eighth line and posts a red flag. What is the distance between both the flags? If Rashmi has to post a blue flag exactly halfway between the line segment joining the two flags, where should she post her flag?



Answer 3:

It can be observed that Niharika posted the green flag at $\frac{1}{4}$ of the distance AD i.e.,

$$\left(\frac{1}{4}\times 100\right)\text{m} = 25$$

metre from the starting point of 2nd line. Therefore, the coordinates of this point G is (2, 25).

Similarly, Preet posted red flag at $\frac{1}{5}$ of the distance AD i.e.,

$$\left(\frac{1}{5}\times 100\right)\text{m} = 20$$

metre from the starting point of 8th line. Therefore, the coordinates of this point R are (8, 20).

Distance between these flags by using distance formula = GR

$$= \sqrt{(8-2)^2 + (25-20)^2} = \sqrt{36+25} = \sqrt{61}\,\text{m}$$

The point at which Rashmi should post her blue flag is the mid-point of the line joining these points. Let this point be A (x, y).

$$x = \frac{2+8}{2},\quad y = \frac{25+20}{2}$$

$$x = \frac{10}{2} = 5,\ y = \frac{45}{2} = 22.5$$

$$\text{Hence, A}(x, y) = (5, 22.5)$$

Therefore, Rashmi should post her blue flag at 22.5m on 5th line

Question 4:

Find the ratio in which the line segment joining the points (− 3, 10) and (6, − 8) is divided by (− 1, 6).

Answer 4:

Let the ratio in which the line segment joining (−3, 10) and (6, −8) is divided by point (−1, 6) be k:1.

$$\text{Therefore, } -1 = \frac{6k-3}{k+1}$$
$$-k-1 = 6k-3$$
$$7k = 2$$
$$k = \frac{2}{7}$$
Therefore, the required ratio is 2 : 7.

Question 5:

Find the ratio in which the line segment joining A (1, − 5) and B (− 4, 5) is divided by the x-axis. Also find the coordinates of the point of division.

Answer 5:

Let the ratio in which the line segment joining A (1, −5) and B (−4, 5) is divided by x - axis be k:1.
Therefore, the coordinates of the point of division is $\left(\frac{-4k+1}{k+1}, \frac{5k-5}{k+1}\right)$.

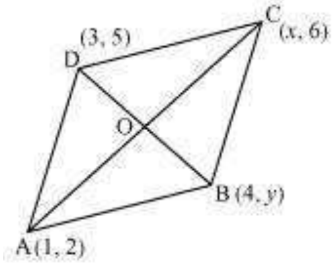
We know that y-coordinate of any point on x-axis is 0.
$$\therefore \frac{5k-5}{k+1} = 0$$
$$k = 1$$
Therefore, x-axis divides it in the ratio 1:1.

Division point = $\left(\frac{-4(1)+1}{1+1}, \frac{5(1)-5}{1+1}\right) = \left(\frac{-4+1}{2}, \frac{5-5}{2}\right) = \left(\frac{-3}{2}, 0\right)$

Question 6:

If (1, 2), (4, y), (x, 6) and (3, 5) are the vertices of a parallelogram taken in order, find x and y.

Answer 6:



Let (1, 2), (4, y), (x, 6), and (3, 5) are the coordinates of A, B, C, D vertices of a parallelogram ABCD. Intersection point O of diagonal AC and BD also divides these diagonals.

Therefore, O is the mid-point of AC and BD.

If O is the mid-point of AC, then the coordinates of O are

$$\left(\frac{1+x}{2}, \frac{2+6}{2}\right) \Rightarrow \left(\frac{x+1}{2}, 4\right)$$

If O is the mid-point of BD, then the coordinates of O are

$$\left(\frac{4+3}{2}, \frac{5+y}{2}\right) \Rightarrow \left(\frac{7}{2}, \frac{5+y}{2}\right)$$

Since both the coordinates are of the same point O,

$$\therefore \frac{x+1}{2} = \frac{7}{2} \text{ and } 4 = \frac{5+y}{2}$$

$$\Rightarrow x+1 = 7 \text{ and } 5+y = 8$$

$$\Rightarrow x = 6 \text{ and } y = 3$$

Question 7:

Find the coordinates of a point A, where AB is the diameter of circle whose centre is (2, − 3) and B is (1, 4)

Answer 7:

Let the coordinates of point A be (x, y).

Mid-point of AB is (2, −3), which is the center of the circle.

$$\therefore (2,-3) = \left(\frac{x+1}{2}, \frac{y+4}{2}\right)$$

$$\Rightarrow \frac{x+1}{2} = 2 \text{ and } \frac{y+4}{2} = -3$$

$$\Rightarrow x+1 = 4 \text{ and } y+4 = -6$$

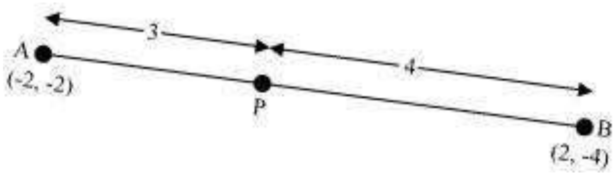
$$\Rightarrow x = 3 \text{ and } y = -10$$

Therefore, the coordinates of A are $(3,-10)$.

Question 8:

If A and B are (− 2, − 2) and (2, − 4), respectively, find the coordinates of P such that $AP = \frac{3}{7}AB$ and P lies on the line segment AB.

Answer 8:



The coordinates of point A and B are (−2, −2) and (2, −4) respectively.

Since $AP = \frac{3}{7}AB$,

Therefore, AP: PB = 3:4

Point P divides the line segment AB in the ratio 3:4.

$$\text{Coordinates of P} = \left(\frac{3\times 2 + 4\times(-2)}{3+4}, \frac{3\times(-4)+4\times(-2)}{3+4}\right)$$

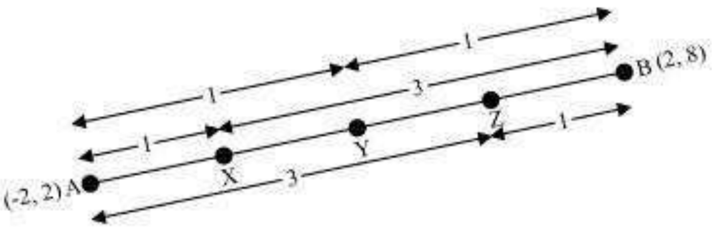
$$= \left(\frac{6-8}{7}, \frac{-12-8}{7}\right)$$

$$= \left(-\frac{2}{7}, -\frac{20}{7}\right)$$

Question 9:

Find the coordinates of the points which divide the line segment joining A (− 2, 2) and B (2, 8) into four equal parts.

Answer 9:



From the figure, it can be observed that points P, Q, R are dividing the line segment in a ratio 1:3, 1:1, 3:1 respectively.

$$\text{Coordinates of P} = \left(\frac{1\times 2+3\times(-2)}{1+3}, \frac{1\times 8+3\times 2}{1+3}\right)$$

$$= \left(-1, \frac{7}{2}\right)$$

$$\text{Coordinates of Q} = \left(\frac{2+(-2)}{2}, \frac{2+8}{2}\right)$$

$$= (0, 5)$$

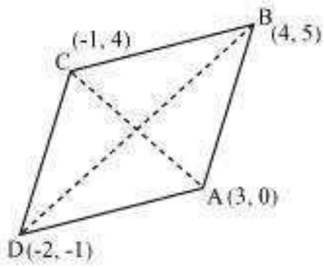
$$\text{Coordinates of R} = \left(\frac{3\times 2+1\times(-2)}{3+1}, \frac{3\times 8+1\times 2}{3+1}\right)$$

$$= \left(1, \frac{13}{2}\right)$$

Question 10:

Find the area of a rhombus if its vertices are (3, 0), (4, 5), (− 1, 4) and (− 2, −1) taken in order. [**Hint:** Area of a rhombus = ½ (product of its diagonals)]

Answer 10:



Let (3, 0), (4, 5), (−1, 4) and (−2, −1) are the vertices A, B, C, D of a rhombus ABCD.

$$\text{Length of diagonal AC} = \sqrt{\left[3-(-1)\right]^2 + (0-4)^2}$$
$$= \sqrt{16+16} = 4\sqrt{2}$$
$$\text{Length of diagonal BD} = \sqrt{\left[4-(-2)\right]^2 + \left[5-(-1)\right]^2}$$
$$= \sqrt{36+36} = 6\sqrt{2}$$
$$\text{Therefore, area of rhombus ABCD} = \frac{1}{2} \times 4\sqrt{2} \times 6\sqrt{2}$$
$$= 24 \text{ square units}$$

Mathematics

(Chapter – 7) (Co-ordinate Geometry)

(Class – X)

Exercise 7.3

Question 1:

Find the area of the triangle whose vertices are:

(i) (2, 3), (– 1, 0), (2, – 4) **(ii)** (– 5, – 1), (3, – 5), (5, 2)

Answer 1:

(i) Area of a triangle is given by

$$\text{Area of a triangle} = \frac{1}{2}\{x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\}$$

$$\text{Area of the given triangle} = \frac{1}{2}\left[2\{0-(-4)\} + (-1)\{(-4)-(3)\} + 2(3-0)\right]$$

$$= \frac{1}{2}\{8+7+6\}$$

$$= \frac{21}{2} \text{ square units}$$

(ii) $\text{Area of the given triangle} = \frac{1}{2}\left[(-5)\{(-5)-(2)\} + 3(2-(-1)) + 5\{-1-(-5)\}\right]$

$$= \frac{1}{2}\{35+9+20\}$$

$$= 32 \text{ square units}$$

Question 2:

In each of the following find the value of '*k*', for which the points are collinear.

(i) (7, – 2), (5, 1), (3, – *k*) **(ii)** (8, 1), (*k*, – 4), (2, – 5)

Answer 2:

(i) For collinear points, area of triangle formed by them is zero.

Therefore, for points (7, −2) (5, 1), and (3, *k*), area = 0

$$\frac{1}{2}\left[7\{1-k\}+5\{k-(-2)\}+3\{(-2)-1\}\right]=0$$
$$7-7k+5k+10-9=0$$
$$-2k+8=0$$
$$k=4$$

(ii) For collinear points, area of triangle formed by them is zero.

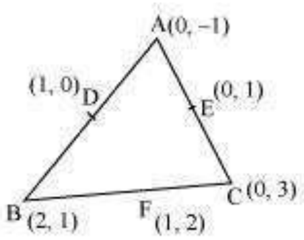
Therefore, for points (8, 1), (*k*, −4), and (2, −5), area = 0

$$\frac{1}{2}\left[8\{-4-(-5)\}+k\{(-5)-(1)\}+2\{1-(-4)\}\right]=0$$
$$8-6k+10=0$$
$$6k=18$$
$$k=3$$

Question 3:

Find the area of the triangle formed by joining the mid-points of the sides of the triangle whose vertices are (0, − 1), (2, 1) and (0, 3). Find the ratio of this area to the area of the given triangle.

Answer 3:



Let the vertices of the triangle be A (0, −1), B (2, 1), C (0, 3).

Let D, E, F be the mid-points of the sides of this triangle. Coordinates of D, E, and F are given by

$$D = \left(\frac{0+2}{2}, \frac{-1+1}{2}\right) = (1, 0)$$

$$E = \left(\frac{0+0}{2}, \frac{3-1}{2}\right) = (0, 1)$$

$$F = \left(\frac{2+0}{2}, \frac{1+3}{2}\right) = (1, 2)$$

$$\text{Area of a triangle} = \frac{1}{2}\{x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\}$$

$$\text{Area of } \Delta DEF = \frac{1}{2}\{1(2-1) + 1(1-0) + 0(0-2)\}$$

$$= \frac{1}{2}(1+1) = 1 \text{ square units}$$

$$\text{Area of } \Delta ABC = \frac{1}{2}\left[0(1-3) + 2\{3-(-1)\} + 0(-1-1)\right]$$

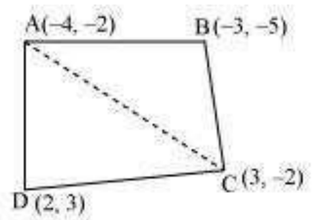
$$= \frac{1}{2}\{8\} = 4 \text{ square units}$$

Therefore, required ratio $= 1:4$

Question 4:

Find the area of the quadrilateral whose vertices, taken in order, are (− 4, − 2), (− 3, − 5), (3, − 2) and (2, 3)

Answer 4:



Let the vertices of the quadrilateral be A (−4, −2), B (−3, −5), C (3, −2), and D (2, 3). Join AC to form two triangles ΔABC and ΔACD.

$$\text{Area of a triangle} = \frac{1}{2}\{x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\}$$

$$\text{Area of } \Delta ABC = \frac{1}{2}\left[(-4)\{(-5)-(-2)\} + (-3)\{(-2)-(-2)\} + 3\{(-2)-(-5)\}\right]$$

$$= \frac{1}{2}(12 + 0 + 9) = \frac{21}{2} \text{square units}$$

$$\text{Area of } \Delta ACD = \frac{1}{2}\left[(-4)\{(-2)-(3)\} + 3\{(3)-(-2)\} + 2\{(-2)-(-2)\}\right]$$

$$= \frac{1}{2}\{20 + 15 + 0\} = \frac{35}{2} \text{ square units}$$

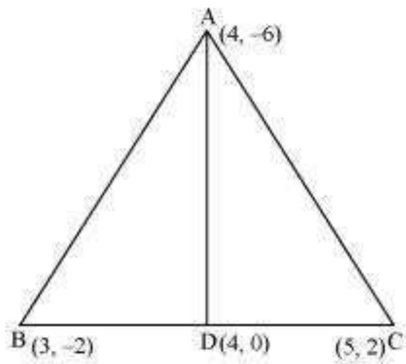
$$\text{Area of } \square ABCD = \text{Area of } \Delta ABC + \text{Area of } \Delta ACD$$

$$= \left(\frac{21}{2} + \frac{35}{2}\right)\text{square units} = 28 \text{ square units}$$

Question 5:

You have studied in Class IX that a median of a triangle divides it into two triangles of equal areas. Verify this result for ΔABC whose vertices are A (4, − 6), B (3, − 2) and C (5, 2)

Answer 5:



Let the vertices of the triangle be A (4, −6), B (3, −2), and C (5, 2).

Let D be the mid-point of side BC of ΔABC. Therefore, AD is the median in ΔABC.

$$\text{Coordinates of point D} = \left(\frac{3+5}{2}, \frac{-2+2}{2}\right) = (4,0)$$

$$\text{Area of a triangle} = \frac{1}{2}\{x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\}$$

$$\text{Area of }\Delta\text{ABD} = \frac{1}{2}\left[(4)\{(-2)-(0)\} + (3)\{(0)-(-6)\} + (4)\{(-6)-(-2)\}\right]$$

$$= \frac{1}{2}(-8+18-16) = -3 \text{ square units}$$

However, area cannot be negative. Therefore, area of ΔABD is 3 square units.

$$\text{Area of }\Delta\text{ADC} = \frac{1}{2}\left[(4)\{0-(2)\} + (4)\{(2)-(-6)\} + (5)\{(-6)-(0)\}\right]$$

$$= \frac{1}{2}(-8+32-30) = -3 \text{ square units}$$

However, area cannot be negative. Therefore, area of ΔADC is 3 square units.

Clearly, median AD has divided ΔABC in two triangles of equal areas.

Mathematics

(Chapter – 7) (Co-ordinate Geometry)

(Class – X)

Exercise 7.4

Question 1:

Determine the ratio in which the line $2x + y - 4 = 0$ divides the line segment joining the points A(2, − 2) and B(3, 7)

Answer 1:

Let the given line divide the line segment joining the points A (2, −2) and B (3, 7) in a ratio *k:1*.

Coordinates of the point of division $= \left(\frac{3k+2}{k+1}, \frac{7k-2}{k+1}\right)$

This point also lies on $2x + y - 4 = 0$

$$\therefore 2\left(\frac{3k+2}{k+1}\right)+\left(\frac{7k-2}{k+1}\right)-4=0$$

$$\Rightarrow \frac{6k+4+7k-2-4k-4}{k+1}=0$$

$$\Rightarrow 9k-2=0$$

$$\Rightarrow k=\frac{2}{9}$$

Therefore, the ratio in which the line $2x + y - 4 = 0$ divides the line segment joining the points A (2, −2) and B (3, 7) is 2:9.

Question 2:

Find a relation between x and y if the points (x, y), (1, 2) and (7, 0) are collinear.

Answer 2:

If the given points are collinear, then the area of triangle formed by these points will be 0.

$$\text{Area of a triangle} = \frac{1}{2}\{x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)\}$$

$$\text{Area} = \frac{1}{2}\left[x(2-0)+1(0-y)+7(y-2)\right]$$
$$0 = \frac{1}{2}[2x-y+7y-14]$$
$$0 = \frac{1}{2}[2x+6y-14]$$
$$2x+6y-14=0$$
$$x+3y-7=0$$

This is the required relation between *x* and *y*.

Question 3:

Find the centre of a circle passing through the points (6, − 6), (3, − 7) and (3, 3).

Answer 3:

Let O (*x*, *y*) be the centre of the circle. And let the points (6, −6), (3, −7), and (3, 3) be representing the points A, B, and C on the circumference of the circle.

$$\therefore \text{OA} = \sqrt{(x-6)^2+(y+6)^2}$$
$$\text{OB} = \sqrt{(x-3)^2+(y+7)^2}$$
$$\text{OC} = \sqrt{(x-3)^2+(y-3)^2}$$
$$\text{However, OA} = \text{OB} \qquad \text{(Radii of the same circle)}$$
$$\Rightarrow \sqrt{(x-6)^2+(y+6)^2} = \sqrt{(x-3)^2+(y+7)^2}$$
$$\Rightarrow x^2+36-12x+y^2+36+12y = x^2+9-6x+y^2+49+14y$$
$$\Rightarrow -6x-2y+14=0$$
$$\Rightarrow 3x+y=7 \qquad \ldots (1)$$
$$\text{Similarly, OA} = \text{OC} \qquad \text{(Radii of the same circle)}$$

$$\Rightarrow \sqrt{(x-6)^2+(y+6)^2}=\sqrt{(x-3)^2+(y-3)^2}$$
$$\Rightarrow x^2+36-12x+y^2+36+12y=x^2+9-6x+y^2+9-6y$$
$$\Rightarrow -6x+18y+54=0$$
$$\Rightarrow -3x+9y=-27 \qquad \ldots (2)$$

On adding equation (1) and (2), we obtain

$10y = -20$ y

$= -2$

From equation (1), we obtain

$3x - 2 = 7$

$3x = 9$ $x =$

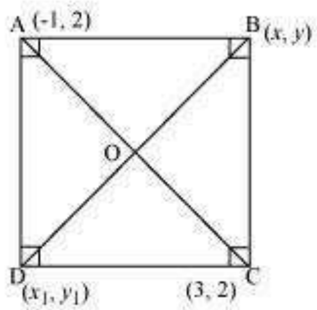
3

Therefore, the centre of the circle is (3, −2).

Question 4:

The two opposite vertices of a square are (− 1, 2) and (3, 2). Find the coordinates of the other two vertices.

Answer 4:



Let ABCD be a square having (−1, 2) and (3, 2) as vertices A and C respectively. Let (x, y), (x_1, y_1) be the coordinate of vertex B and D respectively.

We know that the sides of a square are equal to each other.

∴ AB = BC

$$\Rightarrow \sqrt{(x+1)^2+(y-2)^2}=\sqrt{(x-3)^2+(y-2)^2}$$

$$\Rightarrow x^2+2x+1+y^2-4y+4=x^2+9-6x+y^2+4-4y$$

$$\Rightarrow 8x=8$$

$$\Rightarrow x=1$$

We know that in a square, all interior angles are of 90°.

In ΔABC,

$AB^2 + BC^2 = AC^2$

$$\Rightarrow \left(\sqrt{(1+1)^2+(y-2)^2}\right)^2+\left(\sqrt{(1-3)^2+(y-2)^2}\right)^2=\left(\sqrt{(3+1)^2+(2-2)^2}\right)^2$$

$\Rightarrow 4 + y^2 + 4 - 4y + 4 + y^2 - 4y + 4 = 16$

$\Rightarrow 2y^2 + 16 - 8y = 16$

$\Rightarrow 2y^2 - 8y = 0$

$\Rightarrow y(y - 4) = 0$

$\Rightarrow y = 0$ or 4

We know that in a square, the diagonals are of equal length and bisect each other at 90°. Let O be the mid-point of AC. Therefore, it will also be the mid-point of BD.

$$\text{Coordinate of point O}=\left(\frac{-1+3}{2},\frac{2+2}{2}\right)$$

$$\left(\frac{1+x_1}{2},\frac{y+y_1}{2}\right)=(1,2)$$

$$\frac{1+x_1}{2}=1$$

$$1+x_1=2$$

$$x_1=1$$

$$\text{and }\frac{y+y_1}{2}=2$$

$\Rightarrow y + y_1 = 4$

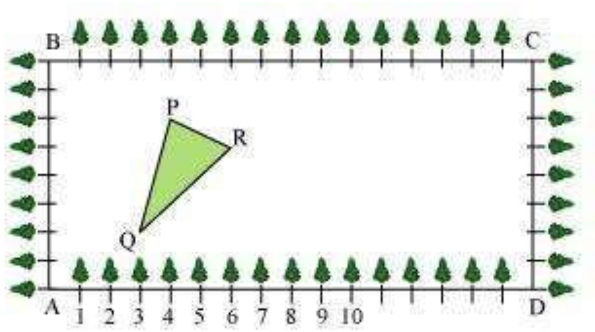
If $y = 0$, $y_1 = 4$

If $y = 4$, $y_1 = 0$

Therefore, the required coordinates are (1, 0) and (1, 4).

Question 5:

The class X students of MRV Public School in Krishna Park have been allotted a rectangular plot of land for their gardening activity. Saplings of Gulmohar are planted on the boundary at a distance of 1 m from each other. There is a triangular grassy lawn in the plot as shown in the following figure. The students are to sow seeds of flowering plants on the remaining area of the plot.



(i) Taking A as origin, find the coordinates of the vertices of the triangle.

(ii) What will be the coordinates of the vertices of Δ PQR if C is the origin?

Also calculate the areas of the triangles in these cases. What do you observe?

Answer 5:

(i) Taking A as origin, we will take AD as x-axis and AB as y-axis. It can be observed that the coordinates of point P, Q, and R are (4, 6), (3, 2), and (6, 5) respectively.

$$\text{Area of triangle PQR} = \frac{1}{2}\left[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)\right]$$
$$= \frac{1}{2}\left[4(2-5)+3(5-6)+6(6-2)\right]$$
$$= \frac{1}{2}[-12-3+24]$$
$$= \frac{9}{2}\text{ square units}$$

(ii) Taking C as origin, CB as *x*-axis, and CD as *y*-axis, the coordinates of vertices P, Q, and R are (12, 2), (13, 6), and (10, 3) respectively.

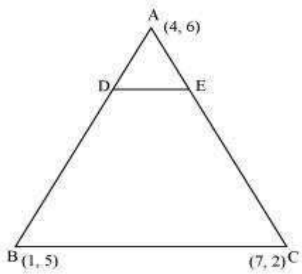
$$\text{Area of triangle PQR} = \frac{1}{2}\left[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1+y_2)\right]$$
$$= \frac{1}{2}\left[12(6-3)+13(3-2)+10(2-6)\right]$$
$$= \frac{1}{2}[36+13-40]$$
$$= \frac{9}{2}\text{ square units}$$

It can be observed that the area of the triangle is same in both the cases.

Question 6:

The vertices of a ΔABC are A (4, 6), B (1, 5) and C (7, 2). A line is drawn to intersect sides AB and AC at D and E respectively, such that $\frac{AD}{AB}=\frac{AE}{AC}=\frac{1}{4}$. Calculate the area of the ΔADE and compare it with the area of ΔABC. (Recall Converse of basic proportionality theorem and Theorem 6.6 related to ratio of areas of two similar triangles)

Answer 6:



Given that:

$$\frac{AD}{AB} = \frac{AE}{AC} = \frac{1}{4}$$

$$\frac{\text{AD}}{\text{AD}+\text{DB}} = \frac{\text{AE}}{\text{AE}+\text{EC}} = \frac{1}{4}$$

$$\frac{\text{AD}}{\text{DB}} = \frac{\text{AE}}{\text{EC}} = \frac{1}{3}$$

Therefore, D and E are two points on side AB and AC respectively such that they divide side AB and AC in a ratio of 1:3.

$$\text{Coordinates of Point D} = \left(\frac{1\times1+3\times4}{1+3}, \frac{1\times5+3\times6}{1+3}\right)$$

$$= \left(\frac{13}{4}, \frac{23}{4}\right)$$

$$\text{Coordinates of point E} = \left(\frac{1\times7+3\times4}{1+3}, \frac{1\times2+3\times6}{1+3}\right)$$

$$= \left(\frac{19}{4}, \frac{20}{4}\right)$$

$$\text{Area of a triangle} = \frac{1}{2}\left[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)\right]$$

$$\text{Area of } \Delta\text{ADE} = \frac{1}{2}\left[4\left(\frac{23}{4}-\frac{20}{4}\right)+\frac{13}{4}\left(\frac{20}{4}-6\right)+\frac{19}{4}\left(6-\frac{23}{4}\right)\right]$$

$$= \frac{1}{2}\left[3-\frac{13}{4}+\frac{19}{16}\right] = \frac{1}{2}\left[\frac{48-52+19}{16}\right] = \frac{15}{32} \text{ square units}$$

$$\text{Area of } \Delta\text{ABC} = \frac{1}{2}\left[4(5-2)+1(2-6)+7(6-5)\right]$$

$$= \frac{1}{2}\left[12-4+7\right] = \frac{15}{2} \text{ square units}$$

Clearly, the ratio between the areas of ΔADE and ΔABC is 1:16.

Alternatively,

We know that if a line segment in a triangle divides its two sides in the same ratio, then the line segment is parallel to the third side of the triangle. These two triangles so formed (here ΔADE and ΔABC) will be similar to each other.

Hence, the ratio between the areas of these two triangles will be the square of the ratio between the sides of these two triangles.

Therefore, ratio between the areas of ΔADE and ΔABC $= \left(\frac{1}{4}\right)^2 = \frac{1}{16}$

Question 7:

Let A (4, 2), B (6, 5) and C (1, 4) be the vertices of ΔABC.

(i) The median from A meets BC at D. Find the coordinates of point D.

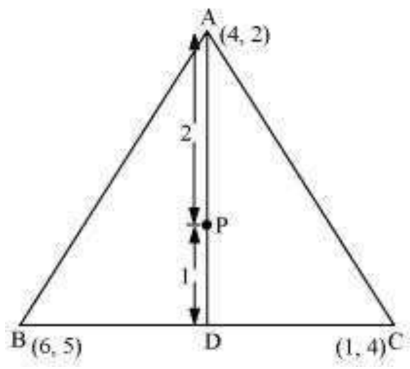
(ii) Find the coordinates of the point P on AD such that AP: PD = 2:1

(iii) Find the coordinates of point Q and R on medians BE and CF respectively such that BQ: QE = 2:1 and CR: RF = 2:1.

(iv) What do you observe?

(v) If A(x_1, y_1), B(x_2, y_2), and C(x_3, y_3) are the vertices of ΔABC, find the coordinates of the centroid of the triangle.

Answer 7:



(i) Median AD of the triangle will divide the side BC in two equal parts.

Therefore, D is the mid-point of side BC.

$$\text{Coordinates of D} = \left(\frac{6+1}{2}, \frac{5+4}{2}\right) = \left(\frac{7}{2}, \frac{9}{2}\right)$$

(ii) Point P divides the side AD in a ratio 2:1.

$$\text{Coordinates of P} = \left(\frac{2\times\frac{7}{2}+1\times 4}{2+1}, \frac{2\times\frac{9}{2}+1\times 2}{2+1}\right) = \left(\frac{11}{3}, \frac{11}{3}\right)$$

(iii) Median BE of the triangle will divide the side AC in two equal parts.

Therefore, E is the mid-point of side AC.

$$\text{Coordinates of E} = \left(\frac{4+1}{2}, \frac{2+4}{2}\right) = \left(\frac{5}{2}, 3\right)$$

Point Q divides the side BE in a ratio 2:1.

$$\text{Coordinates of Q} = \left(\frac{2\times\frac{5}{2}+1\times 6}{2+1}, \frac{2\times 3+1\times 5}{2+1}\right) = \left(\frac{11}{3}, \frac{11}{3}\right)$$

Median CF of the triangle will divide the side AB in two equal parts. Therefore, F is the mid-point of side AB.

$$\text{Coordinates of F} = \left(\frac{4+6}{2}, \frac{2+5}{2}\right) = \left(5, \frac{7}{2}\right)$$

Point R divides the side CF in a ratio 2:1.

$$\text{Coordinates of R} = \left(\frac{2\times 5+1\times 1}{2+1}, \frac{2\times\frac{7}{2}+1\times 4}{2+1}\right) = \left(\frac{11}{3}, \frac{11}{3}\right)$$

(iv) It can be observed that the coordinates of point P, Q, R are the same.

Therefore, all these are representing the same point on the plane i.e., the centroid of the triangle.

(v) Consider a triangle, ΔABC, having its vertices as A(x_1, y_1), B(x_2, y_2), and C(x_3, y_3).

Median AD of the triangle will divide the side BC in two equal parts. Therefore, D is the mid-point of side BC.

$$\text{Coordinates of D} = \left(\frac{x_2+x_3}{2}, \frac{y_2+y_3}{2}\right)$$

Let the centroid of this triangle be O.

Point O divides the side AD in a ratio 2:1.

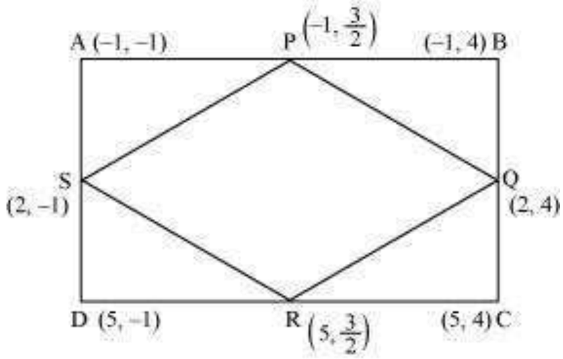
$$\text{Coordinates of O} = \left(\frac{2\times\frac{x_2+x_3}{2}+1\times x_1}{2+1}, \frac{2\times\frac{y_2+y_3}{2}+1\times y_1}{2+1}\right)$$

$$=\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$$

Question 8:

ABCD is a rectangle formed by the points A (−1, −1), B (− 1, 4), C (5, 4) and D (5, −1). P, Q, R and S are the mid-points of AB, BC, CD, and DA respectively. Is the quadrilateral PQRS is a square? a rectangle? or a rhombus? Justify your answer.

Answer 8:



P is the mid-point of side AB.

Therefore, the coordinates of P are $\left(\frac{-1-1}{2}, \frac{-1+4}{2}\right)=\left(-1, \frac{3}{2}\right)$

Similarly, the coordinates of Q, R, and S are $(2, 4)$, $\left(5, \frac{3}{2}\right)$, and $(2, -1)$ respectively.

$$\text{Length of PQ} = \sqrt{(-1-2)^2 + \left(\frac{3}{2}-4\right)^2} = \sqrt{9+\frac{25}{4}} = \sqrt{\frac{61}{4}}$$

$$\text{Length of QR} = \sqrt{(2-5)^2 + \left(4-\frac{3}{2}\right)^2} = \sqrt{9+\frac{25}{4}} = \sqrt{\frac{61}{4}}$$

$$\text{Length of RS} = \sqrt{(5-2)^2 + \left(\frac{3}{2}+1\right)^2} = \sqrt{9+\frac{25}{4}} = \sqrt{\frac{61}{4}}$$

$$\text{Length of SP} = \sqrt{(2+1)^2 + \left(-1-\frac{3}{2}\right)^2} = \sqrt{9+\frac{25}{4}} = \sqrt{\frac{61}{4}}$$

$$\text{Length of PR} = \sqrt{(-1-5)^2 + \left(\frac{3}{2}-\frac{3}{2}\right)^2} = 6$$

$$\text{Length of QS} = \sqrt{(2-2)^2 + (4+1)^2} = 5$$

It can be observed that all sides of the given quadrilateral are of the same measure. However, the diagonals are of different lengths. Therefore, PQRS is a rhombus.

Mathematics

(Chapter – 8) (Introduction to Trigonometry)
(Class X)

Exercise 8.1

Question 1:

In ΔABC right angled at B, AB = 24 cm, BC = 7 m. Determine

(i) sin A, cos A

(ii) sin C, cos C

Answer 1:

Applying Pythagoras theorem for ΔABC, we obtain

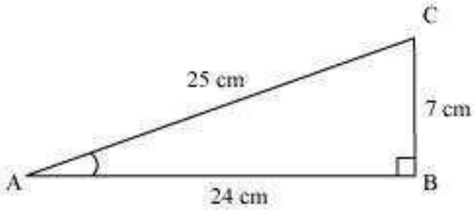
$AC^2 = AB^2 + BC^2$

$= (24 \text{ cm})^2 + (7 \text{ cm})^2$

$= (576 + 49) \text{ cm}^2$

$= 625 \text{ cm}^2$

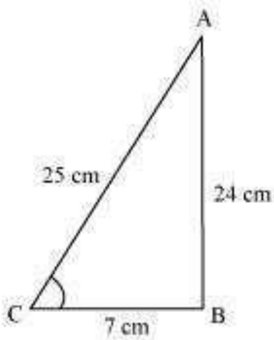
$\therefore AC = \sqrt{625} \text{ cm} = 25 \text{ cm}$



(i) $\sin A = \dfrac{\text{Side opposite to } \angle A}{\text{Hypotenuse}} = \dfrac{BC}{AC} = \dfrac{7}{25}$

$\cos A = \dfrac{\text{Side adjacent to } \angle A}{\text{Hypotenuse}} = \dfrac{AB}{AC} = \dfrac{24}{25}$

(ii)

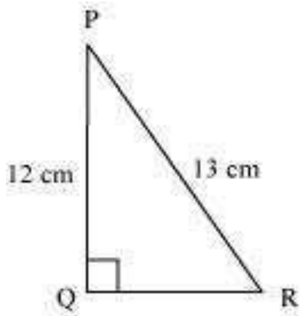


sin C = $\dfrac{\text{Side opposite to } \angle C}{\text{Hypotenuse}} = \dfrac{AB}{AC} = \dfrac{24}{25}$

cos C = $\dfrac{\text{Side adjacent to } \angle C}{\text{Hypotenuse}} = \dfrac{BC}{AC} = \dfrac{7}{25}$

Question 2:

In the given figure find tan P − cot R



Answer 2:

Applying Pythagoras theorem for ΔPQR, we obtain

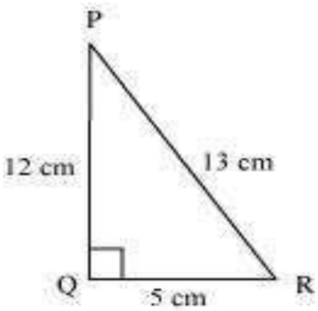
$PR^2 = PQ^2 + QR^2$

$(13\text{ cm})^2 = (12\text{ cm})^2 + QR^2$

$169\text{ cm}^2 = 144\text{ cm}^2 + QR^2$

$25\text{ cm}^2 = QR^2$

QR = 5 cm



$$\tan P = \frac{\text{Side opposite to } \angle P}{\text{Side adjacent to } \angle P} = \frac{QR}{PQ}$$

$$= \frac{5}{12}$$

$$\cot R = \frac{\text{Side adjacent to } \angle R}{\text{Side opposite to } \angle R} = \frac{QR}{PQ}$$

$$= \frac{5}{12}$$

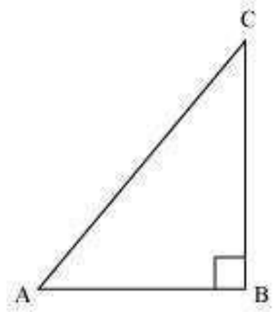
tan P − cot R = $\frac{5}{12} - \frac{5}{12} = 0$

Question 3:

If sin A = $\frac{3}{4}$, calculate cos A and tan A.

Answer 3:

Let ΔABC be a right-angled triangle, right-angled at point B.



Given that,

$$\sin A = \frac{3}{4}$$

$$\frac{BC}{AC} = \frac{3}{4}$$

Let BC be 3*k*. Therefore, AC will be 4*k*, where *k* is a positive integer.

Applying Pythagoras theorem in ΔABC, we obtain

$AC^2 = AB^2 + BC^2$

$(4k)^2 = AB^2 + (3k)^2$

$16k^2 - 9k^2 = AB^2$

$7k^2 = AB^2$

AB = $\sqrt{7}k$

$$\cos A = \frac{\text{Side adjacent to } \angle A}{\text{Hypotenuse}}$$

$$= \frac{AB}{AC} = \frac{\sqrt{7}k}{4k} = \frac{\sqrt{7}}{4}$$

$$\tan A = \frac{\text{Side opposite to } \angle A}{\text{Side adjacent to } \angle A}$$

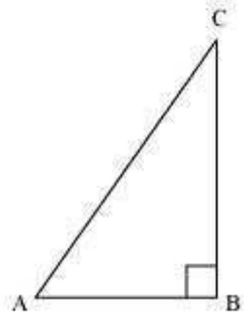
$$= \frac{BC}{AB} = \frac{3k}{\sqrt{7}k} = \frac{3}{\sqrt{7}}$$

Question 4:

Given 15 cot A = 8. Find sin A and sec A

Answer 4:

Consider a right-angled triangle, right-angled at B.



$$\cot A = \frac{\text{Side adjacent to } \angle A}{\text{Side opposite to } \angle A}$$

$$= \frac{AB}{BC}$$

It is given that,

cot A = $\frac{8}{15}$

$$\frac{AB}{BC} = \frac{8}{15}$$

Let AB be $8k$.Therefore, BC will be $15k$, where k is a positive integer.

Applying Pythagoras theorem in ΔABC, we obtain

$AC^2 = AB^2 + BC^2 = (8k)^2 + (15k)^2$

$= 64k^2 + 225k^2$

$= 289k^2$

AC = 17k

$$\sin A = \frac{\text{Side opposite to } \angle A}{\text{Hypotenuse}} = \frac{BC}{AC}$$

$$= \frac{15k}{17k} = \frac{15}{17}$$

$$\sec A = \frac{\text{Hypotenuse}}{\text{Side adjacent to } \angle A}$$

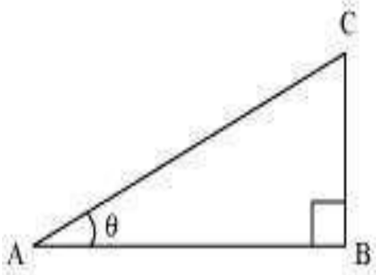
$$= \frac{AC}{AB} = \frac{17}{8}$$

Question 5:

Given sec θ = $\frac{13}{12}$, calculate all other trigonometric ratios.

Answer 5:

Consider a right-angle triangle ΔABC, right-angled at point B.



$$\sec\theta = \frac{\text{Hypotenuse}}{\text{Side adjacent to } \angle\theta}$$

$$\frac{13}{12} = \frac{AC}{AB}$$

If AC is 13k, AB will be 12k, where k is a positive integer.

Applying Pythagoras theorem in ΔABC, we obtain

$(AC)^2 = (AB)^2 + (BC)^2$

$(13k)^2 = (12k)^2 + (BC)^2$

$169k^2 = 144k^2 + BC^2$

$25k^2 = BC^2$

BC = 5k

$$\sin\theta = \frac{\text{Side opposite to } \angle\theta}{\text{Hypotenuse}} = \frac{BC}{AC} = \frac{5k}{13k} = \frac{5}{13}$$

$$\cos\theta = \frac{\text{Side adjacent to } \angle\theta}{\text{Hypotenuse}} = \frac{\text{AB}}{\text{AC}} = \frac{12k}{13k} = \frac{12}{13}$$

$$\tan\theta = \frac{\text{Side opposite to } \angle\theta}{\text{Side adjacent to } \angle\theta} = \frac{\text{BC}}{\text{AB}} = \frac{5k}{12k} = \frac{5}{12}$$

$$\cot\theta = \frac{\text{Side adjacent to } \angle\theta}{\text{Side opposite to } \angle\theta} = \frac{\text{AB}}{\text{BC}} = \frac{12k}{5k} = \frac{12}{5}$$

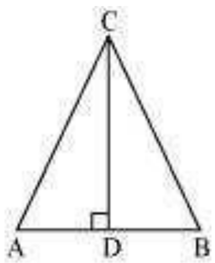
$$\operatorname{cosec}\theta = \frac{\text{Hypotenuse}}{\text{Side opposite to } \angle\theta} = \frac{\text{AC}}{\text{BC}} = \frac{13k}{5k} = \frac{13}{5}$$

Question 6:

If ∠A and ∠B are acute angles such that cos A = cos B, then show that ∠A = ∠B.

Answer 6:

Let us consider a triangle ABC in which CD ⊥ AB.

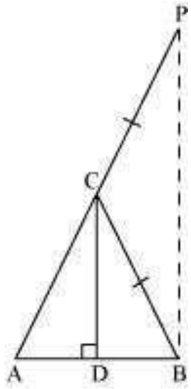


It is given that

cos A = cos B

$$\Rightarrow \frac{\text{AD}}{\text{AC}} = \frac{\text{BD}}{\text{BC}} \quad \text{.....................} \ (1)$$

We have to prove ∠A = ∠B. To prove this, let us extend AC to P such that BC = CP.



From equation (1), we obtain

$$\frac{AD}{BD} = \frac{AC}{BC}$$

$$\Rightarrow \frac{AD}{BD} = \frac{AC}{CP} \qquad \text{(By construction, we have BC = CP)} \qquad \ldots (2)$$

By using the converse of B.P.T,

CD||BP

⇒ ∠ACD = ∠CPB (Corresponding angles) … (3) And,

∠BCD = ∠CBP (Alternate interior angles) … (4)

By construction, we have BC = CP.

∴ ∠CBP **=** ∠CPB (Angle opposite to equal sides of a triangle) … (5)

From equations (3), (4), and (5), we obtain

∠ACD = ∠BCD … (6)

In ΔCAD and ΔCBD,

∠ACD = ∠BCD [Using equation (6)]

∠CDA = ∠CDB [Both 90°]

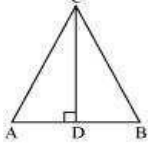
Therefore, the remaining angles should be equal.

∴ ∠CAD = ∠CBD

⇒∠A = ∠B

Alternatively,

Let us consider a triangle ABC in which CD ⊥ AB.



It is given that,

cos A = cos B

$$\Rightarrow \frac{AD}{AC} = \frac{BD}{BC}$$

$$\Rightarrow \frac{AD}{BD} = \frac{AC}{BC}$$

Let $\frac{AD}{BD} = \frac{AC}{BC} = k$

$\Rightarrow$AD = k BD … (1)

And, AC = k BC … (2)

Using Pythagoras theorem for triangles CAD and CBD, we obtain

$CD^2 = AC^2 - AD^2$ … (3)

And, $CD^2 = BC^2 - BD^2$ … (4)

From equations (3) and (4), we obtain

$AC^2 - AD^2 = BC^2 - BD^2$

$\Rightarrow (k\ BC)^2 - (k\ BD)^2 = BC^2 - BD^2$

$\Rightarrow k^2\ (BC^2 - BD^2) = BC^2 - BD^2$

$\Rightarrow k^2 = 1$

$\Rightarrow k = 1$

Putting this value in equation (2), we obtain

AC = BC

$\Rightarrow \angle A = \angle B$(Angles opposite to equal sides of a triangle)

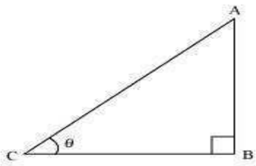
Question 7:

If cot θ = $\frac{7}{8}$, evaluate

(i) $\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)}$ (ii) $\cot^2\theta$

Answer 7:

Let us consider a right triangle ABC, right-angled at point B.



$$\cot\theta = \frac{\text{Side adjacent to } \angle\theta}{\text{Side opposite to } \angle\theta} = \frac{BC}{AB}$$

$$= \frac{7}{8}$$

If BC is 7*k*, then AB will be 8*k*, where *k* is a positive integer.

Applying Pythagoras theorem in ΔABC, we obtain

$AC^2 = AB^2 + BC^2$

$= (8k)^2 + (7k)^2$

$= 64k^2 + 49k^2$

$= 113k^2$

AC = $\sqrt{113}k$

$$\sin\theta = \frac{\text{Side opposite to } \angle\theta}{\text{Hypotenuse}} = \frac{AB}{AC}$$

$$= \frac{8k}{\sqrt{113}k} = \frac{8}{\sqrt{113}}$$

$$\cos\theta = \frac{\text{Side adjacent to } \angle\theta}{\text{Hypotenuse}} = \frac{BC}{AC}$$

$$= \frac{7k}{\sqrt{113}k} = \frac{7}{\sqrt{113}}$$

(i) $\dfrac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)} = \dfrac{(1-\sin^2\theta)}{(1-\cos^2\theta)}$

$$= \frac{1-\left(\frac{8}{\sqrt{113}}\right)^2}{1-\left(\frac{7}{\sqrt{113}}\right)^2} = \frac{1-\frac{64}{113}}{1-\frac{49}{113}}$$

$$= \frac{\frac{49}{113}}{\frac{64}{113}} = \frac{49}{64}$$

(ii) $\cot^2\theta = (\cot\theta)^2 = \left(\frac{7}{8}\right)^2 = \frac{49}{64}$

Question 8:

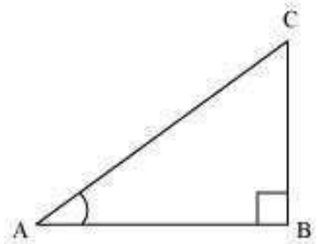
If 3 cot A = 4, Check whether $\frac{1-\tan^2 A}{1+\tan^2 A} = \cos^2 A - \sin^2 A$ or not.

Answer 8:

It is given that 3cot A = 4

Or, cot A = $\frac{4}{3}$

Consider a right triangle ABC, right-angled at point B.



$$\cot A = \frac{\text{Side adjacent to } \angle A}{\text{Side opposite to } \angle A}$$

$$\frac{AB}{BC} = \frac{4}{3}$$

If AB is 4*k*, then BC will be 3*k*, where *k* is a positive integer.

In ΔABC,

$(AC)^2 = (AB)^2 + (BC)^2$

$= (4k)^2 + (3k)^2$

$= 16k^2 + 9k^2$

$= 25k^2$

AC = 5*k*

$$\cos A = \frac{\text{Side adjacent to } \angle A}{\text{Hypotenuse}} = \frac{AB}{AC}$$

$$= \frac{4k}{5k} = \frac{4}{5}$$

$$\sin A = \frac{\text{Side opposite to } \angle A}{\text{Hypotenuse}} = \frac{BC}{AC}$$

$$= \frac{3k}{5k} = \frac{3}{5}$$

$$\tan A = \frac{\text{Side opposite to } \angle A}{\text{Hypotenuse}} = \frac{BC}{AB}$$

$$= \frac{3k}{4k} = \frac{3}{4}$$

$$\frac{1-\tan^2 A}{1+\tan^2 A}=\frac{1-\left(\frac{3}{4}\right)^2}{1+\left(\frac{3}{4}\right)^2}=\frac{1-\frac{9}{16}}{1+\frac{9}{16}}$$

$$=\frac{\frac{7}{16}}{\frac{25}{16}}=\frac{7}{25}$$

$\cos^2 A - \sin^2 A =$ $\left(\frac{4}{5}\right)^2-\left(\frac{3}{5}\right)^2$

$$=\frac{16}{25}-\frac{9}{25}=\frac{7}{25}$$

$$\frac{1-\tan^2 A}{1+\tan^2 A}=\cos^2 A-\sin^2 A$$

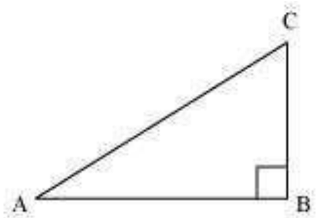
Question 9:

In ΔABC, right angled at B. If $\tan A=\frac{1}{\sqrt{3}}$, find the value of

(i) sin A cos C + cos A sin C

(ii) cos A cos C − sin A sin C

Answer 9:



$$\tan A=\frac{1}{\sqrt{3}}$$

$$\frac{BC}{AB}=\frac{1}{\sqrt{3}}$$

If BC is k, then AB will be $\sqrt{3}k$, where k is a positive integer.

In ΔABC,

$AC^2 = AB^2 + BC^2$

$= \left(\sqrt{3}k\right)^2+(k)^2$

$= 3k^2 + k^2 = 4k^2$

$\therefore$ AC = $2k$

$$\sin A = \frac{\text{Side opposite to } \angle A}{\text{Hypotenuse}} = \frac{BC}{AC} = \frac{k}{2k} = \frac{1}{2}$$

$$\cos A = \frac{\text{Side adjacent to } \angle A}{\text{Hypotenuse}} = \frac{AB}{AC} = \frac{\sqrt{3}k}{2k} = \frac{\sqrt{3}}{2}$$

$$\sin C = \frac{\text{Side opposite to } \angle C}{\text{Hypotenuse}} = \frac{AB}{AC} = \frac{\sqrt{3}k}{2k} = \frac{\sqrt{3}}{2}$$

$$\cos C = \frac{\text{Side adjacent to } \angle C}{\text{Hypotenuse}} = \frac{BC}{AC} = \frac{k}{2k} = \frac{1}{2}$$

(i) sin A cos C + cos A sin C

$$= \left(\frac{1}{2}\right)\left(\frac{1}{2}\right) + \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) = \frac{1}{4} + \frac{3}{4}$$

$$= \frac{4}{4} = 1$$

(ii) cos A cos C − sin A sin C

$$= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{2}\right) - \left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right) = \frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{4} = 0$$

Question 10:

In ΔPQR, right angled at Q, PR + QR = 25 cm and PQ = 5 cm. Determine the values of sin P, cos P and tan P.

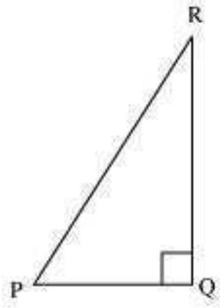
Answer 10:

Given that, PR + QR = 25

PQ = 5

Let PR be x.

Therefore, QR = 25 − x



Applying Pythagoras theorem in ΔPQR, we obtain

$PR^2 = PQ^2 + QR^2$

$x^2 = (5)^2 + (25 - x)^2$

$x^2 = 25 + 625 + x^2 - 50x$

$50x = 650$

$x = 13$

Therefore, PR = 13 cm

QR = (25 − 13) cm = 12 cm

$$\sin P = \frac{\text{Side opposite to } \angle P}{\text{Hypotenuse}} = \frac{QR}{PR} = \frac{12}{13}$$

$$\cos P = \frac{\text{Side adjacent to } \angle P}{\text{Hypotenuse}} = \frac{PQ}{PR} = \frac{5}{13}$$

$$\tan P = \frac{\text{Side opposite to } \angle P}{\text{Side adjacent to } \angle P} = \frac{QR}{PQ} = \frac{12}{5}$$

Question 11:

State whether the following are true or false. Justify your answer.

(i) The value of tan A is always less than 1.

(ii) sec A $= \frac{12}{5}$ for some value of angle A.

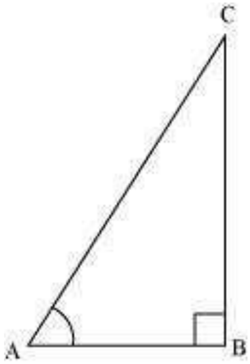
(iii) cos A is the abbreviation used for the cosecant of angle A.

(iv) cot A is the product of cot and A

(v) sin θ = $\frac{4}{3}$, for some angle θ

Answer 11:

(i) Consider a ΔABC, right-angled at B.



$$\tan A = \frac{\text{Side opposite to } \angle A}{\text{Side adjacent to } \angle A}$$

$$= \frac{12}{5}$$

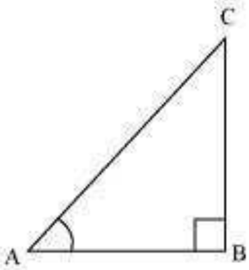
But $\frac{12}{5} > 1$

$\therefore \tan A > 1$

So, $\tan A < 1$ is not always true.

Hence, the given statement is false.

(ii) $\sec A = \frac{12}{5}$



$$\frac{\text{Hypotenuse}}{\text{Side adjacent to } \angle A} = \frac{12}{5}$$

$$\frac{AC}{AB} = \frac{12}{5}$$

Let AC be $12k$, AB will be $5k$, where k is a positive integer.

Applying Pythagoras theorem in ΔABC, we obtain

$AC^2 = AB^2 + BC^2$

$(12k)^2 = (5k)^2 + BC^2$

$144k^2 = 25k^2 + BC^2$

$BC^2 = 119k^2$

BC = 10.9k

It can be observed that for given two sides AC = 12k and AB = 5k,

BC should be such that,

AC − AB < BC < AC + AB

$12k - 5k < BC < 12k + 5k$

$7k < BC < 17k$

However, BC = 10.9k. Clearly, such a triangle is possible and hence, such value of sec A is possible.

Hence, the given statement is true.

(iii) Abbreviation used for cosecant of angle A is cosec A. And cos A is the abbreviation used for cosine of angle A.

Hence, the given statement is false.

(iv) cot A is not the product of cot and A. It is the cotangent of ∠A.

Hence, the given statement is false.

(v) $\sin\theta = \frac{4}{3}$

We know that in a right-angled triangle,

$$\sin\theta = \frac{\text{Side opposite to } \angle\theta}{\text{Hypotenuse}}$$

In a right-angled triangle, hypotenuse is always greater than the remaining two sides. Therefore, such value of sin θ is not possible.

Hence, the given statement is false

Mathematics

(Chapter – 8) (Introduction to Trigonometry)
(Class X)

Exercise 8.2

Question 1:

Evaluate the following

(i) $\sin 60° \cos 30° + \sin 30° \cos 60°$

(ii) $2\tan^2 45° + \cos^2 30° - \sin^2 60°$

(iii) $\dfrac{\cos 45°}{\sec 30° + \operatorname{cosec} 30°}$

(iv) $\dfrac{\sin 30° + \tan 45° - \operatorname{cosec} 60°}{\sec 30° + \cos 60° + \cot 45°}$

(v) $\dfrac{5\cos^2 60° + 4\sec^2 30° - \tan^2 45°}{\sin^2 30° + \cos^2 30°}$

Answer 1:

(i) $\sin 60° \cos 30° + \sin 30° \cos 60°$

$$= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)$$

$$= \frac{3}{4} + \frac{1}{4} = \frac{4}{4} = 1$$

(ii) $2\tan^2 45° + \cos^2 30° - \sin^2 60°$

$$= 2(1)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{\sqrt{3}}{2}\right)^2$$

$$= 2 + \frac{3}{4} - \frac{3}{4} = 2$$

(iii) $\dfrac{\cos 45°}{\sec 30° + \operatorname{cosec} 30°}$

$$= \frac{\frac{1}{\sqrt{2}}}{\frac{2}{\sqrt{3}}+2} = \frac{\frac{1}{\sqrt{2}}}{\frac{2+2\sqrt{3}}{\sqrt{3}}}$$

$$= \frac{\sqrt{3}}{\sqrt{2}\left(2+2\sqrt{3}\right)} = \frac{\sqrt{3}}{2\sqrt{2}+2\sqrt{6}}$$

$$= \frac{\sqrt{3}\left(2\sqrt{6}-2\sqrt{2}\right)}{\left(2\sqrt{6}+2\sqrt{2}\right)\left(2\sqrt{6}-2\sqrt{2}\right)}$$

$$= \frac{2\sqrt{3}\left(\sqrt{6}-\sqrt{2}\right)}{\left(2\sqrt{6}\right)^2-\left(2\sqrt{2}\right)^2} = \frac{2\sqrt{3}\left(\sqrt{6}-\sqrt{2}\right)}{24-8} = \frac{2\sqrt{3}\left(\sqrt{6}-\sqrt{2}\right)}{16}$$

$$= \frac{\sqrt{18}-\sqrt{6}}{8} = \frac{3\sqrt{2}-\sqrt{6}}{8}$$

(iv) $\dfrac{\sin 30^\circ + \tan 45^\circ - \operatorname{cosec} 60^\circ}{\sec 30^\circ + \cos 60^\circ + \cot 45^\circ}$

$$= \frac{\frac{1}{2}+1-\frac{2}{\sqrt{3}}}{\frac{2}{\sqrt{3}}+\frac{1}{2}+1} = \frac{\frac{3}{2}-\frac{2}{\sqrt{3}}}{\frac{3}{2}+\frac{2}{\sqrt{3}}}$$

$$= \frac{\frac{3\sqrt{3}-4}{2\sqrt{3}}}{\frac{3\sqrt{3}+4}{2\sqrt{3}}} = \frac{\left(3\sqrt{3}-4\right)}{\left(3\sqrt{3}+4\right)}$$

$$= \frac{\left(3\sqrt{3}-4\right)\left(3\sqrt{3}-4\right)}{\left(3\sqrt{3}+4\right)\left(3\sqrt{3}-4\right)} = \frac{\left(3\sqrt{3}-4\right)^2}{\left(3\sqrt{3}\right)^2-\left(4\right)^2}$$

$$= \frac{27+16-24\sqrt{3}}{27-16} = \frac{43-24\sqrt{3}}{11}$$

(v) $\dfrac{5\cos^2 60° + 4\sec^2 30° - \tan^2 45°}{\sin^2 30° + \cos^2 30°}$

$$= \frac{5\left(\frac{1}{2}\right)^2 + 4\left(\frac{2}{\sqrt{3}}\right)^2 - (1)^2}{\left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}$$

$$= \frac{5\left(\frac{1}{4}\right) + \left(\frac{16}{3}\right) - 1}{\frac{1}{4} + \frac{3}{4}}$$

$$= \frac{\frac{15+64-12}{12}}{\frac{4}{4}} = \frac{67}{12}$$

Question 2:

Choose the correct option and justify your choice.

(i) $\dfrac{2\tan 30°}{1+\tan^2 30°} =$

(A). sin60° (B). cos60° (C). tan60° (D). sin30°

(ii) $\dfrac{1-\tan^2 45°}{1+\tan^2 45°} =$

(A). tan90° (B). 1 (C). sin45° (D). 0

(iii) sin2A = 2sinA is true when A =

(A). 0° (B). 30° (C). 45° (D). 60°

(iv) $\frac{2\tan 30°}{1-\tan^2 30°} =$

(A). cos60° (B). sin60° (C). tan60° (D). sin30°

Answer 2:

(i) $\frac{2\tan 30°}{1+\tan^2 30°}$

$$= \frac{2\left(\frac{1}{\sqrt{3}}\right)}{1+\left(\frac{1}{\sqrt{3}}\right)^2} = \frac{\frac{2}{\sqrt{3}}}{1+\frac{1}{3}} = \frac{\frac{2}{\sqrt{3}}}{\frac{4}{3}}$$

$$= \frac{6}{4\sqrt{3}} = \frac{\sqrt{3}}{2}$$

Out of the given alternatives, only $\sin 60° = \frac{\sqrt{3}}{2}$

Hence, (A) is correct.

(ii) $\frac{1-\tan^2 45°}{1+\tan^2 45°} = \frac{1-(1)^2}{1+(1)^2} = \frac{1-1}{1+1} = \frac{0}{2} = 0$

Hence, (D) is correct.

(iii)Out of the given alternatives, only A = 0° is correct.

As sin 2A = sin 0° = 0

2 sinA = 2sin 0° = 2(0) = 0

Hence, (A) is correct.

(iv) $\frac{2\tan 30°}{1-\tan^2 30°}$

$$= \frac{2\left(\frac{1}{\sqrt{3}}\right)}{1-\left(\frac{1}{\sqrt{3}}\right)^2} = \frac{\frac{2}{\sqrt{3}}}{1-\frac{1}{3}} = \frac{\frac{2}{\sqrt{3}}}{\frac{2}{3}} = \sqrt{3}$$

Out of the given alternatives, only tan 60° $= \sqrt{3}$

Hence, (C) is correct.

Question 3:

If $\tan(A+B)=\sqrt{3}$ and $\tan(A-B)=\frac{1}{\sqrt{3}}$

0° < A + B ≤ 90°, A > B find A and B.

Answer 3:

$\tan(A+B)=\sqrt{3}$

$\tan(A+B)=\tan 60$

⇒A + B = 60 ………………..… (1)

$\tan(A-B)=\frac{1}{\sqrt{3}}$

⇒tan (A − B) = tan30

⇒A − B = 30 ……………….. (2)

On adding both equations, we obtain

2A = 90

⇒A = 45

From equation (1), we obtain

45 + B = 60

B = 15

Therefore, ∠A = 45° and ∠B = 15°

Question 4:

State whether the following are true or false. Justify your answer.

(i) sin (A + B) = sin A + sin B

(ii) The value of sinθ increases as θ increases

(iii) The value of cos θ increases as θ increases

(iv) sinθ = cos θ for all values of θ

(v) cot A is not defined for A = 0°

Answer 4:

(i) sin (A + B) = sin A + sin B Let A = 30° and B = 60°

sin (A + B) = sin (30° + 60°)

= sin 90° = 1

And sin A + sin B = sin 30° + sin 60°

$$= \frac{1}{2} + \frac{\sqrt{3}}{2} = \frac{1+\sqrt{3}}{2}$$

Clearly, sin (A + B) ≠ sin A + sin B

Hence, the given statement is false.

(ii) The value of sin θ increases as θ increases in the interval of 0° < θ < 90° as sin 0° = 0

$$\sin 30° = \frac{1}{2} = 0.5$$

$$\sin 45° = \frac{1}{\sqrt{2}} = 0.707$$

$$\sin 60° = \frac{\sqrt{3}}{2} = 0.866$$

sin 90° = 1

Hence, the given statement is true.

(iii) cos 0° = 1

$$\cos 30° = \frac{\sqrt{3}}{2} = 0.866$$

$$\cos 45° = \frac{1}{\sqrt{2}} = 0.707$$

$$\cos 60° = \frac{1}{2} = 0.5$$

cos90° = 0

It can be observed that the value of cos θ does not increase in the interval of 0°<θ<90°.

Hence, the given statement is false.

(iv) sin θ = cos θ for all values of θ.

This is true when θ = 45°

As $\sin 45° = \frac{1}{\sqrt{2}}$

$$\cos 45° = \frac{1}{\sqrt{2}}$$

It is not true for all other values of θ.

As $\sin 30° = \frac{1}{2}$ and $\cos 30° = \frac{\sqrt{3}}{2}$

Hence, the given statement is false.

(v) cot A is not defined for A = 0°

As $\cot A = \frac{\cos A}{\sin A}$,

$\cot 0° = \frac{\cos 0°}{\sin 0°} = \frac{1}{0}$ = undefined

Hence, the given statement is true.

Mathematics

(Chapter – 8) (Introduction to Trigonometry)
(Class X)

Exercise 8.3

Question 1:

Evaluate

(I) $\dfrac{\sin 18°}{\cos 72°}$

(II) $\dfrac{\tan 26°}{\cot 64°}$

(III) cos 48° − sin 42°

(IV) cosec 31° − sec 59°

Answer 1:

(I) $\dfrac{\sin 18°}{\cos 72°} = \dfrac{\sin(90° - 72°)}{\cos 72°} = \dfrac{\cos 72°}{\cos 72°} = 1$

(II) $\dfrac{\tan 26°}{\cot 64°} = \dfrac{\tan(90° - 64°)}{\cot 64°} = \dfrac{\cot 64°}{\cot 64°} = 1$

(III) cos 48° − sin 42° = cos (90°− 42°) − sin 42°

= sin 42° − sin 42°

= 0

(iv) cosec 31° − sec 59° = cosec (90° − 59°) − sec 59°

= sec 59° − sec 59°

= 0

Question 2:

Show that

(I) tan 48° tan 23° tan 42° tan 67° = 1

(II) cos 38° cos 52° − sin 38° sin 52° = 0

Answer 2:

(I) tan 48° tan 23° tan 42° tan 67°

= tan (90° − 42°) tan (90° − 67°) tan 42° tan 67°

$= \cot 42° \cot 67° \tan 42° \tan 67°$

$= (\cot 42° \tan 42°) (\cot 67° \tan 67°)$

$= (1) (1)$

$= 1$

(II) $\cos 38° \cos 52° - \sin 38° \sin 52°$

$= \cos (90° - 52°) \cos (90° - 38°) - \sin 38° \sin 52°$

$= \sin 52° \sin 38° - \sin 38° \sin 52°$

$= 0$

Question 3:

If $\tan 2A = \cot (A - 18°)$, where 2A is an acute angle, find the value of A.

Answer 3:

Given that, $\tan 2A = \cot (A - 18°)$

$\cot (90° - 2A) = \cot (A - 18°)$

$90° - 2A = A - 18°$

$108° = 3A$

$A = 36°$

Question 4:

If $\tan A = \cot B$, prove that $A + B = 90°$

Answer 4:

Given that, $\tan A = \cot B$

$\tan A = \tan (90° - B)$

$A = 90° - B$

$A + B = 90°$

Question 5:

If $\sec 4A = \operatorname{cosec} (A - 20°)$, where 4A is an acute angle, find the value of A.

Answer 5:

Given that, $\sec 4A = \operatorname{cosec} (A - 20°)$

$\operatorname{cosec} (90° - 4A) = \operatorname{cosec} (A - 20°)$

$90° - 4A = A - 20°$

$110° = 5A$

$A = 22°$

Question 6:

If A, Band C are interior angles of a triangle ABC then show that

$$\sin\left(\frac{B+C}{2}\right)=\cos\frac{A}{2}$$

Answer 6:

We know that for a triangle ABC,

$\angle A + \angle B + \angle C = 180°$

$\angle B + \angle C = 180° - \angle A$

$$\frac{\angle B+\angle C}{2}=90°-\frac{\angle A}{2}$$

$$\sin\left(\frac{B+C}{2}\right)=\sin\left(90°-\frac{A}{2}\right)$$

$$=\cos\left(\frac{A}{2}\right)$$

Question 7:

Express sin 67° + cos 75° in terms of trigonometric ratios of angles between 0° and 45°.

Answer 7:

sin 67° + cos 75°

= sin (90° − 23°) + cos (90° − 15°)

= cos 23° + sin 15°

Mathematics

(Chapter – 8) (Introduction to Trigonometry)
(Class X)

Exercise 8.4

Question 1:

Express the trigonometric ratios sin A, sec A and tan A in terms of cot A.

Answer 1:

We know that,

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\frac{1}{\operatorname{cosec}^2 A} = \frac{1}{1+\cot^2 A}$$

$$\sin^2 A = \frac{1}{1+\cot^2 A}$$

$$\sin A = \pm \frac{1}{\sqrt{1+\cot^2 A}}$$

Therefore, $\sin A = \frac{1}{\sqrt{1+\cot^2 A}}$

We know that, $\tan A = \frac{\sin A}{\cos A}$

However, $\cot A = \frac{\cos A}{\sin A}$

Therefore, $\tan A = \frac{1}{\cot A}$

Also, $\sec^2 A = 1 + \tan^2 A$

$$= 1 + \frac{1}{\cot^2 A}$$

$$= \frac{\cot^2 A + 1}{\cot^2 A}$$

$$\sec A = \frac{\sqrt{\cot^2 A + 1}}{\cot A}$$

Question 2:

Write all the other trigonometric ratios of ∠A in terms of sec A.

Answer 2:

We know that,

$$\cos A = \frac{1}{\sec A}$$

Also, $\sin^2 A + \cos^2 A = 1$

$\sin^2 A = 1 - \cos^2 A$

$$\sin A = \sqrt{1-\left(\frac{1}{\sec A}\right)^2}$$

$$= \sqrt{\frac{\sec^2 A - 1}{\sec^2 A}} = \frac{\sqrt{\sec^2 A - 1}}{\sec A}$$

$\tan^2 A + 1 = \sec^2 A$

$\tan^2 A = \sec^2 A - 1$

$$\tan A = \sqrt{\sec^2 A - 1}$$

$$\cot A = \frac{\cos A}{\sin A} = \frac{\frac{1}{\sec A}}{\frac{\sqrt{\sec^2 A - 1}}{\sec A}}$$

$$= \frac{1}{\sqrt{\sec^2 A - 1}}$$

$$\operatorname{cosec} A = \frac{1}{\sin A} = \frac{\sec A}{\sqrt{\sec^2 A - 1}}$$

Question 3:

Evaluate

(i) $\dfrac{\sin^2 63^\circ + \sin^2 27^\circ}{\cos^2 17^\circ + \cos^2 73^\circ}$

(ii) $\sin 25^\circ \cos 65^\circ + \cos 25^\circ \sin 65^\circ$

Answer 3:

(i) $$\frac{\sin^2 63^\circ + \sin^2 27^\circ}{\cos^2 17^\circ + \cos^2 73^\circ} = \frac{\left[\sin\left(90^\circ - 27^\circ\right)\right]^2 + \sin^2 27^\circ}{\left[\cos\left(90^\circ - 73^\circ\right)\right]^2 + \cos^2 73^\circ}$$

$$= \frac{\left[\cos 27^\circ\right]^2 + \sin^2 27^\circ}{\left[\sin 73^\circ\right]^2 + \cos^2 73^\circ}$$

$$= \frac{\cos^2 27° + \sin^2 27°}{\sin^2 73° + \cos^2 73°}$$

$$= \frac{1}{1}$$

= 1

(ii) sin25° cos65° + cos25° sin65°

$$= (\sin 25°)\{\cos(90° - 25°)\} + \cos 25°\{\sin(90° - 25°)\}$$

$$= (\sin 25°)(\sin 25°) + (\cos 25°)(\cos 25°)$$

= $\sin^2 25° + \cos^2 25°$

= 1 (As $\sin^2 A + \cos^2 A = 1$)

Question 4:

Choose the correct option. Justify your choice.

(i) 9 $\sec^2$ A − 9 $\tan^2$ A =

(A) 1	(B) 9	(C) 8	(D) 0

(ii) (1 + tan θ + sec θ) (1 + cot θ − cosec θ)

(A) 0	(B)1	(C) 2	(D) −1

(iii) (secA + tanA) (1 − sinA) =

(A) secA	(B) sinA	(C) cosecA	(D) cosA

(iv) $\frac{1 + \tan^2 A}{1 + \cot^2 A}$

(A) $\sec^2$ A	(B) −1	(C) $\cot^2$ A	(D) $\tan^2$ A

Answer 4:

(i) 9 $\sec^2$A − 9 $\tan^2$A

= 9 ($\sec^2$A − $\tan^2$A)

= 9 (1) [As $\sec^2$ A − $\tan^2$ A = 1]

= 9

Hence, alternative (B) is correct.

(ii) (1 + tan θ + sec θ) (1 + cot θ − cosec θ)

$$=\left(1+\frac{\sin\theta}{\cos\theta}+\frac{1}{\cos\theta}\right)\left(1+\frac{\cos\theta}{\sin\theta}-\frac{1}{\sin\theta}\right)$$

$$=\left(\frac{\cos\theta+\sin\theta+1}{\cos\theta}\right)\left(\frac{\sin\theta+\cos\ \theta-1}{\sin\theta}\right)$$

$$=\frac{(\sin\theta+\cos\ \theta)^2-(1)^2}{\sin\theta\cos\theta}$$

$$=\frac{\sin^2\theta+\cos^2\theta+2\sin\theta\ \cos\ \theta-1}{\sin\theta\cos\ \theta}$$

$$=\frac{1+2\sin\theta\cos\ \theta-1}{\sin\theta\cos\ \theta}$$

$$=\frac{2\sin\theta\ \cos\ \theta}{\sin\ \theta\cos\ \theta}=2$$

Hence, alternative (C) is correct.

(iii) (secA + tanA) (1 − sinA)

$$=\left(\frac{1}{\cos A}+\frac{\sin A}{\cos A}\right)(1-\sin A)$$

$$=\left(\frac{1+\sin A}{\cos A}\right)(1-\sin A)$$

$$=\frac{1-\sin^2 A}{\cos A}=\frac{\cos^2 A}{\cos A}$$

= cosA

Hence, alternative (D) is correct.

(iv) $$\frac{1+\tan^2 A}{1+\cot^2 A}=\frac{1+\frac{\sin^2 A}{\cos^2 A}}{1+\frac{\cos^2 A}{\sin^2 A}}=\frac{\frac{\cos^2 A+\sin^2 A}{\cos^2 A}}{\frac{\sin^2 A+\cos^2 A}{\sin^2 A}}=\frac{\frac{1}{\cos^2 A}}{\frac{1}{\sin^2 A}}$$

$$=\frac{\sin^2 A}{\cos^2 A}=\tan^2 A$$

Hence, alternative (D) is correct.

Question 5:

Prove the following identities, where the angles involved are acute angles for which the expressions are defined.

Answer 5:

(i) $(\text{cosec}\theta - \cot\theta)^2 = \frac{1-\cos\theta}{1+\cos\theta}$

$$\text{L.H.S.} = (\text{cosec}\,\theta - \cot\theta)^2$$

$$= \left(\frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta}\right)^2$$

$$= \frac{(1-\cos\theta)^2}{(\sin\theta)^2} = \frac{(1-\cos\theta)^2}{\sin^2\theta}$$

$$= \frac{(1-\cos\theta)^2}{1-\cos^2\theta} = \frac{(1-\cos\theta)^2}{(1-\cos\theta)(1+\cos\theta)} = \frac{1-\cos\theta}{1+\cos\theta}$$

$$= \text{R.H.S.}$$

(ii) $\frac{\cos A}{1+\sin A} + \frac{1+\sin A}{\cos A} = 2\sec A$

$$\text{L.H.S.} = \frac{\cos A}{1+\sin A} + \frac{1+\sin A}{\cos A}$$

$$= \frac{\cos^2 A + (1+\sin A)^2}{(1+\sin A)(\cos A)}$$

$$= \frac{\cos^2 A + 1 + \sin^2 A + 2\sin A}{(1+\sin A)(\cos A)}$$

$$= \frac{\sin^2 A + \cos^2 A + 1 + 2\sin A}{(1+\sin A)(\cos A)}$$

$$= \frac{1+1+2\sin A}{(1+\sin A)(\cos A)} = \frac{2+2\sin A}{(1+\sin A)(\cos A)}$$

$$= \frac{2(1+\sin A)}{(1+\sin A)(\cos A)} = \frac{2}{\cos A} = 2\sec A$$

$$= \text{R.H.S.}$$

(iii) $\frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta} = 1 + \sec\theta\,\text{cosec}\theta$

$$\text{L.H.S.} = \frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta}$$

$$= \frac{\frac{\sin\theta}{\cos\theta}}{1-\frac{\cos\theta}{\sin\theta}} + \frac{\frac{\cos\theta}{\sin\theta}}{1-\frac{\sin\theta}{\cos\theta}}$$

$$= \frac{\frac{\sin\theta}{\cos\theta}}{\frac{\sin\theta-\cos\theta}{\sin\theta}} + \frac{\frac{\cos\theta}{\sin\theta}}{\frac{\cos\theta-\sin\theta}{\cos\theta}}$$

$$= \frac{\sin^2\theta}{\cos\theta(\sin\theta-\cos\theta)} + \frac{\cos^2\theta}{\sin\theta(\sin\theta-\cos\theta)}$$

$$= \frac{1}{(\sin\theta-\cos\theta)}\left[\frac{\sin^2\theta}{\cos\theta} - \frac{\cos^2\theta}{\sin\theta}\right]$$

$$= \left(\frac{1}{\sin\theta-\cos\theta}\right)\left[\frac{\sin^3\theta-\cos^3\theta}{\sin\theta\cos\theta}\right]$$

$$= \left(\frac{1}{\sin\theta-\cos\theta}\right)\left[\frac{(\sin\theta-\cos\theta)(\sin^2\theta+\cos^2\theta+\sin\theta\cos\theta)}{\sin\theta\cos\theta}\right]$$

$$= \frac{(1+\sin\theta\ \cos\theta)}{(\sin\theta\ \cos\theta)}$$

= secθ cosec θ + 1 = R.H.S.

(iv) $\frac{1+\sec A}{\sec A} = \frac{\sin^2 A}{1-\cos A}$

$$\text{L.H.S.} = \frac{1+\sec A}{\sec A} = \frac{1+\frac{1}{\cos A}}{\frac{1}{\cos A}}$$

$$= \frac{\frac{\cos A+1}{\cos A}}{\frac{1}{\cos A}} = (\cos A+1)$$

$$= \frac{(1-\cos A)(1+\cos A)}{(1-\cos A)}$$

$$= \frac{1-\cos^2 A}{1-\cos A} = \frac{\sin^2 A}{1-\cos A} \quad = \text{R.H.S}$$

(v) $\dfrac{\cos A - \sin A + 1}{\cos A + \sin A - 1} = \operatorname{cosec} A + \cot A$

Using the identity cosec $^2 A$ = 1 + cot $^2 A$

L.H.S = $\dfrac{\cos A - \sin A + 1}{\cos A + \sin A - 1}$

$$= \dfrac{\dfrac{\cos A}{\sin A} - \dfrac{\sin A}{\sin A} + \dfrac{1}{\sin A}}{\dfrac{\cos A}{\sin A} + \dfrac{\sin A}{\sin A} + \dfrac{1}{\sin A}}$$

$$= \dfrac{\cot A - 1 + \operatorname{cosec} A}{\cot A + 1 - \operatorname{cosec} A}$$

$$= \dfrac{\{(\cot A) - (1 - \operatorname{cosec} A)\}\{(\cot A) - (1 - \operatorname{cosec} A)\}}{\{(\cot A) + (1 - \operatorname{cosec} A)\}\{(\cot A) - (1 - \operatorname{cosec} A)\}}$$

$$= \dfrac{(\cot A - 1 + \operatorname{cosec} A)^2}{(\cot A)^2 - (1 - \operatorname{cosec} A)^2}$$

$$= \dfrac{\cot^2 A + 1 + \operatorname{cosec}^2 A - 2\cot A - 2\operatorname{cosec} A + 2\cot A \operatorname{cosec} A}{\cot^2 A - (1 + \operatorname{cosec}^2 A - 2\operatorname{cosec} A)}$$

$$= \dfrac{2\operatorname{cosec}^2 A + 2\cot A \operatorname{cosec} A - 2\cot A - 2\operatorname{cosec} A}{\cot^2 A - 1 - \operatorname{cosec}^2 A + 2\operatorname{cosec} A}$$

$$= \dfrac{2\operatorname{cosec} A(\operatorname{cosec} A + \cot A) - 2(\cot A + \operatorname{cosec} A)}{\cot^2 A - \operatorname{cosec}^2 A - 1 + 2\operatorname{cosec} A}$$

$$= \dfrac{(\operatorname{cosec} A + \cot A)(2\operatorname{cosec} A - 2)}{-1 - 1 + 2\operatorname{cosec} A}$$

$$= \dfrac{(\operatorname{cosec} A + \cot A)(2\operatorname{cosec} A - 2)}{(2\operatorname{cosec} A - 2)}$$

= cosec A + cot A

= R.H.S

(vi) $\sqrt{\dfrac{1 + \sin A}{1 - \sin A}} = \sec A + \tan A$

$$\text{L.H.S.} = \sqrt{\frac{1+\sin A}{1-\sin A}}$$

$$= \sqrt{\frac{(1+\sin A)(1+\sin A)}{(1-\sin A)(1+\sin A)}}$$

$$= \frac{(1+\sin A)}{\sqrt{1-\sin^2 A}} \qquad = \frac{1+\sin A}{\sqrt{\cos^2 A}}$$

$$= \frac{1+\sin A}{\cos A} \qquad = \sec A + \tan A$$

$$= \text{R.H.S.}$$

(vii) $\dfrac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta} = \tan\theta$

$$\text{L.H.S.} = \frac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta}$$

$$= \frac{\sin\theta\left(1-2\sin^2\theta\right)}{\cos\theta\left(2\cos^2\theta - 1\right)}$$

$$= \frac{\sin\theta \times \left(1-2\sin^2\theta\right)}{\cos\theta \times \left\{2\left(1-\sin^2\theta\right)-1\right\}}$$

$$= \frac{\sin\theta \times \left(1-2\sin^2\theta\right)}{\cos\theta \times \left(1-2\sin^2\theta\right)}$$

$$= \tan\theta = \text{R.H.S}$$

(viii) $(\sin A + \operatorname{cosec} A)^2 + (\cos A + \sec A)^2 = 7 + \tan^2 A + \cot^2 A$

$$\text{L.H.S} = (\sin A + \operatorname{cosec} A)^2 + (\cos A + \sec A)^2$$

$$= \sin^2 A + \operatorname{cosec}^2 A + 2\sin A \operatorname{cosec} A + \cos^2 A + \sec^2 A + 2\cos A \sec A$$

$$= \left(\sin^2 A + \cos^2 A\right) + \left(\operatorname{cosec}^2 A + \sec^2 A\right) + 2\sin A\left(\frac{1}{\sin A}\right) + 2\cos A\left(\frac{1}{\cos A}\right)$$

$$= (1) + \left(1 + \cot^2 A + 1 + \tan^2 A\right) + (2) + (2)$$

$$= 7 + \tan^2 A + \cot^2 A$$

$$= \text{R.H.S}$$

(ix) $\left(\operatorname{cosec}A-\sin A\right)\left(\sec A-\cos A\right)=\dfrac{1}{\tan A+\cot A}$

$$\begin{aligned}\text{L.H.S} &= \left(\operatorname{cosec}A-\sin A\right)\left(\sec A-\cos A\right)\\ &=\left(\frac{1}{\sin A}-\sin A\right)\left(\frac{1}{\cos A}-\cos A\right)\\ &=\left(\frac{1-\sin^2 A}{\sin A}\right)\left(\frac{1-\cos^2 A}{\cos A}\right)\\ &=\frac{\left(\cos^2 A\right)\left(\sin^2 A\right)}{\sin A\cos A}\\ &=\sin A\cos A\end{aligned}$$

$$\begin{aligned}\text{R.H.S} &=\frac{1}{\tan A+\cot A}\\ &=\frac{1}{\frac{\sin A}{\cos A}+\frac{\cos A}{\sin A}}=\frac{1}{\frac{\sin^2 A+\cos^2 A}{\sin A\cos A}}\\ &=\frac{\sin A\cos A}{\sin^2 A+\cos^2 A}=\sin A\cos A\end{aligned}$$

Hence, L.H.S = R.H.S

(x) $\left(\dfrac{1+\tan^2 A}{1+\cot^2 A}\right)=\left(\dfrac{1-\tan A}{1-\cot A}\right)^2=\tan^2 A$

$$\begin{aligned}\frac{1+\tan^2 A}{1+\cot^2 A} &=\frac{1+\frac{\sin^2 A}{\cos^2 A}}{1+\frac{\cos^2 A}{\sin^2 A}}=\frac{\frac{\cos^2 A+\sin^2 A}{\cos^2 A}}{\frac{\sin^2 A+\cos^2 A}{\sin^2 A}}\\ &=\frac{\frac{1}{\cos^2 A}}{\frac{1}{\sin^2 A}}=\frac{\sin^2 A}{\cos^2 A}\\ &=\tan^2 A\end{aligned}$$

$$\left(\frac{1-\tan A}{1-\cot A}\right)^2 = \frac{1+\tan^2 A - 2\tan A}{1+\cot^2 A - 2\cot A}$$

$$= \frac{\sec^2 A - 2\tan A}{\operatorname{cosec}^2 A - 2\cot A}$$

$$= \frac{\dfrac{1}{\cos^2 A} - \dfrac{2\sin A}{\cos A}}{\dfrac{1}{\sin^2 A} - \dfrac{2\cos A}{\sin A}} = \frac{\dfrac{1-2\sin A\cos A}{\cos^2 A}}{\dfrac{1-2\sin A\cos A}{\sin^2 A}}$$

$$= \frac{\sin^2 A}{\cos^2 A} = \tan^2 A$$

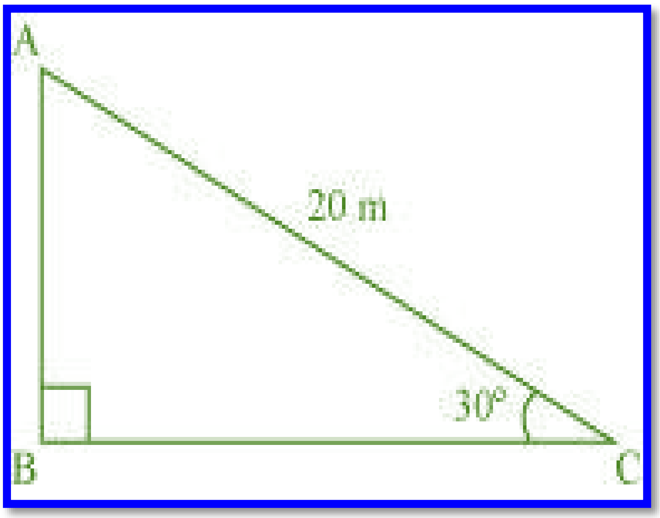
Mathematics

(Chapter – 9) (Some Applications of Trigonometry)

(Class – X)

Exercise 9.1

Question 1:
A circus artist is climbing a 20 m long rope, which is tightly stretched and tied from the top of a vertical pole to the ground. Find the height of the pole, if the angle made by the rope with the ground level is 30 °.



Answer 1:
It can be observed from the figure that AB is the pole.

In ΔABC,

$$\frac{AB}{AC} = \sin 30°$$

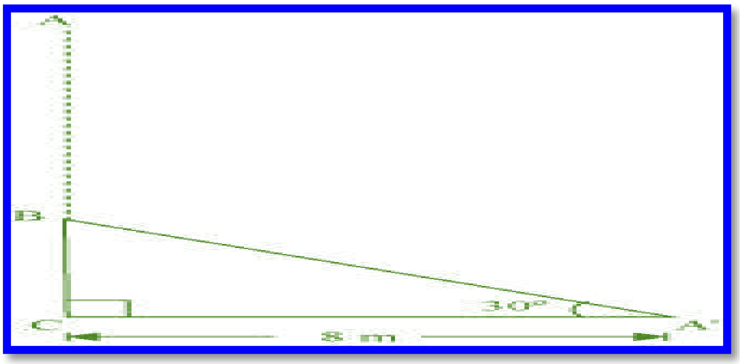
$$\frac{AB}{20} = \frac{1}{2}$$

$$AB = \frac{20}{2} = 10$$

Therefore, the height of the pole is 10 m.

Question 2:
A tree breaks due to storm and the broken part bends so that the top of the tree touches the ground making an angle 30 ° with it. The distance between the foot of the tree to the point where the top touches the ground is 8 m. Find the height of the tree.

Answer 2:



Let AC was the original tree. Due to storm, it was broken into two parts. The broken part $A'B$ is making 30° with the ground.

In $\Delta A'BC$,

$$\frac{BC}{A'C} = \tan 30°$$

$$\frac{BC}{8} = \frac{1}{\sqrt{3}}$$

$$BC = \left(\frac{8}{\sqrt{3}}\right) m$$

$$\frac{A'C}{A'B} = \cos 30°$$

$$\frac{8}{A'B} = \frac{\sqrt{3}}{2}$$

$$A'B = \left(\frac{16}{\sqrt{3}}\right) m$$

Height of the tree = A'B + BC

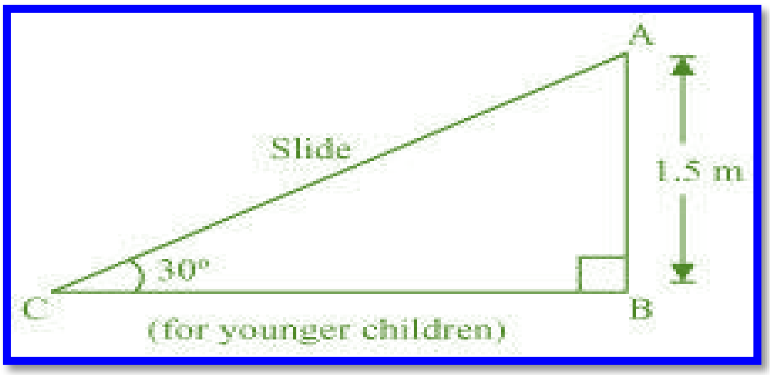
$$= \left(\frac{16}{\sqrt{3}} + \frac{8}{\sqrt{3}}\right) m = \frac{24}{\sqrt{3}} \ m$$

$$= 8\sqrt{3} \ m$$

Hence, the height of the tree is $8\sqrt{3}$ m.

Question 3:
A contractor plans to install two slides for the children to play in a park. For the children below the age of 5 years, she prefers to have a slide whose top is at a height of 1.5 m, and is inclined at an angle of 30 ° to the ground, where as for the elder children she wants to have a steep side at a height of 3 m, and inclined at an angle of 60 ° to the ground. What should be the length of the slide in each case?

Answer 3:
It can be observed that AC and PR are the slides for younger and elder children respectively.

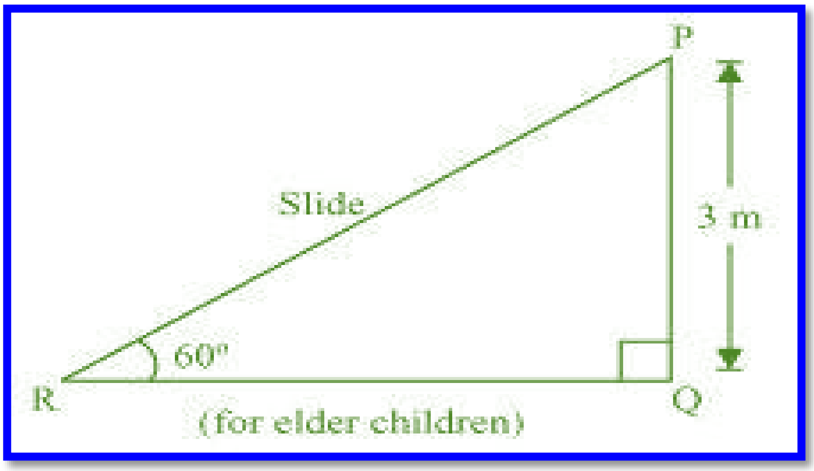


In ΔABC,

$$\frac{AB}{AC} = \sin 30^\circ$$

$$\frac{1.5}{AC} = \frac{1}{2}$$

$$AC = 3 \text{ m}$$



In ΔPQR,

$$\frac{PQ}{PR} = \sin 60$$

$$\frac{3}{PR} = \frac{\sqrt{3}}{2}$$

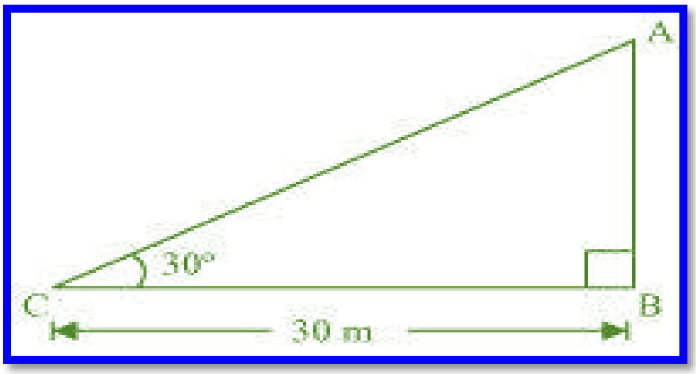
$$PR = \frac{6}{\sqrt{3}} = 2\sqrt{3} \text{ m}$$

Therefore, the lengths of these slides are 3 m and $2\sqrt{3}$ m.

Question 4:

The angle of elevation of the top of a tower from a point on the ground, which is 30 m away from the foot of the tower is 30°. Find the height of the tower.

Answer 4:



Let AB be the tower and the angle of elevation from point C (on ground) is 30°.
In ΔABC,

$$\frac{AB}{BC} = \tan 30°$$

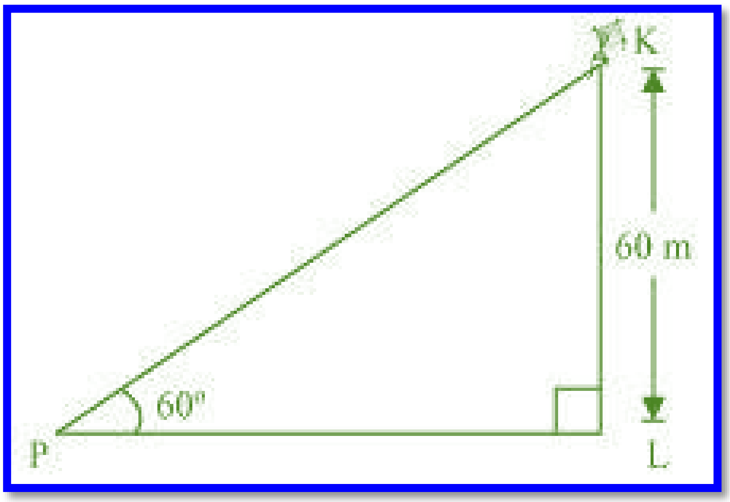
$$\frac{AB}{30} = \frac{1}{\sqrt{3}}$$

$$AB = \frac{30}{\sqrt{3}} = 10\sqrt{3} \text{ m}$$

Therefore, the height of the tower is $10\sqrt{3}$ m.

Question 5:
A kite is flying at a height of 60 m above the ground. The string attached to the kite is temporarily tied to a point on the ground. The inclination of the string with the ground is 60°. Find the length of the string, assuming that there is no slack in the string.

Answer 5:



Let K be the kite and the string is tied to point P on the ground.

In ΔKLP,

$$\frac{KL}{KP} = \sin 60°$$

$$\frac{60}{KP} = \frac{\sqrt{3}}{2}$$

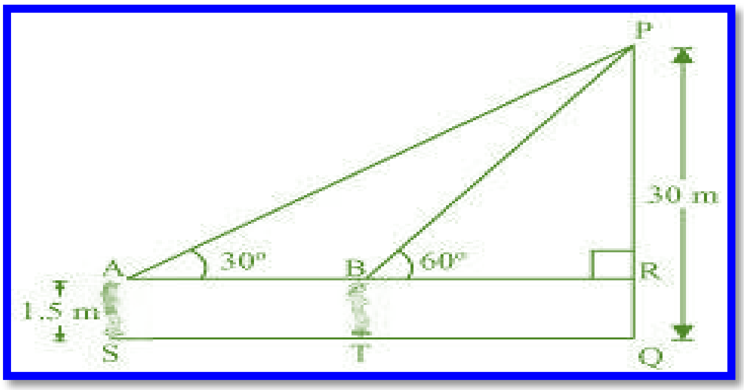
$$KP = \frac{120}{\sqrt{3}} = 40\sqrt{3} \text{ m}$$

Hence, the length of the string is $40\sqrt{3}$ m.

Question 6:

A 1.5 m tall boy is standing at some distance from a 30 m tall building. The angle of elevation from his eyes to the top of the building increases from 30° to 60° as he walks towards the building. Find the distance he walked towards the building.

Answer 6:



Let the boy was standing at point S initially. He walked towards the building and reached at point T. It can be observed that

PR = PQ − RQ

= (30 − 1.5) m = 28.5 m = $\frac{57}{2}$ m

In ΔPAR,

$$\frac{PR}{AR} = \tan 30°$$

$$\frac{57}{2AR} = \frac{1}{\sqrt{3}}$$

$$AR = \left(\frac{57}{2}\sqrt{3}\right) m$$

In ΔPRB,

$$\frac{PR}{BR} = \tan 60°$$

$$\frac{57}{2\,BR} = \sqrt{3}$$

$$BR = \frac{57}{2\sqrt{3}} = \left(\frac{19\sqrt{3}}{2}\right) m$$

ST = AB

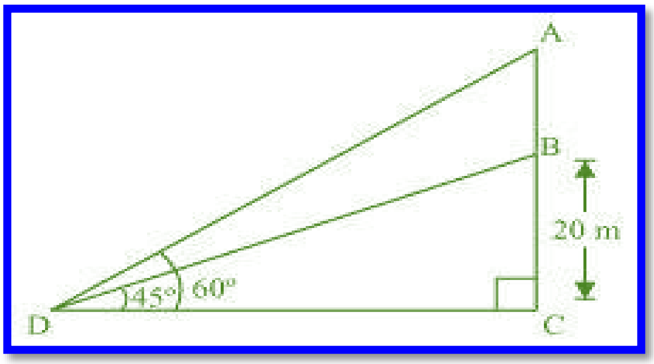
$$= AR - BR = \left(\frac{57\sqrt{3}}{2} - \frac{19\sqrt{3}}{2}\right) m$$

$$= \left(\frac{38\sqrt{3}}{2}\right) m = 19\sqrt{3} \text{ m}$$

Hence, he walked $19\sqrt{3}$ m towards the building.

Question 7:
From a point on the ground, the angles of elevation of the bottom and the top of a transmission tower fixed at the top of a 20 m high building are 45° and 60° respectively. Find the height of the tower.

Answer 7:



Let BC be the building, AB be the transmission tower, and D be the point on the ground from where the elevation angles are to be measured.
In ΔBCD,

$$\frac{BC}{CD} = \tan 45°$$

$$\frac{20}{CD} = 1$$

$$CD = 20 \text{ m}$$

In ΔACD,

$$\frac{AC}{CD} = \tan 60°$$

$$\frac{AB + BC}{CD} = \sqrt{3}$$

$$\frac{AB + 20}{20} = \sqrt{3}$$

$$AB = \left(20\sqrt{3} - 20\right)\text{m}$$

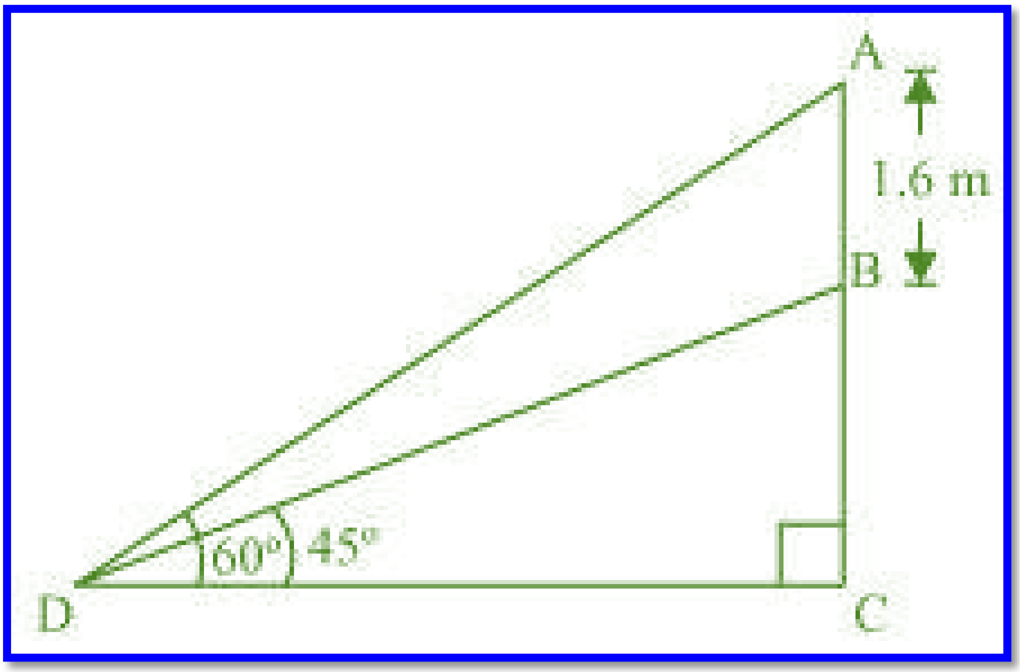
$$= 20\left(\sqrt{3} - 1\right)\text{m}$$

Therefore, the height of the transmission tower is $20(\sqrt{3} - 1)$ m.

Question 8:

A statue, 1.6 m tall, stands on a top of pedestal, from a point on the ground, the angle of elevation of the top of statue is 60° and from the same point the angle of elevation of the top of the pedestal is 45 °. Find the height of the pedestal.

Answer 8:



Let AB be the statue, BC be the pedestal, and D be the point on the ground from where the elevation angles are to be measured.

In ΔBCD,

$$\frac{BC}{CD} = \tan 45^\circ$$
$$\frac{BC}{CD} = 1$$
$$BC = CD$$

In ΔACD,

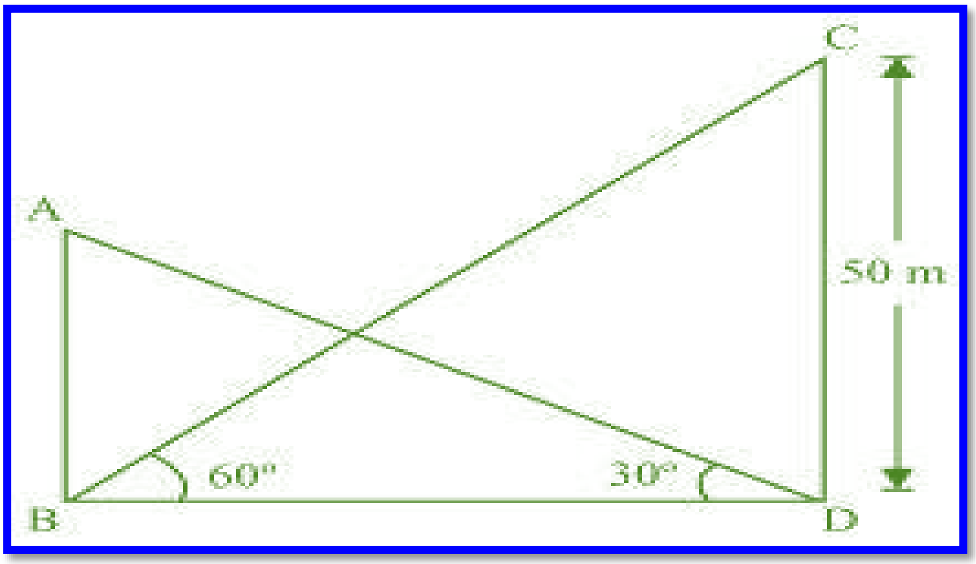
$$\frac{AB + BC}{CD} = \tan 60^\circ$$
$$\frac{AB + BC}{BC} = \sqrt{3}$$
$$1.6 + BC = BC\sqrt{3}$$
$$BC\left(\sqrt{3} - 1\right) = 1.6$$
$$BC = \frac{(1.6)\left(\sqrt{3}+1\right)}{\left(\sqrt{3}-1\right)\left(\sqrt{3}+1\right)}$$
$$= \frac{1.6\left(\sqrt{3}+1\right)}{\left(\sqrt{3}\right)^2 - (1)^2}$$
$$= \frac{1.6\left(\sqrt{3}+1\right)}{2} = 0.8\left(\sqrt{3}+1\right)$$

Therefore, the height of the pedestal is $0.8(\sqrt{3} + 1)$ m.

Question 9:

The angle of elevation of the top of a building from the foot of the tower is 30° and the angle of elevation of the top of the tower from the foot of the building is 60°. If the tower is 50 m high, find the height of the building.

Answer 9:



Let AB be the building and CD be the tower.
In ΔCDB,

$$\frac{CD}{BD} = \tan 60^\circ$$

$$\frac{50}{BD} = \sqrt{3}$$

$$BD = \frac{50}{\sqrt{3}}$$

In ΔABD,

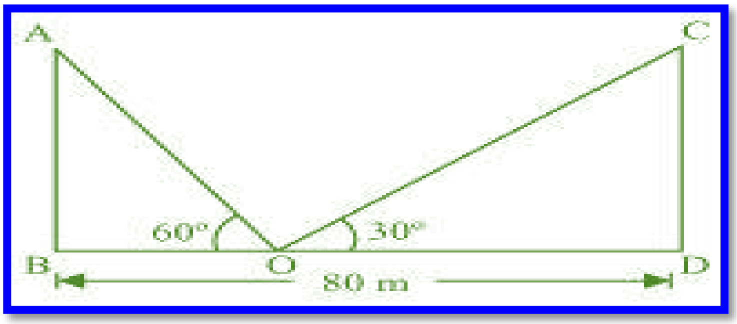
$$\frac{AB}{BD} = \tan 30^\circ$$

$$AB = \frac{50}{\sqrt{3}} \times \frac{1}{\sqrt{3}} = \frac{50}{3} = 16\frac{2}{3}$$

Therefore, the height of the building is $16\frac{2}{3}$ m.

Question 10:
Two poles of equal heights are standing opposite each other and either side of the road, which is 80 m wide. From a point between them on the road, the angles of elevation of the top of the poles are 60° and 30º, respectively. Find the height of poles and the distance of the point from the poles.

Answer 10:



Let AB and CD be the poles and O is the point from where the elevation angles are measured.

In ΔABO,

$$\frac{AB}{BO} = \tan 60°$$

$$\frac{AB}{BO} = \sqrt{3}$$

$$BO = \frac{AB}{\sqrt{3}}$$

In ΔCDO,

$$\frac{CD}{DO} = \tan 30°$$

$$\frac{CD}{80 - BO} = \frac{1}{\sqrt{3}}$$

$$CD\sqrt{3} = 80 - BO$$

$$CD\sqrt{3} = 80 - \frac{AB}{\sqrt{3}}$$

$$CD\sqrt{3} + \frac{AB}{\sqrt{3}} = 80$$

Since the poles are of equal heights,

CD = AB

$$CD\left[\sqrt{3}+\frac{1}{\sqrt{3}}\right]=80$$

$$CD\left(\frac{3+1}{\sqrt{3}}\right)=80$$

$$CD=20\sqrt{3}\text{ m}$$

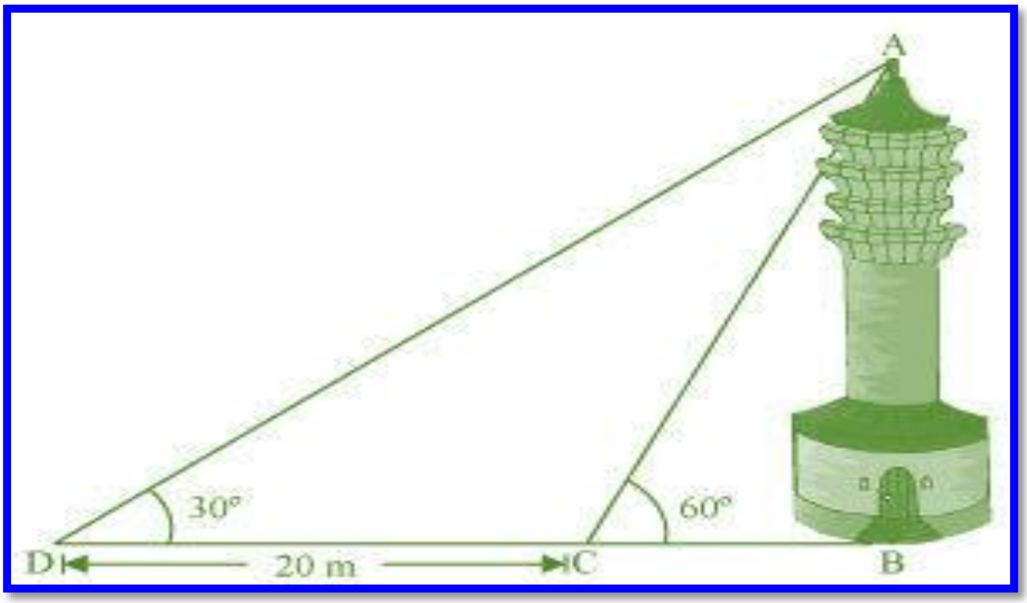
$$BO=\frac{AB}{\sqrt{3}}=\frac{CD}{\sqrt{3}}=\left(\frac{20\sqrt{3}}{\sqrt{3}}\right)\text{m}=20\text{ m}$$

DO = BD − BO = (80 − 20) m = 60 m

Therefore, the height of poles is $20\sqrt{3}$ m and the point is 20 m and 60 m far from these poles.

Question 11:

A TV tower stands vertically on a bank of a canal. From a point on the other bank directly opposite the tower the angle of elevation of the top of the tower is 60°. From another point 20 m away from this point on the line joining this point to the foot of the tower, the angle of elevation of the top of the tower is 30°. Find the height of the tower and the width of the canal.



Answer 11:

In ΔABC,

$$\frac{AB}{BC} = \tan 60°$$

$$\frac{AB}{BC} = \sqrt{3}$$

$$BC = \frac{AB}{\sqrt{3}}$$

In ΔABD,

$$\frac{AB}{BD} = \tan 30°$$

$$\frac{AB}{BC + CD} = \frac{1}{\sqrt{3}}$$

$$\frac{AB}{\frac{AB}{\sqrt{3}} + 20} = \frac{1}{\sqrt{3}}$$

$$\frac{AB\sqrt{3}}{AB + 20\sqrt{3}} = \frac{1}{\sqrt{3}}$$

$$3AB = AB + 20\sqrt{3}$$

$$2AB = 20\sqrt{3}$$

$$AB = 10\sqrt{3} \text{ m}$$

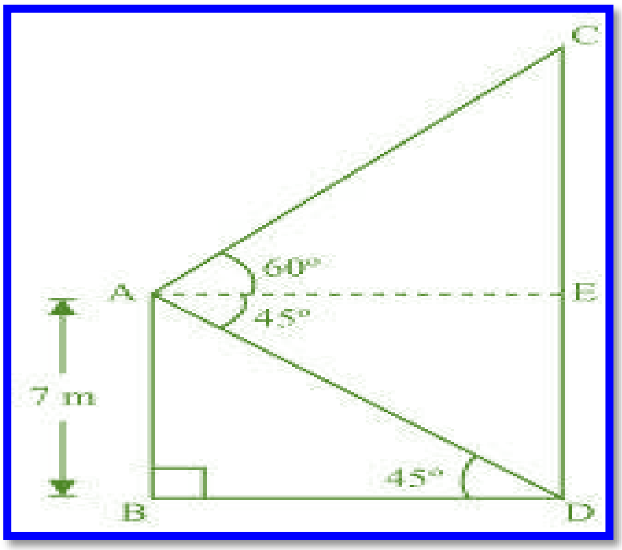
$$BC = \frac{AB}{\sqrt{3}} = \left(\frac{10\sqrt{3}}{\sqrt{3}}\right) \text{m} = 10 \text{ m}$$

Therefore, the height of the tower is $10\sqrt{3}$ m and the width of the canal is 10 m.

Question 12:

From the top of a 7 m high building, the angle of elevation of the top of a cable tower is 60° and the angle of depression of its foot is 45°. Determine the height of the tower.

Answer 12:



Let AB be a building and CD be a cable tower.

In ΔABD,

$$\frac{AB}{BD} = \tan 45°$$

$$\frac{7}{BD} = 1$$

$$BD = 7 \text{ m}$$

In ΔACE,

AC = BD = 7 m

$$\frac{CE}{AE} = \tan 60°$$

$$\frac{CE}{7} = \sqrt{3}$$

$$CE = 7\sqrt{3} \text{ m}$$

$$CD = CE + ED = \left(7\sqrt{3} + 7\right)\text{m}$$

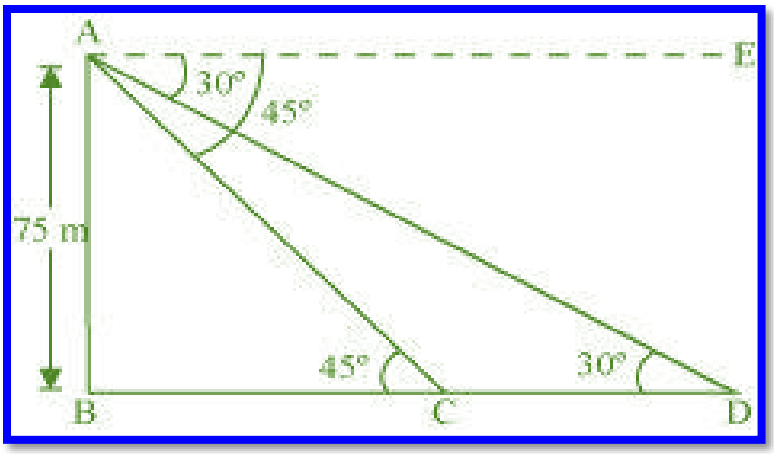
$$= 7\left(\sqrt{3} + 1\right)\text{m}$$

Therefore, the height of the cable tower is $7\left(\sqrt{3}+1\right)$ m.

Question 13:

As observed from the top of a 75 m high lighthouse from the sea-level, the angles of depression of two ships are 30° and 45°. If one ship is exactly behind the other on the same side of the lighthouse, find the distance between the two ships.

Answer 13:



Let AB be the lighthouse and the two ships be at point C and D respectively.

In ΔABC,

$$\frac{AB}{BC} = \tan 45°$$

$$\frac{75}{BC} = 1$$

$$BC = 75 \text{ m}$$

In ΔABD,

$$\frac{AB}{BD} = \tan 30°$$

$$\frac{75}{BC + CD} = \frac{1}{\sqrt{3}}$$

$$\frac{75}{75 + CD} = \frac{1}{\sqrt{3}}$$

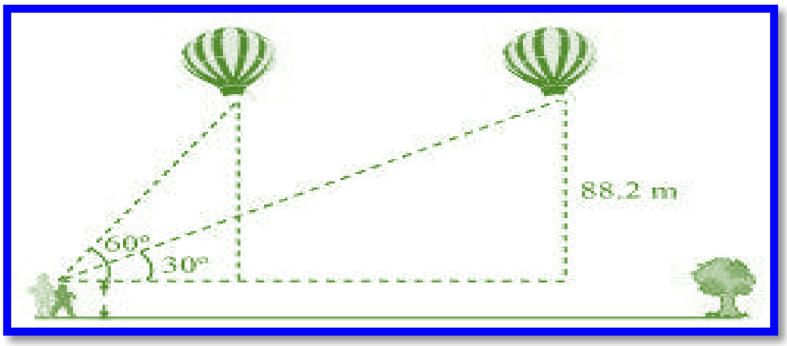
$$75\sqrt{3} = 75 + CD$$

$$75\left(\sqrt{3} - 1\right)\text{m} = CD$$

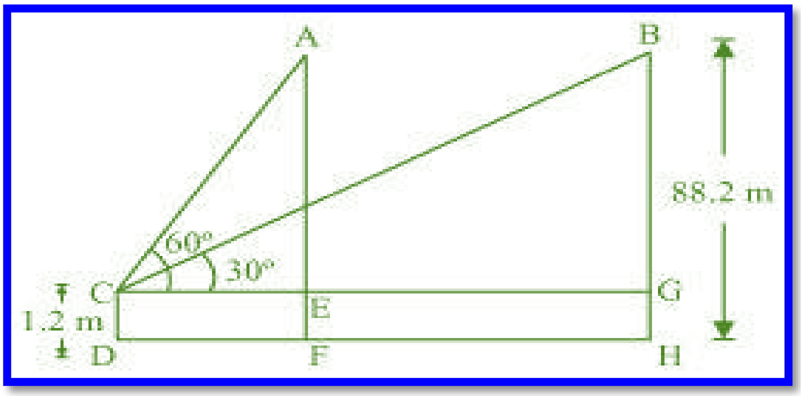
Therefore, the distance between the two ships is $75(\sqrt{3} - 1)$ m.

Question 14:

A 1.2 m tall girl spots a balloon moving with the wind in a horizontal line at a height of 88.2 m from the ground. The angle of elevation of the balloon from the eyes of the girl at any instant is 60°. After some time, the angle of elevation reduces to 30°. Find the distance travelled by the balloon during the interval.



Answer 14:



Let the initial position A of balloon change to B after some time and CD be the girl.

In ΔACE,

$$\frac{AE}{CE} = \tan 60°$$

$$\frac{AF - EF}{CE} = \tan 60°$$

$$\frac{88.2 - 1.2}{CE} = \sqrt{3}$$

$$\frac{87}{CE} = \sqrt{3}$$

$$CE = \frac{87}{\sqrt{3}} = 29\sqrt{3} \text{ m}$$

In ΔBCG,

$$\frac{BG}{CG} = \tan 30°$$

$$\frac{88.2 - 1.2}{CG} = \frac{1}{\sqrt{3}}$$

$$87\sqrt{3} \text{ m} = CG$$

Distance travelled by balloon = EG = CG − CE

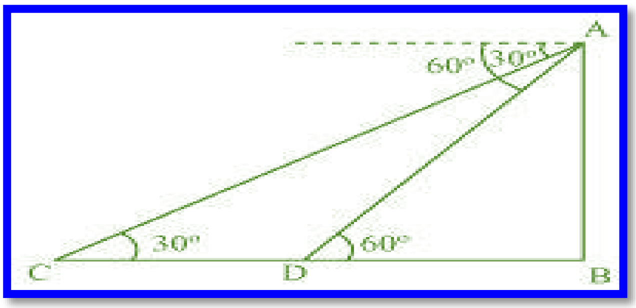
$$= \left(87\sqrt{3} - 29\sqrt{3}\right)\text{m}$$

$$= 58\sqrt{3} \text{ m}$$

Question 15:

A straight highway leads to the foot of a tower. A man standing at the top of the tower observes a car as an angle of depression of 30°, which is approaching the foot of the tower with a uniform speed. Six seconds later, the angle of depression of the car is found to be 60°. Find the time taken by the car to reach the foot of the tower from this point.

Answer 15:



Let AB be the tower.

Initial position of the car is C, which changes to D after six seconds.

In ΔADB,

$$\frac{AB}{DB} = \tan 60^\circ$$

$$\frac{AB}{DB} = \sqrt{3}$$

$$DB = \frac{AB}{\sqrt{3}}$$

In ΔABC,

$$\frac{AB}{BC} = \tan 30^\circ$$

$$\frac{AB}{BD + DC} = \frac{1}{\sqrt{3}}$$

$$AB\sqrt{3} = BD + DC$$

$$AB\sqrt{3} = \frac{AB}{\sqrt{3}} + DC$$

$$DC = AB\sqrt{3} - \frac{AB}{\sqrt{3}} = AB\left(\sqrt{3} - \frac{1}{\sqrt{3}}\right)$$

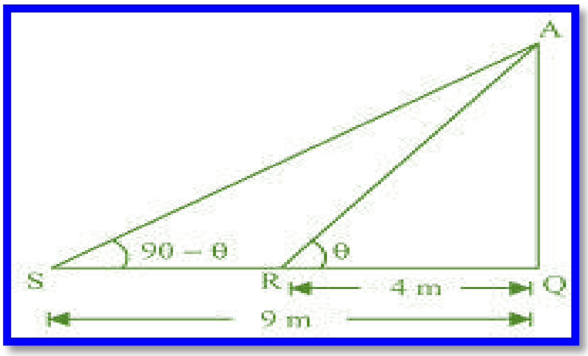
$$= \frac{2AB}{\sqrt{3}}$$

Time taken by the car to travel distance DC= $\left(\text{i.e., } \frac{2AB}{\sqrt{3}}\right)$ 6 seconds Time taken by the car to travel distance DB $\left(\text{i.e., } \frac{AB}{\sqrt{3}}\right) = \frac{6}{\frac{2AB}{\sqrt{3}}} \times \frac{AB}{\sqrt{3}}$

$$= \frac{6}{2} = 3 \text{ seconds}$$

Question 16:
The angles of elevation of the top of a tower from two points at a distance of 4 m and 9 m. from the base of the tower and in the same straight line with it are complementary. Prove that the height of the tower is 6 m.

Answer 16:



Let AQ be the tower and R, S are the points 4m, 9m away from the base of the tower respectively.
The angles are complementary. Therefore, if one angle is θ, the other will be 90 − θ.
In ΔAQR,

$$\frac{AQ}{QR} = \tan\theta$$

$$\frac{AQ}{4} = \tan\theta \qquad \ldots(i)$$

In ΔAQS,

$$\frac{AQ}{SQ} = \tan(90-\theta)$$

$$\frac{AQ}{9} = \cot\theta \qquad \ldots(ii)$$

On multiplying equations (*i*) and (*ii*), we obtain

$$\left(\frac{AQ}{4}\right)\left(\frac{AQ}{9}\right) = (\tan\theta).(\cot\theta)$$

$$\frac{AQ^2}{36} = 1$$

$$AQ^2 = 36$$

$$AQ = \sqrt{36} = \pm 6$$

However, height cannot be negative.
Therefore, the height of the tower is 6 m.

Mathematics

(Chapter – 10) (Circles)
(Class X)

Exercise 10.1

Question 1:
How many tangents can a circle have?

Answer 1:
A circle can have infinite number of tangents because a circle have infinite number of points on it and at every point a tangent can be drawn.

Question 2:
Fill in the blanks:

(i) A tangent to a circle intersects it in _________ point (s).

(ii) A line intersecting a circle in two points is called a _________.

(iii) A circle can have _________ parallel tangents at the most.

(iv) The common point of a tangent to a circle and the circle is called _________.

Answer 2:
(i) A tangent to a circle intersects it in ***one*** point (s).

(ii) A line intersecting a circle in two points is called a ***Secant***.

(iii) A circle can have ***two*** parallel tangents at the most.

(iv) The common point of a tangent to a circle and the circle is called ***point of contact***.

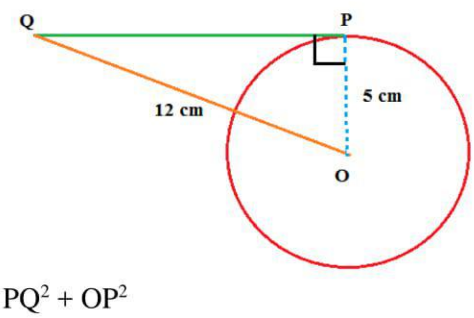
Question 3:
A tangent PQ at a point P of a circle of radius 5 cm meets a line through the centre O at a point Q so that OQ = 12 cm. Length PQ is:

(A) 12 cm (B) 13 cm (C) 8.5 cm (D) $\sqrt{119}$ cm.

Answer 3:
(D) $\sqrt{119}$ cm.

Solution:

In ΔOPQ, angle P is right angle. [Since radius is perpendicular to tangent]



Using Pythagoras theorem, $OQ^2 = PQ^2 + OP^2$

$\Rightarrow 12^2 = PQ^2 + 5^2$

$\Rightarrow 144 = PQ^2 + 25$

$\Rightarrow PQ^2 = 144 - 25 = 119$

$\Rightarrow PQ = \sqrt{119}$

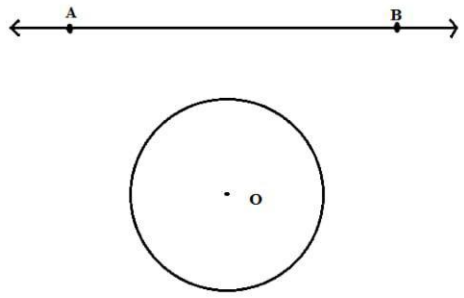
Hence, the option (D) is correct.

Question 4:

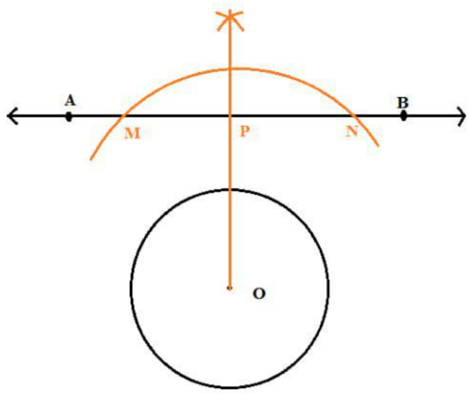
Draw a circle and two lines parallel to a given line such that one is a tangent and the other, a secant to the circle.

Answer 4:

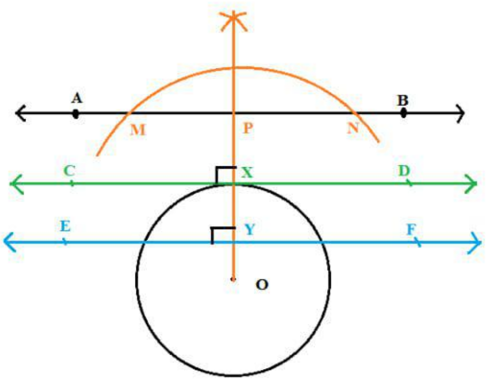
Consider a circle with centre O. Let AB is the given line.



Now draw a perpendicular from O to line AB, which intersect AB at P.



Now take two points on the line PO, one at circle X and another Y inside the circle. Draw lines parallel to AB and passing through X and Y. CD and EF are the required lines.



Mathematics

(Chapter – 10) (Circles)

(Class – X)

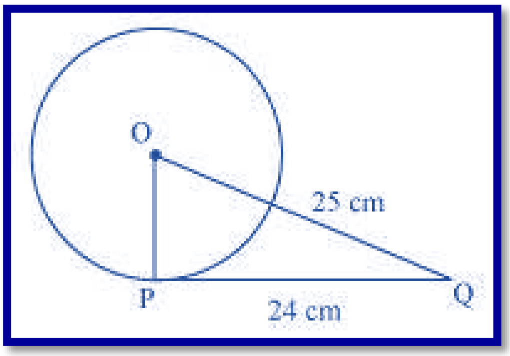
Exercise 10.2

Question 1:

From a point Q, the length of the tangent to a circle is 24 cm and the distance of Q from the centre is 25 cm. The radius of the circle is

(A) 7 cm (B) 12 cm (C) 15 cm (D) 24.5 cm

Answer 1:



Let O be the centre of the circle.

Given that,

OQ = 25cm and PQ = 24 cm

As the radius is perpendicular to the tangent at the point of contact,

Therefore, OP $\perp$ PQ

Applying Pythagoras theorem in ΔOPQ, we obtain

$OP^2 + PQ^2 = OQ^2$

$OP^2 + 24^2 = 25^2$

$OP^2 = 625 - 576$

$OP^2 = 49$

$OP = 7$

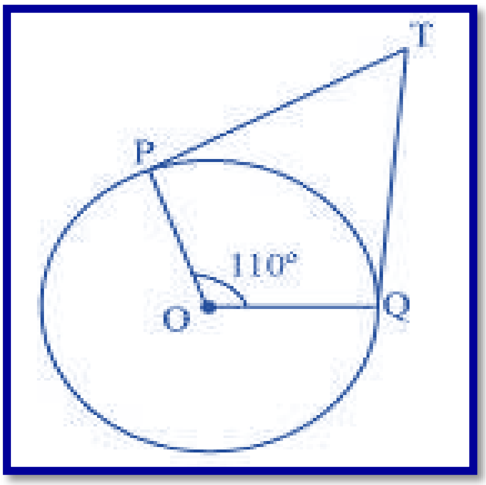
Therefore, the radius of the circle is 7 cm.

Hence, alternative (A) is correct

Question 2:

In the given figure, if TP and TQ are the two tangents to a circle with centre O so that $\angle POQ = 110^\circ$, then $\angle PTQ$ is equal to

(A) 60° (B) 70° (C) 80° (D) 90°



Answer 2:

It is given that TP and TQ are tangents.

Therefore, radius drawn to these tangents will be perpendicular to the tangents.

Thus, $OP \perp TP$ and $OQ \perp TQ$

$\angle OPT = 90^\circ$

$\angle OQT = 90^\circ$

In quadrilateral POQT,

Sum of all interior angles $= 360^\circ$

$\angle OPT + \angle POQ + \angle OQT + \angle PTQ = 360^\circ$

$\Rightarrow 90^\circ + 110^\circ + 90^\circ + \angle PTQ = 360^\circ$

$\Rightarrow \angle PTQ = 70^\circ$

Hence, alternative (B) is correct

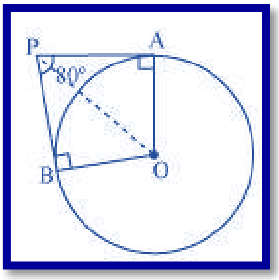
Question 3:

If tangents PA and PB from a point P to a circle with centre O are inclined to each other an angle of 80°, then ∠POA is equal to

(A) 50° (B) 60° (C) 70° (D) 80°

Answer 3:

It is given that PA and PB are tangents.



Therefore, the radius drawn to these tangents will be perpendicular to the tangents.
Thus, OA ⊥ PA and OB ⊥ PB

∠OBP = 90º and ∠OAP = 90º

In AOBP,
Sum of all interior angles = 360°

∠OAP + ∠APB +∠PBO + ∠BOA = 360°

90º + 80º +90º + ∠BOA = 360°

∠BOA = 100°

In ΔOPB and ΔOPA,
AP = BP (Tangents from a point)
OA = OB (Radii of the circle)
OP = OP (Common side)

Therefore, ΔOPB ≅ ΔOPA (SSS congruence criterion)

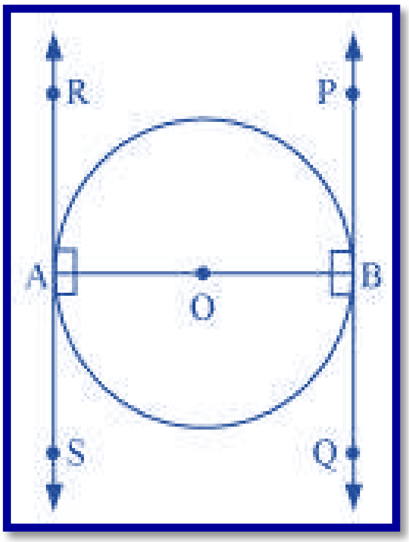
And thus, ∠POB = ∠POA

$$\angle POA = \frac{1}{2}\angle AOB = \frac{100^\circ}{2} = 50^\circ$$

Hence, alternative (A) is correct.

Question 4:
Prove that the tangents drawn at the ends of a diameter of a circle are parallel.

Answer 4:



Let AB be a diameter of the circle. Two tangents PQ and RS are drawn at points A and B respectively.

Radius drawn to these tangents will be perpendicular to the tangents.

Thus, $OA \perp RS$ and $OB \perp PQ$

$\angle OAR = 90^{o}$ $\angle OAS$

$= 90^{o}$

$\angle OBP = 90^{o}$

$\angle OBQ = 90^{o}$

It can be observed that

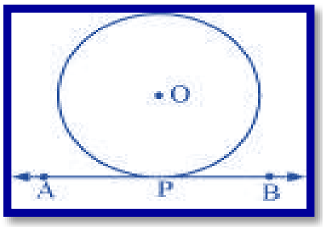
$\angle OAR = \angle OBQ$ (Alternate interior angles)

$\angle OAS = \angle OBP$ (Alternate interior angles)

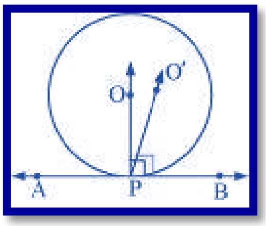
Since alternate interior angles are equal, lines PQ and RS will be parallel.

Question 5:
Prove that the perpendicular at the point of contact to the tangent to a circle passes through the centre.

Answer 5:
Let us consider a circle with centre O. Let AB be a tangent which touches the circle at P.



We have to prove that the line perpendicular to AB at P passes through centre O. We shall prove this by contradiction method.
Let us assume that the perpendicular to AB at P does not pass through centre O. Let it pass through another point O′. Join OP and O′P.



As perpendicular to AB at P passes through O′, therefore,
$\angle O'PB = 90°$ .... (1)

O is the centre of the circle and P is the point of contact. We know the line joining the centre and the point of contact to the tangent of the circle are perpendicular to each other.
$\therefore \angle OPB = 90°$ .... (2)

Comparing equations (1) and (2), we obtain
$\angle O'PB = \angle OPB$ .... (3)

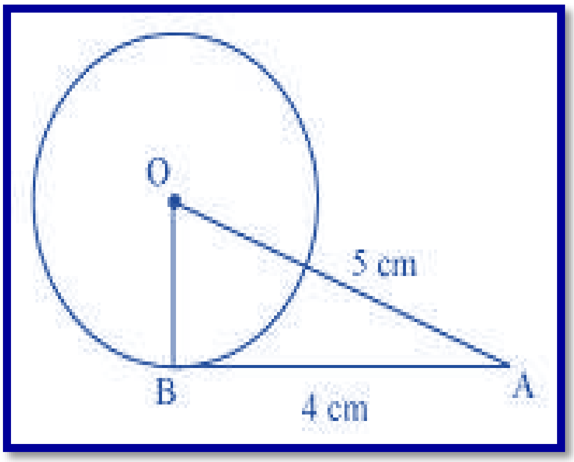
From the figure, it can be observed that,
$\angle O'PB < \angle OPB$ .... (4)

Therefore, $\angle O'PB = \angle OPB$ is not possible. It is only possible, when the line O′P coincides with OP.
Therefore, the perpendicular to AB through P passes through centre O.

Question 6:

The length of a tangent from a point A at distance 5 cm from the centre of the circle is 4 cm. Find the radius of the circle.

Answer 6:



Let us consider a circle centered at point O.

AB is a tangent drawn on this circle from point A.

Given that,

OA = 5cm and AB = 4 cm

In ΔABO,

$OB \perp AB$ (Radius $\perp$ tangent at the point of contact)

Applying Pythagoras theorem in ΔABO, we obtain

$AB^2 + BO^2 = OA^2$

$4^2 + BO^2 = 5^2$

$16 + BO^2 = 25$

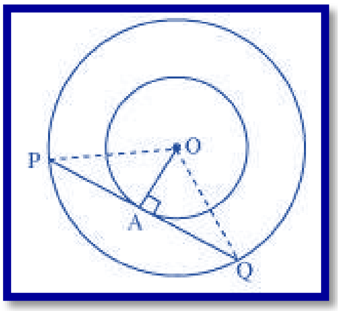
$BO^2 = 9$

$BO = 3$

Hence, the radius of the circle is 3 cm.

Question 7:
Two concentric circles are of radii 5 cm and 3 cm. Find the length of the chord of the larger circle which touches the smaller circle.

Answer 7:



Let the two concentric circles be centered at point O. And let PQ be the chord of the larger circle which touches the smaller circle at point A. Therefore, PQ is tangent to the smaller circle.

$OA \perp PQ$ (As OA is the radius of the circle)

Applying Pythagoras theorem in ΔOAP, we obtain

$OA^2 + AP^2 = OP^2$

$3_2 + AP_2 = 5_2$

$9 + AP^2 = 25$

$AP^2 = 16$

$AP = 4$

In ΔOPQ,

Since $OA \perp PQ$,

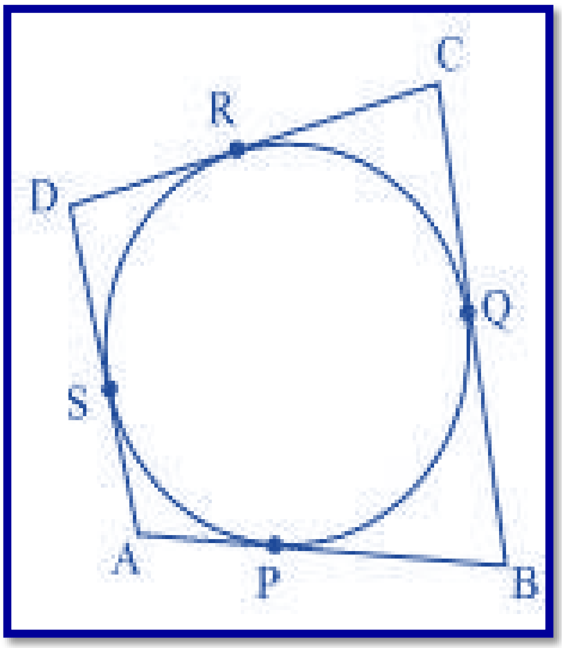
AP = AQ (Perpendicular from the center of the circle bisects the chord)

$\therefore PQ = 2AP = 2 \times 4 = 8$

Therefore, the length of the chord of the larger circle is 8 cm.

Question 8:

A quadrilateral ABCD is drawn to circumscribe a circle (see given figure) Prove that AB + CD = AD + BC



Answer 8:

It can be observed that

DR = DS (Tangents on the circle from point D) ………….. (1)

CR = CQ (Tangents on the circle from point C) …………… (2)

BP = BQ (Tangents on the circle from point B) …………… (3)

AP = AS (Tangents on the circle from point A) …………… (4)

Adding all these equations, we obtain

DR + CR + BP + AP = DS + CQ + BQ + AS

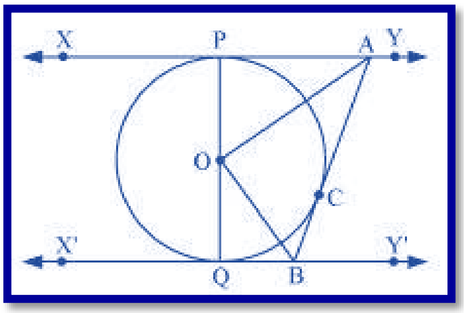
(DR + CR) + (BP + AP) = (DS + AS) + (CQ + BQ)

CD + AB = AD + BC

Question 9:

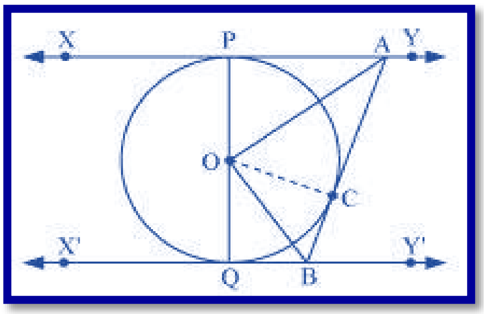
In the given figure, XY and X′Y′ are two parallel tangents to a circle with centre O and another tangent AB with point of contact C intersecting XY at A and X′Y′ at B.

Prove that $\angle AOB = 90^\circ$.



Answer 9:

Let us join point O to C.



In ΔOPA and ΔOCA,

OP = OC (Radii of the same circle)

AP = AC (Tangents from point A)

AO = AO (Common side)

ΔOPA ≅ ΔOCA (SSS congruence criterion)

$\angle POA = \angle COA$ .... (*i*)

Similarly, ΔOQB ≅ ΔOCB

$\angle QOB = \angle COB$ .... (*ii*)

Since POQ is a diameter of the circle, it is a straight line.

Therefore, $\angle POA + \angle COA + \angle COB + \angle QOB = 180^\circ$

From equations (*i*) and (*ii*), it can be observed that

$2\angle COA + 2\angle COB = 180^\circ$

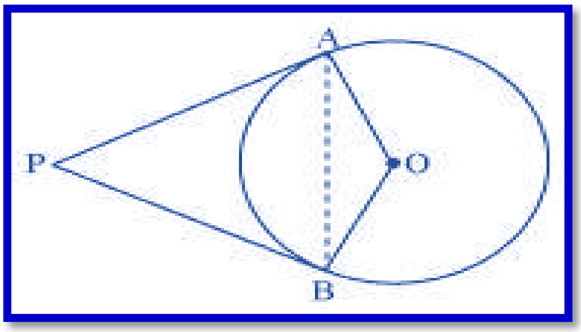
$\angle COA + \angle COB = 90^\circ$

$\angle AOB = 90^\circ$

Question 10:

Prove that the angle between the two tangents drawn from an external point to a circle is supplementary to the angle subtended by the line-segment joining the points of contact at the centre.

Answer 10:



Let us consider a circle centered at point O. Let P be an external point from which two tangents PA and PB are drawn to the circle which are touching the circle at point A and B respectively and AB is the line segment, joining point of contacts A and B together such that it subtends $\angle AOB$ at center O of the circle.

It can be observed that

OA (radius) $\perp$ PA (tangent)

Therefore, $\angle OAP = 90°$

Similarly, OB (radius) $\perp$ PB (tangent)

$\angle OBP = 90°$

In quadrilateral OAPB,

Sum of all interior angles = 360º

$\angle OAP + \angle APB + \angle PBO + \angle BOA = 360°$

$90° + \angle APB + 90° + \angle BOA = 360°$

$\angle APB + \angle BOA = 180°$

Hence, it can be observed that the angle between the two tangents drawn from an external point to a circle is supplementary to the angle subtended by the line segment joining the points of contact at the centre.

Question 11:

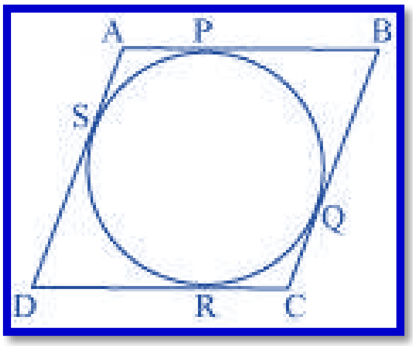
Prove that the parallelogram circumscribing a circle is a rhombus.

Answer 11:

Since ABCD is a parallelogram,

AB = CD ……………………………………(1)

BC = AD …………………………………..(2)



It can be observed that

DR = DS (Tangents on the circle from point D)

CR = CQ (Tangents on the circle from point C)

BP = BQ (Tangents on the circle from point B)

AP = AS (Tangents on the circle from point A)

Adding all these equations, we obtain

DR + CR + BP + AP = DS + CQ + BQ + AS

(DR + CR) + (BP + AP) = (DS + AS) + (CQ + BQ)

CD + AB = AD + BC

On putting the values of equations (1) and (2) in this equation, we obtain

2AB = 2BC

AB = BC ……………………………………(3)

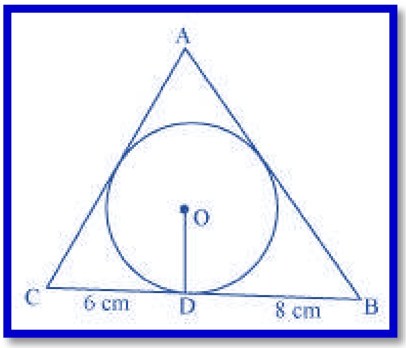
Comparing equations (1), (2), and (3), we obtain

AB = BC = CD = DA

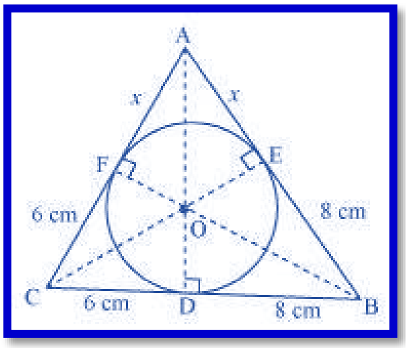
Hence, ABCD is a rhombus.

Question 12:

A triangle ABC is drawn to circumscribe a circle of radius 4 cm such that the segments BD and DC into which BC is divided by the point of contact D are of lengths 8 cm and 6 cm respectively (see given figure). Find the sides AB and AC.



Answer 12:



Let the given circle touch the sides AB and AC of the triangle at point E and F respectively and the length of the line segment AF be x.

In ΔABC,

CF = CD = 6cm (Tangents on the circle from point C)
BE = BD = 8cm (Tangents on the circle from point B)
AE = AF = x (Tangents on the circle from point A)
AB = AE + EB = x + 8
BC = BD + DC = 8 + 6 = 14
CA = CF + FA = 6 + x
2s = AB + BC + CA

$= x + 8 + 14 + 6 + x$

$= 28 + 2x$ $s = 14 + x$

$$\text{Area of } \Delta ABC = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{\{14+x\}\{(14+x)-14\}\{(14+x)-(6+x)\}\{(14+x)-(8+x)\}}$$

$$= \sqrt{(14+x)(x)(8)(6)}$$

$$= 4\sqrt{3(14x+x^2)}$$

Area of ΔOBC = $\frac{1}{2} \times OD \times BC = \frac{1}{2} \times 4 \times 14 = 28$

Area of ΔOCA = $\frac{1}{2} \times OF \times AC = \frac{1}{2} \times 4 \times (6+x) = 12 + 2x$

Area of ΔOAB = $\frac{1}{2} \times OE \times AB = \frac{1}{2} \times 4 \times (8+x) = 16 + 2x$

Area of ΔABC = Area of ΔOBC + Area of ΔOCA + Area of ΔOAB

$$4\sqrt{3(14x+x^2)} = 28 + 12 + 2x + 16 + 2x$$

$$\Rightarrow 4\sqrt{3(14x+x^2)} = 56 + 4x$$

$$\Rightarrow \sqrt{3(14x+x^2)} = 14 + x$$

$$\Rightarrow 3(14x+x^2) = (14+x)^2$$

$$\Rightarrow 42x + 3x^2 = 196 + x^2 + 28x$$

$$\Rightarrow 2x^2 + 14x - 196 = 0$$

$$\Rightarrow x^2 + 7x - 98 = 0$$

$$\Rightarrow x^2 + 14x - 7x - 98 = 0$$

$$\Rightarrow x(x+14) - 7(x+14) = 0$$

$$\Rightarrow (x+14)(x-7) = 0$$

Either $x + 14 = 0$ or $x - 7 = 0$

Therefore, $x = -14$ and 7

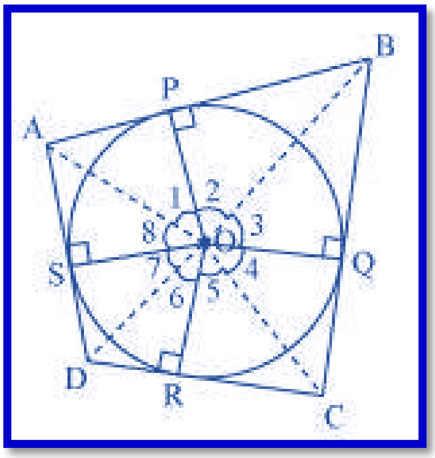
However, $x = -14$ is not possible as the length of the sides will be negative.
Therefore, $x = 7$

Hence, $AB = x + 8 = 7 + 8 = 15$ cm

$CA = 6 + x = 6 + 7 = 13$ cm

Question 13:
Prove that opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.

Answer 13:



Let ABCD be a quadrilateral circumscribing a circle centered at O such that it touches the circle at point P, Q, R, S. Let us join the vertices of the quadrilateral ABCD to the center of the circle.

Consider ΔOAP and ΔOAS,

AP = AS (Tangents from the same point)

OP = OS (Radii of the same circle)

OA = OA (Common side)

ΔOAP ≅ ΔOAS (SSS congruence criterion)

thus, $\angle POA = \angle AOS$

$\angle 1 = \angle 8$

Similarly,

$\angle 2 = \angle 3$

$\angle 4 = \angle 5$

$\angle 6 = \angle 7$

$\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6 + \angle 7 + \angle 8 = 360^o$

$(\angle 1 + \angle 8) + (\angle 2 + \angle 3) + (\angle 4 + \angle 5) + (\angle 6 + \angle 7) = 360^o$

$2\angle 1 + 2\angle 2 + 2\angle 5 + 2\angle 6 = 360^o$

$2(\angle 1 + \angle 2) + 2(\angle 5 + \angle 6) = 360^o$

$(\angle 1 + \angle 2) + (\angle 5 + \angle 6) = 180^o$

$\angle AOB + \angle COD = 180^o$

Similarly, we can prove that BOC + DOA = 180º

Hence, opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.