Additional Practice Question Paper (2023-24)
CLASS-XII
MATHEMATICS (041)

TIME: 3 Hours **MM.80**

General Instructions:

- This Question paper contains - five sections **A, B, C, D** and **E**. Each section is compulsory. However, there are internal choices in some questions.
- **Section A** has **18** MCQ's and **02** Assertion-Reason based questions of **1** mark each.
- **Section B** has **5** Very Short Answer (VSA)-type questions of **2** marks each.
- **Section C** has **6** Short Answer (SA)-type questions of **3** marks each.
- **Section D** has **4** Long Answer (LA)-type questions of **5** marks each.
- **Section E** has **3** source based/case based/passage based/integrated units of assessment of **4** marks each with sub-parts.

	Section –A **(Multiple Choice Questions)** **Each question carries 1 mark**
Q1.	The value of x – y + z from the following equation is $\begin{bmatrix} x+y+z \\ x+z \\ y+z \end{bmatrix} = \begin{bmatrix} 9 \\ 5 \\ 7 \end{bmatrix}$ (a) - 3 (b) - 1 (c) 1 (d) 3
Q2.	If A be a 3 × 3 square matrix such that $A(adj\ A) = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$ then the value of $\lvert Adj\ A \rvert$ is (a) 5 (b) 25 (c) 125 (d) 625
Q3.	If A and B are symmetric matrices of same order, then $\left(AB^T - 2\,BA^T\right)$ is a (a) Skew symmetric matrix (b)Symmetric matrix (c) Neither Symmetric matrix nor Skew symmetric matrix (d) Null matrix
Q4.	In the interval (1,2) the function $f(x) = 2\lvert x-1 \rvert + 3\lvert x-2 \rvert$ is (a) Strictly Increasing (b) Strictly Decreasing (c) Neither Increasing nor Decreasing (d) Remains constant
Q5.	If the set A contains 5 elements and the set B contains 6 elements, then the number of both one-one and onto mapping from A to B is (a) 720 (b) 120 (c) 30 (d) 0
Q6.	The sum of order & degree of the differential equation $\frac{d^3y}{dx^3} = (1 + \frac{dy}{dx})^5$ is (a) 3 (b) 4 (c) 5 (d) 8

Q7.	The solution set of the inequation $3x+2y>3$ is (a) half plane containing the origin (b) half plane not containing the origin (c) the point being on the line $3x+2y=3$ (d) None of these
Q8.	The two vectors $\hat{j}+\hat{k}$ and $3\hat{i}-\hat{j}+4\hat{k}$ represents the two sides AB and AC, respectively of ΔABC. The length of the median through A is (a) $\frac{\sqrt{34}}{2}$ (b) $\frac{\sqrt{48}}{2}$ (c) $\sqrt{18}$ (d) $\sqrt{52}$
Q9.	The value of $\int_{-\pi/2}^{\pi/2} x^3 \sin^4 x \, dx$ is (a) 0 (b) $\frac{\pi}{2}$ (c) π (d) $\frac{\pi^2}{4}$
Q10.	If $A=\begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$ be such that $A^{-1}=kA$, then k is equal to (a) 19 (b) 1/19 (c) -1/19 (d) - 19
Q11.	The corner points of the feasible region for the Linear Programming Problem are (0, 2), (3, 0), (6, 0), (6, 8) and (0, 5). Let the objective function is $Z=4x+6y$ then the minimum value of the objective function occurs at (a) (0, 2) only (b) (3, 0) only (c) The mid-point on the line segment joining the points (0,2) and (3,0) (d) Any point on the line segment joining the points (0,2) and (3,0)
Q12.	If the projection of $\lambda\hat{i}+\hat{j}+4\hat{k}$ on $2\hat{i}+6\hat{j}+3\hat{k}$ is 4 units, then the value of λ is equal to (a) - 9 (b) - 5 (c) 5 (d) 9
Q13.	If $A=\begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix}$ and $B^{-1}=\begin{bmatrix} 4 & -5 \\ -3 & 4 \end{bmatrix}$ then $(AB)^{-1}$ is equal to (a) $\begin{bmatrix} 15 & -19 \\ -26 & 33 \end{bmatrix}$ (b) $\begin{bmatrix} 11 & -14 \\ -29 & 37 \end{bmatrix}$ (c) $\begin{bmatrix} 37 & 14 \\ 29 & 11 \end{bmatrix}$ (d) $\begin{bmatrix} 37 & -14 \\ -29 & 11 \end{bmatrix}$
Q14.	In a hockey match, both teams A and B scored same number of goals up to the end of this game, so to decide the winner, the referee asked both the captains to throw a die alternately and decided that the team, whose captain gets a six first, will be declared the winner. If the captain of team A was asked to start, then probability of B winning the match is. (a) 1 / 6 (b) 5 / 6 (c) 5/11 (d) 6 / 11

Q15.	The value of λ for which the vectors $3\hat{i}-6\hat{j}+\hat{k}$ and $2\hat{i}-4\hat{j}+\lambda\hat{k}$ are parallel is (a) $\frac{2}{3}$ (b) $\frac{3}{2}$ (c) $\frac{5}{2}$ (d) $\frac{2}{5}$
Q16.	The integrating factor of the differential equation $\frac{dy}{dx}(x\log x)+y=2\log x$ is (a) e^x (b) $\log x$ (c) $\log(\log x)$ (d) x
Q17.	The function $f(x)=x^x$ has a stationary point at (a) $x=e$ (b) $x=\frac{1}{e}$ (c) $x=1$ (d) $x=\sqrt{e}$
Q18.	The direction ratios of the line 3x + 1 = 6y − 2 = 1 – z are (a) 3, 6, 1 (b) 3, 6, -1 (c) 2, 1, 6 (d) 2, 1, -6
	ASSERTION-REASON BASED QUESTIONS The following questions consist of two statements – Assertion (A) and Reason (R). Answer these questions selecting the appropriate option given below: (a) Both A and R are true and R is the correct explanation for A. (b) Both A and R are true and R is not the correct explanation for A. (c) A is true but R is false. (d) A is false but R is true.
Q19.	**Assertion (A)** :The Differential coefficient of $\sec(\tan^{-1}x)$ with respect to *x* is $\frac{x}{\sqrt{1+x^2}}$ **Reason (R)** :The Differential coefficient of the function with respect to *x*is the first order derivative of the function .
Q20.	**Assertion (A)** : The vector equation of the line passing through the points (6,-4,5) and (3,4,1) is $\vec{r}=(6\hat{i}-4\hat{j}+5\hat{k})+\lambda(-3\hat{i}+8\hat{j}+4\hat{k})$. **Reason (R)** : The vector equation of the line passing through the points $\vec{a}$ and $\vec{b}$ is $\vec{r}=\vec{a}+\lambda(\vec{b}-\vec{a})$.
	Section – B **(This section comprises of very short answer type questions (VSA) of 2 marks each)**
Q21.	If $\cos^{-1}\alpha+\cos^{-1}\beta+\cos^{-1}\gamma=3\pi$, then find the value of $\alpha(\beta+\gamma)-\beta(\gamma+\alpha)+\gamma(\alpha+\beta)$. **OR** Reduce $\cot^{-1}\left\{\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right\}$ where $\frac{\pi}{2}<x<\pi$ in to simplest form.
Q22.	The two equal sides of an isosceles triangle with fixed base b are decreasing at the rate of 3cm/sec. How fast is the area decreasing when the two equal sides are equal to the base? **OR** The volume of the cube increases at a constant rate. Prove that the increase in its surface area varies inversely as the length of the side.
Q23.	If $\vec{a}+\vec{b}+\vec{c}=0$, $\lvert\vec{a}\rvert=3$, $\lvert\vec{b}\rvert=5$ and $\lvert\vec{c}\rvert=7$ then find the angle between $\vec{a}$ and $\vec{b}$
Q24.	Find the point(s) on the line $\frac{x+2}{3}=\frac{y+1}{2}=\frac{z-3}{2}$ at a distance 5 $units$ from the point (1,3,3).
Q25.	Find the area of the region bounded by the curve $y^2=4x$, y-axis and line $y=3$.

Section – C

(This section comprises of short answer type questions (SA) of 3 marks each)

Q26. Solve the following Linear Programming Problem graphically:

Minimize Z = 3x + 9y

Subject to the constraints

$x + 3y \le 60$

$x + y \ge 10$

$x \le y$

$x \ge 0,\ y \ge 0.$

Q27. Two numbers are selected at random (without replacement) from positive integers 2, 3, 4, 5, 6 and 7. Let X denote the larger of the two numbers obtained, Find the probability distribution of X.

OR

A and B are two independent events. The probability that both A and B occur is 1/6 and the probability that neither of them occur is 1/3. Find the probability of the occurrence of A.

Q28. Find $\int \frac{\cos x}{(1-\sin x)(2-\sin x)}\,dx$

OR

Find $\int \frac{\sin\phi}{\sqrt{\sin^2\phi + 2\cos\phi + 3}}\,d\phi$

Q29. Solve the differential equation $2ye^{\frac{x}{y}}dx + \left(y - 2xe^{\frac{x}{y}}\right)dy = 0$.

OR

Solve the differential equation $\frac{dy}{dx} - 3y\cot x = \sin 2x$

Q30. Draw a rough sketch of the curve $y = 1 + |x+1|$, $x = -3$, $x = 3,\ y = 0$ and find the area of the region bounded by them using integration.

Q31. If $x = a\sin t - b\cos t, y = a\cos t + b\sin t,$ then $prove\, that$ $\frac{d^2y}{dx^2} = -\,(\frac{x^2+y^2}{y^3}).$

Section – D

(This section comprises of long answer type questions (LA) of 5 marks each)

Q32. If A = $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & -3 \\ -3 & 2 & -4 \end{bmatrix}$, find A^{-1} and hence solve the system of equations

$x + 2y - 3z = -4$;$2x + 3y + 2z = 14$; $3x - 3y - 4z = -15$

Q33. Find the equations of the lines through the origin which intersect the line $\frac{x-3}{2} = \frac{y-3}{1} = \frac{z}{1}$ at an angle of $\frac{\pi}{3}$ each.

OR

Find the equation of the line which intersect the lines $\frac{x+2}{1} = \frac{y-3}{2} = \frac{z+1}{4}$ and $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and passes through the point (1, 1, 1).

Q34. Evaluate $\int_0^{\pi} \frac{x\tan x}{\sec x + \tan x}\,dx$.

OR

Evaluate $\int_0^{\pi} \log(1+\cos x)\,dx$

Q35. Consider $f : R_+ \to [-5, \infty)$ given by $f(x) = 9x^2 + 6x - 5$ where R_+ is the set of all non-negative real numbers. Prove that *f* is one- one and onto function.

Section – E

(This section comprises of 3 case- study/passage-based questions of 4 marks each with sub parts. The first two case study questions have three sub parts (i), (ii), (iii) of marks 1,1,2 respectively. The third case study question has two sub parts of 2 marks each.)

Q36. The use of electric vehicles will curb air pollution in the long run. The use of electric vehicles is increasing every year and estimated number of electric vehicles in use at any time t is given by the function

$$V(t) = t^3 - 3t^2 + 3t - 100$$

Where t represents time and t = 1, 2, 3, --------- corresponds to year 2021, 2022, 2023 ------- respectively.

Based on the above information answer the following:

(i) Can the above function be used to estimate number of vehicles in the year 2020? Justify.

(ii) Find the estimated number of vehicles in the year 2040.

(iii) Prove that the function V(t) is an increasing function.

Q37. Senior students tend to stay up all night and therefore are not able to wake up on time in morning. Not only this but their dependence on tuitions further leads to absenteeism in school. Of the students in class XII, it is known that 30% of the students have 100% attendance. Previous year results report that 80% of all students who have 100% attendance attain A grade and 10% irregular students attain A grade in their annual examination. At the end of the year, one student is chosen at random from the class XII.

Using above information answer the following:

(i). Find the conditional probability that a student attains A grade given that he is not 100 % regular student.

(ii) Find the probability of attaining A grade by the students of class XII

.

(iii) Find the probability that student is 100% regular given that he attains A grade.

	OR Find the probability that student is irregular given that he attains A grade.
Q38.	In a park, an open tank is to be constructed using metal sheet with a square base and vertical sides so that it contains 500 cubic meters of water.  Using above information answer the following: (i) Find the minimum surface area of the tank. (ii) Find the percentage increase in volume of the tank, if size of square base of tank become twice and height remains same.

MARKING SCHEME

Additional Practice Question Paper (2023-24)

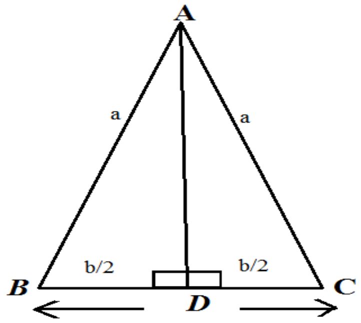
CLASS-XII

MATHEMATICS (041)

	SECTION: A (Solution of MCQs of 1 Mark each)	
Q.No.	**Solution**	**Marks**
1.	$x+y+z=9$, $x+z=5$, $y+z=7$ On solving above equations, we get $x=2$, $y=4$, $z=3$ $\therefore x-y+z=1$ **Correct Answer is Option (c) 1**	1
2.	$\mathrm{A(Adj\,A)}=5\mathrm{I}$ $\mathrm{A(Adj\,A)}=\lvert A\rvert \mathrm{I}$ $\therefore\ \lvert A\rvert=5$ $\lvert Adj\ A\rvert=\lvert A\rvert^2=25$ **Correct Answer is Option (b) 25**	1
3.	Let $\left(AB^T-2BA^T\right)^T=C$ Consider $C^T=\left(AB^T-2BA^T\right)^T=BA^T-2AB^T\neq C\ or\ -C$ $\therefore\ C$ is neither Symmetric matrix nor Skew symmetric matrix **Correct Answer is Option (c) Neither Symmetric matrix nor Skew symmetric matrix**	1
4.	$If\ \ 1<x<2\ then\ f(x)=2(x-1)-3(x-2)=-x+4$ $\therefore\ f'(x)=-1$ Hence $f(x)$ is Strictly decreasing function **Correct Answer is Option (b) Strictly Decreasing**	1
5.	Set A contains 5 elements and the set B contains 6 elements. For one-one function each element in set B is assigned to only one element in set A **Correct Answer is Option (d) 0**	1
6.	Order = 3, Degree = 1 **Correct Answer is Option (b) 4**	1
7.	**Correct Answer is Option (b) half plane not containing the origin**	1
8.	$\overrightarrow{AD}=\frac{3}{2}\hat{i}+\frac{5}{2}\hat{k}$ $AD=\frac{\sqrt{34}}{2}$ **Correct Answer is Option (a)** $\frac{\sqrt{34}}{2}$	1
9.	$f(x)=x^3\sin^4 x$ $f(-x)=(-x)^3[\sin(-x)]^4=-x^3[-\sin x]^4=-x^3\sin^4 x=-f(x)$ $\therefore f(x)\ is\ an\ odd\ \ function$ $\therefore\ \int_{-\pi/2}^{\pi/2} x^3\sin^4 x\ dx=0$ **Correct Answer is Option (a) 0**	1

10.	$A^{-1} = kA$ $\frac{-1}{19}\begin{bmatrix}-2 & -3\\ -5 & 2\end{bmatrix} = k\begin{bmatrix}2 & 3\\ 5 & -2\end{bmatrix}$ $\therefore k = \frac{1}{19}$ **Correct Answer is Option (b) 1/19**	**1**
11.	Corner Points — Value of Z A (0, 2) — Z = 0 + 12 = 12 B (3, 0) — Z = 12 + 0 = 12 C (6, 0) — Z = 24 + 0 = 24 D (6,8) — Z = 24 + 48 = 72 E (0,5) — Z = 0 + 30 = 30 Minimum value of Z = 12 **Correct Answer is Option (d)Any point on the line segment joining the points (0,2) and (3,0)**	**1**
12.	According to the Question $\frac{(\lambda\hat{i} + \hat{j} + 4\hat{k}).(2\hat{i} + 6\hat{j} + 3\hat{k})}{\left\|2\hat{i} + 6\hat{j} + 3\hat{k}\right\|} = 4$ $\frac{2\lambda + 6 + 12}{\sqrt{4+36+9}} = 4$ $\therefore \lambda = 5$ **Correct Answer is Option (c) 5**	**1**
13.	$(AB)^{-1} = B^{-1}A^{-1} = \begin{bmatrix}4 & -5\\ -3 & 4\end{bmatrix}\begin{bmatrix}3 & -1\\ -5 & 2\end{bmatrix} = \begin{bmatrix}37 & -14\\ -29 & 11\end{bmatrix}$ **Correct Answer is Option (d)** $\begin{bmatrix}37 & -14\\ -29 & 11\end{bmatrix}$	**1**
14.	B will win in second attempt or fourth attempt or sixth attempt or so on $\therefore P(B\,winning) = P(\bar{A}B) + P(\bar{A}\ \bar{B}\ \bar{A}\,B) + P(\bar{A}\ \bar{B}\ \bar{A}\bar{B}\ \bar{A}\,B) + ...$ $= \frac{5}{6}.\frac{1}{6} + \frac{5}{6}.\frac{5}{6}.\frac{5}{6}.\frac{1}{6} + \frac{5}{6}.\frac{5}{6}.\frac{5}{6}.\frac{5}{6}.\frac{5}{6}.\frac{1}{6} + ...$ $= \frac{5/36}{1 - 25/36} = \frac{5}{11}$ **Correct Answer is Option (c) 5/11**	**1**
15.	According to the Question $\frac{3}{2} = \frac{-6}{-4} = \frac{1}{\lambda}$ $\therefore \lambda = \frac{2}{3}$ **Correct Answer is Option (a) 2/3**	**1**
16	$Integrating\ Factor = e^{\int \frac{1}{x\log x}dx} = e^{\log(\log x)} = \log x$ **Correct Answer is Option (b)** $\log x$	**1**
17.	$f'(x) = x^x(1 + \log x)$ For Stationary point $f'(x) = 0$ $x^x(1 + \log x) = 0 \Rightarrow \log x = -1 \Rightarrow x = e^{-1} = \frac{1}{e}$	

	Correct Answer is Option (b) 1/e	**1**
18.	$3x+1=6y-2=1-z$ $3(x+1/3)=6(y-1/3)=-(z-1)$ $\frac{x+1/3}{1/3}=\frac{y-1/3}{1/6}=\frac{z-1}{-1}$ $\frac{x+1/3}{2}=\frac{y-1/3}{1}=\frac{z-1}{-6}$ **Correct Answer is Option (d) 2, 1, - 6.**	**1**
19.	**Correct Answer is Option (a)Both A and R true and R is the correct explanation for A.**	**1**
20.	**Correct Answer is Option (d)A is false but R is true.**	**1**
	Section –B **[This section comprises of solution of very short answer type questions (VSA) of 2 marks each]**	
21.	$\cos^{-1}\alpha+\cos^{-1}\beta+\cos^{-1}\gamma=3\pi$ $\Rightarrow cos^{-1}\alpha = \pi, cos^{-1}\beta = \pi \ \& \ cos^{-1}\gamma = \pi$ $\therefore \ \alpha = \beta = \gamma = -1$ $\alpha\left(\beta+\gamma\right) -\beta\left(\gamma+\alpha\right) + \gamma\left(\alpha+\beta\right)$ $= (-1)(-1-1) - (-1)(-1-1) + (-1)(-1-1)$ $= 2 - 2 + 2=2$ **OR** $\cot^{-1}\left\{\frac{\sqrt{\left(\sin\frac{x}{2}+\cos\frac{x}{2}\right)^2}+\sqrt{\left(\sin\frac{x}{2}-\cos\frac{x}{2}\right)^2}}{\sqrt{\left(\sin\frac{x}{2}+\cos\frac{x}{2}\right)^2}-\sqrt{\left(\sin\frac{x}{2}-\cos\frac{x}{2}\right)^2}}\right\}$ $=\cot^{-1}\left\{\frac{\left(\sin\frac{x}{2}+\cos\frac{x}{2}\right)+\left(\sin\frac{x}{2}-\cos\frac{x}{2}\right)}{\left(\sin\frac{x}{2}+\cos\frac{x}{2}\right)-\left(\sin\frac{x}{2}-\cos\frac{x}{2}\right)}\right\}$ $=\cot^{-1}\left(\frac{2\sin\frac{x}{2}}{2\cos\frac{x}{2}}\right)=\cot^{-1}\left(\tan\frac{x}{2}\right)$ $=\cot^{-1}\left(\cot(\frac{\pi}{2}-\frac{x}{2})\right)= \frac{\pi}{2}-\frac{x}{2}$	**1** **1** **1** **1/2** **1/2**

22.	In isosceles ΔABC, let AB = AC = a and BC = b (given), $\frac{da}{dt} = -3$cm/sec. AD = $\sqrt{a^2 - \frac{b^2}{4}}$  A = Area of $\Delta\, ABC = \frac{1}{2}$ (BC) (AD) = $\left(\frac{1}{2}\right)(b)\sqrt{a^2 - \frac{b^2}{4}}$ $\Rightarrow \frac{dA}{dt} = \left(\frac{b}{2}\right)\left(\frac{1}{2}\right)\left(a^2 - \frac{b^2}{4}\right)^{-1/2} . \left(2a\frac{da}{dt}\right)$ $= \frac{b(2a)(-3)}{4\sqrt{a^2 - \frac{b^2}{4}}} = \frac{-3ab}{2\sqrt{a^2 - \frac{b^2}{4}}}$ $\Rightarrow \left(\frac{dA}{dt}\right)_{at\, a=b} = \frac{-3b^2}{2\sqrt{b^2 - \frac{b^2}{4}}} = \frac{-3b^2}{\frac{2\sqrt{3}b}{2}} = -\sqrt{3}b$ $\therefore$ Area is decreasing at the rate of $\sqrt{3}b$ cm/sec. **OR** Let the side of a cube be x unit. Volume of cube = V $= x^3$ $\frac{dV}{dt} = 3x^2\frac{dx}{dt} = k\ (constant)$ $\frac{dx}{dt} = \frac{k}{3x^2}$ Surface area = S = $6x^2$ $\frac{dS}{dt} = 12\, x\frac{dx}{dt}$ $\frac{dS}{dt} = 12x\frac{k}{3x^2} = 4\left(\frac{k}{x}\right)$ Hence, the surface area of the cube varies inversely as length of side	1 1 1 1
23.	$\vec{a} + \vec{b} + \vec{c} = 0$ $\vec{a} + \vec{b} = -\vec{c}$ $(\vec{a} + \vec{b}).(\vec{a} + \vec{b}) = (-\vec{c}).(-\vec{c})$ $\vec{a}.\vec{a} + \vec{a}.\vec{b} + \vec{b}.\vec{a} + \vec{b}.\vec{b} = \vec{c}.\vec{c}$ $\lvert\vec{a}\rvert^2 + 2\vec{a}.\vec{b} + \lvert\vec{b}\rvert^2 = \lvert\vec{c}\rvert^2$ $2\lvert\vec{a}\rvert\ \lvert\vec{b}\rvert\ \cos\theta = \lvert\vec{c}\rvert^2 - \lvert\vec{a}\rvert^2 - \lvert\vec{b}\rvert^2$ $2(3)(5)\cos\theta = 49 - 9 - 25$ $\Rightarrow \cos\theta = \frac{15}{30} = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3}$	1/2 1/2 1/2 1/2

24.

Let $P(3\lambda-2, 2\lambda-1, 2\lambda+3)$ be any point on a line $\frac{x+2}{3}=\frac{y+1}{2}=\frac{z-3}{2}$ which is at a distance of 5 units from the point $Q(1,3,3)$.

According to the Question

$PQ=5$

$(PQ)^2=25$

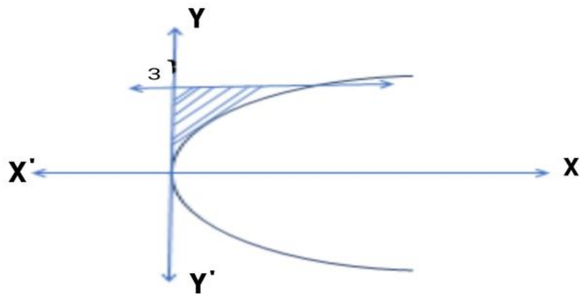
$(3\lambda-2-1)^2+(2\lambda-1-3)^2+(2\lambda+3-3)^2=25$ **1**

$17\lambda^2-34\lambda=0$

$\lambda=0\, or\, 2$

Required Point is (-2,-1,3) *or* (4,3,7) **1**

25.



1 Mark For Correct Figure

Required Area = $\int_0^3 \frac{y^2}{4}\,dy = \left.\frac{y^3}{12}\right]_0^3$ **1/2**

$= \frac{27}{12} - 0 = \frac{9}{4}$ square units **1/2**

Section –C

[This section comprises of solution short answer type questions (SA) of 3 marks each]

26.

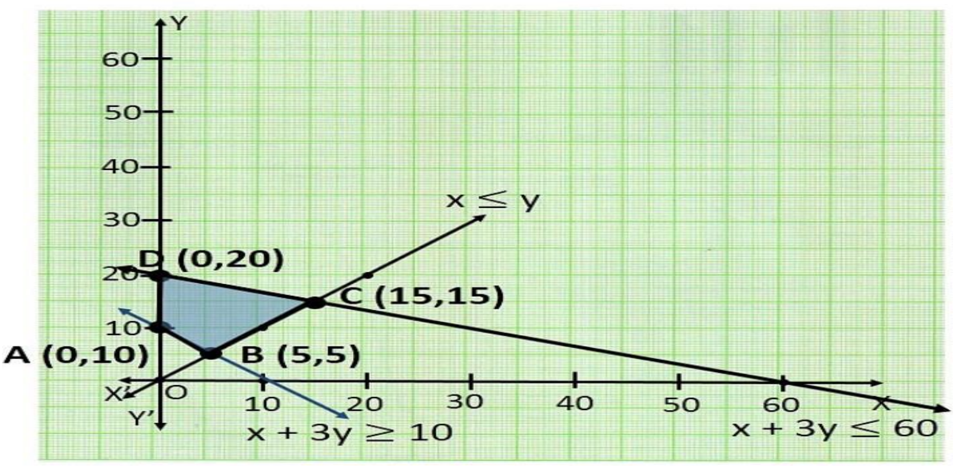
$x+3y\le 60$ $\quad$ $x+y\ge 10$ $\quad$ $x\le y$

$x+3y=60$ $\quad$ $x+y=10$ $\quad$ $x=y$

x	0	60
y	20	0

x	0	10
y	10	0

x	0	10
y	0	10

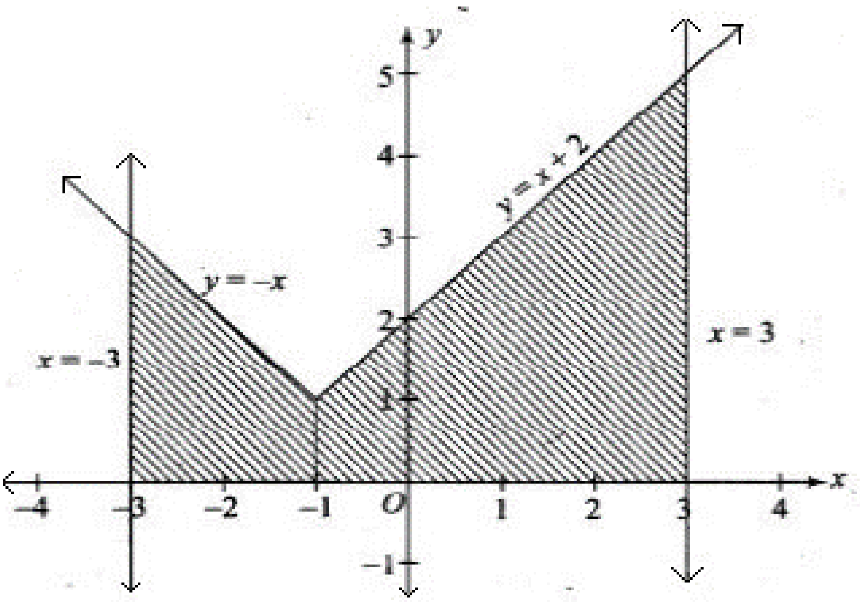


1.5 Marks For Correct Figure

Corner points are A (0, 10), B (5, 5), C (15, 15) and D (0, 20) **1/2**

	Corner Points — Value of Z A (0, 10) — Z = 0 + 90 = 90 B (5, 5) — Z = 15 + 45 = 60 C (15, 15) — Z = 45 + 135 = 180 D (0,20) — Z = 0 + 180 = 180 Minimum value of Z = 60	**1/2** **1/2**
27.	$n(S) = {}^6C_2 = \frac{6\times5}{2\times1} = 15$ Let X denote the larger of the two numbers obtained $\therefore$ X = 3, 4, 5, 6, 7 The Probability Distribution is X — P (X) 3 — 1/15 4 — 2/15 5 — 3/15 6 — 4/15 7 — 5/15 **OR** Let $P(A) = x$ *and* $P(B) = y$ According to the Question $P(A\cap B) = \frac{1}{6}$ *and* $P(A'\cap B') = \frac{1}{3}$ $P(A)P(B) = \frac{1}{6}$ *and* $P(A')P(B') = \frac{1}{3}$ $xy = \frac{1}{6}$ and $(1-x)(1-y) = \frac{1}{3}$ on solving we get $x = \frac{1}{2}$ *or* $\frac{1}{3}$	**1** **1/2** **1.5** **1** **1** **1**
28.	$I = \int \frac{\cos x}{(1-\sin x)(2-\sin x)}dx$ $= \int \frac{\cos x}{(\sin x-1)(\sin x-2)}dx$ — put sin x = t cos x dx = dt $\Rightarrow I = \int \frac{dt}{(t-1)(t-2)}$ $= \int \left(\frac{-1}{t-1}+\frac{1}{t-2}\right)dt$ $= -\log\|t-1\|+\log\|t-2\|$ $= \log\left\|\frac{t-2}{t-1}\right\|+c$ $= \log\left\|\frac{\sin x-2}{\sin x-1}\right\|+c$	**1** **1** **1**

	OR $I=\int \frac{\sin\phi}{\sqrt{\sin^2\phi+2\cos\phi+3}}\,d\phi$ $=\int \frac{\sin\phi}{\sqrt{1-\cos^2\phi+2\cos\phi+3}}\,d\phi$ $=\int \frac{\sin\phi\ d\phi}{\sqrt{4+2\cos\phi-\cos^2\phi}}$ put $\cos\phi = t$ $-\sin\phi\, d\phi = dt$ $=\int \frac{-dt}{\sqrt{4+2t-t^2}} \quad =-\int \frac{dt}{\sqrt{-\left[t^2-2t-4\right]}}$ $=-\int \frac{dt}{\sqrt{-\left[t^2-2t+1-5\right]}} \quad =-\int \frac{dt}{\sqrt{\left(\sqrt{5}\right)^2-\left(t-1\right)^2}}$ $=-\sin^{-1}\left(\frac{t-1}{\sqrt{5}}\right)+c \quad =-\sin^{-1}\left(\frac{\cos\phi-1}{\sqrt{5}}\right)+c$	1 1 1
29.	$2ye^{\frac{x}{y}}dx+\left(y-2xe^{\frac{x}{y}}\right)dy=0$ $2ye^{\frac{x}{y}}dx=\left(2xe^{\frac{x}{y}}-y\right)dy$ $\frac{dx}{dy}=\left(\frac{2xe^{\frac{x}{y}}-y}{2ye^{\frac{x}{y}}}\right)$ Put x = vy $\frac{dx}{dy}=v+y\frac{dv}{dy}$ $v+y\frac{dv}{dy}=\frac{2vye^v-y}{2ye^v}$ $y\frac{dv}{dy}=\frac{2vye^v-y-2vye^v}{2ye^v}$ $\frac{dv}{dy}=\frac{-1}{2ye^v}$ $\int 2e^v dv=\int -\frac{1}{y}dy$ $2e^v=-\log y+c$ $2e^{\frac{x}{y}}+\log y=c$ **OR** $\frac{dy}{dx}-3y\cot x=\sin 2x$ Compare with $\frac{dy}{dx}+Py=Q$ $P=-3\cot x \quad , \quad Q=\sin 2x$	1 1 1 1/2

	$I.F = e^{\int P dx} = e^{\int -3\cot x\, dx} = e^{-3\log \sin x}$ $= e^{\log(\sin x)^{-3}} = (\sin x)^{-3} = \frac{1}{\sin^3 x}$ The solution of given differential equation is $y(I.F) = \int Q(I.F)dx$ $y\left(\frac{1}{\sin^3 x}\right) = \int \sin 2x.\frac{1}{\sin^3 x}dx$ $\frac{y}{\sin^3 x} = \int 2\sin x\cos x.\frac{1}{\sin^3 x}dx$ $\frac{y}{\sin^3 x} = \int 2\cot x\,\text{cosecx}\, dx$ $\frac{y}{\sin^3 x} = -2\text{cosec}\, x + C$ $y = -2\sin^2 x + c\sin^3 x$	1 1 1/2
30.	 Required Area $= \int_{-3}^{-1} -x\,dx + \int_{-1}^{3}(x+2)dx$ $= \left[\frac{-x^2}{2}\right]_{-3}^{-1} + \left[\frac{x^2}{2} + 2x\right]_{-1}^{3}$ $= \frac{-1}{2}(1-9) + \left[(\frac{9}{2}+6) - (\frac{1}{2}-2)\right]$ $= 4 + 12 = 16\, square\, units$	**1.5 Mark For Correct Figure** 1/2 1
31.	$x = a\sin t - b\cos t$ $y = a\cos t + b\sin t$ $\frac{dx}{dt} = a\cos t + b\sin t = y$ $\frac{dy}{dt} = -a\sin t + b\cos t = -x$ $\frac{dy}{dx} = \frac{-x}{y}$	1 1/2

$$\frac{d^2y}{dx^2} = \frac{y(-1)-(-x).\frac{dy}{dx}}{(y)^2}$$ **1**

$$= \frac{-y+x\left(-\frac{x}{y}\right)}{(y)^2}$$

$$= \frac{-y^2-x^2}{y^3} = \quad = \frac{-(y^2+x^2)}{y^3}$$ **1/2**

Section –D

[This section comprises of solution of long answer type questions (LA) of 5 marks each]

32.

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & -3 \\ -3 & 2 & -4 \end{bmatrix}$$

$$|A| = 1(-12+6) - 2(-8-9) + 3(4+9)$$

$$= -6+34+39 = 67 \neq 0$$ **1/2**

$$\text{Adj. } A = \begin{bmatrix} -6 & 14 & -15 \\ 17 & 5 & 9 \\ 13 & -8 & -1 \end{bmatrix}$$ **2**

$$A^{-1} = \frac{1}{67}\begin{bmatrix} -6 & 14 & -15 \\ 17 & 5 & 9 \\ 13 & -8 & -1 \end{bmatrix}$$ **1/2**

The matrix form of the equations is

$$\begin{bmatrix} 1 & 2 & -3 \\ 2 & 3 & 2 \\ 3 & -3 & -4 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -4 \\ 14 \\ -15 \end{bmatrix}$$

$$A^t X = B$$

$$X = \left(A^t\right)^{-1} B$$

$$= \left(A^{-1}\right)^t B$$ **1/2**

$$= \frac{1}{67}\begin{bmatrix} -6 & 17 & 13 \\ 14 & 5 & -8 \\ -15 & 9 & -1 \end{bmatrix}\begin{bmatrix} -4 \\ 14 \\ -15 \end{bmatrix}$$ **1/2**

$$= \frac{1}{67}\begin{bmatrix} 24+238-195 \\ -56+70+120 \\ 60+126+15 \end{bmatrix}$$

$$= \frac{1}{67}\begin{bmatrix} 67 \\ 134 \\ 201 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

x = 1, y = 2, z = 3 **1**

33.	Let $P(2\lambda+3,\lambda+3,\lambda)$ be any point on line $\frac{x-3}{2}=\frac{y-3}{1}=\frac{z}{1}$ Let the line through origin and making an angle of $\frac{\pi}{3}$ with the given line be along OP. Then direction ratios are proportional to $2\lambda+3-0,\lambda+3-0,\lambda-0$ *i.e.* $2\lambda+3,\lambda+3,\lambda$ Also, direction ratios of the given line are proportional to 2,1,1. $\therefore \quad \cos\frac{\pi}{3}=\frac{(2\lambda+3)(2)+(\lambda+3)(1)+(\lambda)(1)}{\sqrt{(2\lambda+3)^2+(\lambda+3)^2+(\lambda)^2}\sqrt{2^2+1^2+1^2}}$ $\Rightarrow \frac{1}{2}=\frac{6\lambda+9}{\sqrt{6\lambda^2+18\lambda+18}\sqrt{6}}$ $\Rightarrow \frac{1}{2}=\frac{3(2\lambda+3)}{6\sqrt{\lambda^2+3\lambda+3}}$ $\Rightarrow \sqrt{\lambda^2+3\lambda+3}=(2\lambda+3)$ Squaring both sides, we get $\lambda^2+3\lambda+3=(2\lambda+3)^2$ $\Rightarrow \lambda^2+3\lambda+2=0$ $\Rightarrow \lambda=-1,-2$ Therefore, the coordinates of point $P(1,2,-1)$ *or* $P(-1,1,-2)$ Hence Equations of required lines are $\frac{x}{1}=\frac{y}{2}=\frac{z}{-1}$ *and* $\frac{x}{-1}=\frac{y}{1}=\frac{z}{-2}$ **OR** The lines are $\frac{x+2}{1}=\frac{y-3}{2}=\frac{z+1}{4}$ *and* $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$ Let $P(\lambda-2,2\lambda+3,4\lambda-1)$ be any point on line (1) and $Q(2\mu+1,3\mu+2,4\mu+3)$ be any point on line (2). Also, the given point is $A(1,1,1)$. For some definite values of λ *and* μ, the required line passes through A, P and Q The direction ratios of AP are $\lambda-3,2\lambda+2,4\lambda-2$ The direction ratios of AQ are $2\mu,3\mu+1,4\mu+2$ $\therefore \frac{\lambda-3}{2\mu}=\frac{2\lambda+2}{3\mu+1}=\frac{4\lambda-2}{4\mu+2}$ $\Rightarrow \frac{\lambda-3}{2\mu}=\frac{2\lambda+2}{3\mu+1}=\frac{2\lambda-1}{2\mu+1}=k \ (let)$ $\Rightarrow \lambda-3=2\mu k$, $2\lambda+2=3\mu k+k$, $2\lambda-1=2\mu k+k$ $\Rightarrow \mu k=\frac{\lambda-3}{2}$, $2\lambda+2=3(\frac{\lambda-3}{2})+k$, $2\lambda-1=2(\frac{\lambda-3}{2})+k$ $\Rightarrow \mu k=\frac{\lambda-3}{2}$, $k=\frac{\lambda+13}{2}$, $k=\lambda+2$	1 1 2 1 1 1 1

	$\therefore\ \dfrac{\lambda+13}{2}=\lambda+2\Rightarrow\lambda=9$ $Also\ k=\lambda+2=11$ Hence The direction ratios of AP are $6,20,34$ i.e. $3,10,17$ Therefore, Equation of required line is $\dfrac{x-1}{3}=\dfrac{y-1}{10}=\dfrac{z-1}{17}$	**1** **1/2** **1/2**
34.	Let $\mathrm{I}=\int_0^{\pi}\dfrac{x\tan x}{\sec x+\tan x}dx$ .... (1) Using $\int_0^a f(x)dx=\int_0^a f(a-x)dx$ $\mathrm{I}=\int_0^{\pi}\dfrac{(\pi-x)\tan(\pi-x)}{\sec(\pi-x)+\tan(\pi-x)}dx$ $\Rightarrow\ \mathrm{I}=\int_0^{\pi}\dfrac{(\pi-x)(-\tan x)}{-\sec x-\tan x}dx$ $\Rightarrow\ \mathrm{I}=\int_0^{\pi}\dfrac{(\pi-x)\tan x}{\sec x+\tan x}dx$ .... (2) $(1)+(2)\Rightarrow 2I=\int_0^{\pi}\dfrac{(x+\pi-x)\tan x}{\sec x+\tan x}dx$ $\Rightarrow\ \mathrm{I}=\dfrac{\pi}{2}\int_0^{\pi}\dfrac{\sin x}{1+\sin x}dx$ $=\dfrac{\pi}{2}\int_0^{\pi}\dfrac{\sin x(1-\sin x)}{(1+\sin x)(1-\sin x)}dx$ $=\dfrac{\pi}{2}\int_0^{\pi}\left(\dfrac{\sin x}{\cos^2 x}-\dfrac{\sin^2 x}{\cos^2 x}\right)dx$ $=\dfrac{\pi}{2}\int_0^{\pi}\left(\tan x\sec x-\tan^2 x\right)dx$ $=\dfrac{\pi}{2}\int_0^{\pi}\left[\sec x\tan x-\sec^2 x+1\right]dx$ $=\dfrac{\pi}{2}\left[\sec x-\tan x+x\right]_0^{\pi}$ $=\dfrac{\pi}{2}\left[(-1-0+\pi)-(1-0+0)\right]$ $\therefore\ \mathrm{I}=\dfrac{\pi}{2}[\pi-2]$ **OR** Let $\mathrm{I}=\int_0^{\pi}\log(1+\cos x)dx$ ...(1) $\Rightarrow\ \mathrm{I}=\int_0^{\pi}\log\left[1+\cos(\pi-x)\right]dx$	 **1** **1** **1** **1** **1**

$$\Rightarrow \quad I = \int_0^{\pi} \log(1-\cos x)\,dx \qquad \text{...(2)}$$

Adding (1) and (2)

$$2I = \int_0^{\pi} \log\left[(1+\cos x)(1-\cos x)\right]dx$$

$$\Rightarrow I = \frac{1}{2}\int_0^{\pi} \log\left(1-\cos^2 x\right)dx$$

$$\Rightarrow \quad I = \frac{1}{2}\int_0^{\pi} \log\sin^2 x\,dx = \int_0^{\pi} \log\sin x\,dx$$

1

Since $\log\left[\sin(\pi - x)\right] = \log\sin x$

$$\therefore \qquad I = 2\int_0^{\pi/2} \log\sin x\,dx \qquad \text{...(3)}$$

$$\Rightarrow \quad I = 2\int_0^{\pi/2} \log\sin\left(\frac{\pi}{2} - x\right)dx$$

$$\Rightarrow \quad I = 2\int_0^{\pi/2} \log\cos x\,dx \qquad \text{...(4)}$$

Adding (3) and (4)

$$2I = 2\int_0^{\pi/2} \log\sin x\cos x\,dx$$

1

$$\Rightarrow \qquad I = \int_0^{\pi/2} \log\left(\frac{\sin 2x}{2}\right)dx$$

$$= \int_0^{\pi/2} \log\sin 2x\,dx - \log 2\int_0^{\pi/2} 1.dx$$

$$\Rightarrow \qquad I = I_1 - \log 2\left[x\right]_0^{\pi/2} = I_1 - \frac{\pi}{2}\log 2 \qquad \text{...(5)}$$

1

Now $\quad I_1 = \int_0^{\pi/2} \log\sin 2x\,dx$

Let $2x = t \Rightarrow 2dx = dt$

$$= \frac{1}{2}\int_0^{\pi} \log\sin t\,dt = \frac{1}{2}\int_0^{\pi} \log\sin x\,dx$$

(Changing t to x)

$$= \frac{1}{2}\times 2\int_0^{\pi/2} \log\sin x\,dx$$

$$\Rightarrow I_1 = \frac{1}{2}I$$

1

From (5) $\quad \Rightarrow \quad I = \frac{1}{2}I - \frac{\pi}{2}\log 2$

	$\Rightarrow \frac{1}{2}\text{I} = -\frac{\pi}{2}\log 2$ $\therefore \quad \text{I} = -\pi \log 2.$	**1**
35.	**One – one**: Let $x_1, x_2 \in R_+$ such that $f(x_1) = f(x_2)$ $9x_1^2 + 6x_1 - 5 = 9x_2^2 + 6x_2 - 5$ $\Rightarrow 9\left(x_1^2 - x_2^2\right) + 6\left(x_1 - x_2\right) = 0$ $\Rightarrow \left(x_1 - x_2\right)\left\{9\left(x_1 + x_2\right) + 6\right\} = 0$ $\Rightarrow \quad x_1 - x_2 = 0 \quad or \quad 9x_1 + 9x_2 + 6 = 0$ which is not possible $\Rightarrow \quad x_1 = x_2$ $\therefore \quad f$ is one-one. **Onto**: Let $y = 9x^2 + 6x - 5$ $\Rightarrow 9x^2 + 6x - (5 + y) = 0$ $\Rightarrow \quad x = \frac{-6 \pm \sqrt{36 + 4(9)(5+y)}}{2(9)} = \frac{-6 \pm \sqrt{36}\sqrt{1+5+y}}{18}$ $x = \frac{6(-1 \pm \sqrt{y+6})}{6(3)} = \frac{-1 \pm \sqrt{y+6}}{3}$ Now, $x \in R_+ \Rightarrow x \geq 0$ and so $x = \frac{-1-\sqrt{y+6}}{3}$ is rejected $\therefore x = \frac{-1+\sqrt{y+6}}{3}$ Now $x \geq 0 \Rightarrow \frac{-1+\sqrt{y+6}}{3} \geq 0$ $\Rightarrow \sqrt{y+6} \geq 1 \Rightarrow y + 6 \geq 1$ $\Rightarrow y \geq -5$ $\therefore R_f = \left\{y : y \in \left[-5, \infty\right)\right\} =$ codomain of f. $\therefore f$ is onto. Hence f is one one and onto function.	**1** **1** **1/2** **1** **1** **1/2**
	Section –E **[This section comprises solution of 3 case- study/passage-based questions of 4 marks each with two sub parts. Solution of the first two case study questions have three sub parts (i),(ii),(iii) of marks 1,1,2 respectively. Solution of the third case study question has two sub parts of 2 marks each.)**	
36.	V(t) = $t^3 - 3t^2 + 3t - 100$ (i) No, the above function cannot be used to estimate number of vehicles in the year 2020 because for 2020 we have t = 0 and V(0) = $0 - 0 + 0 - 100 = -100$ Which is not possible	**1**

	(ii) V(20) = $(20)^3 - 3(20)^2 + 3(20) - 100$ Therefore, the estimated number of vehicles in the year 2040 are 6760. (iii) $V'(t) = 3t^2 - 6t + 3$ $= 3(t^2 - 2t + 1)$ $= 3(t-1)^2 \geq 0.$ Hence $V(t)$ is always increasing function.	**1** **1** **1**
37.	Let E_1 is the event that a student is regular E_2 is the event that a student is irregular A is the event that a student attains grade A $P(E_1) = \frac{30}{100}$, $P(E_2) = \frac{70}{100}$ $P(A/E_1) = \frac{80}{100}$, $P(A/E_2) = \frac{10}{100}$ (i) Required Probability = $P(A/E_2) = \frac{10}{100} = \frac{1}{10}$ (ii) $Required\ Probability = P(A)$ $= P(E_1)P(A/E_1) + P(E_2)P(A/E_2)$ $= \frac{30}{100}.\frac{80}{100} + \frac{70}{100}.\frac{10}{100} = \frac{31}{100}$ (iii) $Required\ Probability = P(E_1/A)$ $= \frac{P(E_1)P(A/E_1)}{P(E_1)P(A/E_1) + P(E_2)P(A/E_2)}$ $= \frac{\frac{30}{100}.\frac{80}{100}}{\frac{30}{100}.\frac{80}{100} + \frac{70}{100}.\frac{10}{100}} = \frac{24}{31}$ **OR** (iii) $Required\ Probability = P(E_2/A)$ $= \frac{P(E_2)P(A/E_2)}{P(E_1)P(A/E_1) + P(E_2)P(A/E_2)}$ $= \frac{\frac{70}{100}.\frac{10}{100}}{\frac{30}{100}.\frac{80}{100} + \frac{70}{100}.\frac{10}{100}} = \frac{7}{31}$	**1** **1** **1** **1** **1** **1**
38.	(i) Let length, breadth and height of the tank are $x, x\ and\ y$ respectively According to the Question $\therefore x^2 y = 500 \Rightarrow y = \frac{500}{x^2}$ Surface Area $= S = x^2 + 4xy$ $S = x^2 + 4x(\frac{500}{x^2}) = x^2 + \frac{2000}{x}$ $\Rightarrow \frac{dS}{dx} = 2x - \frac{2000}{x^2}$	**1/2** **1/2**

	For maxima or minima, $\frac{dS}{dx}=0 \Rightarrow 2x-\frac{2000}{x^2}=0 \Rightarrow x=10\,m$ Now $\frac{d^2S}{dx^2}=2+\frac{4000}{x^3}$ and $\left(\frac{d^2S}{dx^2}\right)_{at\,x=10}=2+\frac{4000}{(10)^3}>0$ $\therefore$ Surface Area is minimum when $x=10\,m$ $\therefore$ Minimum Surface Area $=100+\frac{2000}{10}=300\,m^2$ (ii) $If\ \ x=10\,m\ \ then\ \ y=5\,m$ $and\ Volume\ of\ the$ tank $=x^2y=(10)^2(5)=500\,m^3$ $New\ Volume=(2x)^2y=4x^2y=4(10)^2(5)=2000\,m^3$ $\therefore\ Increase\ in\ Volume\ of\ the$ tank $=2000-500=1500\,m^3$ $\therefore\ \%\ Increase\ in\ Volume\ of\ the$ tank $=300\%$	**1/2** **1/2** **1/2** **1/2** **1/2** **1/2**