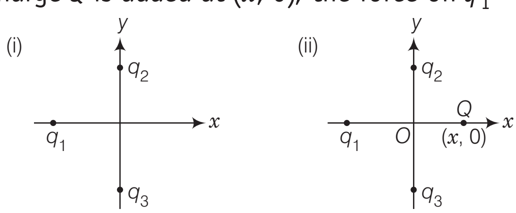
1

Electric Charges and Field

Multiple Choice Questions (MCQs)

Q. 1 In figure two positive charges q_2 and q_3 fixed along the y-axis, exert a net electric force in the + x-direction on a charge q_1 fixed along the x-axis. If a positive charge Q is added at $(x, 0)$, the force on q_1

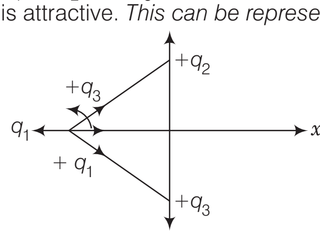


(a) shall increase along the positive x-axis
(b) shall decrease along the positive x-axis
(c) shall point along the negative x-axis
(d) shall increase but the direction changes because of the intersection of Q with q_2 and q_3

K **Thinking Process**

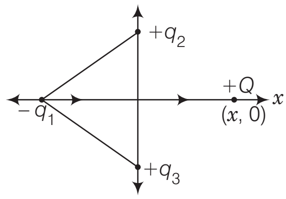
Find the nature of force between $q_1 - q_2$ and $q_1 - q_3$. Nature of force will give the type of charge q_1. Find the nature of force between newly introduced charge and charge q_1.

Ans. *(a)* The net force on q_1 by q_2 and q_3 is along the + x-direction, so nature of force between q_1, q_2 and q_1, q_3 is attractive. *This can be represent by the figure given below*



The attractive force between these charges states that q_1 is a negative charge (since, q_2 and q_3 are positive).

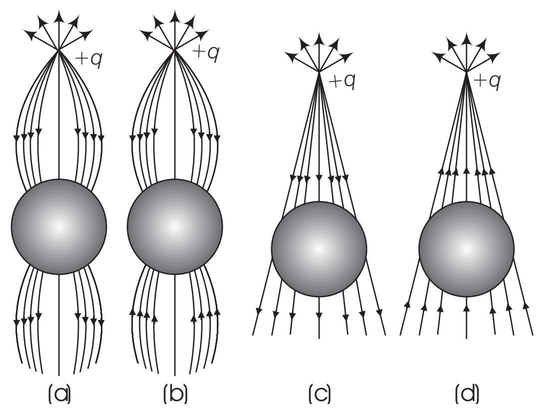
Thus, nature of force between q_1 and newly introduced charge Q (positive) is attractive and net force on q_1 by q_2, q_3 and Q are along the same direction as given in the diagram below



The figure given above clearly shows that the force on q_1 shall increase along the positive x-axis due to the positive charge Q.

Note *Unlike charges repel each other and like charges attract each other.*

Q. 2 A point positive charge is brought near an isolated conducting sphere (figure). The electric field is best given by



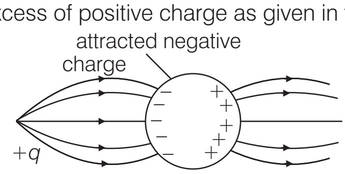
K **Thinking Process**

Bringing the point positive charge towards the conducting sphere, charges the sphere by induction process. Electric field lines passes through a charged body following some rules.

Ans. ***(a)*** When a positive point charge is brought near an isolated conducting sphere without touching the sphere, then the free electrons in the sphere are attracted towards the positive charge. This leaves an excess of positive charge on the rear (right) surface of sphere.

Both kinds of charges are bound in the metal sphere and cannot escape. They, therefore, reside on the surface.

Thus, the left surface of sphere has an excess of negative charge and the right surface of sphere has an excess of positive charge as given in the figure below

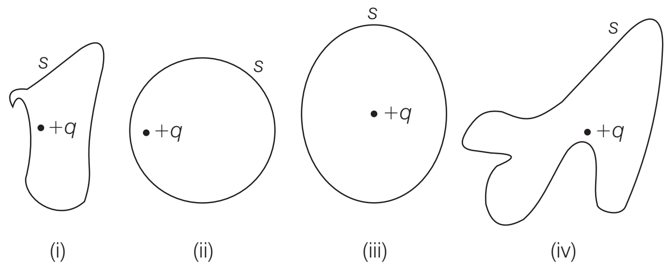


An electric field lines start from positive charge and ends at negative charge (in this case from point positive charge to negative charge created inside the sphere).

Also, electric field line emerges from a positive charge, in case of single charge and ends at infinity.

Here, all these conditions are fulfilled in Fig. (a).

Q. 3 The electric flux through the surface



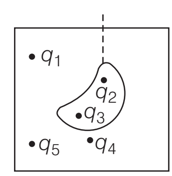
(a) in Fig. (iv) is the largest
(b) in Fig. (iii) is the least
(c) in Fig. (ii) is same as Fig. (iii) but is smaller than Fig. (iv)
(d) is the same for all the figures

Ans. *(d)* Gauss' law of electrostatics state that the total of the electric flux out of a closed surface is equal to the charge enclosed decided by the permittivity *i.e.,* Q electric $= \dfrac{Q}{\varepsilon_0}$.

Thus, electric flux through a surface doesn't depend on the shape, size or area of a surface but it depends on the number of charges enclosed by the surface.

So, here in this question, all the figures same electric flux as all of them has single positive charge.

Q. 4 Five charges q_1, q_2, q_3, q_4, and q_5 are fixed at their positions as shown in Figure, S is a Gaussian surface. The Gauss' law is given by $\int_S E.dS = \dfrac{q}{\varepsilon_0}$. Which of the following statements is correct?



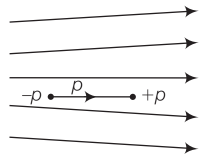
(a) **E** on the LHS of the above equation will have a contribution from q_1, q_5 and q_1, q_5 and q_3 while q on the RHS will have a contribution from q_2 and q_4 only
(b) **E** on the LHS of the above equation will have a contribution from all charges while q on the RHS will have a contribution from q_2 and q_4 only
(c) **E** on the LHS of the above equation will have a contribution from all charges while q on the RHS will have a contribution from q_1, q_3 and q_5 only
(d) Both **E** on the LHS and q on the RHS will have contributions from q_2 and q_4 only

Ans. *(b)* According to Gauss' law, the term q on the right side of the equation $\int_S \text{E.dS} = \dfrac{q}{\varepsilon_0}$ includes the sum of all charges enclosed by the surface.

The charges may be located anywhere inside the surface, if the surface is so chosen that there are some charges inside and some outside, the electric field on the left side of equation is due to all the charges, both inside and outside S.

So, E on LHS of the above equation will have a contribution from all charges while q on the RHS will have a contribution from q_2 and q_4 only.

Q. 5 Figure shows electric field lines in which an electric dipole P is placed as shown. Which of the following statements is correct?



(a) The dipole will not experience any force
(b) The dipole will experience a force towards right
(c) The dipole will experience a force towards left
(d) The dipole will experience a force upwards

K **Thinking Process**

Find the electric field strength on the charges of dipole.

Force varies directly with electric field strength i.e., higher the electric field strength greater the force and vice-versa.

Ans. *(c)* The space between the electric field lines is increasing, here from left to right and its characteristics states that, strength of electric field decreases with the increase in the space between electric field lines. As a result force on charges also decreases from left to right.

Thus, the force on charge $-q$ is greater than force on charge $+q$ in turn dipole will experience a force towards left.

Q. 6 A point charge $+q$ is placed at a distance d from an isolated conducting plane. The field at a point P on the other side of the plane is

(a) directed perpendicular to the plane and away from the plane
(b) directed perpendicular to the plane but towards the plane
(c) directed radially away from the point charge
(d) directed radially towards the point charge

Ans. *(a)* When a point positive charge brought near an isolated conducting plane, some negative charge developes on the surface of the plane towards the charge and an equal positive charge developes on opposite side of the plane. This process is called charging by induction.

Q. 7 A hemisphere is uniformely charged positively. The electric field at a point on a diameter away from the centre is directed

(a) perpendicular to the diameter
(b) parallel to the diameter
(c) at an angle tilted towards the diameter
(d) at an angle tilted away from the diameter

Ans. *(a)* When the point is situated at a point on diameter away from the centre of hemisphere charged uniformly positively, the electric field is perpendicular to the diameter. The component of electric intensity parallel to the diameter cancel out.

Multiple Choice Questions (More Than One Options)

Q. 8 If $\int_s \mathbf{E.dS} = 0$ over a surface, then

(a) the electric field inside the surface and on it is zero
(b) the electric field inside the surface is necessarily uniform
(c) the number of flux lines entering the surface must be equal to the number of flux lines leaving it
(d) all charges must necessarily be outside the surface

K **Thinking Process**

Go through Gauss' law in detail.

Ans. ***(c, d)***

$\oint_s \text{E.dS} = 0$ represents electric flux over the closed surface.

In general, $\oint_s \text{E.dS}$ means the algebraic sum of number of flux lines entering the surface and number of flux lines leaving the surface.

When $\oint_s \text{E.dS} = 0$, it means that the number of flux lines entering the surface must be equal to the number of flux lines leaving it.

Now, from Gauss' law, we know that $\oint_s \text{E.dS} = \frac{q}{\varepsilon_0}$ where q is charge enclosed by the surface. When $\int_s \text{E.dS} = 0$, $q = 0$ *i.e.,* net charge enclosed by the surface must be zero. Therefore, all other charges must necessarily be outside the surface. This is because charges outside because of the fact that charges outside the surface do not contribute to the electric flux.

Q. 9 The electric field at a point is

(a) always continuous
(b) continuous if there is no charge at that point
(c) discontinuous only if there is a negative charge at that point
(d) discontinuous if there is a charge at that point

Ans. ***(b, d)***

The electric field due to a charge Q at a point in space may be defined as the force that a unit positive charge would experience if placed at that point. Thus, electric field due to the charge Q will be continuous, if there is no charge at that point. It will be discontinuous if there is a charge at that point.

Q. 10 If there were only one type of charge in the universe, then

(a) $\oint_s \mathbf{E.\,dS} \neq 0$ on any surface
(b) $\oint_s \mathbf{E.\,dS} = 0$ if the charge is outside the surface
(c) $\oint_s \mathbf{E.\,dS}$ could not be defined
(d) $\oint_s \mathbf{E.\,dS} = \frac{q}{\varepsilon_0}$ if charges of magnitude q were inside the surface

Ans. ***(c, d)***

Gauss' law states that $\oint_s E.dS = \frac{q}{\varepsilon_0}$, where q is the charge enclosed by the surface. If the charge is outside the surface, then charge enclosed by the surface is $q = 0$ and thus, $\oint_s E.dS = 0$. Here, electric flux doesn't depend on the type or nature of charge.

Q. 11 Consider a region inside which there are various types of charges but the total charge is zero. At points outside the region,

(a) the electric field is necessarily zero
(b) the electric field is due to the dipole moment of the charge distribution only
(c) the dominant electric field is $\propto \frac{1}{r_3}$, for large r, where r is the distance from a origin in this regions
(d) the work done to move a charged particle along a closed path, away from the region, will be zero

Ans. ***(c, d)***

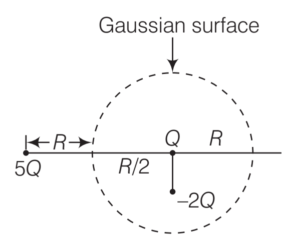
When there are various types of charges in a region, but the total charge is zero, the region can be supposed to contain a number of electric dipoles.

Therefore, at points outside the region (may be anywhere w.r.t. electric dipoles), the dominant electric field $\propto \frac{1}{r_3}$ for large r.

Further, as electric field is conservative, work done to move a charged particle along a closed path, away from the region will be zero.

Q. 12 Refer to the arrangement of charges in figure and a Gaussian surface of radius R with Q at the centre. Then,

(a) total flux through the surface of the sphere is $\frac{-Q}{\varepsilon_0}$
(b) field on the surface of the sphere is $\frac{-Q}{4\pi\varepsilon_0 R^2}$
(c) flux through the surface of sphere due to $5Q$ is zero
(d) field on the surface of sphere due to $-2Q$ is same everywhere



Ans. ***(a, c)***

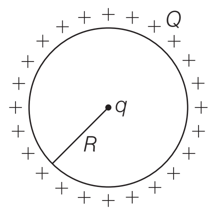
Gauss' law states that total electric flux of an enclosed surface is given by $\frac{q}{\varepsilon_0}$ where q is the charge enclosed by the surface. Thus, from figure,

Total charge inside the surface is $= Q - 2Q = -Q$

$\therefore$ Total flux through the surface of the sphere $= \frac{-Q}{\varepsilon_0}$

Now, considering charge $5Q$. Charge $5Q$ lies outside the surface, thus it makes no contribution to electric flux through the given surface.

Q. 13 A positive charge Q is uniformly distributed along a circular ring of radius R. A small test charge q is placed at the centre of the ring figure. Then,



(a) if $q > 0$ and is displaced away from the centre in the plane of the ring, it will be pushed back towards the centre

(b) if $q < 0$ and is displaced away from the centre in the plane of the ring, it will never return to the centre and will continue moving till it hits the ring

(c) if $q < 0$, it will perform SHM for small displacement along the axis

(d) q at the centre of the ring is in an unstable equilibrium within the plane of the ring for $q > 0$

Ans. ***(a, b, c)***

The positive charge Q is uniformly distributed at the outer surface of the enclosed sphere. Thus, electric field inside the sphere is zero.

So, the effect of electric field on charge q due to the positive charge Q is zero.

Now, the only governing factor is the attractive and repulsive forces between charges

(Q and q) there are two cases arise.

Case I When charge $q > 0$ *i.e.,* q is a positive charge, there creates a repulsive force between charge q and Q.

The repulsive forces of charge Q from all around the charge q will push it towards the centre if it is displaced from the centre of the ring.

Case II When charge $q < 0$ *i.e.,* q is a negative charge then there is an attractive force between charge Q and q.

If q is shifted from the centre, then the positive charges nearer to this charge will attract it towards itself and charge q will never return to the centre.

Very Short Answer Type Questions

Q. 14 An arbitrary surface encloses a dipole. What is the electric flux through this surface?

Ans. From Gauss' law, the electric flux through an enclosed surface is given by $\oint_s \mathbf{E}.d\mathbf{S} = \frac{q}{\varepsilon_0}$.

Here, q is the net charge inside that enclosed surface.

Now, the net charge on a dipole is given by $-q + q = 0$

$\therefore$ Electric flux through a surface enclosing a dipole $= \frac{-q+q}{\varepsilon_0} = \frac{0}{\varepsilon_0} = 0$

Q. 15 A metallic spherical shell has an inner radius R_1 and outer radius R_2. A charge Q is placed at the centre of the spherical cavity. What will be surface charge density on

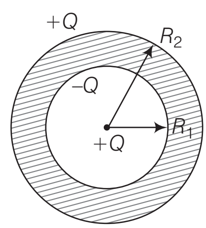
(i) the inner surface (ii) the outer surface?

Thinking Process

Let us draw the diagram as per the given situation. Using the induction process of charging distribute the charge on whole spherical shell.

Now, find the required surface charge density.

Ans. Here, the charge placed at the centre of the spherical cavity is positively charged. So, the charge created at the inner surface of the sphere, due to induction will be $-Q$ and due to this charge created at outer surface of the sphere is $+Q$.



Now, surface charge density on the inner surface $= \frac{-Q}{4\pi R_1^2}$ and

Surface charge density on the inner surface $= \frac{+Q}{4\pi R_2^2}$

Q. 16 The dimensions of an atom are of the order of an Angstrom. Thus, there must be large electric fields between the protons and electrons. Why, then is the electrostatic field inside a conductor zero?

Ans. The protons and electrons are bound into a atom with distinct and independent existence and neutral in charge.

Electrostatic fields are caused by the presence of excess charges.

But there can be no excess charge on the inter surface of an isolated conductor. So, the electrostatic fields inside a conductor is zero despite the fact that the dimensions of an atom are of the order of an Angstrom.

Q. 17 **If the total charge enclosed by a surface is zero, does it imply that the elecric field everywhere on the surface is zero? Conversely, if the electric field everywhere on a surface is zero, does it imply that net charge inside is zero.**

Ans. Gauss' law also implices that when the surface is so chosen that there are some chargas inside and some outside.

The flux in such situation is given by $\oint \text{E.dS} = \frac{q}{\varepsilon_0}$.

In such situations, the electric field in the LHS is due to all the charges both inside and outside the surface. The term q on the right side of the equation given by Gauss' law represent only the total charge inside the surface.

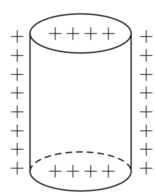
Thus, despite being total charge enclosed by a surface zero, it doesn't imply that the electric field everywhere on the surface is zero, the field may be normal to the surface.

Also, conversely if the electric field everywhere on a surface is zero, it doesn't imply that net charge inside it is zero.

i.e., Putting $E = 0$ in $\oint \text{E.dS} = \frac{q}{\varepsilon_0}$

we get $q = 0$.

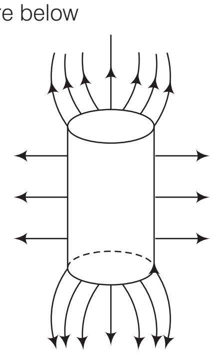
Q. 18 **Sketch the electric field lines for a uniformly charged hollow cylinder shown in figure.**



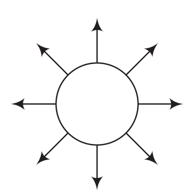
K **Thinking Process**

According to general properties, electric field lines start from positive charges and end at negative charges. If there is a single charge, they may start or end at infinity.

Ans. Thus, the electric field lines will start from positive charges and move towards infinity as given in the figure below



Side view



Top view

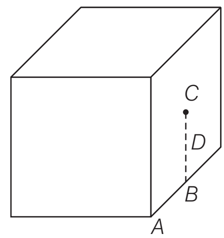
Q. 19 What will be the total flux through the faces of the cube as given in the figure with side of length a if a charge q is placed at?

(a) A a corner of the cube

(b) B mid-point of an edge of the cube

(c) C centre of a face of the cube

(d) D mid-point of B and C



K **Thinking Process**

Imagine logically about a symmetric figure in such a way that placed charge arrives at the centre of imaginated figure. Thus, applying Gauss' theorem to find flux linked with imaginary figure. Thereafter find flux linked with the given figure.

Ans. (a) There are eight corners in a cube so, total charge for the cube is $\frac{q}{8}$.

Thus, electric flux at $A = \frac{q}{8\varepsilon_0}$.

(b) When the charge q is place at B, middle point of an edge of the cube, it is being shared equally by 4 cubes. Therefore, total flux through the faces of the given cube $= q / 4\,\varepsilon_0$.

(c) When the charge q is placed at C, the centre of a face of the cube, it is being shared equally by 2 cubes. Therefore, total flux through the faces of the given cube $= q / 2\,\varepsilon_0$.

(d) Similarly, when charge q is placed at Q, the mid-point of B and C, it is being shared equally by 2 cubes. Therefore, total flux through the faces of the given cube $= q / 2\,\varepsilon_0$.

Short Answer Type Questions

Q. 20 A paisa coin is made up of Al-Mg alloy and weight 0.75g. It has a square shape and its diagonal measures 17 mm. It is electrically neutral and contains equal amounts of positive and negative charges.

K **Thinking Process**

Treating the paisa coins made up of only Al, find the magnitude of equal number of positive and negative charges. What conclusion do you draw from this magnitude?

Ans. Here, given quantities are

Mass of a paisa coin = 0.75g

Atomic mass of aluminium = 26.9815 g

Avogadro's number $= 6.023 \times 10^{23}$

$\therefore$ Number of aluminium atoms in one paisa coin,

$$N = \frac{6.023 \times 10^{23}}{26.9815} \times 0.75 = 1.6742 \times 10^{22}$$

As charge number of Al is 13, each atom of Al contains 13 protons and 13 electrons.

$\therefore$ Magnitude of positive and negative charges in one paisa coin $= N\,z\,e$

$$= 1.6742 \times 10^{22} \times 13 \times 1.60 \times 10^{-19}\,\text{C}$$

$$= 3.48 \times 10^{4}\ \text{C} = 34.8\,\text{kC}$$

This is a very large amount of charge. Thus, we can conclude that ordinary neutral matter contains enormous amount of ± charges.

Q. 21 Consider a coin of Question 20. It is electrically neutral and contains equal amounts of positive and negative charge of magnitude 34.8 kC. Suppose that these equal charges were concentrated in two point charges separated by

(i) 1 cm $\left(\sim \frac{1}{2} \times \text{diagonal of the one paisa coin}\right)$

(ii) 100 m (~ length of a long building)

(iii) 10^6 m (radius of the earth). Find the force on each such point charge in each of the three cases. What do you conclude from these results?

K **Thinking Process**

Force on a point charge $= \frac{|q|^2}{4\pi\varepsilon_0 r^2}$. *Here, q = magnitude of one charge, r = distance between two charges.*

Ans. Here,

$$q = \pm\ 34.8\,\text{RC} = \pm\ 3.48 \times 10^4\,\text{C}$$

$$r_1 = 1\,\text{cm} = 10^{-2}\ \text{m},\ r_2 = 100\,\text{m},\ r_3 = 10^6\ \text{m and } \frac{1}{4\pi\varepsilon_0} = 9 \times 10^9$$

$$F_1 = \frac{|q|^2}{4\pi\varepsilon_0\, r_1^2} = \frac{9 \times 10^9\ (3.48 \times 10^4)^2}{(10^{-2})^2} = 1.09 \times 10^{23}\ \text{N}$$

$$F_2 = \frac{|q|^2}{4\pi\varepsilon_0\, r_2^2} = \frac{9 \times 10^9\ (3.48 \times 10^4)^2}{(100)^2} = 1.09 \times 10^{15}\ \text{N}$$

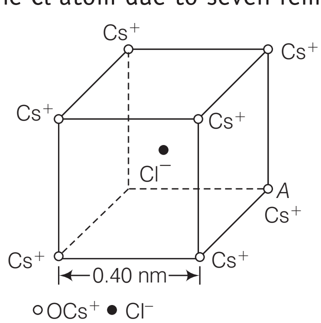
$$F_3 = \frac{|q|^2}{4\pi\varepsilon_0\, r_3^2} = \frac{9 \times 10^9\ (3.48 \times 10^4)^2}{(10^6)^2} = 1.09 \times 10^{7}\ \text{N}$$

Conclusion from this result We observe that when ± charges in ordinary neutral matter are separated as point charges, they exert an enormous force. Hence, it is very difficult to disturb electrical neutrality of matter.

Q. 22 Figure represents a crystal unit of cesium chloride, CsCl. The cesium atoms, represented by open circles are situated at the corners of a cube of side 0.40nm, whereas a Cl atom is situated at the centre of the cube. The Cs atoms are deficient in one electron while the Cl atom carries an excess electron.

(i) What is the net electric field on the Cl atom due to eight Cs atoms?

(ii) Suppose that the Cs atom at the corner *A* is missing. What is the net force now on the Cl atom due to seven remaining Cs atoms?



K **Thinking Process**

(i) Net force on a charge due to two equal and opposite charges will be zero. Also electric field on a charge is given by $E = \frac{F}{q}$ where $E =$ electric field, $F =$ force on charge q due to electric field, $q =$ magnitude of charge q

(ii) If a Cs atom is removed from the corner A then a singly charged negative Cs ion at A will appear.

Ans. (i) From the given figure, we can analyse that the chlorine atom is at the centre of the cube *i.e.,* at equal distance from all the eight corners of cube where cesium atoms are placed. Thus, due to symmetry the forces due to all Cs tons, on Cl atom will cancel out.

Hence, $$E = \frac{F}{q} \text{ where } F = 0$$

$$\therefore \quad E = 0$$

(ii) Thus, net force on Cl atom at A would be,

$$F = \frac{e^2}{4\pi\varepsilon_0 r^2},$$

where, $r =$ distance between Cl ion and Cs ion.

Applying Pythagorous theorem, we get

$$r = \sqrt{(0.20)^2 + (0.20)^2 + (0.20)^2} \times 10^{-9} \text{ m}$$
$$= 0.346 \times 10^{-9} \text{ m}$$

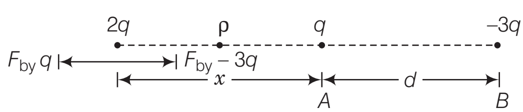
Now, $$F = \frac{q^2}{4\pi\varepsilon_0 r^2} = \frac{e^2}{4\pi\varepsilon_0 r_2}$$
$$= \frac{9\times 10^9 \,(1.6\times 10^{-19})^2}{(0.346\times 10^{-9})^2} = 1.92\times 10^{-9} \text{ N}$$

Q. 23 Two charges q and $-3q$ are placed fixed on x-axis separated by distance d. Where should a third charge $2q$ be placed such that it will not experience any force?

K **Thinking Process**

The force on any charge will be zero only if all forces are balanced i.e., force of attraction is balanced by force of repulsion.

Ans. Here, let us keep the charge $2q$ at a distance r from A.



Thus, charge $2q$ will not experience any force.

When, force of repulsion on it due to q is balanced by force of attraction on it due to $-3q$, at B, where $AB = d$.

Thus, force of attraction by $-3q$ = Force of repulsion by q

$$\Rightarrow \quad \frac{2q \times q}{4\pi\varepsilon_0 x^2} = \frac{2q \times 3q}{4\pi\varepsilon_0 (x+d)^2}$$

$$\Rightarrow \quad (x+d)^2 = 3x^2$$

$$\Rightarrow \quad x^2 + d^2 + 2xd = 3x^2$$

$$\Rightarrow \quad = 2x^2 - d^2$$

$\therefore \quad 2x^2 - 2dx - d^2 = 0$

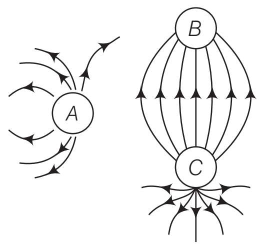
$$x = \frac{d}{2} \pm \frac{\sqrt{3}d}{2}$$

(Negative sign be between q and $-3q$ and hence is unadaptable.)

$$x = -\frac{d}{2} + \frac{\sqrt{3}d}{2}$$

$$= \frac{d}{2}(1 + \sqrt{3}) \text{to the left of } q.$$

Q. 24 Figure shows the electric field lines around three point charges A, B and C



(i) Which charges are positive?

(ii) Which charge has the largest magnitude? Why?

(iii) In which region or regions of the picture could the electric field be zero? Justify your answer.

(a) Near A
(b) Near B
(c) Near C
(d) Nowhere

K **Thinking Process**

(i) *Electric lines of forces always starts from a positive charge and ends at a negative charge. In case of a single charge, electric lines of force start from positive charge ends at infinity.*

(ii) *The magnitude of a charge depends on the number of lines of force enamating from a charge i.e., higher the number of lines of forces, higher the magnitude of charge and vice-versa.*

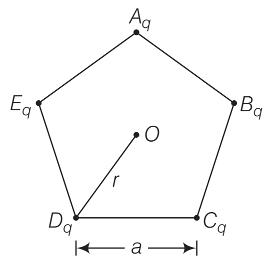
Ans. (i) Here, in the figure, the electric lines of force emanate from A and C. Therefore, charges A and C must be positive.

(ii) The number of electric lines of forces enamating is maximum for charge C here, so C must have the largest magnitude.

(iii) Point between two like charges where electrostatic force is zero is called netural point. So, the neutral point lies between A and C only.

Now the position of neutral point depends on the strength of the forces of charges. Here, more number of electric lines of forces shows higher strength of charge C than A. So, neutral point lies near A.

Q. 25 Five charges, q each are placed at the corners of a regular pentagon of side.



(a) (i) What will be the electric field at O, the centre of the pentagon?

(ii) What will be the electric field at O if the charge from one of the corners (say A) is removed?

(iii) What will be the electric field at O if the charge q at A is replaced by $-q$?

(b) How would your answer to (a) be affected if pentagon is replaced by n-sided regular polygon with charge q at each of its corners?

Thinking Process

Due to symmetry forces by all the charges are cancelled out.

Ans. (a) (i) The point O is equidistant from all the charges at the end point of pentagon. Thus, due to symmetry, the forces due to all the charges are cancelled out. As a result electric field at O is zero.

(ii) When charge q is removed a negative charge will develop at A giving electric field $E = \dfrac{q \times 1}{4\pi\varepsilon_0 r^2}$ along OA.

(iii) If charge q at A is replaced by $-q$, then two negative charges $-2q$ will develop there. Thus, the value of electric field $E = \dfrac{2q}{4\pi\varepsilon_0 r^2}$ along OA.

(b) When pentagon is replaced by n sided regular polygon with charge q at each of its corners, the electric field at O would continue to be zero as symmetricity of the charges is due to the regularity of the polygon. It doesn't depend on the number of sides or the number of charges.

Long Answer Type Questions

Q. 26 In 1959 Lyttleton and Bondi suggested that the expansion of the universe could be explained if matter carried a net charge. Suppose that the universe is made up of hydrogen atoms with a number density N, which is maintained a constant. Let the charge on the proton be $e_p = -(1+y)e$ where e is the electronic charge.

(a) Find the critical value of y such that expansion may start.

(b) Show that the velocity of expansion is proportional to the distance from the centre.

K **Thinking Process**

Expansion of the universe will start if the coulomb repulsion on a hydrogen atom, at R, is larger than the gravitational attraction.

Ans. (a) Let us suppose that universe is a perfect sphere of radius R and its constituent hydrogen atoms are distributed uniformly in the sphere.

As hydrogen atom contains one proton and one electron, charge on each hydrogen atom.

$$e_H = e_P + e = -(1+Y)e + e = -Ye = (Ye)$$

If E is electric field intensity at distance R, on the surface of the sphere, then according to Gauss' theorem,

$$\oint \text{E.ds} = \frac{q}{\varepsilon_0} \quad i.e., \; E\,(4\pi R^2) = \frac{4}{3}\frac{\pi R^3 N|Ye|}{\varepsilon_0}$$

$$E = \frac{1}{3}\frac{N|Ye|R}{\varepsilon_0} \qquad \text{...(i)}$$

Now, suppose, mass of each hydrogen atom $\simeq m_P$ = Mass of a proton, G_R = gravitational field at distance R on the sphere.

Then $$-4\pi R^2 G_R = 4\pi G\, m_P \left(\frac{4}{3}\pi R^3\right)N$$

$$\Rightarrow \quad G_R = \frac{-4}{3}\pi G m_P N R \qquad \text{...(ii)}$$

$\therefore$ Gravitational force on this atom is $F_G = m_P \times G_R = \dfrac{-4\pi}{3} G m_P^2 N R$ ...(iii)

Coulomb force on hydrogen atom at R is $F_C = (Ye)\,E = \dfrac{1}{3}\dfrac{NY^2e^2R}{\varepsilon_0}$ [from Eq. (i)]

Now, to start expansion $F_C > F_G$ and critical value of Y to start expansion would be when

$$F_C = F_G$$

$$\Rightarrow \quad \frac{1}{3}\frac{NY^2e^2R}{\varepsilon_0} = \frac{4\pi}{3} G m_P^2 N R$$

$$\Rightarrow \quad Y^2 = (4\pi\varepsilon_0)\,G\left(\frac{m_P}{e}\right)^2$$

$$= \frac{1}{9\times 10^9} \times (6.67\times 10^{-11})\left(\frac{(1.66\times 10^{-27})^2}{(1.6\times 10^{-19})^2}\right) = 79.8\times 10^{-38}$$

$$\Rightarrow \quad Y = \sqrt{79.8\times 10^{-38}} = 8.9\times 10^{-19} \simeq 10^{-18}$$

Thus, 10^{-18} is the required critical value of Y corresponding to which expansion of universe would start.

(b) Net force experience by the hydrogen atom is given by

$$F = F_C - F_G = \frac{1}{3}\frac{NY^2e^2R}{\varepsilon_0} - \frac{4\pi}{3}Gm_p^2NR$$

If acceleration of hydrogen atom is represent by d^2R/dt^2, then

$$m_p\frac{d^2R}{dt^2} = F = \frac{1}{3}\frac{NY^2e^2R}{\varepsilon_0} - \frac{4\pi}{3}Gm_p^2NR$$

$$= \left(\frac{1}{3}\frac{NY^2e^2}{\varepsilon_0} - \frac{4\pi}{3}Gm_p^2N\right)R$$

$\therefore$
$$\frac{d^2R}{dt^2} = \frac{1}{m_p}\left[\frac{1}{3}\frac{NY^2e^2}{\varepsilon_0} - \frac{4\pi}{3}Gm_p^2N\right]R = \alpha^2R \quad \text{...(iv)}$$

where,
$$\alpha^2 = \frac{1}{m_p}\left[\frac{1}{3}\frac{NY^2e^2}{\varepsilon_0} - \frac{4\pi}{3}Gm_p^2N\right]$$

The general solution of Eq. (iv) is given by $R = Ae^{\alpha t} + Be^{-\alpha t}$. We are looking for expansion, here, so $B = 0$ and $R = Ae^{\alpha t}$.

$\Rightarrow$ Velocity of expansion, $v = \frac{dR}{dt} = Ae^{\alpha t}(\alpha) = \alpha Ae^{\alpha t} = \alpha R$

Hence, $v \propto R$ *i.e.,* velocity of expansion is proportional to the distance from the centre.

Q. 27 Consider a sphere of radius R with charge density distributed as $p(r) = kr$ for $r \le R = 0$ for $r > R$.

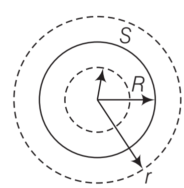
(a) Find the electric field as all points r.

(b) Suppose the total charge on the sphere is $2e$ where e is the electron charge. Where can two protons be embedded such that the force on each of them is zero. Assume that the introduction of the proton does not alter the negative charge distribution.

K **Thinking Process**

According to the given charge density distribution of the sphere of radius R i.e., $p(r) = Kr$ for $r \le R = 0$ for $r > R$ it is obvious that the electric field is radial.

Ans. (a) Let us consider a sphere S of radius R and two hypothetic sphere of radius $r < R$ and $r > R$.



Now, for point $r < R$, electric field intensity will be given by,

$$\oint \text{E.dS} = \frac{1}{\varepsilon_0}\int \rho dV$$

$$\left[\text{For } dV,\ V = \frac{4}{3}\pi r^3 \Rightarrow dV = 3\times\frac{4}{3}\pi r^3\, dr = 4\pi r^2 dr\right]$$

$$\Rightarrow \oint \text{E.dS} = \frac{1}{\varepsilon_0}4\pi K\int_0^r r^3 dr \qquad (\because p(r) = Kr)$$

$$\Rightarrow (E)\,4\pi r^2 = \frac{4\pi K}{\varepsilon_0}\frac{r^4}{4}$$

$$\Rightarrow E = \frac{1}{4\varepsilon_0}Kr^2$$

Here, charge density is positive.

So, direction of E is radially outwards.

For points $r > R$, electric field intensity will be given by

$$\oint \mathbf{E}.d\mathbf{S} = \frac{1}{\varepsilon_0}\int \rho.dV$$

$$\Rightarrow \quad E(4\pi r^2) = \frac{4\pi K}{\varepsilon_0}\int_0^R r^3 dr = \frac{4\pi K}{\varepsilon_0}\frac{R^4}{4}$$

$$\Rightarrow \quad E = \frac{K}{4\varepsilon_0}\frac{R^4}{r^2}$$

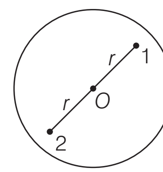
Charge density is again positive. So, the direction of E is radially outward.

(b) The two protons must be on the opposite sides of the centre along a diameter following the rule of symmetry. This can be shown by the figure given below. Charge on the sphere,

$$q = \int_0^R \rho dV = \int_0^R (Kr)\, 4\pi r^2 dr$$

$$q = 4\pi K \frac{R^4}{4} = 2e$$

$$\therefore \quad K = \frac{2e}{\pi R^4}$$



If protons 1 and 2 are embedded at distance r from the centre of the sphere as shown,then attractive force on proton 1 due to charge distribution is

$$F_1 = eE = \frac{-eKr^2}{4\varepsilon_0}$$

Repulsive force on proton 1 due to proton 2 is

$$F_2 = \frac{e^2}{4\pi\varepsilon_0 (2r)^2}$$

Net force on proton 1, $\quad F = F_1 + F_2$

$$F = \frac{-eKr^2}{4\varepsilon_0} + \frac{e^2}{16\pi\varepsilon_0 r^2}$$

So, $\quad F = \left[\frac{-er^2}{4\varepsilon_0}\frac{Ze}{\pi R^4} + \frac{e^2}{16\pi\varepsilon_0 r^4}\right]$

Thus, net force on proton 1 will be zero, when

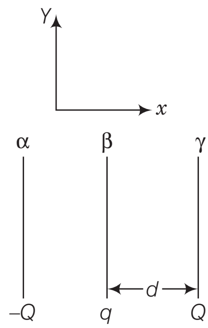
$$\frac{er^2 2e}{4\varepsilon_0 \pi R^4} = \frac{e^2}{16\pi\varepsilon_0 r}$$

$$\Rightarrow \quad r^4 = \frac{R^4}{8}$$

$$\Rightarrow \quad r = \frac{R}{(8)^{1/4}}$$

This is the distance of each of the two protons from the centre of the sphere.

Q. 28 Two fixed, identical conducting plates (α and β), each of surface area S are charged to $-Q$ and q, respectively, where $Q > q > 0$. A third identical plate (γ), free to move is located on the other side of the plate with charge q at a distance d (figure). The third plate is released and collides with the plate β. Assume the collision is elastic and the time of collision is sufficient to redistribute charge amongst β and γ.



(a) Find the electric field acting on the plate γ before collision.
(b) Find the charges on β and γ after the collision.
(c) Find the velocity of the plate γ after the collision and at a distance d from the plate β.

Ans. (a) Net electric field at plate γ before collision is equal to the sum of electric field at plate γ due to plate α and β.

The electric field at plate γ due to plate α is $E_1 = \dfrac{-Q}{S(2\varepsilon_0)}$, to the left.

The electric field at plate γ due to plate β is $E_2 = \dfrac{q}{S(2\varepsilon_0)}$, to the right.

Hence, the net electric field at plate γ before collision.

$$E = E_1 + E_2 = \frac{q - Q}{S\ (2\varepsilon_0)}, \text{ to the left, if } Q > q.$$

(b) During collision, plates β and γ are together. Their potentials become same.

Suppose charge on plate β is q_1 and charge on plate γ is q_2. At any point O, in between the two plates, the electric field must be zero.

Electric field at O due to plate $\alpha = \dfrac{-Q}{S(2\varepsilon_0)}$, to the left

Electric field at O due to plate $\beta = \dfrac{q_1}{S(2\varepsilon_0)}$, to the right

Electric field at O due to plate $\gamma = \dfrac{q_2}{S(2\varepsilon_0)}$, to the left

As the electric field at O is zero, therefore

$$\frac{Q + q_2}{S(2\varepsilon_0)} = \frac{q_1}{S(2\varepsilon_0)}$$

$\therefore$
$$Q + q_2 = q_1$$
$$Q = q_1 - q_2 \qquad \text{...(i)}$$

As there is no loss of charge on collision,

$$Q + q = q_1 + q_2 \qquad \text{...(ii)}$$

On solving Eqs. (i) and (ii), we get

$$q_1 = (Q + q/2) = \text{charge on plate } \beta$$
$$q_2 = (q/2) = \text{charge on plate } \gamma$$

(c) After collision, at a distance d from plate β,

Let the velocity of plate γ be v. After the collision, electric field at plate γ is

$$E_2 = \frac{-Q}{2\varepsilon_0 S} + \frac{(Q + q/2)}{2\varepsilon_0 S} = \frac{q/2}{2\varepsilon_0 S} \text{ to the right.}$$

Just before collision, electric field at plate γ is $E_1 = \dfrac{Q - q}{2\varepsilon_0 S}$.

If F_1 is force on plate γ before collision, then $F_1 = E_1 Q = \dfrac{(Q - q)Q}{2\varepsilon_0 S}$

Total work done by the electric field is round trip movement of plate γ

$$W = (F_1 + F_2)d$$
$$= \frac{[(Q - q)Q + (q/2)^2]d}{2\varepsilon_0 S} = \frac{(Q - q/2)^2 d}{2\varepsilon_0 S}$$

If m is mass of plate γ, the KE gained by plate $\gamma = \dfrac{1}{2}mv^2$

According to work-energy principle, $\dfrac{1}{2}mv^2 = W = \dfrac{(Q - q/2)^2 d}{2\varepsilon_0 S}$

$$\gamma = (Q - q/2)\left(\frac{d}{m\varepsilon_0 S}\right)^{1/2}$$

Q. 29 There is another useful system of units, besides the SI/MKS. A system, called the CGS (Centimeter-Gram-Second) system. In this system, Coulomb's law is given by $\mathbf{F} = \dfrac{Qq}{r^2}\hat{\mathbf{r}}$.

where the distance r is measured in cm $(= 10^{-2}\,\mu)$, F in dynes $(= 10^{-5}\text{N})$ and the charges in electrostatic units (es units), where 1 es unit of charge $= \dfrac{1}{[3]} \times 10^{-9}$C. The number [3] actually arises from the speed of light in vacuum which is now taken to be exactly given by $c = 2.99792458 \times 10^8$ m/s. An approximate value of c, then is $c = 3 \times 10^8$ m/s.

(i) Show that the Coulomb's law in CGS units yields 1 esu of charge $= 1\,(\text{dyne})^{1/2}$ cm. Obtain the dimensions of units of charge in terms of mass M, length L and time T. Show that it is given in terms of fractional powers of M and L.

(ii) Write 1 esu of charge $= xC$, where x is a dimensionless number. Show that this gives $\dfrac{1}{4\pi\varepsilon_0} = \dfrac{10^{-9}}{x^2}\dfrac{\text{Nm}^2}{\text{C}^2}$. With $x = \dfrac{1}{[3]} \times 10^{-9}$, we have $\dfrac{1}{4\pi\varepsilon_0} = [3]^2 \times 10^9 \dfrac{\text{Nm}^2}{\text{C}^2}$, $\dfrac{1}{4\pi\varepsilon_0} = (2.99792458)^2 \times 10^9 \dfrac{\text{Nm}^2}{\text{C}^2}$ (exactly).

Ans. (i) From the relation, $F = \frac{Qq}{r^2} = 1 \text{ dyne} = \frac{[1 \text{ esu of charge}]^2}{[1 \text{ cm}]^2}$

So, 1 esu of charge $= (1 \text{ dyne})^{1/2} \times 1 \text{ cm} = F^{1/2}.L = [\text{MLT}^{-2}]^{1/2}\text{L}$

$\Rightarrow$ 1 esu of charge $= \text{M}^{1/2}\text{L}^{3/2}\text{T}^{-1}$.

Thus, esu of charge is represented in terms of fractional powers $\frac{1}{2}$ of M and $\frac{3}{2}$ of L.

(ii) Let 1 esu of charge $= x\ C$, where x is a dimensionless number. Coulomb force on two charges, each of magnitude 1 esu separated by 1 cm is dyne $= 10^{-5}$N. This situation is equivalent to two charges of magnitude $x\ C$ separated by 10^{-2} m.

$$\therefore \quad F = \frac{1}{4\pi\varepsilon_0} \frac{x^2}{(10^{-2})^2} = 1 \text{ dyne} = 10^{-5}\text{N}$$

$$\therefore \quad \frac{1}{4\pi\varepsilon_0} = \frac{10^{-9}}{x^2} \frac{\text{Nm}^2}{\text{C}^2}$$

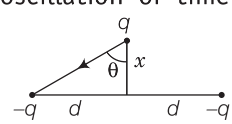
Taking, $$x = \frac{1}{|3| \times 10^9},$$

we get, $$\frac{1}{4\pi\varepsilon_0} = 10^{-9} \times |3|^2 \times 10^{18} \frac{\text{Nm}^2}{\text{C}^2} = 9 \times 10^9 \frac{\text{Nm}^2}{\text{C}^2}$$

If $|3| \to 2.99792458$, we get $\frac{1}{4\pi\varepsilon_0} = 8.98755 \times 10^9 \text{ Nm}^2\text{C}^{-2}$.

Q. 30 **Two charges $-q$ each are fixed separated by distance $2d$. A third charge q of mass m placed at the mid-point is displaced slightly by $x(x << d)$ perpendicular to the line joining the two fixed charged as shown in figure. Show that q will perform simple harmonic oscillation of time period.**

$$T = \left[\frac{8\pi^3\varepsilon_0 md^3}{q^2}\right]^{1/2}$$



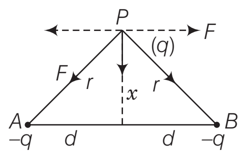
K **Thinking Process**

For motion of charge q to be simple harmonic, force on charge q must be proportional to its distance from the centre O and is directed towards O.

Ans. Let us elaborate the figure first.

Given, two charge $-q$ at A and B

$$AB = AO + OB = 2d$$



x = small distance perpendicular to O.

i.e., $x < d$ mass of charge q is. So, force of attraction at P towards A and B are each $F = \frac{q\ (q)}{4\pi\varepsilon_0\ r^2}$, where $AP = BP = r$

Horizontal components of these forces F_n are cancel out. Vertical components along PO add.

If $\angle APO = O$, the net force on q along PO is $F' = 2F \cos Q$

$$= \frac{2q^2}{4\pi\varepsilon_0 r^2}\left(\frac{x}{r}\right)$$

$$= \frac{2q^2 x}{4\pi\varepsilon_0 (d^2 + x^2)^{3/2}}$$

When, $$x << d, F' = \frac{2q^2 x}{4\pi\varepsilon_0 d^3} = Kx$$

where, $$K = \frac{2q^2}{4\pi\varepsilon_0 d^3}$$

$\Rightarrow$ $$F \propto x$$

i.e., force on charge q is proportional to its displacement from the centre O and it is directed towards O.

Hence, motion of charge q would be simple harmonic, where

$$\omega = \sqrt{\frac{K}{m}}$$

and $$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{K}}$$

$$= 2\pi\sqrt{\frac{m.4\pi\varepsilon_0 d^3}{2q^2}} = \left[\frac{8\pi^3 \varepsilon_0 m d^3}{q^2}\right]^{1/2}$$

Q. 31 **Total charge $-Q$ is uniformly spread along length of a ring of radius R. A small test charge $+q$ of mass m is kept at the centre of the ring and is given a gentle push along the axis of the ring.**

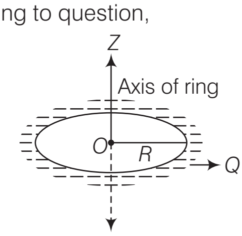
(a) Show that the particle executes a simple harmonic oscillation.

(b) Obtain its time period.

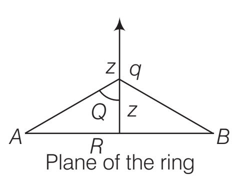
K **Thinking Process**

For simple harmonic oscillation, force on q is proportional to negative of its displacement.

Ans. Let us draw the figure according to question,



A gentle push on q along the axis of the ring gives rise to the situation shown in the figure below.



Taking line elements of charge at A and B, having unit length, then charge on each elements.

$$dF = 2\left(-\frac{Q}{2\pi R}\right)q \times \frac{1}{4\pi\varepsilon_0}\frac{1}{r^2}\cos\theta$$

Total force on the charge q, due to entire ring

$$F = -\frac{Qq}{\pi R}(\pi R)\cdot\frac{1}{4\pi\varepsilon_0}\frac{1}{r^2}\cdot\frac{2}{r}$$

$$F = -\frac{Qqz}{4\pi\varepsilon_0 (Z^2 + R^2)^{3/2}}$$

Here, $Z << R$, $$F = -\frac{Qqz}{4\pi\varepsilon_0 R^3} = -Kz$$

where $$\frac{Qq}{4\pi\varepsilon_0 R^3} = \text{constant}$$

$\Rightarrow$ $$F \propto -Z$$

Clearly, force on q is proportional to negative of its displacement. Therefore, motion of q is simple harmonic.

$$\omega = \sqrt{\frac{K}{m}} \text{ and } T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{K}}$$

$$T = 2\pi\sqrt{\frac{m\, 4\pi\varepsilon_0 R^3}{Qq}}$$

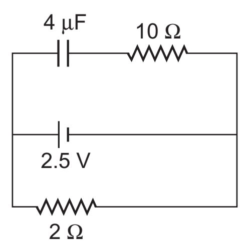
$\Rightarrow$ $$T = 2\pi\sqrt{\frac{4\pi\varepsilon_0\, m R^3}{Qq}}$$

2

Electrostatic Potential and Capacitance

Multiple Choice Questions (MCQs)

Q. 1 A capacitor of 4 μF is connected as shown in the circuit. The internal resistance of the battery is 0.5Ω. The amount of charge on the capacitor plates will be



(a) 0 (b) 4 μ C (c) 16μC (d) 8 μC

Thinking Process

In this problem, the three parallel branches of circuit can be considered in parallel , combination with one-another. Therefore, potential difference across each branch is same. The capacitor offers infinite resistance in DC circuit, therefore no current flows through capacitor and 10Ω resistance, leaving zero potential difference across 10Ω resistance.

Thus, potential difference across lower and middle branch of circuit is equal to the potential difference across capacitor of upper branch of circuit.

Ans. *(d)* Current flows through 2Ω resistance from left to right, is given by

$$I = \frac{V}{R + r} = \frac{2.5\text{V}}{2 + 0.5} = 1\text{A}$$

The potential difference across 2Ω resistance $V = IR = 1 \times 2 = 2\text{ V}$

Since, capacitor is in parallel with 2Ω resistance, so it also has 2V potential difference across it.

The charge on capacitor

$$q = CV = (2\,\mu\text{F}) \times 2\text{V} = 8\mu\text{C}$$

Note *The potential difference across 2Ω resistance solely occurs across capacitor as no potential drop occurs across 10Ω resistance.*

Q. 2 A positively charged particle is released from rest in an uniform electric field. The electric potential energy of the charge

(a) remains a constant because the electric field is uniform
(b) increases because the charge moves along the electric field
(c) decreases because the charge moves along the electric field
(d) decreases because the charge moves opposite to the electric field

K **Thinking Process**

In this problem, the relationship between E and V is actualised .

Ans. *(c)* The direction of electric field is always perpendicular to one equipotential surface maintained at high electrostatic potential to other equipotential surface maintained at low electrostatic potential.

The positively charged particle experiences electrostatic force along the direction of electric field *i.e.*, from high electrostatic potential to low electrostatic potential. Thus, the work is done by the electric field on the positive charge, hence electrostatic potential energy of the positive charge decreases.

Q. 3 Figure shows some equipotential lines distributed in space. A charged object is moved from point *A* to point *B*.

(a) The work done in Fig. (i) is the greatest
(b) The work done in Fig. (ii) is least
(c) The work done is the same in Fig. (i), Fig.(ii) and Fig. (iii)
(d) The work done in Fig. (iii) is greater than Fig. (ii) but equal to that in

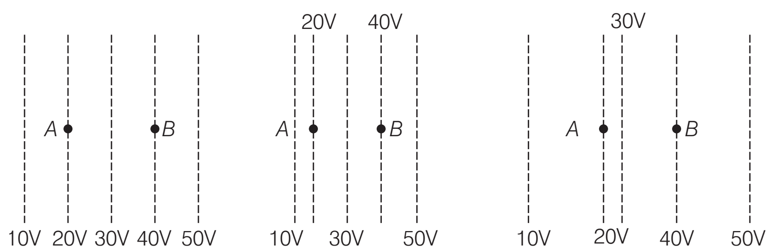


Fig. (i) **Fig. (ii)** **Fig. (iii)**

Ans. *(c)* The work done by a electrostatic force is given by $W_{12} = q(V_2 - V_1)$. Here initial and final potentials are same in all three cases and same charge is moved, so work done is same in all three cases.

Q. 4 The electrostatic potential on the surface of a charged conducting sphere is 100V. Two statements are made in this regard

S_1 at any point inside the sphere, electric intensity is zero.

S_2 at any point inside the sphere, the electrostatic potential is 100V.

Which of the following is a correct statement?

(a) S_1 is true but S_2 is false
(b) Both S_1 and S_2 are false
(c) S_1 is true, S_2 is also true and S_1 is the cause of S_2
(d) S_1 is true, S_2 is also true but the statements are independant

Ans. *(c)* In this problem, the electric field intensity E and electric potential V are related as

$$E = -\frac{dV}{dr}$$

Electric field intensity $E = 0$ suggest that $\frac{dV}{dr} = 0$

This imply that V = constant.

Thus, $E = 0$ inside the charged conducting sphere causes , the same electrostatic potential 100V at any point inside the sphere.

Note *V equals zero does not necessary imply that $E = 0$ e.g., the electric potential at any point on the perpendicular bisector due to electric dipole is zero but E not.*

$E = 0$ does not necessary imply that $V = 0$ e.g., the electric field intensity at any point inside the charged spherical shell is zero but there may exist non-zero electric potential.

Q. 5 Equipotentials at a great distance from a collection of charges whose total sum is not zero are approximately

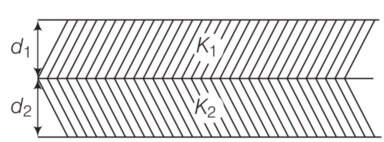
(a) spheres
(b) planes
(c) paraboloids
(d) ellipsoids

Ans. *(a)* In this problem, the collection of charges, whose total sum is not zero, with regard to great distance can be considered as a point charge. The equipotentials due to point charge are spherical in shape as electric potential due to point charge q is given by

$$V = k_e \frac{q}{r}$$

This suggest that electric potentials due to point charge is same for all equidistant points. The locus of these equidistant points, which are at same potential, form spherical surface.

Q. 6 A parallel plate capacitor is made of two dielectric blocks in series. One of the blocks has thickness d_1 and dielectric constant K_1 and the other has thickness d_2 and dielectric constant K_2 as shown in figure. This arrangement can be thought as a dielectric slab of thickness $d\ (= d_1 + d_2)$ and effective dielectric constant K. The K is`



(a) $\frac{K_1 d_1 + K_2 d_2}{d_1 + d_2}$
(b) $\frac{K_1 d_1 + K_2 d_2}{K_1 + K_2}$
(c) $\frac{K_1 K_2 (d_1 + d_2)}{(K_1 d_1 + K_2 d_2)}$
(d) $\frac{2K_1 K_2}{K_1 + K_2}$

K **Thinking Process**

In this problem, the system can be considered as the series combination of two capacitors which are of thicknesses d_1 and filled with dielectric medium of dielectric constant K_1 and thicknesses d_2 and filled with dielectric medium of dielectric constant K_2.

Ans. *(c)* The capacitance of parallel plate capacitor filled with dielectric block has thickness d_1 and dielectric constant K_2 is given by

$$C_1 = \frac{K_1 \varepsilon_0 A}{d_1}$$

Similarly, capacitance of parallel plate capacitor filled with dielectric block has thickness d_2 and dielectric constant K_2 is given by

$$C_2 = \frac{K_2 \varepsilon_0 A}{d_2}$$

Since, the two capacitors are in series combination, the equivalent capacitance is given by

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$$

or

$$C = \frac{C_1 C_2}{C_1 + C_2} = \frac{\frac{K_1 \varepsilon_0 A}{d_1} \frac{K_2 \varepsilon_0 A}{d_2}}{\frac{K_1 \varepsilon_0 A}{d_1} + \frac{K_2 \varepsilon_0 A}{d_2}} = \frac{K_1 K_2 \varepsilon_0 A}{K_1 d_2 + K_2 d_1} \quad \text{...(i)}$$

But the equivalent capacitances is given by

$$C = \frac{K \varepsilon_0 A}{d_1 + d_2}$$

On comparing, we have

$$K = \frac{K_1 K_2 (d_1 + d_2)}{K_1 d_2 + K_2 d_1}$$

Note *For the equivalent capacitance of the combination, thickness is equal to the separation between two plates i.e., $d_1 + d_2$ and dielectric constant K.*

Multiple Choice Questions (More Than One Options)

Q. 7 Consider a uniform electric field in the $\hat{\mathbf{z}}$-direction. The potential is a constant

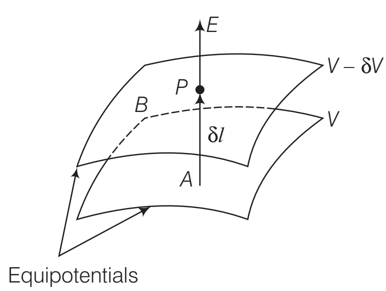
(a) in all space

(b) for any x for a given z

(c) for any y for a given z

(d) on the x-y plane for a given z

Ans. *(b, c, d)*



Here, the figure electric field is always remain in the direction in which the potential decreases steepest. Its magnitude is given by the change in the magnitude of potential per unit displacement normal to the equipotential surface at the point.

The electric field in z-direction suggest that equipotential surfaces are in x-y plane. Therefore the potential is a constant for any x for a given z, for any y for a given z and on the x-y plane for a given z.

Note *The shape of equipotential surfaces depends on the nature and type of distribution of charge e.g., point charge leads to produce spherical surfaces whereas line charge distribution produces cylindrical equipotential surfaces.*

Q. 8 Equipotential surfaces

(a) are closer in regions of large electric fields compared to regions of lower electric fields

(b) will be more crowded near sharp edges of a conductor

(c) will be more crowded near regions of large charge densities

(d) will always be equally spaced

K **Thinking Process**

In this problem, we need a relation between the electric field intensity E and electric potential V given by

$$E = -\frac{dV}{dr}$$

Ans. ***(a,b,c)***

The electric field intensity E is inversely proportional to the separation between equipotential surfaces. So, equipotential surfaces are closer in regions of large electric fields.

Since, the electric field intensities is large near sharp edges of charged conductor and near regions of large charge densities. Therefore, equipotential surfaces are closer at such places.

Q. 9 The work done to move a charge along an equipotential from A to B

(a) cannot be defined as $-\int_A^B \mathbf{E}.d\mathbf{l}$

(b) must be defined as $-\int_A^B \mathbf{E}.d\mathbf{l}$

(c) is zero

(d) can have a non-zero value

Ans. ***(c)*** Work done in displacing a charge particle is given by $W_{12} = q(V_2 - V_1)$ and the line integral of electrical field from point 1 to 2 gives potential difference $V_2 - V_1 = -\int_1^2 E.dl$

For equipotential surface, $V_2 - V_1 = 0$ and $W = 0$.

Note *If displaced charged particle is + 1 C, then and only then option (b) is correct. But the NCERT exemplar book has given (b) as correct options which probably not so under given conditions.*

Q. 10 In a region of constant potential

(a) the electric field is uniform

(b) the electric field is zero

(c) there can be no charge inside the region

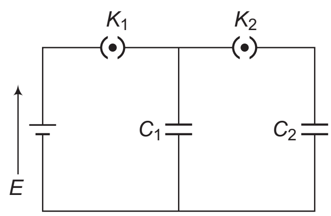
(d) the electric field shall necessarily change if a charge is placed outside the region

Ans. *(b, c)*

The electric field intensity E and electric potential V are related as $E = 0$ and for $V =$ constant, $\frac{dV}{dr} = 0$

This imply that electric field intensity $E = 0$.

Q. 11 In the circuit shown in figure initially key K_1 is closed and key K_2 is open. Then K_1 is opened and K_2 is closed (order is important).
[Take Q_1' and Q_2' as charges on C_1 and C_2 and V_1 and V_2 as voltage respectively.]



Then,

(a) charge on C_1 gets redistributed such that $V_1 = V_2$
(b) charge on C_1 gets redistributed such that $Q_1' = Q_2'$
(c) charge on C_1 gets redistributed such that $C_1V_1 + C_2V_2 = C_1E$
(d) charge on C_1 gets redistributed such that $Q_1' + Q_2' = Q$

K **Thinking Process**

When key K_1 is closed and key K_2 is open, the capacitor C_1 is charged by cell and when K is opened and K_2 is closed, the charge stored by capacitor C_1 gets redistributed between C_1 and C_2.

Ans. *(a, d)*

The charge stored by capacitor C_1 gets redistributed between C_1 and C_2 till their potentials become same *i.e.*, $V_2 = V_1$. By law of conservation of charge, the charge stored in capacitor C_1 when key K_1 is closed and key K_2 is open is equal to sum of charges on capacitors C_1 and C_2 when K_1 is opened and K_2 is closed *i.e.*,

$$Q'_1 + Q'_2 = Q$$

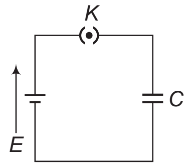
Q. 12 If a conductor has a potential $V \neq 0$ and there are no charges anywhere else outside, then

(a) there must be charges on the surface or inside itself
(b) there cannot be any charge in the body of the conductor
(c) there must be charges only on the surface
(d) there must be charges inside the surface

Ans. *(a, b)*

The charge resides on the outer surface of a closed charged conductor.

Q. 13 A parallel plate capacitor is connected to a battery as shown in figure. Consider two situations.



A. Key K is kept closed and plates of capacitors are moved apart using insulating handle.

B. Key K is opened and plates of capacitors are moved apart using insulating handle.

Choose the correct option(s).

(a) In **A** Q remains same but C changes

(b) In **B** V remains same but C changes

(c) In **A** V remains same and hence Q changes

(d) In **B** Q remains same and hence V changes

K **Thinking Process**

The cell is responsible for maintaining potential difference equal to its emf across connected capacitor in every circumstance. However, charge stored by disconnected charged capacitor remains conserved.

Ans. ***(c , d)***

Case A When key K is kept closed and plates of capacitors are moved apart using insulating handle, the separation between two plates increases which in turn decreases its capacitance $\left(C = \frac{K\varepsilon_0 A}{d}\right)$ and hence, the charge stored decreases as $Q = CV$ (potential continue to be the same as capacitor is still connected with cell).

Case B When key K is opened and plates of capacitors are moved apart using insulating handle, charge stored by disconnected charged capacitor remains conserved and with the decreases of capacitance, potential difference V increases as $V = Q/C$.

Very Short Answer Type Questions

Q. 14 Consider two conducting spheres of radii R_1 and R_2 with $R_1 > R_2$. If the two are at the same potential, the larger sphere has more charge than the smaller sphere. State whether the charge density of the smaller sphere is more or less than that of the larger one.

K **Thinking Process**

The electric potentials on spheres due to their charge need to be written in terms of their charge densities.

Ans. Since, the two spheres are at the same potential, therefore

$$\frac{kq_1}{R_1} = \frac{kq_2}{R_2} \Rightarrow \frac{kq_1R_1}{4\pi R_1^2} = \frac{kq_2R_2}{4\pi R_2^2}$$

or $$\sigma_1 R_1 = \sigma_2 R_2 \Rightarrow \frac{\sigma_1}{\sigma_2} = \frac{R_2}{R_1}$$

$$R_2 > R_1$$

This imply that $\sigma_1 > \sigma_2$.

The charge density of the smaller sphere is more than that of the larger one.

Q. 15 Do free electrons travel to region of higher potential or lower potential?

Ans. The free electrons experiences electrostatic force in a direction opposite to the direction of electric field being is of negative charge. The electric field always directed from higher potential to lower travel.

Therefore, electrostatic force and hence direction of travel of electrons is from lower potential to region of higher potential .

Q. 16 Can there be a potential difference between two adjacent conductors carrying the same charge?

K **Thinking Process**

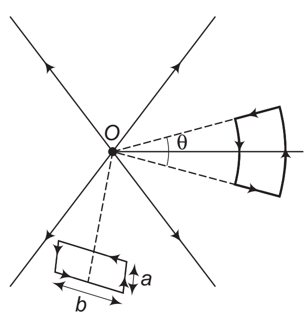
The capacity of conductor depend on its geometry i.e., length and breadth . For given charge potential $V \propto 1/C$, so two adjacent conductors carrying the same charge of different dimensions may have different potentials.

Ans. *Yes*, if the sizes are different.

Q.17 Can the potential function have a maximum or minimum in free space?

Ans. *No*, The absence of atmosphere around conductor prevents the phenomenon of electric discharge or potential leakage and hence, potential function do not have a maximum or minimum in free space.

Q. 18 A test charge q is made to move in the electric field of a point charge Q along two different closed paths [figure first path has sections along and perpendicular to lines of electric field. Second path is a rectangular loop of the same area as the first loop. How does the work done compare in the two cases?



Ans. As electric field is conservative, work done will be zero in both the cases.

Note *Conservative forces (like electrostatic force or gravitational force) are those forces, work done by which depends only on initial position and final position of object viz charge, but not on the path through which it goes from initial position to final position.*

Short Answer Type Questions

Q. 19 Prove that a closed equipotential surface with no charge within itself must enclose an equipotential volume.

K **Thinking Process**

In this problem, we need to know that the electric field intensity E and electric potential V are related as $E = -\frac{dV}{dr}$ *and the field lines are always perpendicular to one equipotential surface maintained at high electrostatic potential to other equipotential surface maintained at low electrostatic potential.*

Ans. Let's assume contradicting statement that the potential is not same inside the closed equipotential surface. Let the potential just inside the surface is different to that of the surface causing in a potential gradient $\left(\frac{dV}{dr}\right)$. Consequently electric field comes into existence, which is given by as $E = -\frac{dV}{dr}$.

Consequently field lines pointing inwards or outwards from the surface. These lines cannot be again on the surface, as the surface is equipotential. It is possible only when the other end of the field lines are originated from the charges inside.

This contradict the original assumption. Hence, the entire volume inside must be equipotential.

Q. 20 A capacitor has some dielectric between its plates and the capacitor is connected to a DC source. The battery is now disconnected and then the dielectric is removed . State whether the capacitance, the energy stored in it, electric field, charge stored and the voltage will increase, decrease or remain constant.

K **Thinking Process**

Here, the charge stored by the capacitor remains conserved after its disconnection from battery.

Ans. The capacitance of the parallel plate capacitor, filled with dielectric medium of dielectric constant K is given by

$$C = \frac{K\varepsilon_0 A}{d}, \text{ where signs are as usual.}$$

The capacitance of the parallel plate capacitor decreases with the removal of dielectric medium as for air or vacuum $K = 1$.

After disconnection from battery charge stored will remain the same due to conservation of charge.

The energy stored in an isolated charge capacitor $= \frac{q^2}{2C}$; as q is constant, energy stored $\propto$ $1/C$ and C decreases with the removal of dielectric medium, therefore energy stored increases. Since q is constant and $V = q/C$ and C decreases which in turn increases V and therefore E increases as $E = V/d$.

Note *One of the very important questions with the competitive point of view.*

Q. 21 Prove that, if an insulated, uncharged conductor is placed near a charged conductor and no other conductors are present, the uncharged body must intermediate in potential between that of the charged body and that of infinity.

K **Thinking Process**

The electric field $E = -\frac{dV}{dr}$ suggest that electric potential decreases along the direction of electric field.

Ans. Let us take any path from the charged conductor to the uncharged conductor along the direction of electric field. Therefore, the electric potential decrease along this path.

Now, another path from the uncharged conductor to infinity will again continually lower the potential further. This ensures that the uncharged body must be intermediate in potential between that of the charged body and that of infinity.

Q. 22 Calculate potential energy of a point charge $-q$ placed along the axis due to a charge $+Q$ uniformly distributed along a ring of radius R. Sketch PE, as a function of axial distance z from the centre of the ring. Looking at graph, can you see what would happen if $-q$ is displaced slightly from the centre of the ring (along the axis)?

K **Thinking Process**

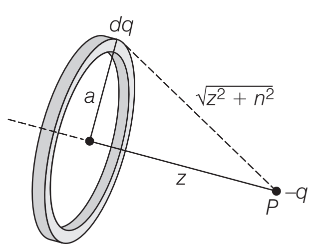
The work done or PE stored in a system of charges can be obtained
$U = W = q \times$ potential difference

Ans. Let us take point P to be at a distance x from the centre of the ring, as shown in figure. The charge element dq is at a distance x from point P. Therefore, V can be written as

$$V = k_e \int \frac{dq}{r} = k_e \int \frac{dq}{\sqrt{z^2 + a^2}}$$

where, $k = \frac{1}{4\pi\varepsilon_0}$, since each element dq is at the same distance from point P, so we have net potential

$$V = \frac{k_e}{\sqrt{z^2 + a^2}} \int dq = \frac{k_e Q}{\sqrt{z^2 + a^2}}$$



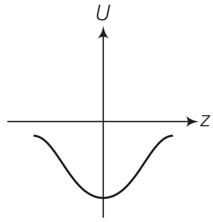
Considering $-q$ charge at P, the potential energy is given by

$$U = W = q \times \text{potential difference}$$

$$U = \frac{k_e Q(-q)}{\sqrt{z^2 + a^2}}$$

or $$U = \frac{1}{4\pi\varepsilon_0} \frac{-Qq}{\sqrt{z^2 + a^2}}$$

$$= \frac{1}{4\pi\varepsilon_0 a} \frac{-Qq}{\sqrt{1 + \left(\frac{z}{a}\right)^2}}$$



This is the required expression.

The variation of potential energy with z is shown in the figure. The charge $-q$ displaced would perform oscillations.

Nothing can be concluded just by looking at the graph.

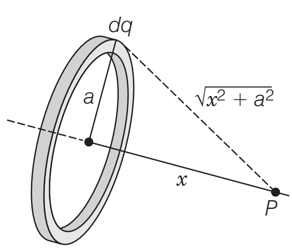
Q. 23 Calculate potential on the axis of a ring due to charge Q uniformly distributed along the ring of radius R.

Ans. Let us take point P to be at a distance x from the centre of the ring,as shown in figure. The charge element dq is at a distance x from point P. Therefore, V can be written as

$$V = k_e \int \frac{dq}{r} = k_e \int \frac{dq}{\sqrt{x^2 + a^2}}$$

where, $k_e = \frac{1}{4\pi\varepsilon_0}$, since each element dq is at the same distance from point P, so we have net potential

$$V = \frac{k_e}{\sqrt{x^2 + a^2}} \int dq = \frac{k_e Q}{\sqrt{x^2 + a^2}}$$



The net electric potential $$V = \frac{1}{4\pi\varepsilon_0} \frac{Q}{\sqrt{x^2 + a^2}}$$

Long Answer Type Questions

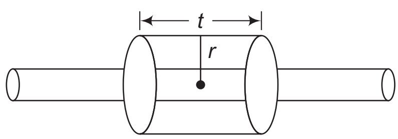
Q. 24 Find the equation of the equipotentials for an infinite cylinder of radius r_0 carrying charge of linear density λ.

K **Thinking Process**

The electric field due to line charge need to be obtained in order to find the potential at distance r from the line charge. As line integral of electric field gives potential difference between two points.

$$V(r) - V(r_0) = -\int_{r_0}^{r} \mathbf{E}.d\mathbf{l}$$

Ans. Let the field lines must be radically outward. Draw a cylindrical Gaussian surface of radius r and length l. Then, applying Gauss' theorem



$$\int \mathbf{E}.d\mathbf{S} = \frac{1}{\varepsilon_0}\lambda l$$

or $$E_r 2\pi r l = \frac{1}{\varepsilon_0}\lambda l \Rightarrow E_r = \frac{\lambda}{2\pi\varepsilon_0 r}$$

Hence, if r_0 is the radius, $$V(r) - V(r_0) = -\int_{r_0}^{r} \mathbf{E}.d\mathbf{l} = \frac{\lambda}{2\pi\varepsilon_0}\ln\frac{r_0}{r}$$

Since, $$\int_{r_0}^{r} \frac{\lambda}{2\pi\varepsilon_0 r}dr = \frac{\lambda}{2\pi\varepsilon_0}\int_{r_0}^{r}\frac{1}{r}dr = \frac{\lambda}{2\pi\varepsilon_0}\ln\frac{r}{r_0}$$

For a given V,

$$\ln\frac{r}{r_0} = -\frac{2\pi\varepsilon_0}{\lambda}[V(r) - V(r_0)]$$

$\Rightarrow$ $$r = r_0 e^{-2\pi\varepsilon_0 V r_0/\lambda} e + 2\pi\varepsilon_0 V(r)/\lambda$$

$$r = r_0 e^{-2\pi\varepsilon_0 [V(r) - V(r_0)]/\lambda}$$

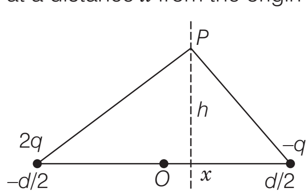
The equipotential surfaces are cylinders of radius.

Q. 25 Two point charges of magnitude $+q$ and $-q$ are placed at $(-d/2, 0, 0)$ and $(d/2, 2, 0)$, respectively. Find the equation of the equipotential surface where the potential is zero.

K **Thinking Process**

The net electric potential at any point due to system of point charges is equal to the algebraic sum of electric potential due to each individual charges.

Ans. Let the required plane lies at a distance x from the origin as shown in figure.



The potential at the point P due to charges is given by

$$\frac{1}{4\pi\varepsilon_0}\frac{q}{[(x+d/2)^2+h^2]^{1/2}}-\frac{1}{4\pi\varepsilon_0}\frac{q}{[(x-d/2)^2+h^2]^{1/2}}$$

If net electric potential is zero, then

$$\frac{1}{[(x+d/2)^2+h^2]^{1/2}}=\frac{1}{[(x-d/2)+h^2]^{1/2}}$$

Or $$(x-d/2)^2+h^2=(x+d/2)^2+h^2$$

$\Rightarrow$ $$x^2-dx+d^2/4=x^2+dx+d^2/4$$

Or $$2dx=0 \;\Rightarrow\; x=0$$

The equation of the required plane is $x = 0$ *i.e.,* y-z plane.

Q. 26 **A parallel plate capacitor is filled by a dielectric whose relative permittivity varies with the applied voltage (U) as $\varepsilon = \alpha U$ where $a = 2V^{-1}$. A similar capacitor with no dielectric is charged to $U_0 = 78$ V. It is then connected to the uncharged capacitor with the dielectric. Find the final voltage on the capacitors.**

K **Thinking Process**

In this problem, the dielectric of variable permittivity is used which gives new insight in the ordinary problem.

Ans. Assuming the required final voltage be U. If C is the capacitance of the capacitor without the dielectric, then the charge on the capacitor is given by $Q_1 = CU$

Since, the capacitor with the dielectric has a capacitance εC. Hence, the charge on the capacitor is given by

$$Q_2 = \varepsilon CU = (\alpha U)CU = \alpha CU$$

The initial charge on the capacitor is given by

$$Q_0 = CU_0$$

From the conservation of charges, $Q_0 = Q_1 + Q_2$

Or $$CU_0 = CU + \alpha CU^2$$

$\Rightarrow$ $$\alpha U^2 + U - U_0 = 0$$

$\therefore$ $$U = \frac{-1 \pm \sqrt{1+4\alpha U_0}}{2\alpha}$$

On solving for $U_0 = 78\,V$ and $a = 2/V$, we get

$$U = 6\text{V}$$

Q. 27 **A capacitor is made of two circular plates of radius R each, separated by a distance $d << R$. The capacitor is connected to a constant voltage. A thin conducting disc of radius $r << R$ and thickness $t << r$ is placed at a centre of the bottom plate. Find the minimum voltage required to lift the disc if the mass of the disc is m.**

K **Thinking Process**

The disc will be lifted when weight is balanced by electrostatic force.

Ans. Assuming initially the disc is in touch with the bottom plate, so the entire plate is a equipotential.

The electric field on the disc, when potential difference V is applied across it, given by

$$E = \frac{V}{d}$$

Let charge q' is transferred to the disc during the process,

Therefore by Gauss' theorem,

$$\therefore \qquad q' = -\varepsilon_0 \frac{V}{d} \pi r^2$$

Since, Gauss theorem states that

$$\phi = \frac{q}{\varepsilon_0} \text{ or } q = \frac{\varepsilon_0}{\phi}$$

$$= \varepsilon EA = \frac{\varepsilon_0 V}{d} A$$

The force acting on the disc is

$$-\frac{V}{d} \times q' = \varepsilon_0 \frac{V^2}{d^2} \pi r^2$$

If the disc is to be lifted, then

$$\varepsilon_0 \frac{V^2}{d^2} \pi r^2 = mg \quad \Rightarrow \quad V = \sqrt{\frac{mgd^2}{\pi \varepsilon_0 r^2}}$$

This is the required expression.

Q. 28 (a) In a quark model of elementary particles, a neutron is made of one up quarks [charge $(2/3)\ e$] and two down quarks [charges $-(1/3)\ e$]. Assume that they have a triangle configuration with side length of the order of 10^{-15} m. Calculate electrostatic potential energy of neutron and compare it with its mass 939 MeV.

(b) Repeat above exercise for a proton which is made of two up and one down quark.

Ans. This system is made up of three charges. The potential energy of the system is equal to the algebraic sum of PE of each pair. So,

$$U = \frac{1}{4\pi\varepsilon_0}\left\{\frac{q_d q_d}{r} - \frac{q_u q_d}{r} - \frac{q_u q_d}{r}\right\}$$

$$= \frac{9 \times 10^9}{10^{-15}} (1.6 \times 10^{-19})^2 \ [\{(1/3)^2 - (2/3)(1/3) - (2/3)(1/3)\}]$$

$$= 2.304 \times 0^{-13} \left\{\frac{1}{9} - \frac{4}{9}\right\} = -7.68 \times 10^{-14} \text{J}$$

$$= 4.8 \times 10^5 \text{ eV} = 0.48 \text{ meV} = 5.11 \times 10^{-4} (m_n c^2)$$

Q. 29 Two metal spheres, one of radius R and the other of radius $2R$, both have same surface charge density σ. They are brought in contact and separated. What will be new surface charge densities on them?

Ans. The charges on two metal spheres, before coming in contact, are given by

$$Q = \sigma.4\pi R^2$$

$$Q_2 = \sigma.4\pi(2R^2)$$

$$= 4\,(\sigma.4\pi R^2) = 4Q_1$$

Let the charges on two metal spheres , after coming in contact becomes Q_1' and Q_2'.

Now applying law of conservation of charges is given by

$$Q_1' + Q_2' = Q_1 + Q_2 = 5Q_1$$
$$= 5\,(\sigma.4\pi R^2)$$

After coming in contact, they acquire equal potentials. Therefore, we have

$$\frac{1}{4\pi\varepsilon_0}\frac{Q_1'}{R} = \frac{1}{4\pi\varepsilon_0}\frac{Q_2'}{R}$$

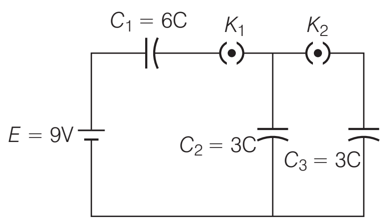
On solving, we get

$\therefore$ $$Q_1' = \frac{5}{3}(\sigma.4\pi R^2) \text{ and } Q_2' = \frac{10}{3}(\sigma.4\pi R^2)$$

$\therefore$ $$\sigma_1 = 5/3\sigma \text{ and}$$

$\therefore$ $$\sigma_2 = \frac{5}{6}\sigma$$

Q. 30 In the circuit shown in figure, initially K_1 is closed and K_2 is open. What are the charges on each capacitors?
Then K_1 was opened and K_2 was closed (order is important), what will be the charge on each capacitor now? $[C = 1\mu F]$



Ans. In the circuit, when initially K_1 is closed and K_2 is open, the capacitors C_1 and C_2 acquires potential difference V_1 and V_2 respectively. So, we have

$$V_1 + V_2 = E$$

and $$V_1 + V_2 = 9V$$

Also, in series combination , $$V \propto 1/C$$

$$V_1 : V_2 = 1/6 : 1/3$$

On solving

$\Rightarrow$ $$V_1 = 3V \text{ and } V_2 = 6V$$

$\therefore$ $$Q_1 = C_1V_1 = 6C \times 3 = 18\,\mu C$$

$$Q_2 = 9\,\mu C \text{ and } Q_3 = 0$$

Then, K_1 was opened and K_2 was closed, the parallel combination of C_2 and C_3 is in series with C_1.

$$Q_2 = Q_2' + Q_3$$

and considering common potential of parallel combination as V, then we have

$$C_2V + C_3V = Q_2$$

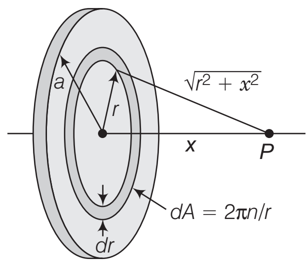
$\Rightarrow$ $$V = \frac{Q_2}{C_2 + C_3} = (3/2)V$$

On solving, $$Q_2' = (9/2)\,\mu C$$

and $$Q_3 = (9/2)\,\mu C$$

Q. 31 Calculate potential on the axis of a disc of radius R due to a charge Q uniformly distributed on its surface.

Ans. Let the point P lies at a distance x from the centre of the disk and take the plane of the disk to be perpendicular to the $\mathbf{x}$-axis. Let the disc is divided into a number of charged rings as shown in figure.



The electric potential of each ring, of radius r and width dr, have charge dq is given by

$$\sigma\, dA = \sigma 2\pi r dr$$

and potential is given by

(*Refer the solution of Q.* 23)

$$dV = \frac{k_e dq}{\sqrt{r^2 + x^2}} = \frac{k_e \sigma 2\pi r\, dr}{\sqrt{r^2 + x^2}}$$

where $k_e = \dfrac{1}{4\pi\varepsilon_0}$ the total electric potential at P, is given by

$$V = \pi k_e \sigma \int_0^a \frac{2r\, dr}{\sqrt{r^2 + x^2}} = \pi k_e \sigma \int_0^a (r^2 + x^2)^{-1/2} 2r\, dr$$

$$V = 2\pi k_e \sigma\, [(x^2 + a^2)^{1/2} - x]$$

So, we have by substring $\quad k_e = \dfrac{1}{4\pi\varepsilon_0}$

$$V = \frac{1}{4\pi\varepsilon_0} \frac{2Q}{a^2} [\sqrt{x^2 + a^2} - x]$$

Note *You may take $a = R$ in this problem.*

Q. 32 Two charges q_1 and q_2 are placed at (0, 0, d) and (0, 0, $-d$) respectively. Find locus of points where the potential is zero.

Thinking Process

Here, 3-dimensional imagination is required to actualise the problem. Also, the net electric potential at any point due to system of point charges is equal to the algebraic sum of electric potential due to each individual charges.

Ans. Let any arbitrary point on the required plane is (x, y, z) .The two charges lies on z-axis at a separation of $2d$.

The potential at the point P due to two charges is given by

$$\frac{q_1}{\sqrt{x^2 + y^2 + (z - d)^2}} + \frac{q_2}{\sqrt{x^2 + y^2 + (z + d)^2}} = 0$$

$$\therefore \quad \frac{q_1}{\sqrt{x^2 + y^2 + (z - d)^2}} = \frac{-q_2}{\sqrt{x^2 + y^2 + (z + d)^2}}$$

On squaring and simplifying, we get

$$x^2 + y^2 + z^2 + \left[\frac{(q_1/q_2)^2 + 1}{(q_1/q_2)^2 - 1}\right](2zd) + d^2 - 0$$

This is the equation of a sphere with centre at

$$\left(0, 0, -2d\left[\frac{q_1^2 + q_2^2}{q_1^2 - q_2^2}\right]\right)$$

Note *The centre and radius of sphere $(x-a)^2 + (y-b)^2 + (z-c)^2 = r^2$ is (a, b, c) and r respectively.*

Q. 33 **Two charges $-q$ each are separated by distance $2d$. A third charge $+q$ is kept at mid-point O. Find potential energy of $+q$ as a function of small distance x from O due to $-q$ charges. Sketch PE Vs/x and convince yourself that the charge at O is in an unstable equilibrium.**

Ans. Let third charge $+q$ is slightly displaced from mean position towards first charge. So, the total potential energy of the system is given by

$$U = \frac{1}{4\pi\varepsilon_0}\left\{\frac{-q^2}{(d-x)} + \frac{-q^2}{(d+x)}\right\}$$

$$U = \frac{-q^2}{4\pi\varepsilon_0}\frac{2d}{(d^2 - x^2)}$$

$$\frac{dU}{dx} = \frac{-q^2 2d}{4\pi\varepsilon_0} \cdot \frac{2x}{(d^2 - x^2)^2}$$

The system will be in equilibrium, if

$$F = -\frac{dU}{dx} = 0$$

On solving, $x = 0$. So for, $+q$ charge to be in stable/unstable equilibrium, finding second derivative of PE.

$$\frac{d^2U}{dx^2} = \left(\frac{-2dq^2}{4\pi\varepsilon_0}\right)\left[\frac{2}{(d^2 - x^2)^2} - \frac{8x^2}{(d^2 - x^2)^3}\right]$$

$$= \left(\frac{-2dq^2}{4\pi\varepsilon_0}\right)\frac{1}{(d^2 - x^2)^3}[2(d^2 - x^2)^2 - 8x^2]$$

At $x = 0$

$$\frac{d^2U}{dx^2} = \left(\frac{-2dq^2}{4\pi\varepsilon_0}\right)\left(\frac{1}{d^6}\right)(2d^2), \text{ which is } < 0$$

This shows that system will be unstable equilibrium.

Note *For function $y = f(x)$, on solving $\frac{dy}{dx} = 0$ gives critical points i.e., points of local maxima or local minima. If for any critical point, this imply that y acquires maximum value at $x = x_1$, $x = x_1$*

$\frac{d^2y}{dx^2} > 0$ this imply that y acquires minimum value at $x = x_1$ and for $\frac{d^2y}{dx^2} < 0$

3

Current Electricity

Multiple Choice Questions (MCQs)

Q. 1 Consider a current carrying wire (current I) in the shape of a circle.

(a) source of emf

(b) electric field produced by charges accumulated on the surface of wire

(c) the charges just behind a given segment of wire which push them just the right way by repulsion

(d) the charges ahead

Ans. *(b)* Current per unit area (taken normal to the current), I/A, is called current density and is denoted by j. The SI units of the current density are A/m^2. The current density is also directed along E and is also a vector and the relationship is given by

$$j = sE$$

The j changes due to electric field produced by charges accumulated on the surface of wire.

Note *That as the currrent progresses along the wire, the direction of j (current density) changes in an exact manner, while the current I remain unaffected. The agent that is essentially responsible for this.*

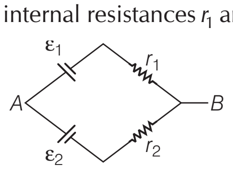
Q. 2 Two batteries of emf ε_1 and $\varepsilon_2 (\varepsilon_2 > \varepsilon_1)$ and internal resistances r_1 and r_2 respectively are connected in parallel as shown in figure.

(a) Two equivalent emf ε_{eq} of the two cells is between ε_1 and ε_2, *i.e.*, $\varepsilon_1 < \varepsilon_{eq} < \varepsilon_2$

(b) The equivalent emf ε_{eq} is smaller than ε_1

(c) The ε_{eq} is given by $\varepsilon_{eq} = \varepsilon_1 + \varepsilon_2$ always

(d) ε_{eq} is independent of internal resistances r_1 and r_2



Ans. *(a)* The equivalent emf of this combination is given by

$$\varepsilon_{eq} = \frac{\varepsilon_2 r_1 + \varepsilon_1 r_2}{r_1 + r_2}$$

This suggest that the equivalent emf ε_{eq} of the two cells is given by

$$\varepsilon_1 < \varepsilon_{eq} < \varepsilon_2$$

Q. 3 A resistance R is to be measured using a meter bridge, student chooses the standard resistance S to be 100Ω. He finds the null point at $l_1 = 2.9$ cm. He is told to attempt to improve the accuracy. Which of the following is a useful way?

(a) He should measure l_1 more accurately

(b) He should change S to 1000Ω and repeat the experiment

(c) He should change S to 3Ω and repeat the experiment

(d) He should given up hope of a more accurate measurement with a meter bridge

Thinking Process

Here, the concept of accurate balanced Wheatstone bridge is to be used.

Ans. *(c)* The percentage error in R can be minimised by adjusting the balance point near the middle of the bridge, *i.e.*, when l_1 is close to 50 cm. This requires a suitable choice of S.

Since,
$$\frac{R}{S} = \frac{R\, l_1}{R\,(100 - l_1)} = \frac{l_1}{100 - l_1}$$

Since here, $R : S :: 2.9 : 97.1$ imply that the S is nearly 33 times to that of R. In orded to make this ratio 1:1, it is necessary to reduce the value of S nearly $\frac{1}{33}$ times *i.e.*, nearly 3Ω.

Q. 4 Two cells of emfs approximately 5 V and 10 V are to be accurately compared using a potentiometer of length 400 cm.

(a) The battery that runs the potentiometer should have voltage of 8V

(b) The battery of potentiometer can have a voltage of 15 V and R adjusted so that the potential drop across the wire slightly exceeds 10 V

(c) The first portion of 50 cm of wire itself should have a potential drop of 10 V

(d) Potentiometer is usually used for comparing resistances and not voltages

Thinking Process

The potential drop across wires of potentiometer should be greater than emfs of primary cells.

Ans. *(b)* In a potentiometer experiment, the emf of a cell can be measured, if the potential drop along the potentiometer wire is more than the emf of the cell to be determined. Here, values of emfs of two cells are given as 5V and 10V, therefore, the potential drop along the potentiometer wire must be more than 10V.

Q. 5 A metal rod of length 10 cm and a rectangular cross-section of 1cm x $\frac{1}{2}$ cm is connected to a battery across opposite faces. The resistance will be

(a) maximum when the battery is connected across 1 cm x $\frac{1}{2}$ cm faces

(b) maximum when the battery is connected across 10 cm x 1 cm faces

(c) maximum when the battery is connected across 10 cm x $\frac{1}{2}$ cm faces

(d) same irrespective of the three faces

Thinking Process

The resistance of wire depends on its geometry l (length of the rod). Here, the metallic rod behaves as a wire.

Ans. ***(a)*** The resistance of wire is given by

$$R = \rho \frac{l}{A}$$

For greater value of R, l must be higher and A should be lower and it is possible only when the battery is connected across 1 cm x $\left(\frac{1}{2}\right)$ cm (area of cross-section A).

Q. 6 Which of the following characteristics of electrons determines the current in a conductor?

(a) Drift velocity alone
(b) Thermal velocity alone
(c) Both drift velocity and thermal velocity
(d) Neither drift nor thermal velocity

Ans. ***(a)*** The relationship between current and drift speed is given by

$$I = neAv_d$$

Here, I is the current and v_d is the drift velocity.

So, $$I \propto v_d$$

Thus, only drift velocity determines the current in a conductor.

Multiply Choice Questions (More Than One Options)

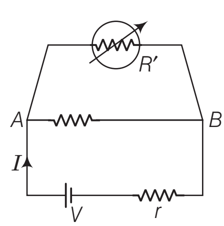
Q. 7 Kirchhoff's junction rule is a reflection of

(a) conservation of current density vector
(b) conservation of charge
(c) the fact that the momentum with which a charged particle approaches a junction is unchanged (as a vector) as the charged particle leaves the junction
(d) the fact that there is no accumulation of charges at a junction

Ans. ***(b, d)***

Kirchhoff's junction rule is also known as Kirchhoff's current law which states that the algebraic sum of the currents flowing towards any point in an electric network is zero. *i.e.,* charges are conserved in an electric network.
So, Kirchhoff's junction rule is the reflection of conservation of charge

Q. 8 Consider a simple circuit shown in figure stands for a variable resistance R'.R' can vary from R_0 to infinity. r is internal resistance of the battery (r<<R<<R,).



(a) Potential drop across AB is nearly constant as R' is varied

(b) Current through $R'$is nearly a constant as R' is varied

(c) Current I depends sensitively on R'

(d) $I \geq \frac{V}{r+R}$ always

Ans. ***(a, d)***

Here, the potential drop is taking place across AB and r. Since the equivalent resistance of parallel combination of R and R' is always less than R, therefore $I \geq \frac{V}{r+R}$ always.

Note *In parallel combination of resistances, the equivalent resistance is smaller than smallest resistance present in combination.*

Q. 9 Temperature dependence of resistivity $\rho(T)$ of semiconductors, insulators and metals is significantly based on the following factors

(a) number of charge carriers can change with temperature T

(b) time interval between two successive collisions can depend on T

(c) length of material can be a function of T

(d) mass of carriers is a function of T

Ans. ***(a, b)***

The resistivity of a metallic conductor is given by,

$$e = \frac{m}{ne^2\tau}$$

where n is number of charge carriers per unit volume which can change with temperature T and τ is time interval between two successive collisions which decreases with the increase of temperature.

Q. 10 The measurement of an unknown resistance R is to be carried out using Wheatstones bridge as given in the figure below. Two students perform an experiment in two ways. The first students takes $R_2 = 10\Omega$ and $R_1 = 5\Omega$. The other student takes $R_2 = 1000\Omega$ and $R_1 = 500\Omega$. In the standard arm, both take $R_3 = 5\Omega$.

Both find $R = \frac{R_2}{R_1}, R_3 = 10\Omega$ within errors.

(a) The errors of measurement of the two students are the same

(b) Errors of measurement do depend on the accuracy with which R_2 and R_1 can be measured

(c) If the student uses large values of R_2 and R_1 the currents through the arms will be feeble. This will make determination of null point accurately more difficult

(d) Wheatstone bridge is a very accurate instrument and has no errors of measurement

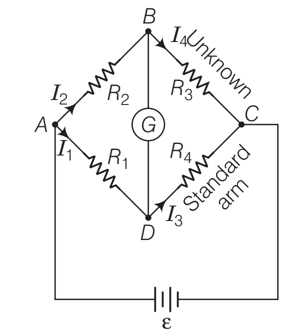
Ans. ***(b, c)***

Given, for first student, $R_2 = 10\ \Omega, R_1 = 5\ \Omega, R_3 = 5\ \Omega$

For second student, $R_1 = 500\ \Omega, R_3 = 5\ \Omega$

Now, according to Wheatstone bridge rule,

$$\frac{R_2}{R} = \frac{R_1}{R_3} \Rightarrow R = R_3 \times \frac{R_2}{R_1} \quad ...(i)$$



Now putting all the values in Eq. (i), we get $R = 10\,\Omega$ for both students. Thus, we can analyse that the Wheatstone bridge is most sensitive and accurate if resistances are of same value.

Thus, the errors of measurement of the two students depend on the accuracy and sensitivity of the bridge, which inturn depends on the accuracy with which R_2 and R_1 can be measured.

When R_2 and R_1 are larger, the currents through the arms of bridge is very weak. This can make the determination of null point accurately more difficult.

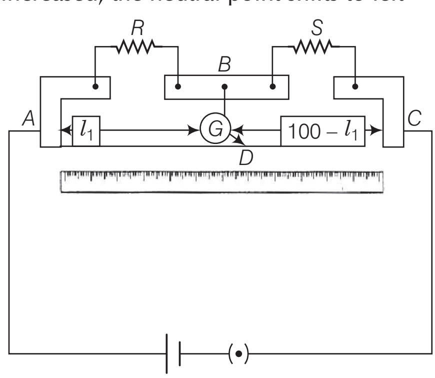
Q. 11 In a meter bridge, the point *D* is a neutral point (figure).

(a) The meter bridge can have no other neutral. A point for this set of resistances

(b) When the jockey contacts a point on meter wire left of *D*, current flows to *B* from the wire

(c) When the jockey contacts a point on the meter wire to the right of *D*, current flows from *B* to the wire through galvanometer

(d) When *R* is increased, the neutral point shifts to left



Ans. ***(a, c)***

At neutral point, potential at *B* and neutral point are same. When jockey is placed at to the right of *D*, the potential drop across *AD* is more than potential drop across *AB*, which brings the potential of point *D* less than that of *B*, hence current flows from *B* to *D*.

Very Short Answer Type Questions

Q. 12 Is the motion of a charge across junction momentum conserving? Why or why not?

Ans. When an electron approaches a junction, in addition to the uniform electric field **E** facing it normally. It keep the drift velocity fixed as drift velocity depend on E by the relation drift velocity

$$v_d = \frac{eE\tau}{m}$$

This result into accumulation of charges on the surface of wires at the junction. These produce additional electric field. These fields change the direction of momentum.

Thus, the motion of a charge across junction is not momentum conserving.

Q. 13 The relaxation time τ is nearly independent of applied E field whereas it changes significantly with temperature T. First fact is (in part) responsible for Ohm's law whereas the second fact leads to variation of p with temperature. Elaborate why?

K **Thinking Process**

The higher drift velocities of electrons make collisions more frequent which in turn decreases the time interval between two successive collision.

Ans. Relaxation time is inversely proportional to the velocities of electrons and ions. The applied electric field produces the insignificant change in velocities of electrons at the order of 1mm/s, whereas the change in temperature (T) , affects velocities at the order of 10^2m/s.

This decreases the relaxation time considerably in metals and consequently resistivity of metal or conductor increases as .

$$\rho = \frac{1}{\sigma} = \frac{m}{ne^2\tau}$$

Q. 14 What are the advantages of the null-point method in a Wheatstone bridge? What additional measurements would be required to calculate R_{unknown} by any other method?

Ans. The advantage of null point method in a Wheatstone bridge is that the resistance of galvanometer does not affect the balance point, there is no need to determine current in resistances and the internal resistance of a galvanometer.

It is easy and convenient method for observer.

The R_{unknown} can be calculated applying Kirchhoff's rules to the circuit. We would need additional accurate measurement of all the currents in resistances and galvanometer and internal resistance of the galvanometer.

Note *The necessary and sufficient condition for balanced Wheatstone bridge is*

$$\frac{P}{Q} = \frac{R}{S}$$

where P and Q are ratio arms and R is known resistance and S is unknown resistance.

Q. 15 What is the advantage of using thick metallic strips to join wires in a potentiometer?

Ans. In potentiometer, the thick metallic strips are used as they have negligible resistance and need not to be counted in the length l_1 of the null point of potentiometer. It is for the convenience of experimenter as he measures only their lengths along the straight wires each of lengths 1 m.

This measurements is done with the help of centimetre scale or metre scale with accuracy.

Q. 16 For wiring in the home, one uses Cu wires or Al wires. What considerations are involved in this?

K **Thinking Process**

The availability, conductivity and the cost of the metal are main criterion for the selection of metal for wiring in home.

Ans. The Cu wires or Al wires are used for wiring in the home.

The main considerations are involved in this process are cost of metal, and good conductivity of metal.

Q. 17 Why are alloys used for making standard resistance coils?

Ans. Alloys have small value of temperature coefficient of resistance with less temperature sensitivity.

This keeps the resistance of the wire almost constant even in small temperature change. The alloy also has high resistivity and hence high resistance, because for given length and cross-section area of conductor. (L and A are constant)

$$R \propto \rho$$

Q. 18 Power P is to be delivered to a device *via* transmission cables having resistance R_c. If V is the voltage across R and I the current through it, find the power wasted and how can it be reduced.

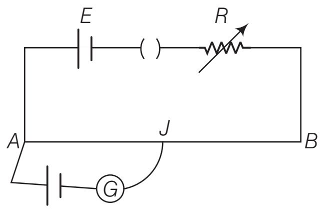
Ans. The power consumption in transmission lines is given by $P = i^2 R_c$, where R_c is the resistance of transmission lines. The power is given by

$$P = VI$$

The given power can be transmitted in two ways namely (i) at low voltage and high current or (ii) high voltage and low current. In power transmission at low voltage and high current more power is wasted as $P \propto i^2$ whereas power transmission at high voltage and low current facilitates the power transmission with minimal power wastage.

The power wastage can be reduced by transmitting power at high voltage.

Q. 19 AB is a potentiometer wire (figure). If the value of R is increased, in which direction will the balance point J shift?



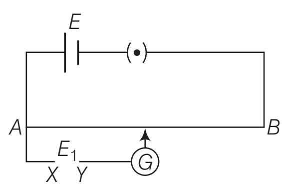
Ans. With the increase of R, the current in main circuit decreases which in turn , decreases the potential difference across AB and hence potential gradient(k) across AB decreases.

Since, at neutral point, for given emf of cell, I increases as potential gradient (k) across AB has decreased because

$$E = kI$$

Thus, with the increase of I, the balance point neutral point will shift towards B.

Q. 20 While doing an experiment with potentiometer (figure) it was found that the deflection is one sided and (i) the deflection decreased while moving from one and A of the wire, to the end R; (ii) the deflection increased, while the jockey was moved towards the end D.



(i) Which terminal positive or negative of the cell E_1 is connected at X in case (i) and how is E_1, related to E?

(ii) Which terminal of the cell E_1 is connected at X in case (1 in 1)?

Ans. **(i)** The deflection in galvanometer is one sided and the deflection decreased, while moving from one end 'A' of the wire to the end 'B', thus imply that current in auxiliary circuit (lower circuit containing primary cell) decreases, while potential difference across A and jockey increases.

This is possible only when positive terminal of the cell E_1, is connected at X and $E_1 > E$.

(ii) The deflection in galvanometer is one sided and the deflection increased, while moving from one end A of the wire to the end B, this imply that current in auxiliary circuit (lower circuit containing primary cell) increases, while potential difference across A and jockey increases.

This is possible only when negative terminal of the cell E_1, is connected at X.

Q. 21 A cell of emf E and internal resistance r is connected across an external resistance R. Plot a graph showing the variation of potential differential across R, *versus* R.

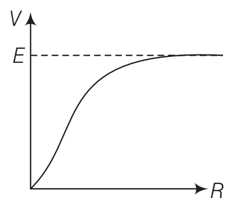
K **Thinking Process**

When the cell of emf E and internal resistance r is connected across an external resistance R, the relationship between the voltage across R is given by

$$V = \frac{E}{1 + \frac{r}{R}}$$

With the increase of R , V approaches closer to E and when E is infinite , V reduces to 0.

Ans. *The graphical relationship between voltage across R and the resistance R is given below*



Short Answer Type Questions

Q. 22 First a set of n equal resistors of R each are connected in series to a battery of emf E and internal resistance R, A current I is observed to flow. Then, the n resistors are connected in parallel to the same battery. It is observed that the current is increased 10 times. What is 'n' ?

K **Thinking Process**

Here, in series combination of resistors, the equivalent resistance of series combination is in series with the internal resistance R of battery resistors whereas in parallel combination of resistors , the equivalent resistance of parallel combination is in series with the internal resistance of battery.

Ans. In series combination of resistors, current I is given by $I = \frac{E}{R + nR'}$

whereas in parallel combination current 10 I is given by

$$\frac{E}{R + \frac{R}{n}} = 10I$$

Now, according to problem,

$$\frac{1+n}{1+\frac{1}{n}} \Rightarrow 10 = \left(\frac{1+n}{n+1}\right)n \Rightarrow n = 10$$

Q. 23 Let there be n resistors $R_1 .... R_n$ with $R_{\max} = \max(R_1 .... R_n)$ and $R_{\min} = \min\{R_1 .... R_n\}$. Show that when they are connected in parallel, the resultant resistance $R_p = R_{\min}$ and when they are connected in series, the resultant resistance $R_s > R_{\max}$. Interpret the result physically.

Ans. When all resistances are connected in parallel, the resultant resistance R_p is given by

$$\frac{1}{R_p} = \frac{1}{R_1} + ... + \frac{1}{R_n}$$

On multiplying both sides by $R_{\min}$ we have

$$\frac{R_{\min}}{R_p} = \frac{R_{\min}}{R_1} + \frac{R_{\min}}{R_2} + ... + \frac{R_{\min}}{R_n}$$

Here, in RHS, there exist one term $\frac{R_{\min}}{R_{\min}} = 1$ and other terms are positive, so we have

$$\frac{R_{\min}}{R_p} = \frac{R_{\min}}{R_1} + \frac{R_{\min}}{R_2} + ... + \frac{R_{\min}}{R_n} > 1$$

This shows that the resultant resistance $R_p < R_{\min}$.

Thus, in parallel combination, the equivalent resistance of resistors is less than the minimum resistance available in combination of resistors . Now, in series combination, the equivalent resistant is given by

$$R_s = R_1 + ... + R_n$$

Here, in RHS, there exist one term having resistance $R_{\max}$.

So, we have

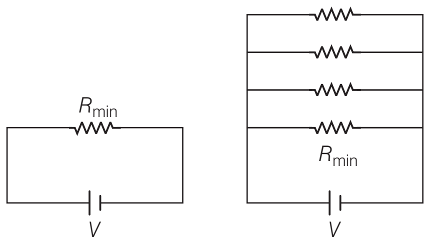
or $$R_s = R_1 + ... + R_{\max} + .. + ..+ R_n$$

$$R_s = R_1 + ... + R_{\max} ... + R_n = R_{\max} + ... (R_1 + ...+) R_n$$

or $$R_s \geq R_{\max}$$

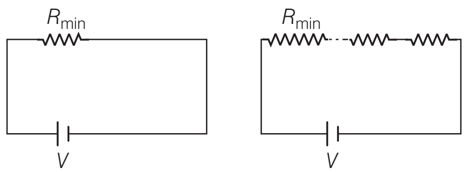
$$R_s = R_{\max} \; (R_1 + \; ... + R_n)$$

Thus, in series combination, the equivalent resistance of resistors is greater than the maximum resistance available in combination of resistors. Physical interpretation



(a) (b)

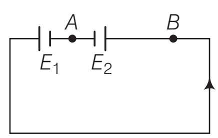
In Fig. (b), R_{min} provides an equivalent route as in Fig. (a) for current. But in addition there are (n–1) routes by the remaining (n –1) resistors. Current in Fig. (b) is greater than current in Fig. (a). Effective resistance in Fig. (b) $< R_{min}$. Second circuit evidently affords a greater resistance.



(c) (d)

In Fig. (d), R_{max} provides an equivalent route as in Fig. (c) for current. Current in Fig. (d) < current in Fig. (c). Effective resistance in Fig. (d) $> R_{max}$. Second circuit evidently affords a greater resistance.

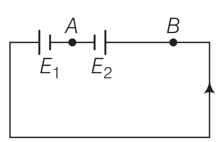
Q. 24 **The circuit in figure shows two cells connected in opposition to each other. Cell E_1 is of emf 6V and internal resistance 2Ω the cell E_2 is of emf 4V and internal resistance 8Ω. Find the potential difference between the points A and B.**



K **Thinking Process**

Here, after finding the electric current flow in the circuit by using Kirchhoff's law or Ohm's law, the potential difference across AB can be obtained.

Ans. Applying Ohm's law.



Effective resistance $= 2\,\Omega + 8\Omega = 10\Omega$ and effective emf of two cells $= 6 - 4 = 2$V, so the electric current is given by

$$I = \frac{6-4}{2+8} = 0.2\text{A}$$

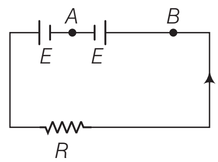
along anti-clockwise direction, since $E_1 > E_2$.

The direction of flow of current is always from high potential to low potential. Therefore $V_B > V_A$.

$\Rightarrow$ $$V_B - 4V - (0.2) \times 8 = V_A$$

Therefore, $$V_B - V_A = 3.6\text{V}$$

Q. 25 Two cells of same emf E but internal resistance r_1 and r_2 are connected in series to an external resistor R (figure). What should be the value of R so that the potential difference across the terminals of the first cell becomes zero?



K **Thinking Process**

Here, after finding the electric current flow in the circuit by using Kirchhoff 's law or Ohm's law, the potential difference across first cell can be obtained.

Ans. Applying Ohm's law,

Effective resistance $= R + r_1 + r_2$ and effective emf of two cells $= E + E = 2E$, so the electric current is given by

$$I = \frac{E + E}{R + r_1 + r_2}$$

The potential difference across the terminals of the first cell and putting it equal to zero.

$$V_1 = E - Ir_1 = E - \frac{2E}{r_1 + r_2 + R} r_1 = 0$$

or
$$E = \frac{2Er_1}{r_1 + r_2 + R} \Rightarrow 1 = \frac{2r_1}{r_1 + r_2 + R}$$

$$r_1 + r_2 + R = 2r_1 \Rightarrow R = r_1 - r_2$$

This is the required relation.

Q. 26 Two conductors are made of the same material and have the same length. Conductor A is a solid wire of diameter 1mm. Conductor B is a hollow tube of outer diameter 2mm and inner diameter 1mm. Find the ratio of resistance R_A to R_B.

K **Thinking Process**

The resistance of wire is given by $R = \rho \frac{l}{A}$

where A is cross-sectional area of conductor.

Ans. The resistance of first conductor

$$R_A = \frac{\rho l}{\pi (10^{-3} \times 0.5)^2}$$

The resistance of second conductor,

$$R_B = \frac{\rho l}{\pi [(10^{-3})^2 - (0.5 \times 10^{-3})^2]}$$

Now, the ratio of two resistors is given by

$$\frac{R_A}{R_B} = \frac{(10^{-3})^2 - (0.5 \times 10^{-3})^2}{(0.5 \times 10^{-3})^2} = 3 : 1$$

Q. 27 Suppose there is a circuit consisting of only resistances and batteries. Suppose one is to double (or increase it to n-times) all voltages and all resistances. Show that currents are unaltered. Do this for circuit of Examples 3,7 in the NCERT Text Book for Class XII.

Thinking Process

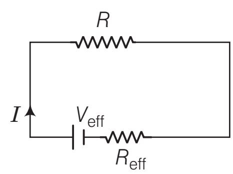
The electric current in two cases is obtained and then shown equal to each other

Ans. Let the effective internal resistance of the battery is R_{eff}, the effective external resistance R and the effective voltage of the battery is V_{eff}.

Applying Ohm's law,

Then current through R is given by

$$I = \frac{V_{eff}}{R_{eff} + R}$$



If all the resistances and the effective voltage are increased n-times, then we have

$$V_{eff}^{new} = nV_{eff}, R_{eff}^{new} = nR_{eff}$$

and $$R^{new} = nR$$

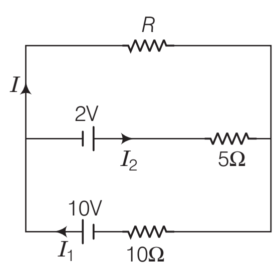
Then, the new current is given by

$$I' = \frac{nV_{eff}}{nR_{eff} + nR} = \frac{n(V_{eff})}{n(R_{eff} + R)} = \frac{(V_{eff})}{(R_{eff} + R)} = I$$

Thus, current remains the same.

Long Answer Type Questions

Q. 28 Two cells of voltage 10V and 2V and 10Ω internal resistances 10Ω and 5Ω respectively, are connected in parallel with the positive end of 10V battery connected to negative pole of 2V battery (figure). Find the effective voltage and effective resistance of the combination.



Thinking Process

The question can be solved by using Kirchhoff's voltage rule/ loop rule.

Ans. Applying Kirchhoff's junction rule, $I_1 = I + I_2$

Applying Kirchhoff's II law / loop rule applied in outer loop containing 10V cell and resistance R, we have

$$10 = IR + 10I_1 \quad \text{...(i)}$$

Applying Kirchhoff II law / loop rule applied in outer loop containing 2V cell and resistance R, we have

$$2 = 5I_2 - RI = 5(I_1 - I) - RI$$

or $$4 = 10I_1 - 10I - 2RI \quad \text{...(ii)}$$

Solving Eqs. (i) and (ii), gives

$\Rightarrow$ $$6 = 3RI + 10I$$

$$2 = I\left(R + \frac{10}{3}\right)$$

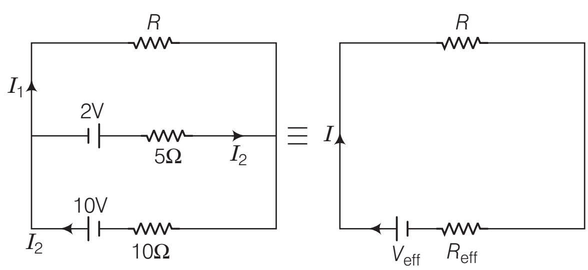
Also, the external resistance is R . The Ohm's law states that

$$V = I(R + R_{eff})$$

On comparing, we have $V = 2V$ and effective internal resistance

$$(R_{eff}) = \left(\frac{10}{3}\right)\Omega$$

Since, the effective internal resistance (R_{eff}) of two cells is $\left(\frac{10}{3}\right)\Omega$, being the parallel combination of 5Ω and 10Ω. *The equivalent circuit is given below*



Q. 29 A room has AC run for 5 a day at a voltage of 220V. The wiring of the room consists of Cu of 1 mm radius and a length of 10m. Power consumption per day is 10 commercial units. What fraction of it goes in the joule heating in wires? What would happen if the wiring is made of aluminium of the same dimensions?

$$[\rho_{Cu} = 11.7 \times 10^{-8}\ \Omega\text{m},\ \rho_{Al} = 2.7 \times 10^{-8}\ \Omega\text{m}]$$

K **Thinking Process**

The power consumption in a current carrying resistor is given by $P = I^2R$

Ans. Power consumption in a day *i.e.,* in 5 = 10 units

Or power consumption per hour = 2units

Or power consumption = 2units = 2kW = 2000J/s

Also, we know that power consumption in resistor,

$$P = V \times I$$

$\Rightarrow$ $$2000\text{W} = 220\text{V} \times I \text{ or } I \approx 9\text{ A}$$

Now, the resistance of wire is given by $R = \rho\dfrac{l}{A}$

where, A is cross-sectional area of conductor.

Power consumption in first current carrying wire is given by

$$P = I^2 R$$

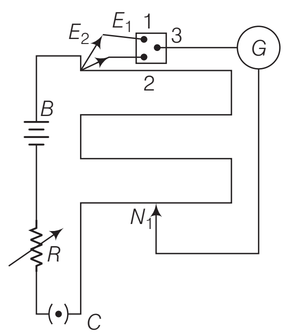
$$\rho \frac{l}{A} I^2 = 1.7 \times 10^{-8} \times \frac{10}{\pi \times 10^{-6}} \times 81 \text{ J/s} \approx 4 \text{ J/s}$$

The fractional loss due to the joule heating in first wire $= \frac{4}{2000} \times 100 = 0.2\%$

Power loss in Al wire $= 4 \frac{\rho_{Al}}{\rho_{Cu}} = 1.6 \times 4 = 6.4 \text{ J/s}$

The fractional loss due to the joule heating in second wire $= \frac{6.4}{2000} \times 100 = 0.32\%$

Q. 30 In an experiment with a potentiometer, $V_B = 10\text{V}$. R is adjusted to be 50Ω (figure). A student wanting to measure voltage E_1 of a battery (approx. 8V) finds no null point possible. He then diminishes R to 10Ω and is able to locate the null point on the last (4th) segment of the potentiometer. Find the resistance of the potentiometer wire and potential drop per unit length across the wire in the second case.



K **Thinking Process**

The null point is obtained only when emf of primary cell is less than the potential difference across the wires of potentiometer.

Ans. Let R' be the resistance of the potentiometer wire.

Effective resistance of potentiometer and variable resistor ($R = 50\Omega$) is given by $= 50\Omega + R'$

Effective voltage applied across potentiometer $= 10\text{V}$.

The current through the main circuit,

$$I = \frac{V}{50\Omega + R} = \frac{10}{50\Omega + R}$$

Potential difference across wire of potentiometer,

$$IR' = \frac{10R'}{50\Omega + R}$$

Since with $50\,\Omega$ resistor, null point is not obtained it's possible only when

$$\frac{10 \times R'}{50 + R} < 8$$

$$\Rightarrow \qquad 10R' < 400 + 8R'$$

$$2R' < 400 \text{ or } R' < 200\Omega.$$

Similarly with 10Ω resistor , null point is obtained its possible only when

$$\frac{10 \times R'}{10 + R'} > 8$$

$\Rightarrow$ $$2R' > 80$$

$\Rightarrow$ $$R' > 40$$

$$\frac{10 \times \frac{3}{4} R'}{10 + R'} < 8$$

$\Rightarrow$ $$7.5R' < 80 + 8R'$$

$$R' > 160$$

$\Rightarrow$ $$160 < R' < 200.$$

Any R' between 160Ω and 200 Ω wlll achieve.

Since, the null point on the last (4th) segment of the potentiometer, therefore potential drop across 400 cm of wire > 8V.

This imply that potential gradient

$$k \times 400\,\text{cm} > 8\text{V}$$

or $$k \times 4\,\text{m} > 8\text{V}$$

$$k > 2\,\text{V}/\text{m}$$

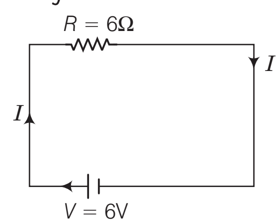
Similarly, potential drop across 300 cm wire < 8V.

$$k \times 300\text{cm} < 8\text{V}$$

or $$k \times 3\text{m} < 8\text{V}, \; k < 2\frac{2}{3}\,\text{V}/\text{m}$$

Thus, $$2\frac{2}{3}\,\text{V}/\text{m} > k > 2\text{V}/\text{m}$$

Q. 31 (a) Consider circuit in figure. How much energy is absorbed by electrons from the initial state of no current (Ignore thermal motion) to the state of drift velocity ?



(b) Electrons give up energy at the rate of RI^2 per second to the thermal energy. What time scale would number associate with energy in problem (a)? n = number of electron/volume = 10^{29} / m^3. Length of circuit = 10 cm, cross-section= $A = (1\text{ mm})^2$.

K **Thinking Process**

The current in a conductor and drift velocity of electrons are related as $i = neAv_d$, where v_d is drift speed of electrons and n is number density of electrons.

Ans. (a) By Ohm's law, current I is given by

$$I = 6\text{V} / 6\Omega = 1\text{A}$$

But, $$I = net\,A\,v_d \text{ or } v_d = \frac{i}{neA}$$

On substituting the values

For, n = number of electron/volume = 10^{29}/m^3

length of circuit = 10cm, cross-section $= A = (1\text{mm})^2$

$$v_d = \frac{1}{10^{29} \times 1.6 \times 10^{-19} \times 10^{-6}}$$

$$= \frac{1}{1.6} \times 10^{-4} \text{ m/s}$$

Therefore, the energy absorbed in the form of KE is given by

$$\text{KE} = \frac{1}{2} m_e \ v_d{}^2 \times nAI$$

$$= \frac{1}{2} \times 9.1 \times 10^{31} \times \frac{1}{2.56} \times 10^{20} \times 10^{8} \times 10^{6} \times 10^{1}$$

$$= 2 \times 10^{-17} \text{J}$$

(b) Power loss is given by $P = I^2 \ R = 6 \times 1^2 = 6\text{W} = 6\text{J/s}$

Since, $$P = \frac{E}{t}$$

Therefore, $$E = P \times t$$

or $$t = \frac{E}{P} = \frac{2 \times 10^{-17}}{6} \approx 10^{-17} \text{ s}$$

4

Moving Charges and Magnetism

Multiple Choice Questions (MCQs)

Q. 1 Two charged particles traverse identical helical paths in a completely opposite sense in a uniform magnetic field $\mathbf{B} = B_0 \hat{\mathbf{k}}$.

(a) They have equal z-components of momenta
(b) They must have equal charges
(c) They necessarily represent a particle, anti-particle pair
(d) The charge to mass ratio satisfy

$$\left(\frac{e}{m}\right)_1 + \left(\frac{e}{m}\right)_2 = 0$$

K **Thinking Process**

The uniqueness of helical path is determined by its pitch which is given by

$$\text{Pitch} = \frac{2\pi m v \cos\theta}{qB}$$

Ans. *(d)* For given pitch d correspond to charge particle, we have

$$\frac{q}{m} = \frac{2\pi v \cos\theta}{qB} = \text{constant}$$

Since, charged particles traverse identical helical paths in a completely opposite sense in a uniform magnetic field **B**, LHS for two particles should be same and of opposite sign. Therefore,

$$\left(\frac{e}{m}\right)_1 + \left(\frac{e}{m}\right)_2 = 0$$

Note *Consider e in place of q in solution.*

Q. 2 Biot-Savart law indicates that the moving electrons (velocity v) produce a magnetic field **B** such that

(a) B is perpendicular of
(b) B is parallel to v
(c) it obeys inverse cube law
(d) it is along the line joining the electron and point of observation

K **Thinking Process**

Here use of Biot-Savart law play vital role.

Ans. *(a)* In Biot-Savart's law, magnetic field $\mathbf{B} \| \mathbf{i}dl \times \mathbf{r}$ and $\mathbf{idl}$ due to flow of electron is in opposite direction of $\mathbf{v}$ and by direction of cross product of two vectors

$$\mathbf{B} \perp \mathbf{v}$$

Q. 3 A current carrying circular loop of radius R is placed in the x-y plane with centre at the origin. Half of the loop with $x > 0$ is now bent so that it now lies in the y-z plane.

(a) The magnitude of magnetic moment now diminishes
(b) The magnetic moment does not change
(c) The magnitude of B at $(0, 0, z)$, $z > R$ increases
(d) The magnitude of B at $(0, 0, z)$, $z >> R$ is unchanged

K **Thinking Process**

The magnetic moment of circular loop and the net magnitudes of magnetic moment of each semicircular loop of radius R lie in the x-y plane and the y-z plane are compared.

Ans. *(a)* The direction of magnetic moment of circular loop of radius R is placed in the x-y plane is along z-direction and given by $M = I(\pi r^2)$, when half of the loop with $x > 0$ is now bent so that it now lies in the y-z plane, the magnitudes of magnetic moment of each semicircular loop of radius R lie in the x-y plane and the y-z plane is $M' = I(\pi r^2)/4$ and the direction of magnetic moments are along z-direction and x-direction respectively.

Their resultant

$$M_{\text{net}} = \sqrt{M'^2 + M'^2} = \sqrt{2}\, M' = \sqrt{2}\, I(\pi r^2)/4$$

So, $M_{\text{net}} < M$ or M diminishes.

Q. 4 An electron is projected with uniform velocity along the axis of a current carrying long solenoid. Which of the following is true?

(a) The electron will be accelerated along the axis
(b) The electron path will be circular about the axis
(c) The electron will experience a force at 45° to the axis and hence execute a helical path
(d) The electron will continue to move with uniform velocity along the axis of the solenoid

K **Thinking Process**

Here, magnetic lorentz force comes into existence when a charge moves in uniform magnetic field produces by current carrying long solenoid.

Ans. *(d)* Magnetic Lorentz force electron is projected with uniform velocity along the axis of a current carrying long solenoid $F = -evB\sin 180° = 0\ (\theta = 0°)$ as magnetic field and velocity are parallel. The electron will continue to move with uniform velocity along the axis of the solenoid.

Q. 5 In a cyclotron, a charged particle

(a) undergoes acceleration all the time
(b) speeds up between the dees because of the magnetic field
(c) speeds up in a dee
(d) slows down within a dee and speeds up between dees

K **Thinking Process**

Here, understanding of working of cyclotron is needed.

Ans. *(a)* *The charged particle undergoes acceleration as*

(i) speeds up between the dees because of the oscillating electric field and

(ii) speed remain the same inside the dees because of the magnetic field but direction undergoes change continuously.

Q.6 A circular current loop of magnetic moment M is in an arbitrary orientation in an external magnetic field **B.** The work done to rotate the loop by 30° about an axis perpendicular to its plane is

(a) MB
(b) $\sqrt{3}\,\frac{MB}{2}$
(c) $\frac{MB}{2}$
(d) zero

K **Thinking Process**

The rotation of the loop by 30° about an axis perpendicular to its plane imply that the axis of the loop still continues to inclined with the same angle with the direction of magnetic field .

Ans. *(a)* The rotation of the loop by 30° about an axis perpendicular to its plane make no change in the angle made by axis of the loop with the direction of magnetic field, therefore, the work done to rotate the loop is zero.

Note *The work done to rotate the loop in magnetic field $W = MB(\cos\theta_1 - \cos\theta_2)$. where signs are as usual.*

Q.7 The gyro-magnetic ratio of an electron in an H-atom, according to Bohr model, is

(a) independent of which orbit it is in
(b) negative
(c) positive
(d) increases with the quantum number n.

K **Thinking Process**

The gyro-magnetic ratio of an electron in an H-atom is equal to the ratio of the magnetic moment and the angular momentum of the electron.

Ans. *(a)* If I is the magnitude of the angular momentum of the electron about the central nucleus (orbital angular momentum). Vectorially,

$$\mu_l = -\frac{e}{2m_e} l.$$

The negative sign indicates that the angular momentum of the electron is opposite in direction to the magnetic moment.

Multiple Choice Questions (More Than One Options)

Q. 8 Consider a wire carrying a steady current, I placed in a uniform magnetic field **B** perpendicular to its length. Consider the charges inside the wire. It is known that magnetic forces do no work. This implies that,

(a) motion of charges inside the conductor is unaffected by **B**, since they do not absorb energy
(b) some charges inside the wire move to the surface as a result of **B**
(c) If the wire moves under the influence of **B**, no work is done by the force
(d) if the wire moves under the influence of **B**, no work is done by the magnetic force on the ions, assumed fixed within the wire.

Ans. *(b, d)*

Magnetic forces on a wire carrying a steady current, I placed in a uniform magnetic field B, a wire carrying a steady current, I placed in a uniform magnetic field **B** perpendicular to its length is given by

$$F = IlB$$

The direction of force is given by Fleming's left hand rule and F is perpendicular to the direction of magnetic field **B**. Therefore, work done by the magnetic force on the ions is zero.

Q. 9 Two identical current carrying coaxial loops, carry current I in an opposite sense. A simple amperian loop passes through both of them once. Calling the loop as C,

(a) $\oint \mathbf{B}.\,d\mathbf{l} = m\mu_0 I$
(b) the value of $\oint \mathbf{B}.\,d\mathbf{l} = \mp 2\mu_0 I$ is independent of sense of C
(c) there may be a point on C where, **B** and **dl** are perpendicular
(d) B vanishes everywhere on C

K **Thinking Process**

The Ampere's circuital law is to be applied on given situation.

Ans. *(b, c)*

Applying the Ampere's circuital law, we have

$$\oint_C \mathbf{B}.d\mathbf{l} = \mu_0 (I - I) = 0 \quad \text{(because current is in opposite sense.)}$$

Also, there may be a point on C where **B** and **dl** are perpendicular and hence,

$$\oint_C \mathbf{B}.d\mathbf{l} = 0$$

Q. 10 A cubical region of space is filled with some uniform electric and magnetic fields. An electron enters the cube across one of its faces with velocity v and a positron enters *via* opposite face with velocity $-v$. At this instant,

(a) the electric forces on both the particles cause identical accelerations
(b) the magnetic forces on both the particles cause equal accelerations
(c) both particles gain or loose energy at the same rate
(d) the motion of the Centre of Mass (CM) is determined by **B** alone

Thinking Process

The Lorentz force is experienced by the single moving charge in space is filled with some uniform electric and magnetic fields is given by $\mathbf{F} = \mathbf{qE} + q(\mathbf{v} \times \mathbf{B})$.

Ans. ***(b, c, d)***

The magnetic forces $\mathbf{F} = q(\mathbf{v} \times \mathbf{B})$, on charge particle is either zero or $\mathbf{F}$ is perpendicular to $\mathbf{v}$ (or component of $\mathbf{v}$) which in turn revolves particles on circular path with uniform speed. In both the cases particles have equal accelerations.

Both the particles gain or loss energy at the same rate as both are subjected to the same electric force ($\mathbf{F} = q\mathbf{E}$) in opposite direction.

Since, there is no change of the Centre of Mass (CM) of the particles , therefore the motion of the Centre of Mass (CM) is determined by $\mathbf{B}$ alone.

Q. 11 A charged particle would continue to move with a constant velocity in a region wherein,

(a) $\mathbf{E} = 0, \mathbf{B} \neq 0$
(b) $\mathbf{E} \neq 0, \mathbf{B} \neq 0$
(c) $\mathbf{E} \neq 0, \mathbf{B} = 0$
(d) $\mathbf{E} = 0, \mathbf{B} = 0$

Thinking Process

The Lorentz force is experienced by the single moving charge in space is filled with some uniform electric and magnetic fields is given by $\mathbf{F} = q\mathbf{E} + q(\mathbf{v} \times \mathbf{B})$

Ans. ***(a, b, d)***

Here, force on charged particle due to electric field $\mathbf{F}_E = q\mathbf{E}$.

Force on charged particle due to magnetic field, $\mathbf{F}_m = q(\mathbf{v} \times \mathbf{B})$

Now, $F_E = 0$ if $E = 0$ and $F_m = 0$ if $\sin\theta = 0$ or $\theta° = 0°$ or $180°$

Hence, $B \neq 0$.

Also, $E = 0$ and $B = 0$ and the resultant force $q\,\mathbf{E} + q\,(\mathbf{v} \times \mathbf{B}) = 0$. In this case $E \neq 0$ and $B \neq 0$

Very Short Answer Type Questions

Q. 12 Verify that the cyclotron frequency $\omega = eB/m$ has the correct dimensions of $[T]^{-1}$.

Ans. For a charge particle moving perpendicular to the magnetic field, the magnetic Lorentz forces provides necessary centripetal force for revolution.

$$\frac{mv^2}{R} = qvB$$

On simplifying the terms, we have

$$\therefore \quad \frac{qB}{m} = \frac{v}{R} = \omega$$

Finding the dimensional formula of angular frequency

$$\therefore \quad [\omega] = \left[\frac{qB}{m}\right] = \left[\frac{v}{R}\right] = [T]^{-1}$$

Q. 13 Show that a force that does no work must be a velocity dependent force.

Ans. Let no work is done by a force , so we have

$$dW = \mathbf{F}.\mathbf{dl} = 0$$

$\Rightarrow$ $\mathbf{F}.\mathbf{v}\, dt = 0$ (Since, $\mathbf{dl} = \mathbf{v}$ dt and d$\mathbf{t} \neq 0$)

$\Rightarrow$ $\mathbf{F}.\mathbf{v} = 0$

Thus, F must be velocity dependent which implies that angle between F and v is 90°. If v changes (direction), then (directions) F should also change so that above condition is satisfied,

Q. 14 The magnetic force depends on v which depends on the inertial frame of reference. Does then the magnetic force differ from inertial frame to frame? Is it reasonable that the net acceleration has a different value in different frames of reference?

Ans. *Yes*, the magnetic force differ from inertial frame to frame. The magnetic force is frame dependent.

The net acceleration which comes into existing out of this is however, frame independent (non -relativistic physics) for inertial frames.

Q. 15 Describe the motion of a charged particle in a cyclotron if the frequency of the radio frequency (rf) field were doubled.

K **Thinking Process**

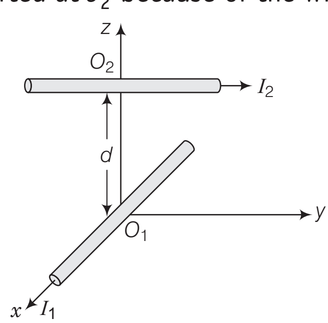
The relationship of radio frequency and charge particle frequency must be equal in order to accelerate the charge particle between the dees in cyclotron.

Ans. Here, the condition of magnetic resonance is violated.

When the frequency of the radio frequency (rf) field were doubled, the time period of the radio frequency (rf) field were halved. Therefore, the duration in which particle completes half revolution inside the dees, radio frequency completes the cycle.

Hence, particle will accelerate and decelerate alternatively. So, the radius of path in the dees will remain same.

Q. 16 Two long wires carrying current I_1 and I_2 are arranged as shown in figure. The one carrying current I_1 is along is the x-axis. The other carrying current I_2 is along a line parallel to the y-axis given by $x = 0$ and $z = d$. Find the force exerted at o_2 because of the wire along the x-axis.



K **Thinking Process**

Here, the understanding of application of the rule of finding directions of magnetic field and magnetic force on current carrying wire placed in magnetic field is beautifully tested.

Ans. In Biot- Savart law, magnetic field **B** is parallel to $i d\mathbf{l} \times \mathbf{r}$ and $i\mathbf{dl}$ have its direction along the direction of flow of current.

Here, for the direction of magnetic field, At O_2, due to wire carrying I_1 current is

$$\mathbf{B} \,||\, \text{parallel } \; i\mathbf{dl} \times r \text{ or } \hat{\mathbf{i}} \times \hat{\mathbf{k}}, \text{ but } \hat{\mathbf{i}} \times \hat{\mathbf{k}} = -\hat{\mathbf{j}}$$

So, the direction at O_2 is along Y- direction.

The direction of magnetic force exerted at O_2 because of the wire along the, x-axis.

$$\mathbf{F} = Il \times \mathbf{B} \approx \hat{\mathbf{j}} \times (-\hat{\mathbf{j}}) = 0$$

So, the magnetic field due to l_1 is along the y-axis. The second wire is along the y-axis and hence, the force is zero.

Short Answer Type Questions

Q. 17 A current carrying loop consists of 3 identical quarter circles of radius R, lying in the positive quadrants of the x-y, y-z and z-x planes with their centres at the origin, joined together. Find the direction and magnitude of **B** at the origin.

K **Thinking Process**

The magnetic field due to arc of current carrying coil which subtends an angle θ *at centre is given by* $\mathbf{B} = \dfrac{\mu_0}{4\pi}\dfrac{I\,\theta}{R}$.

Ans. For the current carrying loop quarter circles of radius R, lying in the positive quadrants of the x-y plane

$$\mathbf{B}_1 = \frac{\mu_0}{4\pi}\frac{I\,(\pi/2)}{R}\hat{\mathbf{k}} = \frac{\mu_0}{4}\frac{I}{2R}\hat{\mathbf{k}}$$

For the current carrying loop quarter circles of radius R, lying in the positive quadrants of the y-z plane

$$\mathbf{B}_2 = \frac{\mu_0}{4}\frac{I}{2R}\hat{\mathbf{i}}$$

For the current carrying loop quarter circles of radius R, lying in the positive quadrants of the z-x plane

$$\mathbf{B}_3 = \frac{\mu_0}{4}\frac{I}{2R}\hat{\mathbf{i}}$$

Current carrying loop consists of 3 identical quarter circles of radius R, lying in the positive quadrants of the x-y, y-y and z-z planes with their centres at the origin, joined together is equal to the vector sum of magnetic field due to each quarter and given by

$$\mathbf{B} = \frac{1}{4\pi}(\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}})\frac{\mu_0 I}{2R}.$$

Q. 18 A charged particle of charge e and mass m is moving in an electric field **E** and magnetic field **B**. Construct dimensionless quantities and quantities of dimension $[T]^{-1}$.

Ans. No dimensionless quantity can be constructed using given quantities.

For a charge particle moving perpendicular to the magnetic field, the magnetic Lorentz forces provides necessary centripetal force for revolution.

$$\frac{mv^2}{R} = qvB$$

On simplifying the terms, we have

$$\therefore \qquad \frac{qB}{m} = \frac{v}{R} = \omega$$

Finding the dimensional formula of angular frequency

$$\therefore \qquad [\omega] = \left[\frac{qB}{m}\right] = \left[\frac{v}{R}\right] = [T]^{-1}$$

This is the required expression.

Q. 19 An electron enters with a velocity $\mathbf{v} = v_0\hat{\mathbf{i}}$ into a cubical region (faces parallel to coordinate planes) in which there are uniform electric and magnetic fields. The orbit of the electron is found to spiral down inside the cube in plane parallel to the x-y plane. Suggest a configuration of fields **E** and **B** that can lead to it.

Thinking Process

The magnetic field revolves the charge particle in uniform circular motion in x-y plane and electric field along x-direction increases the speed, which in turn increases the radius of circular path and hence, particle traversed on spiral path.

Ans. Considering magnetic field $\mathbf{B} = B_0\hat{\mathbf{k}}$, and an electron enters with a velocity $\mathbf{v} = v_0\hat{\mathbf{i}}$ into a cubical region (faces parallel to coordinate planes).

The force on electron, using magnetic Lorentz force, is given by

$$\mathbf{F} = -\mathbf{e}\,(\mathbf{v}_0\hat{\mathbf{i}} \times \mathbf{B}_0\hat{\mathbf{k}}) = \mathbf{e}\,\mathbf{v}_0\mathbf{B}_0\,\hat{\mathbf{i}}$$

which revolves the electron in x-y plane.

The electric force $\mathbf{F} = -\mathbf{e}\mathbf{E}_0\,\hat{\mathbf{k}}$ accelerates e along z-axis which in turn increases the radius of circular path and hence particle traversed on spiral path.

Q. 20 Do magnetic forces obey Newton's third law. Verify for two current elements $\mathbf{dl}_1 = \mathbf{dl}\,\hat{\mathbf{i}}$ located at the origin and $\mathbf{dl}_2 = \mathbf{dl}\,\hat{\mathbf{j}}$ located at $(0, R, 0)$. Both carry current I.

Thinking Process

Here, the understanding of application of the rules of finding directions of magnetic field and magnetic force on current carrying wire placed in magnetic field is needed.

Ans. In Biot-Savart's law, magnetic field **B** is parallel (II) to $i\mathbf{dl} \times \mathbf{r}$ and $i$dl have its direction along the direction of flow of current.

Here, for the direction of magnetic field, At $\mathbf{dl}_2$, located at (0, R, 0)due to wire d_1 is given by $B \,||\, i\mathbf{dl} \times \mathbf{r}$ or $\hat{\mathbf{i}} \times \hat{\mathbf{j}}$ (because point (0, R, 0) lies on y-axis), but $\hat{\mathbf{i}} \times \hat{\mathbf{j}} = \hat{\mathbf{k}}$

So, the direction of magnetic field at d_2 is along z-direction.

The direction of magnetic force exerted at d_2 because of the first wire along the x-axis.

$F = i\ (I \times B)$ *i.e.*, $F \parallel (i \times k)$ or along $-\hat{\mathbf{j}}$ direction.

Therefore, force due to $\mathbf{dl}_1$ on $\mathbf{dl}_2$ is non-zero.

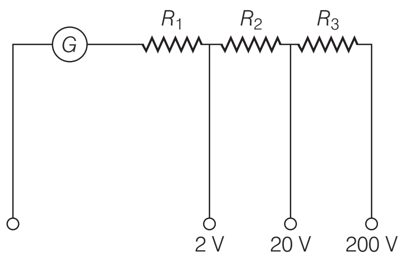
Now, for the direction of magnetic field, At d_1, located at (0, 0, 0) due to wire d_2 is given by $B \parallel i\mathbf{dl} \times \mathbf{r}$ or $\hat{\mathbf{j}} \times -\hat{\mathbf{j}}$ (because origin lies on y-direction w.r.t. point (0, R, 0).), but $\mathbf{j} \times -\mathbf{j} = 0$.

So, the magnetic field at d_1 does not exist.

Force due to $\mathbf{dl}_2$ on $\mathbf{dl}_1$ is zero.

So, magnetic forces do not obey Newton's third law.

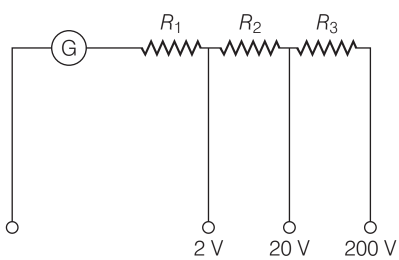
Q. 21 A multirange voltmeter can be constructed by using a galvanometer circuit as shown in figure. We want to construct a voltmeter that can measure 2V, 20V and 200V using a galvanometer of resistance 10Ω and that produces maximum deflection for current of 1 mA. Find R_1, R_2 and R_3 that have to be used.



K Thinking Process

A galvanometer can be converted into voltmeter by connecting a very high resistance wire connected in series with galvanometer. The relationship is given by $I_g (G + R) = V$ where I_g is range of galvanometer, G is resistance of galvanometer and R is resistance of wire connected in series with galvanometer.

Ans.

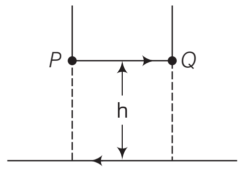


Applying expression in different situations

For	$i_G\ (G + R_1) = 2$	for 2V range
For	$i_G\ (G + R_1 + R_2) = 20$	for 20V range
and For	$i_G\ (G + R_1 + R_2 + R_3) = 200$	for 200V range

On solving, we get $R_1 = 1990\,\Omega$, $R_2 = 18\text{k}\,\Omega$ and $R_3 = 180\,\text{k}\,\Omega$.

Q. 22 A long straight wire carrying current of 25A rests on a table as shown in figure. Another wire *PQ* of length 1m, mass 2.5 g carries the same current but in the opposite direction. The wire *PQ* is free to slide up and down. To what height will *PQ* rise?



K **Thinking Process**

The force applied on PQ by long straight wire carrying current of 25A *rests on a table must balance the weight of small current carrying wire.*

Ans. The magnetic field produced by long straight wire carrying current of 25A rests on a table on small wire

$$B = \frac{\mu_0 I}{2\pi h}$$

The magnetic force on small conductor is

$$F = BIl \sin\theta = BIl$$

Force applied on *PQ* balance the weight of small current carrying wire.

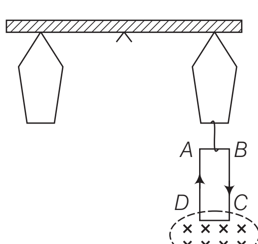
$$F = mg = \frac{\mu_0 I^2 l}{2\pi h}$$

$$h = \frac{\mu_0 I^2 l}{2\pi mg} = \frac{4\pi \times 10^{-7} \times 25 \times 25 \times 1}{2\pi \times 2.5 \times 10^{-3} \times 9.8} = 51 \times 10^{-4}$$

$$h = 0.51\text{cm}$$

Long Answer Type Questions

Q. 23 A 100 turn rectangular coil *ABCD* (in *X-Y*plane) is hung from one arm of a balance figure. A mass 500g is added to the other arm to balance the weight of the coil. A current 4.9 A passes through the coil and a constant magnetic field of 0.2 T acting inward (in x-z plane) is switched on such that only arm *CD* of length 1 cm lies in the field. How much additional mass *m* must be added to regain the balance?



K **Thinking Process**

The magnetic force applied on CD by magnetic field must balance the weight.

Ans. For equilibrium/ balance, net torque should also be equal to zero.

When the field is off $\sum t = 0$ considering the separation of each hung from mid-point be l.

$$Mgl = W_{\text{coil}}\ l$$
$$500\ g\ l = W_{\text{coil}}\ l$$
$$W_{\text{coil}} = 500 \times 9.8\,\text{N}$$

Taking moment of force about mid-point, we have the weight of coil

When the magnetic field is switched on

$$Mgl + mgl = W_{\text{coil}}\ l + IBL \sin 90° I$$
$$mgl = BIL\ l$$
$$m = \frac{BIL}{g} = \frac{0.2 \times 4.9 \times 1 \times 10^{-2}}{9.8} = 10^{-3}\ \text{kg} = 1\,\text{g}$$

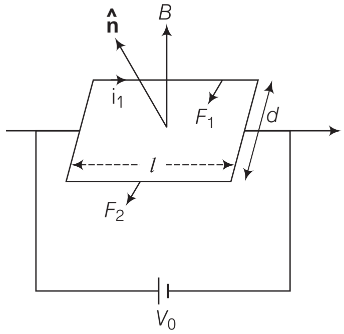
Thus, 1g of additional mass must be added to regain the balance.

Q. 24 A rectangular conducting loop consists of two wires on two opposite sides of length l joined together by rods of length d. The wires are eachof the same material but with cross-sections differing by a factor of 2. The thicker wire has a resistance R and the rods are of low resistance, which in turn are connected to a constant voltage source V_0. The loop is placed in uniform a magnetic field **B** at 45° to its plane. Find τ, the torque exerted by the magnetic field on the loop about an axis through the centres of rods.

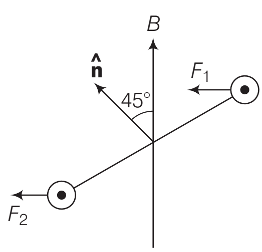
K **Thinking Process**

After finding current in both wires, magnetic forces and torques need to be calculated for finding the net torque.

Ans.



Front view



Side view

The thicker wire has a resistance R, then the other wire has a resistance $2R$ as the wires are of the same material but with cross-sections differing by a factor 2.

Now, the force and hence, torque on first wire is given by

$$F_1 = i_1\ lB = \frac{V_0}{2R}\ lB,\ \tau_1 = \frac{d}{2\sqrt{2}} F_1 = \frac{V_0 l d B}{2\sqrt{2}R}$$

Similarly, the force hence torque on other wire is given by

$$F_2 = i_2 lB = \frac{V_0}{2R}\ lB,\ \tau_2 = \frac{d}{2\sqrt{2}}\ F_2 = \frac{V_0 l d B}{4\sqrt{2}\ R}$$

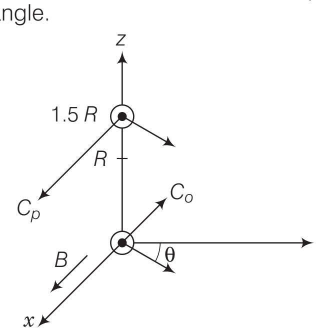
So, net torque,
$$\tau = \tau_1 - \tau_2$$
$$\tau = \frac{1}{4\sqrt{2}} \frac{V_0 l d B}{R}$$

Q.25 An electron and a positron are released from (0, 0, 0) and (0, 0, 1.5R) respectively, in a uniform magnetic field $\mathbf{B} = B_0\hat{\mathbf{i}}$, each with an equal momentum of magnitude $p = eBR$. Under what conditions on the direction of momentum will the orbits be non-intersecting circles?

K **Thinking Process**

The circles of the electron and a positron shall not overlap if the distance between the two centers are greater than 2*R*.

Ans. Since, B is along the x-axis, for a circular orbit the momenta of the two particles are in the y-z plane. Let p_1 and p_2 be the momentum of the electron and positron, respectively. Both traverse a circle of radius R of opposite sense. Let p_1 make an angle θ with the y-axis p_2 must make the same angle.



The centres of the respective circles must be perpendicular to the momenta and at a distance R. Let the centre of the electron be at C_e and of the positron at C_p. The coordinates of C_e is

$$C_e \equiv (0, -R\sin\theta, R\cos\theta)$$

The coordinates of C_p is

$$C_p \equiv (0, -R\sin\theta, \frac{3}{2}R - R\cos\theta)$$

The circles of the two shall not overlap if the distance between the two centers are greater than 2R.

Let d be the distance between C_p and C_e.

Let d be the distance between C_p and C_e .

Then,
$$d^2 = (2R\sin\theta)^2 + \left(\frac{3}{2}R - 2R\cos\theta\right)^2$$

$$= 4R^2\sin^2\theta + \frac{9^2}{4}R - 6R^2\cos\theta + 4R^2\cos^2\theta$$

$$= 4R^2 + \frac{9}{4}R^2 - 6R^2\cos\theta$$

Since, d has to be greater than 2R

$$d^2 > 4R^2$$

$\Rightarrow$
$$4R^2 + \frac{9}{4}R^2 - 6R^2\cos\theta > 4R^2$$

$\Rightarrow$
$$\frac{9}{4} > 6\cos\theta$$

or,
$$\cos\theta < \frac{3}{8}$$

Q. 26 A uniform conducting wire of length $12a$ and resistance R is wound up as a current carrying coil in the shape of (i) an equilateral triangle of side a, (ii) a square of sides a and, (iii) a regular hexagon of sides a. The coil is connected to a voltage source V_0. Find the magnetic moment of the coils in each case.

K **Thinking Process**

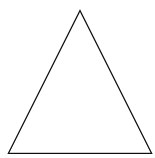
The different shapes forms figures of different area and hence, there magnetic moments varies.

Ans. We know that magnetic moment of the coils $m = nIA$.

Since, the same wire is used in three cases with same potentials, therefore, same current flows in three cases.

(i) for an equilateral triangle of side a,

$n = 4$ as the total wire of length $= 12a$

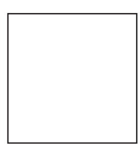


$n = 3$

Magnetic moment of the coils $m = nIA = 4I\left(\frac{\sqrt{3}}{4}a^2\right)$

$\therefore \qquad m = Ia^2\sqrt{3}$

(ii) For a square of sides a,



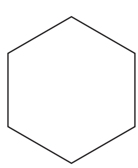
$n = 4$

$$A = a^2$$

$n = 3$ as the total wire of length $= 12a$

Magnetic moment of the coils $m = nIA = 3\,I\,(a^2) = 3\,I\,a^2$

(iii) For a regular hexagon of sides a,



$n = 2$

$n = 2$ as the total wire of length $= 12a$

Magnetic moment of the coils $m = nIA = 2\;1\left(\frac{6\sqrt{3}}{4}a^2\right)$

$$m = 3\sqrt{3}a^2 I$$

m is in a geometric series.

Q. 27 Consider a circular current-carrying loop of radius R in the x-yplane with centre at origin. Consider the line integral

$$\Im(L) = \left| \int_{-L}^{L} \mathbf{B} \cdot d\mathbf{l} \right|$$

taken along z-axis.

(a) Show that $\Im(L)$ monotonically increases with L

(b) Use an appropriate amperian loop to show that $\Im(\infty) = \mu_0 I$. where I is the current in the wire

(c) Verify directly the above result

(d) Suppose we replace the circular coil by a square coil of sides R carrying the same current I.

What can you say about $\Im(L)$ and $\Im(\infty)$?

Thinking Process

This question revolves around the application of Ampere circuital law.

Ans. (a) B (z) points in the same direction on z-axis and hence, $J(L)$ is a monotonically function of L.

Since, B and $\mathbf{dl}$ along the same direction, therefore $\mathbf{B} \cdot \mathbf{dl} = \mathbf{B} \cdot \mathbf{dl}$ as $\cos 0 = 1$

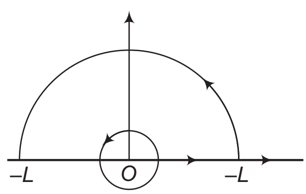
(b) $J(L)$ + contribution from large distance on contour $C = \mu_0 I$

$\therefore$ as $L \to \infty$

Contribution from large distance $\to 0$ (as $B \propto 1/r^3$)

$$J(\infty) - \mu_0 I$$

(c) The magnetic field due to circular current-carrying loop of radius R in the x-y plane with centre at origin at any point lying at a distance of from origin.



$$B_z = \frac{\mu_0 I R^2}{2(z^2 + R^2)^{3/2}}$$

$$\int_{-\infty}^{\infty} B_z dz = \int_{-\infty}^{\infty} \frac{\mu_0 I R^2}{2(z^2 + R^2)^{3/2}} dz$$

Put $z = R \tan \theta_1$

$\Rightarrow$ $dz = R \sec^2 \theta d\theta$

$\therefore$ $$\int_{-\infty}^{\infty} B_z dz = \frac{\mu_0 I}{2} \int_{-\pi/2}^{\pi/2} \cos\theta d\theta = \mu_0 I$$

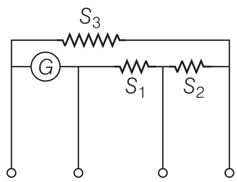
(d) $B(z)_{\text{square}} < B(z)_{\text{circular coil}}$

$\therefore$ $$\Im(L)_{\text{square}} < \Im(L)_{\text{circular coil}}$$

But by using arguments as in (b)

$$\Im(\infty)_{\text{square}} = \Im(\infty)_{\text{circular}}$$

Q. 28 A multirange current meter can be constructed by using a galvanometer circuit as shown in figure. We want a current meter that can measure 10mA, 100mA and 1mA using a galvanometer of resistance 10Ω and that produces maximum deflection for current of 1mA. Find S_1, S_2 and S_3 that have to be used.



K **Thinking Process**

A galvanometer can be converted into ammeter by connecting a very low resistance wire (shunt S) connected in parallel with galvanometer. The relationship is given by $I_g G = (I - I_g) S$, where I_g is range of galvanometer, G is resistance of galvanometer.

Ans.

$$I_G . G = (I_1 - I_G)(S_1 + S_2 + S_3) \quad \text{for } I_1 = 10\text{ mA}$$

$$I_G (G + S_1) = (I_2 - I_G)(S_2 + S_3) \quad \text{for } I_2 = 100\text{ mA}$$

and $$I_G (G + S_1 + S_2) = (I_3 - I_G)(S_3) \text{ for } I_3 = 1\text{ A}$$

gives $$S_1 = 1\text{ W}, S_2 = 0.1\text{ W}$$

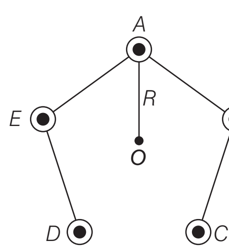
and $$S_3 = 0.01\text{ W}$$

Q. 29 Five long wires A, B, C, D and E, each carrying current I are arranged to form edges of a pentagonal prism as shown in figure. Each carries current out of the plane of paper.

(a) What will be magnetic induction at a point on the axis O? Axis is at a distance R from each wire.

(b) What will be the field if current in one of the wires (say A) is switched off?

(c) What if current in one of the wire (say A) is reversed?



K **Thinking Process**

The vector sum of magnetic field produced by each wire at O is equal to 0.

Ans. **(a)** Suppose the five wires A, B, C, D and E be perpendicular to the plane of paper at locations as shown in figure.

Thus, magnetic field induction due to five wires will be represented by various sides of a closed pentagon in one order, lying in the plane of paper. So, its value is zero.

(b) Since, the vector sum of magnetic field produced by each wire at O is equal to 0. Therefore, magnetic induction produced by one current carrying wire is equal in magnitude of resultant of four wires and opposite in direction.

Therefore, the field if current in one of the wires (say A) is switched off is $\frac{\mu_0}{2\pi}\frac{i}{R}$ perpendicular to AO towards left.

(c) If current in wire A is reversed, then

total magnetic field induction at O

= Magnetic field induction due to wire A + magnetic field induction due to wires B, C, D and E

$$= \frac{\mu_0}{4\pi R}\frac{2I}{R}$$

(acting perpendicular to AO towards left) + $\frac{\mu_0}{\pi}\frac{2I}{R}$ (acting perpendicular AO towards left)

$$= \frac{\mu_0 I}{\pi R} \text{ acting perpendicular } AO \text{ towards left.}$$

5

Magnetism and Matter

Multiple Choice Questions (MCQs)

Q. 1 A toroid of n turns, mean radius R and cross-sectional radius a carries current I. It is placed on a horizontal table taken as xy-plane. Its magnetic moment **m**

(a) is non-zero and points in the z-direction by symmetry

(b) points along the axis of the toroid ($\mathbf{m} = m\,\phi$)

(c) is zero, otherwise there would be a field falling as $\frac{1}{r^3}$ at large distances outside the toroid

(d) is pointing radially outwards

K **Thinking Process**

Toroid is a hollow circular ring on which a large number of turns of a wire are closely wound. Thus, in such a case magnetic field is only confined inside the body of toroid.

Ans. *(c)* In case of toroid, the magnetic field is only confined inside the body of toroid in the form of concentric magnetic lines of force and there is no magnetic field outside the body of toroid. This is because the loop encloses no current. Thus, the magnetic moment of toroid is zero.

In general, if we take r as a large distance outside the toroid, then $m \propto \frac{1}{r^3}$. But this case is not possible here.

Q. 2 The magnetic field of the earth can be modelled by that of a point dipole placed at the centre of the earth. The dipole axis makes an angle of 11.3° with the axis of the earth. At Mumbai, declination is nearly zero. Then,

(a) the declination varies between 11.3° W to 11.3° E

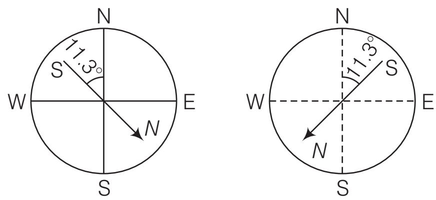
(b) the least declination is 0°

(c) the plane defined by dipole axis and the earth axis passes through Greenwich

(d) declination averaged over the earth must be always negative

Ans. *(a)* For the earth's magnetism, the magnetic field lines of the earth resemble that of a hypothetical magnetic dipole located at the centre of the earth.

The axis of the dipole does not coincide with the axis of rotation of the earth but is presently tilted by approxmately 11.3° with respect to the later. This results into two situations as given in the figure ahead.



Hence, the declination varies between 11.3° W to 11.3° E.

Q. 3 In a permanent magnet at room temperature,

(a) magnetic moment of each molecule is zero

(b) the individual molecules have non-zero magnetic moment which are all perfectly aligned

(c) domains are partially aligned

(d) domains are all perfectly aligned

Thinking Process

Permanent magnet at room temperature behave as a ferromagnetic substance for a long period of time.

Ans. *(d)* As we know a permanent magnet is a substance which at room temperature retain ferromagnetic property for a long period of time.

The individual atoms in a ferromagnetic material possess a dipole moment as in a paramagnetic material.

However, they interact with one another in such a way that they spontaneously align themselves in a common direction over a macroscopic volume called domain. Thus, we can say that in a permanent magnet at room temperature, domains are all perfectly aligned.

Q. 4 Consider the two idealised systems (i) a parallel plate capacitor with large plates and small separation and (ii) a long solenoid of length $L >> R$, radius of cross-section. In (i) **E** is ideally treated as a constant between plates and zero outside. In (ii) magnetic field is constant inside the solenoid and zero outside. These idealised assumptions, however, contradict fundamental laws as below

(a) case (i) contradicts Gauss' law for electrostatic fields

(b) case (ii) contradicts Gauss' law for magnetic fields

(c) case (i) agrees with $\oint \mathbf{E}.d\mathbf{l} = 0$.

(d) case (ii) contradicts $\oint \mathbf{H}.d\mathbf{l} = I_{en}$

Thinking Process

The electric field lines, do not form a continuous closed path while the magnetic field lines form the closed paths.

Ans. *(b)* As Gauss' law states, $\oint_S \mathbf{E} \cdot d\mathbf{s} = \frac{q}{\varepsilon_0}$ for electrostatic field. It does not contradict for electrostatic fields as the electric field lines do not form continuous closed path.

According to Gauss' law in magnetic field,

$$\oint_S \mathbf{E} \cdot d\mathbf{s} = 0$$

It contradicts for magnetic field, because there is a magnetic field inside the solenoid and no field outside the solenoid carrying current but the magnetic field lines form the closed path.

Q. 5 A paramagnetic sample shows a net magnetisation of 8 Am^{-1} when placed in an external magnetic field of 0.6 T at a temperature of 4 K. When the same sample is placed in an external magnetic field of 0.2 T at a temperature of 16 K, the magnetisation will be

(a) $\frac{32}{3}\ \text{Am}^{-1}$ (b) $\frac{2}{3}\ \text{Am}^{-1}$ (c) 6 Am^{-1} (d) 2.4 Am^{-1}

K **Thinking Process**

From Curie law, we know that magnetisation is directly proportional to the magnetic field induction and inversely proportional to the temperature in kelvin.

Ans. *(b)* As Curie law explains, we can deduce a formula for the relation between magnetic field induction, temperature and magnetisation.

i.e., $I \text{ (magnetisation)} \propto \frac{B \text{ (magnetic field induction)}}{t \text{ (temperature in kelvin)}}$

$\Rightarrow \frac{I_2}{I_1} = \frac{B_2}{B_1} \times \frac{t_1}{t_2}$

Let us suppose, here $I_1 = 8\ \text{Am}^{-1}$

$B_1 = 0.6\text{T},\ t_1 = 4\text{ K}$

$B_2 = 0.2\text{ T},\ t_2 = 16\text{ K}$

$I_2 = ?$

$\Rightarrow \frac{0.2}{0.6} \times \frac{4}{16} = \frac{I_2}{8}$

$\Rightarrow I_2 = 8 \times \frac{1}{12} = \frac{2}{3}\ \text{Am}^{-1}$

Multiple Choice Questions (More Than One Options)

Q. 6 S is the surface of a lump of magnetic material.

(a) Lines of **B** are necessarily continuous across S

(b) Some lines of **B** must be discontinuous across S

(c) Lines of **H** are necessarily continuous across S

(d) Lines of **H** cannot all be continuous across S

K **Thinking Process**

According to the properties of magnetic field lines (B), for any magnet, it forms continuous closed loops. This is unlike the electric dipole where these field lines begin from a positive charge and end on the negative charge or escape to infinity.

Also, magnetic intensity (H) outside any magnet is $H = B/\mu_0$ and for inside the magnet $H = B/\mu_0\mu_r$, where μ_r is the relative permeability of material (magnetic).

Ans. *(a, d)*

Magnetic field lines for magnetic induction (**B**) form continuous lines. So, lines of **B** are necessarily continuous across S.

Also, magnetic intensity (**H**) varies for inside and outside the lump. So, lines of **H** cannot all be continuous across S.

Q. 7 The primary origin (s) of magnetism lies in

(a) atomic currents

(b) Pauli exclusion principle

(c) polar nature of molecules

(d) intrinsic spin of electron

Ans *(a, d)*

The primary origin of magnetism lies in the fact that the electrons are revolving and spinning about nucleus of an atom, which gives rise to current called atomic current.

This atomic currents gives rise to magnetism. The revolving and spinning about nucleus of an atom is called intrinsic spin of electron.

Q. 8 A long solenoid has 1000 turns per metre and carries a current of 1 A. It has a soft iron core of $\mu_r = 1000$. The core is heated beyond the Curie temperature, T_c.

(a) The **H** field in the solenoid is (nearly) unchanged but the **B** field decreases drastically

(b) The **H** and **B** fields in the solenoid are nearly unchanged

(c) The magnetisation in the core reverses direction

(d) The magnetisation in the core diminishes by a factor of about 10^8

K **Thinking Process**

*The magnetic intensity **H** field $= nI$, where n = number of turns per metre of a solenoid and I = current and $B = \mu_0 \mu_r nI$.*

Also, at normal temperature, a solenoid behave as a ferromagnetic substand and at the temperature beyond the Curie temperature, it behaves as a paramagnetic substance.

Ans. *(a, d)*

Here, for solenoid **H** $= nI$.

$\Rightarrow$ $$H = 1000 \times 1 = 1000 \text{ Am}$$

Thus, **H** is a constant ,so it is nearly unchanged.

But $$B = \mu_0 \mu_r n I$$
$$= \mu_0 n I \mu_r$$
$$= k \text{ (constant) } \mu_r.$$

Thus, from above equation, we find that **B** varies with the variation in μ_r.

Now, for magnetisation in the core, when temperature of the iron core of solenoid is raised beyond Curie temperature, then it behave as paramagnetic material, where

and $$(\chi_m)_{\text{Fero}} \approx 10^3$$

and $$(\chi_m)_{\text{Para}} \approx 10^{-5}$$

$$\Rightarrow \quad \frac{(\chi_m)_{\text{Fero}}}{(\chi_m)_{\text{Para}}} = \frac{10^3}{10^{-5}} = 10^8$$

Q. 9 Essential difference between electrostatic shielding by a conducting shell and magnetostatic shielding is due to

(a) electrostatic field lines can end on charges and conductors have free charges

(b) lines of **B** can also end but conductors cannot end them

(c) lines of **B** cannot end on any material and perfect shielding is not possible

(d) shells of high permeability materials can be used to divert lines of **B** from the interior region

Ans. ***(a, c, d)***

Electrostatic shielding is the phenomenon to block the effects of an electric field. The conducting shell can block the effects of an external field on its internal content or the effect of an internal field on the outside environment.

Magnetostatic shielding is done by using an enclosure made of a high permeability magnetic material to prevent a static magnetic field outside the enclosure from reaching objects inside it or to confine a magnetic field within the enclosure.

Q. 10 Let the magnetic field on the earth be modelled by that of a point magnetic dipole at the centre of the earth. The angle of dip at a point on the geographical equator

(a) is always zero (b) can be zero at specific points

(c) can be positive or negative (d) is bounded

K **Thinking Process**

Angle of inclination or dip is the angle that the total magnetic field of the earth makes with the surface of the earth.

Ans. ***(b, c, d)***

If the total magnetic field of the earth is modelled by a point magnetic dipole at the centre, then it is in the same plane of geographical equator, thus the angle of dip at a point on the geographical equator is bounded in a range from positive to negative value.

Very Short Answer Type Questions

Q. 11 A proton has spin and magnetic moment just like an electron. Why then its effect is neglected in magnetism of materials?

K **Thinking Process**

Mass of a proton is very larger than the mass of an electron, so its spinning is negligible as compared to that of electron spin.

Ans. The comparison between the spinning of a proton and an electron can be done by comparing their magnetic dipole moment which can be given by

$$M = \frac{eh}{4\pi m} \text{ or } M \propto \frac{1}{m} \qquad \left(\because \frac{eh}{4\pi} = \text{constant}\right)$$

$$\therefore \quad \frac{M_p}{M_e} = \frac{m_e}{m_p}$$

$$= \frac{M_e}{1837 M_e} \qquad (\because M_p = 1837 m_e)$$

$$\Rightarrow \quad \frac{M_p}{M_e} = \frac{1}{1837} << 1$$

$$\Rightarrow \quad M_p << M_e$$

Thus, effect of magnetic moment of proton is neglected as compared to that of electron.

Q. 12 A permanent magnet in the shape of a thin cylinder of length 10 cm has $M = 10^6$ A/m. Calculate the magnetisation current I_M.

Ans. Given, M (intensity of magnetisation) = 10^6 A/m.

$$l \text{ (length)} = 10\text{ cm} = 10 \times 10^{-2}\text{ m} = 0.1\text{ m}$$

and $\quad I_M$ = magnetisation current

We know that $\quad M = \dfrac{I_M}{l}$

$$\Rightarrow \quad I_M = M \times l$$

$$= 10^6 \times 0.1 = 10^5\text{ A}$$

Note *Here, M = intensity of magnetisation as its unit is given as A/m.*

Q. 13 Explain quantitatively the order of magnitude difference between the diamagnetic susceptibility of N_2 ($\sim 5 \times 10^{-9}$) (at STP) and Cu($\sim 10^{-5}$).

K **Thinking Process**

Magnetic susceptibility is a measure of how a magnetic material responds to an external field.

Ans. We know that

$$\text{Density of nitrogen } \rho_{N_2} = \frac{28\text{ g}}{22.4\text{ L}} = \frac{28\text{ g}}{22400\text{ cc}}$$

Also, $$\text{density of copper } \rho_{Cu} = \frac{8\text{g}}{22.4 L} = \frac{8\text{g}}{22400\text{ cc}}$$

Now, comparing both densities

$$\frac{\rho_{N_2}}{\rho_{Cu}} = \frac{28}{22400} \times \frac{1}{8} = 1.6 \times 10^{-4}$$

Also given $$\frac{\chi_{N_2}}{\chi_{Cu}} = \frac{5 \times 10^{-9}}{10^{-5}} = 5 \times 10^{-4}$$

We know that, $$\chi = \frac{\text{Magnetisation } (M)}{\text{Magnetic intensity } (H)}$$

$$= \frac{\text{Magnetic moment } (M) \,/\, \text{Volume } (V)}{H}$$

$$= \frac{M}{HV} = \frac{M}{H\,(\text{mass}/\text{density})} = \frac{M\rho}{Hm}$$

$\therefore$ $$\chi \propto \rho \qquad \left(\because \frac{M}{Hm} = \text{constant}\right)$$

Hence, $$\frac{\chi_{N_2}}{\chi_{Cu}} = \frac{\rho_{N_2}}{\rho_{Cu}} = 1.6 \times 10^{-4}$$

Thus, we can say that magnitude difference or major difference between the diamagnetic susceptibility of N_2 and Cu.

Q. 14 From molecular view point, discuss the temperature dependence of susceptibility for diamagnetism, paramagnetism and ferromagnetism.

Ans. Susceptibility of magnetic material $\chi = \frac{I}{H}$, where I is the intensity of magnetisation induced in the material and H is the magnetising force.

Diamagnetism is due to orbital motion of electrons in an atom developing magnetic moments opposite to applied field. Thus, the resultant magnetic moment of the diamagnetic material is zero and hence, the susceptibility χ of diamagnetic material is not much affected by temperature.

Paramagnetism and ferromagnetism is due to alignments of atomic magnetic moments in the direction of the applied field. As temperature is raised, the alignment is disturbed, resulting decrease in susceptibility of both with increase in temperature.

Q. 15A ball of superconducting material is dipped in liquid nitrogen and placed near a bar magnet.

(i) In which direction will it move?

(ii) What will be the direction of its magnetic moment?

K **Thinking Process**

A superconducting material and nitrogen are diamagnetic in nature.

Ans. When a diamagnetic material is dipped in liquid nitrogen, it again behaves as a diamagnetic material. Thus, superconducting material will again behave as a diamagnetic material. When this diamagnetic material is placed near a bar magnet, it will be feebly magnetised opposite to the direction of magnetising field.

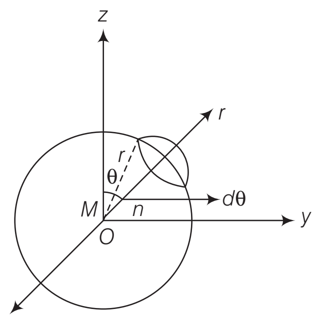
(i) Thus, it will be repelled.

(ii) Also its direction of magnetic moment will be opposite to the direction of magnetic field of magnet.

Short Answer Type Questions

Q. 16 Verify the Gauss's law for magnetic field of a point dipole of dipole moment **m** at the origin for the surface which is a sphere of radius *R*.

Ans. Let us draw the figure for given situation,



We have to prove that $\oint \mathbf{B} \cdot d\mathbf{S} = 0$. This is called Gauss's law in magnetisation.

According to question,
Magnetic moment of dipole at origin *O* is

$$\mathbf{M} = M\hat{k}$$

Let *P* be a point at distance *r* from *O* and *OP* makes an angle θ with *z*-axis. Component of **M** along $OP = M\cos\theta$.
Now, the magnetic field induction at *P* due to dipole of moment $\mathbf{M}\cos\theta$ is

$$\mathbf{B} = \frac{\mu_0}{4\pi}\frac{2M\cos\theta}{r^3}\hat{\mathbf{r}}$$

From the diagram, *r* is the radius of sphere with centre at *O* lying in *yz*-plane. Take an elementary area $d\mathbf{S}$ of the surface at *P*. Then,

$$d\mathbf{S} = r(r\sin\theta\ d\theta)\hat{\mathbf{r}} = r^2\sin\theta\, d\theta\hat{\mathbf{r}}$$

$$\oint \mathbf{B} \cdot d\mathbf{S} = \oint \frac{\mu_0}{4\pi}\frac{2M\cos\theta}{r^3}\hat{\mathbf{r}}\,(r^2\sin\theta\, d\theta\hat{\mathbf{r}})$$

$$= \frac{\mu_0}{4\pi}\frac{M}{r}\int_0^{2\pi} 2\sin\theta.\cos\theta\, d\theta$$

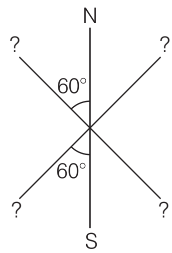
$$= \frac{\mu_0}{4\pi}\frac{M}{r}\int_0^{2\pi} \sin 2\theta d\theta$$

$$= \frac{\mu_0}{4\pi}\frac{M}{r}\left(\frac{-\cos 2\theta}{2}\right)_0^{2\pi}$$

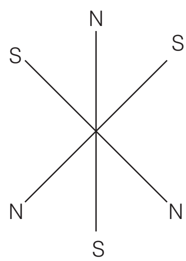
$$= -\frac{\mu_0}{4\pi}\frac{M}{2r}[\cos 4\pi - \cos 0]$$

$$= \frac{\mu_0}{4\pi}\frac{M}{2r}[1 - 1] = 0$$

Q. 17 Three identical bar magnets are rivetted together at centre in the same plane as shown in figure. This system is placed at rest in a slowly varying magnetic field. It is found that the system of magnets does not show any motion. The north-south poles of one magnet is shown in the figure. Determine the poles of the remaining two.



Ans. The system will be in stable equilibrium if the net force on the system is zero and net torque on the system is also zero. This is possible only when the poles of the remaining two magnets are as given in the figure.



Q. 18 Suppose we want to verify the analogy between electrostatic and magnetostatic by an explicit experiment. Consider the motion of (i) electric dipole **p** in an electrostatic field **E** and (ii) magnetic dipole **M** in a magnetic field **B**. Write down a set of conditions on **E**, **B**, **p**, **M** so that the two motions are verified to be identical. (Assume identical initial conditions).

Thinking Process

$E(r) = cB(r)$, suppose the angle between **p** *and* **E** *is θ. Torque on electric dipole of moment* **p** *in electric field* **E**, $\tau = pE\sin\theta$.

Ans. Now, suppose that the angle between **M** and B is θ.

Torque on magnetic dipole moment **M** in magnetic field **B**,

$$\tau' = MB\sin\theta$$

Two motions will be identical, if

$$pE\sin\theta = MB\sin\theta$$

$\Rightarrow$ $$pE = MB \quad \ldots(i)$$

But, $$E = cB$$

$\therefore$ Putting this value in Eq. (i),

$$pcB = MB$$

$\Rightarrow$ $$p = \frac{M}{c}$$

Q. 19 A bar magnet of magnetic moment M and moment of inertia I (about centre, perpendicular to length) is cut into two equal pieces, perpendicular to length. Let T be the period of oscillations of the original magnet about an axis through the mid-point, perpendicular to length, in a magnetic field B. What would be the similar period T' for each piece?

Thinking Process

$$T = 2\pi\sqrt{\frac{I}{MB}}$$

where, T = time period
I = moment of inertia
m = mass of magnet
B = magnetic field

Ans. Given, I = moment of inertia of the bar magnet
m = mass of bar magnet
l = length of magnet about an any passing through its centre and perpendicular to its length
M = magnetic moment of the magnet
B = uniform magnetic field in which magnet is oscillating, we get time period of oscillation is,

$$T = 2\pi\sqrt{\frac{I}{MB}}$$

Here,

$$I = \frac{ml^2}{12}.$$

When magnet is cut into two equal pieces, perpendicular to length, then moment of inertia of each piece of magnet about an axis perpendicular to length passing through its centre is

$$I' = \frac{m}{2}\frac{(l/2)^2}{12} = \frac{ml^2}{12} \times \frac{1}{8} = \frac{I}{8}$$

Magnetic dipole moment $M' = M/2$

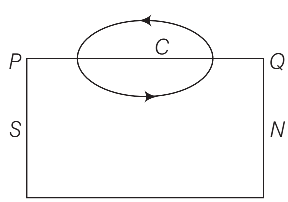
Its time period of oscillation is

$$T' = 2\pi\sqrt{\frac{I'}{M'B}} = 2\pi\sqrt{\frac{I/8}{(M/2)B}} = \frac{2\pi}{2}\sqrt{\frac{I}{MB}}$$

$$T' = \frac{T}{2}.$$

Q. 20 Use (i) the Ampere's law for H and (ii) continuity of lines of **B**, to conclude that inside a bar magnet, (a) lines of **H** run from the N-pole to S- pole, while (b) lines of **B** must run from the S-pole to N-pole.

Ans. Consider a magnetic field line of **B** through the bar magnet as given in the figure below.



The magnetic field line of B through the bar magnet must be a closed loop.

Let C be the amperian loop. Then,

$$\int_Q^P \mathbf{H.dl} = \int_Q^P \frac{\mathbf{B}}{m_0}.\mathbf{dl}$$

We know that the angle between **B** and **dl** is less than 90° inside the bar magnet. So, it is positive.

i.e.,
$$\int_Q^P \mathbf{H.dl} = \int_Q^P \frac{\mathbf{B}}{\mu_0}.\mathbf{dl} > 0$$

Hence, the lines of B must run from south pole(S) to north pole (N) inside the bar magnet.

According to Ampere's law,

$$\therefore \quad \oint_{PQP} \mathbf{H.dl} = 0$$

$$\therefore \quad \oint_{PQP} \mathbf{H.dl} = \int_P^Q \mathbf{H.dl} + \int_Q^P \mathbf{H.dl} = 0$$

As
$$\int_Q^P \mathbf{H.dl} > 0, \text{ so, } \int_P^Q \mathbf{H.dl} < 0 \qquad (\textit{i.e., } \text{negative})$$

It will be so if angle between **H** and **dl** is more than 90°, so that $\cos\theta$ is negative. It means the line of **H** must run from N-pole to S-pole inside the bar magnet.

Long Answer Type Questions

Q. 21 Verify the Ampere's law for magnetic field of a point dipole of dipole moment $\mathbf{M} = M\hat{\mathbf{k}}$. Take C as the closed curve running clockwise along

(i) the z-axis from $z = a > 0$ to $z = R$,

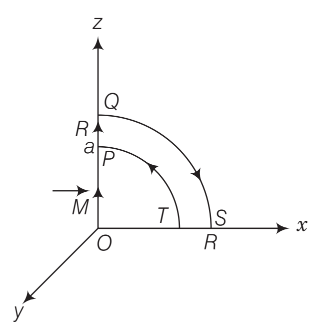
(ii) along the quarter circle of radius R and centre at the origin in the first quadrant of xz-plane,

(iii) along the x-axis from $x = R$ to $x = a$, and

(iv) along the quarter circle of radius a and centre at the origin in the first quadrant of xz-plane

Thinking Process

Let us consider the figure below



Ans. From P to Q, every point on the z-axis lies at the axial line of magnetic dipole of moment $\mathbf{M}$. Magnetic field induction at a point distance z from the magnetic dipole of moment is

$$|\mathbf{B}| = \frac{\mu_0}{4\pi}\frac{2|\mathbf{M}|}{z^3} = \frac{\mu_0 M}{2\pi z^3}$$

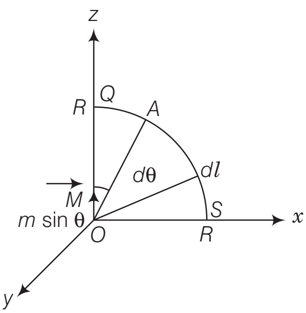
(i) Along z-axis from P to Q.

$$\int_P^Q \mathbf{B.dl} = \int_P^Q \mathbf{B.dl}\cos 0° = \int_a^R \mathbf{B}\, dz$$

$$= \int_a^R \frac{\mu_0}{2\pi}\frac{M}{z^3}dz = \frac{\mu_0 M}{2\pi}\left(\frac{-1}{2}\right)\left(\frac{1}{R^2} - \frac{1}{a^2}\right)$$

$$= \frac{\mu_0 M}{4\pi}\left(\frac{1}{a^2} - \frac{1}{R^2}\right)$$

(ii) Along the quarter circle QS of radius R as given in the figure below



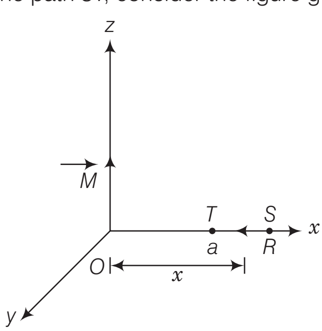
The point A lies on the equatorial line of the magnetic dipole of moment $M\sin\theta$. Magnetic field at point A on the circular arc is

$$B = \frac{\mu_0}{4\pi}\frac{M\sin\theta}{R^3};\ \mathbf{dl} = Rd\theta$$

$$\therefore \qquad \int \mathbf{B.dl} = \int \mathbf{B.dl}\cos\theta = \int_0^{\frac{\pi}{2}} \frac{\mu_0}{4\pi}\frac{M\sin\theta}{R^3}Rd\theta$$

Circular arc $= \dfrac{\mu_0}{4\pi}\dfrac{M}{R}(-\cos\theta)_0^{\pi/2} = \dfrac{\mu_0}{4\pi}\dfrac{M}{R^2}$

(iii) Along x-axis over the path ST, consider the figure given ahead

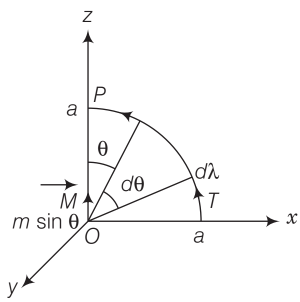


From figure, every point lies on the equatorial line of magnetic dipole. Magnetic field induction at a point distance x from the dipole is

$$B = \frac{\mu_0}{4\pi}\frac{M}{x^3}$$

$$\therefore \qquad \int_S^T \mathbf{B.dl} = \int_R^a -\frac{\mu_0 \mathbf{M}}{4\pi x^3}.\mathbf{dl} = 0 \quad [\because \text{angle between } (-\mathbf{M}) \text{ and } \mathbf{dl} \text{ is } 90°]$$

(iv) Along the quarter circle TP of radius a. Consider the figure given below



From case (ii), we get line integral of **B** along the quarter circle TP of radius a is

circular arc TP $$\int \mathbf{B.dl} = \int_{\pi/2}^{0} \frac{\mu_0}{4\pi} \frac{\mathbf{M}\sin\theta}{a^3} a\,d\theta$$

$$= \frac{\mu_0}{4\pi} \frac{M}{a^2} \int_{\pi/2}^{0} \sin\theta\, d\theta = \frac{\mu_0}{4\pi} \frac{M}{a^2} [-\cos\theta]_{\pi/2}^{0}$$

$$= \frac{-\mu_0}{4\pi} \frac{M}{a^2}$$

$\therefore$ $$\oint_{PQST} \mathbf{B.dl} = \int_P^Q \mathbf{B.dl} + \int_Q^S \mathbf{B.dl} + \int_S^T \mathbf{B.dl} + \int_T^P \mathbf{B.dl}$$

$$= \frac{\mu_0 M}{4}\left[\frac{1}{a^2} - \frac{1}{R^2}\right] + \frac{\mu_0}{4\pi} \frac{M}{R^2} + 0 + \left(-\frac{\mu_0}{4\pi} \frac{M}{a^2}\right) = 0$$

Q. 22 What are the dimensions of χ, the magnetic susceptibility? Consider an H-atom. Gives an expression for χ, upto a constant by constructing a quantity of dimensions of χ, out of parameters of the atom e, m, v, R and μ_0. Here, m is the electronic mass, v is electronic velocity, R is Bohr radius. Estimate the number so obtained and compare with the value of $|\chi| \sim 10^{-5}$ for many solid materials.

Thinking Process

Magnetic susceptibility is a measure of how a magnetic material responds to an external field. i.e., magnetic susceptibility

$$\chi_m = \frac{I}{H} = \frac{\text{(Intensity of magnetisation)}}{\text{(Magnetising force)}}$$

Ans. As I and H both have same units and dimensions, hence, χ has no dimensions. Here, in this question, χ is to be related with e, m, v, R and μ_0. We know that dimensions of $\mu_0 = [\text{ML}\theta^{-2}]$

From Biot-Savart's law,

$$dB = \frac{\mu_0}{4\pi} \frac{I dl \sin\theta}{r^2}$$

$\Rightarrow$ $$\mu_0 = \frac{4\pi r^2 dB}{I dl \sin\theta} = \frac{4\pi r^2}{I dl \sin\theta} \times \frac{f}{qv\sin\theta} \qquad \left[\because\ dB = \frac{F}{qv\sin\theta}\right]$$

$\therefore$ $$\text{Dimensions of } \mu_0 = \frac{\text{L}^2 \times (\text{MLT}^{-2})}{(\text{QT}^{-1})\,(\text{L}) \times 1 \times (\text{Q})\,(\text{LT}^{-1}) \times (1)} = [\text{MLQ}^{-2}]$$

where Q is the dimension of charge.

As χ is dimensionless, it should have no involvement of charge Q in its dimensional formula. It will be so if μ_0 and e together should have the value $\mu_0 e^2$, as e has the dimensions of charge.

Let $$\chi = \mu_0 e^2 m^a v^b R^c \quad \ldots(i)$$

where a, b, c are the power of m, v and R respectively, such that relation (i) is satisfied.

Dimensional equation of (i) is

$$[M^0L^0T^0Q^0] = [MLQ^{-2}] \times [Q^2][M^a] \times (LT^{-1})^b \times [L]^c$$
$$= [M^{1+a} + L^{1+b+c}T^{-b}Q^0]$$

Equating the powers of M, L and T, we get

$$0 = 1 + a \Rightarrow a = -1, \; 0 = 1 + b + c \quad \ldots(ii)$$
$$0 = -b \Rightarrow b = 0, \; 0 = 1 + 0 + c \text{ or } c = -1$$

Putting values in Eq. (i), we get

$$\chi = \mu_0 e^2 m^{-1} v^2 R^{-1} = \frac{\mu_0 e^2}{mR} \quad \ldots(iii)$$

Here,
$$\mu_0 = 4\pi \times 10^{-7} \text{Tm A}^{-1}$$
$$e = 1.6 \times 10^{-19} \text{ C}$$
$$m = 9.1 \times 10^{-31} \text{ kg}, \; R = 10^{-10} \text{m}$$
$$\chi = \frac{(4\pi \times 10^{-7}) \times (1.6 \times 10^{-19})^2}{(9.1 \times 10^{-31}) \times 10^{-10}} \approx 10^{-4}$$

$$\therefore \quad \frac{\chi}{\chi_{\text{(given solid)}}} = \frac{10^{-4}}{10^{-5}} = 10$$

Q. 23 Assume the dipole model for the earth's magnetic field B which is given by B_V = vertical component of magnetic field $= \dfrac{\mu_0}{4\pi} \dfrac{2m\cos\theta}{r^3}$

B_H = horizontal component of magnetic field $= \dfrac{\mu_0}{4\pi} \dfrac{\sin\theta m}{r^3}$

$\theta = 90° -$ lattitude as measured from magnetic equator.

Find loci of points for which (a) $|B|$ is minimum (b) dip angle is zero and (c) dip angle is 45°.

Ans. (a)
$$B_V = \frac{\mu_0}{4\pi} \frac{2m\cos\theta}{r^3} \quad \ldots(i)$$
$$B_H = \frac{\mu_0}{4\pi} \frac{\sin\theta m}{r^3} \quad \ldots(ii)$$

Squaring both the equations and adding, we get

$$B_V^2 + B_H^2 = \left(\frac{\mu_0}{4\pi}\right) \frac{m^2}{r^6} [4\cos^2\theta + \sin^2\theta]$$
$$B = \sqrt{B_V^2 + B_H^2} = \frac{\mu_0}{4\pi} \frac{m}{r^3} [3\cos^2\theta + 1]^{1/2} \quad \ldots(iii)$$

From Eq. (iii), the value of B is minimum, if $\cos\theta = \dfrac{\pi}{2}$

$\theta = \dfrac{\pi}{2}$. Thus, the magnetic equator is the locus.

(b) Angle of dip,

$$\tan\delta = \frac{B_V}{B_H} = \frac{\frac{\mu_0}{4\pi}\cdot\frac{2m\cos\theta}{r^3}}{\frac{\mu_0}{4\pi}\cdot\frac{\sin\theta\cdot m}{r^3}} = 2\cot\theta \qquad \ldots\text{(iv)}$$

$$\tan\delta = 2\cot\theta$$

For dip angle is zero *i.e.,* $\delta = 0$

$$\cot\theta = 0$$

$$\theta = \frac{\pi}{2}$$

It means that locus is again magnetic equator.

(c) $\tan\delta = \dfrac{B_V}{B_H}$

Angle of dip *i.e.,* $\delta = \pm 45$

$$\frac{B_V}{B_H} = \tan(\pm 45^\circ)$$

$$\frac{B_V}{B_H} = 1$$

$$2\cot\theta = 1 \qquad \text{[From Eq. (iv)]}$$

$$\cot\theta = \frac{1}{2}$$

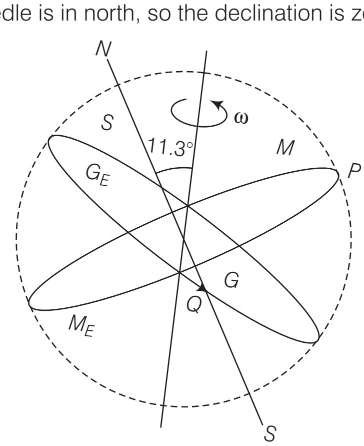
$$\tan\theta = 2$$

$$\Rightarrow \qquad \theta = \tan^{-1}(2)$$

Thus, $\theta = \tan^{-1}(2)$ is the locus.

Q. 24 **Consider the plane S formed by the dipole axis and the axis of earth. Let P be point on the magnetic equator and in S. Let Q be the point of intersection of the geographical and magnetic equators. Obtain the declination and dip angles at P and Q.**

Ans. P is in the plane S, needle is in north, so the declination is zero.



P is also on the magnetic equator, so the angle of dip = 0, because the value of angle of dip at equator is zero. Q is also on the magnetic equator, thus the angle of dip is zero.

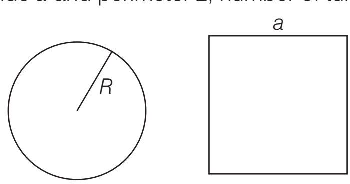
As earth tilted on its axis by 11.3°, thus the declination at Q is 11.3°.

Q. 25 There are two current carrying planar coil made each from identical wires of length L. C_1 is circular (radius R) and C_2 is square (side a). They are so constructed that they have same frequency of oscillation when they are placed in the same uniform **B** and carry the same current. Find a in terms of R.

Ans. C_1 = circular coil of radius R, length L, number of turns per unit length

$$n_1 = \frac{L}{2\pi R}$$

C_2 = square of side a and perimeter L, number of turns per unit length $n_2 = \dfrac{L}{4a}$



Magnetic moment of C_1

$\Rightarrow$ $$m_1 = n_1 I A_1$$

Magnetic moment of C_2

$\Rightarrow$ $$m_2 = n_2 I A_2$$

$$m_1 = \frac{L \cdot I \cdot \pi R^2}{2\pi R}$$

$$m_2 = \frac{L}{4a} \cdot I \cdot a^2$$

$$m_1 = \frac{LIR}{2} \quad \text{...(i)}$$

$$m_2 = \frac{LIa}{4} \quad \text{...(ii)}$$

Moment of inertia of $C_1 \Rightarrow I_1 = \dfrac{MR^2}{2}$ ...(iii)

Moment of inertia of $C_2 \Rightarrow I_2 = \dfrac{Ma^2}{12}$ ...(iv)

Frequency of $C_1 \Rightarrow f_1 = 2\pi\sqrt{\dfrac{I_1}{m_1 B}}$

Frequency of $C_2 \Rightarrow f_2 = 2\pi\sqrt{\dfrac{I_2}{m_2 B}}$

According to question, $f_1 = f_2$

$$2\pi\sqrt{\frac{I_1}{m_1 B}} = 2\pi\sqrt{\frac{I_2}{m_2 B}}$$

$$\frac{I_1}{m_1} = \frac{I_2}{m_2} \text{ or } \frac{m_2}{m_1} = \frac{I_2}{I_1}$$

Plugging the values by Eqs. (i), (ii), (iii) and (iv)

$$\frac{LIa \cdot 2}{4 \times LIR} = \frac{Ma^2 \cdot 2}{12 \cdot MR^2}$$

$$\frac{a}{2R} = \frac{a^2}{6R^2}$$

$$3R = a$$

Thus, the value of a is $3R$.

6

Electromagnetic Induction

Multiple Choice Questions (MCQs)

Q. 1 A square of side L metres lies in the xy-plane in a region, where the magnetic field is given by $\mathbf{B} = B_0(2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 4\hat{\mathbf{k}})$ T, where B_0 is constant. The magnitude of flux passing through the square is

(a) $2B_0L^2$ Wb (b) $3B_0L^2$ Wb (c) $4B_0L^2$ Wb (d) $\sqrt{29}B_0L^2$ Wb

Thinking Process

The magnetic flux linked with uniform surface of area A in uniform magnetic field is given by

$$\phi = \mathbf{B.A}$$

Ans. *(c)* Here, $A = L^2\hat{\mathbf{k}}$ and $\mathbf{B} = B_0(2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 4\hat{\mathbf{k}})$ T

$$\phi = \mathbf{B.A} = B_0(2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 4\hat{\mathbf{k}}).L^2\hat{\mathbf{k}} = 4B_0L^2 \text{ Wb}$$

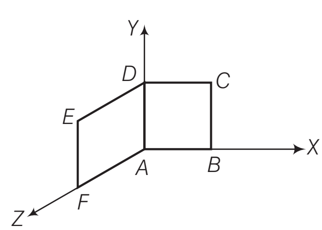
Q. 2 A loop, made of straight edges has six corners at A (0, 0, 0), B (L, 0, 0), C (L, L, 0), D (0, L, 0), E (0, L, L) and F (0, 0, L). A magnetic field $\mathbf{B} = B_0(\hat{\mathbf{i}} + \hat{\mathbf{k}})$ T is present in the region. The flux passing through the loop $ABCDEFA$ (in that order) is

(a) B_0L^2 Wb (b) $2B_0L^2$ Wb (c) $\sqrt{2}B_0L^2$ Wb (d) $4B_0L^2$ Wb

Thinking Process

Here, loop ABCDA lies in x-y plane whose area vector $\mathbf{A}_1 = L^2\hat{\mathbf{k}}$ *whereas loop ADEFA lies in y-z plane whose area vector* $\mathbf{A}_2 = L^2\hat{\mathbf{i}}$.

Ans. *(b)* Also, the magnetic flux linked with uniform surface of area A in uniform magnetic field is given by



$$\phi = \mathbf{B.A}$$

$$\mathbf{A} = \mathbf{A}_1 + \mathbf{A}_2 = (L^2\hat{\mathbf{k}} + L^2\hat{\mathbf{i}})$$

and $\mathbf{B} = B_0(\hat{\mathbf{i}} + \hat{\mathbf{k}})T$

Now, $\phi = \mathbf{B.A} = B_0(\hat{\mathbf{i}} + \hat{\mathbf{k}}).(L^2\hat{\mathbf{k}} + L^2\hat{\mathbf{i}})$

$$= 2B_0L^2 \text{ Wb}$$

Q. 3 A cylindrical bar magnet is rotated about its axis. A wire is connected from the axis and is made to touch the cylindrical surface through a contact. Then,

(a) a direct current flows in the ammeter A

(b) no current flows through the ammeter A

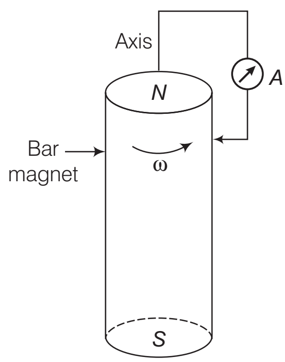
(c) an alternating sinusoidal current flows through the ammeter A with a time period $T = \frac{2\pi}{\omega}$

(d) a time varying non-sinusoidal current flows through the ammeter A

K Thinking Process

The problem is associated with the phenomenon of electromagnetic induction.

Ans. *(b)* When cylindrical bar magnet is rotated about its axis, no change in flux linked with the circuit takes place, consequently no emf induces and hence, no current flows through the ammeter A.



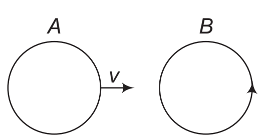
Q. 4 There are two coils A and B as shown in figure. A current starts flowing in B as shown, when A is moved towards B and stops when A stops moving. The current in A is counter clockwise. B is kept stationary when A moves. We can infer that

(a) there is a constant current in the clockwise direction in A

(b) there is a varying current in A

(c) there is no current in A

(d) there is a constant current in the counter clockwise direction in A



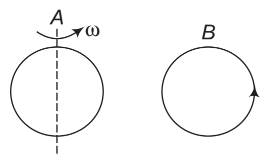
K Thinking Process

The induced emf in B is due to the variation of magnetic flux in it.

Ans. *(d)* When the A stops moving the current in B become zero, it possible only if the current in A is constant. If the current in A would be variable, there must be an induced emf (current) in B even if the A stops moving.

Q. 5 Same as problem 4 except the coil A is made to rotate about a vertical axis (figure). No current flows in B if A is at rest. The current in coil A, when the current in B (at $t = 0$) is counter-clockwise and the coil A is as shown at this instant, $t = 0$, is

(a) constant current clockwise
(b) varying current clockwise
(c) varying current counter clockwise
(d) constant current counter clockwise



K **Thinking Process**

Here, the application of Lenz's law is tested through this problem.

Ans. ***(a)*** When the current in B (at $t = 0$) is counter-clockwise and the coil A is considered above to it. The counterclockwise flow of the current in B is equivalent to north pole of magnet and magnetic field lines are emanating upward to coil A. When coil A start rotating at $t = 0$, the current in A is constant along clockwise direction by Lenz's rule.

Q. 6 The self inductance L of a solenoid of length l and area of cross-section A, with a fixed number of turns N increases as

(a) l and A increase
(b) l decreases and A increases
(c) l increases and A decreases
(d) both l and A decrease

K **Thinking Process**

The self inductance L of a solenoid depends on its geometry (i.e., length, cross-sectional area, number of turns etc.) and on the permeability of the medium.

Ans. ***(b)*** The self-inductance of a long solenoid of cross-sectional area A and length l, having n turns per unit length, filled the inside of the solenoid with a material of relative permeability (*e.g.*, soft iron, which has a high value of relative permeability) is given by

$$L = \mu_r \mu_0 n^2 A l$$

where, $$n = N / l$$

Note *The capacitance, resistance, self and mutual inductance depends on the geometry of the devices as well as permittivity/permeability of the medium.*

Multiple Choice Questions (More Than One Options)

Q. 7 A metal plate is getting heated. It can be because

(a) a direct current is passing through the plate
(b) it is placed in a time varying magnetic field
(c) it is placed in a space varying magnetic field, but does not vary with time
(d) a current (either direct or alternating) is passing through the plate

K **Thinking Process**

This problem is associated with the heating effect of current as well as the phenomenon of electromagnetic induction and eddy currents.

Ans. ***(a, b, d)***

A metal plate is getting heated when a DC or AC current is passed through the plate, known as heating effect of current. Also, when metal plate is subjected to time varying magnetic field, the magnetic flux linked with the plate changes and eddy currents comes into existence which make the plate hot.

Q. 8 An emf is produced in a coil, which is not connected to an external voltage source. This can be due to

(a) the coil being in a time varying magnetic field
(b) the coil moving in a time varying magnetic field
(c) the coil moving in a constant magnetic field
(d) the coil is stationary in external spatially varying magnetic field, which does not change with time

K **Thinking Process**

This problem is associated with the phenomenon of electromagnetic induction.

Ans. ***(a, b, c)***

Here, magnetic flux linked with the isolated coil change when the coil being in a time varying magnetic field, the coil moving in a constant magnetic field or in time varying magnetic field.

Note *When magnetic flux linked with the coil change, an emf is used in the coil. This is known as electromagnetic induction.*

Q. 9 The mutual inductance M_{12} of coil 1 with respect to coil 2

(a) increases when they are brought nearer
(b) depends on the current passing through the coils
(c) increases when one of them is rotated about an axis
(d) is the same as M_{21} of coil 2 with respect to coil 1

K **Thinking Process**

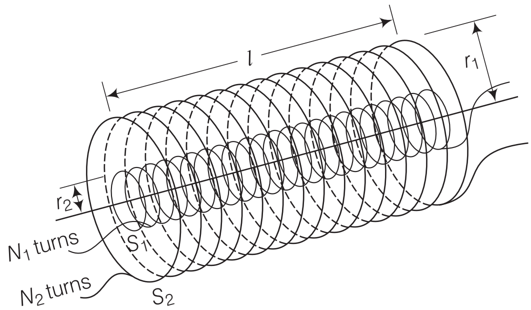
Here, it is important to know that the mutual inductance of a pair of coils, solenoids, etc., depends on their separation, their relative orientation as well as the geometry of pair of coils, solenoids, etc.

Ans. ***(a, d)***

The mutual inductance M_{12} of coil increases when they are brought nearer and is the same as M_{21} of coil 2 with respect to coil 1.

M_{12} *i.e.,* mutual inductance of solenoid S_1 with respect to solenoid S_2 is given by

$$M_{21} = \mu_0 n_1 n_2 \pi r_1^2 l$$



where signs are as usual.

Also, M_{12} *i.e.*, mutual inductance of solenoid S_2 with respect to solenoid S_1 is given by

$$M_{21} = \mu_0 n_1 n_2 \pi r_1^2 l$$

So, we have $$M_{12} = M_{21} = M$$

Q. 10 A circular coil expands radially in a region of magnetic field and no electromotive force is produced in the coil. This can be because

(a) the magnetic field is constant

(b) the magnetic field is in the same plane as the circular coil and it may or may not vary

(c) the magnetic field has a perpendicular (to the plane of the coil) component whose magnitude is decreasing suitably

(d) there is a constant magnetic field in the perpendicular (to the plane of the coil) direction

K **Thinking Process**

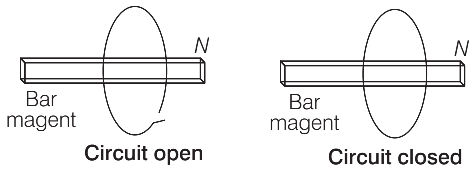
The various arrangement are to be thought of in such a way that the magnetic flux linked with the coil do not change even if coil is placed and expanding in magnetic field.

Ans. ***(b, c)***

When circular coil expands radially in a region of magnetic field such that the magnetic field is in the same plane as the circular coil or the magnetic field has a perpendicular (to the plane of the coil) component whose magnitude is decreasing suitably in such a way that the cross product of magnetic field and surface area of plane of coil remain constant at every instant.

Very Short Answer Type Questions

Q. 11 Consider a magnet surrounded by a wire with an on/off switch S (figure). If the switch is thrown from the off position (open circuit) to the on position (closed circuit), will a current flow in the circuit? Explain.



K **Thinking Process**

The magnetic flux linked with uniform surface of area A in uniform magnetic field is given by

$$\phi = \mathbf{B} . \mathbf{A} = BA \cos \theta$$

So, flux linked will change only when either B, or A or the angle between B and A change.

Ans. When the switch is thrown from the off position (open circuit) to the on position (closed circuit), then neither *B*, nor *A* nor the angle between *B* and *A* change. Thus, no change in magnetic flux linked with coil occur, hence no electromotive force is produced and consequently no current will flow in the circuit.

Q. 12 A wire in the form of a tightly wound solenoid is connected to a DC source, and carries a current. If the coil is stretched so that there are gaps between successive elements of the spiral coil, will the current increase or decrease? Explain.

K **Thinking Process**

Here, the application of Lenz's law is tested through this problem.

Ans. When the coil is stretched so that there are gaps between successive elements of the spiral coil *i.e.,* the wires are pulled apart which lead to the flux leak through the gaps. According to Lenz's law, the emf produced must oppose this decrease, which can be done by an increase in current. So, the current will increase.

Q. 13 A solenoid is connected to a battery so that a steady current flows through it. If an iron core is inserted into the solenoid, will the current increase or decrease? Explain.

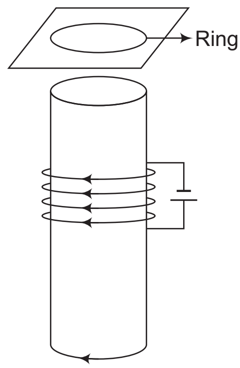
K **Thinking Process**

Here, the application of Lenz's law is tested through this problem.

Ans. When the iron core is inserted in the current carrying solenoid, the magnetic field increase due to the magnetisation of iron core and consequently the flux increases.

According to Lenz's law, the emf produced must oppose this increase in flux, which can be done by making decrease in current. So, the current will decrease.

Q. 14 Consider a metal ring kept on top of a fixed solenoid (say on a cardboard) (figure). The centre of the ring coincides with the axis of the solenoid. If the current is suddenly switched on, the metal ring jumps up. Explain



K **Thinking Process**

Here, the application of Lenz's law is tested through this problem.

Ans. When the current is switched on, magnetic flux is linked through the ring. Thus, increase in flux takes place. According to Lenz's law, this increase in flux will be opposed and it can happen if the ring moves away from the solenoid.

This happen because the flux increases will cause a counter clockwise current (as seen from the top in the ring in figure.) *i.e.,* opposite direction to that in the solenoid.

This makes the same sense of flow of current in the ring (when viewed from the bottom of the ring) and solenoid forming same magnetic pole infront of each other. Hence, they will repel each other and the ring will move upward.

Q. 15 Consider a metal ring kept (supported by a cardboard) on top of a fixed solenoid carrying a current I (see figure of Question 14). The centre of the ring coincides with the axis of the solenoid. If the current in the solenoid is switched off, what will happen to the ring?

K **Thinking Process**

This problem is based on the application of Lenz's law.

Ans. *(b)* When the current is switched off, magnetic flux linked through the ring decreases. According to Lenz's law, this decrease in flux will be opposed and the ring experience downward force towards the solenoid.

This happen because the flux *i* decrease will cause a clockwise current (as seen from the top in the ring in figure) *i.e.,* the same direction to that in the solenoid. This makes the opposite sense of flow of current in the ring (when viewed from the bottom of the ring) and solenoid forming opposite magnetic pole infront of each other.

Hence, they will attract each other but as ring is placed at the cardboard it could not be able to move downward.

Q. 16 Consider a metallic pipe with an inner radius of 1 cm. If a cylindrical bar magnet of radius 0.8 cm is dropped through the pipe, it takes more time to come down than it takes for a similar unmagnetised cylindrical iron bar dropped through the metallic pipe. Explain.

K **Thinking Process**

This problem is based on the concept of eddy current and application of Lenz's law.

Ans. When cylindrical bar magnet of radius 0.8 cm is dropped through the metallic pipe with an inner radius of 1 cm, flux linked with the cylinder changes and consequently eddy currents are produced in the metallic pipe. According to Lenz's law, these currents will oppose the (cause) motion of the magnet.

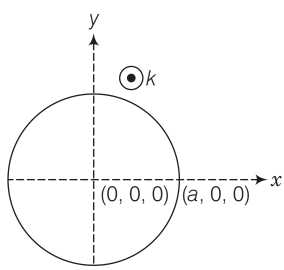
Therefore, magnet's downward acceleration will be less than the acceleration due to gravity g. On the other hand, an unmagnetised iron bar will not produce eddy currents and will fall with an acceleration due to gravity g.

Thus, the magnet will take more time to come down than it takes for a similar unmagnetised cylindrical iron bar dropped through the metallic pipe.

Short Answer Type Questions

Q. 17 A magnetic field in a certain region is given by $\mathbf{B} = B_0 \cos(\omega t)\,\hat{\mathbf{k}}$ and *a* coil of radius a with resistance R is placed in the x–y plane with its centre at the origin in the magnetic field (figure). Find the magnitude and the direction of the current at $(a, 0, 0)$ at

$$t = \frac{\pi}{2\omega},\ t = \frac{\pi}{\omega} \text{ and } t = \frac{3\pi}{2\omega}$$



K **Thinking Process**

This problem requires application of Faraday's law of EMI and finding mathematical values of emf at different instants.

Ans. At any instant, flux passes through the ring is given by

$$\phi = \mathbf{B}.\mathbf{A} = BA\cos\theta = BA \qquad (\because \theta = 0)$$

or $$\phi = B_0(\pi a^2)\cos\omega t$$

By Faraday's law of electromagnetic induction.,

Magnitude of induced emf is given by

$$\varepsilon = B_0(\pi a^2)\,\omega\sin\omega t$$

This causes flow of induced current, which is given by

$$I = B_0(\pi a^2)\,\omega\sin\omega t\,/R$$

Now, finding the value of current at different instants, so we have current at

$$t = \frac{\pi}{2\omega}$$

$$I = \frac{B_0(\pi a^2)\omega}{R} \text{ along } \hat{\mathbf{j}}$$

Because $$\sin\omega t = \sin\left(\omega \frac{\pi}{2\omega}\right) = \sin\frac{\pi}{2} = 1$$

$$t = \frac{\pi}{\omega},\ I = \frac{B(\pi a^2)\omega}{R}$$

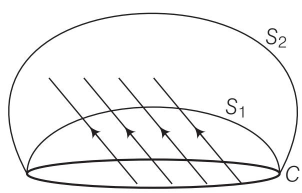
Here, $$\sin\omega t = \sin\left(\omega \frac{\pi}{\omega}\right) = \sin \pi = 0$$

$$t = \frac{3}{2}\frac{\pi}{\omega}$$

$$I = \frac{B(\pi a^2)\omega}{R} \text{ along } -\hat{\mathbf{j}}$$

$$\sin\omega t = \sin\left(\omega \frac{3\pi}{2\omega}\right) = \sin\frac{3\pi}{2} = -1$$

Q. 18 Consider a closed loop C in a magnetic field (figure). The flux passing through the loop is defined by choosing a surface whose edge coincides with the loop and using the formula $\phi = \mathbf{B}_1\, d\mathbf{A}_1,\ \mathbf{B}_2\, d\mathbf{A}_2$ ..... . Now, if we choose two different surfaces S_1 and S_2 having C as their edge, would we get the same answer for flux. Justify your answer.

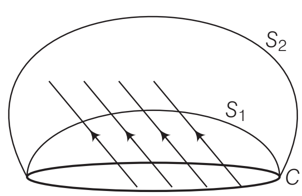


K **Thinking Process**

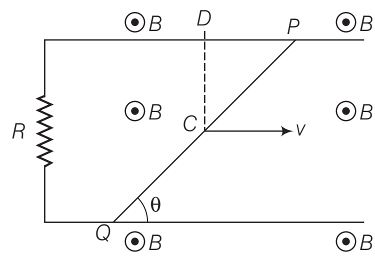
This problem underline the concept of continuity of magnetic field lines. They can neither be originated nor be destroyed in space.

Ans. The magnetic flux linked with the surface can considered as the number of magnetic field lines passing through the surface. So, let $d\phi = \mathbf{BA}$ represents magnetic lines in an area A to B.

By the concept of continuity of lines B cannot end or start in space, therefore the number of lines passing through surface S_1 must be the same as the number of lines passing through the surface S_2. Therefore, in both the cases we gets the same answer for flux.



Q. 19 Find the current in the wire for the configuration shown in figure. Wire *PQ* has negligible resistance. *B*, the magnetic field is coming out of the paper. θ is a fixed angle made by *PQ* travelling smoothly over two conducting parallel wires separated by a distance *d*.



K **Thinking Process**

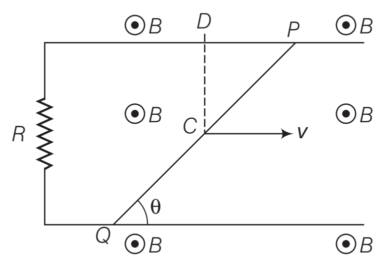
The emf induced across PQ due to its motion or change in magnetic flux linked with the loop change due to change of enclosed area.

Ans. The motional electric field E along the dotted line CD ($\wedge$ to both $\mathbf{v}$ and $\mathbf{B}$ and along $\mathbf{V} \times \mathbf{B}$) $= vB$

Therefore, the motional emf along PQ = (length PQ) $\times$ (field along PQ)

$$= (\text{length } PQ) \times (vB \sin\theta)$$

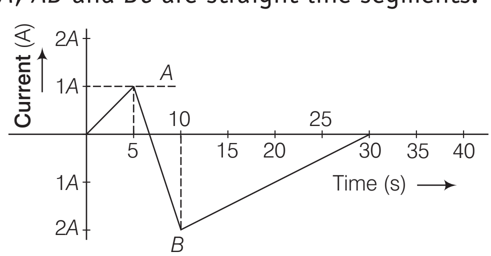
$$= \left(\frac{d}{\sin\theta}\right) \times (vB\sin\theta) = vBd$$



This induced emf make flow of current in closed circuit of resistance R.

$$I = \frac{dvB}{R} \text{ and is independent of } q.$$

Q. 20 A (current *versus* time) graph of the current passing through a solenoid is shown in figure. For which time is the back electromotive force (u) a maximum. If the back emf at $t = 3$ s is e, find the back emf at $t = 7$ s, 15s and 40 s. *OA*, *AB* and *BC* are straight line segments.



K **Thinking Process**

When the current is varied, the flux linked with the coil also changes and an emf is induced in the coil. The induced emf is given by

$$\varepsilon = -\frac{d(N\phi_B)}{dt}$$

$$\varepsilon = -L\frac{dI}{dt}$$

Thus, the self-induced emf always opposes any change (increase or decrease) of current in the coil.

Ans. The back electromotive force in solenoid is (u) a maximum when there is maximum rate of change of current. This occurs is in AB part of the graph. So maximum back emf will be obtained between $5\text{s} < t < 10\text{ s}$.

Since, the back emf at $t = 3\text{s}$ is e,

Also,

the rate of change of current at $t = 3$, s = slope of OA from $t = 0\text{s}$ to $t = 5\text{ s} = 1/5$ A/s.

So, we have

If $u = L\ 1/5 \left(\text{for } t = 3\text{s}, \frac{dI}{dt} = 1/5\right)$ (L is a constant). Applying $\varepsilon = -L\frac{dI}{dt}$

Similarly, we have for other values

For $5\text{s} < t < 10\text{s}$ $\quad u_1 = -L\frac{3}{5} = -\frac{3}{5}L = -3e$

Thus, $\quad$ at $t = 7$ s, $u_1 = -3e$

For $10\text{s} < t < 30\text{ s}$

$$u_2 = L\frac{2}{20} = \frac{L}{10} = \frac{1}{2}e$$

For $t > 30$ s, $u_2 = 0$

Thus, the back emf at $t = 7$ s, 15 s and 40 s are $-3e$, $e/2$ and 0 respectively.

Q. 21 There are two coils A and B separated by some distance. If a current of 2A flows through A, a magnetic flux of 10^{-2} Wb passes through B (no current through B). If no current passes through A and a current of 1 A passes through B, what is the flux through A?

K **Thinking Process**

A current I_1 is passed through the coil A and the flux linkage with coil B is,

$$N_2\phi_2 = M_{21}I_1$$

where, M_{21} is called the mutual inductance of coil A with respect to coil B and $M_{21} = M_{12}$
And M_{12} is called the mutual inductance of coil B with respect to coil A.

Ans. Applying the mutual inductance of coil A with respect to coil B

$$M_{21} = \frac{N_2\phi_2}{I_1}$$

Therefore, we have

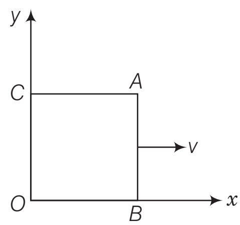
$$\text{Mutual inductance} = \frac{10^{-2}}{2} = 5\text{ mH}$$

Again applying this formula for other case

$$N_1\phi_1 = M_{12}I_2 = 5\text{ mH} \times 1\text{A} = 5\text{ mWb}.$$

Long Answer Type Questions

Q. 22 A magnetic field $\mathbf{B} = B_0 \sin(\omega t)\hat{\mathbf{k}}$ covers a large region where a wire AB slides smoothly over two parallel conductors separated by a distance d (figure). The wires are in the x-y plane. The wire AB (of length d) has resistance R and the parallel wires have negligible resistance. If AB is moving with velocity v, what is the current in the circuit. What is the force needed to keep the wire moving at constant velocity?



K **Thinking Process**

The emf induced across AB due to its motion and change in magnetic flux linked with the loop change due to change of magnetic field.

Ans. Let us assume that the parallel wires at are $y = 0$ *i.e.,* along x-axis and $y = d$. At $t = 0$, AB has $x = 0$, *i.e.,* along y-axis and moves with a velocity v. Let at time t, wire is at $x(t) = vt$.

Now, the motional emf across AB is

$$= (B_0 \sin\omega t) vd(-\hat{\mathbf{j}})$$

emf due to change in field (along $OBAC$)

$$= -B_0 \omega \cos\omega t \, x(t) \, d$$

Total emf in the circuit = emf due to change in field (along $OBAC$) + the motional emf across AB

$$= -B_0 d[\omega x \cos(\omega t) + v \sin(\omega t)]$$

Electric current in clockwise direction is given by

$$= \frac{B_0 d}{R}(\omega x \cos\omega t + v \sin\omega t)$$

The force acting on the conductor is given by $F = ilB \sin 90° = ilB$

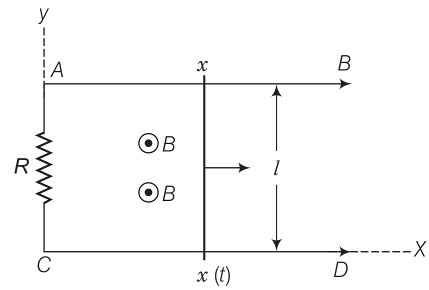
Substituting the values, we have

$$\text{Force needed along } \mathbf{i} = \frac{B_0 d}{R}(\omega x \cos\omega t + v \sin\omega t) \times d \times B_0 \sin\omega t$$

$$= \frac{B_0^2 d^2}{R}(\omega x \cos\omega t + v \sin\omega t) \sin\omega t$$

This is the required expression for force.

Q. 23 A conducting wire XY of mass m and negligible resistance slides smoothly on two parallel conducting wires as shown in figure. The closed circuit has a resistance R due to AC. AB and CD are perfect conductors. There is a magnetic field $\mathbf{B} = B(t)\hat{\mathbf{k}}$



(i) Write down equation for the acceleration of the wire XY.

(ii) If **B** is independent of time, obtain $v(t)$, assuming $v(0) = u_0$

(iii) For (ii), show that the decrease in kinetic energy of XY equals the heat lost in .

K **Thinking Process**

This problem relates EMI, magnetic force, power consumption and mechanics.

Ans. Let us assume that the parallel wires at are $y = 0$, *i.e.*, along x-axis and $y = l$. At $t = 0$, XY has $x = 0$ *i.e.*, along y-axis.

(i) Let the wire be at $x = x(t)$ at time t.

The magnetic flux linked with the loop is given by

$$\phi = \mathbf{B.A} = BA\cos 0 = BA$$

at any instant t Magnetic flux $= B(t)(l \times x(t))$

Total emf in the circuit = emf due to change in field (along $XYAC$) + the motional emf across XY

$$E = -\frac{d\phi}{dt} = -\frac{dB(t)}{dt}lx(t) - B(t)lv(t) \qquad \text{[second term due to motional emf]}$$

Electric current in clockwise direction is given by

$$I = \frac{1}{R}E$$

The force acting on the conductor is given by $F = ilB\sin 90° = ilB$

Substituting the values, we have

$$\text{Force} = \frac{IB(t)}{R}\left[-\frac{dB(t)}{dt}I\,x(t) - B(t)Iv(t)\right]\hat{\mathbf{i}}$$

Applying Newton's second law of motion,

$$m\frac{d^2x}{dt^2} = -\frac{I^2B(t)}{R}\frac{dB}{dt}x(t) - \frac{I^2B^2(t)}{R}\frac{dx}{dt} \qquad \text{...(i)}$$

which is the required equation.

(ii) If **B** is independent of time *i.e.*, B = Constant

Or $$\frac{dB}{dt} = 0$$

Substituting the above value in Eq (i), we have

$$\frac{d^2x}{dt^2} + \frac{I^2B^2}{mR}\frac{dx}{dt} = 0$$

or

$$\frac{dv}{dt} + \frac{I^2B^2}{mR}v = 0$$

Integrating using variable separable form of differential equation, we have

$$v = A\exp\left(\frac{-I^2B^2t}{mR}\right)$$

Applying given conditions, at $t = 0$, $v = u_0$

$$v(t) = u_0 \exp(-I^2B^2t / mR)$$

This is the required equation.

(iii) Since the power consumption is given by $P = I^2R$

Here,

$$I^2R = \frac{B^2I^2v^2(t)}{R^2} \times R$$

$$= \frac{B^2I^2}{R}u_0^2 \exp(-2I^2B^2t / mR)$$

Now, energy consumed in time interval dt is given by energy consumed $= Pdt = I^2Rdt$

Therefore, total energy consumed in time t

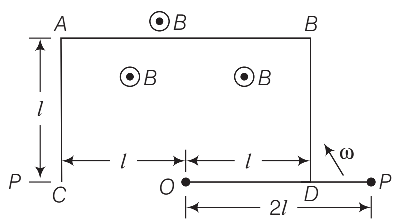
$$= \int_0^t I^2Rdt = \frac{B^2I^2}{R}u_0^2 \frac{mR}{2I^2B^2}\left[1 - e^{-(l^2B^2t/mr)}\right]$$

$$= \frac{m}{2}u_0^2 - \frac{m}{2}v^2(t)$$

$=$ decrease in kinetic energy.

This proves that the decrease in kinetic energy of XY equals the heat lost in R.

Q. 24 *ODBAC* is a fixed rectangular conductor of negligible resistance (*CO* is not connected) and *OP* is a conductor which rotates clockwise with an angular velocity ω (figure). The entire system is in a uniform magnetic field **B** whose direction is along the normal to the surface of the rectangular conductor *ABDC*. The conductor *OP* is in electric contact with *ABDC*. The rotating conductor has a resistance of λ per unit length. Find the current in the rotating conductor, as it rotates by 180°.

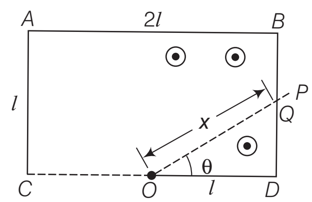


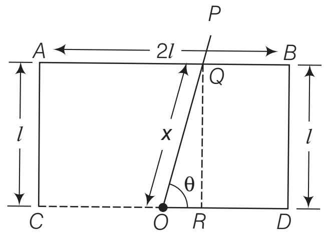
Thinking Process

The pattern of rate of change of area (hence flux) can be considered uniform from $0 < \theta < \frac{\pi}{4}$; $\frac{\pi}{4} < \theta < \frac{3\pi}{4}$ *and* $\frac{3\pi}{4} < \theta < \frac{\pi}{2}$. *Hence, forth finding emf and current.*

Ans. Let us consider the position of rotating conductor at time interval

$$t = 0 \text{ to } t = \frac{\pi}{4\omega} \text{ (or } T/8)$$





the rod OP will make contact with the side BD. Let the length OQ of the contact at sometime t such that $0 < t < \frac{\pi}{4\omega}$ or $0 < t < \frac{T}{8}$ be x. The flux through the area ODQ is

$$\phi = B\frac{1}{2}QD \times OD = B\frac{1}{2}l\tan\theta \times l$$

$$= \frac{1}{2}Bi^2 \tan\theta, \text{ where } \theta = \omega t$$

Applying Faraday's law of EMI,

Thus, the magnitude of the emf generated is $\varepsilon = \frac{d\phi}{dt} = \frac{1}{2}Bl^2\omega\sec^2\omega t$

The current is $I = \frac{\varepsilon}{R}$ where R is the resistance of the rod in contact.

where, $R \propto \lambda$

$$R = \lambda x = \frac{\lambda l}{\cos\omega t}$$

$\therefore$
$$I = \frac{1}{2}\frac{Bl^2\omega}{\lambda l}\sec^2\omega t\cos\omega t = \frac{Bl\omega}{2\lambda\cos\omega t}$$

Let the length OQ of the contact at some time t such that $\frac{\pi}{4\omega} < t < \frac{3\pi}{4\omega}$ or $\frac{T}{8} < t < \frac{3T}{8}$ be x. The rod is in contact with the side AB. The flux through the area $OQBD$ is

$$\phi = \left(l^2 + \frac{1}{2}\frac{l^2}{\tan\theta}\right)B$$

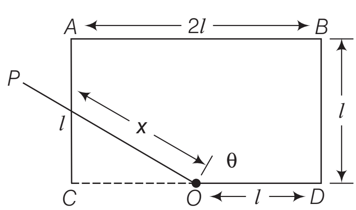
Where, $\theta = \omega t$

Thus, the magnitude of emf generated in the loop is

$$\varepsilon = \frac{d\phi}{dt} = \frac{1}{2}Bl^2\omega\frac{\sec^2\omega t}{\tan^2\omega t}$$

The current is $I = \frac{\varepsilon}{R} = \frac{\varepsilon}{\lambda x} = \frac{\varepsilon\sin\omega t}{\lambda l} = \frac{1}{2}\frac{Bl\omega}{\lambda\sin\omega t}$

Similarly for $\frac{3\pi}{4\omega} < t < \frac{\pi}{\omega}$ or $\frac{3T}{8} < t < \frac{T}{2}$, the rod will be in touch with AC.



The flux through *OQABD* is given by

$$\phi = \left(2l^2 - \frac{l^2}{2\tan\omega t}\right)B$$

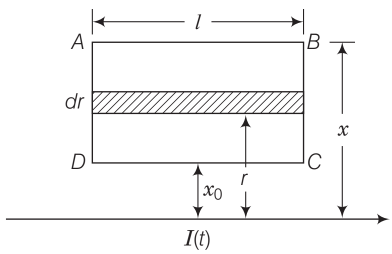
And the magnitude of emf generated in loop is given by

$$\varepsilon = \frac{d\phi}{dt} = \frac{B\omega l^2 \sec^2 \omega t}{2\tan^2 \omega t}$$

$$l = \frac{\varepsilon}{R} = \frac{\varepsilon}{\lambda x} = \frac{1}{2}\frac{Bl\omega}{\lambda \sin\omega t}$$

These are the required expressions.

Q. 25 Consider an infinitely long wire carrying a current $I(t)$, with $\frac{dI}{dt} = \lambda$ = constant. Find the current produced in the rectangular loop of wire *ABCD* if its resistance is *R* (figure).



K **Thinking Process**

This question need the use of integration in order to find the total magnetic flux linked with the loop.

Ans. Let us consider a strip of length l and width dr at a distance r from infinite long current carrying wire. The magnetic field at strip due to current carrying wire is given by

$$\text{Field } B(r) = \frac{\mu_o I}{2\pi r} \text{ (out of paper)}$$

Total flux through the loop is

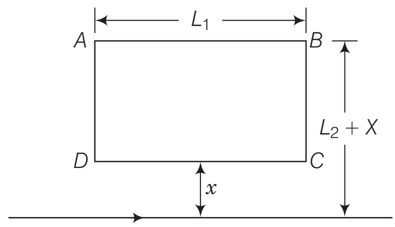
$$\text{Flux} = \frac{\mu_0 I}{2\pi} l \int_{x_0}^{x} \frac{dr}{r} = \frac{\mu_o I}{2\pi} \ln \frac{x}{x_0} \qquad \text{...(i)}$$

The emf induced can be obtained by differentiating the eq. (i) wrt t and then applying Ohm's law

$$\frac{\varepsilon}{R} = I$$

We have, induced current $= \frac{1}{R}\frac{d\phi}{dt} = \frac{\varepsilon}{R} = \frac{\mu_0 I}{2\pi}\frac{\lambda}{R}\ln\frac{x}{x_0}$ $\qquad \left(\because \frac{dI}{dt} = \lambda\right)$

Q. 26 A rectangular loop of wire $ABCD$ is kept close to an infinitely long wire carrying a current $I(t) = I_0(1 - t/T)$ for $0 \le t \le T$ and $I(0) = 0$ for $t > T$ (figure.). Find the total charge passing through a given point in the loop, in time T. The resistance of the loop is R.



K **Thinking Process**

The charge passes through the circuit can be obtained by finding the relation between instantaneous current and instantaneous magnetic flux linked with it.

Ans. The emf induced can be obtained by differentiating the expression of magnetic flux linked wrt t and then applying Ohm's law

$$I = \frac{E}{R} = \frac{1}{R}\frac{d\phi}{dt}$$

We know that electric current

$$I(t) = \frac{dQ}{dt} \quad \text{or} \quad \frac{dQ}{dt} = \frac{1}{R}\frac{d\phi}{dt}$$

Integrating the variable separable form of differential equation for finding the charge Q that passed in time t, we have

$$Q(t_1) - Q(t_2) = \frac{1}{R}[\phi(t_1) - \phi(t_2)]$$

$$\phi(t_1) = L_1 \frac{\mu_0}{2\pi}\int_x^{L_2 + x}\frac{dx'}{x'} I(t_1) \qquad \text{[Refer to the Eq. (i) of answer no.25]}$$

$$= \frac{\mu_0 L_1}{2\pi} I(t_1) \ln \frac{L_2 + x}{x}$$

The magnitude of charge is given by,

$$= \frac{\mu_0 L_1}{2\pi} \ln \frac{L_2 + x}{x}[I_0 + 0]$$

$$= \frac{\mu_0 L_1}{2\pi} I_1 \ln\left(\frac{L_2 + x}{x}\right)$$

This is the required expression.

Q. 27 A magnetic field **B** is confined to a region $r \le a$ and points out of the paper (the z-axis), $r = 0$ being the centre of the circular region. A charged ring (charge $= Q$) of radius b, $b > a$ and mass m lies in the x-y plane with its centre at the origin. The ring is free to rotate and is at rest. The magnetic field is brought to zero in time Δt. Find the angular velocity ω of the ring after the field vanishes.

K **Thinking Process**

The decrease in magnetic field causes induced emf and hence, electric field around ring. The torque experienced by the ring produces change in angular momentum.

Ans. Since, the magnetic field is brought to zero in time Δt, the magnetic flux linked with the ring also reduces from maximum to zero. This, in turn, induces an emf in ring by the phenomenon of EMI. The induces emf causes the electric field E generation around the ring.

The induced emf = electric field $E \times (2\pi b)$ (Because $V = E \times d$) ...(i)

By Faraday's law of EMI

The induced emf = rate of change of magnetic flux

$$= \text{rate of change of magnetic field} \times \text{area}$$

$$= \frac{B\pi a^2}{\Delta t} \quad \text{...(ii)}$$

From Eqs. (i) and (ii), we have

$$2\pi bE = emf = \frac{B\pi a^2}{\Delta t}$$

Since, the charged ring experienced a electric force = QE

This force try to rotate the coil, and the torque is given by

$$\text{Torque} = b \times \text{Force}$$

$$= QEb = Q\left[\frac{B\pi a^2}{2\pi b\Delta t}\right]b$$

$$= Q\frac{Ba^2}{2\Delta t}$$

If ΔL is the change in angular momentum

$$\Delta L = \text{Torque} \times \Delta t = Q\frac{Ba^2}{2}$$

Since, initial angular momentum = 0

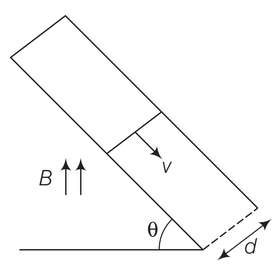
Now, since Torque $\times \Delta t$ = Change in angular momentum

Final angular momentum = $mb^2\omega = \dfrac{QBa^2}{2}$

$$\omega = \frac{QBa^2}{2mb^2}$$

On rearranging the terms, we have the required expression of angular speed.

Q. 28 A rod of mass m and resistance R slides smoothly over two parallel perfectly conducting wires kept sloping at an angle θ with respect to the horizontal (figure). The circuit is closed through a perfect conductor at the top. There is a constant magnetic field **B** along the vertical direction. If the rod is initially at rest, find the velocity of the rod as a function of time.



K **Thinking Process**

This problem combines the mechanics, EMI, magnetic force and linear differential equation.

Ans. Here, the component of magnetic field perpendicular the plane = $B\cos\theta$

Now, the conductor moves with speed v perpendicular to $B\cos\theta$ component of magnetic field. This causes motional emf across two ends of rod, which is given by = $v(B\cos\theta)d$

This makes flow of induced current $i = \frac{v(B\cos\theta)d}{R}$ where, R is the resistance of rod. Now, current carrying rod experience force which is given by $F = iBd$ (horizontally in backward direction). Now, the component of magnetic force parallel to incline plane along upward direction $= F\cos\theta = iBd\cos\theta = \left(\frac{v(B\cos\theta)d}{R}\right)Bd\cos\theta$ where, $v = \frac{dx}{dt}$

Also, the component of weight (mg) parallel to incline plane along downward direction $= mg\sin\theta$.

Now, by Newton's second law of motion

$$m\frac{d^2x}{dt^2} = mg\sin\theta - \frac{B\cos\theta d}{R}\left(\frac{dx}{dt}\right) \times (Bd)\cos\theta$$

$$\frac{dv}{dt} = g\sin\theta - \frac{B^2d^2}{mR}(\cos\theta)^2 v$$

$$\frac{dv}{dt} + \frac{B^2d^2}{mR}(\cos\theta)^2 v = g\sin\theta$$

But, this is the linear differential equation.

On solving, we get

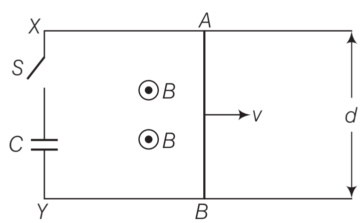
$$v = \frac{g\sin\theta}{\frac{B^2d^2\cos^2\theta}{mR}} + A\exp\left(-\frac{B^2d^2}{mR}(\cos^2\theta)t\right)$$

A is a constant to be determine by initial conditions.

The required expression of velocity as a function of time is given by

$$= \frac{mgR\sin\theta}{B^2d^2\cos^2\theta}\left(1 - \exp\left(-\frac{B^2d^2}{mR}(\cos^2\theta)t\right)\right)$$

Q. 29 **Find the current in the sliding rod AB (resistance $= R$) for the arrangement shown in figure. B is constant and is out of the paper. Parallel wires have no resistance, v is constant. Switch S is closed at time $t = 0$.**



K **Thinking Process**

This problem combines the concept of EMI, charging of capacitor and linear differential equation.

Ans. The conductor of length d moves with speed v, perpendicular to magnetic field B as shown in figure. This produces motional emf across two ends of rod, which is given by $= vBd$. Since, switch S is closed at time $t = 0$. capacitor is charged by this potential difference. Let $Q(t)$ is charge on the capacitor and current flows from A to B.

Now, the induced current

$$I = \frac{vBd}{R} - \frac{Q}{RC}$$

On rearranging the terms, we have

$$\frac{Q}{RC} + \frac{dQ}{dt} = \frac{vBd}{R}$$

This is the linear differential equation. On solving, we get

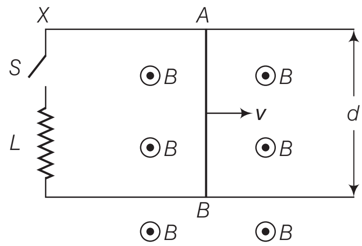
$$Q = vBdC + Ae^{-t/RC}$$

$\Rightarrow$ $$Q = vBdC\,[1 - e^{-t/RC}] \quad \text{(At time } t = 0,\ Q = 0 = A = -\ vBdc).$$

Differentiating, we get $I - \dfrac{vBd}{R} e^{-t/RC}$

This is the required expression of current.

Q. 30 Find the current in the sliding rod AB (resistance $= R$) for the arrangement shown in figure. **B** is constant and is out of the paper. Parallel wires have no resistance, **v** is constant. Switch S is closed at time $t = 0$.



K **Thinking Process**

This problem combines the concept of EMI, growth of current in inductor and linear differential equation.

Ans. The conductor of length d moves with speed v, perpendicular to magnetic field **B** as shown in figure. This produces motional emf across two ends of rod, which is given by $= vBd$.

Since, switch S is closed at time $t = 0$. current start growing in inductor by the potential difference due to motional emf.

Applying Kirchhoff's voltage rule, we have

$$-L\frac{dI}{dt} + vBd = IR \text{ or } L\frac{dI}{dt} + IR = vBd$$

This is the linear differential equation. On solving, we get

$$I = \frac{vBd}{R} + Ae^{-Rt/2}$$

$$\text{At } t = 0 \quad I = 0$$

$\Rightarrow$ $$A = -\frac{vBd}{R} \Rightarrow I = \frac{vBd}{R}(1 - e^{-Rt/L}).$$

This is the required expression of current.

Q. 31 A metallic ring of mass m and radius l (ring being horizontal) is falling under gravity in a region having a magnetic field. If z is the vertical direction, the z-component of magnetic field is $B_z = B_0\,(1 + \lambda\, z)$. If R is the resistance of the ring and if the ring falls with a velocity v, find the energy lost in the resistance. If the ring has reached a constant velocity, use the conservation of energy to determine v in terms of m, B, λ and acceleration due to gravity g.

K **Thinking Process**

This problem establishes a relationship between induced current, power lost and velocity acquired by freely falling ring.

Ans. The magnetic flux linked with the metallic ring of mass m and radius l falling under gravity in a region having a magnetic field whose z-component of magnetic field is $B_z = B_0 (1 + \lambda z)$ is

$$\phi = B_z (\pi l^2) = B_0 (1 + \lambda z)(\pi l^2)$$

Applying Faraday's law of EMI, we have emf induced given by $\frac{d\phi}{dt}$ = rate of change of flux

Also, by Ohm's law

$$B_0 (\pi l^2)\lambda \frac{dz}{dt} = IR$$

On rearranging the terms, we have $I = \frac{\pi l^2 B_0 \lambda}{R} v$

$$\text{Energy lost/second} = I^2 R = \frac{(\pi l^2 \lambda)^2 B_0^2 v^2}{R}$$

This must come from rate of change in PE $= mg \frac{dz}{dt} = mgv$

[as kinetic energy is constant for $v =$ constant]

Thus, $mgv = \frac{(\pi l^2 \lambda B_0)^2 v^2}{R}$ or $v = \frac{mgR}{(\pi l^2 \lambda B_0)^2}$

This is the required expression of velocity.

Q. 32 A long solenoid S has n turns per meter, with diameter a. At the centre of this coil, we place a smaller coil of N turns and diameter b (where $b < a$). If the current in the solenoid increases linearly, with time, what is the induced emf appearing in the smaller coil. Plot graph showing nature of variation in emf, if current varies as a function of $mt^2 + C$.

K **Thinking Process**

This problem require an insight to magnetic field due to current carrying solenoid having varying current which induces emf in coil of radius B.

Ans. Magnetic field due to a solenoid S, $B = \mu_0 nI$ where signs are as usual.

Magnetic flux in smaller coil $\phi = NBA$, where

$$A = \pi b^2$$

Applying Faraday's law of EMI, we have

So, $e = \frac{-d\phi}{dt} = \frac{-d}{dt}(NBA)$

$$= -N\pi b^2 \frac{d(B)}{dt}$$

where, $B = \mu_0 Ni$

$$= -N\pi b^2 \mu_0 n \frac{dI}{dt}$$

$$= -Nn\pi\mu_0 b^2 \frac{d}{dt}(mt^2 + C) = -\mu_0 Nn\pi b^2 2mt$$

Since, current varies as a function of $mt^2 + C$.

$$e = -\mu_0 Nn\pi b^2 2mt$$

7

Alternating Current

Multiple Choice Questions (MCQs)

Q. 1 If the rms current in a 50 Hz AC circuit is 5 A, the value of the current 1/300 s after its value becomes zero is

(a) $5\sqrt{2}$ A (b) $5\sqrt{3/2}$ A

(c) $5/6$ A (d) $5/\sqrt{2}$ A

Ans. *(b)* Given,

$$\nu = 50\text{ Hz}, \; I_{\text{rms}} = 5\text{A}$$

$$t = \frac{1}{300}\text{s}$$

We have to find $I(t)$

$$I_0 = \text{Peak value} = \sqrt{2}, \; I_{\text{rms}} = \sqrt{2} \times 5$$

$$= 5\sqrt{2}\text{ A}$$

$$I = I_0 \sin \omega t = 5\sqrt{2} \sin 2\pi \nu t = 5\sqrt{2} \sin 2\pi \times 50 \times \frac{1}{300}$$

$$= 5\sqrt{2} \sin \frac{\pi}{3} = 5\sqrt{2} \times \frac{\sqrt{3}}{2} = 5\sqrt{3/2}\text{ A}$$

Q. 2 An alternating current generator has an internal resistance R_g and an internal reactance X_g. It is used to supply power to a passive load consisting of a resistance R_g and a reactance X_L. For maximum power to be delivered from the generator to the load, the value of X_L is equal to

(a) zero (b) X_g

(c) $-X_g$ (d) R_g

Ans. *(c)* For delivering maximum power from the generator to the load, total internal reactance must be equal to conjugate of total external reactance.

Hence, $X_{\text{int}} = {}^{*}X_{\text{ext}}$

$\Rightarrow$ $X_g = (X_L)^* = -X_L$

$\Rightarrow$ $X_L = -X_g$

Q. 3 When a voltage measuring device is connected to AC mains, the meter shows the steady input voltage of 220 V. This means

(a) input voltage cannot be AC voltage, but a DC voltage

(b) maximum input voltage is 220 V

(c) the meter reads not v but $<v^2>$ and is calibrated to read $\sqrt{<v^2>}$

(d) the pointer of the meter is stuck by some mechanical defect

Ans. *(c)* The voltmeter connected to AC mains reads mean value ($<v^2>$) and is calibrated in such a way that it gives value of $<v^2>$, which is multiplied by form factor to give rms value.

Q. 4 To reduce the resonant frequency in an *L-C-R* series circuit with a generator

(a) the generator frequency should be reduced

(b) another capacitor should be added in parallel to the first

(c) the iron core of the inductor should be removed

(d) dielectric in the capacitor should be removed

Ans. *(b)* We know that resonant frequency in an *L-C-R* circuit is given by

$$v_0 = \frac{1}{2\pi\sqrt{LC}}$$

Now to reduce v_0 either we can increase L or we can increase C.

To increase capacitance, we must connect another capacitor parallel to the first.

Q. 5 Which of the following combinations should be selected for better tuning of an *L-C-R* circuit used for communication?

(a) $R = 20\ \Omega$, $L = 1.5$ H, $C = 35\ \mu$F

(b) $R = 25\ \Omega$, $L = 2.5$ H, $C = 45\ \mu$F

(c) $R = 15\ \Omega$, $L = 3.5$ H, $C = 30\ \mu$F

(d) $R = 25\ \Omega$, $L = 1.5$ H, $C = 45\ \mu$F

K **Thinking Process**

For better tuning of an L-C-R circuit used for communication, quality factor of the circuit must be as high as possible.

Ans. *(c)* Quality factor (Q) of an *L-C-R* circuit is given by,

$$Q = \frac{1}{R}\sqrt{\frac{L}{C}}$$

where R is resistance, L is inductance and C is capacitance of the circuit. To make Q high,

R should be low, L should be high and C should be low.

These conditions are best satisfied by the values given in option (c).

Note *We should be careful while writing formula for quality factor, because we are considering series L-C-R circuit.*

Q. 6 An inductor of reactance 1Ω and a resistor of 2Ω are connected in series to the terminals of a 6V (rms) AC source. The power dissipated in the circuit is

(a) 8 W
(b) 12 W
(c) 14.4 W
(d) 18 W

Ans. *(c)* Given, $X_L = 1\Omega, R = 2\Omega$

$E_{rms} = 6\,V, P_{av} = ?$

Average power dissipated in the circuit

$$P_{av} = E_{rms} I_{rms} \cos\phi \qquad \text{...(i)}$$

$$I_{rms} = \frac{I_0}{\sqrt{2}} = \frac{E_{rms}}{Z}$$

$$Z = \sqrt{R^2 + X_L^2}$$

$$= \sqrt{4+1} = \sqrt{5}$$

$$I_{rms} = \frac{6}{\sqrt{5}}\,A$$

$$\cos\phi = \frac{R}{Z} = \frac{2}{\sqrt{5}}$$

$$P_{av} = 6 \times \frac{6}{\sqrt{5}} \times \frac{2}{\sqrt{5}} \qquad \text{[from Eq. (i)]}$$

$$= \frac{72}{\sqrt{5}\,\sqrt{5}} = \frac{72}{5} = 14.4\,W$$

Q. 7 The output of a step-down transformer is measured to be 24 V when connected to a 12 W light bulb. The value of the peak current is

(a) $1/\sqrt{2}$ A
(b) $\sqrt{2}$ A
(c) 2 A
(d) $2\sqrt{2}$ A

Ans. *(a)* Secondary voltage $V_S = 24V$

Power associated with secondary $P_S = 12$ W

$$I_S = \frac{P_S}{V_S} = \frac{12}{24}$$

$$= \frac{1}{2}\,A = 0.5\,A$$

Peak value of the current in the secondary

$$I_0 = I_S\sqrt{2}$$

$$= (0.5)\,(1.414) = 0.707 = \frac{1}{\sqrt{2}}\,A$$

Multiple Choice Questions (More Than One Options)

Q. 8 As the frequency of an AC circuit increases, the current first increases and then decreases. What combination of circuit elements is most likely to comprise the circuit?

(a) Inductor and capacitor
(b) Resistor and inductor
(c) Resistor and capacitor
(d) Resistor, inductor and capacitor

K **Thinking Process**

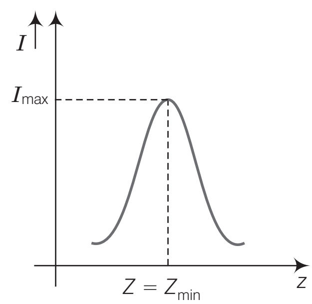
We can decide the elements, comprising the given circuit by predicting the variation in their reactances with frequency.

Ans. ***(a,d)***

Reactance of an inductor of inductance L is, $X_L = 2\pi\nu L$

where v is frequency of the AC circuit.

$$X_c = \text{Reactance of the capacitive circuit}$$
$$= \frac{1}{2\pi fC}$$



On increasing frequency ν, clearly X_L increases and X_C decreases.

For a L-C-R circuit,

$$Z = \text{Impedance of the circuit}$$
$$= \sqrt{R^2 + (X_L - X_C)^2}$$
$$= \sqrt{R^2 + \left(2\pi\nu L - \frac{1}{2\pi\nu C}\right)^2}$$

As frequency (ν) increases, Z decreases and at certain value of frequency know as resonant frequency (ν_0), impedance Z is minimum that is $Z_{min} = R$ current varies inversely with impedance and at Z_{min} current is maximum.

Q. 9 In an alternating current circuit consisting of elements in series, the current increases on increasing the frequency of supply. Which of the following elements are likely to constitute the circuit?

(a) Only resistor
(b) Resistor and an inductor
(c) Resistor and a capacitor
(d) Only a capacitor

Ans. ***(c, d)***

According to the question, the current increases on increasing the frequency of supply. Hence, the reactance of the circuit must be decreases as increasing frequency.

For a capacitive circuit,

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi fC}$$

Clearly when frequency increases, X_C decreases.

For R-C circuit, $\quad X = \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}$

when frequency increases X decreases.

Q. 10 Electrical energy is transmitted over large distances at high alternating voltages. Which of the following statements is (are) correct?

(a) For a given power level, there is a lower current

(b) Lower current implies less power loss

(c) Transmission lines can be made thinner

(d) It is easy to reduce the voltage at the receiving end using step-down transformers

Thinking Process

Power loss due to transmission lines having resistance (R) and rms current flowing I_{rms} is $I^2_{rms}R$.

Ans. ***(a, b, d)***

We have to transmit energy (power) over large distances at high alternating voltages, so current flowing through the wires will be low because for a given power (P).

$$P = E_{rms} I_{rms}, \ I_{rms} \text{ is low when } E_{rms} \text{ is high.}$$

$$\text{Power loss} = I^2_{rms} R = \text{low} \qquad (\because I_{rms} \text{ is low})$$

Now at the receiving end high voltage is reduced by using step-down transformers.

Q. 11 For a *L-C-R* circuit, the power transferred from the driving source to the driven oscillator is $P = I^2 Z \cos\phi$.

(a) Here, the power factor $\cos\phi \geq 0, P \geq 0$

(b) The driving force can give no energy to the oscillator $(P = 0)$ in some cases

(c) The driving force cannot syphon out $(P < 0)$ the energy out of oscillator

(d) The driving force can take away energy out of the oscillator

Ans. ***(a, b, c)***

According to question power transferred,

$$P = I^2 Z \cos\phi$$

where I is the current, Z = Impedance and $\cos\phi$ is power factor

As power factor, $\cos\phi = \frac{R}{Z}$

where $R > 0$ and $Z > 0$

$\Rightarrow \cos\phi > 0 \Rightarrow P > 0$

Q. 12 When an AC voltage of 220 V is applied to the capacitor C

(a) the maximum voltage between plates is 220 V

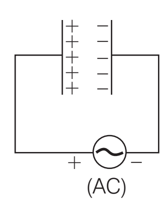
(b) the current is in phase with the applied voltage

(c) the charge on the plates is in phase with the applied voltage

(d) power delivered to the capacitor is zero

Ans. ***(c, d)***

When the AC voltage is applied to the capacitor, the plate connected to the positive terminal will be at higher potential and the plate connected to the negative terminal will be at lower potential.



The plate with positive charge will be at higher potential and the plate with negative charge will be at lower potential. So, we can say that the charge is in phase with the applied voltage.

Power applied to a circuit is $P_{av} = V_{rms} I_{rms} \cos\phi$

For capacitive circuit, $\phi = 90°$

$\Rightarrow \cos\phi = 0$

$\Rightarrow P_{av} = \text{Power delivered} = 0$

Q. 13 The line that draws power supply to your house from street has

(a) zero average current

(b) 220 V average voltage

(c) voltage and current out of phase by $90°$

(d) voltage and current possibly differing in phase ϕ such that $|\phi| < \frac{\pi}{2}$

Ans. *(a, d)*

For house hold supplies, AC currents are used which are having zero average value over a cycle.

The line is having some resistance so power factor $\cos\phi = \frac{R}{Z} \neq 0$

so, $\phi \neq \pi/2 \Rightarrow \phi, < \pi/2$

i.e., phase lies between 0 and $\pi/2$.

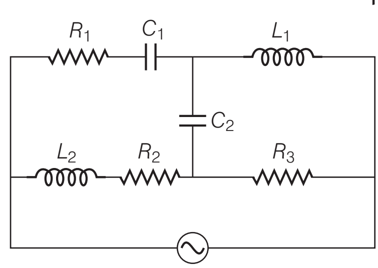
Very Short Answer Type Questions

Q. 14 If a *L-C* circuit is considered analogous to a harmonically oscillating springblock system, which energy of the *L-C* circuit would be analogous to potential energy and which one analogous to kinetic energy?

Ans. If we consider a *L-C* circuit analogous to a harmonically oscillating springblock system.

The electrostatic energy $\frac{1}{2}CV^2$ is analogous to potential energy and energy associated with moving charges (current) that is magnetic energy $\left(\frac{1}{2}LI^2\right)$ is analogous to kinetic energy.

Q. 15 Draw the effective equivalent circuit of the circuit shown in figure, at very high frequencies and find the effective impedance.



Thinking Process

The component with infinite resistance will be considered as open circuit and the component with zero resistance will be considered as short circuited.

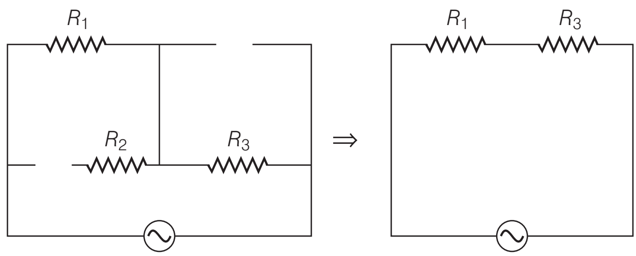
Ans. We know that inductive reactance $X_L = 2\pi fL$

and capacitive reactance $X_C = \dfrac{1}{2\pi fC}$

For very high frequencies $(f \to \infty)$, $X_L \to \infty$ and $X_C \to 0$

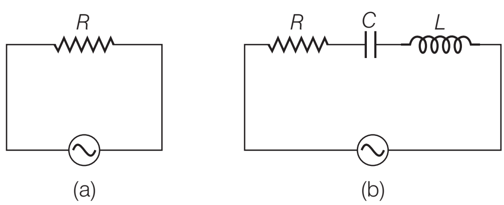
When reactance of a circuit is infinite it will be considered as open circuit. When reactance of a circuit is zero it will be considered as short circuited.

So, $C_1, C_2 \to$ shorted and $L_1, L_2 \to$ opened.



So, effective impedance $= R_{eq} = R_1 + R_3$

Q. 16 **Study the circuits (a) and (b) shown in figure and answer the following questions.**



(a) Under which conditions would the rms currents in the two circuits be the same?

(b) Can the rms current in circuit (b) be larger than that in (a)?

Ans. Let,

$$(I_{rms})\, a = \text{rms current in circuit (a)}$$
$$(I_{rms})\, b = \text{rms current in circuit (b)}$$
$$(I_{rms})\, a = \frac{V_{rms}}{R} = \frac{V}{R}$$
$$(I_{rms})\, b = \frac{V_{rms}}{Z} = \frac{V}{\sqrt{R^2 + (X_L - X_C)^2}}$$

(a) When $(I_{rms})\, a = (I_{rms})\, b$

$$R = \sqrt{R^2 + (X_L - X_C)^2}$$

$\Rightarrow$ $X_L = X_C$, resonance condition

(b) As $Z \geq R$

$$\Rightarrow \quad \frac{(I_{rms})a}{(I_{rms})b} = \frac{\sqrt{R^2 + (X_L - X_C)^2}}{R}$$
$$= \frac{Z}{R} \geq 1$$

$\Rightarrow$ $(I_{rms})\, a \geq (I_{rms})b$

No, the rms current in circuit (b), cannot be larger than that in (a).

Q. 17 Can the instantaneous power output of an AC source ever be negative? Can the average power output be negative?

Ans. Let the applied emf

$$E = E_0 \sin(\omega t)$$

and current developed is

$$I = I_0 \sin(\omega t \pm \phi)$$

Instantaneous power output of the AC source

$$P = EI = (E_0 \sin \omega t) \qquad [I_0 \sin(\omega t \pm \phi)]$$

$$= E_0 I_0 \sin \omega t \cdot \sin(\omega t + \phi)$$

$$= \frac{E_0 I_0}{2} [\cos \phi - \cos(2\omega t + \phi)] \qquad \text{...(i)}$$

Average power $\quad P_{av} = \frac{V_0}{\sqrt{2}} \frac{I_0}{\sqrt{2}} \cos \phi$

$$= V_{rms} \, I_{rms} \cos \phi \qquad \text{...(ii)}$$

where ϕ is the phase difference.

Clearly, from Eq. (i)

when $\quad \cos \phi < \cos(2\omega t + \phi)$

$$P < 0$$

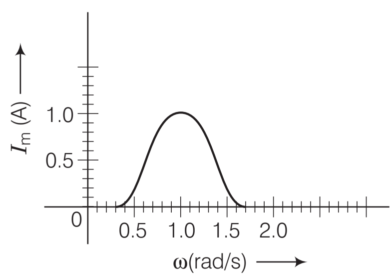
Yes, the instantaneous power output of an AC source can be negative

From Eq. (ii) $\quad P_{av} > 0$

Because $\quad \cos \phi = \frac{R}{Z} > 0$

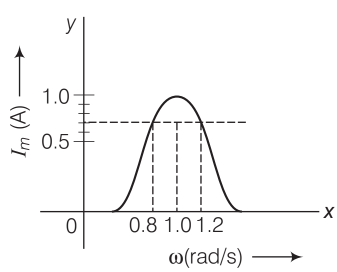
No, the average power output of an AC source cannot be negative.

Q. 18 In series *LCR* circuit, the plot of I_{max} *versus* ω is shown in figure. Find the bandwidth and mark in the figure.



Ans. Consider the diagram .

Bandwidth $= \omega_2 - \omega_1$



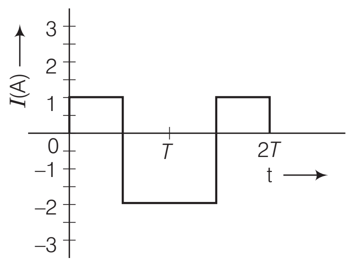
where ω_1 and ω_2 corresponds to frequencies at which magnitude of current is $\frac{1}{\sqrt{2}}$ times of maximum value.

$$I_{rms} = \frac{I_{max}}{\sqrt{2}} = \frac{1}{\sqrt{2}} \approx 0.7 \text{ A}$$

Clearly from the diagram, the corresponding frequencies are 0.8 rad/s and 1.2 rad/s.

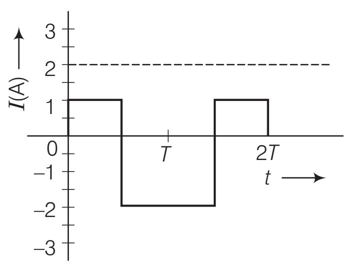
$$\Delta\omega = \text{Bandwidth} = 1.2 - 0.8 = 0.4 \text{ rad/s}$$

Q. 19 **The alternating current in a circuit is described by the graph shown in figure. Show rms current in this graph.**



Ans. I_{rms} = rms current

$$= \sqrt{\frac{1^2 + 2^2}{2}} = \sqrt{\frac{5}{2}} = 1.58\text{A} \approx 1.6\text{A}$$



The rms value of the current $(I_{rms}) = 1.6$ A is indicated in the graph.

Q. 20 **How does the sign of the phase angle ϕ, by which the supply voltage leads the current in an *L-C-R* series circuit, change as the supply frequency is gradually increased from very low to very high values.**

Ans. The phase angle (ϕ) by which voltage leads the current in *L-C-R* series circuit is given by

$$\tan\phi = \frac{X_L - X_C}{R} = \frac{2\pi\nu L - \frac{1}{2\pi\nu C}}{R}$$

$$\tan\phi < 0 \text{ (for } \nu < \nu_0)$$

$$\tan\phi > 0 \text{ (for } \nu > \nu_0)$$

$$\tan\phi = 0 \qquad \left(\text{for } \nu = \nu_0 = \frac{1}{2\pi\sqrt{2C}}\right)$$

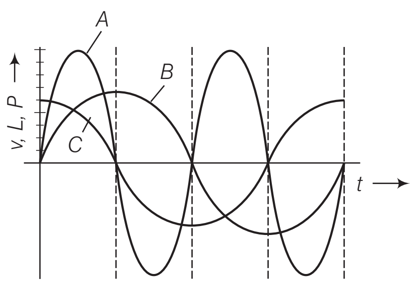
Short Answer Type Questions

Q. 21 A device 'X' is connected to an AC source. The variation of voltage, current and power in one complete cycle is shown in figure.

(a) Which curve shows power consumption over a full cycle?

(b) What is the average power consumption over a cycle?

(c) Identify the device X.

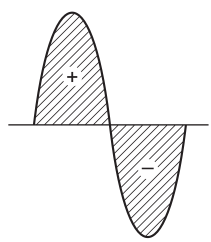


Ans. (a) We know that

$$\text{Power} = P = VI$$

that is curve of power will be having maximum amplitude, equals to multiplication of amplitudes of voltage (V) and current (I) curve. So, the curve will be represented by A.

(b) As shown by shaded area in the diagram, the full cycle of the graph consists of one positive and one negative symmetrical area.



Hence, average power over a cycle is zero.

(c) As the average power is zero, hence the device may be inductor (L) or capacitor (C) or the series combination of L and C.

Q. 22 Both alternating current and direct current are measured in amperes. But how is the ampere defined for an alternating current?

Ans. For a Direct Current (DC),

$$1 \text{ ampere} = 1 \text{ coulomb/sec}$$

An AC current changes direction with the source frequency and the attractive force would average to zero. Thus, the AC ampere must be defined in terms of some property that is independent of the direction of current.

Joule's heating effect is such property and hence it is used to define rms value of AC.

Q. 23 A coil of 0.01H inductance and 1Ω resistance is connected to 200 V, 50Hz AC supply. Find the impedance of the circuit and time lag between maximum alternating voltage and current.

Ans. Given, inductance $L = 0.01\,\text{H}$

resistance $R = 1\Omega$, voltage $(V) = 200\,\text{V}$

and frequency $(f) = 50$ Hz.

Impedance of the circuit $Z = \sqrt{R^2 + X_L^2} = \sqrt{R^2 + (2\pi fL)^2}$

$$= \sqrt{1^2 + (2 \times 3.14 \times 50 \times 0.01)^2}$$

or $Z = \sqrt{10.86} = 3.3\Omega$

$$\tan\phi = \frac{\omega L}{R} = \frac{2\pi fL}{R} = \frac{2 \times 3.14 \times 50 \times 0.01}{1} = 3.14$$

$$\phi = \tan^{-1}(3.14) \approx 72°$$

Phase difference $\phi = \dfrac{72 \times \pi}{180}$ rad.

Time lag between alternating voltage and current

$$\Delta t = \frac{\phi}{\omega} = \frac{72\pi}{180 \times 2\pi \times 50} = \frac{1}{250}\,\text{s}$$

Q. 24 A 60 W load is connected to the secondary of a transformer whose primary draws line voltage. If a current of 0.54 A flows in the load, what is the current in the primary coil? Comment on the type of tansformer being used.

Ans. Given, $P_S = 60\,\text{W}$, $I_S = 0.54$ A

Current in the primary $I_p = ?$

Taking line voltage as 220 V.

We can write Since,

$\Rightarrow$ $P_L = 60\text{W}, I_L = 0.54\text{A}$

$\Rightarrow$ $V_L = \dfrac{60}{0.54} = 110\text{V}.$ ...(i)

Voltage in the secondary (E_S) is less than voltage in the primary (E_P).

Hence, the transformer is step down transformer.

Since, the transformation ratio

$$r = \frac{V_s}{V_p} = \frac{I_p}{I_s}$$

Substituting the values, $\dfrac{110\text{V}}{220\text{V}} = \dfrac{I_p}{0.54\text{A}}$

On solving $I_p = 0.27\text{A}$

Q. 25 Explain why the reactance provided by a capacitor to an alternating current decreases with increasing frequency.

Ans. A capacitor does not allow flow of direct current through it as the resistance across the gap is infinite. When an alternating voltage is applied across the capacitor plates, the plates are alternately charged and discharged. The current through the capacitor is a result of this changing voltage (or charge).

Thus, a capacitor will pass more current through it if the voltage is changing at a faster rate, *i.e.* if the frequency of supply is higher. This implies that the reactance offered by a capacitor is less with increasing frequency.

Mathematically, the reactance can be written as $X_C = \frac{1}{\omega C}$.

Q. 26 **Explain why the reactance offered by an inductor increases with increasing frequency of an alternating voltage.**

Ans. An inductor opposes flow of current through it by developing a back emf according to Lenz's law. The induced voltage has a polarity so as to maintain the current at its present value. If the current is decreasing, the polarity of the induced emf will be so as to increase the current and *vice -versa*.

Since, the induced emf is proportional to the rate of change of current, it will provide greater reactance to the flow of current if the rate of change is faster, *i.e.,* if the frequency is higher. The reactance of an inductor, therefore, is proportional to the frequency. Mathematically, the reactance offered by the inductor is given by $X_L = \omega L$.

Long Answer Type Questions

Q. 27 **An electrical device draws 2 kW power from AC mains (voltage 223 V (rms) = $\sqrt{50000}$ V). The current differs (lags) in phase by $\phi \left(\tan\phi = \frac{-3}{4} \right)$ as compared to voltage. Find (a) R, (b) $X_C - X_L$ and (c) I_M. Another device has twice the values for R, X_C and X_L. How are the answers affected?**

K **Thinking Process**

We have to apply the formula for phase relation, net reactance as well as instantaneous power associate with the circuit in terms of voltage and current.

Ans. Given, power drawn $= P = 2\text{kW} = 2000$ W

$$\tan\phi = -\frac{3}{4},\ I_M = I_0 = ?,\ R = ?,\ X_C - X_L = ?$$

$$V_{\text{rms}} = V = 223\,\text{V}$$

$$\text{Power } P = \frac{V^2}{Z}$$

$$\Rightarrow \qquad Z = \frac{V^2}{P} = \frac{223 \times 223}{2 \times 10^3} = 25$$

$$\text{Impedance } Z = 25\ \Omega$$

$$\text{Impedance } Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$\Rightarrow \qquad 25 = \sqrt{R^2 + (X_L - X_C)^2}$$

or $$625 = R^2 + (X_L - X_C)^2 \qquad \ldots\text{(i)}$$

Again, $$\tan\phi = \frac{X_L - X_C}{R} = \frac{3}{4}$$

or $$X_L - X_C = \frac{3R}{4} \qquad \ldots\text{(ii)}$$

From Eq. (ii), we put $X_L - X_C = \frac{3R}{4}$ in Eq. (i), we get

$$625 = R^2 + \left(\frac{3R}{4}\right)^2 = R^2 + \frac{9R^2}{16}$$

or $$625 = \frac{25R^2}{16}$$

(a) Resistance $R = \sqrt{25 \times 16} = \sqrt{400} = 20\Omega$

(b) $X_L - X_C = \frac{3R}{4} = \frac{3}{4} \times 20 = 15\Omega$

(c) Main current $I_M = \sqrt{2}I = \sqrt{2}\frac{V}{Z} = \frac{223}{25} \times \sqrt{2} = 12.6$ A

As R, X_C, X_L are all doubled, $\tan\phi$ does not change. Z is doubled, current is halved. So, power is also halved.

Q. 28 1 MW power is to be delivered from a power station to a town 10 km away. One uses a pair of Cu wires of radius 0.5 cm for this purpose. Calculate the fraction of ohmic losses to power transmitted if

(i) power is transmitted at 220V. Comment on the feasibility of doing this.

(ii) a step-up transformer is used to boost the voltage to 11000V, power transmitted, then a step-down transformer is used to bring voltage to 220 V. ($\rho_{cu} = 1.7 \times 10^{-8}$ SI unit)

Ans. (i) The town is 10 km away, length of pair of Cu wires used, $L = 20$ km $= 20000$ m.

Resistance of Cu wires, $$R = \frac{l}{A} = \frac{l}{\pi (r)^2}$$

$$= \frac{1.7 \times 10^{-8} \times 20000}{3.14\,(0.5 \times 10^{-2})^2} = 4\Omega$$

I at 220 V $$VI = 10^6 \text{ W};\ I = \frac{10^6}{220} = 0.45 \times 10^4 \text{ A}$$

$$RI^2 = \text{power loss}$$

$$= 4 \times (0.45)^2 \times 10^8 \text{ W}$$

$$> 10^6 \text{ W}$$

Therefore, this method cannot be used for transmission.

(ii) When power $P = 10^6$ W is transmitted at 11000 V.

$$V'I' = 10^6 \text{ W} = 11000\,I'$$

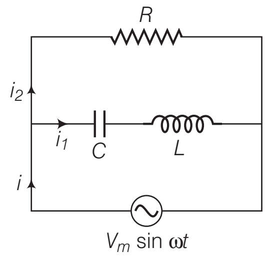
Current drawn, $$I' = \frac{1}{1.1} \times 10^2$$

Power loss $$= RI^2 = \frac{1}{1.21} \times 4 \times 10^4$$

$$= 3.3 \times 10^4 \text{ W}$$

Fraction of power loss $$= \frac{3.3 \times 10^4}{10^6} = 3.3\%$$

Q. 29 Consider the *L-C-R* circuit shown in figure. Find the net current i and the phase of i. Show that $i = \dfrac{V}{Z}$. Find the impedance Z for this circuit.



K **Thinking Process**

The circuit consists of inductor (L) and capacitor (C) connected in series and the combination is connected parallel with a resistance R. Due to this combination there is oscillation of electromagnetic energy.

Ans. In the given figure i is the total current from the source. It is divided into two parts i_1 through R and i_2 through series combination of C and L.

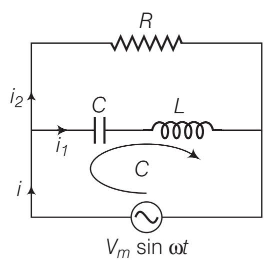
So, we can write $i = i_1 + i_2$

As, $$V_m \sin \omega t = R\, i_1 \quad \text{[from the circuit diagram]}$$

$\Rightarrow$ $$i_1 = \frac{V_m \sin \omega t}{R} \quad \text{...(i)}$$

If q_2 is charge on the capacitor at any time t, then for series combination of C and L.

Applying KVL in the Lower circuit as shown,



$$\frac{q_2}{C} + \frac{L\,di_2}{dt} - V_m \sin \omega t = 0$$

$\Rightarrow$ $$\frac{q_2}{C} + \frac{L\,d^2 q_2}{dt^2} = V_m \sin \omega t \quad \left[\because i_2 = \frac{dq_2}{dt}\right] \text{... (ii)}$$

Let $$q_2 = q_m \sin(\omega t + \phi) \quad \text{... (iii)}$$

$\therefore$ $$\frac{dq_2}{dt} = q_m \omega \cos(\omega t + \phi)$$

$\Rightarrow$ $$\frac{d^2 q_2}{dt^2} = -q_m \omega^2 \sin(\omega t + \phi)$$

Now putting these values in Eq. (ii), we get

$$q_m \left[\frac{1}{C} + L(-\omega^2)\right] \sin(\omega t + \phi) = V_m \sin \omega t$$

If $\phi = 0$ and $\left(\frac{1}{C} - L\omega^2\right) > 0$,

then $$q_m = \frac{V_m}{\left(\frac{1}{C} - L\omega^2\right)} \quad \text{...(iv)}$$

From Eq. (iii), $$i_2 = \frac{dq_2}{dt} = \omega q_m \cos(\omega t + \phi)$$

using Eq. (iv), $$i_2 = \frac{\omega V_m \cos(\omega t + \phi)}{\frac{1}{C} - L\omega^2}$$

$$\text{Taking } \phi = 0;\ i_2 = \frac{V_m \cos(\omega t)}{\left(\frac{1}{\omega C} - L\omega\right)} \quad \text{...(v)}$$

From Eqs. (i) and (v), we find that i_1 and i_2 are out of phase by $\frac{\pi}{2}$.

Now, $$i_1 + i_2 = \frac{V_m \sin \omega t}{R} + \frac{V_m \cos \omega t}{\left(\frac{1}{\omega C} - L\omega\right)}$$

Put $$\frac{V_m}{R} = A = C\cos\phi \ \text{ and } \ \frac{V_m}{\left(\frac{1}{\omega C} - L\omega\right)} = B = C\sin\phi$$

∴ $$i_1 + i_2 = C\cos\phi \sin\omega t + C\sin\phi\cos\omega t$$
$$= C\sin(\omega t + \phi)$$

where $$C = \sqrt{A^2 + B^2}$$

and $$\phi = \tan^{-1}\frac{B}{A} C = \left[\frac{V_m^2}{R^2} + \frac{V_m^2}{\left(\frac{1}{\omega C} - L\omega\right)}\right]^{1/2}$$

and $$\phi = \tan^{-1}\frac{R}{\left(\frac{1}{\omega C} - L\omega\right)}$$

Hence, $$i = i_1 + i_2 = \left[\frac{V_m^2}{R^2} + \frac{V_m^2}{\left(\frac{1}{\omega C} - L\omega\right)^2}\right]^{1/2} \sin(\omega t + \phi)$$

or $$\frac{i}{V_m} = \frac{1}{Z} = \left[\frac{1}{R^2} + \frac{1}{\left(\frac{1}{\omega C} - L\omega\right)^2}\right]^{1/2}$$

This is the expression for impedance Z of the circuit.

Note *In this problem, we should not apply the formulae of L-C-R series circuit directly.*

Q. 30 For a *L-C-R* circuit driven at frequency ω, the equation reads

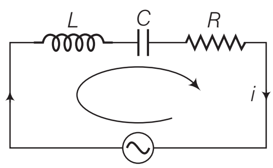
$$L\frac{di}{dt} + Ri + \frac{q}{C} = V_i = V_m \sin \omega t$$

(a) Multiply the equation by i and simplify where possible.
(b) Interpret each term physically.
(c) Cast the equation in the form of a conservation of energy statement.
(d) Intergrate the equation over one cycle to find that the phase difference between V and i must be acute.

Thinking Process

Apply KVL for the given L-C-R series circuit and find the required relations. Also find energy loss through the resistors to know net loss of energy through the circuit.

Ans. Consider the *L-C-R* circuit. Applying KVL for the loop, we can write



$V = V_m \sin \omega t$

$$\Rightarrow \quad L\frac{di}{dt} + \frac{q}{C} + iR = V_m \sin \omega t \quad \text{.... (i)}$$

Multiplying both sides by i, we get

$$Li\frac{di}{dt} + \frac{q}{C}i + i^2R = (V_m i)\sin \omega t = Vi \quad \text{..... (ii)}$$

where $Li\dfrac{di}{dt} = \dfrac{d}{dt}\left(\dfrac{1}{2}Li^2\right)$ = rate of change of energy stored in an inductor.

Ri^2 = joule heating loss

$\dfrac{q}{C}i = \dfrac{d}{dt}\left(\dfrac{q^2}{2C}\right)$ = rate of change of energy stored in the capacitor.

Vi = rate at which driving force pours in energy. It goes into (i) ohmic loss and (ii) increase of stored energy.

Hence Eq. (ii) is in the form of conservation of energy statement. Integrating both sides of Eq. (ii) with respect to time over one full cycle $(0 \rightarrow T)$ we may write

$$\int_0^T \frac{d}{dt}\left(\frac{1}{2}Li^2 + \frac{q^2}{2C}\right)dt + \int_0^T Ri^2 dt = \int_0^T Vi\, dt$$

$$\Rightarrow \quad 0 + (+ve) = \int_0^T Vi\, dt$$

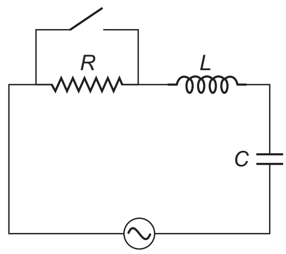
$\Rightarrow \int_0^T Vi\, dt > 0$ if phase difference between V and i is a constant and acute angle.

Q. 31 In the L-C-R circuit, shown in figure the AC driving voltage is $V = V_m$ $\sin \omega t$.

(a) Write down the equation of motion for $q(t)$.

(b) At $t = t_0$, the voltage source stops and R is short circuited. Now write down how much energy is stored in each of L and C.

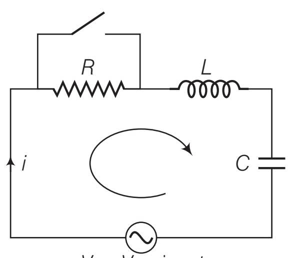
(c) Describe subsequent motion of charges.



K **Thinking Process**

We have to apply KVL write the equations in the form of current and charge double differentiate the equation with respect to time and find the required relations.

Ans. **(a)** Consider the R-L-C circuit shown in the adjacent diagram.



$V = V_m \sin \omega t$

Given $\qquad V = V_m \sin \omega t$

Let current at any instant be i

Applying KVL in the given circuit

$$iR + L\frac{di}{dt} + \frac{q}{C} - V_m \sin \omega t = 0 \qquad \ldots(i)$$

Now, we can write $\qquad i = \frac{dq}{dt} \Rightarrow \frac{di}{dt} = \frac{d^2q}{dt^2}$

From Eq. (i) $\qquad \frac{dq}{dt} R + L\frac{d^2q}{dt^2} + \frac{q}{C} = V_m \sin \omega t$

$\Rightarrow \qquad L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{q}{C} = V_m \sin \omega t$

This is the required equation of variation (motion) of charge.

(b) Let $\quad q = q_m \sin(\omega t + \phi) = -q_m \cos(\omega t + \phi)$

$$i = i_m \sin(\omega t + \phi) = q_m \omega \sin(\omega t + \phi)$$

$$i_m = \frac{V_m}{Z} = \frac{V_m}{\sqrt{R^2 + (X_C - X_L)^2}}$$

$$\phi = \tan^{-1}\left(\frac{X_C - X_L}{R}\right)$$

When R is short circuited at $t = t_0$, energy is stored in L and C.

$$U_L = \frac{1}{2} Li^2 = \frac{1}{2} L \left[\frac{V_m}{\sqrt{(R^2 + X_C - X_L)^2}}\right]^2 \sin^2(\omega t_0 + \phi)$$

and
$$U_C = \frac{1}{2} \times \frac{q^2}{C} = \frac{1}{2C} [q^2 m \cos^2(\omega t_0 + \phi)]$$

$$= \frac{1}{2C}\left[\frac{V_m}{\sqrt{R^2 + (X_C - X_L)^2}}\right]^2$$

$$= \frac{1}{2C} \times \left(\frac{i_m}{\omega}\right)^2 \cos^2(\omega t_0 + \phi)$$

$$= \frac{i^2 m}{2C\omega^2} \cos^2(\omega t_0 + \phi) \qquad [\because i_m = q_m \omega]$$

$$= \frac{1}{2C}\left[\frac{V_m}{\sqrt{R^2 + (X_C - X_L)^2}}\right]^2 \frac{\cos^2(\omega t_0 + \phi)}{\omega^2}$$

$$= \frac{1}{2C\omega^2}\left[\frac{V_m}{\sqrt{R^2 + (X_C - X_L)^2}}\right]^2 \cos^2(\omega t_0 + \phi)$$

(c) When R is short circuited, the circuit becomes an L-C oscillator. The capacitor will go on discharging and all energy will go to L and back and forth. Hence, there is oscillation of energy from electrostatic to magnetic and magnetic to electrostatic.