CBSE

Additional Practice Questions

Subject: Mathematics (041)

Class: XII 2023-24

Time Allowed: 3 Hours **Maximum Marks: 80**

General Instructions:

1. This Question paper contains - five sections **A, B, C, D and E**. Each section is compulsory. However, there are internal choices in some questions.
2. **Section A** has 18 MCQs and 02 Assertion-Reason based questions of 1 mark each.
3. **Section B** has 5 Very Short Answer (VSA)-type questions of 2 marks each.
4. **Section C** has 6 Short Answer (SA)-type questions of 3 marks each.
5. **Section D** has 4 Long Answer (LA)-type questions of 5 marks each.
6. **Section E** has 3 source based/case based/passage based/integrated units of assessment (4 marks each) with sub parts.

SECTION A

(This section comprises of Multiple-choice questions (MCQ) of 1 mark each.)

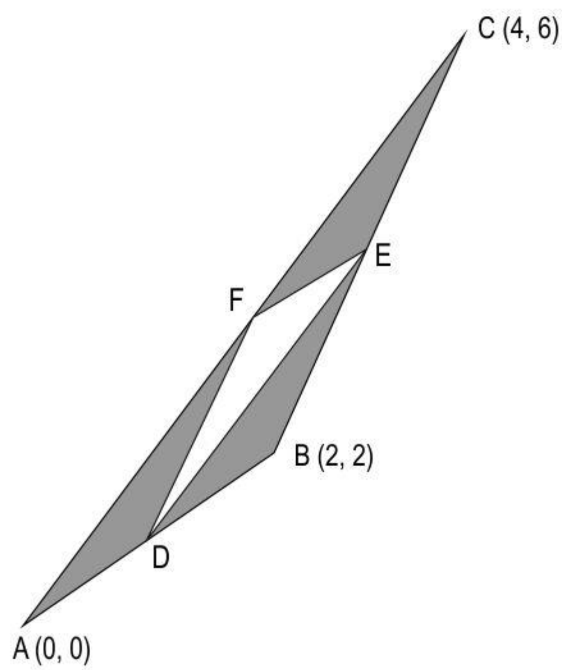
Serial No.	Question	Marks
1	For any 2 × 2 matrix P, which of the following matrices can be Q such that PQ = QP? (a) $\begin{bmatrix} 1 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ (d) (No such matrix exists as matrix multiplication is not commutative.)	1

2 V is a matrix of order 3 such that |adj V| = 7. 1

Which of these could be |V|?

(a) 7^2
(b) 7
(c) $\sqrt{7}$
(d) $\sqrt[3]{7}$

3 The points D, E and F are the mid-points of AB, BC and CA respectively. 1



(Note: The figure is not to scale.)

What is the area of the shaded region?

(a) 2 sq units
(b) $\frac{3}{2}$ sq units
(c) $\frac{1}{2}$ sq units
(d) $(2\sqrt{26} - 1)$ sq units

4 If $f(x) = \cos^{-1}\sqrt{x}$, $0 < x < 1$, which of the following is equal to $f'(x)$? 1

(a) $\frac{-1}{\sqrt{1-x}}$

(b) $\frac{1}{\sqrt{1-x}}$

(c) $\frac{1}{2\sqrt{x(1-x)}}$

(d) $\frac{-1}{2\sqrt{x(1-x)}}$

5 A function f: R -> R is defined by: 1

$$f(x) = \begin{cases} e^{-2x}, & x < \ln\frac{1}{2} \\ 4 \quad , & \ln\frac{1}{2} \leq x \leq 0 \\ e^{-2x}, & x > 0 \end{cases}$$

Which of the following statements is true about the function at the point $x = \ln\frac{1}{2}$?

(a) $f(x)$ is not continuous but differentiable.
(b) $f(x)$ is continuous but not differentiable.
(c) $f(x)$ is neither continuous nor differentiable.
(d) $f(x)$ is both continuous as well as differentiable.

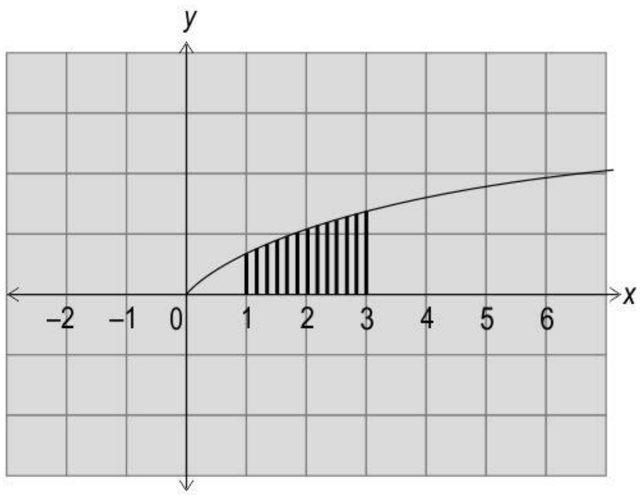
6 In which of these intervals is the function $f(x) = 3x^2 - 4x$ strictly decreasing? 1

(a) $(-\infty, 0)$
(b) $(0, 2)$
(c) $\left(\frac{2}{3}, \infty\right)$
(d) $(-\infty, \infty)$

7 Which of these is equal to $\int e^{(x\log 5)} e^x dx$, where C is the constant of integration? 1

(a) $\frac{(5e)^x}{\log 5e} + C$
(b) $\log 5^x + x + C$
(c) $5^x e^x + C$
(d) $(5e)^x \log x + C$

8 Shown below is the curve defined by the equation $y = \log(x + 1)$ for $x \geq 0$. 1



Which of these is the area of the shaded region?

(a) 6log(2) - 2
(b) 6log(2) - 6
(c) 6log(2)
(d) 5log(2)

9 In which of the following differential equations is the degree equal to its order? 1

(a) $x^3\left(\frac{dy}{dx}\right) - \frac{d^3y}{dx^3} = 0$

(b) $\left(\frac{d^3y}{dx^3}\right)^3 + \sin\left(\frac{dy}{dx}\right) = 0$

(c) $x^2\left(\frac{dy}{dx}\right)^4 + \sin y - \left(\frac{d^2y}{dx^2}\right)^2 = 0$

(d) $\left(\frac{dy}{dx}\right)^3 + x\left(\frac{d^2y}{dx^2}\right) - y^3\left(\frac{d^3y}{dx^3}\right) + y = 0$

10 Kapila is trying to find the general solution of the following differential equations. 1

(i) $xe^{\frac{x}{y}}dx - ye^{\frac{3x}{y}}dy = 0$

(ii) $(2x+1)\frac{dy}{dx} = 3 - 2y$

(iii) $\frac{dy}{dx} = \sin x - \cos y$

Which of the above become variable separable by substituting $y = b.x$, where b is a variable?

(a) only (i)
(b) only (i) and (ii)
(c) all - (i), (ii) and (iii)
(d) None of the above

11 For which of these vectors is the projection on the y-axis zero? 1

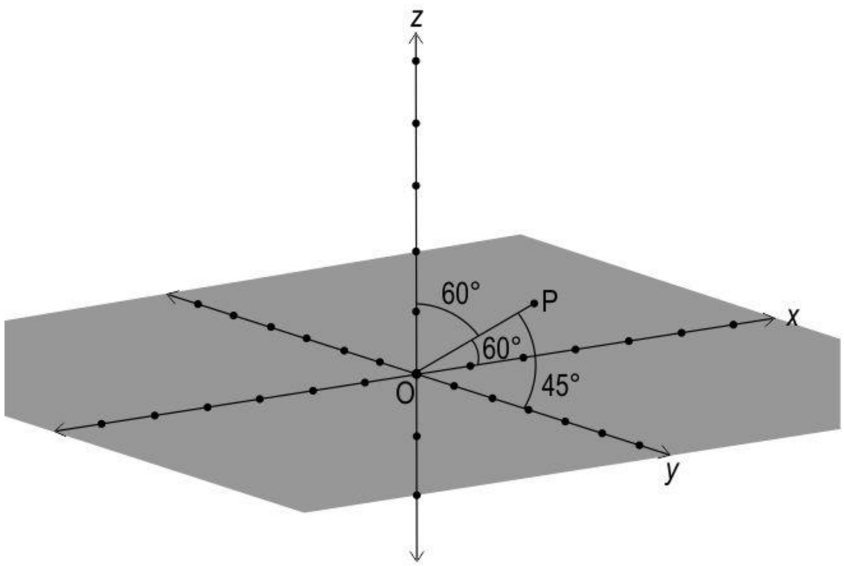
(i) $2\hat{j}$
(ii) $-5\hat{k}$
(iii) $\hat{i} - 4\hat{k}$

(a) only (i)
(b) only (ii)
(c) only (i) and (ii)
(d) only (ii) and (iii)

12 If $\left(\hat{i} + \lambda\hat{j}\right) \times \left(5\hat{i} + 3\hat{j} + \sigma\hat{k}\right) = 0$, what are the values of λ and σ? 1

(a) $\lambda = \frac{3}{5}, \sigma = 0$
(b) $\lambda = \frac{5}{3}, \sigma = 5$
(c) $\lambda = 3, \sigma = 0$
(d) (cannot be found as there are two unknowns and only one equation)

13 A line $\overrightarrow{OP}$ in space, represented by the figure below, has a magnitude of $2\sqrt{2}$ units. 1



Which of these are the direction ratios of $\overrightarrow{OP}$?

(a) $(2, \sqrt{2}, 2)$
(b) $(\sqrt{2}, 2, \sqrt{2})$
(c) $\left(\frac{1}{2}, \frac{1}{\sqrt{2}}, \frac{1}{2}\right)$
(d) $(2\sqrt{2}, 2\sqrt{2}, 2\sqrt{2})$

14 A line *m* passes through the point (-4, 2, -3) and is parallel to line *n*, given by: 1

$$\frac{-x-2}{4} = \frac{y+3}{-2} = \frac{2z-6}{3}$$

The vector equation of line *m* is given by:

$$\vec{r} = (-4\hat{i} + 2\hat{j} - 3\hat{k}) + \lambda(p\hat{i} + q\hat{j} + r\hat{k}), \text{ where } \lambda \in \mathbf{R}$$

Which of the following could be the possible values for *p*, *q* and *r*?

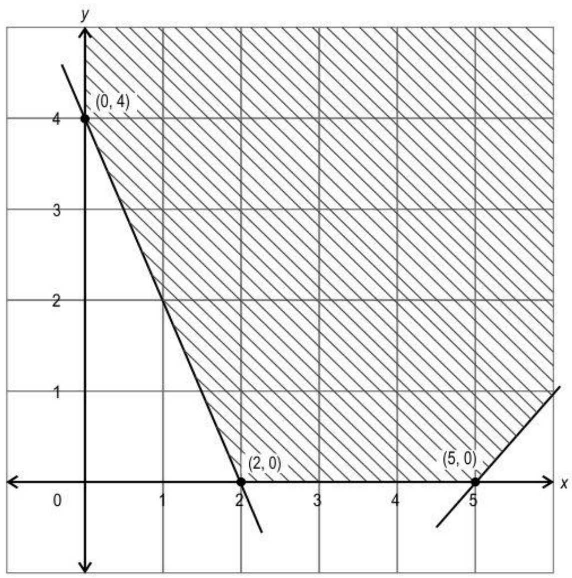
(a) $p = 4$, $q = (-2)$, $r = 3$
(b) $p = (-4)$, $q = (-2)$, $r = 3$
(c) $p = (-2)$, $q = 3$, $r = (-6)$
(d) $p = 8$, $q = 4$, $r = (-3)$

15 L_1 and L_2 are two skew lines. 1

How many lines joining L_1 and L_2 can be drawn such that the line is perpendicular to both L_1 and L_2 ?

(a) exactly one
(b) exactly two
(c) infinitely many
(d) (there cannot be a line joining two skew lines such that it is perpendicular to both)

16 A linear programming problem (LPP) along with the graph of its constraints is shown below. The corresponding objective function is Minimize: $Z = 3x + 2y$. The minimum value of the objective function is obtained at the corner point (2, 0). 1



The optimal solution of the above linear programming problem __________.

(a) does not exist as the feasible region is unbounded.

(b) does not exist as the inequality $3x + 2y < 6$ does not have any point in common with the feasible region.

(c) exists as the inequality $3x + 2y > 6$ has infinitely many points in common with the feasible region.

(d) exists as the inequality $3x + 2y < 6$ does not have any point in common with the feasible region.

17 The feasible region of a linear programming problem is bounded. The corresponding objective function is $Z = 6x - 7y$. 1

The objective function attains __________ in the feasible region.

(a) only minimum
(b) only maximum
(c) both maximum and minimum
(d) either maximum or minimum but not both

18 M and N are two events such that P(M ∩ N) = 0. 1

Which of the following is equal to P(M|(M ∪ N))?

(a) $\frac{P(M)}{P(N)}$

(b) $\frac{P(M \cup N)}{P(M \cup N)}$

(c) $\frac{P(M)}{P(M)+P(N)}$

(d) $\frac{P(M)}{P(M)\times P(N)}$

19 X = {0, 2, 4, 6, 8}. 1
P is a relation on X defined by P = {(0, 2), (4, 2), (4, 6), (8, 6), (2, 4), (0, 4)}.

Based on the above information, two statements are given below - one labelled Assertion (A) and the other labelled Reason (R). Read the statements carefully and choose the option that correctly describes statements (A) and (R).

Assertion (A): The relation P on set X is a transitive relation.

Reason (R): The relation P has a subset of the form {(a, b), (b, c), (a, c)}, where $a, b, c \in$ X.

(a) Both (A) and (R) are true and (R) is the correct explanation for (A).
(b) Both (A) and (R) are true but (R) is not the correct explanation for (A).
(c) (A) is true but (R) is false.
(d) (A) is false but (R) is true.

20 Two statements are given below - one labelled Assertion (A) and the other labelled Reason (R). Read the statements carefully and choose the option that correctly describes statements (A) and (R). 1

Assertion (A): The maximum value of the function $f(x) = x^5$, $x \in [-1, 1]$, is attained at its critical point, $x = 0$.

Reason (R): The maximum of a function can only occur at points where derivative is zero.

(a) Both (A) and (R) are true and (R) is the correct explanation for (A).
(b) Both (A) and (R) are true but (R) is not the correct explanation for (A).
(c) (A) is false but (R) is true.
(d) Both (A) and (R) are false.

SECTION B

(This section comprises of very short answer type-questions (VSA) of 2 marks each.)

Serial No.	Question	Marks
21	Find the domain of the function $y = \cos^{-1}(\lvert x - 1\rvert)$. Show your steps.	2
	OR	
	Draw the graph of the following function: $y = 2\sin^{-1}(x)$, $-\pi \le y \le \pi$	2
22	The sum of a matrix and its transpose is $\begin{bmatrix} 6 & -1 \\ -1 & 4 \end{bmatrix}$. Find one such matrix for which this holds true. Show your work.	2
23	If $x = \cot t$ and $y = \text{cosec}^2 t$, find: i) $\frac{dy}{dx}$ ii) $\frac{d^2y}{dx^2}$	2

Show your steps.

24 Iqbal, a data analyst in a social media platform is tracking the number of active users on their site between 5 pm and 6 pm on a particular day. 2

The user growth function is modelled by $N(t) = 1000e^{0.1t}$, where $N(t)$ represents the number of active users at time t minutes during that period.

Find how fast the number of active users are increasing or decreasing at 10 minutes past 5 pm. Show your steps.

OR

The population of rabbits in a forest is modelled by the function below: 2

$P(t) = \frac{2000}{1+e^{-0.5t}}$, where P represents the population of rabbits in t years

Determine whether the rabbit population is increasing or not, and justify your answer.

25 Solve the integral: 2

$\mathrm{I} = \int x(\mathrm{k} - x)^{23} dx$, where k is a constant

Show your steps.

SECTION C

(This section comprises of short answer type questions (SA) of 3 marks each)

Serial No.	Question	Marks
26		3

Solve the integral:

$$I = \int \frac{3x+5}{x^2+4x+7}dx$$

Show your work.

27 Evaluate the integral: 3

$$\int_0^{\frac{\pi}{2}} \frac{sin\ \theta\ d\theta}{(25+cos\ \theta)(26+cos\ \theta)}$$

Show your steps.

OR

Using the properties of definite integrals, prove the following: 3

$\int_0^{\pi} h(\sin x)\, dx = 2\int_0^{\frac{\pi}{2}} h(\sin x)dx$, where $h(\sin x)$ is a function of $\sin x$.

State the property used.

28 When an object is thrown vertically upward, it is under the effect of gravity and air resistance. For small objects, the force due to air resistance is numerically equal to some constant k times v, where v is the velocity of the object (in m/s) at time t (s). 3

This situation can be modelled as the differential equation shown below.

$$m\frac{dv}{dt} = -F_R - mg$$

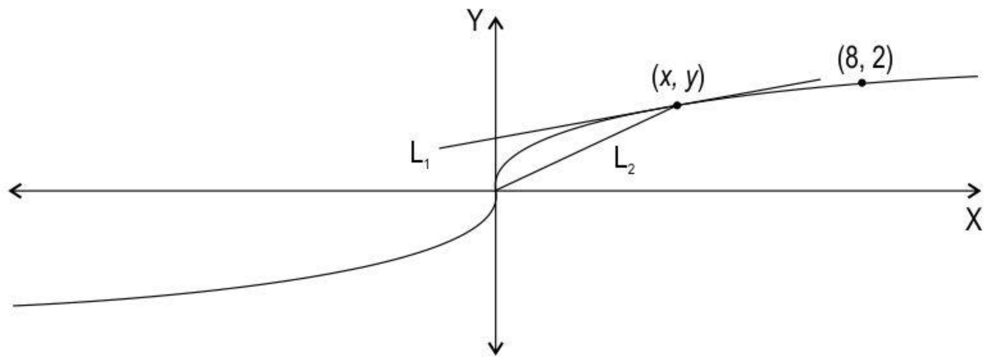
where,
m is the mass of the object in kg.
$\frac{dv}{dt}$ is the acceleration of the object in m/s^2.
F_R is the force due to air resistance.
g is the acceleration due to gravity ($10\ m/s^2$).

A tennis ball of mass 0.050 kg is hit upwards with a velocity of 10 m/s. An air resistance numerically equal to $0.4v$ acts on the ball.

(i) Model the above situation using a differential equation.
(ii) Write an expression for the velocity of the ball in terms of the time.

Show your work.

29 Shown below is a curve. 3



L_1 is the tangent to any point (x, y) on the curve.
L_2 is the line that connects the point (x, y) to the origin.

The slope of L_1 is one third of the slope of L_2.

Find the equation of the curve. Show your work.

OR

Given $x + (y+1)\frac{dy}{dx} = 2$. 3

(i) Solve the differential equation and show that the solution represents a family of circles.

(ii) Find the radius of a circle belonging to the above family that passes through the origin.

Show your work.

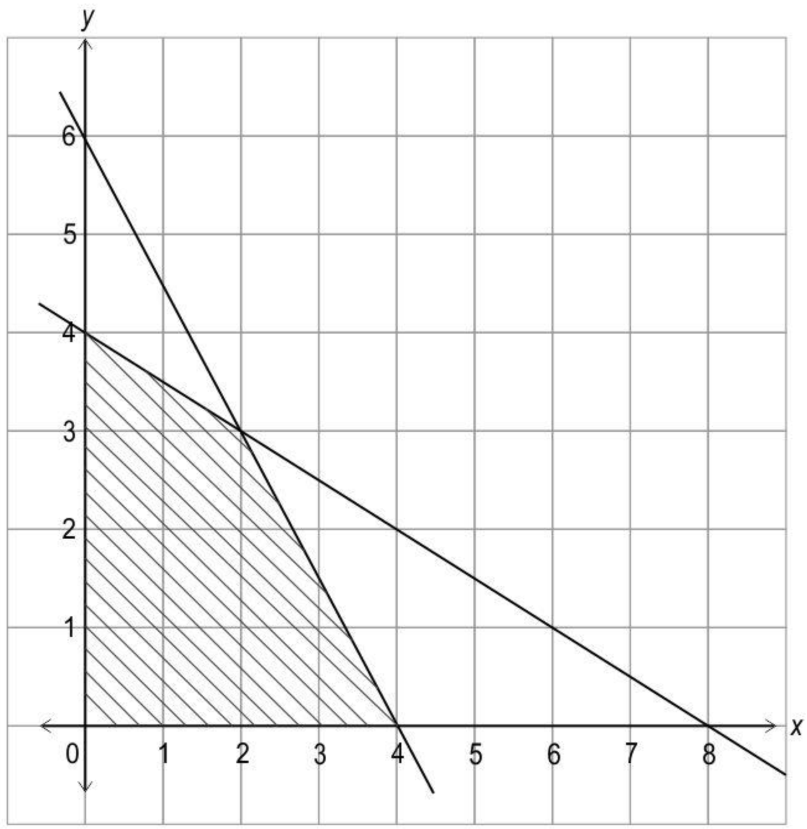
30 Each unit of Product A that a company produces, is sold for Rs 100 with a production cost of Rs 60 and each unit of Product B is sold for Rs 150 with production cost of Rs 90. On a given day, the company has a budget of Rs 8000 to spend on production. The production process makes it such that they can only produce a maximum of 100 units each day. Also, the number of product B produced cannot be more than twice as many of Product A. 3

Frame a linear programming problem to determine how many units of Product A and B should the company produce in a day in order to maximize their profit?

(Note: No need to find the feasible region and optimal solution.)

OR

Shown below is the feasible region of a maximisation problem whose objective function is given by Z = $5x + 3y$. 3



i) List all the constraints the problem is subjected to.
ii) Find the optimal solution of the problem.

Show your work.

31 A company follows a model of bifurcating the tasks into the categories shown below. 3

	URGENT	NOT URGENT
IMPORTANT	urgent and important	not urgent but important
NOT IMPORTANT	urgent but not important	not urgent and not important

At the beginning of a financial year, it was noticed that:

♦ 40% of the total tasks were urgent and the rest were not.
♦ half of the urgent tasks were important, and
♦ 30% of the tasks that were not urgent, were not important

What is the probability that a randomly selected task that is not important is urgent? Use Bayes' theorem and show your steps.

SECTION D

(This section comprises of long answer-type questions (LA) of 5 marks each)

Serial No.	Question	Marks
32	The Earth has 24 time zones, defined by dividing the Earth into 24 equal longitudinal segments. These are the regions on Earth that have the same standard time. For example, USA and India fall in different time zones, but Sri Lanka and India are in the same time zone. A relation R is defined on the set U = {All people on the Earth} such that R = $\{(x, y)|$ the time difference between the time zones x and y reside in is 6 hours}. i) Check whether the relation R is reflexive, symmetric and transitive. ii) Is relation R an equivalence relation? Show your work.	5

OR

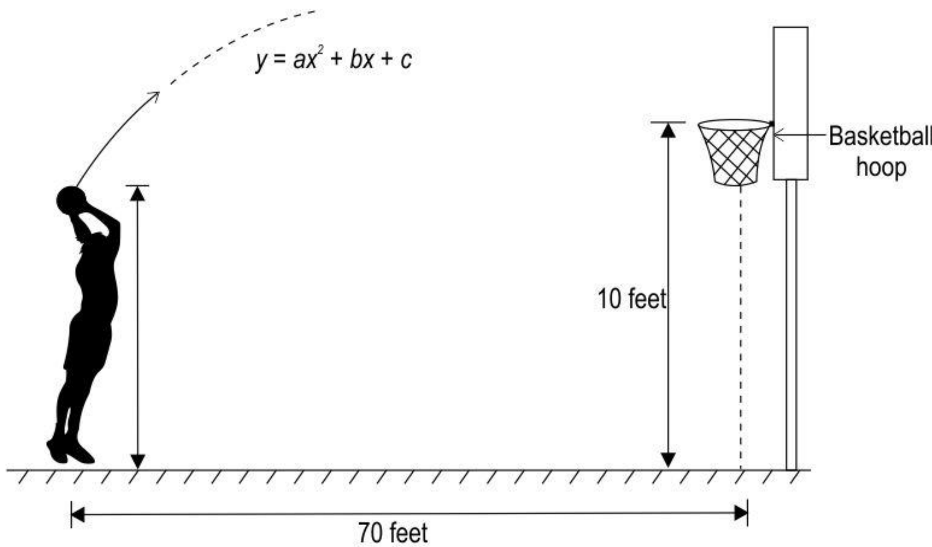
A function f : R - {-1, 1} -> R is defined by: 5

$$f(x) = \frac{x}{x^2 - 1}$$

i) Check if f is one-one.
ii) Check if f is onto.

Show your work.

33 Abdul threw a basketball in the direction of the basketball hoop which traversed a parabolic path in a vertical plane as shown below. 5



(Note: The image is for representation purpose only.)

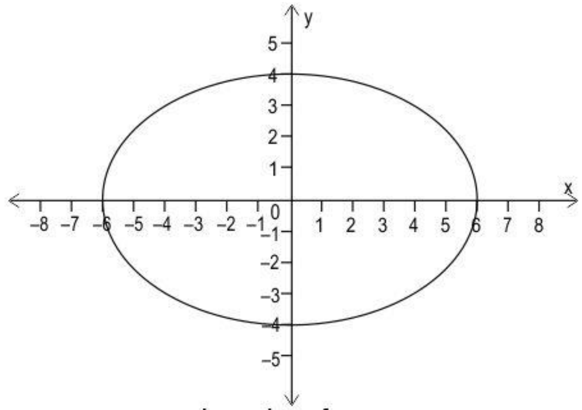
The equation of the path traversed by the ball is $y = ax^2 + bx + c$ with respect to a xy-coordinate system in the vertical plane. The ball traversed through the points (10, 16), (20, 22) and (30, 25). The basketball hoop is at a horizontal distance of 70 feet from Abdul. The height of the basketball hoop is 10 feet from the floor to the top edge of the rim.

Did the ball successfully go through the hoop? Justify your answer.

(Hint: Consider the point where Abdul is standing as the origin of the xy-coordinate system.)

34 Shown below are concrete elliptical water pipes, each 10 feet in length. 5

Concrete elliptical pipes



Inner circumference of the pipe

The graph given above represents the inner circumference of the elliptical pipe, where x and y are in feet. Assume that the water flows uniformly and fully covers the inner cross-sectional area of the pipe.

Find the volume of water in the pipe at a given instant of time, in terms of π. Use the integration method and show your steps.

(Note: Volume = Area of the base × Height)

35 i) Find the vector and cartesian equations of the straight line passing through the point (-5, 7, -4) and in the direction of (3, -2, 1). 5

ii) Find the point where this straight line crosses the *xy*-plane.

Show your work.

OR

Given below are two lines L_1 and L_2: 5

L_1: $2x = 3y = -z$
L_2: $6x = -y = -4z$

i) Find the angle between the two lines.
ii) Find the shortest distance between the two lines.

Show your work.

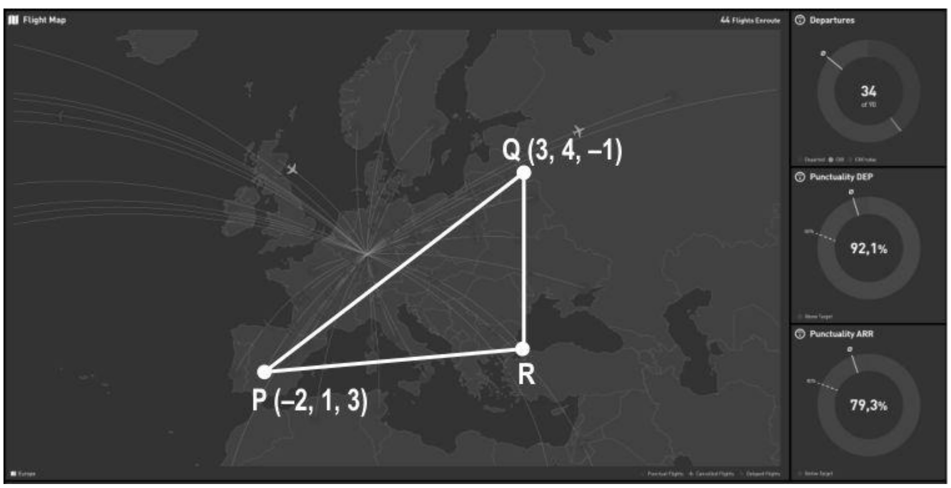
SECTION E

(This section comprises of 3 case-study/passage-based questions of 4 marks each with two sub-questions. First two case study questions have three sub questions of marks 1, 1, 2 respectively. The third case study question has two sub questions of 2 marks each.)

Serial No.	Question	Marks

36 **Answer the questions based on the given information.**

The flight path of two airplanes in a flight simulator game are shown below. The coordinates of the airports P and Q are given.



Airplane 1 flies directly from P to Q.

Airplane 2 has a layover at R and then flies to Q.

The path of Airplane 2 from P to R can be represented by the vector $5\hat{i} + \hat{j} - 2\hat{k}$.

(Note: Assume that the flight path is straight and fuel is consumed uniformly throughout the flight.)

i) Find the vector that represents the flight path of Airplane 1. Show your steps. 1

ii) Write the vector representing the path of Airplane 2 from R to Q. Show your steps. 1

iii) What is the angle between the flight paths of Airplane 1 and Airplane 2 just after takeoff? Show your work. 2

OR

iii) Consider that Airplane 1 started the flight with a full fuel tank. 2

Find the position vector of the point where a third of the fuel runs out if the entire fuel is required for the flight. Show your work.

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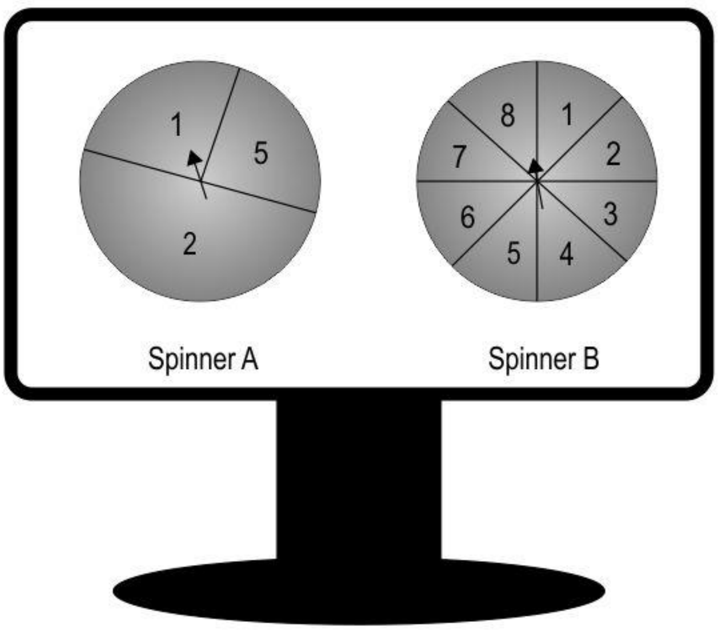
37 **Answer the questions based on the given information.**

Rubiya, Thaksh, Shanteri, and Lilly entered a spinning zone for a fun game, but there is a twist: they don't know which spinner will appear on their screens until it is their turn to play. They may encounter one of the following spinners, or perhaps even both:



Different combinations of numbers will lead to exciting prizes. Below are some of the rewards they can win:

♦ Get the number '5', from Spinner A and '8' from Spinner B, and you'll win a music player!

♦ You win a photo frame if Spinner A lands on a value greater than that of Spinner B!

i) Thaksh spun both the spinners, A and B in one of his turns. 1

What is the probability that Thaksh wins a music player in that turn? Show your steps.

ii) Lilly spun spinner B in one of her turns. 1

What is the probability that the number she got is even given that it is a multiple of 3? Show your steps.

iii) Rubiya spun both the spinners. 2

What is the probability that she wins a photo frame? Show your work.

OR

iii) As Shanteri steps up to the screen, the game administrator reveals that for her turn, the probability of seeing Spinner A on the screen is 65%, while that of Spinner B is 35%. 2

What is the probability that Shanteri gets the number '2'? Show your steps.

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38 **Answer the questions based on the given information.**

Two metal rods, R_1 and R_2, of lengths 16 m and 12 m respectively, are insulated at both the ends. Rod R_1 is being heated from a specific point while rod R_2 is being cooled from a specific point.

The temperature (T) in Celsius within both rods fluctuates based on the distance (x) measured from either end. The temperature at a particular point along the rod is determined by the equations $T = (16 - x)x$ and $T = (x - 12)x$ for rods R_1 and R_2 respectively, where the distance x is measured in meters from one of the ends.

i) Find the rate of change of temperature at the mid point of the rod that is being heated. Show your steps. 2

ii) Find the minimum temperature attained by the rod that is being cooled. Show your work. 2

CBSE

Additional Practice Questions-Marking Scheme

Subject: Mathematics (041)

Class: XII 2023-24

Time Allowed: 3 Hours **Maximum Marks: 80**

SECTION A

Multiple Choice Questions of 1 mark each.

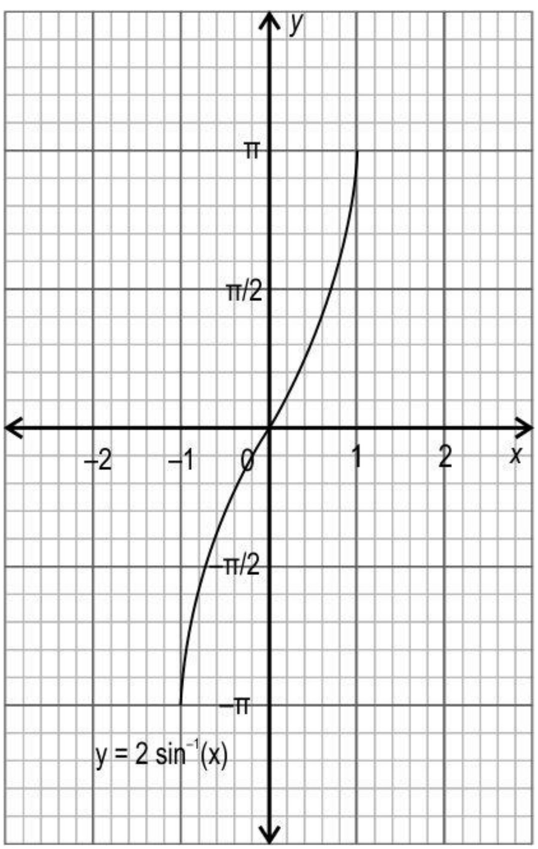
Q No.	Answer/Solution	Marks
1	(b) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	1
2	(c) $\sqrt{7}$	1
3	(b) $\frac{3}{2}$ sq units	1
4	(d) $\frac{-1}{2\sqrt{x(1-x)}}$	1
5	(b) $f(x)$ is continuous but not differentiable.	1
6	(a) $(-\infty, 0)$	1
7	(a) $\frac{(5e)^x}{\log 5e} + C$	1
8	(a) $6\log(2) - 2$	1
9	(c)	1

	$x^2\left(\frac{dy}{dx}\right)^4 + \sin y - \left(\frac{d^2y}{dx^2}\right)^2 = 0$	
10	(a) only (i)	1
11	(d) only (ii) and (iii)	1
12	(a) $\lambda = \frac{3}{5}, \sigma = 0$	1
13	(b) $(\sqrt{2}, 2, \sqrt{2})$	1
14	(d) $p = 8$, $q = 4$, $r = (-3)$	1
15	(a) exactly one	1
16	(d) exists as the inequality $3x + 2y < 6$ does not have any point in common with the feasible region.	1
17	(c) both maximum and minimum	1
18	(c) $\frac{P(M)}{P(M)+P(N)}$	1
19	(d) (A) is false but (R) is true.	1
20	(d) Both (A) and (R) are false.	1

SECTION B

Very short answer questions of 2 marks each.

Q No.	Answer/Solution	Marks

21	Since the domain of inverse of cosine function is [-1, 1], finds the domain of the given function as follows: $-1 \leq x - 1 \leq 1$ So, $0 \leq x \leq 2$ And, $-1 \leq 1 - x \leq 1$ $\Rightarrow 1 \geq x - 1 \geq -1$ So, $2 \geq x \geq 0$ Concludes the domain of $\cos^{-1}(\lvert x - 1 \rvert)$ as [0, 2]. OR	1.5 0.5
	Draws the graphs of $y = 2\sin^{-1}(x)$ as shown below. 	2.0

22	Considers one such matrix as $\begin{bmatrix} x & y \\ z & w \end{bmatrix}$ and frames the following relationship: $\begin{bmatrix} x & y \\ z & w \end{bmatrix} + \begin{bmatrix} x & z \\ y & w \end{bmatrix} = \begin{bmatrix} 6 & -1 \\ -1 & 4 \end{bmatrix}$ Obtains $x = 3$ and $w = 2$ using the relationship obtained in the previous step as follows: $2x = 6 \Rightarrow x = 3$ $2w = 4 \Rightarrow w = 2$ Writes any value of y and z that satisfies the third relationship obtained in the first step. For example, $y = 0$ and $z = -1$. Writes one such matrix as $\begin{bmatrix} 3 & 0 \\ -1 & 2 \end{bmatrix}$. (Award full marks for any other matrix that satisfies the relationship.)	0.5 0.5 0.5 0.5
23	i) Finds $\frac{dx}{dt}$ as $(-cosec^2 t)$. Finds $\frac{dy}{dt}$ as $2cosec\ t\ (-cosec\ t\ cot\ t) = -2cosec^2 t\ cot\ t$. Finds $\frac{dy}{dx}$ as: $\frac{\frac{dy}{dt}}{\frac{dx}{dt}} = 2cot\ t$ *ii*) Finds $\frac{d^2y}{dx^2}$ as: $\frac{d}{dx}(2cot\ t)$ $= \frac{d}{dx}(2x)$ $= 2$ (Award full marks if any alternate method is used.)	0.5 0.5 0.5 0.5

24	Writes that the rate at which the number of active users is increasing or decreasing at a given time is given by $\frac{d}{dt}N(t)$. Finds the derivative of $N(t)$ as: $\frac{d}{dt}N(t) = 1000\ (0.1)\ e^{0.1t}$ Finds the rate of change of active users at 10 minutes past 5 pm as: $\frac{d}{dt}N(10) = 1000(0.1)e^{(0.1)(10)} = 100e$ Concludes that the number of active users are increasing at a rate of $100e$ people per minute at 5:10 pm on that day. OR Finds the derivative of the given function as: $P'(t) = 2000 \times \frac{(-1)}{(1+e^{-0.5t})^2} \times e^{-0.5t} \times (-\frac{1}{2}) = \frac{1000(e^{-0.5t})}{(1+e^{-0.5t})^2}$ Writes that the above quantity is greater than 0 for any value of t. Concludes that the rabbit population is increasing.	0.5 1.0 0.5 1.0 0.5 0.5
25	Substitutes $(k - x)$ by u to get $dx = -du$ and rewrites the given integral as: $I = \int (k-u)(u)^{23}(-du)$ Integrates the expression in above step as: $I = \frac{-ku^{24}}{24} + \frac{u^{25}}{25} + C$ where, C is the constant of integration. Substitutes $u = (k - x)$ back in the above expression and writes: $I = \frac{(-k)(k-x)^{24}}{24} + \frac{(k-x)^{25}}{25} + C$ where, C is the constant of integration.	0.5 1.0 0.5

SECTION C

Short answer questions of 3 marks each.

Q No.	Answer/Solution	Marks
26	Rewrites the numerator of the given integral as: $3x + 5 = A\frac{d}{dx}(x^2 + 4x + 7) + B$ => $3x + 5 = A(2x + 4) + B$	0.5
	Finds the values of A and B by comparing the coefficients of like terms as: $2A = 3 \Rightarrow A = \frac{3}{2}$ $4A + B = 5 \Rightarrow B = -1$	1.0
	Substitutes the values of A and B in the given integral and integrates the same as: $I = \int \frac{\frac{3}{2}(2x+4)-1}{x^2+4x+7}\,dx$ $\Rightarrow I = \frac{3}{2}\int \frac{2x+4}{x^2+4x+7}\,dx - \int \frac{1}{(x+2)^2+(\sqrt{3})^2}\,dx$ $\Rightarrow I = \frac{3}{2}\log\lvert x^2+4x+7\rvert - \frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{x+2}{\sqrt{3}}\right) + C$ where, C is the constant of integration.	1.5
27	Takes $u = (25 + \cos\theta)$. Finds du as: $du = -\sin\theta\, d\theta$	0.5
	Finds the change in limit when $\theta = 0$ and $\theta = \frac{\pi}{2}$ to $u = 26$ and $u = 25$ respectively.	0.5
	Rewrites the given integral using the above substitution and integrates the same as:	

	$\int_0^{\frac{\pi}{2}} \frac{\sin\theta\ d\theta}{(25+\cos\theta)(26+\cos\theta)}$ $= \int_{26}^{25} \frac{-du}{u(1+u)}$ $= \int_{25}^{26} \frac{du}{u(1+u)}$ $= \int_{25}^{26} \frac{(1+u)-u}{u(1+u)} du$ $= \int_{25}^{26} \frac{1}{u} du - \int_{25}^{26} \frac{1}{1+u} du$ $= [\log u - \log(1+u)]_{25}^{26}$ Applies the limit to find the value of the given definite integral as $\log \frac{26\times 26}{25\times 27}$. (Award full marks if the problem is solved correctly by taking $u = 26 + \cos\theta$.) OR States the property that is going to be used as: If $f(2a - x) = f(x)$, then $\int_0^{2a} f(x)dx = 2\int_0^a f(x)dx$ Takes $2a = \pi$ and $f(x) = h(\sin x)$. Finds $f(2a - x)$ as: $f(2a - x) = f(\pi - x) = h(\sin(\pi - x)) = h(\sin x) = f(x)$ Thus confirms that the property listed in the above step can be applied to the given integral. Hence concludes that: $\int_0^{\pi} h(\sin x)\,dx = 2\int_0^{\frac{\pi}{2}} h(\sin x)dx$	1.5 0.5 1.0 2.0
28	(i) Models the situation and rearranges terms to form a linear differential equation as follows: $0.05\frac{dv}{dt} = -0.4v - 0.5$ $\Rightarrow \frac{dv}{dt} + 8v = -10$	0.5

	(ii) Considering the obtained equation as linear of the form $\frac{dy}{dx} + Py = Q$ with $P = 8$ and hence takes the integrating factor as: $e^{\int 8dt} = e^{8t}$ Multiplies the differential equation by the integrating factor as follows: $e^{8t}\frac{dv}{dt} + 8ve^{8t} = -10e^{8t}$ $\Rightarrow \frac{d}{dt}\left(ve^{8t}\right) = -10e^{8t}$ Integrates both sides to obtain the general solution of the differential equation as follows: $\int \frac{d}{dt}\left(ve^{8t}\right) = -10\int e^{8t}$ $\Rightarrow ve^{8t} = \frac{-10}{8}e^{8t} + C$ $\Rightarrow v = -1.25 + Ce^{-8t}$ where C is the constant of integration. Uses the initial condition $v(0) = 10$ m/s to find the value of C as follows: $10 = -1.25 + C$ $\Rightarrow C = 11.25$ Hence, writes the expression for the velocity of the ball as a function of time as follows: $v = -1.25 + 11.25\ e^{-8t}$	0.5 0.5 1.0 0.5
29	Frames the differential equation using the given conditions as follows: $\frac{dy}{dx} = \frac{1}{3} \times \frac{y}{x}$ $\Rightarrow \frac{dy}{dx} = \frac{y}{3x}$ Rearranges the terms to separate the variables as follows: $\frac{dy}{y} = \frac{dx}{3x}$ Integrates both sides to obtain the general solution of the curve. The working may look as follows:	1.0 0.5

	$\int \frac{dy}{y} = \frac{1}{3}\int \frac{dx}{x}$ $\Rightarrow \log y = \frac{1}{3}\log x + \log C_1$ $\Rightarrow y = C_2 x^{\frac{1}{3}}$ where C_1 and C_2 are constants of integration. Substitutes (8, 2) to obtain the equation of the curve as follows: $2 = C_2(8)^{\frac{1}{3}}$ $\Rightarrow C_2 = 1$ $\therefore y = x^{\frac{1}{3}}$ OR (i) Separates the variables and rearranges the terms of the differential equation as follows: $x + (y+1)\frac{dy}{dx} = 2$ $\Rightarrow (y+1)dy = (2-x)dx$ Integrates both sides to obtain the following: $\int (y+1)dy = \int (2-x)dx$ $\Rightarrow \frac{y^2}{2} + y = 2x - \frac{x^2}{2} + C_1$ $\Rightarrow x^2 + y^2 - 4x + 2y + C_2 = 0$ where C_1 and C_2 are constants of integration. Writes that the solution is a general solution of a circle and hence it represents a family of circles. (ii) Substitutes $x = 0$ and $y = 0$ into the general solution to obtain $C_2 = 0$ and writes the particular solution as: $x^2 + y^2 - 4x + 2y = 0$ Rearranges the terms to rewrite the particular solution as $(x-2)^2 + (y+1)^2 = 5$ to find the radius as $\sqrt{5}$ units. The working may look as follows: $x^2 + y^2 - 4x + 2y = 0$ Adding 5 to both sides and rearranging terms, $\Rightarrow (x^2 - 4x + 4) + (y^2 + 2y + 1) = 5$ $\Rightarrow (x-2)^2 + (y+1)^2 = 5$	1.0 0.5 0.5 1.0 0.5 0.5 0.5

<table>
<tr>
<td>30</td>
<td>Finds the profit on selling the products as:

Profit for each unit of Product A sold = 100 - 60 = Rs 40

Profit for each unit of Product B sold = 150 - 90 = Rs 60.

Takes x and y to be the numbers of Product A and Product B to be produced in a day respectively and frames the objective function as:

Maximise Z = $40x + 60y$

Writes the constraints of the given linear programming problem as:

$60x + 90y \leq 8000$
$x + y \leq 100$
$y \leq 2x$ or $-2x + y \leq 0$
$x, y \geq 0$

OR

i) Uses the graph of the feasible region and lists the constraints of the given maximisation problem as:

$3x + 2y \leq 12$
$x + 2y \leq 8$
$x, y \geq 0$

ii) Finds the value of the objective function at corner points as:</td>
<td>0.5

1.0

1.5

1.5</td>
</tr>
<tr>
<td></td>
<td><table><tr><td>Corner point</td><td>$z = 5x + 3y$</td></tr><tr><td>(0, 0)</td><td>0</td></tr><tr><td>(0, 4)</td><td>12</td></tr><tr><td>(2, 3)</td><td>19</td></tr><tr><td>(4, 0)</td><td>20</td></tr></table>
Concludes that the objective function attains maximum value at (4, 0) and hence (4, 0) is the optimal solution.</td>
<td>1.0

0.5</td>
</tr>
<tr>
<td>31</td>
<td>Applies Bayes' theorem and writes:</td>
<td></td>
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	P(Urgent|Not Important) = $$\frac{P(Urgent)\times P(Not\ Important\|Urgent)}{P(Urgent)\times P(Not\ Important\|Urgent)+P(Not\ Urgent)\times P(Not\ Important\|Not\ Urgent)}$$	1.0
	Substitutes respective probabilities in the expression obtained above to find the required probability as follows: P(Urgent|Not Important) $$=\frac{\left(\frac{40}{100}\times\frac{1}{2}\right)}{\left(\frac{40}{100}\times\frac{1}{2}\right)+\left(\frac{60}{100}\times\frac{30}{100}\right)}$$	1.5
	Simplifies the above expression to get the probability that a randomly selected task that is not important is urgent as $\frac{10}{19}$ or 52.63%.	0.5

SECTION D

Long answer questions of 5 marks each.

Q No.	Answer/Solution	Marks

32	i) Writes that for no $x \in$ U, $(x, x) \in$ R as the difference in time between x & x is 0 hours. Concludes that R is not reflexive. Writes that, whenever the difference in time between x and y is 6 hours, the difference in time between y and x is also 6 hours. That is, $(x, y) \in$ R => $(y, x) \in$ R. Concludes that R is symmetric. Writes that, if the difference in time between x and y is 6 hours, and the difference in time between y and z is also 6 hours, then the difference in time between x and z could be either 0 hours or 12 hours. That is, $(x, y) \in$ R & $(y, z) \in$ R but $(x, z) \notin$ R. Concludes that R is not transitive. ii) From the above steps, concludes that R is not an equivalence relation. OR i) Assumes $f(x) = f(y)$ and evaluates the same as: $\frac{x}{x^2-1} = \frac{y}{y^2-1}$ $\Rightarrow x(y^2-1) = y(x^2-1)$ $\Rightarrow xy^2 - x - yx^2 + y = 0$ $\Rightarrow (y-x)(xy+1) = 0$ Uses the above step to conclude that $x = y$ or $xy = -1$. Takes a pair of numbers x and y such that $xy = -1$ to show that f is not one-one. For example, for $x = \frac{1}{2}$ and $y = -2$, $f(x) = -\frac{2}{3}$ and $f(y) = -\frac{2}{3}$.	 1.5 1.5 1.5 0.5 1.5 0.5 1.0

	ii) Equates $f(x)$ to y and solves the same to express x in terms of y as: $\frac{x}{x^2-1} = y$ $\Rightarrow x = yx^2 - y$ $\Rightarrow yx^2 - x - y = 0$ $\Rightarrow x = \frac{1 \pm \sqrt{1+4y^2}}{2y}$ Since $1 + 4y^2 > 0$, real root exists and also they are not ± 1 $\Rightarrow x = \frac{1 \pm \sqrt{1+4y^2}}{2y} \in R - \{-1, +1\}$	1.5
	Writes that for any $y \in$ R (codomain), there exists $x \in$ R - {-1, 1} (domain) such that $f(x) = y$. Hence concludes that f is onto.	0.5
33	Writes the system of equations as: $100a + 10b + c = 16$ $400a + 20b + c = 22$ $900a + 30b + c = 25$	0.5
	Writes the above system of equations in the form AX = B as: $\begin{pmatrix} 100 & 10 & 1 \\ 400 & 20 & 1 \\ 900 & 30 & 1 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 16 \\ 22 \\ 25 \end{pmatrix}$	0.5
	Finds |A| as 1(18000 - 12000) - 1(3000 - 9000) + 1(2000 - 4000) = -2000 and writes that A^{-1} exists as |A| $\neq 0$.	0.5
	Finds A^{-1} as: $A^{-1} = \begin{pmatrix} \frac{1}{200} & \frac{-1}{100} & \frac{1}{200} \\ \frac{-1}{4} & \frac{2}{5} & \frac{-3}{20} \\ 3 & -3 & 1 \end{pmatrix}$ (Award 1 mark if only all the cofactors are found correctly.)	1.5

	Finds the values of a, b and c as $-\frac{3}{200}, \frac{21}{20}$ and 7 respectively by solving X = A^{-1}B as: $$X = A^{-1}B = \begin{pmatrix} \frac{1}{200} & \frac{-1}{100} & \frac{1}{200} \\ \frac{-1}{4} & \frac{2}{5} & \frac{-3}{20} \\ 3 & -3 & 1 \end{pmatrix} \times \begin{pmatrix} 16 \\ 22 \\ 25 \end{pmatrix} = \begin{pmatrix} \frac{-3}{200} \\ \frac{21}{20} \\ 7 \end{pmatrix}$$	1.0
	Finds the equation of the path traversed by the ball as: $y = -\frac{3}{200}x^2 + \frac{21}{20}x + 7.$	0.5
	Writes that when x = 70 feet, y = 7 feet. So, the ball went by 7 feet above the floor that means 3 feet below the basketball hoop. So, the ball did not go through the hoop.	0.5
34	Finds the equation of the ellipse as: $\frac{x^2}{36} + \frac{y^2}{16} = 1$	0.5
	Expresses y in terms of x as: $y = \pm\frac{4}{6}\sqrt{36 - x^2}$	0.5
	Integrates the above equation with respect to x from limit 0 to 6, that gives the area of one quarter of the ellipse. The working may look as follows: $\int_0^6 \frac{4}{6}\sqrt{36 - x^2}dx$	0.5
	Applies the formula of integration and simplifies as: $\frac{4}{6}\left[\frac{x}{2}\sqrt{6^2 - x^2} + \frac{6^2}{2}\sin^{-1}\left(\frac{x}{6}\right)\right]_0^6$	1.0
	Applies the limit and solves further as: $\frac{4}{6}\left[\frac{6}{2} \times 0 + \frac{6^2}{2}\sin^{-1}(1) - 0\right]$	0.5
	Simplifies the above expression to get the area of one-quarter of the base as 6π sq feet.	1.0
	Finds the area of the whole ellipse as 4 × 6π = 24π sq feet.	0.5
	Finds the volume of water as 24π × 10 = 240π cubic feet.	0.5

35	i) Takes $\vec{a} = -5\hat{i} + 7\hat{j} - 4\hat{k}$ and $\vec{b} = 3\hat{i} - 2\hat{j} + \hat{k}$ Writes the vector equation of the given straight line as: $\vec{r} = (-5\hat{i} + 7\hat{j} - 4\hat{k}) + \lambda(3\hat{i} - 2\hat{j} + \hat{k})$ Writes the cartesian equation of the given straight line as: $\frac{x+5}{3} = \frac{y-7}{-2} = \frac{z+4}{1}$ ii) Simplifies the vector form obtained in step 2 as: $\vec{r} = (-5 + 3\lambda)\hat{i} + (7 - 2\lambda)\hat{j} + (\lambda - 4)\hat{k}$ Writes that at the point where the line crosses *xy*-plane, its *z*-coordinate is zero and equates the *z*-coordinate of the above equation to zero as: $\lambda - 4 = 0$ $\Rightarrow \lambda = 4$ Substitutes $\lambda = 4$ in the vector form to get the required point as (7, -1, 0). OR Rewrites the equation of L_1 in cartesian form as: $\frac{x}{3} = \frac{y}{2} = \frac{z}{-6}$ Rewrites the equation of L_2 in cartesian form as: $\frac{x}{2} = \frac{y}{-12} = \frac{z}{-3}$ i) Identifies the direction cosines of both the lines as (3, 2, -6) and (2, -12, -3). Finds the cosine of the angle between the two lines as: $\cos\theta = \left\vert\frac{6-24+18}{\sqrt{49}\sqrt{157}}\right\vert = 0$ (Award 0.5 marks if only the formula of the cosine of the angle between the two lines is written correctly.) Concludes that the angle between the two lines is 90°.	0.5 1.0 1.0 1.0 0.5 1.0 0.5 0.5 0.5 1.5 0.5

	ii) Rewrites the equations of L_1 and L_2 in vector form as: $\vec{r}_1 = (0\hat{i} + 0\hat{j} + 0\hat{k}) + \lambda(3\hat{i} + 2\hat{j} - 6\hat{k})$, where $\lambda \in R$ $\vec{r}_2 = (0\hat{i} + 0\hat{j} + 0\hat{k}) + \lambda(2\hat{i} - 12\hat{j} - 3\hat{k})$, where $\lambda \in R$ Writes that both the lines pass through the origin hence intersect at the origin. (Award full marks if the inference about both lines passing through the origin is drawn without writing the vector forms.) Writes that since both the lines intersect at the origin, the shortest distance between the two lines is 0 units.	1.0 0.5

SECTION E

Case-based questions of 4 marks each.

Q No.	Answer/Solution	Marks
36 i)	Writes the vectors for points P and Q as follows: $\overrightarrow{OP} = -2\hat{i} + \hat{j} + 3\hat{k}$ $\overrightarrow{OQ} = 3\hat{i} + 4\hat{j} - \hat{k}$ Finds the vector representing the flight path of Airplane 1 as: $\overrightarrow{PQ} = \overrightarrow{OQ} - \overrightarrow{OP}$ $= (3\hat{i} + 4\hat{j} - \hat{k}) - (-2\hat{i} + \hat{j} + 3\hat{k})$ $= 5\hat{i} + 3\hat{j} - 4\hat{k}$	0.5 0.5
36 ii)	Uses vector subtraction to find the vector representing the flight path from R to Q as: $\overrightarrow{RQ} = \overrightarrow{PQ} - \overrightarrow{PR}$ $= (5\hat{i} + 3\hat{j} - 4\hat{k}) - (5\hat{i} + \hat{j} - 2\hat{k})$ $= 2\hat{j} - 2\hat{k}$	1.0

36 iii)	Finds the cosine of the angle between the vectors representing the flight paths of Airplane 1 and Airplane 2 as: $\cos\theta = \frac{\overrightarrow{PQ}\cdot\overrightarrow{PR}}{\lvert\overrightarrow{PQ}\rvert\cdot\lvert\overrightarrow{PR}\rvert}$ $= \frac{(5\hat{i}+3\hat{j}-4\hat{k})\cdot(5\hat{i}+\hat{j}-2\hat{k})}{\sqrt{50}\cdot\sqrt{30}}$ $= \frac{18}{5\sqrt{15}}$ Finds the angle between the flight paths as: $\theta = \cos^{-1}\left(\frac{18}{5\sqrt{15}}\right)$ OR Considers a point S which divides PQ internally in the ratio 1:2. Finds the position vector of point S as: $\overrightarrow{OS} = \frac{1(\overrightarrow{OQ})+2(\overrightarrow{OP})}{1+2}$ $= \frac{1(3\hat{i}+4\hat{j}-\hat{k})+2(-2\hat{i}+\hat{j}+3\hat{k})}{3}$ $= -\frac{1}{3}\hat{i} + 2\hat{j} + \frac{5}{3}\hat{k}$ (Award 0.5 marks if only the formula is written correctly.)	1.5 0.5 0.5 1.5
37 i)	Finds the required probability as: P(5 from spinner A) $\cap$ P(8 from spinner B) $= \frac{1}{4} \times \frac{1}{8}$ $= \frac{1}{32}$	1.0
37 ii)	Uses the conditional probability and finds the required probability as follows: P(Even|Multiple of 3) = P(Even $\cap$ Multiple of 3) $\div$ P(Multiple of 3) $= \frac{\frac{1}{8}}{\frac{2}{8}}$ $= \frac{1}{2}$	1.0

37 iii)	Finds the probability of getting 2 from spinner A and getting 1 from spinner B as: $P_1 = \frac{1}{2} \times \frac{1}{8} = \frac{1}{16}$ Finds the probability of getting 5 from spinner A and getting either 1, 2, 3 or 4 from spinner B as: $P_2 = \frac{1}{4} \times \left[\frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8}\right]$ $= \frac{1}{4} \times \frac{4}{8}$ $= \frac{1}{8}$ Writes that P_1 and P_2 are mutually exclusive and hence, finds the probability that she wins a photo frame as: $P_1 + P_2 = \frac{1}{16} + \frac{1}{4}$ $= \frac{5}{16}$ OR Uses the theorem of total probability and writes: P(getting 2) = [P(Spinner A) × P(Getting 2|Spinner A)] + [P(Spinner B) × P(Getting 2|Spinner B)] Finds the required probability by substituting the required probability as: $\left[\frac{65}{100} \times \frac{1}{2}\right] + \left[\frac{35}{100} \times \frac{1}{8}\right]$ $= \frac{59}{160}$	0.5 1.0 0.5 0.5 1.5
38 i)	Identifies that the rod being heated is R_1 and finds the rate of change of temperature at any distance from one end of R_1 as: $\frac{dT}{dx} = \frac{d}{dx}(16 - x)x = \frac{d}{dx}(16x - x^2) = 16 - 2x$ Finds the mid-point of the rod as $x = 8$ m.	1.0 0.5

	Finds the rate of change of temperature at the mid point of R_1 as: $\frac{dT}{dx}_{(\text{at } x=8)} = 16 - 2(8) = 0$	0.5
38 ii)	Identifies that the rod being cooled is R_2 and finds the rate of change of temperature at any distance x m as: $\frac{dT}{dx} = \frac{d}{dx}(x - 12)x = \frac{d}{dx}(x^2 - 12x) = 2x - 12$ Equates $\frac{dT}{dx}$ to 0 to get the critical point as $x = 6$. Finds the second derivative of T as: $\frac{d^2T}{dx^2} = 2$ And concludes that at $x = 6\ m$, the rod has minimum temperature as $\frac{d^2T}{dx^2}_{(\text{at } x=6)} = 2 > 0$. Finds the minimum temperature attained by the rod R_2 as T(6) = (6 - 12)6 = -36 °C.	1.0 0.5 0.5