Cylindrical Coordinates:-

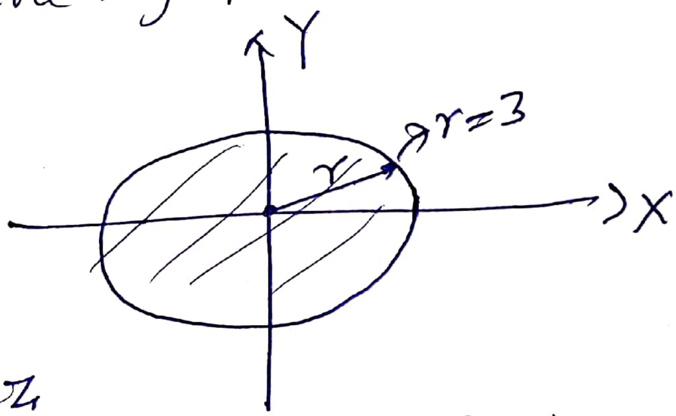
$$\iiint_Q f(r,\theta,z)\,dV = \int_\alpha^\beta \int_{g_1(\theta)}^{g_2(\theta)} \int_{K_1(r,\theta)}^{K_2(r,\theta)} f(r,\theta,z)\, r\,dz\,dr\,d\theta$$

$\boxed{x = r\cos\theta,\ y = r\sin\theta;\ z.}$ $\cancel{(x,y,z)} \to (r,\theta,z)$.

Volume in cylindrical coordinates (r,θ,z).

$$V = \iiint_V r\,dr\,d\theta\,dz.$$

① Evaluate $\iiint_Q e^{x^2+y^2}\,dV$, where Q is the solid bounded by the cylinder $x^2+y^2=9$, the xy-plane and the plane $z=5$.



Solution:- $r: 0 \to 3$

$\theta: 0 \to 2\pi$; $z: 0 \to 5$

$$I = \int_{\theta=0}^{2\pi}\int_{r=0}^{3}\int_{z=0}^{5} e^{r^2}\cdot r\,dr\,d\theta\,dz$$

$$= 5\int_0^{2\pi}\left[\int_{r=0}^{3} e^{r^2}\cdot r\,dr\right]d\theta$$

Put $r^2 = t$

$2r\,dr = dt$

$r\,dr = \frac{dt}{2}$

$t: 0 \to 9$

$$= \frac{5}{2}\int_0^{2\pi}\left[\int_0^9 e^t\,dt\right]d\theta$$

$$= \frac{5}{2}(2\pi)(e^9-1) = 5\pi(e^9-1).$$ Ans

(2) Equation of a cylinder in Cylindrical coordinates

$$x^2+y^2=16 \Rightarrow r^2=16 \Rightarrow \boxed{r=4}$$

(3) Eqn of a Cone in Cylindrical coordinates

$$z^2=x^2+y^2 \Rightarrow z^2=r^2 \Rightarrow \boxed{z=r}$$

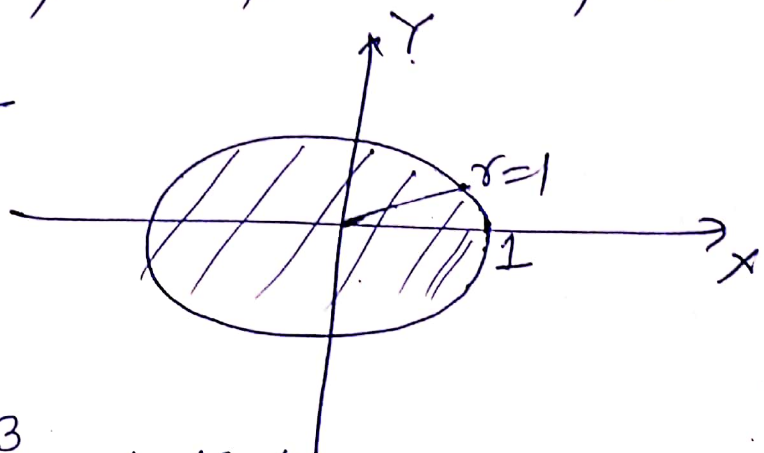
Prob ④ By using changing from rectangular to Cylindrical coordinates, Evaluate the triple integral

$$\int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}}\int_{x^2+y^2}^{2-x^2-y^2}(x^2+y^2)^{3/2}\,dz\,dy\,dx.$$

Sol^n:- $x^2+y^2=2-x^2-y^2 \Rightarrow 2(x^2+y^2)=2 \Rightarrow x^2+y^2=1$

$z=x^2+y^2 \Rightarrow z=r^2$

$z=2-x^2-y^2 \Rightarrow z=2-r^2$



$r: 0 \to 1$

$\theta: 0 \to 2\pi$

and $z: r^2 \to 2-r^2$

$$\therefore I=\int_0^{2\pi}\int_0^1\int_{r^2}^{2-r^2} r^3.dz\,r\,dr\,d\theta$$

$$=\int_0^{2\pi}\int_0^1 r^4\left[\int_{r^2}^{2-r^2}dz\right]dr\,d\theta$$

$$=\int_0^{2\pi}\int_0^1 r^4(2-r^2-r^2)\,dr\,d\theta=2\int_0^{2\pi}\int_0^1(r^4-r^6)\,dr\,d\theta$$

$$=2\times 2\pi\times\left[\frac{r^5}{5}-\frac{r^7}{7}\right]_0^1=4\pi\left[\frac{1}{5}-\frac{1}{7}\right]=\frac{4\pi(7-5)}{35}$$

$$=\frac{8\pi}{35}.$$ Ans

⑤ Use a triple integral to find the volume of the solid Q bounded by the graph of $y = 4 - x^2 - z^2$ and the xz-plane.

Sol^n:- $y = 4 - x^2 - z^2 \Rightarrow z = \pm\sqrt{4 - x^2 - y}$

$\therefore z : -\sqrt{4-x^2-y} \to \sqrt{4-x^2-y}$

with $z = 0$, $y = 4 - x^2$

i.e. $y : 0 \to 4 - x^2$ with $y = 0 \Rightarrow x^2 = 4$, $x = \pm 2$

$\therefore x : -2 \to 2$

Required volume $V = \iiint_Q dV = \int_{-2}^{2}\int_{0}^{4-x^2}\int_{-\sqrt{4-x^2-y}}^{\sqrt{4-x^2-y}} dz\,dy\,dx$

$= 2\int_{-2}^{2}\int_{0}^{4-x^2} \sqrt{4-x^2-y}\,dy\,dx = -2 \times \frac{2}{3}\int_{-2}^{2} \left.(4-x^2-y)^{3/2}\right|_0^{4-x^2} dx$

$= -\frac{4}{3}\int_{-2}^{2} -(4-x^2)^{3/2} dx = \frac{4}{3}\int_{-2}^{2} (4-x^2)^{3/2} dx = 8\pi$

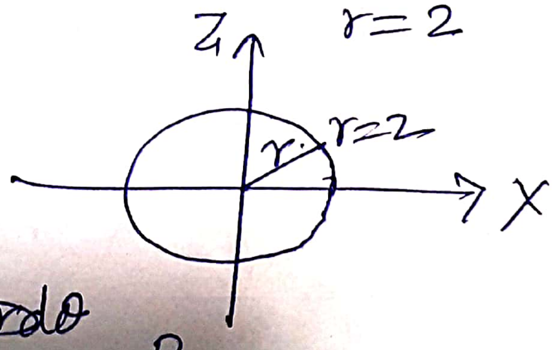
2nd method $\quad y = 4 - x^2 - z^2$ with $y = 0 \Rightarrow x^2 + z^2 = 4$, $r^2 = 4$, $r = 2$

we take $x = r\cos\theta$, $z = r\sin\theta$

then $y = 4 - r^2$

$r : 0 \to 2$

$\theta : 0 \to 2\pi$



$\cancel{V = \int_0^{2\pi}\int_0^{2}\int_0^{4-x^2-z^2} dz\, r\,dr\,d\theta}$

$V = \int_0^{2\pi}\int_0^{2} y\, r\,dr\,d\theta = \int_0^{2\pi}\int_0^{2} (4 - r^2)\, r\,dr\,d\theta$

$= 8\pi$