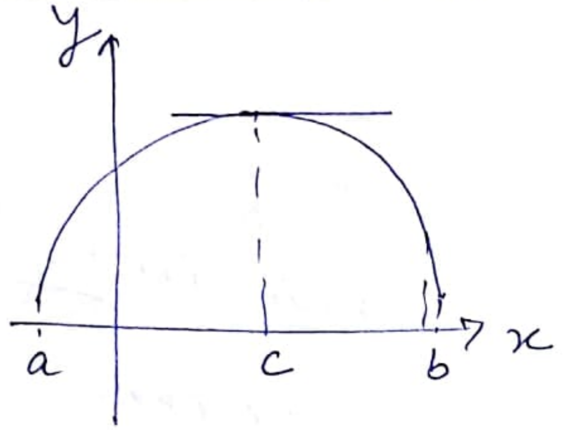
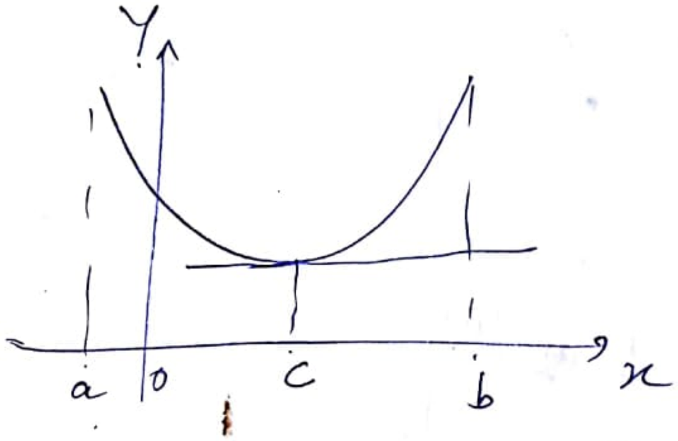
Module – II

Lecture 7

Rolle's Theorem:- If f is continuous on the interval $[a,b]$, differentiable on the interval (a,b) and $f(a)=f(b)$. Then there exist a number $c\in(a,b)$ such that $f'(c)=0$.

Geometrical interpretation





Problem:- Find a value of c satisfying the conclusion of Rolle's theorem for
$f(x)=x^3-3x^2+2x+2$ on the interval $[0,1]$.

Sol^n:- $\because f(x)$ is polynomial and all polynomials are continuous and differentiable everywhere.

Hence $f(x)$ is cts on $[0,1]$
& $f(x)$ is diff^n on $(0,1)$.

Next, $f(0)=2$, $f(1)=1-3+2+2=2$

i.e. $f(0)=f(1)$.

Now, $f'(x)=3x^2-6x+2$

By Rolle's thm, $f'(c)=0$ where $c\in(0,1)$

i.e. $3c^2-6c+2=0 \Rightarrow c=1\pm\frac{\sqrt{3}}{3}$

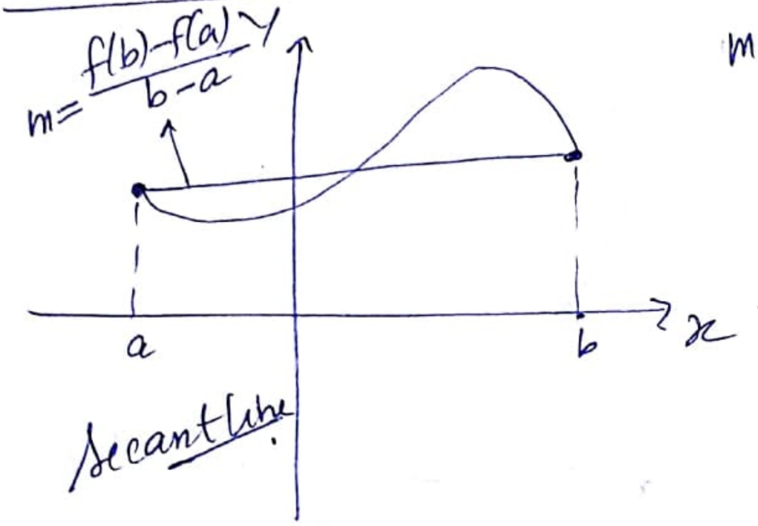
$\times$ $c=1.5774$ or $\boxed{c=0.42265}$ Ans

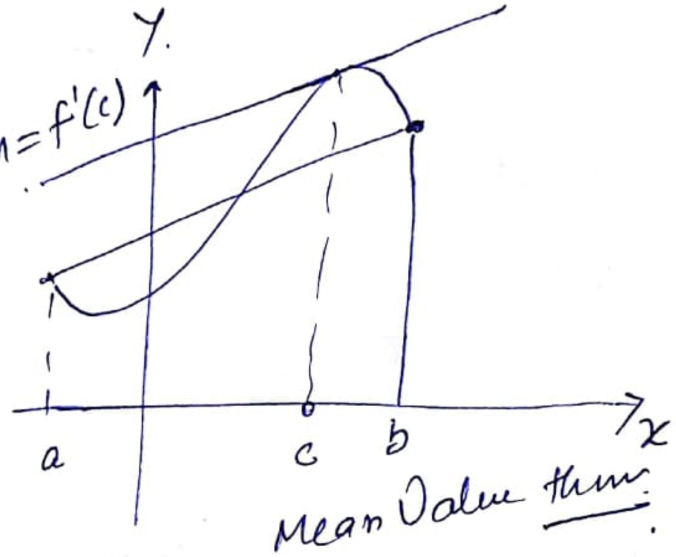
Scanned by CamScanner

Mean Value Theorem If f is continuous on the interval $[a,b]$ and differentiable on (a,b). Then there exist a number $c \in (a,b)$ such that

$$f'(c) = \frac{f(b)-f(a)}{b-a}.$$

Geometrical interpretation





Problem: Find the value of c satisfying the conclusion of the Mean value theorem for $f(x) = x^3 - x^2 - x + 1$ on the interval $[0,2]$.

Soln: f is cts and diffn obviously

By MV Theorem, $\exists\ c \in [0,2]$ for which

$$f'(c) = \frac{f(2)-f(0)}{2-0} = \frac{3-1}{2} = 1 \quad \text{--- (1)}$$

To find c:

$$f'(x) = 3x^2-2x-1$$

$$f'(c) = 3c^2-2c-1$$

from (1); $3c^2-2c-1 = 1 \Rightarrow 3c^2-2c-2c=0$

$$c = \frac{1 \pm \sqrt{7}}{3} = \frac{1+\sqrt{7}}{3} \in [0,2]$$

$$= \frac{1-\sqrt{7}}{3} \notin [0,2]$$

Hence, $\boxed{c = \frac{1+\sqrt{7}}{3}}$.