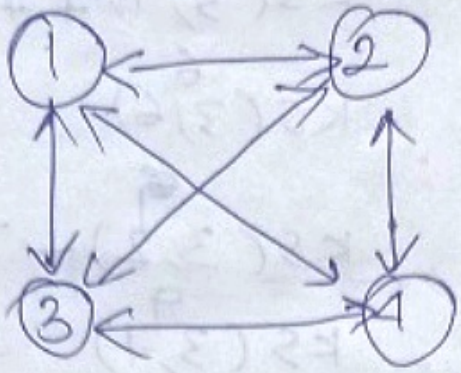
Travelling Salesman Problem



	1	2	3	4
1	0	6	1	3
2	4	0	2	1
3	1	2	0	8
4	3	1	7	0

1 2 3 4 1

1 3 2 4 1

Hamiltonian cycle.

(Start & ending ~~are~~ is same with other vertices visit atmost one in the given graph)

Bruteforce method.

1 2 3 4 1

1 3 2 4 1

3!

n = 4

(4-1)!

3!

n = 5

(5-1)

4!

(n-1)! = O(n!) = O(n^n)

Dynamic programing: Optimal substructure

Tsp (1, {2, 3, 4})

$$\min \begin{cases} = C(1,2) + TSP(2, \{3,4\}) \\ = C(1,3) + TSP(3, \{2,4\}) \\ = C(1,4) + TSP(4, \{2,3\}) \end{cases}$$

(7)

$T(1,\{2,3,4\})$

$(1,2)+T(2,\{3,4\}) = 15$ (6, 9) $\quad (1,3)+T(3,\{2,4\}) = 7$ (1, 6) $\quad (1,4)+T(4,\{2,3\}) = 7$ (3, 4)

$(2,3)+T(3,\{4\})$ (2, 11) $\quad (2,4)+T(4,\{3\})$ (1, 8) $\quad (3,2)+T(2,\{4\})$ (2, 4) $\quad (3,4)+T(4,\{2\})$ (8, 5) $\quad (4,2)+T(2,\{3\})$ (1, 3) $\quad (4,3)+T(3,\{2\})$ (7, 6)

$(3,4)+T(4,\{\phi\})$ (8, 3) $\quad (4,3)+T(3,\{\phi\})$ (7, 1) $\quad (2,4)+T(4,\{\phi\})$ (1, 8) $\quad (4,2)+T(2,\{\phi\})$ (1, 4) $\quad (2,3)+T(3,\{\phi\})$ (2, 1) $\quad (3,2)+T(2,\{\phi\})$ (2, 1)

$(4,1)=3 \quad (3,1)=1 \quad (4,1)=3 \quad (2,1)=4 \quad (3,1)=1 \quad (2,1)=4$

$TSP(i,S) = \min_{k \in S} C(i,k) + TSP(k, S-\{k\})$

$C(i,\perp) \quad S=\phi$

$1 \to 3 \to 2 \to 4 \to 1 = 7$

$1 \to 4 \to 2 \to 3 \to 1 = 7$

$(n-1) \times 2^{n-2}$

$\Rightarrow$ recursive relation

$n=4$

$3 \times 2^2 = 12$

$= O(n \cdot 2^n) < O(n^n)$

DPA BFSA + total distinct fn call