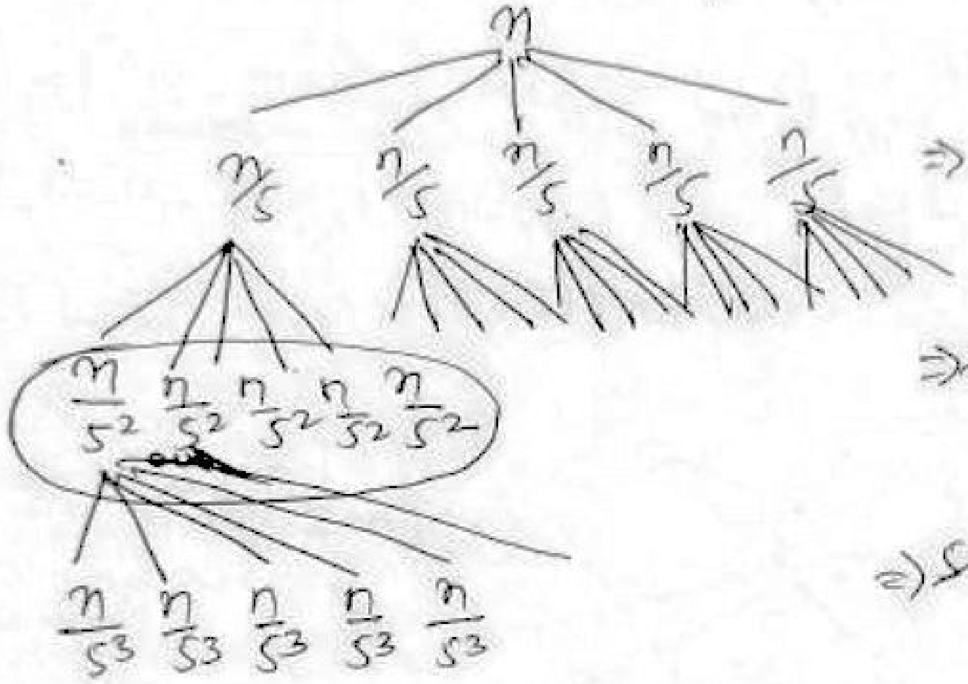
Recursive Tree method

$$T(n) = \begin{cases} 1 & \text{if } n = 1 \\ 5T(n/5) + cn \end{cases}$$

5 ⇓ No of subproblem

$n/5$ ⇓ Size of each sub problem



$\Rightarrow cn$

$\Rightarrow \cancel{5} \times \dfrac{c \cdot n}{\cancel{5}} \Rightarrow cn$

$\Rightarrow \cancel{5 \times 5}\, \dfrac{c \cdot n}{\cancel{5^2}} \Rightarrow c \cdot n$

$\underbrace{cn + cn + cn + \cdots + cn}$

total term = height of tree

$\left(\dfrac{n}{5^k}\right) = 1 \Rightarrow \boxed{k = \log_5 n}$

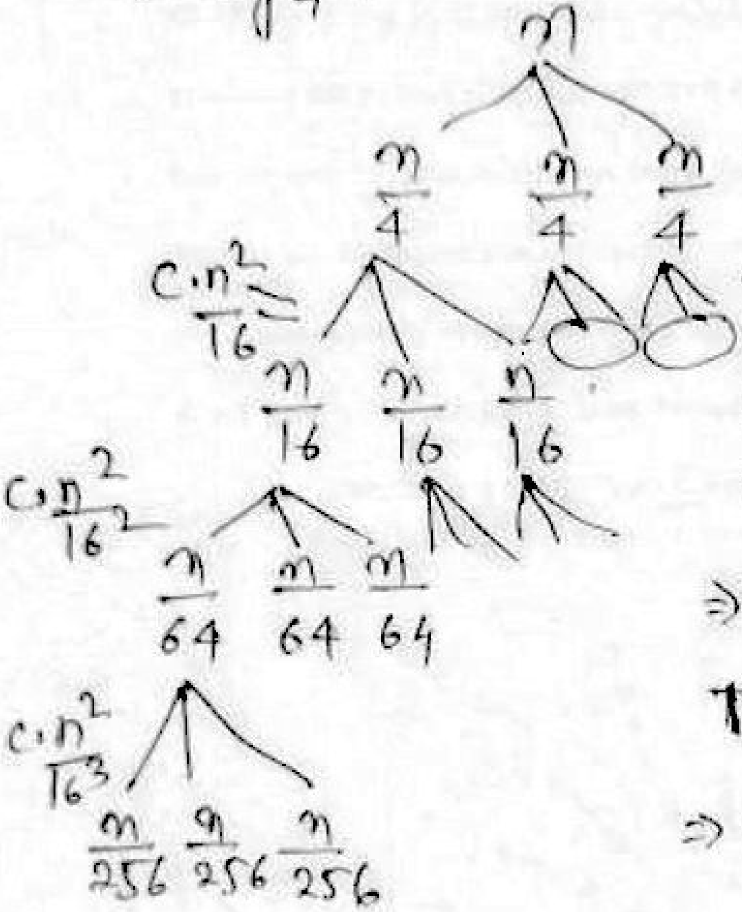
$k \cdot cn$

$= n \log_5 n.$

Recursive tree Method

$$T(n) = \begin{cases} 1 & \text{if } n = 1 \\ 3T\left(\left\lfloor \frac{n}{4} \right\rfloor\right) + c \cdot n^2 \end{cases}$$

$k = \log_4 n$



$\Rightarrow c \cdot n^2$

$T(n/4) = 3T\left(\frac{n}{16}\right) + \frac{c \cdot n^2}{16}$

$\Rightarrow c \cdot \frac{3}{16} n^2$

$T(n/16) = 3T\left(\frac{n}{64}\right) + \frac{c \cdot n^2}{16^2}$

$\Rightarrow c \cdot \frac{9}{16^2} n^2$

$T(n/64) = 3T\left(\frac{n}{256}\right) + \frac{c \cdot n^2}{16^3}$

$\Rightarrow c \cdot \frac{27}{16^3} n^2$

$$= cn^2 + c \cdot \frac{3}{16} n^2 + c \cdot \frac{9}{16^2} n^2 + c \cdot \frac{27}{16^3} n^2 + \cdots$$

$$= cn^2 \left[1 + \frac{3}{16} + \left(\frac{3}{16}\right)^2 + \left(\frac{3}{16}\right)^3 + \cdots \right]$$

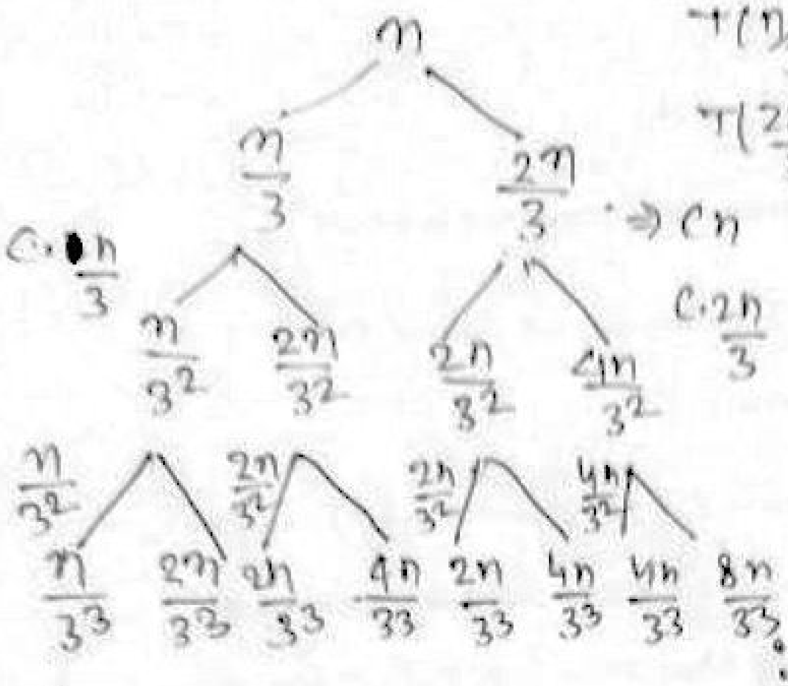
$$\leq c \cdot n^2 \left[\frac{1}{1 - \frac{3}{16}} \right] = c \cdot n^2 \frac{16}{13} = O(n^2)$$

Recursive tree method

$$T(n) = T(n/3) + T\left(\frac{2n}{3}\right) + cn$$

$$T(n/3) = T\left(\frac{n}{3^2}\right) + T\left(\frac{2n}{3^2}\right) + \frac{c \cdot n}{3}$$

$$T\left(\frac{2n}{3}\right) = T\left(\frac{2n}{3^2}\right) + \left(\frac{4n}{3^2}\right) + \frac{c \cdot 2n}{3}$$



$c \cdot \frac{n}{3}$ $\quad \frac{2n}{3} \Rightarrow cn$

$$\frac{c \cdot 2n}{3} \Rightarrow \frac{cn}{3} + \frac{c \cdot 2n}{3} \Rightarrow c \cdot n$$

$$\frac{n}{3^2} + \frac{2n}{3^2} + \frac{2n}{3^2} + \frac{4n}{3^2} = \frac{9n}{3^2}$$

$c \cdot n$

$$T\left(\frac{n}{3^2}\right) = T\left(\frac{n}{3^3}\right) + T\left(\frac{2n}{3^3}\right) + \frac{cn}{3^2}$$

$$T\left(\frac{2n}{3^2}\right) = T\left(\frac{2n}{3^3}\right) + T\left(\frac{4n}{3^3}\right) + \frac{c \cdot 2n}{3^2}$$

$$\left(\frac{n}{3^K}\right) = 1$$

$K = \log_3 n$

$$n \Rightarrow \frac{n}{(3/2)} \Rightarrow \frac{n}{(3/2)^2} \Rightarrow \frac{n}{(3/2)^3} \cdots (1)$$

$$\frac{n}{(3/2)^k} = (1)$$

$k = \log_{3/2} n$

After $\left(\frac{n}{3^k}\right)$ this $\leq c \cdot n$

$$= c \cdot n \cdot \log_{3/2} n$$

$$= O(n \cdot \log_{3/2} n)$$