

Finding Maximum and Minimum element using Divide and Conquer

Iterative method

Find max_min(A, n)

{ max = min = A[0];

for (i = 1; i < n; i++) $\Rightarrow n$.

{ if (max < A[i])

max = A[i]

elseif (min > A[i])

min = A[i]

}

return (max, min);

}

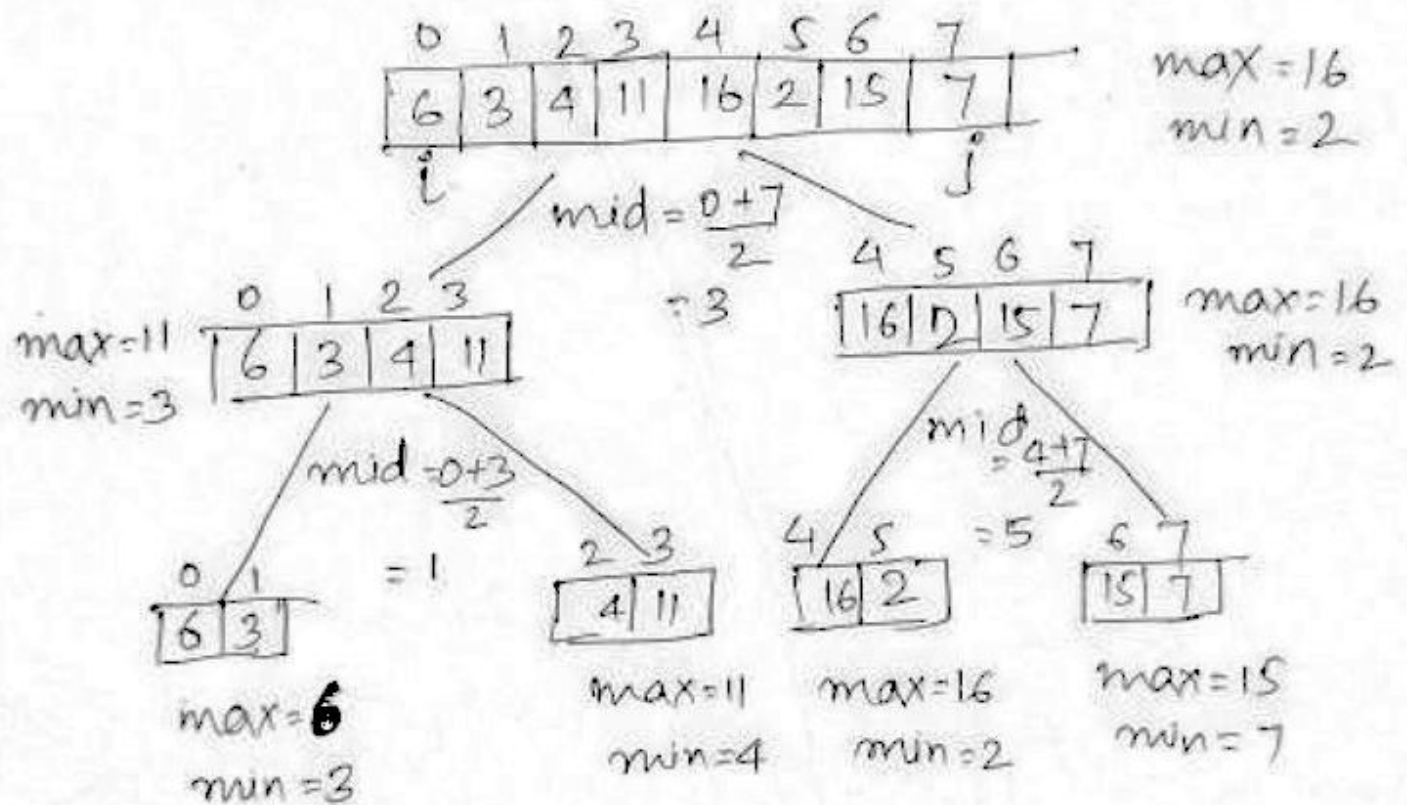
$$T(n) = O(n)$$

A	6	3	4	11	16	2	15	7	8
	0	1	2	3	4	5	6	7	8
	i	i	i	i	i	i	i	i	i

$\max = 8 \neq 16$

$\min = 8 \neq 2$

Divide and Conquer Approach



$$\text{mid} = \frac{i+j}{2}$$

Small $n=2$

DAC max_min(A, i, j, max, min)

{ int mid

if (i == j)

max = min = A[i];

elseif (i == j - 1)

{ if (A[i] < A[j])

max = A[j]; min = A[j];

else

max = A[i]; min = A[j];

} else {

med = $\frac{i+j}{2}$;

DAC max_min(A, i, med, max, min)

DAC max_min(A, med+1, max, min)

if (max₁ < max₂)

max = max₂;

else max = max₁;

if (min₁ < min₂)

min = min₁;

else

min = min₂;

$$T(n) = \begin{cases} 0 & n=1 \\ 1 & n=2 \\ T\left(\frac{n}{2}\right) + T\left(\frac{n}{2}\right) + 2 & n > 2 \end{cases}$$

$$T(n) = 2T\left(\frac{n}{2}\right) + 2$$

$$= 2[2T\left(\frac{n}{2^2}\right) + 2] + 2$$

$$= 2^2 T\left(\frac{n}{2^2}\right) + 2^2 + 2$$

$$= 2^2 [2T\left(\frac{n}{2^3}\right) + 2] + 2^2 + 2$$

$$= 2^3 T\left(\frac{n}{2^3}\right) + 2^3 + 2^2 + 2$$

$$= 2^k T\left(\frac{n}{2^k}\right) + 2^k + 2^{k-1} + \dots + 2^2 + 2$$

$$= 2^k \times 1 + (2^k + 2^{k-1} + \dots + 2^2 + 2^1)$$

$$= 2^k + \frac{2(2^k - 1)}{2 - 1}$$

$$= 2^k + 2^{k+1} - 2$$

$$= \frac{n}{2} + n - 2$$

$$= \frac{3n}{2} - 2 = \boxed{1.5n - 2} = O(n)$$

$$\boxed{\begin{array}{l} \frac{n}{2^k} = 2 \Rightarrow \frac{n}{2} = 2^k \\ 2^{k+1} = n \end{array}}$$