BMA 101 (Multivariable Calculus)

Lecture 1

Module 1

Sequence :- An infinite sequence (or sequence) of numbers is a function whose domain is the set of integers greater than or equal to some integers n_0.

$\{f_n\} : N \longrightarrow R$ is called sequence of real number.

Examples:-

$$a(n) = \sqrt{n}$$
$$a(n) = (-1)^{n+1} \frac{1}{n}$$

where $a(n)$ is the nth term of sequence

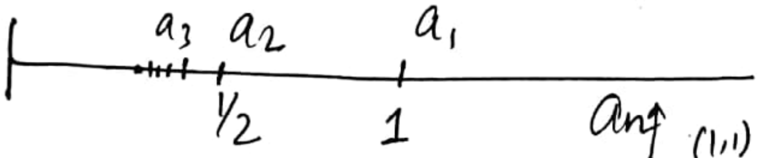
Example ① $1, \sqrt{2}, \sqrt{3}, \sqrt{4}, \dots, \sqrt{n} \dots$

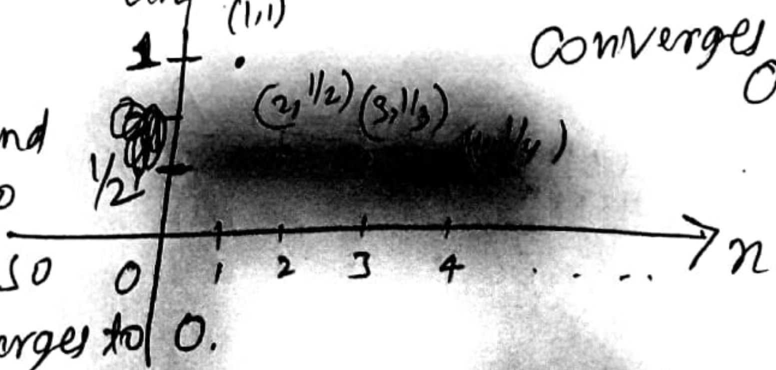
we write as sequence $a_n = \sqrt{n}$

Convergence of sequence (Graphically)

$a_n = 1/n$ sequence $\{\frac{1}{n}\}$

$\{\sqrt{n}\}$ diverges





converges to 0.

The terms $a_n = 1/n$ decreases steadily and get arbitrary close to 0 as n increases, so the sequence $\{a_n\}$ converges to 0.

Problem:- By graphically show that $(-1)^{n+1}\left(\frac{n-1}{n}\right)$

Non-decreasing:- A sequence $\{a_n\}$ with the prop that $a_n \leq a_{n+1}$ $\forall n$ is called a nondecreasing sequence.

eg:- (a) $1, 2, 3, \dots, n, \dots$

(b) $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \dots, \frac{n}{n+1}, \dots$

(c) The constant sequence $\{3\}$.

Non-increasing:- A sequence $\{a_n\}$ with the property that $a_n \geq a_{n+1}$ $\forall n$, is called a non-increasing sequence.

Limit of Sequences:- Let $\{a_n\}$ and $\{b_n\}$ be sequences of real numbers and let A and B be real numbers. The following rules hold if $\lim_{n\to\infty} a_n = A$ and $\lim_{n\to\infty} b_n = B$.

1. $\lim_{n\to\infty}(a_n \pm b_n) = A \pm B$
2. $\lim_{n\to\infty}(a_n \cdot b_n) = A \cdot B$
3. $\lim_{n\to\infty}(K \cdot b_n) = K \cdot B$ (Any number K)
4. $\lim_{n\to\infty} \frac{a_n}{b_n} = \frac{A}{B}$ if $B \neq 0$

ples

$\lim_{n\to\infty}\left(-\frac{1}{n}\right) = -1 \cdot \lim_{n\to\infty}\left(\frac{1}{n}\right) = -1\times 0 = 0$

$\lim_{n\to\infty}\left(\frac{n-1}{n}\right) = \lim_{n\to\infty}\left(1-\frac{1}{n}\right) = \lim_{n\to\infty} 1 - \lim_{n\to\infty}\frac{1}{n}$
$= 1-0 = 1$

$\lim_{n\to\infty}\frac{5}{n^2} = 5\lim_{n\to\infty}\frac{1}{n^2} = 5\times 0 = 0$

$\lim_{n\to\infty}\frac{4-7n^6}{n^6+3} = \lim_{n\to\infty}\left(\frac{\frac{4}{n^6}-7}{1+3/n^6}\right) = \frac{0-7}{1+0} = -7$

Sandwich theorem :- Let $\{a_n\}$, $\{b_n\}$, and $\{c_n\}$ be sequences of real numbers. If $a_n \le b_n \le c_n$ hold $\forall n$ and if $\lim_{n\to\infty} a_n = \lim_{n\to\infty} c_n = L$, then $\lim_{n\to\infty} b_n = L$ also

eg:- ② (i) $\frac{\cos n}{n} \to 0$. because
$\left|\frac{\cos n}{n}\right| = \frac{|\cos n|}{n} \le \frac{1}{n}$

(ii) $\frac{1}{2^n} \to 0$ as $\frac{1}{2^n} \le \frac{1}{n}$

(iii) $(-1)^n\frac{1}{n} \to 0$ as $\left|(-1)^n\frac{1}{n}\right| \le \frac{1}{n}$

Theorem:- If $a_n \to L$ and if f is continuous at L, then $f(a_n) \to f(L)$

Scanned by CamScanner

Problem:- Show that $\sqrt{\frac{n+1}{n}} \to 1$

Solⁿ, Solⁿ:- ~~Taking $f(x) = \sqrt{x}$~~ Taking $f(x) = \sqrt{}$

$$\lim_{n\to\infty} \sqrt{\frac{n+1}{n}} = \lim_{n\to\infty} \sqrt{1+\frac{1}{n}}$$

Now, $\lim_{n\to\infty} \left(\frac{n+1}{n}\right) = \lim_{n\to\infty}\left(1+\frac{1}{n}\right) = 1+0 = 1$

Hence, $\lim_{n\to\infty} \sqrt{\frac{n+1}{n}} = \sqrt{1} = 1.$

Problem:- Find the limit of following ~~function~~ sequence

(i) $\{2^{1/n}\}$ (ii) $\left\{\frac{\ln n}{n}\right\}$ (iii) $\left\{\frac{2^n}{5n}\right\}$

Formula:- (i) $\lim_{n\to\infty} \frac{\ln n}{n} = 0$

(ii) $\lim_{n\to\infty} \sqrt[n]{n} = 1$

(iii) $\lim_{n\to\infty} x^{1/n} = 1$ $(x>0)$

(iv) $\lim_{n\to\infty} x^n = 0$ $(|x|<1)$

(v) $\lim_{n\to\infty} \left(1+\frac{x}{n}\right)^n = e^x$

(vi) $\lim_{n\to\infty} \frac{x^n}{n!} = 0$

Problem:- Does the sequence whose nth term is $a_n = \left(\frac{n+1}{n-1}\right)^n$ converges? If so, find $\lim_{n\to\infty} a_n$.

Solⁿ:- 1^∞ form $\log a_n = \log\left(\frac{n+1}{n-1}\right)^n = n\log\left(\frac{n+1}{n-1}\right)$

$$\log a_n = \frac{\log\left(\frac{n+1}{n-1}\right)}{1/n} = \lim_{n\to\infty} \frac{-2/(n^2-1)}{-1/n^2} = \lim_{n\to\infty} \frac{2n^2}{n^2-1} = 2$$

Hence $a_n = e^2$ Yes.

$\{a_n\}$ converges to. e^2.

Prem:- Find the limit of following n^{th} term of sequence

1) $\dfrac{\ln(n^2)}{n}$

b) $\sqrt[n]{n^2}$

c) $\sqrt[n]{3n}$

d) $\left(-\frac{1}{2}\right)^n$

e) $\left(\frac{n-2}{n}\right)^n$

(f) $\dfrac{100^n}{n!}$

Infinite series Given a sequence of numbers $\{a_n\}$, an expression of the form

$$a_1 + a_2 + a_3 + \cdots + a_n + \cdots$$

is an infinite series. The number a_n is the n^{th} term of the series

Sequence of Partial sum:- The sequence $\{S_n\}$ defined by

$$S_1 = a_1$$
$$S_2 = a_1 + a_2$$
$$\vdots$$
$$S_n = a_1 + a_2 + \cdots + a_n = \sum_{k=1}^{n} a_k$$

[illegible] series, is the sequence of partia[illegible] n^{th} partial sum. the number S_n being the

If the sequence of partial sums converges to a limit L, we say that the series converges and that its sum is L. In this case, we also define

$$a_1 + a_2 + \cdots + a_n + \cdots = \sum_{n=1}^{\infty} a_n = L.$$