Surface integral

Flux:- The flux of a vector field $\vec{V} = V_1\hat{i} + V_2\hat{j} + V_3\hat{k}$ through surface S is given by

$$\text{flux} = \iint_S (\vec{V} \cdot \hat{n})\, dA$$

where $\hat{n}$ is a unit vector normal to the surface.

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37. Flux of $\vec{F}$ through S $= \iint_S \vec{F} \cdot n\, dA$

$\vec{F} = 6z\hat{i} + 6\hat{j} + 3y\hat{k}$; where S is the portion of the plane $2x + 3y + 4z = 12$, which is in the first octant

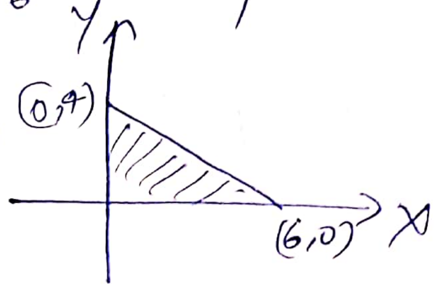
let $f(x,y,z) = 2x + 3y + 4z - 12 = 0$ be the surface.

Then grad $f = \nabla f = 2\hat{i} + 3\hat{j} + 4\hat{k}$,

normal unit vector to the surface

$$\hat{n} = \frac{\text{grad} f}{|\text{grad} f|} = \frac{2\hat{i} + 3\hat{j} + 4\hat{k}}{\sqrt{4+9+16}} = \frac{1}{\sqrt{29}}(2\hat{i} + 3\hat{j} + 4\hat{k}),$$

Now, $dA = \frac{dx\,dy}{\hat{n} \cdot \hat{k}}$ {projection of S on the xy plane}



$$dA = \frac{dx\,dy}{4/\sqrt{29}} = \frac{\sqrt{29}}{4}\, dx\,dy$$

Therefore, $\iint_S \vec{F}\cdot\hat{n}\,dA = \iint_S \frac{(12z+18+12y)}{\sqrt{29}}\,dA$ —— (1)

For $2x+3y+4z=12$

$\Rightarrow 4z = 12-2x-3y$

For $z=0$; $12-2x-3y=0 \Rightarrow 3y = 12-2x$

$y = \frac{12-2x}{3}$

For $y=0$; $12-2x=0 \Rightarrow x=6$

Limits are :- $x: 0 \to 6$

$y: 0 \to \frac{12-2x}{3}$

From (1);

$\iint_S \vec{F}\cdot\hat{n}\,dA = \frac{1}{4}\iint_R (54-6x+3y)\,dx\,dy$ [Put values of z]

$= \frac{1}{4}\int_0^6\int_0^{\frac{12-2x}{3}} (54-6x+3y)\,dy\,dx$

$= \frac{1}{6}\int_0^6 (360-102x+7x^2)\,dx = 138.$

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