

Total Differential:- If ~~$f(x, y)$ is a function~~

If $z = f(x, y)$ is a function of two variables x and y , then we define total differential as

$$dz = \frac{\partial f}{\partial x} dx + \frac{\partial f}{\partial y} dy.$$

Prob (1) Determine the total differential of the function (i) $f(x, y) = 5x^2y^3$ (ii) $f(x, y) = y e^x + \sin x$
(iii) $f(x, y, z) = 3x^3 - 2y^4 + 5z$ (iv) $f(x, y, z) = x e^{x^2y/z} - z \sqrt{\ln(x+y)}$

Solution (i) let $z = f(x, y) = 5x^2y^3$

Then total differential is

$$dz = \frac{\partial f}{\partial x} dx + \frac{\partial f}{\partial y} dy = 10xy^3 dx + 15x^2y^2 dy \quad \underline{\text{Ans}}$$

(ii) $z = f(x, y) = y e^x + \sin x$

Then total differential is

$$dz = \frac{\partial f}{\partial x} dx + \frac{\partial f}{\partial y} dy = (y e^x + \cos x) dx + e^x dy \quad \underline{\text{Ans}}$$

(iii) Assume that $w = f(x, y, z) = 3x^3 - 2y^4 + 5z$

Then total differential is

$$\begin{aligned} dw &= \frac{\partial f}{\partial x} dx + \frac{\partial f}{\partial y} dy + \frac{\partial f}{\partial z} dz \\ &= 9x^2 dx - 8y^3 dy + 5dz \quad \underline{\text{Ans}} \end{aligned}$$

(iv) let $w = f(x, y, z) = x e^{x^2y/z} - z \sqrt{\ln(x+y)}$

Then total differential is

$$dw = \frac{\partial f}{\partial x} dx + \frac{\partial f}{\partial y} dy + \frac{\partial f}{\partial z} dz \quad \text{--- (1)}$$

$$\begin{aligned} \frac{\partial f}{\partial x} &= e^{x^2y/z} + x e^{x^2y/z} \times \frac{2xy}{z} - \frac{z}{2} \frac{x}{\sqrt{\ln(x+y)}} \cdot \frac{1}{x+y} \\ &= (1+x) e^{x^2y/z} + \frac{2xy}{z} - \frac{z}{2(x+y)\sqrt{\ln(x+y)}} \end{aligned}$$

$$\frac{\partial f}{\partial y} = x e^{x^2 y / z} \times \frac{x^2}{z} - \frac{z}{2 \sqrt{\ln(x+y)}} \cdot \frac{1}{x+y} = \frac{x}{z} e^{x^2 y / z} - \frac{1}{2(x+y) \sqrt{\ln(x+y)}}$$

$$\frac{\partial f}{\partial z} = x e^{x^2 y / z} \times \left(-\frac{x^2 y}{z^2}\right) - \sqrt{\ln(x+y)} = -\frac{x^3 y}{z^2} e^{x^2 y / z} - \sqrt{\ln(x+y)}$$

Substituting all values in eqⁿ (i); we get

$$dw =$$

Problem:- Find the total differential of the following functions at indicated point

(i) $f(x, y) = x^3 y^4$; (1, 1)

(ii) $f(x, y, z) = \sqrt{x^2 + y^2 + z^2}$; (1, 0, 0)

Solⁿ (i) Total differential of $f(x, y)$ is

$$df = \frac{\partial f}{\partial x} dx + \frac{\partial f}{\partial y} dy$$

$$df = 3x^2 y^4 dx + 4x^3 y^3 dy$$

At (1, 1) $df = 3 dx + 4 dy$

(ii) Total differential of $f(x, y, z)$ is

$$df = \frac{\partial f}{\partial x} dx + \frac{\partial f}{\partial y} dy + \frac{\partial f}{\partial z} dz$$

$$df = \frac{1}{\sqrt{x^2 + y^2 + z^2}} (x dx + y dy + z dz)$$

At (1, 0, 0)

$$df = dx \quad \underline{\text{Ans}}$$