3-Unit

Q 10 $\int_0^{2\pi} \sin x \, dx$

$a = 0 \quad b = 2\pi \quad n = 8$

$h = \frac{b-a}{n} = \frac{2\pi}{8} = \frac{\pi}{4}$

$x_0 = 0$

$x_1 = \frac{2\pi}{4}$

$x_2 = 2\pi/4$

$x_3 = 3\pi/4$

$x_4 = 4\pi/4$

$x_5 = \frac{5\pi}{4}$

$x_6 = \frac{6\pi}{4}$

$x_7 = 7\pi/4$

$x_8 = \frac{8\pi}{4}$

$y_0 = \sin x_0 = 0$

$y_1 = 0.707106$

$y_2 = 1$

$y_3 = 0.707108$

$y_4 = 0$

$y_5 = -0.707106$

$y_6 = -1$

$y_7 = -0.707106$

$y_8 = 0$

$$\int_0^{2\pi} \sin x \, dx = h\left[\frac{y_0 + y_8}{2} + y_1 + y_2 + y_3 + y_4 + y_5 + y_6 + y_7\right]$$

$$\frac{\pi}{4}\left[0 + 0.707106 + \cancel{1} + 0.707106 + 0 - 0.707106 - \cancel{1} - 0.707106\right]$$

$$= \underline{\underline{0}}$$

(17) Given pts on the curve $f(x)$ are

$P(0,3)$ $Q(0.5,4)$ & $R(1,5)$

Let

x	0	0.5	1
y	3	4	5

Trapezoidal rule

$$\int_0^1 f(x)\,dx = h\left(\frac{y_0+y_n}{2} + y_1 + y_2 \cdots\right)$$

[diff b/w x value is 0.5 so $h = 0.5$

$$= 0.5\left(\frac{y_0+y_2}{2} + y_1\right)$$

$$= 0.5\left(\frac{0+1}{2}\right.$$

$$= 0.5\left[\left(\frac{3+5}{2}\right) + 4\right]$$

$$= 4$$

Simpsons $\frac{1}{3}$ rule

$$I = \frac{h}{3}\left[(y_0+y_n) + 4(y_1+y_3 \cdots) + 2(y_2+y_4 \cdots y_{n-1})\right]$$

$= \frac{0.5}{3}\left[(y_0+y_2) + 4y_1\right]$

$= \frac{0.5}{3}\left[(3+5) + 16\right] = 4$

$\therefore$ Difference in the result $= 4-4 = \underline{\underline{0}}$

(18) $\int_3^7 x^2 \log x \, dx$

$n=4 \quad a=3 \quad b=7$

$h = \frac{7-3}{4} = 1$

$x_0 = 3 \qquad y_0 = x_0^2 \log_e x_0 = 9.8875106$

$x_1 = 4 \qquad y_1 = 22.1807098$

$x_2 = 5 \qquad y_2 = 40.2359478$

$x_3 = 6 \qquad y_3 = 64.5033409$

$x_4 = 7 \qquad y_4 = 67.9284236$

Weddle's rule : $\frac{3h}{10}\left[y_0 + 5y_1 + y_2 + 6y_3 + y_4\right]$

$= \frac{3}{10}\left[9.8875106 + 110.903549 + 40.2359478 + 387.020045 + 67.9284236\right]$

$$\frac{3}{10}(615.975476)$$

$$= 184.792643$$

(20) $\int_0^1 \frac{dx}{4+3x}$ using simpson's 3/8 th.

$a = 0 \quad b = 1 \quad n = 6$ let

$$h = \frac{b-a}{n} = \frac{1}{6}$$

$x_0 = 0 \qquad y_0 = \frac{1}{4+3x_0} = \frac{1}{4} = 0.25$

$x_1 = 1/6 \qquad y_1 = \frac{1}{4+3x_1} = \frac{2}{9} = 0.222222$

$x_2 = 2/6 \qquad y_2 = 1/5 = 0.2$

$x_3 = 3/6 \qquad y_3 = \frac{2}{11} = 0.181818$

$x_4 = 4/6 \qquad y_4 = \frac{1}{6} = 0.166667$

$x_5 = 5/6 \qquad y_5 = 2/13 = 0.153846$

$x_6 = 1 \qquad y_6 = 1/7 = 0.142857$

$$\int_0^1 \frac{dx}{4+3x} = \frac{3h}{8}\left[(y_0+y_6) + 3(y_1+y_2+y_4+y_5) + 2(y_3 + y_6)\right]$$

$$= \frac{3}{8} \times \frac{1}{6}\left[(0.25+0.142857)+3(0.222222 + 0.2+0.166667+0.153846) + 2(0.181818)\right]$$

$$0.0625\left[0.392857 + 2.228205 + 0.363636\right]$$

$$= 0.186543625$$ Approximate value

Exact value

$$\int_0^1 \frac{dx}{4+3x}$$

let $4+3x = t$

$$3 = \frac{dt}{dx}, \quad dx = \frac{dt}{3}$$

$$\int_0^1 \frac{1}{t}\cdot\frac{1}{3}dt = \frac{1}{3}\int_0^1 \frac{1}{t}\,dt$$

$$\frac{1}{3}(\log t)_0^1 = \frac{1}{3}\log(1) + 0$$

$$\frac{1}{3}\left[(\log(4+3x))_0^1\right]$$

$$\frac{1}{3}\left[\log(7) - \log(4)\right] = \frac{1}{3}\left[0.84509804 - 0.6020599991\right]$$

$$= 0.08101 \simeq 0.1$$

both values are equal

Q 21

$a=-1 \quad b=1 \quad n=3$

$h = \frac{b-a}{n} = \frac{2}{3}$

$x_0 = -1$ $\qquad y_0 = |-1| = 1$

$x_1 = x_0 + h = -1 + \frac{2}{3} = \frac{-1}{3}$ $\qquad y_1 = \frac{1}{3}$

$x_2 = \frac{-1}{3} + \frac{2}{3} = \frac{1}{3}$ $\qquad y_2 = \frac{1}{3}$

$y_3 = 1$

$x_3 = 1$

Trapezoidal rule

$$\int_{-1}^{1} |x|\,dx = h\left[\frac{y_0+y_n}{2} + y_1 + y_2 + y_3 \cdots y_{n-1}\right]$$

$$= h\left[\frac{1+1}{2} + \frac{1}{3} + \frac{1}{3}\right]$$

$$= \frac{2}{3}\left[1 + \frac{2}{3}\right] = 1.11111$$

Q 2

$x_0 =$

x_1

x_2

x_3

x_4

x_5

Q 23

$$\int_{2.5}^{4} \ln(x)\, dx$$

$a = 2.5 \quad b = 4 \quad n = 5$

$$h = \frac{4-2.5}{5} = 0.3$$

$x_0 = 2.5 \quad y_0 = \ln(x_0) = 0.916290$

$x_1 = 2.8 \quad y_1 = 1.029619$

$x_2 = 3.1 \quad y_2 = 1.131402$

$x_3 = 3.4 \quad y_3 = 1.223775$

$x_4 = 3.7 \quad y_4 = 1.308332$

$x_5 = 4.0 \quad y_5 = 1.386294$

Trapezoidal rule: $h\left[\frac{y_0 + y_n}{2} + y_1 + y_2 + y_3 \cdots\right]$

$$= 0.3\left[\frac{0.916290 + 1.386294}{2} + 1.029619 + 1.131402 + 1.223775 + 1.308332\right]$$

$0.3\ (1.151292 + 4.693128)$

1.753326

(24) $\int_1^3 \frac{1}{x}\, dx$

$a = 1 \quad b = 3 \quad h = 1$ (given)

$x_0 = 1 \qquad y_0 = \frac{1}{x_0} = 1$

$x_1 = 2 \qquad y_1 = \frac{1}{2} = 0.5$

$x_2 = 3 \qquad y_2 = \cancel{0.666667}\ 0.333333$

$x_3 = 4 \qquad y_3 = \cancel{0.599998}\ 0.25$

$x_4 = 5 \qquad y_4 = \cancel{0.166667}\ 0.2$

$x_5 = 6 \qquad y_5 = \cancel{5.99998}\ 0.166667$

$x_6 = 7 \qquad y_6 = 0.142857$

trapezoidal rule

$$\int_{x_0}^{x_n} f(x)\, dx = \frac{h}{2}\left[(y_0 + y_n) + 2(y_1 + y_2 + \cdots)\right]$$

$$= \frac{1}{2}\left[(1 + 5.99998) + 2(0.5 + 0.66667)\right]$$

$$= 0.5\,[6.9999 + 2.33334]$$

$\frac{1}{2}\left[1+0.142857+2(0.5+0.333333+0.25+0.2+0.166667)\right]$

$\frac{1}{2}\left[1.142857+2.9\right]$

(24) $\int_1^3 \frac{1}{x}\,dx$

$a=1$ $b=3$ $h=1$ (given), $\frac{3-a}{h}=n$

$n=\underline{2}$

$x_0=1$ $y_0=1$

$x_1=2$ $y_1=\frac{1}{2}$

$x_2=3$ $y_3=\frac{1}{3}$

By trapezoidal rule

$$\int_1^3 \frac{1}{x}\,dx = \frac{h}{2}\left[y_0+y_n+2(y_1+y_2\cdots y_{n-1})\right]$$

$$=\frac{1}{2}\left[1+\frac{1}{3}+2\left(\frac{1}{2}\right)\right]$$

$$=\frac{1}{2}\cdot\frac{7}{3} = \underline{\underline{1.1667}}$$

(25)

$$\int_1^3 \frac{1}{x}\,dx$$

$a=1 \quad b=3 \quad n=2 \quad h=1$

$x_0 = 1 \qquad y_0 = \frac{1}{x_0} = 1$

$x_1 = 2 \qquad y_1 = \frac{1}{x_1} = \frac{1}{2} = 0.5$

$x_2 = 3 \qquad y_2 = \frac{1}{x_2} = \frac{1}{3} = 0.3333$

Simpson's 1/3 rule $= \frac{1}{3}h\left[y_0 + y_n + 4(y_1 + y_3 + \cdots y_{n-1}) + 2(y_2 + y_4 + \cdots y_{n-1})\right]$

$$= \frac{h}{3}\left[y_0 + y_2 + 4y_1\right]$$

$$= \frac{1}{3}\left[1 + 0.3333 + 2\right]$$

$$= \frac{3.3333}{3} = 1.1111$$

(26)

$$\int_0^{\pi/2} \sin x\,dx$$

$a=0 \quad b=\pi/2 \quad n=10$

$h = \frac{\pi}{20}$

$x_0 = 0$ $\quad y_0 = \sin 0 = 0$

$x_1 = \frac{\pi}{20}$ $\quad y_1 = \sin \pi/20 = 0.1564344$

$x_2 = 2\pi/20$ $\quad y_2 = \sin 2\pi/20 = 0.3090169$

$x_3 = 3\pi/20$ $\quad y_3 = \sin 3\pi/20 = 0.4539905$

$x_4 = 4\pi/20$ $\quad y_4 = 0.58778525$

$x_5 = 5\pi/20$ $\quad y_5 = 0.7071067$

$x_6 = 6\pi/20$ $\quad y_6 = 0.8090169$

$x_7 = 7\pi/20$ $\quad y_7 = 0.8910065$

$x_8 = 8\pi/20$ $\quad y_8 = 0.9510565$

$x_9 = 9\pi/20$ $\quad y_9 = 0.9876883$

$x_{10} = 10\pi/20$ $\quad y_{10} = 1$

Trapezoidal rule $= h\left(\frac{y_0 + y_n}{2} + y_1 + y_2 + \cdots y_{n-1}\right)$

$$= \frac{\pi}{20}\left(\frac{0+1}{2} + 0.1564344 + 0.3090169 + 0.4539905 + 0.5877852 + 0.7071067 + 0.8090169 + 0.8910065 + 0.9510565 + 0.9876883\right)$$

$= 0.9979429$

(31) using simpson's one third rule

$$\frac{h}{3}\left[y_0+y_n+4(y_1+y_3+\cdots y_{n-1})+2(y_2+y_4\cdots+y_{n-1})\right]$$

$h = 0.5$

$$= \frac{0.5}{3}\left[1.0+25.0+4(2.875+14.125)+2(7.0)\right]$$

$$\frac{0.5}{3}\left[1.0+25.0+68+14.0\right]$$

$$= \underline{18}$$

(32) Evaluate $I = \int_4^{5.2} \log x \, dx$

$h = 0.2$ (given)

$$n = \frac{5.2-4}{0.2} = 6$$

$x_0 = 4$

$x_1 = 10$

$x_2 = 16$

$x_3 = 22$

$x_4 = 28$

$x_0 = 4$ $\quad y_0 = \log x_0 = 1.386294$

$x_1 = 4.2$ $\quad y_1 = 1.435084$

$x_2 = 4.4$ $\quad y_2 = 1.481604$

$x_3 = 4.6$ $\quad y_3 = 1.526056$

$x_4 = 4.8$ $\quad y_4 = 1.568615$

$x_5 = 5.0$ $\quad y_5 = \cancel{1.410786}$ 1.609437

$x_6 = 5.2$ $\quad y_6 = \cancel{1.415853}$ 1.648658

Trapezoidal rule $= h\left(\frac{y_0 + y_n}{2} + y_1 + y_2 + y_3 + y_{n-1}\right)$

$$0.2\left[\frac{1.386294 + 1.648658}{2} + 1.435084 + 1.481604 + 1.526056 + 1.568615 + 1.609437\right]$$

$$0.2\,(1.517476 + 7.620796)$$

$$= \cancel{1.7646837} \quad 1.8276544$$

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$$I = \int_3^7 \log x \, dx$$

$n = 8 \quad a = 3 \quad b = 7$

$h = \dfrac{7-3}{8} = 0.5$

$x_0 = 3 \qquad y_0 = \log x_0 = 1.098612$

$x_1 = 3.5 \qquad y_1 = 1.252762$

$x_2 = 4.0 \qquad y_2 = 1.386294$

$x_3 = 4.5 \qquad y_3 = 1.504077$

$x_4 = 5 \qquad y_4 = 1.609437$

$x_5 = 5.5 \qquad y_5 = 1.704748$

$x_6 = 6 \qquad y_6 = 1.791759$

$x_7 = 6.5 \qquad y_7 = 1.871802$

$x_8 = 7 \qquad y_8 = 1.945910$

Trapezoidal rule : $h\left(\dfrac{y_0 + y_n}{2} + y_1 + y_2 \cdots y_{n-1}\right)$

$$= 0.5\left[\frac{1.098612 + 1.945910}{2} + 1.252762 + 1.386294 + 1.504077 + 1.609437 + 1.704748 + 1.791759 + 1.871802\right]$$

$0.5\ (1.522261 + 11.120879)$

$= 6.32157$

(34) $I = \int_0^6 \frac{1}{1+x^2}\, dx$ $\quad$ 3/8 rule

$a = 0 \quad b = 6 \qquad n = 6$ let

$h = \frac{b-a}{n} = 1$

$x_0 = 0 \qquad y_0 = 1$

$x_1 = 1 \qquad y_1 = 0.5$

$x_2 = 2 \qquad y_2 = 0.20$

$x_3 = 3 \qquad y_3 = 0.10$

$x_4 = 4 \qquad y_4 = 0.588235$

$x_5 = 5 \qquad y_5 = 0.0384615$

$x_6 = 6 \qquad y_6 = 0.0270270$

$$\int_0^6 \frac{1}{1+x^2} = \frac{3}{8}\Big(y_0 + y_n + 3(y_1 + y_2 + \cdots y_{n-1}) + 2(y_3 + y_6 + \cdots y_{n-3})\Big]$$

$$= \frac{3}{8}\Big[1 + 0.0270270 + 3(0.5 + 0.20 + 0.588235 + 0.0384615) + 2(0.10)\Big]$$

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+

2]

$\frac{3}{8}\left(1.027027 + 3.9800895 + 0.2\right)$

$= 1.952668$

(35) $f(x) = \int_0^{0.8} \frac{\sin x}{x}$

$a = 0 \quad b = 0.8 \quad n = 4$

$h = \frac{0.8}{4} = 0.2$

$x_0 = 0$	$y_0 = 0$
$x_1 = 0.2$	$y_1 = 0.0174532$
$x_2 = 0.4$	$y_2 = 0.0174531$
$x_3 = 0.6$	$y_3 = 0.0174529$
$x_4 = 0.8$	$y_4 = 0.8966951$

(a) Trapezoidal rule

$$\int_0^{0.8} \frac{\sin x}{x} = h\left[\frac{y_0 + y_n}{2} + y_1 + y_2 + \cdots y_{n-1}\right]$$

$$0.2\left[\frac{0.8966951}{2} + 0.0174532 + \right.$$

$$\left[0.0174531 + 0.0174529\right]$$

$0.2\left[0.44834755 + 0.0523592\right]$

$= 0.10014135$

(b) Simpson 1/3 rule

$$\int_0^{0.8} \frac{\sin x}{x} = \frac{h}{3}\left[y_0 + y_n + 4(y_1 + y_3 + \dots y_{n-1}) + 2(y_2 + y_4 + \dots y_{n-1})\right]$$

$$= \frac{0.2}{3}\left[0.8966951 + 4(0.0174532 + 0.0174529) + 2(0.0174531)\right]$$

$$= (0.8966951 + 0.1396244 + 0.0349062)\frac{0.2}{3}$$

$= 0.0714150 \simeq 0.1$

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$$\int_0^4 x^5$$

$a = 0 \quad b = 4$

case 1: $h = \frac{0+4}{2} = 2$

(37) $\int_0^1 \sqrt{\sin x + \cos x}\, dx$

$a=0 \quad b=1 \quad n=6$ [ordinate mean 1 less]

$h = \frac{1}{6}$

$x_0 = 0 \qquad y_0 = 1$

$x_1 = 1/6 \qquad y_1 = 1.05392978$

$x_2 = 2/6 \qquad y_2 = 1.076257$

$x_3 = 3/6 \qquad y_3 = 1.093377$

$x_4 = 4/6 \qquad y_4 = 1.107799$

$x_5 = 5/6 \qquad y_5 = 1.120492$

$x_6 = 1 \qquad y_6 = 1.131955$

using simpson's $\frac{1}{3}$ rule :

$= \frac{h}{3}\left[y_0 + y_n + 4(y_1 + y_3 + y_5) + 2(y_2 + y_4)\right]$

$= \frac{1}{18}\left[1 + 1.131955 + 4(1.05392978 + 1.093377 + 1.120492) + 2(1.076257 + 1.107799)\right]$

$= \frac{1}{18}\left[2.131955 + 13.0711951 + 4.368112\right]$

$= 1.08729 \simeq 1.1$

(38) $\int_{0.2}^{1.4} (\sin x - \log_e x + e^x)$

$a = 0.2 \quad b = 1.4 \quad n = 6 \quad h = 0.2$

x :	0.2	0.4	0.6	0.8	1.0	1.2	1.4
y :	3.0295	2.7975	2.8976	3.1660	3.5597	4.0698	4.4042

- By Simpson's 3/8 rule

$$\int_{0.2}^{1.4} y\,dx = \frac{3h}{8}\left[(y_0+y_6) + 2(y_3) + 3(y_1+y_2+y_4 + y_5)\right]$$

$$= \frac{3}{8}(0.2)\left[7.7336 + 2(3.1660) + 3(13.3247)\right]$$

$$= \underline{\underline{4.053}}$$

Boole's rule

$$\int_{0.2}^{1.4} y\,dx = \frac{2h}{45}\left[7(y_0+y_n) + 32(y_1+y_3+y_5+\cdots) + 12(y_2+y_6+y_{10}+\cdots) + 14(y_4+y_8+\cdots)\right]$$

$$= \frac{2\times 0.2}{45}\left[7(3.0295 + 4.4042) + 32(2.7975 + 3.1660 + 4.0698) + 12(2.8976) + 14(3.5597)\right]$$

$$\frac{0.4}{45}\left[52.0359 + 321.0656 + 34.7712 + 49.8358\right]$$

$$= \underline{\underline{4.06852}}$$

(39) $I = \int_0^1 \frac{x^2}{1+x^3}\,dx$, $\frac{1}{3}$ rule $h = 0.25$

$a = 0 \quad b = 1$

$n = \frac{1}{0.25} = 4$

$h = 0.25$

$x_0 = 0 \qquad y_0 = 0$

$x_1 = 0.25 \qquad y_1 = \frac{0.0625}{1.015625} = 0.061538$

$x_2 = 0.5 \qquad y_2 = \frac{0.25}{1.125} = 0.222222$

$x_3 = 0.75 \qquad y_3 = \frac{0.5625}{1.421875} = 0.395604$

$x_4 = 1 \qquad y_4 = 0.5$

$\frac{h}{3}\left[y_0 + y_4 + 4(y_1 + y_3) + 2(y_2)\right]$

$\frac{0.25}{3}\left[0 + 0.5 + 4(0.061538 + 0.395604) + 2(0.222222)\right]$

$\frac{0.25}{3}\left[0.5 + 1.828568 + 0.444444\right]$

$= 0.231084$

(38)

x :

y :

• By