Comput $\int_0^1 \int_0^{y^2} 3y^3 e^{xy}\, dx\, dy$

$\int_0^1$

(58) $f(x,y) = x + y + 2z$ on $x^2+y^2+z^2 = 3$. — (I)

$F = (x+y+2z) + \lambda(x^2+y^2+z^2-3)$

$\partial F = 0$

$$\Rightarrow \left.\begin{aligned} \frac{\partial F}{\partial x} &= 1 + \lambda(2x) = 0 \Rightarrow x = -\frac{1}{2\lambda} \\ \frac{\partial F}{\partial y} &= 1 + \lambda(2y) = 0 \Rightarrow y = -\frac{1}{2\lambda} \\ \frac{\partial F}{\partial z} &= 2 + \lambda(2z) = 0 \Rightarrow z = -\frac{1}{\lambda} \end{aligned}\right\} \text{(II)}$$

$\Rightarrow$ putting all values from (II) to (I) -

$$\left(\frac{-1}{2\lambda}\right)^2 + \left(\frac{-1}{2\lambda}\right)^2 + \left(\frac{-1}{\lambda}\right)^2 = 3$$

$$\Rightarrow \frac{1+1+4}{4\lambda^2} = 3 \Rightarrow \frac{6}{4\lambda^2} = 3$$

$$\Rightarrow \lambda^2 = \frac{6}{4\times 3} = \frac{1}{2} \Rightarrow \lambda = \frac{\pm 1}{\sqrt{2}}$$

So $\lambda = \frac{1}{\sqrt{2}}$ and $\frac{-1}{\sqrt{2}}$

$x = \frac{-1}{2}(\sqrt{2}) = -\frac{1}{\sqrt{2}}$

$\Rightarrow$ ~~$x = \frac{-1}{2\sqrt{2}}$, $y = \frac{-1}{2\sqrt{2}}$, $z =$~~

$\Rightarrow (x, y, z) = \left(\frac{-1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}, -\sqrt{2}\right)$

and $(x, y, z) = \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, \sqrt{2}\right)$

are two extrema of $f(x,y,z)$

and extreme values of f are

$$f_{\left(\frac{-1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}, -\sqrt{2}\right)} = \frac{-1}{\sqrt{2}} + \left(\frac{-1}{\sqrt{2}}\right) - 2\sqrt{2} = \frac{-2}{\sqrt{2}} - 2\sqrt{2} = -\sqrt{2} - 2\sqrt{2}$$
$$= -3\sqrt{2}$$

$f\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, \sqrt{2}\right) = 3\sqrt{2}$.