## ⇒ (Geomatrical Transformation)
## UNIT-II



* change the appear way of objects
  - size
  - Position
  - orientation

Implementation is called transformation.
↓
change

$[X][T] = [X^*]$ ← Transformed object Matrix

$[X]$ ← origional Matrix

$[T]$ ← transformation Matrix

various op$^n$
- Rotation ③.
- Translation ①.
- scaling ②.
- Reflection ④.
- Shearing ⑤

⇒ Type of Transformation
- Geometric (object moves and co-ordinate are fixed)
- Co-ordinate (object – stationary co-ordinate system – Moves)

## Representation of object/point in Matrix form:

2D-Coordinate system :- Any point represented in x & y co-ordinates

Points can be converted into matrix by two ways

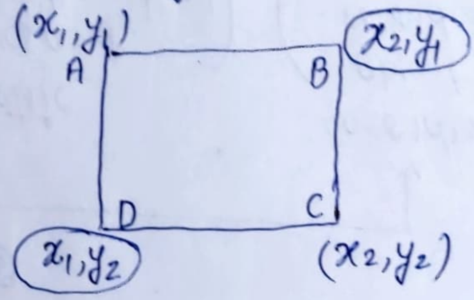
① Row-major matrix $[x\ y]_{1\times 2}$

② Column-major matrix $\begin{bmatrix} x \\ y \end{bmatrix}_{2\times 1}$

} Position vector

Series of position vector stored in matrix/array

eg:- opposite vertices of a rectangle $(x_1,y_1)$ & $(x_2,y_2)$.




$$\begin{bmatrix} A \\ B \\ C \\ D \end{bmatrix} = \begin{bmatrix} x_1 & y_1 \\ x_2 & y_1 \\ x_2 & y_2 \\ x_1 & y_2 \end{bmatrix}$$

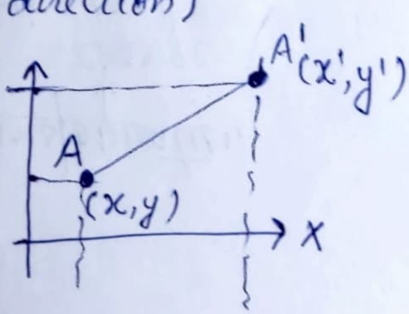
## Geometric Transformations:-

1. **Translation:** Shift of object parallel to itself in any direction (Shift → x & y direction)

$x' = x + t_x$ ← amount of x-shift

$y' = y + t_y$

↑ amount of y-shift

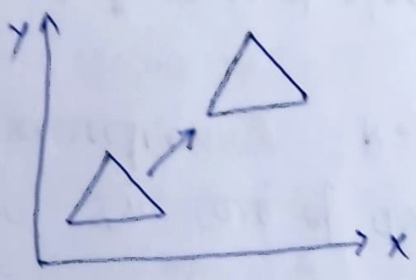



$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} t_x \\ t_y \end{bmatrix} \qquad P' = P + T$$

Applied on rigid-body.

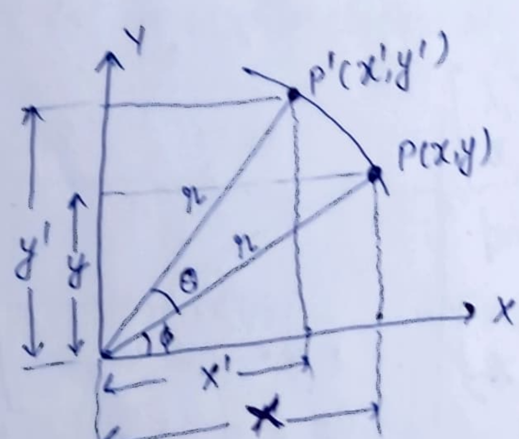
⇒ Translate each point with same value (x, y) and connect the points.

$$x' = x + t_x$$
$$y' = y + t_y$$




Translate a polygon with coordinates A(2,5), B(7,10) and C(10,2) by 3 unit in x direction and 4 unit in y direction.

2. Rotation:




Repositioning object along a circular path in xy place by an angle θ about the origion

clockwise direction rotation ← negative value

anticlockwise / counterclockwise " ← positive "

$x = r\cos\phi$ $\quad x' = r\cos(\phi+\theta) = r\cos\theta\cos\phi - r\sin\theta\sin\phi$

$y = r\sin\phi$ $\quad y' = r\sin(\theta+\phi) = r\sin\theta\cos\phi + r\cos\theta\sin\phi$

put value of x and y

$$x' = x\cos\theta - y\sin\theta$$
$$y' = x\sin\theta + y\cos\theta$$

Matrix by Row-Major

$$[x' \ y'] = [x \ y][T]$$

put value of x' and y'

$$[x\cos\theta - y\sin\theta \quad x\sin\theta + y\cos\theta] = [x \ y][T]$$

So $[T] = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$

$|T| = \cos^2\theta + \sin^2\theta = 1$

$\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 2\times2$ ...

$x\cos\theta + y\sin\theta \quad x\sin\theta \quad y\cos\theta$

⇒ Determinant of a rotation matrix is always +1.

In column-major order

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

Suppose we rotate P' to P (Back) to perform inverse transformation or rotation so angle is $\underline{-\theta}$

$[T] = [$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} x\cos\theta - y\sin\theta \\ x\sin\theta + y\cos\theta \end{bmatrix}$$

put $(-\theta)$ in place of $(\theta)$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos(-\theta) & -\sin(-\theta) \\ \sin(-\theta) & \cos(-\theta) \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$=$

So In case of anticlockwise/Counterclockwise direction $R = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$

and in case of clockwise direction

$$R = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$