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Course = B.TECH in CS
ADMNO = 21SCSE1011675
Sub - BBS01T1001 (Multi variable calculus)
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(1) $f(x,y) = \frac{2x}{y-x^2}$

Domain $\in R$

$y - x^2 \neq 0$

$y - x^2 > 0$

domain $\boxed{y > x^2}$

Range:

$y = \frac{2x}{y+x^2}$

$y^2 + yx^2 = 2x$

$y^2 - yx^2 - 2x = 0$

$yx^2 - y^2 + 2x = 0$

$x(yx+2) = y^2$

$x = \frac{y^2}{yx+2}$

(2) $\int_0^2 \int_0^x 2x \, dy \, dx$

$= 2\int_0^2 \left[(xy) \right]_0^2 dx$

$= 2\int_0^2 [(2x - 0) dx$

$= 2\int_0^2 2x dx$

$= 4\left[\frac{x^2}{2}\right]_0^2$

$= \frac{4}{2}\left[4 - 0\right]$

$= 8$

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(3) $$\lim_{(x,y)\to(0,0)} \frac{2xy}{x^2+y^2}$$

Let $x = r\cos\theta$
$y = r\sin\theta$

$(x,y) \to (0,0)$, we have $r \to 0$

$$\lim_{r\to 0} \frac{2(r\cos\theta)(r\sin\theta)}{(r\cos\theta)^2+(r\sin\theta)^2} = \lim_{r\to 0} \frac{2r^2\sin\theta\cos\theta}{(r\cos\theta)^2+(r\sin\theta)^2}$$

$$\lim_{r\to 0} \frac{2r^2\sin\theta\cos\theta}{r^2(\cos^2\theta+\sin^2\theta)}$$

$$\lim_{r\to 0} 2\sin\theta\cos\theta \quad (\because \cos^2\theta+\sin^2\theta=1).$$

$$\lim_{r\to 0} \sin 2\theta = 0$$

(4) $\iint_R f(x,y)\,dA$ for $f(x,y) = 100-6x^2y$ and $R: 0 \le x \le 2, -1 \le y \le 1$

Sol) $$I = \int_0^2 (100-6x^2y)\,dy\,dx \Rightarrow \int_0^2\int_{-1}^1 100\,dy\,dx - \int_0^2\int_{-1}^1 6x^2y\,dy\,dx$$

$$I_1 = \int_0^2\int_{-1}^1 100\,dy\,dx \Rightarrow 100\int_0^2 dx\,[y]_{-1}^{+1} \Rightarrow 100\int_0^2 (1+1)\,dx$$

$$\Rightarrow 100\times 2\int_0^2 dx \Rightarrow 200[x]_0^2$$

$$\Rightarrow 200\times 2 \Rightarrow 400$$

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$$\text{II} \quad \int_0^2 \int_{-1}^{1} 6x^2y \, dy \, dx \Rightarrow 6 \int_0^2 x^2 dx \left[\frac{y^2}{2} \right]_{-1}^{1}$$

$$\Rightarrow \frac{6}{2} \int_0^2 x^2 dx \left(1^2 - (-1)^2 \right)$$

$$\text{II} = 3 \int_0^2 x^2 dx \times 0 \Rightarrow 0$$

$$\therefore \int_0^2 \int_{-1}^{1} (100 - 6x^2y) \, dy \, dx$$

$$I_1 - I_2 \Rightarrow 400 - 0$$

$$400$$

(5) $f(x,y) = -3x^2 + 3y^2 + 6xy - 2y^3$

$$f: \; F_x = -6x + 6y$$
$$F_x = 6(-x+y)$$
$$0 = 6(-x+y)$$
$$\boxed{x = y}$$

$$F_y = 6y + 6x - 6y^2$$
$$F_y = 6(y + x - y^2)$$
$$0 = 6(y + x - y^2)$$
$$0 = y + x - y^2$$
$$0 = 2y - y^2$$
$$2y = y^2$$
$$\boxed{y = 2}$$

$$\boxed{x = y = 2}$$

$$P(2,2)$$

$$\boxed{F_{xx} = -6} \qquad F_{yy} = 6 - 12y = -18$$

$$D = F_{xx} \cdot F_{yy} - F_{xy} = -6 \times -18$$

$$\boxed{D = 108} > 0$$

$D > 0$

$f_{xx} < 0$

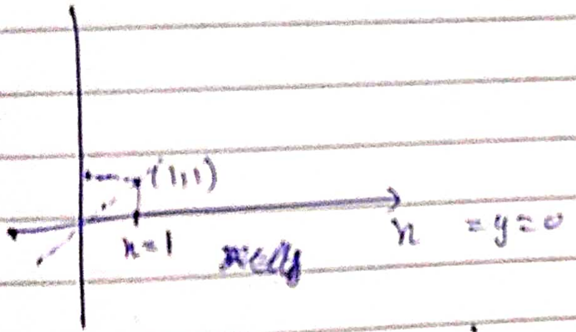
$$f(2,2) = -3(2)^2 + 3(2)^2 + 6(2)(2) - 2(2)^3$$

$$= \cancel{-3\times 4} + \cancel{3\times 4} + 24 - 16$$

$$f(2,2) = 8$$

Value of local min is 8

(Q) (8) $\displaystyle\int_{0=y}^{1-y}\int_{y=x}^{1-x} x^2 e^{xy}\,dx\,dy$



$$= \int_0^1 \left[\frac{e^{xy}}{y}\left(\frac{x^3}{3}\right)_x\right]_y^1 dy$$

$$= \frac{1}{3}\int_0^1 \left[\frac{e^{xy}}{y}\left(1 - y^3\right) dy\right]$$

$$= \frac{1}{3}\left[\frac{e^{xy}}{yx}\left(y - \frac{y^4}{4}\right)\right]_0^1$$

$$= \frac{1}{3}\cdot\frac{e^{xy}}{yx}\left(1 - \frac{1}{4}\right) = \frac{1}{4}\frac{e^{xy}}{xy} + C$$