Lecture 9

Form $0.\infty$

$$f(x)\cdot g(x) = \frac{f(x)}{\frac{1}{g(x)}}$$

$\infty-\infty$ Form

$$f(x)-g(x) = \frac{\frac{1}{g(x)} - \frac{1}{f(x)}}{\frac{1}{f(x)g(x)}}$$

To form of $1^\infty, 0^0, \infty^0$

$$y = f(x)^{g(x)}$$

Taking log,

$$\ln y = g(x)\ln f(x) = K \left(\lim_{x\to a}\right).$$

$$\Rightarrow \boxed{y = e^K}$$

Problem:- ① Evaluate

a) $\lim_{x\to 1} \log(1-x)\cot\frac{\pi x}{2}$ (b) $\lim_{x\to a} (a-x)\cdot\tan\left(\frac{\pi x}{2a}\right)$

Sol^n ⓐ $\lim_{x\to 1} \log(1-x)\cot\frac{\pi x}{2}$: $0.\infty$ form

$$= \lim_{x\to 1} \frac{\log(1-x)}{\tan\frac{\pi x}{2}} \quad : \frac{\infty}{\infty} \text{ form}$$

$$= \lim_{x\to 1} \frac{\frac{-1}{1-x}}{\frac{\pi}{2}\sec^2\left(\frac{\pi x}{2}\right)} \quad : \frac{\infty}{\infty} \text{ form}$$

$$= \lim_{x\to 1} -\frac{2}{\pi}\frac{\cos^2(\pi x/2)}{1-x} = \lim_{x\to 1} \frac{-2}{\pi}\,\frac{2\cdot\frac{\pi}{2}\cos\left(\frac{\pi x}{2}\right)\sin(\pi x/2)}{-1}$$

$$= -2\lim_{x\to 1} \cos\left(\frac{\pi x}{2}\right)\sin\left(\frac{\pi x}{2}\right) = 0.$$

ⓑ $\frac{2a}{\pi}$.

Problem (2) Evaluate

(i) $\lim_{x\to\frac{\pi}{2}}(\sec x-\tan x)$ (ii) $\lim_{x\to 1}\left[\frac{x}{x-1}-\frac{1}{\ln x}\right]$ (1/2) Ans

(iii) $\lim_{x\to 0}\left(\frac{1}{x^2}-\cot^2 x\right)$ (1) Ans.

Sol^n (i) $\lim_{x\to\frac{\pi}{2}}(\sec x-\tan x)$: $\infty-\infty$ form

$=\lim_{x\to\pi/2}\frac{1}{\cos x}-\frac{\sin x}{\cos x}=\lim_{x\to\pi/2}\frac{1-\sin x}{\cos x}$: $\frac{0}{0}$

$=\lim_{x\to\frac{\pi}{2}}\frac{-\cos x}{-\sin x}=0.$

Problem (3) Evaluate

(i) $\lim_{x\to 0}x^x$ (ii) $\lim_{x\to\pi/2}(\cos x)^{(\pi/2-x)}$ Ans 1

(iii) $\lim_{x\to 0}(x+\sin x)^{\tan x}$ Ans 1.

Sol^n (i) $\lim_{x\to 0}x^x$: 0^0 form, Take $y=x^x\Rightarrow \log y=x\log x$

$\therefore \lim_{x\to 0}\log y=\lim_{x\to 0}x\log x$: $(0.\infty$ form$)$

$=\lim_{x\to 0}\frac{\log x}{1/x}$: $\frac{\infty}{\infty}$ form

$=\lim_{x\to 0}\frac{1/x}{-1/x^2}=\lim_{x\to 0}(-x)=0$

$\therefore \lim_{x\to 0}\log y=0 \Rightarrow \lim_{x\to 0}y=e^0$

Hence, $\boxed{\lim_{x\to 0}x^x=1}$ Ans

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Problem ④ Evaluate (i) $\lim_{x\to\infty}(e^x+x)^{1/x}$ (ii) $\lim_{x\to\frac{\pi}{2}}(\tan x)^{\tan 2x}$

ⓔ Ans.

(iii) $\lim_{x\to\infty}(\cosh x)^{1/x}$

Solⁿ (ii) $y=(\tan x)^{\tan 2x} \Rightarrow \ln y = \tan 2x \log \tan x$

$$\lim_{x\to\pi/2} \ln y = \lim_{x\to\pi/2} \frac{\log \tan x}{\cot 2x} = 0.$$

$$\therefore \lim_{x\to\pi/2} (\tan x)^{\tan 2x} = e^0 = 1 \quad \text{Ans.}$$

(iii) $y=(\cosh x)^{1/x} \Rightarrow \log y = \frac{1}{x}\log(\cosh x).$

$$\lim_{x\to\infty} \log y = \lim_{x\to\infty} \frac{\log(\cosh x)}{x} = \lim_{x\to\infty} \frac{\sinh x}{\cosh x} \; : \frac{\infty}{\infty}$$

$$= \lim_{x\to\infty} \frac{\frac{e^x-e^{-x}}{2}}{\frac{e^x+e^{-x}}{2}} = \lim_{x\to\infty} \frac{e^x-e^{-x}}{e^x+e^{-x}} = \lim_{x\to\infty}\frac{e^{2x}-1}{e^{2x}+1} \; : \frac{\infty}{\infty}$$

$$= \lim_{x\to\infty} \frac{2e^{2x}}{2e^{2x}} = 1$$

Hence. $\lim_{x\to\infty}(\cosh x)^{1/x} = e^1 = e.$ Ans

Problem ⑤ Evaluate (i) $\lim_{x\to 1}\left(\tan \frac{\pi x}{4}\right)^{\tan\left(\frac{\pi x}{2}\right)}$

(ii) $\lim_{x\to 0}\left(\frac{2x+1}{x+1}\right)^{1/x}$ (iii) $\lim_{x\to 0}\left(\frac{\sinh x}{x}\right)^{1/x^2}$ Ans $e^{1/6}$

(iv) $\lim_{x\to 0}\left(\frac{a^x+b^x+c^x}{3}\right)^{1/x}$

Solⁿ (i) $\frac{1}{e}$ (ii) e^1

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