Solution of Question Bank (Unit-4)

① Compute $I = \int_0^2 \int_0^2 2x \, dy \, dx$

Sol^n $I = \int_0^2 2x\,dx \int_0^2 1\,dy \Rightarrow 2\int_0^2 x\,dx\,[y]_0^2 \Rightarrow 2\int_0^2 x(2-0)\,dx \Rightarrow 4\int_0^2 x\,dx$

$\Rightarrow 4\left[\frac{x^2}{2}\right]_0^2 \Rightarrow \frac{4}{2}[2^2-0] \Rightarrow \frac{4\times\cancel{2}}{\cancel{2}} = 4$ ✻

② Compute
$I = \int_0^3 \int_0^2 (4-y^2)\,dy\,dx$

Sol^n $\Rightarrow \int_0^3 \int_0^2 4\,dy\,dx - \int_0^3 \int_0^2 y^2\,dy\,dx$

$\Rightarrow 4\int_0^3 dx\,[y]_0^2 - \int_0^3 dx\left[\frac{y^3}{3}\right]_0^2$

$\Rightarrow 4\int_0^3 (2-0)\,dx - \int_0^3 \left(\frac{2^3}{3}\right) dx$

$\Rightarrow 8\int_0^3 dx - \frac{2^3}{3}\int_0^3 dx$

$\Rightarrow 8[x]_0^3 - \frac{2^3}{3}[x]_0^3 \Rightarrow 8\times 3 - \frac{8\times\cancel{3}}{\cancel{3}} \Rightarrow 24-8=16$ ✻

③ Compute $I = \int_{-1}^0 \int_{-1}^1 (x+y+1)\,dy\,dx$

Sol^n $I = \int_{-1}^0 \int_{-1}^1 x\,dy\,dx + \int_{-1}^0 \int_{-1}^1 y\,dy\,dx + \int_{-1}^0 \int_{-1}^1 1\,dy\,dx$

$\Rightarrow +\int_{-1}^0 x\,dx\,[y]_{-1}^1 + \int_{-1}^0 dx\left[\frac{y^2}{2}\right]_{-1}^1 + \int_{-1}^0 dx\,[y]_{-1}^1$

$\Rightarrow \int_{-1}^0 x(1+1)\,dx + \frac{1}{2}\int_{-1}^0 (1^2-(-1)^2)\,dx + \int_{-1}^0 (1+1)\,dx$

$\Rightarrow 2\int_{-1}^0 x\,dx + 0 + 2\int_{-1}^0 dx$

$\Rightarrow \cancel{2}\left[\frac{x^2}{\cancel{2}}\right]_{-1}^0 + 2[x]_{-1}^0 \Rightarrow (0-(-1)^2) + 2(0+1)$

$\Rightarrow -1+2 \Rightarrow 1$ ✻

(4) Compute $I = \int_1^2 \int_0^4 2xy \, dy \, dx$

Sol^n $I = 2\int_1^2 x\,dx \int_0^4 y\,dy \Rightarrow \cancel{2}\int_1^2 x\,dx \left[\frac{y^2}{\cancel{2}}\right]_0^4 \Rightarrow 4^2 \int_1^2 x\,dx$

$\Rightarrow \overset{8}{\cancel{16}} \left[\frac{x^2}{\cancel{2}}\right]_1^2 \Rightarrow 8(2^2-1^2) \Rightarrow (4-1)\times 8 \Rightarrow 3\times 8 \Rightarrow 24$

(5) $R: 0 \le x \le 1,\ 0 \le y \le 2,\ \iint_R (6y^2 - 2x)\,dA$.

Sol^n $I = \int_0^1 \int_0^2 (6y^2 - 2x)\,dy\,dx$

$\Rightarrow \int_0^1 \int_0^2 6y^2\,dy\,dx - \int_0^1 \int_0^2 2x\,dy\,dx$

$\Rightarrow 2\cancel{6}\int_0^1 dx \cdot \left[\frac{y^3}{\cancel{3}}\right]_0^2 - \int_0^1 2x\,dx\,[y]_0^2$

$\Rightarrow 2\times 2^3 \int_0^1 dx - 2\,2\int_0^1 x\,dx$

$\Rightarrow 2^4\,[x]_0^1 - \overset{2}{\cancel{4}}\left[\frac{x^2}{\cancel{2}}\right]_0^1 \Rightarrow 16\times 1 - 2\times 1 \Rightarrow 14$ A.

(6) $\iint_R f(x,y)\,dA$ for $f(x,y) = 100 - 6x^2y$, $R: 0 \le x \le 2,\ -1 \le y \le 1$.

Sol^n $I = \int_0^2 \int_{-1}^1 (100 - 6x^2y)\,dy\,dx \Rightarrow \underbrace{\int_0^2 \int_{-1}^1 100\,dy\,dx}_{I_1} - \underbrace{\int_0^2 \int_{-1}^1 6x^2y\,dy\,dx}_{II}$

$I_1 = \int_0^2 \int_{-1}^1 100\,dy\,dx \Rightarrow 100\int_0^2 dx\,[y]_{-1}^1 \Rightarrow 100\int_0^2 (+1+1)\,dx$

$\Rightarrow 100\times 2\int_0^2 dx \Rightarrow 200\,[x]_0^2 \Rightarrow 200\times 2 \Rightarrow 400$

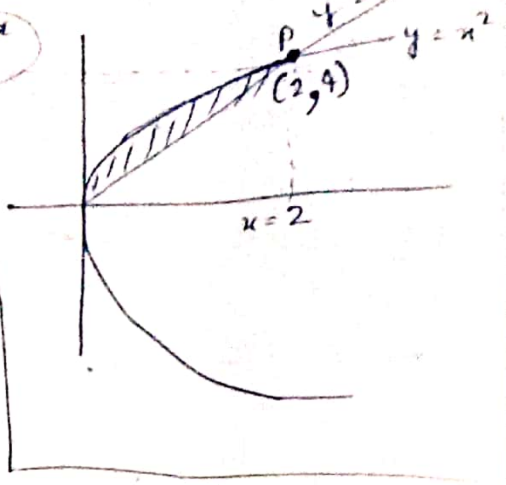
$II = \int_0^2 \int_{-1}^1 6x^2y\,dy\,dx \Rightarrow 6\int_0^2 x^2\,dx \left[\frac{y^2}{2}\right]_{-1}^1 \Rightarrow \frac{6}{2}\int_0^2 x^2\,dx\,(1^2 - (-1)^2)$

$II = 3\int_0^2 x^2\,dx \times 0 \Rightarrow 0$

$\therefore \int_0^2 \int_{-1}^1 (100 - 6x^2y)\,dy\,dx = I_1 - II \Rightarrow 400 - 0 \Rightarrow 400$

$x=0,\ x=2,\ y=x^2,\ y=2x$

At P, $2x = x^2$, $x=2$, $y=4$



(7) $I = \int_0^2 \int_{x^2}^{2x} (4x+2)\, dy\, dx$

Soln

$$\int_0^2 \int_{x^2}^{2x} 4x\, dy\, dx + 2\int_0^2 \int_{x^2}^{2x} dy\, dx$$

$$\Rightarrow 4\int_0^2 x\,dx \left[y\right]_{x^2}^{2x} + 2\int_0^2 \left[y\right]_{x^2}^{2x} dx$$

$$\Rightarrow 4\int_0^2 x(2x - x^2)\,dx + 2\int_0^2 (2x - x^2)\,dx$$

$$\Rightarrow 4\left[\int_0^2 2x^2 - \int_0^2 x^3\,dx\right] + 2\left[\int_0^2 2x\,dx - \int_0^2 x^2\,dx\right]$$

$$\Rightarrow 4\left\{2\left[\frac{x^3}{3}\right]_0^2 - \left[\frac{x^4}{4}\right]_0^2\right\} + 2\left\{\cancel{2}\cdot\left[\frac{x^2}{\cancel{2}}\right]_0^2 - \left[\frac{x^3}{3}\right]_0^2\right\}$$

$$\Rightarrow 4\left[2\times\frac{2^3}{3} - \frac{2^4}{4}\right] + 2\left[2^2 - \frac{2^3}{3}\right]$$

$$\Rightarrow 8\cdot\frac{2^3}{3} - \frac{\cancel{4}\times 2^4}{\cancel{4}} + 2^3 - 2\cdot\frac{2^3}{3}$$

$$\Rightarrow \frac{2^3}{3}(8-2) - 16 + 8 \Rightarrow \frac{8}{\cancel{3}}\times\cancel{6}^{2} - 8 \Rightarrow 16 - 8 \Rightarrow 8 \ \#$$

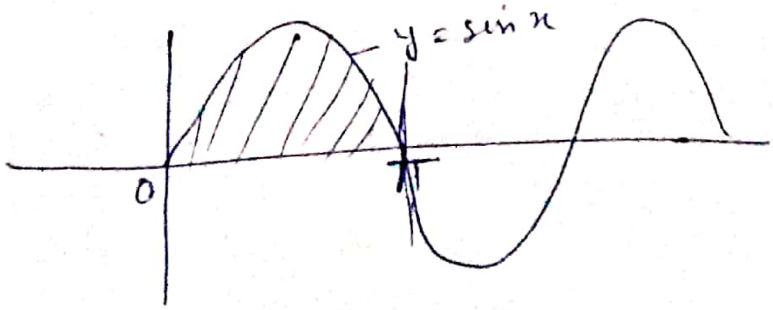
(8) $I = \int_0^{\pi} \int_0^{\sin x} dy\, dx$

Soln

$$\int_0^{\pi} \left[y\right]_0^{\sin x} dx \Rightarrow \int_0^{\pi} [\sin x - 0]\,dx \Rightarrow \int_0^{\pi} \sin x\, dx$$

$$\Rightarrow \left[-\cos x\right]_0^{\pi} \Rightarrow -\left[\cos x\right]_0^{\pi} \Rightarrow -\left[\cos\pi - \cos 0\right]$$

$$\Rightarrow -\left[-1-1\right] \Rightarrow -\times -2 \Rightarrow 2 \ \#$$



(9) $I = \int_0^{\pi} \int_0^{x} x \sin y \, dy \, dx$

$\Rightarrow \int_0^{\pi} x \, dx \int_0^{x} \sin y \cdot dy \Rightarrow \int_0^{\pi} x \, [-\cos y]_0^{x} \, dx \Rightarrow -\int_0^{\pi} \underset{\mathrm{I}}{x} \, \underset{\mathrm{II}}{\cos x} \, dx.$

$\Rightarrow -\left\{ [x \cdot \sin x]_0^{\pi} - \int_0^{\pi} 1 \cdot \sin x \cdot dx. \right\}$

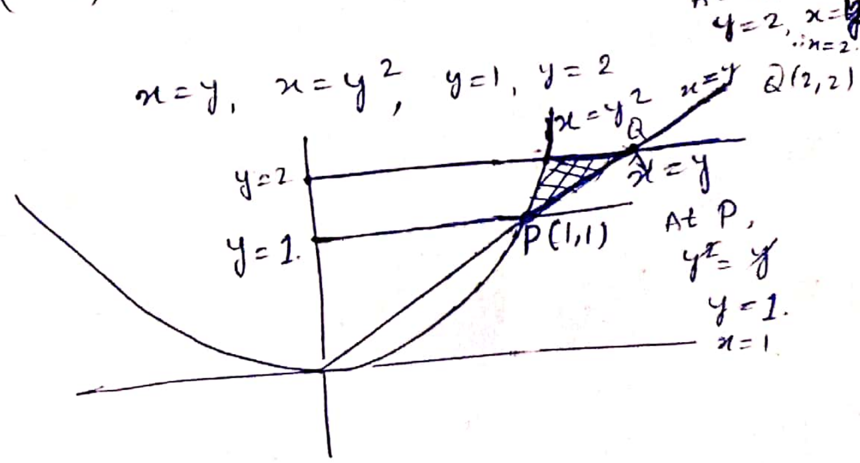
$\Rightarrow -\left\{ (\pi \sin \pi - 0 \times \sin 0) - [-\cos x]_0^{\pi} \right\}$

$\Rightarrow -\{ 0 + (\cos \pi - \cos 0) \}$

$\Rightarrow -(-1-1) \Rightarrow -(-2) = 2$ ※

(10) $I = \int_1^2 \int_y^{y^2} dx \, dy.$

$x = y, \; x = y^2, \; y = 1, \; y = 2$



$\Rightarrow \int_1^2 dy \, [x]_y^{y^2}$

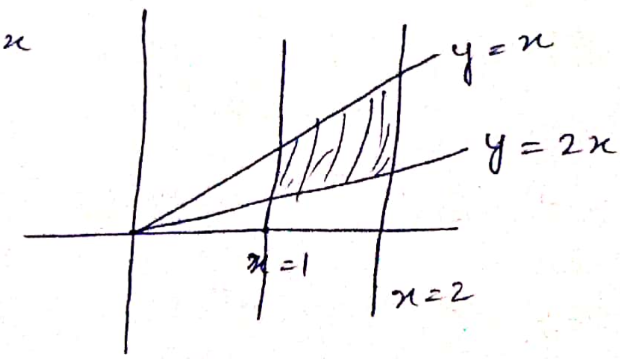
$\Rightarrow \int_1^2 (y^2 - y) \, dy$

$\Rightarrow \left[\frac{y^3}{3} - \frac{y^2}{2} \right]_1^2 \Rightarrow \left[\left(\frac{2^3}{3} - \frac{2^{\cancel{2}}}{\cancel{2}} \right) - \left(\frac{1^3}{3} - \frac{1^2}{2} \right) \right]$

$\Rightarrow \frac{8}{3} - \frac{1}{3} - \frac{4}{2} + \frac{1}{2} \Rightarrow \frac{7}{3} - \frac{3}{2} \Rightarrow \frac{14-9}{6} \Rightarrow \frac{5}{6}$ ※

(11) $I = \int_1^2 \int_x^{2x} \frac{x}{y} \, dy \, dx$,

$y = x, \; y = 2x$
$x = 1, \; x = 2$



$\Rightarrow \int_1^2 x \cdot dx \int_x^{2x} \frac{dy}{y}.$

$\Rightarrow \int_1^2 x \, dx \cdot [\log y]_x^{2x}$

$\Rightarrow \int_1^2 x \cdot [\ln(2x) - \ln x] \, dx. \Rightarrow \underbrace{\int_1^2 \underset{\mathrm{II}}{x} \, \underset{\mathrm{I}}{\ln 2x} \, dx}_{I_1} - \underbrace{\int_1^2 \underset{\mathrm{II}}{x} \, \underset{\mathrm{I}}{\ln x} \, dx.}_{I_2}$

$I \Rightarrow \left[\ln 2x \cdot \left(\frac{x^2}{2} \right) \right]_1^2 - \int_1^2 \frac{\cancel{2}}{\cancel{2x}} \cdot \frac{x^{\cancel{2}}}{\cancel{2}} \cdot dx \Rightarrow \left[\frac{x^2}{2} \ln 2x \right]_1^2 - \frac{1}{2} \left[\frac{x^2}{2} \right]_1^2$

LATE

$$I_1 \Rightarrow \frac{2^2}{2}\ln(4) - \frac{1}{2}\ln 2 - \frac{1}{4}(2^2-1^2)$$

$$\Rightarrow \frac{4}{2}\ln 2^2 - \frac{1}{2}\ln 2 - \frac{1}{4}(4-1)$$

$$\Rightarrow \frac{\cancel{2}\times 4}{\cancel{2}}\ln 2 - \frac{1}{2}\ln 2 - \frac{3}{4}$$

$$\Rightarrow \ln 2\left(4-\frac{1}{2}\right) - \frac{3}{4} \Rightarrow \ln 2\left(\frac{7}{2}\right) - \frac{3}{4}$$

$$I_1 = \frac{7}{2}\ln 2 - \frac{3}{4}$$

$$I_2 = \int_1^2 \underset{II}{x}\,\underset{I}{\ln x}\,dx$$

$$= \left[\ln x \cdot \frac{x^2}{2}\right]_1^2 - \int_1^2 \frac{1}{\cancel{x}}\cdot\frac{x^{\cancel{2}}}{2}\,dx$$

$$= \left(\frac{4}{2}\ln 2 - \frac{1}{2}\ln 1\right) - \frac{1}{2}\left[\frac{x^2}{2}\right]_1^2$$

$$\Rightarrow (2\ln 2 - 0) - \frac{1}{4}(2^2-1^2)$$

$$\Rightarrow 2\ln 2 - \frac{3}{4}$$

$$\therefore \int_1^2\int_x^{2x} \frac{x}{y}\,dy\,dx = I_1 - I_2 \Rightarrow \frac{7}{2}\ln 2 - \cancel{\frac{3}{4}} - 2\ln 2 + \cancel{\frac{3}{4}}$$

$$\Rightarrow \ln 2\left(\frac{7}{2}-2\right) \Rightarrow \frac{3}{2}\ln 2$$

(12) Let $I = \int_0^1\int_0^{y^2} 3y^3 e^{xy}\,dx\,dy$

Soln $I = 3\int_0^1 y^{\cancel{3}2}\left[\frac{e^{xy}}{y}\right]_0^{y^2} dy$

$$= 3\int_0^1 y^2\left[e^{xy}\right]_0^{y^2} dy \Rightarrow 3\int_0^1 y^2\left(e^{y^2\cdot y} - \underset{1}{e^0}\right) dy$$

or

$$I = 3\int_0^1 y^2(e^{y^3}-1)\,dy \Rightarrow \underbrace{3\int_0^1 y^2 e^{y^3}\,dy}_{I_1} - \underbrace{3\int_0^1 y^2\,dy}_{I_2}$$

Let $I_1 = 3\int_0^1 y^2 e^{y^3}\,dy$ Let $y^3 = t$

$3y^2\,dy = dt \Rightarrow y^2\,dy = \frac{dt}{3}$

$y=0,\ t=0$

$y=1,\ t=1$

$$\therefore I_1 = \frac{\cancel{3}}{\cancel{3}}\int_0^1 e^t\,dt \Rightarrow [e^t]_0^1$$

$$\Rightarrow (e^1 - e^0) = (e-1)$$

Scanned with CamScanner

$I_2 = 3 \cdot \int_0^1 y^2 \, dy \Rightarrow \cancel{3}\left[\frac{y^3}{\cancel{3}}\right]_0^1 \Rightarrow 1.$

$\therefore I = I_1 - I_2$

$\Rightarrow e - 1 - 1 \Rightarrow (e-2)$ ##

(13) $$I = \int_0^2 \int_{y^2/4}^{(y+2)/4} (16 - x^2 - y^2) \, dx \, dy.$$

$$\Rightarrow \int_0^2 \left[16x - \frac{x^3}{3} - y^2 \cdot x\right]_{y^2/4}^{(y+2)/4} dy$$

$$\Rightarrow \int_0^2 \left[\left(\overset{4}{\cancel{16}} \frac{(y+2)}{\cancel{4}} - \frac{1}{3} \cdot \left(\frac{y+2}{4}\right)^3 - y^2 \cdot \frac{(y+2)}{4}\right) - \left(\overset{4}{\cancel{16}} \frac{y^2}{\cancel{4}} - \frac{1}{3}\left(\frac{y^2}{4}\right)^3 - y^2 \cdot \frac{y^2}{4}\right)\right] dy$$

$$\Rightarrow \int_0^2 \left(4(y+2) - \frac{(y+2)^3}{3 \times 4^3} - \frac{y^2(y+2)}{4} - 4y^2 + \frac{y^6}{3 \times 4^3} + \frac{y^4}{4}\right) dy.$$

$$\Rightarrow \int_0^2 \left(4y + 8 - \frac{(y^3 + 8 + 6y^2 + 12y)}{3 \times 64} - \frac{y^3}{4} - \frac{\cancel{2}y^2}{\cancel{4}_2} - 4y^2 + \frac{y^6}{3 \times 64} + \frac{y^4}{4}\right) dy.$$

$$\Rightarrow \int_0^2 4y + 8 - \frac{y^3}{3 \times 64} - \frac{\cancel{8}}{3 \times \cancel{64}_8} - \frac{\cancel{6}^2 y^2}{\cancel{3} \times \cancel{64}_{32}} - \frac{\cancel{12}^4 y}{\cancel{3} \times \cancel{64}_{16}} - \frac{y^3}{4} - \frac{y^2}{2} - 4y^2 + \frac{y^6}{3 \times 64} + \frac{y^4}{4}) dy$$

$$\Rightarrow \int_0^2 \left[\frac{y^6}{192} + \frac{y^4}{4} - \left(\frac{1}{192} + \frac{1}{4}\right) \cdot y^3 - \left(\frac{1}{32} + \frac{16}{32} + 4\right) y^2 + \left(4 - \frac{1}{16}\right) y + \left(8 - \frac{1}{24}\right)\right] dy.$$

$$\Rightarrow \left[\frac{y^7}{192 \times 7} + \frac{y^5}{5 \times 4} - \frac{196}{192 \times 4} \frac{y^4}{4} - \left(\frac{17 + 128}{32}\right) \cdot \frac{y^3}{3} + \left(\frac{64-1}{16}\right) \frac{y^2}{2} + \frac{192-1}{24} y\right]_0^2$$

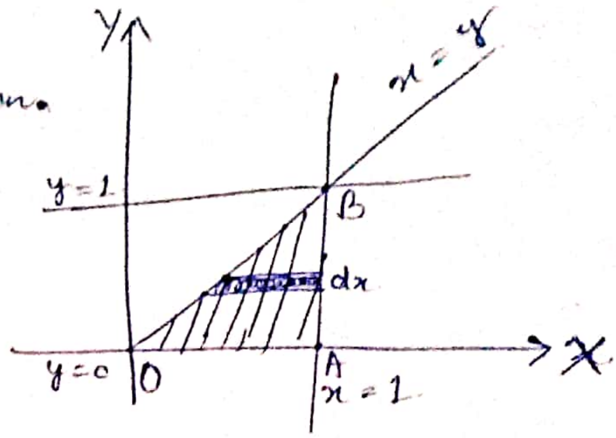
$$\Rightarrow \frac{2^7}{7 \times \cancel{192}_3} + \frac{2^5}{5 \times 4} - \frac{\cancel{196}^{49} \times \cancel{2^4}}{\cancel{192}_{48} \times \cancel{4} \times \cancel{4}} - \frac{145 \times \cancel{2^3}}{\cancel{32}_4 \times 3} + \frac{63}{8\,\cancel{16} \times \cancel{2}} \times \cancel{2^2} + \frac{191}{\cancel{24}_{12}} \times \cancel{2}$$

$$\Rightarrow \frac{2}{21} + \frac{8 \times \cancel{4}}{5 \times \cancel{4}} - \frac{49}{48} - \frac{145}{12} + \frac{63}{8} + \frac{191}{12}.$$

(14) and (15)

$$I = \int_0^1 \int_y^1 \frac{\sin x}{x}\, dx\, dy$$

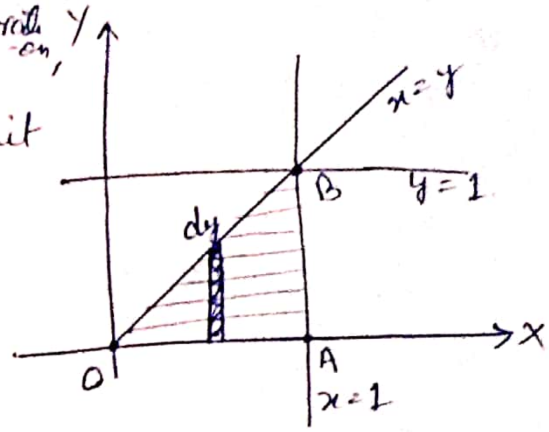
$x = y$, $x = 1$
$y = 0$, $y = 1$



ΔOAB is the region of integration.

Reverse the order of integration

For reversing the order of integration, we will first think for the limit of y and then for limit of x.



$y = 0$, $y = x$.
$x = 0$, $x = 1$.

$$I = \int_0^1 \int_0^x \frac{\sin x}{x}\, dy\, dx.$$

Sol (15)

Let $I = \int_0^1 \int_0^x \frac{\sin x}{x}\, dy\, dx.$

Soln

$$I = \int_0^1 \frac{\sin x}{x} \cdot [y]_0^x\, dx \Rightarrow \int_0^1 \frac{\sin x}{\cancel{x}} (\cancel{x} - 0)\, dx$$

$$= \int_0^1 \sin x\, dx.$$

$$= -[\cos x]_0^1 \Rightarrow -[\cos 1 - \cos 0]$$

$$\Rightarrow -\cos 1 + 1 \Rightarrow 1 - \cos 1 \qquad (\cos(1) = 0.54)$$

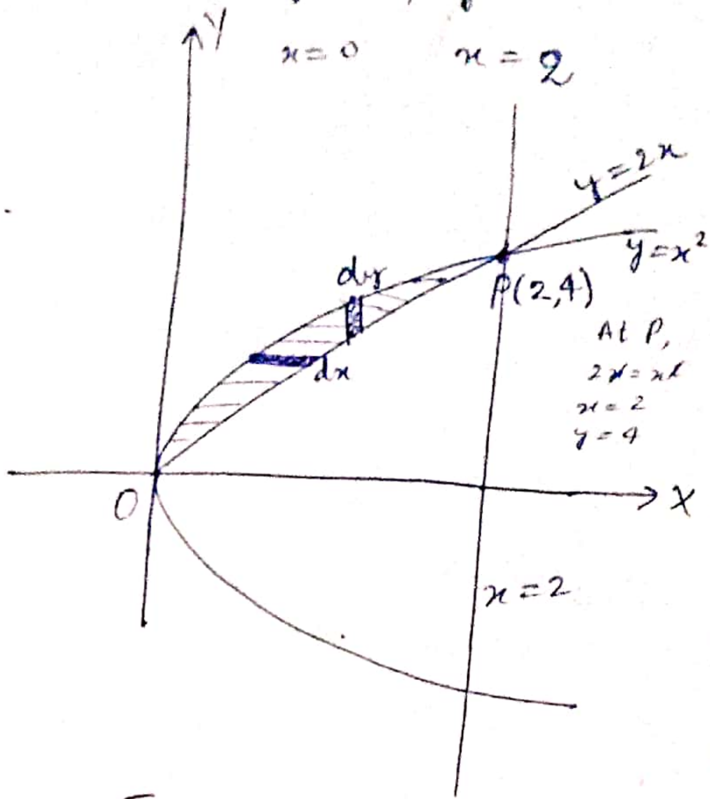
$$\Rightarrow 1 - 0.54 \Rightarrow 0.46$$ #

Scanned with CamScanner

(16) $\int_0^2 \int_{x^2}^{2x} (4x+2)\,dy\,dx$

$y = x^2, \quad y = 2x$

$x = 0 \quad x = 2$



To, change the order of integration. first, we think the limit for x, we have

$x = \sqrt{y}, \quad x = y/2$

$y = 0, \quad y = 4$

$$\int_0^4 \int_{y/2}^{\sqrt{y}} (4x+2)\,dx\,dy$$

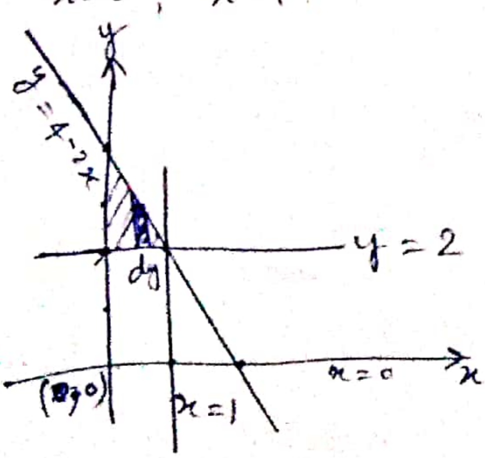
$$\int_0^2 \int_{x^2}^{2x} (4x+2)\,dy\,dx = \int_0^4 \int_{y/2}^{\sqrt{y}} (4x+2)\,dx\,dy = +8$$

or $$\int_0^2 \int_{2x}^{x^2} (4x+2)\,dy\,dx = \int_0^4 \int_{\sqrt{y}}^{y/2} (4x+2)\,dx\,dy = -8$$

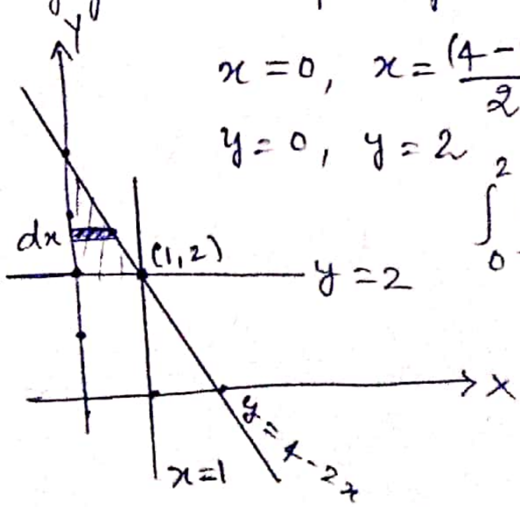
(17) $\int_0^1 \int_2^{4-2x} dy\,dx$

$y = 2, \quad y = 4-2x$

$x = 0, \quad x = 1$



changing the order of integration:



$x = 0, \quad x = \frac{(4-y)}{2}$

$y = 0, \quad y = 2$

$$\int_0^2 \int_0^{\frac{(4-y)}{2}} dx\,dy$$

$$\int_0^1 \int_2^{4-2x} dy\,dx = \int_0^2 \int_0^{\frac{(4-y)}{2}} dx\,dy$$