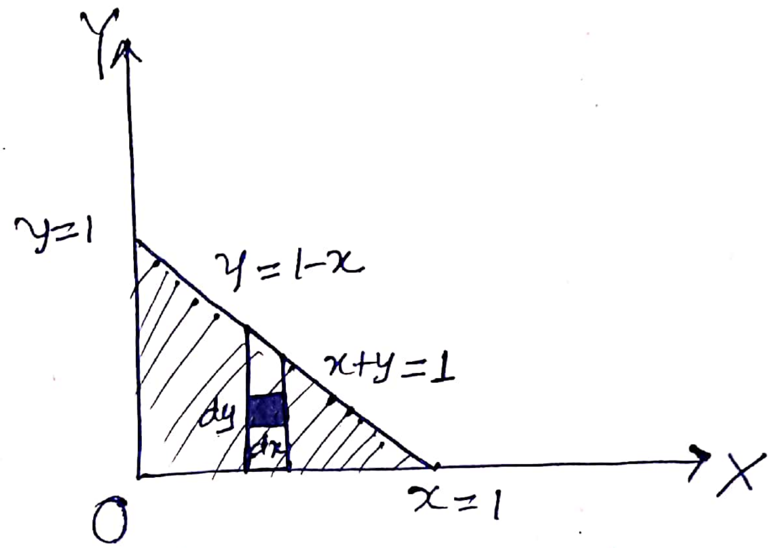
To Calculate the Integral Over a Given Region

(1) Evaluate $\iint xy\,dx\,dy$ over the region in the +ve quadrant for which $x+y \le 1$.

Sol^n:- $x+y=1$

$y: 0 \to 1-x$

$x: 0 \to 1$



Required integral

$$= \int_0^1 x\,dx \int_0^{1-x} y\,dy$$

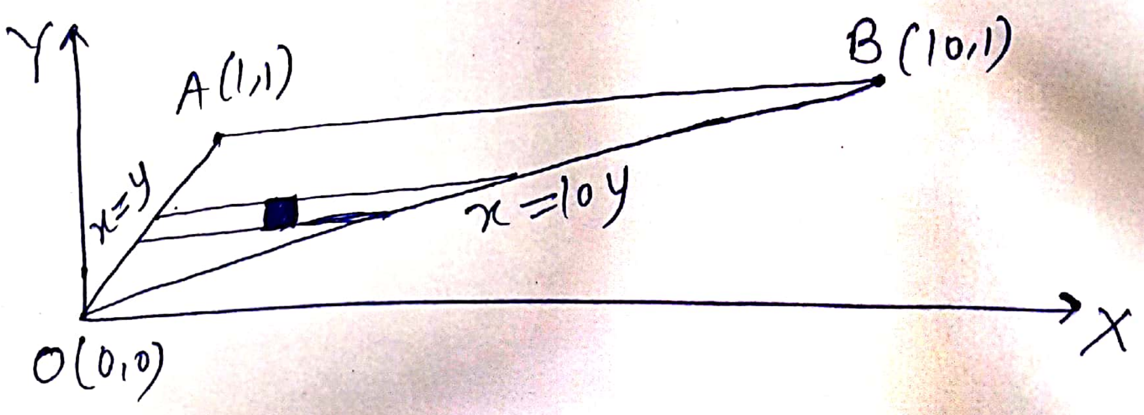
$$= \int_0^1 x\,dx \left[\frac{y^2}{2}\right]_0^{1-x}$$

$$= \frac{1}{2}\int_0^1 x(1-x)^2\,dx = \frac{1}{2}\int_0^1 (x-2x^2+x^3)\,dx$$

$$= \frac{1}{2}\left[\frac{x^2}{2} - \frac{2}{3}x^3 + \frac{x^4}{4}\right]_0^1 = \frac{1}{2}\left[\frac{1}{2} - \frac{2}{3} + \frac{1}{4}\right] = \frac{1}{24}.$$

(2) Evaluate $\iint_S \sqrt{xy-y^2}\,dy\,dx$

where S is a triangle with vertices $(0,0)$, $(10,1)$ and $(1,1)$.



$y \le x \le 10y$, $\quad 0 \le y \le 1$

$I = \int_0^1 dy \int_y^{10y} (xy - y^2)^{1/2} dx$ Put ... $y\,dx = 2t\,dt$

$dx = \frac{2t\,dt}{y}$

$t : 0 \to 3y$

$= 2\int_0^1 \frac{dy}{y} \int_0^{3y} t^2\, dt$

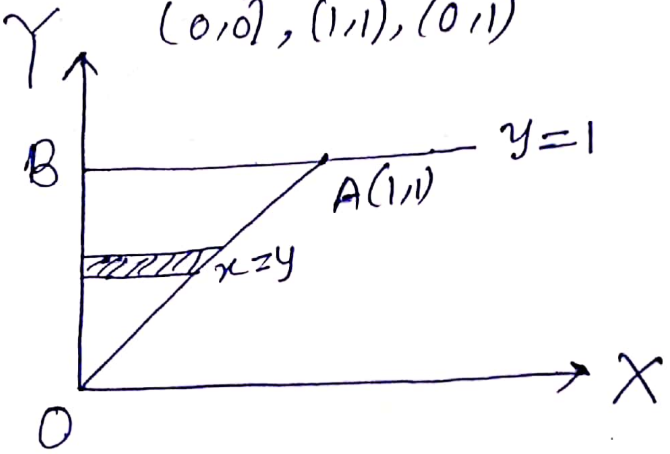
$= \frac{2}{3} \int_0^1 \frac{dy}{y} (3y)^3 = 18\int_0^1 y^2 dy$

$= 6$

(3) $I = \iint_R \frac{2xy^5}{\sqrt{1+x^2y^2-y^4}}\, dx\,dy$; R is region of the triangle whose vertices are $(0,0), (1,1), (0,1)$

Region is $x : 0 \to y$

$y : 0 \to 1$



$I = \int_0^1 \int_0^y \frac{2y^5 \cdot x}{\sqrt{(1-y^4)+x^2y^2}}\, dx\,dy$

$= \int_0^1 \int_0^y \frac{1}{y} \frac{2y^5 \cdot x}{\sqrt{\frac{1-y^4}{y^2}+x^2}}\, dx\,dy$

$= \int_0^1 2y^4 \left[\sqrt{\frac{1-y^4}{y^2}+x^2}\right]_0^y dy$

$= 2\int_0^1 y^4 \left[\frac{1}{y} - \frac{\sqrt{1-y^4}}{y}\right] dy = 2\int_0^1 \left[y^3 - \sqrt{1-y^4}\cdot y^3\right] dy$

$= 2\left[\frac{y^4}{4} + \frac{1}{4}\frac{(1-y^4)^{3/2}}{3/2}\right]_0^1$

$= 2\left[\frac{1}{4} - \frac{1}{4}\cdot\frac{2}{3}\right] = \frac{1}{2} - \frac{1}{3} = \frac{1}{6}.$

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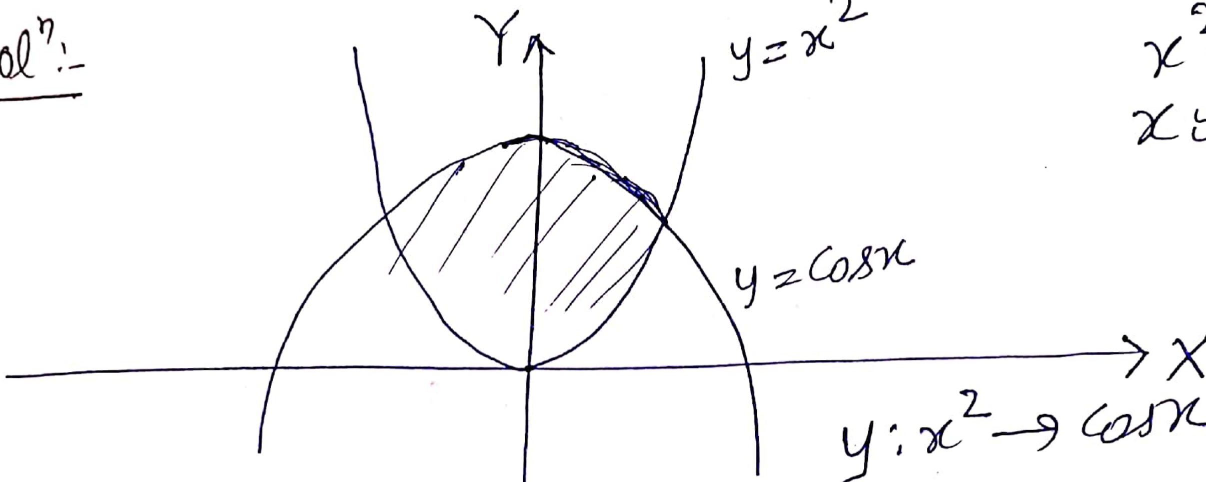
Prob ②:- Evaluate $\iint_R (x^2+6y)\,dA$, where R is the region bounded by the graphs of $y=\cos x$ and $y=x^2$.

Sol^n:-



$x^2 = \cos x$

$x \simeq \pm 0.82413$

$y: x^2 \to \cos x$

$x: -0.82413 \to 0.82413$

$$\iint_R (x^2+6y)\,dA \simeq \int_{-0.82413}^{0.82413} \int_{x^2}^{\cos x} (x^2+6y)\,dy\,dx \simeq 3.659765$$

Prob ③:- Write $\iint_R f(x,y)\,dA$ as an iterated integral, where R is the region bounded by the graphs of $x=y^2$ and $x=2-y$.

Prob ④:- ~~Evaluate the iterated integral~~

$$\cancel{\int_0^1 \int_y^1 e^{x^2}\,dx\,dy.}$$

~~Ans~~

Prob ① Let R be the region bounded by the graphs of $y = x, y = 0$ and $x = 4$. Evaluate

$$\iint_R \left(4e^{x^2} - 5\sin y\right) dA.$$

Solⁿ:- $\iint_R (4e^{x^2} - 5\sin y)\, dA = \int_0^4 \int_0^x (4e^{x^2} - 5\sin y)\, dy\, dx$

$$= 1.78 \times 10^7.$$

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