Question Bank 4

**Q.12:** Compute $\int_0^1 \int_0^{y^2} 3y^3 e^{xy}\, dx\, dy \Rightarrow$ let $I = \int_0^1 \int_0^{y^2} 3y^3 e^{xy}\, dx\, dy$

**Solution:**

$$I = \int_0^1 \left( 3y^3 \frac{e^{xy}}{y} \right)\Big|_0^{y^2} dy$$

$$= \int_0^1 3y^2 e^{xy}\Big|_0^{y^2} dy$$

$$= \int_0^1 3y^2 (e^{y^3} - 1)\, dy$$

$$= \int_0^1 3y^2 e^{y^3} dy - 3\int_0^1 y^2 dy = I_1 - I_2$$

where $I_1 = \int_0^1 3y^2 e^{y^3} dy$

let $y^3 = t \Rightarrow 3y^2 dy = dt$

& when $y = 0 \Rightarrow t = 0$

& $y = 1 \Rightarrow t = 1$

$\Rightarrow I_1 = \int_0^1 e^t dt = e^t\Big|_0^1 \Rightarrow e - 1$

and $I_2 = 3\int_0^1 y^2 dy = 3\,\frac{y^3}{3}\Big|_0^1 = 1$

$\Rightarrow I = I_1 - I_2 = e - 1 - 1 \Rightarrow \boxed{I = e - 2}$