

Strassen's Matrix Multiplication

Formula :-

$$A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \quad B = \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix} \quad C = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}$$

Formula :-

$$P = (A_{11} + A_{22}) (B_{11} + B_{22})$$

$$Q = B_{11} (A_{21} + A_{22})$$

$$R = A_{11} (B_{12} - B_{22})$$

$$S = A_{22} (B_{21} - B_{11})$$

$$T = B_{22} (A_{11} + A_{12})$$

$$U = (A_{21} - A_{11}) (B_{11} + B_{12})$$

$$V = (B_{21} + B_{22}) (A_{12} - A_{22})$$

$$\begin{array}{c} \xrightarrow{T \oplus} \\ \ominus \uparrow \left[\begin{array}{cc} 11 & 12 \\ 21 & 22 \end{array} \right] \downarrow \ominus \\ S \quad \quad \quad R \\ \xrightarrow{Q \oplus} \end{array}$$

$$\underline{BAAB}$$

$$B_{11} \quad A_{11} \quad A_{22} \quad B_{22}$$

P Q R S T U V

R A T

$$C_{11} = P + S - T + V$$

$$C_{12} = R + T$$

$$C_{21} = Q + S$$

$$C_{22} = P + R - Q + U$$

$$Q: A = \begin{bmatrix} 5 & 6 \\ -4 & 3 \end{bmatrix} ; B = \begin{bmatrix} -7 & 6 \\ 5 & 9 \end{bmatrix}$$

$$P = (A_{11} + A_{22})(B_{11} + B_{22}) = (5 + 3)(-7 + 9) = 16$$

$$Q = B_{11}(A_{21} + B_{22}) = -7(-4 + 3) = 7$$

$$R = A_{11}(B_{12} - B_{22}) = 5(6 - 9) = -15$$

$$S = A_{22}(B_{21} - B_{11}) = 3(5 + 7) = 36$$

$$T = B_{22}(A_{11} + A_{12}) = 9(5 + 6) = 99$$

$$U = (A_{21} - A_{11})(B_{11} + B_{12}) = (-4 - 5)(-7 + 6) = 9$$

$$V = (B_{21} + B_{22})(A_{12} - A_{22}) = (5 + 9)(6 - 3) = 42$$

$$C_{11} = P + S - T + V = 16 + 36 - 99 + 42 = -5$$

$$C_{12} = R + T = -15 + 99 = 84$$

$$C_{21} = Q + S = 7 + 36 = 43$$

$$C_{22} = P + R - Q + U = 16 - 15 - 7 + 9 = 3$$

$$= \begin{bmatrix} -5 & 84 \\ 43 & 3 \end{bmatrix}$$