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$$\boxed{\quad f(D)}$$

Complete Solution :-

$$\boxed{y = C.F. + P.I.}$$

form → ① Linear Diff^n eq^n with Constant Coefficient

② Homogeneous Linear diff^n eq^n with Constant Coefficient

↓

Can be solved by CF & PI

$$\underbrace{\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y}_{LHS} = \underbrace{x}_{R.HS}$$

Complementary Function (C.F.)

complementary function is the solution of the given diff^n eq^n, when its Right Hand Side (R.H.S) is replaced by zero.

To find C.F., first we find Auxiliary Equation (A.E.).

Auxiliary Equation (A.E.) :- It is an algebric equation in terms of m of degree n.

Working Rule to find Complementary function (C.F.) :-

Step 1 :- Consider the given diff^n eq^n in 'D' operator like as

$$f(D)y = Q$$

terms of m of degree n.

Working Rule to find Complementary function (C.F.) :-

Step 1 :- Consider the given diffⁿ eqⁿ in 'D' operator like as

$$f(D)y = Q$$

Step 2 :- For C.F., Q is replaced by zero, then step ① become

$$f(D)\,y = 0$$

Step 3 :- To find A.E., put $D = m$ and $y = 1$ in step ②, we get an algebric equation in terms of m.

Step 4 :- Now find the roots of A.E. (Values of m).

... the roots

Step 4 :- Now find the roots of

Step 5 :- C.F. depand upon the nature of the roots.

(i) When roots of A.E. are real & Distinct →

let m_1, m_2 and m_3 be the three roots of A.E., then

$$\boxed{C.F. = c_1 e^{m_1 x} + c_2 e^{m_2 x} + c_3 e^{m_3 x}}$$

(ii) When roots of A.E. are equal →

(a) let $m_1 = m_2 = m$, m_3 be the roots of A.E. , then

$$\boxed{C.F. = (c_1 + c_2 x) e^{mx} + c_3 e^{m_3 x}}$$

$c_1, c_2, c_3 \rightarrow$ arbitrary constant.

(ii) When roots of A.E. are equal $\rightarrow$

(a) let $m_1 = m_2 = m$, m_3 be the roots of A.E., then

$$\boxed{C.F. = (c_1 + c_2 x)e^{mx} + c_3 e^{m_3 x}}$$

(b) let $m_1 = m_2 = m_3 = m$ be the roots of A.E., then

$$\boxed{C.F. = (c_1 + c_2 x + c_3 x^2)e^{mx}}$$

(iii) when roots of A.E. are imaginary $\rightarrow$

let $m_1 = \alpha + i\beta$ and $m_2 = \alpha - i\beta$ be the roots of A.E., then

(iii) when roots of A.E. are imaginary →

let $m_1 = \alpha + i\beta$ and $m_2 = \alpha - i\beta$ be the roots of A.E., then

$$\boxed{C.F. = e^{\alpha x}\left[c_1 \cos\beta x + c_2 \sin\beta x\right]}$$

(iv) When roots of A.E. are repeated imaginary →

let $m_1 = m_2 = \alpha + i\beta$ and $m_3 = m_4 = \alpha - i\beta$ be the roots of A.E., then

$$\boxed{C.F. = e^{\alpha x}\left[(c_1 + c_2 x)\cos\beta x + (c_3 + c_4 x)\sin\beta x\right]}$$

$C.F. = e \ldots [\ldots$

$$CF = e^{\alpha x}\left[(c_1+c_2x+c_3x^2)\cos\beta x + (c_4+c_5x+c_6x^2)\sin\beta x\right] \rightarrow \text{Three equal roots}$$

(v) When roots of A.E. are irrational →

Let $m_1 = \alpha + \sqrt{\beta}$ and $m_2 = \alpha - \sqrt{\beta}$ be the roots of A.E, then

$$\boxed{C.F. = e^{\alpha x}\left[c_1 \cosh\sqrt{\beta}\,x + c_2 \sinh\sqrt{\beta}\,x\right]}$$

(vi) When roots of A.E are repeated irrational →

Let $m_1 = m_2 = \alpha + \sqrt{\beta}$ and $m_3 = m_4 = \alpha - \sqrt{\beta}$ be the roots of A.E, then

$$\boxed{C.F. = e^{\alpha x}\left[(c_1+c_2x)\cosh\sqrt{\beta}\,x + (c_3+c_4x)\sinh\sqrt{\beta}\,x\right]}$$

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Numerical Practice :-

① Solve $(D^4 - n^4)y = 0$, where $D = \frac{d}{dx}$

② Solve the diffⁿ eqⁿ $\frac{d^2 i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{i}{LC} = 0$, where $R^2 C = 4L$ and R, C, L are circuit constant.

③ Solve $(D^2+1)^3 (D^2 + D + 1)^2 y = 0$, where $D = \frac{d}{dx}$

④ Solve $\frac{d^2 x}{dt^2} - 3\frac{dx}{dt} + 2x = 0$, given that $x(0) = 0$ and $x'(0) = 2$.

Home work Numerical for Practice →

14:47 / 43:12

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dx^3 dx

② Solve $(D^4+n^4)y=0$, where $D=\frac{d}{dx}$

③ Solve $\frac{d^5y}{dx^5}-\frac{d^3y}{dx^3}=0$

④ Solve the diffⁿ eqⁿ $\frac{d^2y}{dx^2}+y=0$, given that $y(0)=2$, $y(\frac{\pi}{2})=-2$

⑤ Solve $(D^3+6D^2+12D+8)y=0$, where $D=\frac{d}{dt}$, under the Condition

$y(0)=0$, $y'(0)=0$, $y''(0)=2$.

5 Solve $(D^3+6D^2+12D+8)y=0$, where $D=\frac{d}{dt}$, under the Condition

$y(0)=0$, $y'(0)=0$, $y''(0)=2$.

Ex ① - Solve $(D^4-n^4)y=0$ where $D=\frac{d}{dx}$ → x → independent Variable.

Ans → in terms of x.

Solve → Complete Soln $\boxed{y=CF+PI}$

$f(D)y=0$

Put $D=m$, $y=1$, We get

AE ⇒ $m^4-n^4=0$

$y(0)=0$, $y(0)=0$, $y(0)=2$.

Ex① - Solve $(D^4-n^4)y=0$ where $D=\frac{d}{dx}$ → x → independent Variable.

$f(D)y=0$

Ans → in terms of x.

Solve → Complete Sol^n $\boxed{y=CF+PI}$

Put $D=m$, $y=1$, We get

AE ⇒ $m^4-n^4=0$

$(A^2-B^2)=(A-B)(A+B)$

$(m^2)^2-(n^2)^2=0$

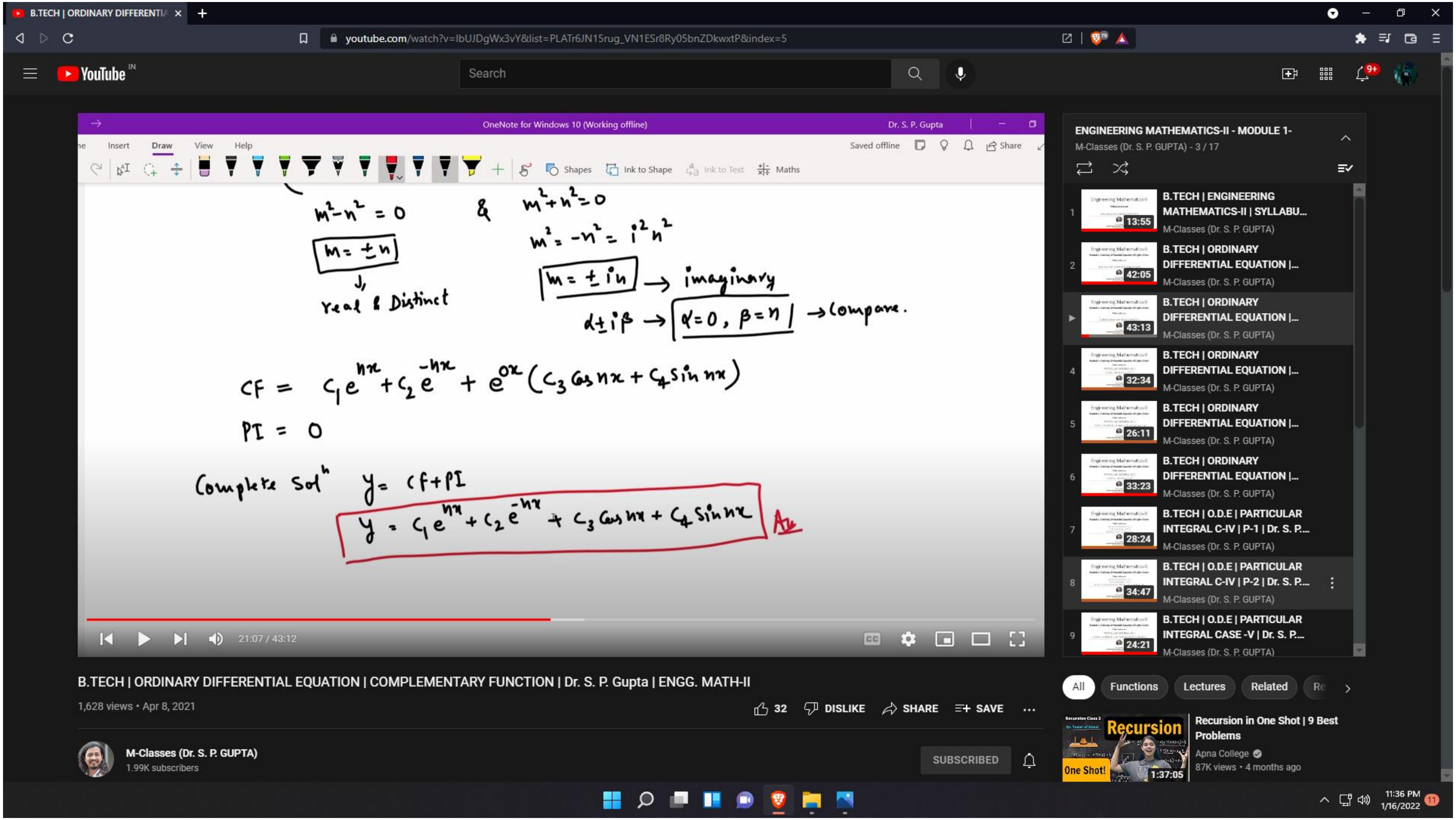
$(m^2-n^2)(m^2+n^2)=0$

$m^2-n^2=0$ & $m^2+n^2=0$

$\boxed{m=\pm n}$

$m^2=-n^2=i^2n^2$

$\boxed{m=\pm in}$



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$m^2-n^2=0$ & $m^2+n^2=0$
$\boxed{m=\pm n}$ Real & Distinct
$m^2=-n^2=i^2n^2$
$\boxed{m=\pm in} \rightarrow$ imaginary
$\alpha\pm i\beta \rightarrow \boxed{\alpha=0,\ \beta=n} \rightarrow$ compare.
$CF = c_1e^{nx}+c_2e^{-nx}+e^{0x}(c_3\cos nx+c_4\sin nx)$
$PI = 0$
Complete Sol^n $y = CF+PI$
$\boxed{y = c_1e^{nx}+c_2e^{-nx}+c_3\cos nx+c_4\sin nx}$ Ans
21:07 / 43:12
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Ex-2 –

$$\frac{d^2 i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{i}{LC} = 0$$

$R^2 C = 4L$

$$\left(D^2 + \frac{R}{L}D + \frac{1}{LC}\right) i = 0$$

[form $f(D)y = Q$]

Put $D = m$, $i = 1$

AE → $m^2 + \frac{R}{L}m + \frac{1}{LC} = 0$

23:49 / 43:12

Put $D = m$, $i = 1$

AE $\rightarrow$ $m^2 + \frac{R}{L} m + \frac{1}{LC} = 0$

$a = 1$, $b = \frac{R}{L}$, $c = \frac{1}{LC}$

$$m = \frac{-\frac{R}{L} \pm \sqrt{\frac{R^2}{L^2} - \frac{4}{LC}}}{2}$$

$R^2 C = 4L$

$$\boxed{R^2 = \frac{4L}{C}}$$

$$m = \frac{-\frac{R}{L} \pm \sqrt{\frac{4L}{L^2 C} - \frac{4}{LC}}}{2}$$

$$2 .$$

$$\boxed{R^2 = \frac{4L}{C}}$$

$$m = \frac{-\frac{R}{L} \pm \sqrt{\frac{4\cancel{L}}{L^{\cancel{2}}C} - \frac{4}{LC}}}{2}$$

$$m = \frac{-\frac{R}{L} \pm 0}{2}$$

+ve $\quad m = -\frac{R}{2L}$

−ve $\quad m = -\frac{R}{2L}$

$$m = -\frac{R}{2L}, -\frac{R}{2L} \text{ (equal roots)}$$

$$m = -\frac{R}{2L}, -\frac{R}{2L} \quad \text{(equal roots)}$$

$$CF = (c_1 + c_2 t)\, e^{-\frac{R}{2L}t}$$

$$PI = 0$$

Complete Solⁿ $i = CF + PI$

$$\boxed{i = (c_1 + c_2 t)\, e^{-\frac{R}{2L}t}}$$

AE- $(m^2+1)^3(m^2+m+1)^2=0$

Consider $m^2+1=0$ & $m^2+m+1=0$

$\boxed{m=\pm i}$ & $m=\frac{-1\pm\sqrt{1-4}}{2}=\frac{-1\pm\sqrt{3}\,i}{2}$

$$\boxed{m=\underbrace{-\frac{1}{2}}_{\alpha}\pm\underbrace{\frac{\sqrt{3}}{2}}_{\beta}i}$$

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$$\boxed{m = -\frac{1}{2} \pm \frac{\sqrt{3}}{2} i}$$

$\alpha \qquad \beta$

$\therefore$ The roots of the given diff^n eq^n. are.

$$m = \underbrace{\pm i, \pm i, \pm i}_{\alpha = 0,\ \beta = 1},\ \underbrace{-\frac{1}{2} \pm \frac{\sqrt{3}}{2} i,\ -\frac{1}{2} \pm \frac{\sqrt{3}}{2} i}_{\alpha = -\frac{1}{2},\ \beta = \frac{\sqrt{3}}{2}} \quad \text{(imaginary)}$$

$$\therefore CF = e^{0x}\left[(c_1 + c_2 x + c_3 x^2)\cos x + (c_4 + c_5 x + c_6 x^2)\sin x\right] +$$

$$e^{-1/2 x}\left[(c_7 + c_8 x)\cos \frac{\sqrt{3}}{2} x + (c_9 + c_{10} x)\sin \frac{\sqrt{3}}{2} x\right]$$

$$PI = 0$$

$$y = CF + PI$$

$\alpha = 0,\ \beta = 1$ $\qquad$ $\alpha = -\frac{1}{2},\ \beta = \frac{\sqrt{3}}{2}$

$$\therefore\ CF = e^{0x}\left[(c_1 + c_2 x + c_3 x^2)\cos x + (c_4 + c_5 x + c_6 x^2)\sin x\right] +$$

$$e^{-1/2 x}\left[(c_7 + c_8 x)\cos \frac{\sqrt{3}}{2}x + (c_9 + c_{10} x)\sin \frac{\sqrt{3}}{2}x\right]$$

$$PI = 0$$

C.S. $\quad y = CF + PI$

$$y = (c_1 + c_2 x + c_3 x^2)\cos x + (c_4 + c_5 x + c_6 x^2)\sin x + e^{-1/2 x}\left[(c_7 + c_8 x)\cos \frac{\sqrt{3}}{2}x + (c_9 + c_{10} x)\sin \frac{\sqrt{3}}{2}x\right]$$

Ans

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