Module-1 (Ordinary Differential Equation) **Introduction**

Differential Equation:- A differential equation is an equation which involve differentials or differential Coefficient. $\left(\frac{d}{dx}, \frac{d}{dx^2}\right)$

eg → $$\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + y = 0 \longrightarrow (1) \rightarrow \text{diff}^n \text{ eq}^n.$$

$\boxed{\text{Differential Equation}}$

$\boxed{\text{(1) Ordinary differential equation}}$

" A diff^n eq^n which involves Only One Independent Variable is Called "Ordinary diff^n equation"."

eg- $$\frac{d^3y}{dx^3} + 3\frac{d^2y}{dx^2} + 6\frac{dy}{dx} - y = \sin x$$

$\boxed{\text{(2) Partial diff}^n \text{ eq}^n.}$

" A diff^n eq^n which involve two or more independent variables and Partial derivative w.r.to them is Called Partial diff^n eq^n."

eg - $$x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = z$$

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2 Marks.

Order of a D.E. :- The order of a differential equation is the order of the highest ordered derivative occurring in the diffn eqn.

Degree of a D.E :- The degree of a differential equation is the degree of the highest ordered derivative present in the diffn eqn when it is made free from radical sign and fractional power.

Examples- ① $\frac{dy}{dx} = \cot x$, Order = 1, Degree = 1

② $\frac{d^2y}{dx^2} + y = 0$, order = 2, Degree = 1

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③ $y = x\dfrac{dy}{dx} + \left(\dfrac{dy}{dx}\right)^3$, Order = 1, Degree = 3

Non-Linear D.E.

④ $x\dfrac{\partial z}{\partial x} + y\dfrac{\partial z}{\partial y} = z$, Order = 1, Degree = 1

⑤ $\dfrac{d^2y}{dx^2} + \sqrt{1+\left(\dfrac{dy}{dx}\right)^3} = 0$, Order = 2, Degree = 2

⑥ $\rho = \dfrac{\left\{1+\left(\dfrac{dy}{dx}\right)^2\right\}^{3/2}}{\dfrac{d^2y}{dx^2}}$, Order = 2, Degree = 2

$\Rightarrow \rho \cdot \frac{d^2y}{dx^2} = \left\{1+\left(\frac{dy}{dx}\right)^2\right\}^{3/2}$, squaring on both side $\Rightarrow$

non-linear D.E.

Solution of D.E. :- A Solution (Integral) of a diff^n eq^n is a relation free from derivatives, between the variables which satisfy the given equation is called the solution of a Differential Equation.

eg $\rightarrow$ solve $\frac{d^2y}{dx^2} = \sin x$

Sol^n -

General Solution :- If in a solution of D.E.

$\Rightarrow \rho \cdot \frac{d^2y}{dx^2} = \left\{1+\left(\frac{dy}{dx}\right)^2\right\}^{3/2}$, squaring on both side $\Rightarrow$

non-linear D.E.

Solution of D.E. :- A Solution (Integral) of a diffⁿ eqⁿ is a relation free from derivatives, between the variables which satisfy the given equation is called the solution of a Differential Equation.

2nd

eg → solve $\frac{d^2y}{dx^2} = \sin x$

Solⁿ :- Integrating on both side.

$$\frac{dy}{dx} = -\cos x + c_1$$

again Integrating on both side

Solution of D.E. ← $\boxed{y = -\sin x + c_1 x + c_2}$

higher

eg - Solve $\frac{d^2y}{dx^2} + y = e^x$

Solⁿ :- Integrating on both side

$$\frac{dy}{dx} + \int y\,dx = e^x$$

direct Integration method ✗

General Solution :- If in a solution of D.E.

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Where $f(D) = D + 3D + 4D + \ldots$

Form of Differential Equation :-

① Linear Differential Equation with Constant Coefficient :-

An equation is of the form

$$a_0 \frac{d^n y}{dx^n} + a_1 \frac{d^{n-1} y}{dx^{n-1}} + a_2 \frac{d^{n-2} y}{dx^{n-2}} + \cdots \cdots + a_n y = Q$$

Where $a_0, a_1, a_2, \ldots a_n$ all are constant and Q is a function of x only.

is called L.D.E. with constant coefficient.

② Homogeneous Linear Differential Equation with Constant Coefficient :-

(Euler-Cauchy Equation)

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(2) Homogeneous Linear Differential Equation with Constant Coefficient :-

(Euler-Cauchy Equation)

An equation is of the form

$$a_0 x^n \frac{d^n y}{dx^n} + a_1 x^{n-1} \frac{d^{n-1} y}{dx^{n-1}} + a_2 x^{n-2} \frac{d^{n-2} y}{dx^{n-2}} + \cdots\cdots + a_n y = Q$$

where $a_0, a_1, a_2, \cdots a_n$ all are Constant and Q is a function of x-only is called Homogeneous L.D.E. with Constant Coefficient.

(3) Linear Differential Equation of Second Order with Variable Coefficient :-

A diffⁿ eqⁿ is of the form $\frac{d^2y}{dx^2} + P\frac{dy}{dx} + Qy = R$

An equation of the form

$$a_0 x^n \frac{d^n y}{dx^n} + a_1 x^{n-1} \frac{d^{n-1} y}{dx^{n-1}} + a_2 x^{n-2} \frac{d^{n-2} y}{dx^{n-2}} + \cdots\cdots + a_n y = Q$$

where $a_0, a_1, a_2, \ldots a_n$ "all are Constant" and Q is a function of x-only is Called Homogeneous L.D.E. with Constant Coefficient.

③ Linear Differential Equation of Second Order with Variable Coefficient :-

A diff^n eq^n is of the form $\frac{d^2y}{dx^2} + P\frac{dy}{dx} + Qy = R$

where P, Q, R are the function of x-only is Called L.D.E. of Second order with variable Coefficient.

Complementary Function (C.F.) :- Consider the D.E. $f(D)y = Q$, then

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Complementary Function (C.F.) :- Consider the D.E. $f(D)y = Q$, then C.F. is the solution of the diffⁿ eqⁿ, when its R.H.S member i.e. Q is replaced by zero. To find C.F., we first find Auxiliary Equation (A.E.)

Particular Integral (P.I.) :- Consider the D.E. $f(D)y = Q$, then

$$\boxed{PI = \frac{1}{f(D)} Q}$$

Complete Solution :-

$$\boxed{y = C.F. + P.I.}$$