38. Surface integral over Cylindrical

$\vec{F} = z^2 \hat{i} + xy\hat{j} - y^2\hat{k}$; $x^2+y^2=36$, $0 \le z \le 4$ in first octant

let $f(x,y,z) = x^2+y^2-36 = 0$ be the surface.

Then $\text{grad} f = 2x\hat{i} + 2y\hat{j}$, $n = \dfrac{\text{grad} f}{|\text{grad} f|} = \frac{1}{6}(x\hat{i}+y\hat{j})$

The projection of S on x-y plane cannot be considered.

Project S on the yz plane. Hence $dA = \dfrac{dy\,dz}{\hat{n}\cdot\hat{i}} = \dfrac{dy\,dz}{x/6}$

Therefore, $\iint\limits_S \vec{F}\cdot\hat{n}\,dA = \iint\limits_S \frac{1}{6}(z^2 x + xy^2)\,dA$

$= \int_0^4 \int_0^6 \frac{x}{6}(y^2+z^2)\dfrac{dy\,dz}{x/6} = 416$

For $x=0$
$y^2 = 36$
$\boxed{y=6}$
$\therefore y: 0 \to 6$ ✓