Master theorem

$$T(n) = aT\left(\frac{n}{b}\right) + \theta(n^k \log^p n)$$

$a \geq 1$, $b > 1$, $k \geq 0$ and p is a real no.

(1)

if $a > b^k$ then $T(n) = \theta(n^{\log_b a})$

(2) if $a = b^k$ then

a) $p > -1$ then $T(n) = \theta(n^{\log_b a} \log^{p+1} n)$

b) $p = -1$ then $T(n) = \theta(n^{\log_b a} \log\log n)$

c) $p < -1$ then $T(n) = \theta(n^{\log_b a})$

(3) if $a < b^k$ then

a) if $p \geq 0$ then $T(n) = \theta(n^k \log^p n)$

b) if $p < 0$ then $T(n) = \theta(n^k)$

(1) $T(n) = 4T(n/2) + n$

$T(n) = 4T(n/2) + n \log^0 n$

$a = 4, \; b = 2, \; k = 1, \; p = 0$

$4 > 2^1$

$= \Theta(n^{\log_b a})$

$= \Theta(n^{\log_2 4})$

$= \Theta(n^2)$

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(2)(a) $T(n) = 2T(n/2) + n \log^0$

$T(n) = 2T(n/2) + n \log^0 n)$

$a = 2 \quad b = 2 \quad k = 1 \quad p = 0$

$a = b^k$

$2 = 2^1$

$p > -1$

$0 > -1$

$= \theta(n^{\log_b a} \log^{k+1} n)$

$= \theta(n^{\log_2 2} \log^{0+1} n)$

$= \theta(n \cdot \log n)$

(b) $T(n) = 2T(n/2) + \frac{n}{\log n}$

$T(n) = 2T(n/2) + n \log^{-1} n$

$a = 2 \quad b = 2 \quad k = 1 \quad p = -1$

$a = 2^k \qquad p = -1$

$2 = 2^1 \qquad -1 = -1$

$$= \Theta(n^{\log_b a} \log\log n)$$

$$= \Theta(n^{\log_2 2} \log\log n)$$

$$= \Theta(n \cdot \log\log n)$$

2 (c) $T(n) = 2T(n/2) + \frac{n}{\log^2 n}$

$T(n) = 2T\left(\frac{n}{2}\right) + n \cdot \log^{-2} n$

$a = 2 \quad b = 2 \quad k = 1 \quad p = -2$

$2 = 2^1$ $p < -1$

$2 < -1$

~~$\Theta = \Theta(n^{\log_b a})$~~

$= \Theta(n^{\log_2 2})$

$= \Theta(n)$

3 (a) $T(n) = 2T(n/2) + n^2$

$T(n) = 2T\left(\frac{n}{2}\right) + n^2 \log^0 n$

$a = 2 \quad b = 2 \quad k = 2 \quad p = 0$

$a < 2^k$

$2 < 2^2$

$p \geq 0$

$0 \geq 0$

$= \Theta(n^k \log^p n)$

$= \Theta(n^2 \log^0 n) = \Theta(n^2)$

(3) (b) $T(n) = 2T(n/2) + \frac{n^2}{\log n}$

$T(n) = 2T(n/2) + n^2 \times \log^{-1} n$

$a = 2, \quad b = 2 \quad k = 2, \quad p = -1$

$a < b^k \qquad p < 0$

$2 < 2^2 \qquad -1 < 0$

$= \Theta(n^k)$

$= \Theta(n^2)$

Some Equation/function's where Master theorem is not applicable

$T(n) = 2^n T(n/2) + n$

$T(n) = 64 T(n/2) - n^2$

$T(n) = T(n/2) + \sin n$

$T(n) = 2T(n/2) + \frac{1}{n}$

$T(n) = 0.5\, T(n/2) + n$

$T(n) = \sqrt{2}\, T(n/2) + \log n$