Strassen's Matrix multiplication

DAC.

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \times \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} = \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix}$$

A 2×2 B 2×2 C

$n \times n$

$c_{11} = a_{11} * b_{11} + a_{12} * b_{21}$

$c_{12} = a_{11} * b_{12} + a_{12} * b_{22}$

$c_{21} = a_{21} * b_{11} + a_{22} * b_{21}$

$c_{22} = a_{21} * b_{12} + a_{22} * b_{22}$

$= 8$

$= 2^3$

no. of multiplication

$= 2^2$

no. of Addition

```
for (i=0; i<n; i++)
{
  for (j=0; j<n; j++)
  {
    c[i][j]=0          ⇒ O(n^3)
    for(k=0; k<n; k++)
    {
    c[i][j]= c[i][j] + A[i][k] + B[i][k];
    }
  }
}
```

$A_{p \times q} \cdot B_{q \times r} = C_{p \times r}$

$$A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} = \left[\begin{array}{cc|cc} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ \hline a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{array}\right] \quad B = \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix} = \left[\begin{array}{cc|cc} b_{11} & b_{12} & b_{13} & b_{14} \\ b_{21} & b_{22} & b_{23} & b_{24} \\ \hline b_{31} & b_{32} & b_{33} & b_{44} \\ b_{41} & b_{42} & b_{43} & b_{44} \end{array}\right]$$

A_{21} A_{22} $\frac{n}{2} \times \frac{n}{2}$ B_{21} B_{22} $\frac{n}{2} \times \frac{n}{2}$

$$C_{11} = A_{11} * B_{11} + A_{12} * B_{21}$$

$$C_{12} = A_{11} * B_{12} + A_{12} * B_{22}$$

$$C_{21} = A_{21} * B_{11} + A_{22} * B_{21}$$

$$C_{22} = A_{21} * B_{12} + A_{22} * B_{22}$$

```
MM (A, B, n)
{ if (n ≤ 2)
  {
    c11 = a11 * b11 + a12 * b21
    c12 = a11 * b12 + a12 * b22
    c21 = a21 * b11 + a22 * b21
    c22 = a21 * b12 + a22 * b22
  }
else
```

$$T(n) = 8T(n/2) + 4n^2$$

Divide A & B with order n×n into 4 submatrix each with order $n/2 \times n/2$

A B

$MM(A_{11}, B_{11}, n/2) + MM(A_{12}, B_{21}, n/2)$

$MM(A_{11}, B_{12}, n/2) + MM(A_{12}, B_{22}, n/2)$

$MM(A_{21}, B_{11}, n/2) + MM(A_{22}, B_{21}, n/2)$

$MM(A_{21}, B_{12}, n/2) + MM(A_{22}, B_{22}, n/2)$

n value always be taken 2 power

eg. 4×4, 8×8, 16×16 ...etc.

$C_{11} = P + S - T + V$

$C_{12} = R + T$

$C_{21} = Q + S$

$C_{22} = P + R - Q + U$

$T(n) = 7T(n/2) + 18n^2$

$= O(n^{2.81})$

$P = (A_{11} + A_{21})(B_{11} + B_{22})$

$Q = (A_{21} + A_{22}) B_{11}$

$R = A_{11}(B_{12} - B_{22})$

$S = A_{22}(B_{21} - B_{11})$

$T = (A_{11} + A_{12}) B_{22}$

$U = (A_{21} - A_{11})(B_{11} + B_{12})$

$V = (A_{12} - A_{22})(B_{21} + B_{22})$