

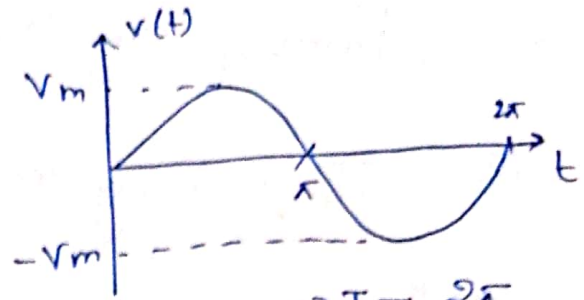
STEADY STATE AC ckt:-

$$v(t) = V_m \sin \omega t$$

$V_m \rightarrow$ maximum
or
peak value.

$\omega \rightarrow$ angular frequency (rad/sec)

$\omega t \rightarrow$ argument \Rightarrow unit (rad).



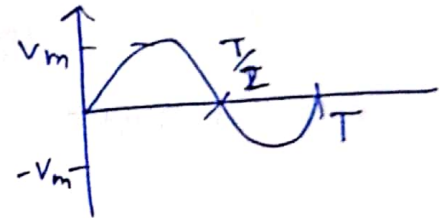
$$\omega T = 2\pi$$

$$\omega = \frac{2\pi}{T}$$

$$T = \frac{2\pi}{\omega}$$

$$T = \frac{1}{f}$$

$$\left[f = \frac{\omega}{2\pi} \right] \text{ Hz}$$

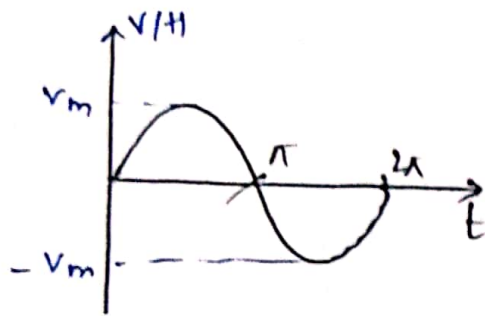


Advantage of sinusoidal waveform:-

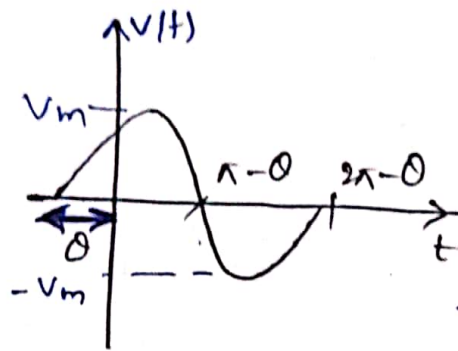
- ① It is easy to handle mathematically (differentiation of the sine function and integral of the sine function can be rewrite in terms of sine function).
- ② The natural phenomenon like motion of simple pendulum and response of undamped system exhibits sinusoidal characters.

③ Any periodic waveform can be expressed in terms of sine function by using fourier analysis.

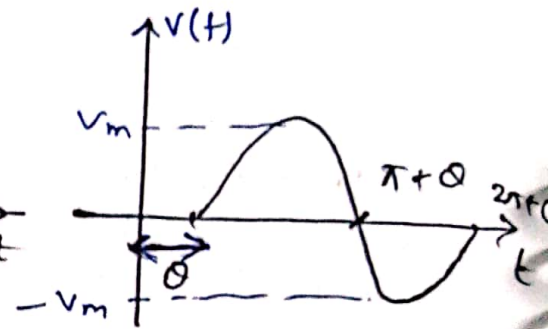
④ It is easy to generate in the laboratory.



$V(t)$



Lead



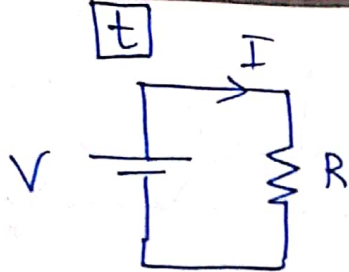
Lag

$$V(t) = V_m \sin \omega t$$

$$V(t) = V_m \sin(\omega t + \theta) \quad \dots \quad V(t) = V_m \sin(\omega t - \theta)$$

⇒ Rms value is defined based on heating effect of waveform.

⇒ The voltage at which heat dissipation in ac ckt. is equal to heat dissipation in dc ckt. is called as V_{rms} . provided both ac. ckt and dc ckt. have equal value of resistance and operate at equal duration of time.

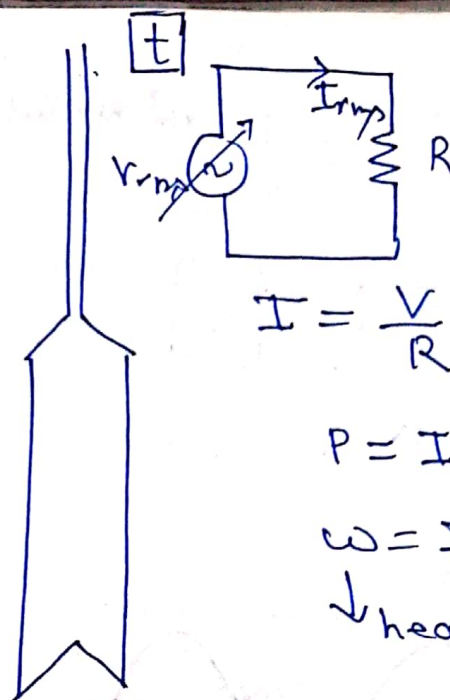


$$I = \frac{V}{R}$$

$$P = I^2 R$$

$$W = I^2 R t$$

↳ heat



$$I = \frac{V}{R}$$

$$P = I^2 R$$

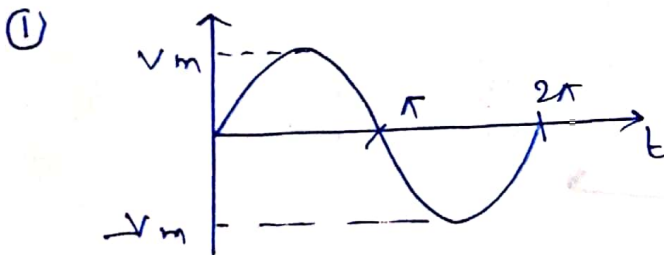
$$W = I^2 R t$$

↓ heat

$$W_{AC} = W_{DC}$$

$$V_{rms} = \sqrt{\frac{1}{T} \int_0^T V^2 dt}$$

$$= \sqrt{\frac{1}{2\pi} \int_0^{2\pi} V^2 dt}$$



$$V_{rms}^2 = \frac{1}{T} \int_0^T V_m^2 \sin^2 \omega t dt$$

$$= \frac{V_m^2}{T} \left[\frac{1}{2} T \right]$$

$$V_{rms} = \frac{V_m}{\sqrt{2}}$$

$$V_{rms}^2 = \frac{1}{T} \int_0^T V_m^2 \sin^2 \omega t dt$$

$$= \frac{V_m^2}{2\pi} \left[\frac{1}{2} (2\pi) \right]$$

$$= \frac{V_m^2}{2}$$

$$\cos 2\theta = \sin^2 \theta - \cos^2 \theta$$

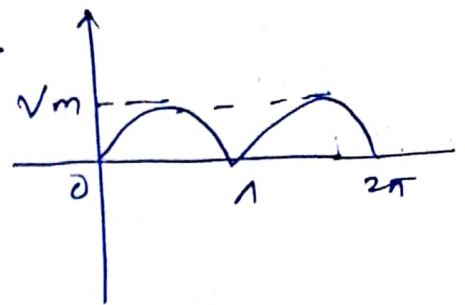
$$1 + \cos 2\theta = 2 \sin^2 \theta - 1$$

②

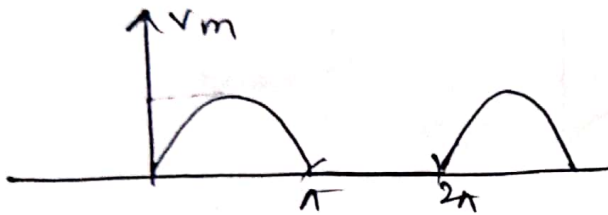
$$V_{rms}^2 = \frac{1}{T} \int_0^T v_m^2 \sin^2 \omega t \, dt$$

$$= \frac{V_m^2}{T} \left[\frac{1}{2} \right]$$

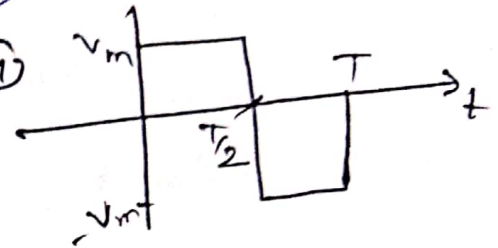
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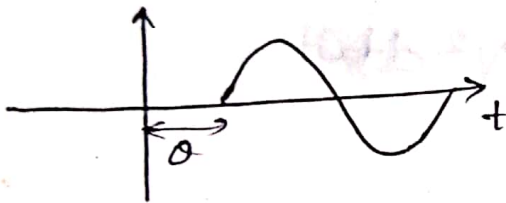
③



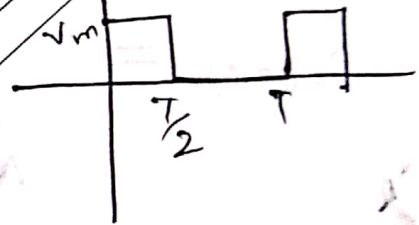
④



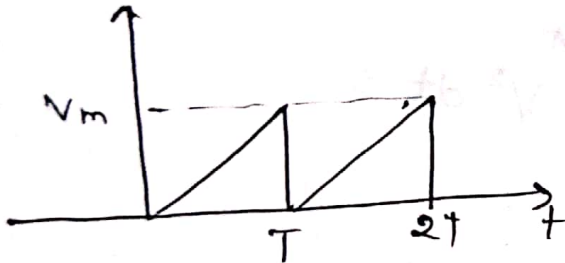
⑤



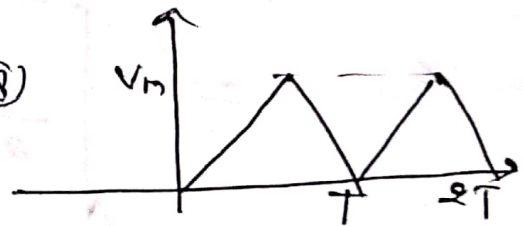
⑥



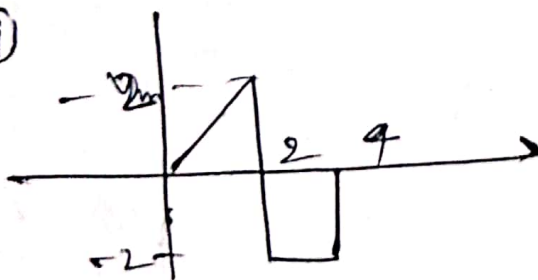
⑦



⑧



⑨



①

$$V_{rms} = \sqrt{\frac{1}{2\pi} \int_0^{2\pi} (V_m \sin \omega t)^2 d\omega t}$$

$$V_{rms} = \frac{V_m}{\sqrt{2}}$$

$$\sin^2 \omega t \leftrightarrow \frac{1 - \cos 2\omega t}{2}$$

②

$$V_{rms} = \sqrt{\frac{1}{\pi} \int_0^{\pi} (V_m \sin \omega t)^2 d\omega t}$$

$$V_{rms} = \frac{V_m}{\sqrt{2}}$$

Full wave rectifier:-

③

Half wave rectifier:-

$$V_{rms} = \frac{V_m}{2}$$

$$V_{rms} = \sqrt{\frac{V_m^2}{2\pi} \int_0^{\pi} \left(\frac{1 - \cos 2\omega t}{2} \right) d\omega t}$$

④

$$V(t) = V_m \sin(\omega t - \theta)$$

$$V_{rms} = \frac{V_m}{\sqrt{2}} \quad \left(\text{it does not change with shifting} \right)$$

Note:- RMS value is independent of position of starting of waveform. but it depends on shape of the waveform.

⑥

$$V_{rms} = \sqrt{\frac{1}{T} \int_0^T (V_m)^2 dt}$$

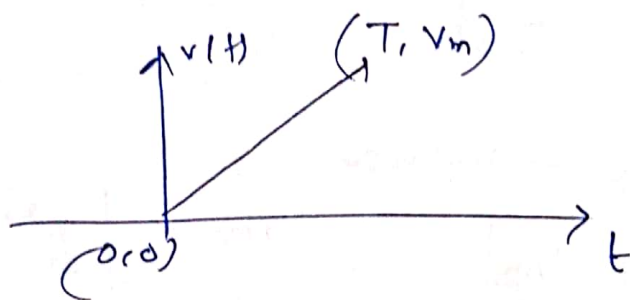
$$= \sqrt{\frac{V_m^2}{T} T} = \sqrt{V_m^2} = V_m$$

⑥

$$V_{rms} = \sqrt{\frac{1}{T} \int_0^{T/2} V_m^2 dt + \int_{T/2}^T 0 dt}$$

$$= \frac{V_m^2 \cdot T/2}{T} = \frac{V_m}{\sqrt{2}}$$

⑦



$$y - 0 = \frac{V_m}{T} (x - 0)$$

$$y = \frac{V_m}{T} x$$

$$v(t) = \frac{V_m}{T} t \quad \text{--- (1)}$$

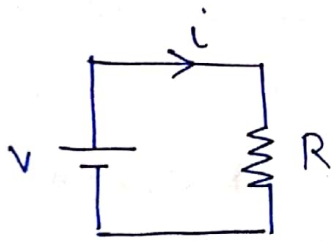
$$V_{rms} = \sqrt{\frac{1}{T} \int_0^T \left(\frac{V_m}{T} t\right)^2 dt}$$

$$= \sqrt{\frac{V_m^2}{T^3} \left| \frac{t^3}{3} \right|_0^T} = \frac{V_m}{\sqrt{3}}$$

Avg Value :-

① Avg value is defined based on charge transfer in the ckt

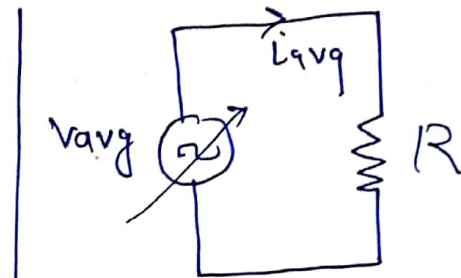
② The voltage at which charge transfer in ac ckt is equal to charge transfer in dc ckt is called as V_{avg} , provided that both ac ckt and dc ckt have equal value of resistance and operated for same time.



$$i = \frac{V}{R}$$

It operates for time t sec.

$$Q_{dc} = it$$



$$i = \frac{V}{R}$$

$$Q_{ac} = i t$$

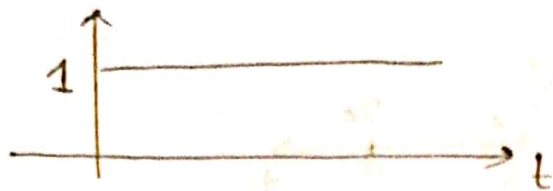
$$Q_{ac} = Q_{dc}$$

$$R = R$$

Then, Voltage is called V_{avg} :

when $Q_{ac} = Q_{dc}$ at $R = R$.

Problem: ①



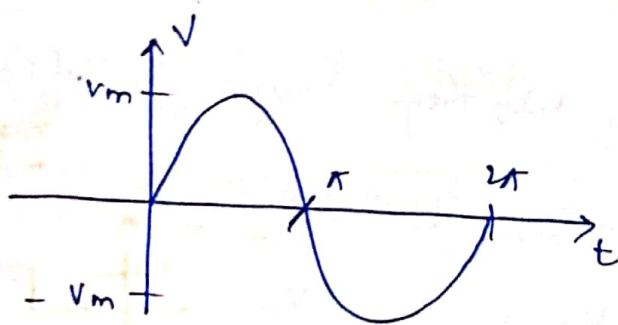
$$V_{avg} = \frac{1}{T} \int_0^T V(t) dt$$

$$= \frac{1}{T} \int$$

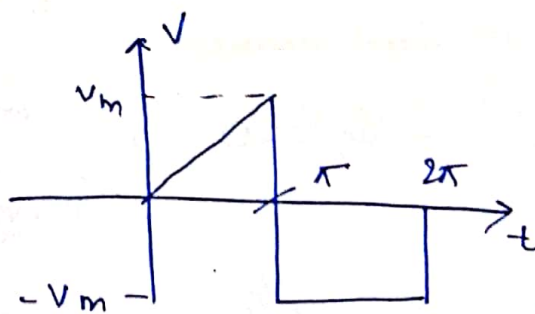
⇒ Arg values of complete cycle of symmetrical wave is equal to '0'

⇒ For analysis purpose while finding avg value of symmetrical wave. only +ve half cycle is considered.

⇒ While finding avg value of unsymmetrical wave angle made by complete cycle is considered.



Symmetrical wave.



Unsymmetrical wave

$$V_{avg} = \frac{1}{\pi} \int_0^{\pi} V dt$$

$$V_{avg} = \frac{1}{2\pi} \left[\int_0^{\pi} V dt + \int_{\pi}^{2\pi} V dt \right]$$

⇒ FORM FACTOR is a Ratio of:
rms value of waveform to avg. or d.c
value of waveform

$$\text{Form Factor} = \frac{V_{\text{rms}}}{V_{\text{avg}}} = \frac{I_{\text{rms}}}{I_{\text{avg}}}$$

⊗) PEAK FACTOR: — Ratio of max value
of waveform to rms value of waveform.

$$\text{P.F} = \frac{V_m}{V_{\text{rms}}} = \frac{I_m}{I_{\text{rms}}}$$

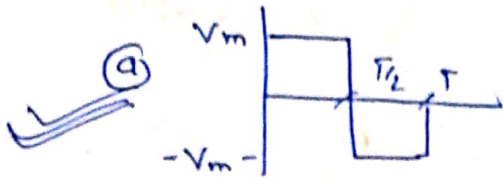
Note: — To justify about shape of waveform
F.F and P.F concept are used.

For sine wave. (ideally)

$$\left[\begin{array}{l} \text{FF} = 1.11 \\ \text{PF} = \sqrt{2} \end{array} \right]$$

If o/p waveform
have these
value then least
distortion is present

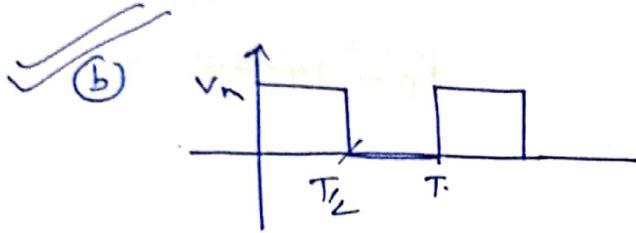
Prob: ① Which of following waveform have $FF=PF$



$$\Rightarrow V_{avg} = V_{rms} = V_m$$

$$F.F = \frac{V_m}{V_m} = 1$$

$$P.F = \frac{V_m}{V_m} = 1$$

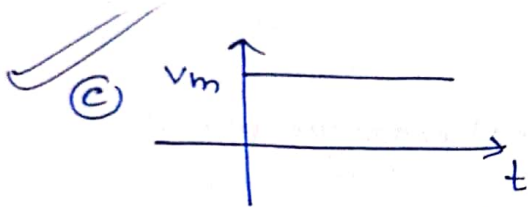


$$\Rightarrow V_{rms} = \frac{V_m}{\sqrt{2}}$$

$$V_{avg} = \frac{V_m}{2}$$

$$F.F = \frac{V_m^2}{\frac{V_m^2}{2}} = \sqrt{2}$$

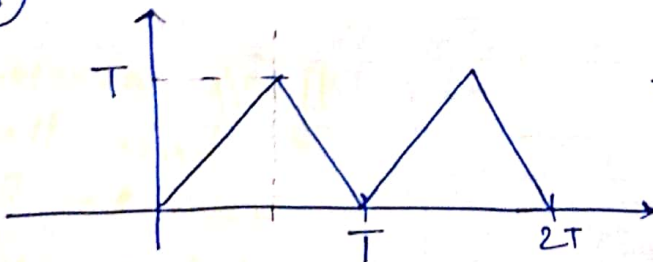
$$P.F = \frac{V_m \sqrt{2}}{V_m} = \sqrt{2}$$



$$\Rightarrow V_{rms} = V_{avg} = V$$

$$F.F = P.F = 1$$

X (d)



$$\Rightarrow V_{avg} = \frac{V_m}{2}$$

$$V_{rms} = \frac{V_m}{\sqrt{3}}$$

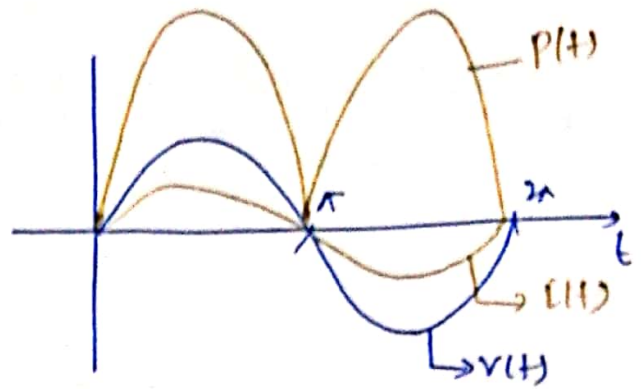
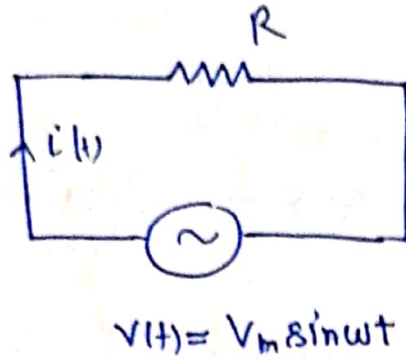
$$F.F = \frac{V_m^2}{\frac{V_m^2}{3}} = \sqrt{3}$$

$$P.F = \frac{V_m \sqrt{3}}{V_m} = \sqrt{3}$$

Not

Case-1

AC SOURCE ACROSS RESISTOR



$$f = 50 \text{ Hz}$$

$$f_p = 100 \text{ Hz}$$

$$i(t) = \frac{v(t)}{R}$$

$$i(t) = \frac{V_m}{R} \sin \omega t$$

$$i(t) = I_m \sin \omega t \quad \text{--- (1)}$$

$$P(t) = v(t) i(t) \rightarrow \text{instantaneous power}$$

$$P(t) = V_m \sin \omega t \cdot I_m \sin \omega t$$

$$P(t) = \frac{V_m I_m}{2} (1 - \cos 2\omega t)$$

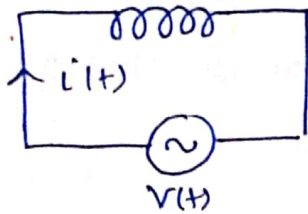
avg. power

$$P_{av} = \frac{I_m V_m}{2} = \frac{I_m}{\sqrt{2}} \frac{V_m}{\sqrt{2}} = V_{rms} \cdot I_{rms}$$

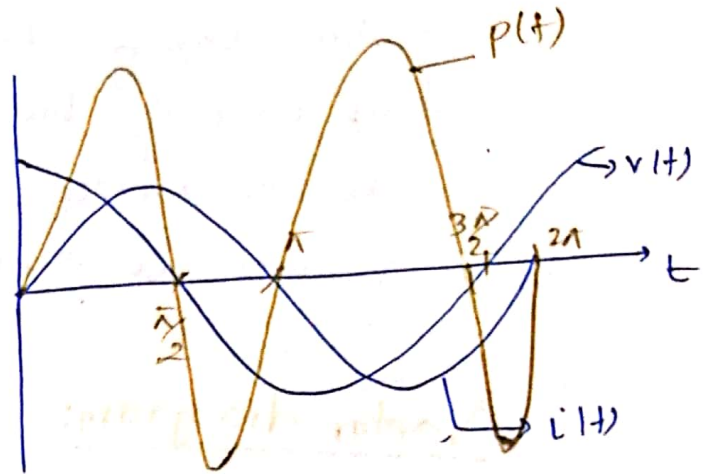
$$P_{av} = \frac{1}{2\pi} \int_0^{2\pi} P(t) dt$$

Prob 7 CASE-2

AC source across inductor:-



$i(t) = I_m \sin \omega t$
 ~~$i(t) = I_m \sin \omega t$~~



$$V = L \frac{di}{dt}$$

$f = 50 \text{ Hz}$
 $f_p = 100 \text{ Hz}$

$$v(t) = L I_m \omega \cos \omega t$$

$$v(t) = \omega L I_m \cos \omega t$$

$$v(t) = X_L I_m \sin(90^\circ + \omega t)$$

 $[V \text{ leads by } 90^\circ]$
 in L

Instantaneous power $P(t)$:

$$P(t) = v(t) i(t)$$

$$= X_m \sin \omega t I_m \sin \omega t$$

$$P(t) = \frac{V_m I_m \sin 2\omega t}{2}$$

Avg. power

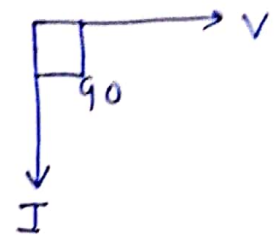
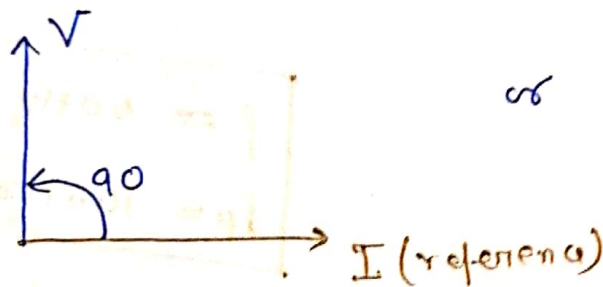
$$P_{av} = \frac{1}{2\pi} \int_0^{2\pi} P(t) dt$$

$P_{av} = 0$

CONCLUSION:-

- ① In +ve half cycle of power inductor takes energy from the source and in -ve half cycle of power inductor deliver energy to the source. \therefore Net power taken from source is equal to '0'.

Vector diagram:



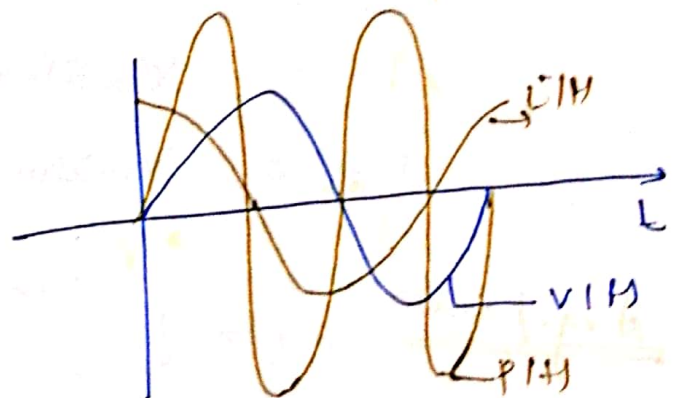
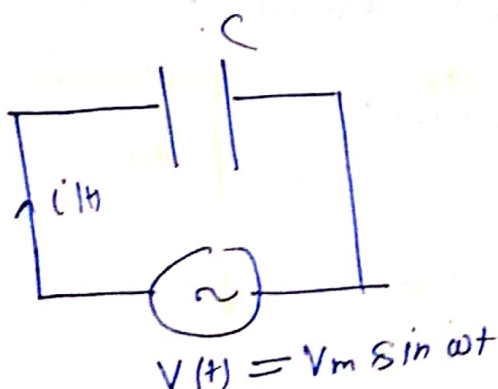
Series: ckt we consider this

parallel ckt and we consider this.

Both same but used for our convenience.

Cap P. ③

Ac. source across Capacitor



$$f = 50 \text{ Hz}$$

$$f_p = 100 \text{ Hz}$$

$$\dot{V} = \frac{e dv}{dt}$$

$$V = L \frac{di}{dt}$$

$$i = \omega e V_m \cos \omega t$$

$$i = \frac{V_m}{X_{\omega C}} \sin(\omega t + 90^\circ) \quad (X_C = \frac{1}{\omega C})$$

$$i = I_m \sin(\omega t + 90^\circ) \quad I_m = \frac{V_m}{(\frac{1}{\omega C})}$$

Instantaneous power:-

$$P(t) = V(t) i(t)$$

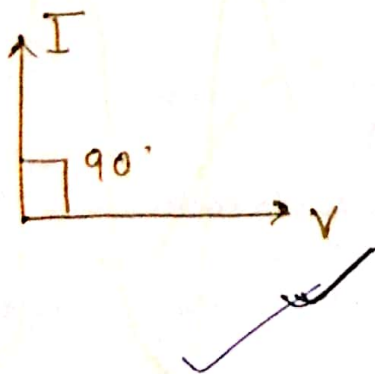
$$P(t) = \frac{V_m I_m}{2} \sin 2\omega t$$

$\left[\begin{array}{l} I \text{ Leads} \\ V \text{ by } 90^\circ \end{array} \right]$

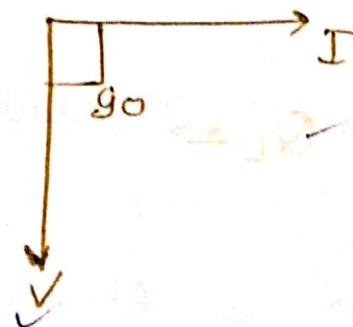
$$P_{avg} = \frac{1}{2\pi} \int_0^{2\pi} P(t) dt$$

$$P_{avg} = 0$$

Vector diagram:



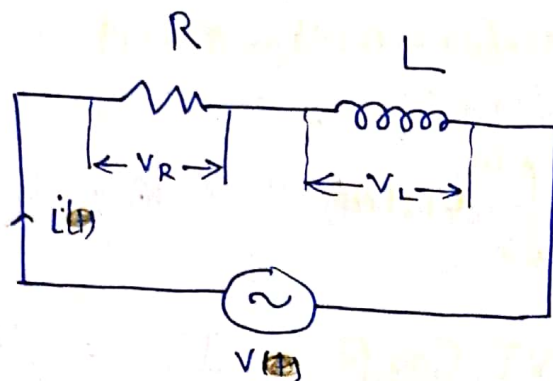
or



Ques. (4)

R-L Series ckt

$$V = L \frac{di}{dt}$$



By KVL

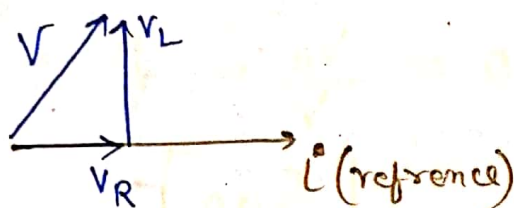
$$V = V_R + V_L$$

$$V = IR + IX_L \angle 90^\circ$$

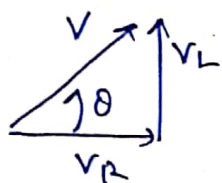
$$IZ = I(R + jX_L)$$

$$Z = R + jX_L$$

Vector diagram:-



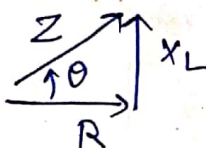
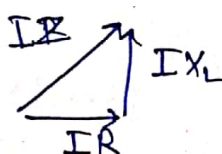
Voltage triangle:-



$$V = \sqrt{V_R^2 + V_L^2}$$

$$\theta = \tan^{-1}\left(\frac{V_L}{V_R}\right)$$

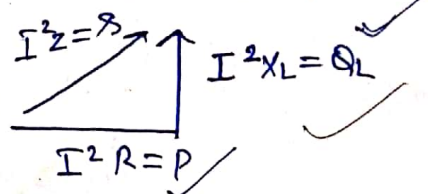
Impedance triangle



$$Z = \sqrt{R^2 + X_L^2}$$

$$\theta = \tan^{-1}\left(\frac{X_L}{R}\right)$$

Power triangle:



$$S = \sqrt{Q_L^2 + P^2}$$

$$\theta = \tan^{-1}\left(\frac{Q_L}{P}\right)$$

$P \rightarrow$ Active power,
or
true power
or
Real power
or
Avg power
or
effective power

$Q_L \Rightarrow$ inductive reactive power
[VAR] (Volt Amp. reactive)

$S \Rightarrow$ Apparent power or
complex power. (VA)
(Volt-ampere)

[Unit: watt]

$$P(t) = v(t) i(t)$$

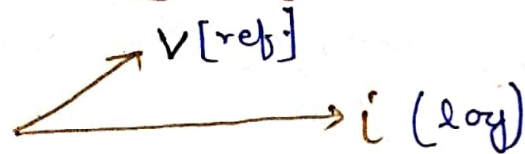
$$P(t) = V_m \sin(\omega t + \theta) I_m \sin \omega t$$

$$P_{av} = \frac{1}{2\pi} \int_0^{2\pi} P(t) dt$$

$$P_{avg} = VI \cos \theta$$

$$\text{power factor} = \cos \theta = \frac{V_R}{V} = \frac{R}{Z} = \frac{P}{S} \quad \left(\frac{B}{H} \right)$$

[lag]



Note:- Power factor angle indicates angle made by the current w.r.t voltage.

② while defining power factor for any combination ofckt voltage vector is taken as a reference.

① In the real time system only independent voltage source exist (source voltage = const)

② In the real time system Load are connected in the parallel (Load voltage = Constant)

Case (5)

$$\begin{aligned} P_{av} &= I^2 R = VI \cos \theta \\ Q_L &= I^2 X_L = VI \sin \theta \\ S &= I^2 Z = VI^* \end{aligned}$$

Ex: -

$$V = 5 \angle 20^\circ$$

$$i = 3 \angle 10^\circ$$

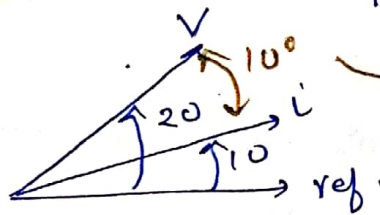
$$S = Vi$$

$$S = 15 \angle 30^\circ$$

$$S = VI^*$$

$$S = 15 \angle 10^\circ \text{ VA}$$

Correct Answer.

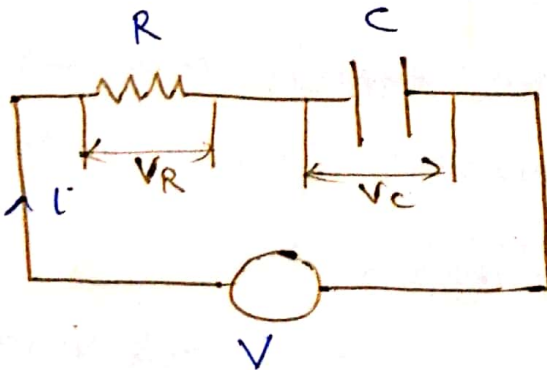


$$S = VI^*$$

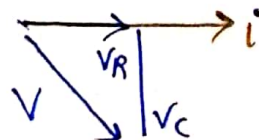
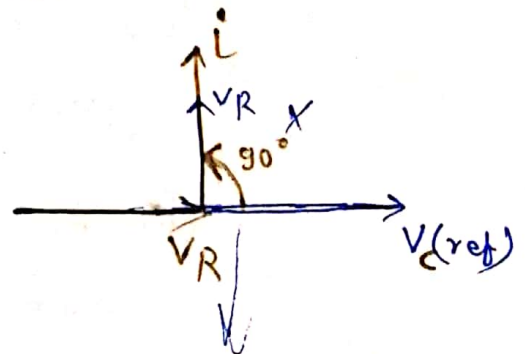
Complex power

Case (5)

RC Series ckt:



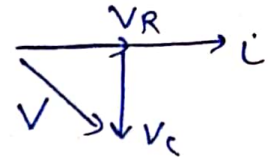
$$i = C \frac{dV}{dt}$$



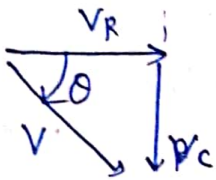
$$V = V_R + V_C$$

$$I_Z = I_R + I_{X_L} \angle -90^\circ \quad \therefore \boxed{V \text{ Lagging}}$$

$$\boxed{Z = R - jX_C}$$



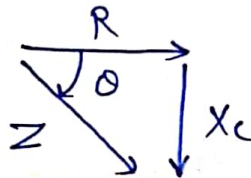
Voltage triangle



$$V = \sqrt{V_R^2 + V_C^2}$$

$$\theta = \tan^{-1} \left(\frac{-V_C}{V_R} \right)$$

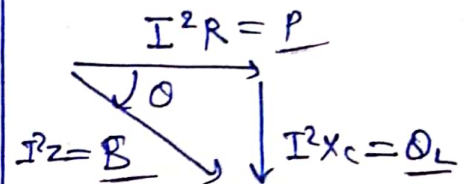
Impedance triangle



$$Z = \sqrt{R^2 + X_C^2}$$

$$\theta = \tan^{-1} \left(\frac{-X_C}{R} \right)$$

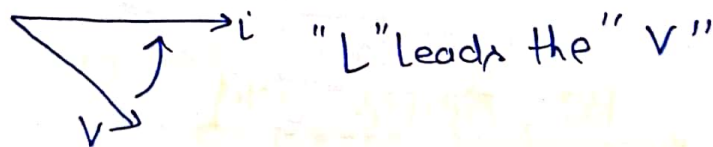
Power triangle



$$S = \sqrt{P^2 + Q_L^2}$$

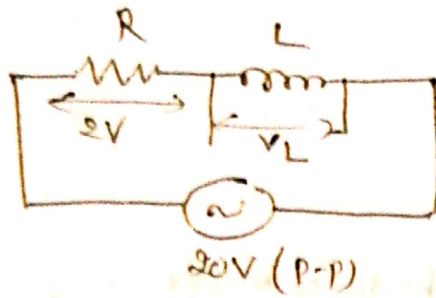
$$\theta = \tan^{-1} \left(\frac{-Q_L}{P} \right)$$

Power factor: - $\cos \theta = \frac{V_R}{V} = \frac{R}{Z} = \frac{P}{S}$ [lead]



Prob ①

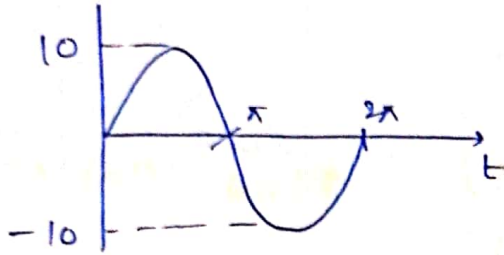
Find Voltage across inductor: -



Can't applied.

$$20 = 2V + V_L$$

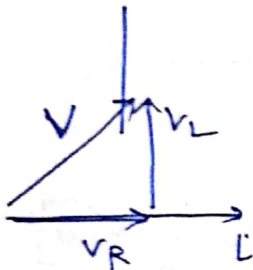
$$V_L =$$



Actual m-I

$$V_m = 10V \Rightarrow V_{rms} = \frac{V_m}{\sqrt{2}} = \frac{10}{\sqrt{2}}$$

$$V = \sqrt{V_R^2 + V_L^2}$$



$$\left(\frac{10}{\sqrt{2}}\right)^2 = V_R^2 + V_L^2$$

$$\frac{100}{2} = V_R^2 + V_L^2$$

$$\frac{100}{2} = (2V)^2 + V_L^2$$

Voltage triangle:

$$V_L = \sqrt{46}$$

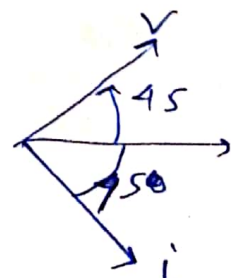
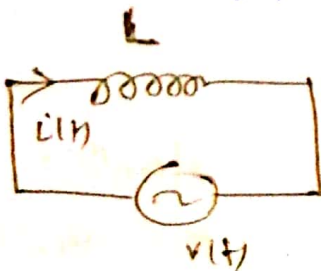
Prob ②

Find ckt element for given voltage and current equation

sol

$$v(t) = 9 \sin(t + 45^\circ)$$

$$i(t) = 3 \sin(t - 45^\circ)$$



V leads i .

Inductor.

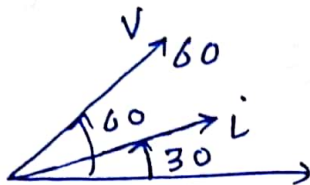
$$X_L = \frac{V_{rms}}{I_{rms}} = \frac{\frac{V_m}{\sqrt{2}}}{\frac{I_m}{\sqrt{2}}} = \frac{V_m}{I_m} = \frac{9}{3} = 3$$

$$X_L = \omega L = 3 \quad \therefore \quad \omega = 1$$

$$\boxed{L = 3H.}$$

Prob. (3) $V(t) = 9 \sin(2t + 60)$
 $i(t) = 3 \sin(t + 30)$

Find ckt element



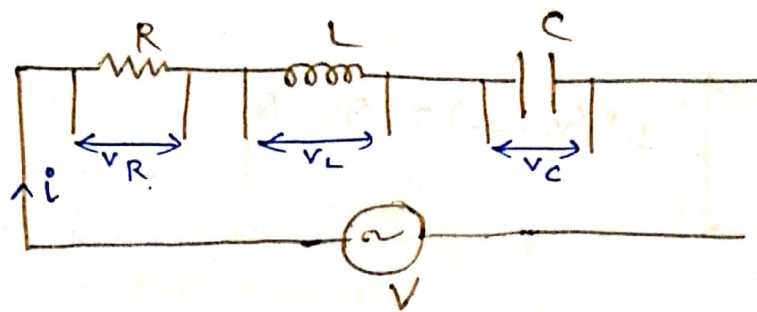
Note:— When voltage and

current equation have different

frequency it is not possible to find ckt element

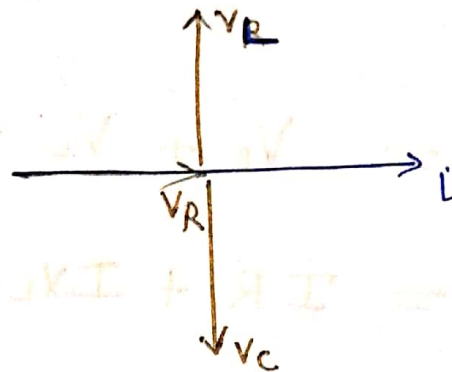
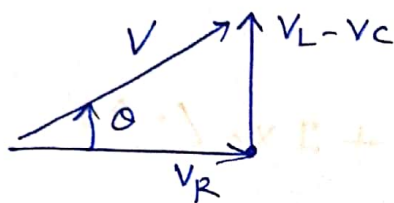
Ques: 6

RLC series ckt



Vector diagram:-

Ques: - ① $V_L > V_C$

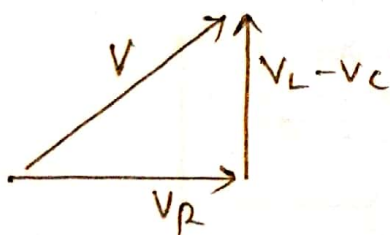


$\angle 90^\circ \quad \angle -90^\circ$

$$V = V_R + V_L + V_C$$

$$Z = R + j(X_L - X_C)$$

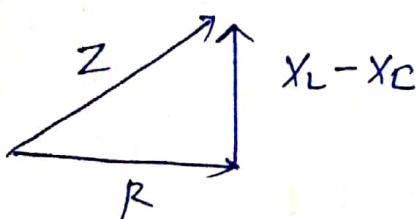
\Rightarrow Voltage triangle.



$$V = \sqrt{V_R^2 + (V_L - V_C)^2}$$

$$\theta = \tan^{-1} \left(\frac{V_L - V_C}{V_R} \right)$$

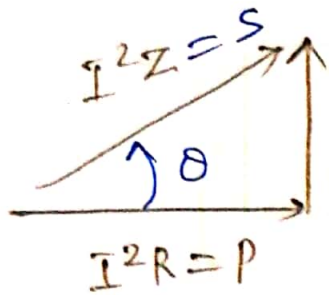
\Rightarrow Impedance triangle.



$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$\theta = \tan^{-1} \left(\frac{X_L - X_C}{R} \right)$$

Power triangle: -



$$I^2 (X_L - X_C) = Q_L - Q_C =$$

$$S = \sqrt{P^2 + (Q_L - Q_C)^2}$$

$$\theta = \tan^{-1} \left(\frac{Q_L - Q_C}{P} \right)$$

Imp:
Concept:-

$$V = V_R + V_L + V_C$$

$$IZ = IR + IX_L \angle 90^\circ + IX_C \angle -90^\circ$$

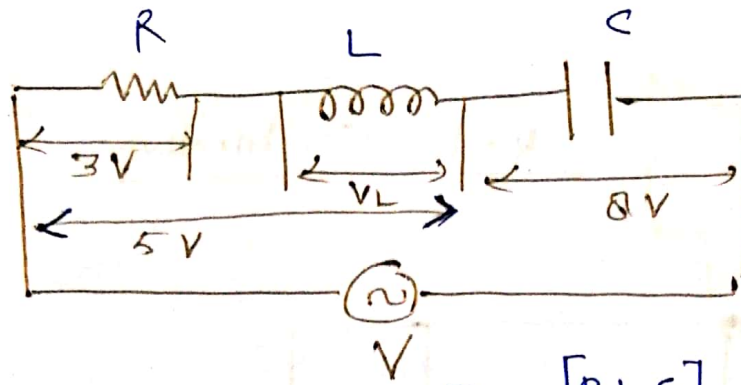
$$Z = R + jX_L - jX_C$$

$$Z = R + j(X_L - X_C)$$

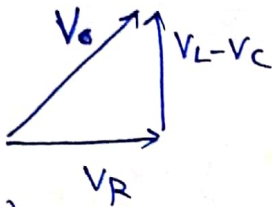
$$|Z| = \sqrt{R^2 + (X_L - X_C)^2}$$

$$\cos \theta = \text{p.f.} = \frac{R}{|Z|}$$

Prob: ①



Find V_L and V .



For $[RLC]$

$$V = \sqrt{V_R^2 + (V_L - V_C)^2} \quad \text{--- ①}$$

First (R-L)

$$5 = \sqrt{9 + V_L^2}$$

$$V = \sqrt{9 + (4 - 8)^2}$$

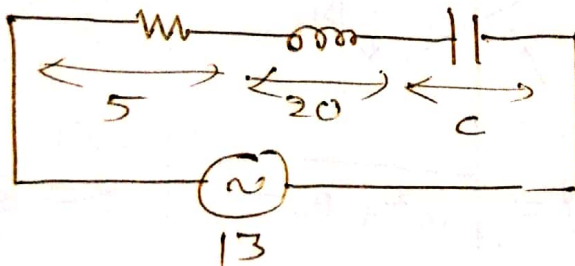
~~25 + 16 = 41~~

$$V = 5$$

$$V_L = 4$$

$$V_C = 8V$$

Prob: ② Find voltage across capacitor of ckt



Both: 8, and 32

$$V = \sqrt{5^2 + (20 - V_C)^2}$$

$$169 - 25 = (20 - V_C)^2$$

$$\text{If } V_L > V_C \quad \pm 12 = 20 - V_C$$

$$V_C = 8$$

$$\text{If } V_C > V_L$$

$$12 = V_C - 20$$

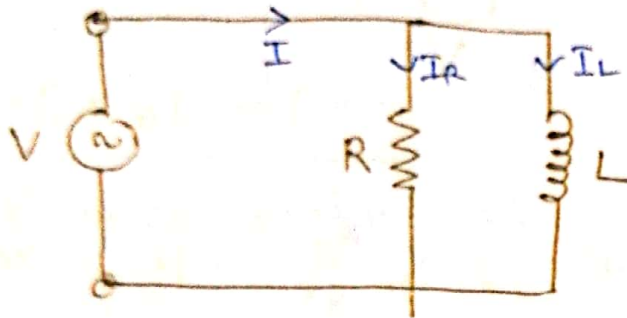
$$V_C = 32$$

PARALLEL COMBINATION:

#

Con- ①

R-L Combination



$$I = I_R + I_L$$

$$\frac{V}{Z} = \frac{V}{R} + \frac{V}{X_L} \angle -90^\circ$$

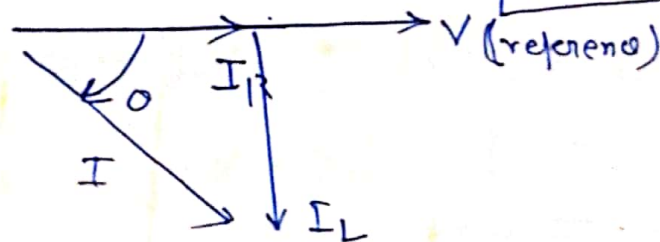
$$\frac{V}{Z} =$$

$$Y = G - jB_L$$

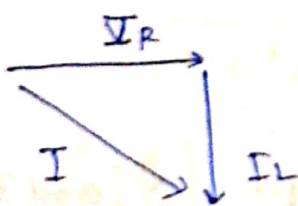
$$Y = G - jB_L$$

$$B_L = \frac{1}{\omega L}$$

$$G = \frac{1}{R}$$



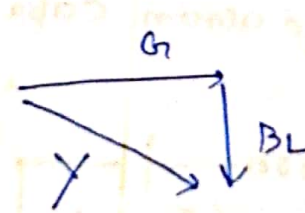
I - Tri.



$$I = \sqrt{I_R^2 + I_L^2}$$

$$\theta = \tan^{-1} \left(\frac{-I_L}{I_R} \right)$$

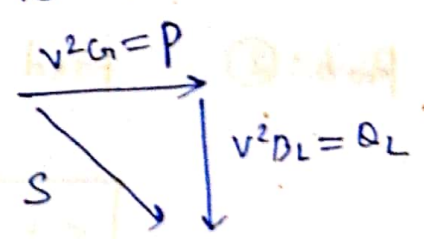
Z - tri.



$$Y = \sqrt{G^2 + B_L^2}$$

$$\theta = \tan^{-1} \left(\frac{-B_L}{G} \right)$$

P - tri.

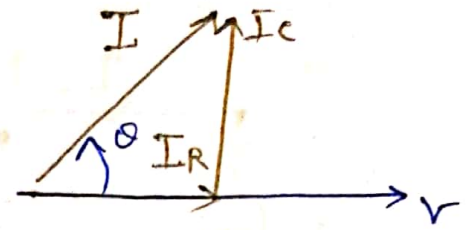
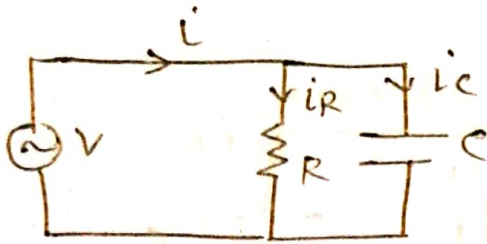


$$S = \sqrt{P^2 + Q_L^2}$$

$$\tan \theta = -\frac{Q_L}{P}$$

#2

R-C parallel ckt:



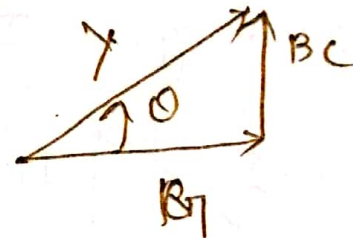
$$I = I_R + I_C$$

$$\frac{V}{Z} = \frac{V}{R} + \frac{V}{X_C} \angle 90^\circ$$

$$Y = G + jB_C$$

$$\therefore B_C = \omega C$$

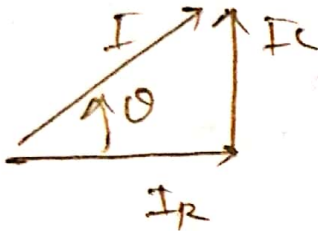
impedance or admittance triangle



$$Y = \sqrt{G^2 + B_C^2}$$

$$\theta = \tan^{-1} \left(\frac{B_C}{G} \right)$$

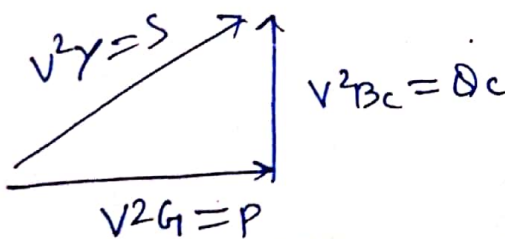
$\Rightarrow I - \text{tri}$



$$I = \sqrt{I_R^2 + I_C^2}$$

$$\theta = \tan^{-1} \left(\frac{I_C}{I_R} \right)$$

\Rightarrow Power triangle:



$$S = \sqrt{P^2 + Q_C^2}$$

$$\theta = \tan^{-1} \left(\frac{Q_C}{P} \right)$$