Task 1: Graph Analysis

Observations

To build a good strategy for the task at hand we must look at the basic features governing the graphs. And that would be the first and the last elements of the Places that we would assign as nodes in our graphs. The Winning strategy in case of only two players can be analyzed through some fundamental Graph theory concepts.

Let's take a scenario where Two players Alice and Bob are playing the game and we need to find an optimal strategy for Alice. Bob can follow the strategy but his strategy should not impact our approach towards the game.

There are 2 ways to play the game: some Offensive and some Defensive leveraging a specific set of rules.

Offensive Approach

- **Outdegree**: Outdegree can be exploited for a strategic edge in this scenario as an Offensive approach going for Nodes with the least outgoing edges.
  - Example: In `501.txt`, cities like `Pune`, `Porto Alegre`, `Port-au-Prince`, and `Xuzhou` have very low outdegrees (3), making them strategic choices for offensive play.

Defensive Approach

- **Density**: Density can be leveraged for a defensive approach as you could play defensively and just always choose the option which lets you stay in the Dense region so leaving you with no shortage of possible alternatives.
  - Example: In `502.txt`, cities like `Adelaide`, `Guangzhou`, `Gwangju`, and `Ganzhou` have high densities with a ratio of 0.96, making them ideal for defensive play.

Bridge Nodes

- **Bridges**: One could also leverage Bridges which link across communities with the average cycle parity being odd and hence exhausting any edge to leave the community hence stochastically increasing your chances to win.
  - Example: In `502.txt`, cities like `Athens` with high inter-community connections can act as bridges.

Node Centrality

- **Node Centrality**: Using Node centrality we could also identify important countries which could help us use other features better.
  - Example: In `501.txt`, cities like `New York City` and `Niamey` have high centrality with significant in-degrees and out-degrees, making them crucial nodes in the graph.

Across different Datasets

Analysis of Graph Theory Features Across Datasets

The approach defined remains consistent across all datasets, helping identify better strategies for all three cases. However, some methods lose effectiveness as the dataset size increases. Here's an analysis of which graph theory features work well across all datasets and which ones struggle with larger datasets:

Features That Work Well Across All Datasets:

1. **Node Centrality**: High centrality nodes consistently provide strategic advantages by controlling the game flow. This feature remains effective regardless of dataset size.
2. **Bridge Nodes**: Nodes that act as bridges between communities are valuable for strategic transitions and remain effective across all datasets.
3. **Outdegree**: Targeting nodes with the least outgoing edges for offensive strategies works well in all datasets, limiting the opponent's options effectively.

Features That Struggle with Larger Datasets:

1. **Density**: While density is useful for defensive strategies in smaller datasets, its effectiveness diminishes in larger datasets due to the increased complexity and number of connections.
2. **Diameter**: The diameter of the graph becomes less practical to leverage in larger datasets as the distance between nodes increases, making it harder to use this feature for strategic advantage.

By understanding which features maintain their effectiveness and which ones do not, players can adapt their strategies accordingly as the domain space of the game increases.

Conclusion

Based on the analysis, the following strategies can be derived:

1. **Offensive Strategy**: Target nodes with the least outgoing edges to limit the opponent's options.
2. **Defensive Strategy**: Stay within dense regions to maximize available moves.
3. **Bridge Nodes**: Utilize bridge nodes to transition between communities and exhaust opponent's moves.
4. **Node Centrality**: Focus on high centrality nodes to control the game flow.

By leveraging these graph properties and metrics, players can gain a competitive advantage in the game Atlas.