Probability - Combinatorics
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1. Evaluate 5P1×5P5

$Using\ nPr = \dfrac{n!}{(n-r)!}$

$5P1 \times 5P5 = \dfrac{5!}{(5-1)!} \times \dfrac{5!}{(5-5)!}$

$\qquad\qquad = \dfrac{5!}{4!} \times \dfrac{5!}{0!}$

$\qquad\qquad = 600$

$\qquad\qquad$ _or just use your calculator_

2. Evaluate $101P2 \times 99P2 \times 97P2 \times ... \times 5P2 \times 3P2$

$101P2 \times 99P2 \times 97P2 \times ... \times 5P2 \times 3P2 = \dfrac{101!}{(101-2)!} \times \dfrac{99!}{(99-2)!} \times \dfrac{97!}{(97-2)!} \times ... \times \dfrac{5!}{(5-2)!} \times \dfrac{3!}{(3-2)!}$

$\qquad\qquad = \dfrac{101!}{99!} \times \dfrac{99!}{97!} \times \dfrac{97!}{95!} \times ... \times \dfrac{5!}{3!} \times \dfrac{3!}{1!}$

$\qquad\qquad = \dfrac{101!}{1!}$

$\qquad\qquad = 101!$

3. On a team of 10 businessmen and 8 doctors, how many ways can we choose 3 businessmen and 3 doctors for the team?

$Using\ nCr = \dfrac{n!}{r!\,(n-r)!}$

$10C3 \times 8C3 = \dfrac{10!}{3!\,(10-3)!} \times \dfrac{8!}{3!\,(8-3)!}$

$\qquad\qquad = \dfrac{10!}{3!\,(7)!} \times \dfrac{8!}{3!\,(5)!}$

$\qquad\qquad = 6720$

$\qquad\qquad$ _or just use your calculator_

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4. What is the probability that if 6 random people are selected from 10 businessmen and 8 doctors, there are exactly 3 businessmen and 3 doctors?

$Using\ nCr = \dfrac{n!}{r!\,(n-r)!}\ \ and\ P(A) = \dfrac{n(A)}{n(S)}$

$$\frac{10C3 \times 8C3}{18C6} = \frac{\dfrac{10!}{3!\,(10-3)!} \times \dfrac{8!}{3!\,(8-3)!}}{\dfrac{18!}{6!\,(18-6)!}}$$

$$= \frac{\dfrac{10!}{3!\,(7)!} \times \dfrac{8!}{3!\,(5)!}}{\dfrac{18!}{6!\,(12)!}}$$

$$= \frac{80}{221} \approx 0.36$$

5. How many ways can the word ABACUS be arranged such that the vowels are put beside one another?

$Combine\ the\ vowels\ into\ 1\ letter{:}V \rightarrow \quad VBCS$
$Now\ this\ is\ a\ permutation\ question\ where\ you\ have\ to\ arrange\ the\ letters\ in\ an\ order$

$Using\ nPr = \dfrac{n!}{(n-r)!}$
$4P4 = \dfrac{4!}{(4-4)!}$
$\quad = 24$

$However, you\ still\ have\ to\ multply\ by\ \dfrac{3!}{2!}\ since\ within\ V, you\ can\ rearrange\ the\ three\ vowels,$
$where\ two\ of\ them\ are\ identical$

$24 \times \dfrac{3!}{2!} = 72$

6. How many 4 letter words can be created from the word TENNESSEE?

$TENNESSEE = EEEE\ NN\ SS\ T$

4 alike	3 alike, 1 different	2 alike, 2 alike	2 alike, 2 different	all different
$1C1 \times \dfrac{4!}{4!} = 1$	$1C1 \times 3C1 \times \dfrac{4!}{3!} = 12$	$3C2 \times \dfrac{4!}{2!\,2!} = 18$	$3C1 \times 3C1 \times \dfrac{4!}{2!} = 108$	$4C4 \times 4! = 24$

$Total = 1 + 12 + 18 + 108 + 24 = 163$