Combinatorics

Math Club

Basic Counting

1) How many ways can you arrange 4 people in a row?

$$\underline{\quad} \ \underline{\quad} \ \underline{\quad} \ \underline{\quad}$$

$$4 \times 3 \times 2 \times 1 = 24$$

2) How many 10 digit numbers that end with 2 are there? No repeating digits.

$$\underline{8} \times \underline{8} \times \underline{7} \times \underline{6} \times \underline{5} \times \underline{4} \times \underline{3} \times \underline{2} \times \underline{1} \times \underline{1}$$

Step 2

8 possibilities since the first

digit can't be 0 or 2

Start over here!

There's only one possibility: 2

Fill in the rest like the example above

3) How many possible postal codes are there?

Permutations

- The number of possible ways to order R out of N items
- Written as nPr
- nPr = n!/(n-r)!   Or   n*(n-1)*(n-2)*...*(n-r+1)

Example 1: How many 3 letter words can be made from ABCD where each letter can be used at most once?

nPr = 4P3 = 4!/(4-3)!

= 4!

= 24

Why? Because we have 4 options for the first letter, 3 from the second, and 2 for the third (4*3*2)

Permutations

- If the set of *n* items has *a* identical items, then there are n!/a! ways to arrange them

Example 2: How many ways can you stack 4 Advanced Functions textbooks, 5 Calculus textbooks, and 2 Statistics textbooks?

4 + 5 + 2 = 11 total textbooks

$$\frac{11!}{4!\ 5!\ 2!}$$
= 6930

Combinations

- The number of possible ways to pick R items from N, where order doesn't matter
- Same as permutations, but this time, order is irrelevant
- nCr = n!/(r!(n-r)!)   or   nCr = nPr/r!
- nPr/r! because there are r! ways to arrange the items, but this time order doesn't matter

Example 3: How many ways can we choose 3 math club members out of 10 to become executives?

        10C3 = 10!/(3!*7!)

              = 120

Permutations vs. Combinations

Example 4: How many ways can we choose 3 math club members out of 10 to become math club president, vice president, and secretary?

        10P3 = 720

Combination and Permutation Questions

1. Evaluate 5P1 $\times$ 5P5
2. Evaluate 101P2 $\times$ 99P2 $\times$ 97P2 $\times$ ... $\times$ 5P2 $\times$ 3P2
3. On a team of 10 businessmen and 8 doctors, how many ways can we choose 3 businessmen and 3 doctors for the team?
4. What is the probability that if 6 random people are selected from 10 businessmen and 8 doctors, there are exactly 3 businessmen and 3 doctors?
5. How many ways can the word ABACUS be arranged such that the vowels are put beside one another?
6. How many 4 letter words can be created from the word TENNESSEE?