Date: __.__.__

Parameter Evaluation

Name → Aryan Gupta
Roll no → 102103140
Group → 3C05

Q1)

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

$x_1, x_2, x_3, \ldots x_n \Rightarrow$ sample size of n

$L(x_1, x_2, x_3 \ldots x_n) = f(x_1) \cdot f(x_2) \cdots f(x_n)$

$$\Rightarrow \left(\frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x_1-\mu)^2}{2\sigma^2}}\right) \cdot \left(\frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x_2-\mu)^2}{2\sigma^2}}\right)$$

taking ln on both sides

$$\ln(L) = -\frac{n}{2} \ln(2\pi\sigma^2) + \sum_{i=1}^{n} \left(\frac{(x_i-\mu)^2}{2\sigma^2}\right) \quad -(1)$$

Partial derivative w.r.t μ

$$\frac{\partial \ln(L)}{\partial \mu} = 0 + \sum_{i=1}^{n} -\left(\frac{2(x_i-\mu)}{2\sigma^2}\right) = 0$$

$$= \sum_{i=1}^{n} (x_i - \mu) = 0$$

$$n\bar{x} - n\mu = 0$$

$$\bar{x} = \mu$$

hence $\theta_1 = \bar{x}$ is therefore sample mean.

Date: ___.___.___

Taking derivative w.r.t σ^2 (equ 1)

$$\frac{\partial \ln(L)}{\partial \sigma^2} = \frac{-n}{2\sigma^2} + \sum_{i=1} \frac{-(x_i-\mu)^2}{2\sigma^2} = 0$$

$$\Rightarrow -n + \sum_{i=1}^{n} \frac{-(x_i-\mu)^2}{\sigma^2} = 0$$

$$\Rightarrow n = \sum_{i=1} \frac{(x_i-\mu)^2}{\sigma^2}$$

$$\sigma^2 = \frac{1}{n} \sum_{i=1}^{n} (x_i-\mu)^2$$

hence $\theta_2 = \frac{1}{n} \sum_{i=1}^{n} (x_i-\mu)^2$

Q2 Binomial $\rightarrow {}^nC_{x_i}\, \theta^{x_i} (1-\theta)^{n-x_i}$

$$L = \prod_{i=1}^{n} {}^nC_{x_i}\, \theta^{x_i} (1-\theta)^{n-x_i}$$

log on both sides

$$\log L = \sum_{i=1}^{n} \left(\log ({}^nC_{x_i}) + \log \theta^{x_i} + \log (1-\theta)^{n-x_i}\right)$$

$$\log L = \sum_{i=1}^{n} \log ({}^nC_{x_i}) + \log \theta \sum_{i=1}^{n} x_i + \log(1-\theta) \sum_{i=1}^{n} (n-x_i)$$

differinciate θ

$$\frac{d \log(L)}{d\theta} = 0$$

$$\frac{1}{\theta} \sum x_i - \frac{1}{1-\theta} \sum (n-x_i) = 0$$

$$\frac{1}{\theta} \sum x_i - \frac{n^2}{1-\theta} + \frac{1}{1-\theta} \sum x_i = 0$$

$$\frac{1}{\theta(1-\theta)} \sum x_i = \frac{n^2}{1-\theta}$$

$$\boxed{\theta = \frac{\sum x_i}{n}}$$