# ★ ARITHMETIC PROGRESSIONS ★

An arithmetic progression is a list of numbers in which each term is obtained by adding a fixed number to the preceding term except the first term.

→ The fixed number is called the common difference (d) $\boxed{d = a_{k+1} - a_k}$

↳ $n^{th}$ term of A.P: $a_n = a + (n-1)d$ (a: first term; d: common diff.)

▫ Reverse: $(a_n)_{last} = l - (n-1)d$ (l: last term)

↳ SUM OF FIRST n TERMS OF AN AP: $S = \frac{n}{2}[2a + (n-1)d]$

$S = \frac{n}{2}[a + l]$ (l: last term)

NOTE: If a, b and c are in AP,

Arithmetic mean; $b = \frac{a+c}{2}$

→ The sum of first n positive integers: $S_n = \frac{n(n+1)}{2}$

⇒ DERIVATION OF SUM FORMULA:

$S = (a + (n-1)d) + (a + (n-2)d) + \dots\dots (a+d) + a$

→ Summing by pairing;

$S = [(2a + (n-1)d) + (2a + (n-1)d) + \dots\dots (2a + (n-1)d)]$

$= \frac{n}{2} * [2a + (n-1)d]$

∴ $\boxed{S_n = \frac{n}{2}(2a + (n-1)d)}$

Q1) Which term of the AP 3, 8, 13, 18 --- will be 55 more than its 20th term.

A) $d = 8-3 \Rightarrow 5$

$a = 3$

$a_n = a + (n-1)d$

$a_{20} = 3 + (20-1)d$

$= 3 + (19 \times 5)$

$= 98$

$a_n = 98 + 55$

$= 153$

$153 = 3 + (n-1)5$

$n-1 = \frac{150}{5}$

$\boxed{n = 31}$

Q2) If the 10th term of an AP is 52 and the 17th term is 20 more than the 13th term. Find the AP

A) $a_{10} = 52$

$a_{17} = a_{13} + 20$

$a + 16d = a + 12d + 20$

$4d = 20$

$d = 5$

$52 = a + 9(5)$

$a = 52 - 45$

$a = 7$

$\therefore$ AP = 7, 12, 17, 22, 27 - - -

Q3) Find the middle term of the AP

6, 13, 20, - - - - - , 216

A) $d = 7$

$a = 6$

$a_n = 216$

$216 = 6 + (n-1)7$

$210 = (n-1)7$

$n = 31$

middle $= \frac{31+1}{2} \Rightarrow 16^{th}$ term

$a_{16} = 6 + (16-1)(7)$

$= 6 + 105 \Rightarrow \boxed{111}$

③

Q) Find the 6th term from the end.

17, 14, 11, ----, -40

A) $(a_6)_{end} = -40 - (6-1)(-3)$

$= -40 - (-15)$

$= -40 + 15$

$= \boxed{-25}$

Q) Is -150 a term of AP 11, 8, 5, 2, ----?

A) $-150 = 11 + (n-1)(-3)$

$-150 - 11 = -3n + 3$

$-164 = -3n$

$n = \frac{164}{3}$ ∴ As n is a decimal, -150 is not a term of the given AP.

Q) If the sixth term of an AP is zero then show that its 33rd term is 3 times its 15th term.

A) $a_6 = 0$

$a + 5d = 0$

$a = -5d$

$a + 14d = -5d + 14d$

$= 9d$

$a_{33} = a + 32d$

$= -5d + 32d$

$= 27d$

$a_{33} : a_{12} = 27d : 9d$

$= 3 : 1$

∴ $a_{33} = 3 \times a_{15}$

④

Q) If 10 times the 10th term of an AP is equal to 15 times the 15th term, show that its 25th term is zero.

A) $10\,a_{10} = 15\,a_{15}$

$10(a+9d) = 15(a+14d)$

$a + 9d = \frac{3}{2}a + 21d$

$a - \frac{3}{2}a = 12d$

$-\frac{1}{2}a = 12d$

$a = -24d$

$a_{25} = a + 24d$

$= -24d + 24d$

$= 0$

$\therefore \boxed{a_{25} = 0}$

Q) A sum of Rs 700 is to be used to give seven cash prizes to students of a school for their overall performance. If each prize is 20 less than the preceding one, find the value of each of the prizes.

A) $S_7 = 700$

$n = 7$

$a = ?$

$d = -20$

$S = \frac{n}{2}[2a + (n-1)d]$

$700 = \frac{7}{2}[2a + 6(-20)]$

$200 = 2a - 120$

$a = 160$

$\therefore$ AP: 160, 140, 120, 100, 80, 60, 40