PROOF:

A method for ascertaining the truth.

↳ Experimentation and Observation
↳ Sampling and counter examples
↳ Listening to Authority
↳ Conviction

1 - Introduction and proofs

⇒ A proposition is a statement that is either True or False.

Eg.) $\forall n \in \mathbb{N}$, $n^2 + n + 41$ is a prime number.

↳ A predicate is a proposition whose truth depends on the value of a variable (n).

A mathematical proof is a verification of a proposition by chain of logical deductions from a set of axioms.

True for the first 39 natural numbers but then we notice that this is untrue

THERE ARE MULTIPLE PROPOSITIONS LIKE THAT WHICH HAVE BEEN PROVEN WRONG.

MORE CASES:

$\boxed{1}$ EULER'S THEOREM: $a^4 + b^4 + c^4 = d^4$ has no +ve int. solution

⇒ This was disproved later

$a = 95,800$ $b = 217,519$ $c = 414,560$

$d = 422,481$

$\boxed{2}$ ELLIPTIC CURVE: $313(x^3 + y^3) = z^3$ has no +ve int. solution

⇒ This is false but the smallest example has thousands of digits.

$\boxed{3}$ GOLDBACH'S CONUNDRUM: Every even +ve integer but 2 is the sum of 2 primes

IMPLIES (⇒)

An implication of $p \Rightarrow q$ is true if p is False or q is True.

p	q	$p \Rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

$\forall n \in \mathbb{Z}, n \geq 2 \iff n^2 \geq 4$ [False for $n = (-3)$]

↳ iff

p	q	p ⇒ q	q ⇒ p	p ⇔ q
T	T	T	T	T
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

Axioms: An axiom is a proposition that is assumed to be true.

Eg.) $a = b$ & $b = c$ $\therefore$ $a = c$

Contradictory Axiom Example: Given a line l and a point P not on Line L, there is exactly one line through P parallel to L.

Eucledian (Planar) geometry

Given a line l and a point P not on Line L, there is $\boxed{\text{no}}$ line through P parallel to L.

Spherical Geometry

Given a line l and a point P not on Line L, there $\boxed{\text{infinitely many}}$ lines through P parallel to L.

Hyperbolic geometry

- Axioms should be:

1) **Consistent:** If no proposition can be proved to be True and False.
2) **Complete:** If it can be used to prove every proposition is either true or false.

↳ Consistent and complete axioms are hard to find.