EXERCISE - 01 CHECK YOUR GRASP

SELECT THE CORRECT ALTERNATIVE (ONLY ONE CORRECT ANSWER)

1. The expression $\dfrac{\tan\left(x-\dfrac{\pi}{2}\right).\cos\left(\dfrac{3\pi}{2}+x\right)-\sin^3\left(\dfrac{7\pi}{2}-x\right)}{\cos\left(x-\dfrac{\pi}{2}\right).\tan\left(\dfrac{3\pi}{2}+x\right)}$ simplifies to -

(A) $(1 + \cos^2 x)$ (B) $\sin^2 x$ (C) $-(1 + \cos^2 x)$ (D) $\cos^2 x$

2. Exact value of $\cos^2 73 + \cos^2 47 - \sin^2 43 + \sin^2 107$ is equal to -

(A) 1/2 (B) 3/4 (C) 1 (D) none

3. The expression $\dfrac{\sin 22^\circ \cos 8^\circ + \cos 158^\circ \cos 98^\circ}{\sin 23^\circ \cos 7^\circ + \cos 157^\circ \cos 97^\circ}$ when simplified reduces to -

(A) 1 (B) -1 (C) 2 (D) none

4. The two legs of right triangle are $\sin\theta + \sin\left(\dfrac{3\pi}{2}-\theta\right)$ and $\cos\theta - \cos\left(\dfrac{3\pi}{2}-\theta\right)$. The length of its hypotenuse is

(A) 1 (B) $\sqrt{2}$ (C) 2 (D) some function of θ

5. If $\tan\theta = \sqrt{\dfrac{a}{b}}$ where a, b are positive reals then the value of $\sin\theta \sec^7\theta + \cos\theta \operatorname{cosec}^7\theta$ is -

(A) $\dfrac{(a+b)^3(a^4+b^4)}{(ab)^{7/2}}$ (B) $\dfrac{(a+b)^3(a^4-b^4)}{(ab)^{7/2}}$ (C) $\dfrac{(a+b)^3(b^4-a^4)}{(ab)^{7/2}}$ (D) $-\dfrac{(a+b)^3(a^4+b^4)}{(ab)^{7/2}}$

6. The expression $\dfrac{\sin(\alpha+\theta)-\sin(\alpha-\theta)}{\cos(\beta-\theta)-\cos(\beta+\theta)}$ is –

(A) independent of α (B) independent of β (C) independent of θ (D) independent of α and β

7. The tangents of two acute angles are 3 and 2. The sine of twice their difference is -

(A) 7/24 (B) 7/48 (C) 7/50 (D) 7/25

8. If $\dfrac{\sin 2\alpha - \sin 3\alpha + \sin 4\alpha}{\cos 2\alpha - \cos 3\alpha + \cos 4\alpha} = \tan k\alpha$ is an identity then the value of k is equal to -

(A) 2 (B) 3 (C) 4 (D) 6

9. Exact value of $\cos 20 + 2\sin^2 55 - \sqrt{2}\sin 65$ is -

(A) 1 (B) $\dfrac{1}{\sqrt{2}}$ (C) $\sqrt{2}$ (D) zero

10. If $\cos(\theta + \phi) = m\cos(\theta - \phi)$, then $\tan\theta$ is equal to -

(A) $\left(\dfrac{1+m}{1-m}\right)\tan\phi$ (B) $\left(\dfrac{1-m}{1+m}\right)\tan\phi$ (C) $\left(\dfrac{1-m}{1+m}\right)\cot\phi$ (D) $\left(\dfrac{1+m}{1-m}\right)\cot\phi$

11. If $\sin\theta + \operatorname{cosec}\theta = 2$, then the value of $\sin^8\theta + \operatorname{cosec}^8\theta$ is equal to -

(A) 2 (B) 2^8 (C) 2^4 (D) none of these

12. If the expression $4\sin 5\alpha \cos 3\alpha \cos 2\alpha$ is expressed as the sum of three sines then two of them are $\sin 4\alpha$ and $\sin 10\alpha$. The third one is -

(A) $\sin 8\alpha$ (B) $\sin 6\alpha$ (C) $\sin 5\alpha$ (D) $\sin 12\alpha$

13. The expression, $3\left[\sin^4\left(\dfrac{3\pi}{2}-\alpha\right)+\sin^4(3\pi+\alpha)\right]-2\left[\sin^6\left(\dfrac{\pi}{2}+\alpha\right)+\sin^6(5\pi-\alpha)\right]$ when simplified is equal to -

(A) 0 (B) 1 (C) 3 (D) $\sin 4\alpha + \cos 6\alpha$

14. If $\cos\theta = \dfrac{1}{2}\left(a+\dfrac{1}{a}\right)$ then $\cos 3\theta$ in terms of 'a' =

(A) $\dfrac{1}{4}\left(a^3+\dfrac{1}{a^3}\right)$ (B) $4\left(a^3+\dfrac{1}{a^3}\right)$ (C) $\dfrac{1}{2}\left(a^3+\dfrac{1}{a^3}\right)$ (D) none

15. $\frac{1}{\cos 290°} + \frac{1}{\sqrt{3}\sin 250°} =$

(A) $\frac{2\sqrt{3}}{3}$ (B) $\frac{4\sqrt{3}}{3}$ (C) $\sqrt{3}$ (D) none

16. The product cot 123 . cot 133 . cot 137 . cot 147 , when simplified is equal to -

(A) – 1 (B) tan 37 (C) cot 33 (D) 1

17. Given $\sin B = \frac{1}{5}\sin(2A + B)$ then, $\tan(A + B) = k\tan A$, where k has the value equal to -

(A) 1 (B) 2 (C) 2/3 (D) 3/2

18. If $A + B + C = \pi$ & $\sin\left(A + \frac{C}{2}\right) = k\sin\frac{C}{2}$, then $\tan\frac{A}{2}\tan\frac{B}{2} =$

(A) $\frac{k-1}{k+1}$ (B) $\frac{k+1}{k-1}$ (C) $\frac{k}{k+1}$ (D) $\frac{k+1}{k}$

19. The value of the expression $\frac{1 - 4\sin 10°\sin 70°}{2\sin 10°}$ is –

(A) 1/2 (B) 1 (C) 2 (D) none of these

20. Which of the following number (s) is / are rational ?

(A) sin15 (B) cos15 (C) sin15 cos15 (D) sin15 cos75

21. If α and β are two positive acute angles satisfying $\alpha - \beta = 15$ and $\sin\alpha = \cos 2\beta$ then the value of $\alpha + \beta$ is equal to-

(A) 35 (B) 55 (C) 65 (D) 85

22. If $\alpha + \beta + \gamma = 2\pi$, then -

(A) $\tan\frac{\alpha}{2} + \tan\frac{\beta}{2} + \tan\frac{\gamma}{2} = \tan\frac{\alpha}{2}\tan\frac{\beta}{2}\tan\frac{\gamma}{2}$ (B) $\tan\frac{\alpha}{2}\tan\frac{\beta}{2} + \tan\frac{\beta}{2}\tan\frac{\gamma}{2} + \tan\frac{\gamma}{2}\tan\frac{\alpha}{2} = 1$

(C) $\tan\frac{\alpha}{2} + \tan\frac{\beta}{2} + \tan\frac{\gamma}{2} = -\tan\frac{\alpha}{2}\tan\frac{\beta}{2}\tan\frac{\gamma}{2}$ (D) $\tan\frac{\alpha}{2}\tan\frac{\beta}{2} + \tan\frac{\beta}{2}\tan\frac{\gamma}{2} + \tan\frac{\gamma}{2}\tan\frac{\alpha}{2} = 0$

23. The value of sin10 + sin20 + sin30 +......+ sin360 is -

(A) 1 (B) 0 (C) –1 (D) none of these

24. If A and C are two angles such that $A + C = \frac{3\pi}{4}$, then (1+cotA)(1+cotC) equals -

(A) 1 (B) 2 (C) -1 (D) -2

25. $\log_{t_1}(4\sin 9°\cos 9°)$; where t_1= 4sin63 cos63 , equals -

(A) $\frac{\sqrt{5}+1}{4}$ (B) $\frac{\sqrt{5}-1}{4}$ (C) 1 (D) none of these

26. $l = \left(\frac{\cot^2 x.\cos^2 x}{\cot^2 x - \cos^2 x}\right)^2$ and $m = a^{\log_{\sqrt{a}}\left[2\cos\frac{y}{2}\right]}$, at $y = 4\pi$, then $l^2 + m^2$ is equal to -

(A) 4 (B) 16 (C) 17 (D) none of these

27. If $(a + b)\tan(\theta - \phi) = (a - b)\tan(\theta + \phi)$, then $\frac{\sin(2\theta)}{\sin(2\phi)}$ is equal to -

(A) ab (B) $\frac{a}{b}$ (C) $\frac{b}{a}$ (D) a^2b^2

<u>SELECT THE CORRECT ALTERNATIVES (ONE OR MORE THAN ONE CORRECT ANSWERS)</u>

28. If θ is internal angle of n sided regular polygon, then $\sin\theta$ is equal to -

(A) $\sin\frac{\pi}{n}$ (B) $\sin\frac{2\pi}{n}$ (C) $\sin\frac{\pi}{2n}$ (D) $\sin\frac{n}{\pi}$

29. If $\sin\theta+\sqrt{\sin\theta+\sqrt{\sin\theta+\sqrt{\sin\theta+......\infty}}}=\sec^4\alpha$, then $\sin\theta$ is equal to -

(A) $\sec^2\alpha\ \tan^2\alpha$ (B) $2\dfrac{(1-\cos 2\alpha)}{(1+\cos 2\alpha)^2}$ (C) $2\dfrac{(1+\cos 2\alpha)}{(1-\cos 2\alpha)^2}$ (D) $\cot^2\alpha\ \mathrm{cosec}^2\alpha$

30. If $\tan\dfrac{\theta}{2}=\mathrm{cosec}\theta-\sin\theta$, then -

(A) $\sin^2\dfrac{\theta}{2}=2\sin^2 18°$

(B) $\cos 2\theta+2\cos\theta+1=0$

(C) $\sin^2\dfrac{\theta}{2}=4\sin^2 18°$

(D) $\cos 2\theta + 2\cos\theta - 1 = 0$

31. If $\cos(A - B) = \dfrac{3}{5}$ & $\tan A\tan B = 2$, then -

(A) $\cos A\cos B=\dfrac{1}{5}$ (B) $\sin A\sin B=-\dfrac{2}{5}$ (C) $\cos(A+B)=-\dfrac{1}{5}$ (D) $\sin A\sin B=\dfrac{2}{5}$

32. Factors of $\cos 4\theta - \cos 4\phi$ are -

(A) $(\cos\theta + \cos\phi)$ (B) $(\cos\theta - \cos\phi)$ (C) $(\cos\theta + \sin\phi)$ (D) $(\cos\theta - \sin\phi)$

33. For the equation $\sin 3\theta+\cos 3\theta=1-\sin 2\theta$ -

(A) $\tan\theta=1$ is possible (B) $\cos\theta=0$ is possible (C) $\tan\dfrac{\theta}{2}=-1$ is possible (D) $\cos\dfrac{\theta}{2}=0$ is possible

34. If $2\tan 10 + \tan 50 = 2x$, $\tan 20 + \tan 50 = 2y$, $2\tan 10 + \tan 70 = 2w$ and $\tan 20 + \tan 70 = 2z$, then which of the following is/are true -

(A) $z > w > y > x$ (B) $w = x + y$ (C) $2y = z$ (D) $z + x = w + y$

35. If $(3 - 4\sin^2 1)(3 - 4\sin^2 3)(3 - 4\sin^2 3^2)$ ...... $(3 - 4\sin^2(3^{n-1})) = \sin a/\sin b$, where $n \in N$ & a, b are integers in radian, then the digit at the unit place of (a + b) may be-

(A) 4 (B) 0 (C) 8 (D) 2

CHECK YOUR GRASP — **ANSWER KEY** — EXERCISE-1

Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans.	B	C	A	B	A	C	D	B	A	C	A	B	B	C	B
Que.	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
Ans.	D	D	A	B	C	C	A	B	B	D	C	B	B	A,B	A,D
Que.	31	32	33	34	35										
Ans.	A,C,D	A,B,C,D	A,B,C	A,B,C,D	A,B,C,D										

EXERCISE - 02

BRAIN TEASERS

SELECT THE CORRECT ALTERNATIVES (ONE OR MORE THAN ONE CORRECT ANSWERS)

1. Let m = tan 3 and n = sec 6, then which of the following statement(s) does/do not hold good ?

(A) m & n both are positive(B) m & n both are negative

(C) m is positive and n is negative (D) m is negative and n is positive

2. If $\sqrt{\dfrac{1-\sin A}{1+\sin A}}+\dfrac{\sin A}{\cos A} = \dfrac{1}{\cos A}$, for all permissible values of A, then A belongs to -

(A) first quadrant (B) second quadrant (C) third quadrant (D) fourth quadrant

3. If $\pi < 2\theta < \dfrac{3\pi}{2}$, then $\sqrt{2+\sqrt{2+2\cos 4\theta}}$ equals -

(A) $-2\cos\theta$ (B) $-2\sin\theta$ (C) $2\cos\theta$ (D) $2\sin\theta$

4. $\dfrac{\sin^3\theta-\cos^3\theta}{\sin\theta-\cos\theta} - \dfrac{\cos\theta}{\sqrt{1+\cot^2\theta}} - 2\tan\theta\cot\theta = -1$ if -

(A) $\theta \in \left(0,\dfrac{\pi}{2}\right)$ (B) $\theta \in \left(\dfrac{\pi}{2},\pi\right)$ (C) $\theta \in \left(\pi,\dfrac{3\pi}{2}\right)$ (D) $\theta \in \left(\dfrac{3\pi}{2},2\pi\right)$

5. If sec A = $\dfrac{17}{8}$ and cosecB = $\dfrac{5}{4}$ then sec (A + B) can have the value equal to -

(A) $\dfrac{85}{36}$ (B) $-\dfrac{85}{36}$ (C) $-\dfrac{85}{84}$ (D) $\dfrac{85}{84}$

6. Which of the following when simplified reduces to unity ?

(A) $\dfrac{1-2\sin^2\alpha}{2\cot\left(\dfrac{\pi}{4}+\alpha\right)\cos^2\left(\dfrac{\pi}{4}-\alpha\right)}$ (B) $\dfrac{\sin(\pi-\alpha)}{\sin\alpha-\cos\alpha\tan\dfrac{\alpha}{2}}+\cos(\pi-\alpha)$

(C) $\dfrac{1}{4\sin^2\alpha\cos^2\alpha}$ + $\dfrac{(1-\tan^2\alpha)^2}{4\tan^2\alpha}$ (D) $\dfrac{1+\sin 2\alpha}{(\sin\alpha+\cos\alpha)^2}$

7. It is known that $\sin\beta = \dfrac{4}{5}$ & $0 < \beta < \pi$ then the value of $\dfrac{\sqrt{3}\sin(\alpha+\beta)-\dfrac{2}{\cos\frac{\pi}{6}}\cos(\alpha+\beta)}{\sin\alpha}$ is -

(A) independent of α for all β in $(0, \pi)$ (B) $\dfrac{5}{\sqrt{3}}$ for $\tan\beta < 0$

(C) $\dfrac{\sqrt{3}(7+24\cot\alpha)}{15}$ for $\tan\beta > 0$ (D) none

8. In a triangle ABC, angle A is greater than angle B. If the measures of angles A and B satisfy the equation $2\tan x - k(1+\tan^2 x) = 0$, where $k \in (0, 1)$, then the measure of the angle C is -

(A) $\dfrac{\pi}{6}$ (B) $\dfrac{\pi}{3}$ (D) $\dfrac{5\pi}{12}$ (D) $\dfrac{\pi}{2}$

9. If $\dfrac{\sin 3\theta}{\sin\theta} = \dfrac{11}{25}$ then $\tan\dfrac{\theta}{2}$ can have the value equal to -

(A) 2 (B) 1/2 (C) -2 (D) $-1/2$

10. The expression $\left(\dfrac{\cos A+\cos B}{\sin A-\sin B}\right)^m + \left(\dfrac{\sin A+\sin B}{\cos A-\cos B}\right)^m$ where m $\in$ N, has the value -

(A) $2\cot^m\left(\dfrac{A-B}{2}\right)$, if m is odd
(B) 0, if m is odd
(C) $2\cot^m\left(\dfrac{A-B}{2}\right)$, if m is even
(D) 0, if m is even

11. If cos(A – B) = 3/5, and tanA tanB = 2, then -

(A) $\cos A\cos B=\dfrac{1}{5}$ (B) $\sin A\sin B=\dfrac{-2}{5}$ (C) $\cos(A+B)=\dfrac{-1}{5}$ (D) none of these

12. If $A+B=\dfrac{\pi}{3}$ and cosA + cosB = 1, then -

(A) cos(A – B) = 1/3
(B) | cosA – cosB | = $\sqrt{\dfrac{2}{3}}$
(C) $\cos(A-B)=-\dfrac{1}{3}$
(D) $|\cos A-\cos B|=\dfrac{1}{2\sqrt{3}}$

13. If A and B are acute positive angles satisfying the equations $3\sin^2A + 2\sin^2B = 1$ and 3sin2A – 2sin2B = 0 then A + 2B is-

(A) $\dfrac{\pi}{3}$ (B) $\dfrac{\pi}{2}$ (C) $\dfrac{2\pi}{3}$ (D) none

14. If A + B – C = 3π, then sinA + sinB – sinC is equal to -

(A) $4\sin\dfrac{A}{2}\sin\dfrac{B}{2}\cos\dfrac{C}{2}$ (B) $-4\sin\dfrac{A}{2}\sin\dfrac{B}{2}\cos\dfrac{C}{2}$ (C) $4\cos\dfrac{A}{2}\cos\dfrac{B}{2}\cos\dfrac{C}{2}$ (D) $-4\cos\dfrac{A}{2}\cos\dfrac{B}{2}\cos\dfrac{C}{2}$

15. 2 sin11 15' is equal to –

(A) $\sqrt{2-\sqrt{2+\sqrt{2}}}$ (B) $\sqrt{2-\sqrt{2-\sqrt{2}}}$ (C) $\sqrt{\dfrac{2+\sqrt{2-\sqrt{2}}}{2}}$ (D) $\sqrt{\dfrac{2+\sqrt{2+\sqrt{2}}}{2}}$

16. If $\tan^3\theta+\cot^3\theta=52$, then the value of $\tan^2\theta+\cot^2\theta$ is equal to -

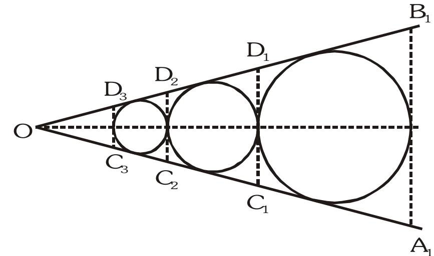
(A) 14 (B) 15 (C) 16 (D) 17

17. If 60 + α & 60 – α are the roots of $\sin^2x$ + bsinx + c = 0, then -

(A) $4b^2 + 3 = 12c$ (B) 4b + 3 = 12 c (C) $4b^2 - 3 = -12c$ (D) $4b^2 - 3 = 12c$

18. If $\angle B_1OA_1$ = 60 & radius of biggest circle is r. According to figure trapezium $A_1B_1D_1C_1$, $C_1D_1D_2C_2$, $C_2D_2D_3C_3$........ and so on are obtained. Sum of areas of all the trapezium is -

(A) $\dfrac{r^2}{2\sqrt{3}}$ (B) $\dfrac{9r^2}{2\sqrt{3}}$ (C) $\dfrac{9r^2}{\sqrt{3}}$ (D) $\dfrac{r^2}{9\sqrt{3}}$



19. If θ & ϕ are acute angles & $\sin\theta=\dfrac{1}{2}$, $\cos\phi=\dfrac{1}{3}$, then the value of $\theta+\phi$ belongs to the interval -

(A) $\left(\dfrac{\pi}{3},\dfrac{\pi}{2}\right]$ (B) $\left(\dfrac{\pi}{2},\dfrac{2\pi}{3}\right]$ (C) $\left(\dfrac{2\pi}{3},\dfrac{5\pi}{6}\right]$ (D) $\left(\dfrac{5\pi}{6},\pi\right]$

20. The maximum value of $\log_{20}(3\sin x - 4\cos x + 15)$ -
(A) 1 (B) 2 (C) 3 (D) 4

21. If $x^2 + y^2 = 9$ & $4a^2 + 9b^2 = 16$, then maximum value of $4a^2x^2 + 9b^2y^2 - 12abxy$ is -
(A) 81 (B) 100 (C) 121 (D) 144

22. Let A,B,C are 3 angles such that $\cos A + \cos B + \cos C = 0$ and if $\cos A \cos B \cos C = \lambda(\cos 3A + \cos 3B + \cos 3C)$, then λ is equal to -
(A) $\frac{1}{3}$ (B) $\frac{1}{6}$ (C) $\frac{1}{9}$ (D) $\frac{1}{12}$

23. $f(x) = \frac{\sin x}{\sqrt{1+\tan^2 x}} + \frac{\cos x}{\sqrt{1+\cot^2 x}}$ is constant in which of following interval -
(A) $\left(0, \frac{\pi}{2}\right)$ (B) $\left(\frac{\pi}{2}, \pi\right)$ (C) $\left(\pi, \frac{3\pi}{2}\right)$ (D) $\left(\frac{3\pi}{2}, 2\pi\right)$

24. Let n be an odd integer. If $\sin n\theta = \sum_{r=0}^{n} b_r \sin^r \theta$, for every value of θ, then -
(A) $b_0 = 1,\ b_1 = 3$ (B) $b_0 = 0,\ b_1 = n$
(C) $b_0 = -1,\ b_1 = n$ (D) $b_0 = 0,\ b_1 = n^2 - 3n + 3$

25. For a positive integer n, let $f_n(\theta) = \left(\tan\frac{\theta}{2}\right)(1+\sec\theta)(1+\sec 2\theta)(1+\sec 4\theta)....(1+\sec 2^n\theta)$. Then **[JEE 99, 3M]**
(A) $f_2\left(\frac{\pi}{16}\right) = 1$ (B) $f_3\left(\frac{\pi}{32}\right) = 1$ (C) $f_4\left(\frac{\pi}{64}\right) = 1$ (D) $f_5\left(\frac{\pi}{128}\right) = 1$

BRAIN TEASERS — ANSWER KEY — EXERCISE-2

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	A,B,C	A,D	D	B	A,B,C,D	A,B,D	D	D	A,B,C,D	B,C
Que.	11	12	13	14	15	16	17	18	19	20
Ans.	A,C	B,C	B	D	A	A	D	C	B	A
Que.	21	22	23	24	25					
Ans.	D	D	B,D	B	A,B,C,D					

EXERCISE - 03 — MISCELLANEOUS TYPE QUESTIONS

TRUE / FALSE

1. If A + B + C = π, then cos2A + cos2B + cos2C + 4cosA cosB cosC is positive.

2. $(\tan 20 \ \ \tan 40 \ \ \tan 80)^2$ is a prime number.

3. ....$\sin^8\theta \le \sin^6\theta \le \sin^4\theta \le \sin^2\theta \le 1$ also ....$\cos^8\theta \le \cos^6\theta \le \cos^4\theta \le \cos^2\theta \le 1$.

FILL IN THE BLANKS

1. If tan α = 2 and $\alpha \in \left(\pi, \dfrac{3\pi}{2}\right)$ then the value of the expression $\dfrac{\cos\alpha}{\sin^3\alpha + \cos^3\alpha}$ is equal to ....

2. The expression $\dfrac{\sin^4 t + \cos^4 t - 1}{\sin^6 t + \cos^6 t - 1}$ when simplified reduces to ....

3. Exact value of tan 200 (cot 10 − tan 10) is ....

4. $96\sqrt{3}\ \sin\dfrac{\pi}{48}\cos\dfrac{\pi}{48}\cos\dfrac{\pi}{24}\cos\dfrac{\pi}{12}\cos\dfrac{\pi}{6}$ has the value = ...

5. If $[1 - \sin(\pi + \alpha) + \cos(\pi + \alpha)]^2 + \left[1 - \sin\left(\dfrac{3\pi}{2} + \alpha\right) + \cos\left(\dfrac{3\pi}{2} - \alpha\right)\right]^2 = a + b\sin 2\alpha$ then the value of 'a' & 'b' are.................. & .... respectively.

6. The least value of the expression $\dfrac{\cot 2x - \tan 2x}{1 + \sin\left(\dfrac{5\pi}{2} - 8x\right)}$ for $0 < x < \dfrac{\pi}{8}$ is..................

MATCH THE COLUMN

Following question contains statements given in two columns, which have to be matched. The statements in **Column-I** are labelled as A, B, C and D while the statements in **Column-II** are labelled as p, q, r and s. Any given statement in **Column-I** can have correct matching with **ONE** statement in **Column-II**.

1.

	Column-I		Column-II
(A)	cosec 10 − $\sqrt{3}$ sec 10 =	(p)	$-\dfrac{1}{2}$
(B)	4 cos 20 − $\sqrt{3}$ cot 20 =	(q)	−1
(C)	$\dfrac{2\cos 40^\circ - \cos 20^\circ}{\sin 20^\circ}$ =	(r)	$\sqrt{3}$
(D)	$2\sqrt{2}\sin 10^\circ\left[\dfrac{\sec 5^\circ}{2} + \dfrac{\cos 40^\circ}{\sin 5^\circ} - 2\sin 35^\circ\right]$ =	(s)	4

Following question contains statements given in two columns, which have to be matched. The statements in **Column-I** are labelled as A, B, C and D while the statements in **Column-II** are labelled as p, q, r, s and t. Any given statement in **Column-I** can have correct matching with **ONE OR MORE** statement(s) in **Column-II**.

2. If maximum and minimum values of expression are λ and μ respectively then match the columns :

	Column-I		Column-II
(A)	$\sin^6\theta + \cos^6\theta$ for all θ	(p)	$\lambda + \mu = 2$
(B)	$\log_{\sqrt{5}}\left[\sqrt{2}(\sin\theta - \cos\theta) + 3\right]$ for all θ	(q)	$\lambda + \mu = 6$
(C)	$\dfrac{7 + 6\tan\theta - \tan^2\theta}{(1 + \tan^2\theta)}$ for all real values of $\theta \sim \dfrac{\pi}{2}$	(r)	$\lambda - \mu = 10$
(D)	$5\cos\theta + 3\cos(\theta + \dfrac{\pi}{3}) + 3$ for all real	(s)	$\lambda - \mu = 14$
	values of θ	(t)	$\lambda + \mu = \dfrac{5}{4}$

ASSERTION & REASON

These questions contains, Statement I (assertion) and Statement II (reason).

(A) Statement-I is true, Statement-II is true ; Statement-II is correct explanation for Statement-I.

(B) Statement-I is true, Statement-II is true ; Statement-II is NOT a correct explanation for Statement-I.

(C) Statement-I is true, Statement-II is false.

(D) Statement-I is false, Statement-II is true.

1. **Statement-I** : $\tan 5\theta - \tan 3\theta - \tan 2\theta = \tan 5\theta \tan 3\theta \tan 2\theta$

Because

Statement-II : $x = y + z \Rightarrow \tan x - \tan y - \tan z = \tan x \tan y \tan z$.

(A) A (B) B (C) C (D) D

2. **Statement-I** : If $\sin\theta + \operatorname{cosec}\theta = 2$, then $\sin^n\theta + \operatorname{cosec}^n\theta = 2^n$.

Because

Statement-II : If $a + b = 2$, $ab = 1$, then $a = b = 1$

(A) A (B) B (C) C (D) D

3. **Statement-I** : $\sec^2\theta = \dfrac{4xy}{(x+y)^2}$ is positive for all real values of x and y only when $x = y$

Because

Statement-II : $t^2 \geq 0 \ \forall \ t \in R$

(A) A (B) B (C) C (D) D

4. **Statement-I** : If A is obtuse angle in ΔABC, then $\tan B \tan C < 1$

Because

Statement-II : In ΔABC, $\tan A = \dfrac{\tan B + \tan C}{\tan B \tan C - 1}$

(A) A (B) B (C) C (D) D

5. **Statement-I** : $\cos^3\alpha + \cos^3\left(\alpha + \dfrac{2\pi}{3}\right) + \cos^3\left(\alpha + \dfrac{4\pi}{3}\right) = 3\cos\alpha\cos\left(\alpha + \dfrac{2\pi}{3}\right)\cos\left(\alpha + \dfrac{4\pi}{3}\right)$

Because

Statement-II : If $a + b + c = 0 \Leftrightarrow a^3 + b^3 + c^3 = 3abc$

(A) A (B) B (C) C (D) D

COMPREHENSION BASED QUESTIONS

Comprehension # 1

Continued product $\cos\alpha\cos 2\alpha\cos 2^2\alpha$ .... $\cos 2^{n-1}\alpha$

$$= \begin{cases} \dfrac{\sin 2^n\alpha}{2^n \sin\alpha}, & \text{if } \alpha \neq n\pi \\ \dfrac{1}{2^n}, & \text{if } \alpha = \dfrac{\pi}{2^n+1} \quad \text{i.e. } 2^n\alpha = \pi - \alpha \\ -\dfrac{1}{2^n}, & \text{if } \alpha = \dfrac{\pi}{2^n-1} \quad \text{i.e. } 2^n\alpha = \pi + \alpha \end{cases}$$

Where, $n \in I$ (Integer)

On the basis of above information, answer the following questions :

1. The value of $\cos\dfrac{2\pi}{7}\cos\dfrac{4\pi}{7}\cos\dfrac{6\pi}{7}$ is -

(A) $-1/2$ (B) $1/2$ (C) $1/4$ (D) $1/8$

2. If $\alpha = \dfrac{\pi}{15}$, then the value of $\prod_{r=1}^{7}\cos r\alpha$ is -

(A) $\dfrac{1}{128}$ (B) $-\dfrac{1}{128}$ (C) $\dfrac{1}{64}$ (D) $\dfrac{1}{32}$

3. The value of $\sin\left(\frac{\pi}{14}\right)\sin\left(\frac{3\pi}{14}\right)\sin\left(\frac{5\pi}{14}\right)\sin\left(\frac{7\pi}{14}\right)\sin\left(\frac{9\pi}{14}\right)\sin\left(\frac{11\pi}{14}\right)\sin\left(\frac{13\pi}{14}\right)$ is -

(A) 1 (B) $\frac{1}{8}$ (C) $\frac{1}{32}$ (D) $\frac{1}{64}$

Comprehension # 2

The measure of an angle in degrees, grades and radians be D, G and C respectively, then the relation between them

$$\frac{D}{90} = \frac{G}{100} = \frac{2C}{\pi} \text{ but } 1^c = \left(\frac{180}{\pi}\right)^\circ$$

$$\simeq 57°,17', 44.8''$$

and sum of interior angles of a n-sided regular polygon is $(2n-4)\pi/2$

On the basis of above information, answer the following questions :

1. Which of the following are correct -

(A) $\sin 1° < \sin 1$ (B) $\cos 1° > \cos 1$ (C) $\cos 1° < \cos 1$ (D) $\sin 1° < \frac{\pi}{180}\sin 1$

2. The angles between the hour hand and minute hand of a clock at half past three is -

(A) $\frac{\pi}{3}$ (B) $\frac{\pi}{4}$ (C) $\frac{5\pi}{12}$ (D) $\frac{7\pi}{12}$

3. The number of sides of two regular polygon are as 5 : 4 and the difference between their angles is $\frac{\pi}{20}$, then the number of sides in the polygons respectively are-

(A) 25, 20 (B) 20, 16 (C) 15, 12 (D) 10, 8

4. One angle of a triangle is $\frac{4x}{3}$ grades and another is 3x degrees, while the third is $\frac{2\pi x}{75}$ radians. Then the angles in degrees are-

(A) 20 , 60 , 100 (B) 24 , 60 , 96 (C) 36 , 60 , 84 (D) 20 , 40 , 120

MISCELLANEOUS TYPE QUESTION **ANSWER KEY** EXERCISE -3

- ***True / False***

1. F 2. T 3. T

- ***Fill in the Blanks***

1. $\frac{5}{9}$ 2. $\frac{2}{3}$ 3. 2 4. 9 5. a = 4 & b = – 2 6. 2

- ***Match the Column***

1. (A)→(s), (B)→(q), (C)→(r), (D)→(s) 2. (A)→(t), (B)→(p), (C)→(q,r), (D)→(q,s)

- ***Assertion & Reason***

1. A 2. D 3. B 4. A 5. C

- ***Comprehension Based Questions***

Comprehension # 1 : 1. D 2. A 3. D

Comprehension # 2 : 1. A,B 2. C 3. D 4. B

EXERCISE - 04 [A] CONCEPTUAL SUBJECTIVE EXERCISE

1. If cos (y − z) + cos (z − x) + cos (x − y) = $-\frac{3}{2}$, prove that cos x + cos y + cos z = 0 = sin x + siny + sinz.

2. Prove that, $\cos 2\alpha = 2\sin\beta + 4\cos(\alpha+\beta)\sin\alpha\ \sin\beta + \cos 2(\alpha+\beta)$

3. For all values of α, β, γ prove that :

$$\cos\alpha + \cos\beta + \cos\gamma + \cos(\alpha+\beta+\gamma) = 4\cos\frac{\alpha+\beta}{2}\cdot\cos\frac{\beta+\gamma}{2}\cdot\cos\frac{\gamma+\alpha}{2}.$$

4. If $\cos(\alpha+\beta) = \frac{4}{5}$; $\sin(\alpha-\beta) = \frac{5}{13}$ & α, β lie between 0 & $\frac{\pi}{4}$, then find the value of $\tan 2\alpha$

5. Prove that :

(a) $\sin^4\frac{\pi}{16} + \sin^4\frac{3\pi}{16} + \sin^4\frac{5\pi}{16} + \sin^4\frac{7\pi}{16} = \frac{3}{2}$

(b) sin 6 . sin 42 . sin 66 . sin 78 = cos 6 . cos 42 . cos 66 . cos 78 = $\frac{1}{16}$

6. If $\cos\theta = \frac{\cos\alpha - e}{1 - e\cos\alpha}$, prove that $\tan\frac{\theta}{2} = \pm\sqrt{\frac{1+e}{1-e}}\ \tan\frac{\alpha}{2}$.

7. Prove that, $\cot 7\frac{1^\circ}{2}$ or $\tan 82\frac{1^\circ}{2} = \left(\sqrt{3}+\sqrt{2}\right)\left(\sqrt{2}+1\right)$ or $\sqrt{2}+\sqrt{3}+\sqrt{4}+\sqrt{6}$

8. Prove that : $\text{cosec}\,\theta + \text{cosec}\,2\theta + \text{cosec}\,2^2\theta + \ldots\ldots + \text{cosec}\,2^{n-1}\theta = \cot(\theta/2) - \cot 2^{n-1}\theta$

9. If $\alpha + \beta = c$ where $\alpha, \beta > 0$ each lying between 0 and $\pi/2$ and c is a constant, find the maximum or minimum value of −

(a) $\sin\alpha + \sin\beta$ (b) $\sin\alpha\,\sin\beta$ (c) $\tan\alpha + \tan\beta$

10. (a) Find the maximum & minimum values of $27^{\cos 2x}.81^{\sin 2x}$.

(b) Find the smallest positive values of x & y satisfying, $x - y = \frac{\pi}{4}$, $\cot x + \cot y = 2$

CONCEPTUAL SUBJECTIVE EXERCISE — **ANSWER KEY** — EXERCISE-4(A)

4. $\frac{56}{33}$

9. (a) max.= $2\sin c/2$ (b) max.= $\sin^2 c/2$ (c) min.= $2\tan c/2$

10. (a) Minimum Value = 3^{-5} ; Maximum Value = 3^5 (b) $x = \frac{5\pi}{12}$, $y = \frac{\pi}{6}$

EXERCISE - 04 [B] BRAIN STORMING SUBJECTIVE EXERCISE

1. Prove that :

(a) In an acute angled triangle ABC, the least values of $\Sigma \sec A$ and $\Sigma \tan^2 A$ are 6 and 9 respectively.

(b) In triangle ABC, the least values of $\Sigma \operatorname{cosec}\left(\frac{A}{2}\right)$ and $\Sigma \sec^2\left(\frac{A}{2}\right)$ are 6 and 4 respectively.

2. Prove that ; $\operatorname{cosec} x \, . \, \operatorname{cosec} 2x \, . \, \sin 4x \, . \, \cos 6x \, . \, \operatorname{cosec} 10x$

$$= \frac{\cos 3x}{\sin 2x \sin 4x} + \frac{\cos 5x}{\sin 4x \sin 6x} + \frac{\cos 7x}{\sin 6x \sin 8x} + \frac{\cos 9x}{\sin 8x \sin 10x} \; .$$

3. If $\tan\alpha = p/q$ where $\alpha = 6\beta$, α being an acute angle, prove that ;

$$\frac{1}{2}(p \operatorname{cosec} 2\beta - q \sec 2\beta) = \sqrt{p^2 + q^2}$$

4. If $\tan\left(\frac{\pi}{4} + \frac{y}{2}\right) = \tan^3\left(\frac{\pi}{4} + \frac{x}{2}\right)$ prove that $\sin y = \sin x\left[\frac{3 + \sin^2 x}{1 + 3\sin^2 x}\right]$

5. Prove that from the equality $\frac{\sin^4 \alpha}{a} + \frac{\cos^4 \alpha}{b} = \frac{1}{a+b}$ follows the relation ;

$$\frac{\sin^8 \alpha}{a^3} + \frac{\cos^8 \alpha}{b^3} = \frac{1}{(a+b)^3}$$

6. If $P = \cos\frac{\pi}{19} + \cos\frac{3\pi}{19} + \cos\frac{5\pi}{19} + \ldots\ldots + \cos\frac{17\pi}{19}$ and

$Q = \cos\frac{2\pi}{21} + \cos\frac{4\pi}{21} + \cos\frac{6\pi}{21} + \ldots\ldots + \cos\frac{20\pi}{21}$, then find P – Q

7. Prove that : $4 \sin 27 = \left(5 + \sqrt{5}\right)^{1/2} - \left(3 - \sqrt{5}\right)^{1/2}$.

8. If $A+B+C = \pi$; prove that $\tan\frac{A}{2} + \tan\frac{B}{2} + \tan\frac{C}{2} \geq 1$.

9. If $\alpha + \beta = \gamma$, prove that $\cos\alpha + \cos\beta + \cos\gamma = 1 + 2\cos\alpha\cos\beta\cos\gamma$.

10. Prove that the triangle ABC is equilateral iff , $\cot A + \cot B + \cot C = \sqrt{3}$.

BRAIN STORMING SUBJECTIVE EXERCISE	ANSWER KEY	EXERCISE-4(B)
6. 1		

EXERCISE - 05 [A] JEE-[MAIN] : PREVIOUS YEAR QUESTIONS

1. Period of $f(x) = \sin^4 x + \cos^4 x$ is - [AIEEE-2002]

(1) π (2) $\frac{\pi}{2}$ (3) 2π (4) None of these

2. Period of $\sin^2 \theta$ is- [AIEEE-2002]

(1) π^2 (2) π (3) 2π (4) $\frac{\pi}{2}$

3. If $y = \sec^2 \theta + \cos^2 \theta,\ \theta \neq 0$, then- [AIEEE-2002]

(1) $y = 0$ (2) $y \leq 2$ (3) $y \geq -2$ (4) $y > 2$.

4. The value of $\frac{1-\tan^2 15°}{1+\tan^2 15°} =$ [AIEEE-2002]

(1) 1 (2) $\sqrt{3}$ (3) $\frac{\sqrt{3}}{2}$ (4) 2

5. If α is a root of $25 \cos^2 \theta + 5 \cos \theta - 12 = 0$, $\frac{\pi}{2} < \alpha < \pi$, then $\sin 2\alpha =$ [AIEEE-2002]

(1) $\frac{24}{25}$ (2) $-\frac{24}{25}$ (3) $\frac{13}{18}$ (4) $-\frac{13}{18}$

6. If $\sin(\alpha + \beta) = 1$, $\sin(\alpha-\beta) = \frac{1}{2}$, then $\tan(\alpha + 2\beta)\tan(2\alpha + \beta) =$ [AIEEE-2002]

(1) 1 (2) -1 (3) zero (4) None of these

7. If $\tan \theta = -\frac{4}{3}$, then $\sin \theta$ is- [AIEEE-2002]

(1) $-\frac{4}{5}$ but not $\frac{4}{5}$ (2) $-\frac{4}{5}$ or $\frac{4}{5}$ (3) $\frac{4}{5}$ but not $-\frac{4}{5}$ (4) None of these

8. $\sec^2 \theta = \frac{4xy}{(x+y)^2}$ is true if and only if - [AIEEE-2003]

(1) $x + y \neq 0$ (2) $x = y, x \neq 0$ (3) $x = y$ (4) $x \neq 0,\ y \neq 0$

9. If $u = \sqrt{a^2 \cos^2 \theta + b^2 \sin^2 \theta} + \sqrt{a^2 \sin^2 \theta + b^2 \cos^2 \theta}$ then the difference between the maximum and minimum values of u^2 is given by- [AIEEE-2004]

(1) $2(a^2 + b^2)$ (2) $2\sqrt{a^2+b^2}$ (3) $(a + b)^2$ (4) $(a - b)^2$

10. Let α,β be such that $\pi < \alpha-\beta < 3\pi$.

If $\sin \alpha + \sin \beta = -\frac{21}{65}$ and $\cos \alpha + \cos \beta = -\frac{27}{65}$, then the value of $\cos \frac{\alpha-\beta}{2}$ is- [AIEEE-2004]

(1) $-\frac{3}{\sqrt{130}}$ (2) $\frac{3}{\sqrt{130}}$ (3) $\frac{6}{65}0$ (4) $\frac{-6}{65}$

11. If $0 < x < \pi$, and $\cos x + \sin x = \frac{1}{2}$, then $\tan x$ is- [AIEEE-2006]

(1) $(4-\sqrt{7})/3$ (2) $-(4+\sqrt{7})/3$ (3) $(1+\sqrt{7})/4$ (4) $(1-\sqrt{7})/4$

12. Let $\cos(\alpha + \beta) = \frac{4}{5}$ and let $\sin(\alpha - \beta) = \frac{5}{13}$, where $0 \leq \alpha, \beta \leq \frac{\pi}{4}$. Then $\tan 2\alpha =$ [AIEEE-2010]

(1) $\frac{25}{16}$ (2) $\frac{56}{33}$ (3) $\frac{19}{12}$ (4) $\frac{20}{7}$

13. If A = $\sin^2 x + \cos^4 x$, then for all real x :- **[AIEEE-2011]**

(1) $1 \le A \le 2$ (2) $\frac{3}{4} \le A \le \frac{13}{16}$ (3) $\frac{3}{4} \le A \le 1$ (4) $\frac{13}{16} \le A \le 1$

14. In a ΔPQR, if $3 \sin P + 4 \cos Q = 6$ and $4 \sin Q + 3 \cos P = 1$, then the angle R is equal to : **[AIEEE-2012]**

(1) $\frac{3\pi}{4}$ (2) $\frac{5\pi}{6}$ (3) $\frac{\pi}{6}$ (4) $\frac{\pi}{4}$

PREVIOUS YEARS QUESTIONS — **ANSWER KEY** — EXERCISE-5 [A]

Que.	1	2	3	4	5	6	7	8	9	10
Ans.	2	2	4	3	2	1	2	2	4	1
Que.	11	12	13	14						
Ans.	2	2	3	3						

EXERCISE - 05 [B] JEE-[ADVANCED] : PREVIOUS YEAR QUESTIONS

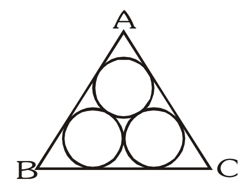
1. If $\alpha+\beta=\frac{\pi}{2}$ and $\beta+\gamma=\alpha$ then $\tan\alpha$ equals - [JEE 2001 Screening, 1M out of 35M]

(A) $2(\tan\beta+\tan\gamma)$ (B) $\tan\beta+\tan\gamma$ (C) $\tan\beta+2\tan\gamma$ (D) $2\tan\beta+\tan\gamma$

2. If θ and ϕ are acute angles satisfying $\sin\theta=\frac{1}{2}$, $\cos\phi=\frac{1}{3}$, then $\theta+\phi\in$ [JEE 2004 Screening]

(A) $\left(\frac{\pi}{3},\frac{\pi}{2}\right]$ (B) $\left(\frac{\pi}{2},\frac{2\pi}{3}\right)$ (C) $\left(\frac{2\pi}{3},\frac{5\pi}{6}\right)$ (D) $\left(\frac{5\pi}{6},\pi\right)$

3. In an equilateral triangle, 3 coins of radii 1 unit each are kept so that they touch each other and also the sides of the triangle. Area of the triangle is -



(A) $4+2\sqrt{3}$ (B) $6+4\sqrt{3}$ [JEE 2005 Screening]

(C) $12+\frac{7\sqrt{3}}{4}$ (D) $3+\frac{7\sqrt{3}}{4}$

4. Let $\theta\in\left(0,\frac{\pi}{4}\right)$ and $t_1=(\tan\theta)^{\tan\theta}, t_2=(\tan\theta)^{\cot\theta}, t_3=(\cot\theta)^{\tan\theta}, t_4=(\cot\theta)^{\cot\theta}$, then -

[JEE 06,3M,–1M]

(A) $t_1>t_2>t_3>t_4$ (B) $t_4>t_3>t_1>t_2$ (C) $t_3>t_1>t_2>t_4$ (D) $t_2>t_3>t_1>t_4$

One or more than one is/are correct : [Q.5(a) & (b)]

5.(a) If $\frac{\sin^4 x}{2}+\frac{\cos^4 x}{3}=\frac{1}{5}$, then [JEE 2009, 4 + 4]

(A) $\tan^2 x=\frac{2}{3}$ (B) $\frac{\sin^8 x}{8}+\frac{\cos^8 x}{27}=\frac{1}{125}$

(C) $\tan^2 x=\frac{1}{3}$ (D) $\frac{\sin^8 x}{8}+\frac{\cos^8 x}{27}=\frac{2}{125}$

(b) For $0<\theta<\frac{\pi}{2}$, the solution(s) of $\sum_{m=1}^{6}\operatorname{cosec}\left(\theta+\frac{(m-1)\pi}{4}\right)\operatorname{cosec}\left(\theta+\frac{m\pi}{4}\right)=4\sqrt{2}$ is (are) -

(A) $\frac{\pi}{4}$ (B) $\frac{\pi}{6}$ (C) $\frac{\pi}{12}$ (D) $\frac{5\pi}{12}$

6.(a) The maximum value of the expression $\frac{1}{\sin^2\theta+3\sin\theta\cos\theta+5\cos^2\theta}$ is

(b) Two parallel chords of a circle of radius 2 are at a distance $\sqrt{3}+1$ apart. If the chords subtend at the center, angles of $\frac{\pi}{k}$ and $\frac{2\pi}{k}$ where k > 0, then the value of [k] is -

[**Note** : [k] denotes the largest integer less than or equal to k] [JEE 2010,3+3]

7. Let $P=\{\theta:\sin\theta-\cos\theta=\sqrt{2}\cos\theta\}$ and $Q=\{\theta:\sin\theta+\cos\theta=\sqrt{2}\sin\theta\}$ be two sets. Then

(A) $P\subset Q$ and $Q-P\neq\varnothing$ (B) $Q\not\subset P$

(C) $P\not\subset Q$ (D) P = Q [JEE 2011,3]

PREVIOUS YEARS QUESTIONS	ANSWER KEY	EXERCISE-5 [B]

1. C	**2.** B	**3.** B	**4.** B	**5.** (a) A,B; (b) C,D	**6.** (a) 2; (b) k = 3	**7.** D