SETS

Definition: - A set is well defined collection of objects. It is collections of objects of similar kind.

Some of the collections are:

(i) Natural number
(ii) Prime number.
(iii)The vowels in English alphabet.
(iv) Solution of $x^2 - 5x + 4 = 0$, viz 1 and 4.
(v) The rivers of India.

- Sets are usually denoted by capital letters, $A, B, C, X, \ldots$ etc.
- Objects are also called elements or members.
- Elements are generally denoted by small letters, $a, \mathrm{b}, \mathrm{c}, \ldots$ etc.
- If a is an element of a set A, we say that "a belongs to A", and written as $a \in A$.
- $\in$ is Greek symbol (epsilon).
- $\in$ is read as "belongs to".
- If a is not an element of a set A, we say that "a not belongs to A", and written as $a \notin A$.

Some general sets used in mathematics

$\mathbf{N}$: Set of all-natural numbers
$\mathbf{W}$: set of all whole numbers
$\mathbf{Z}$ or $\mathbf{I}$: Set of all integers
$\mathbf{Z}^+$: Set of positive integers
$\mathbf{Z}^-$: Set of negative integers
$\mathbf{Z}_0$: Set of non-zero integers
$\mathbf{Q}$: Set of rational number
$\mathbf{Q}^+$: Set of positive rational numbers
$\mathbf{R}$: Set of real numbers
$\mathbf{R}^+$: Set of positive real numbers
$\mathbf{R}_0$: Set of non-zero real numbers
$\mathbf{Q}^c$ or $\mathbf{R} - \mathbf{Q}$: Set of irrational number
$\mathbf{C}$: Set of complex numbers

Methods of representing set

1. Roster or tabular form: In this method a set is described by listing elements, elements of set are separated by commas and are enclosed by curly brackets. $\{\ \}$

- While writing the set-in roster form an element is not generally repeated.
  e.g.: -
  (i) The set of odd natural number less than 10 can be described as $\{1, 3, 5, 7, 9\}$.
  (ii) The set of all vowels in English alphabet is $\{a, e, i, o, u\}$
  (iii) The set of letters forming the word 'SCHOOL' is $\{S, C, H, O, L\}$.

2. Set-builder or Property set: In this method, we write down a property or rule which gives us all the element of the set.
   e.g.: -
   (i) The set of natural number less than 7 can be described as
   $\{x : x \text{ is a natural number less than } 7\}$
   (ii) $B = \{y : y \text{ is a vowel in English alphabet}\}$

(iii) $C = \{z : z \in N \text{ and } z = 2n \text{ for } n \in N\}$

Example 1. Write the solution set of the equation $x^2 + x - 2 = 0$ in roster form.

Example 2. Write the set $A = \{1, 4, 9, 16, 25, \ldots\}$ in set builder form.

Example 3. Write the following set in roster form

(i) $\{x : x \text{ is a positive integer and } x^2 < 40\}$

(ii) $\{x : x \text{ is a positive integer less than 10 and } 2^x - 1 \text{ is an odd number}\}$

(iii) $\{x : x^2 + 7x - 8 = 0, x \in R\}$

(iv) $\{x : x \in R, x^2 = 16 \text{ and } 2x = 6\}$

Example 4. State which of the following statements are true and which are false.

(i) $37 \notin \{x \mid x \text{ has exactly two positive factors}\}$

(ii) $28 \in \{y \mid \text{the sum of all positive factors of } y \text{ is } 2y\}$

(iii) $7,747 \in \{t \mid t \text{ is a multiple of } 37\}$

Example 5. Given that $E = \{1, 2, 3, 4, 5, 6\}$. If n represents any member of E, then, write the following sets containing all numbers represented by

(i) $n + 1$

(ii) n^2

(iii) $n \in E$ but $2n \notin E$

(iv) $n + 5 = 8$

(v) n is greater than 3

Example 6. Match column I with column II.

Column I (Roster form)		**Column II** (Set-builder form)	
(i)	$\{P, R, I, N, C, A, L\}$	(a)	$\{x : x \text{ is a positive integer and is divisor of } 18\}$
(ii)	$\{0\}$	(b)	$\{x : x \text{ is an integer and } x^2 - 9 = 0\}$
(iii)	$\{1, 2, 3, 6, 9, 18\}$	(c)	$\{x : x \text{ is an integer and } x + 1 = 1\}$
(iv)	$\{3, -3\}$	(d)	$\{x : x \text{ is a letter of the word PRINCIPAL}\}$

Types of sets

1. **Null or empty or void set**: A set having no element is called empty set.
   - It is denoted by $\{\ \}$ or ϕ.
   - $\{\phi\}$ is not empty set.

   e.g.: - $A = \{x : x \le 0, x \in N\}$

2. **Singleton set**: A set containing only one element is called singleton.
   - $\{\phi\}$ has one element, i.e. ϕ. Therefore, it is singleton set.

3. **Finite set**: A set which has finite number of elements is called finite set.
   e.g.: -
   (i) $A = \{a, e, i, o, u\}$
   (ii) $B = \{x : x \text{ is rivers of India}\}$

4. **Cardinal number or Order of finite set**: The number of elements in a finite set is called cardinal number.
   - It is denoted by $n(A)$.

5. **Infinite set**: A set which has infinite number of elements is called infinite set.
    - We represent some infinite set in the roster form by writing a few elements which clearly indicate the structure of the set followed by three dots.

e.g.: -

(i) $A = \{x : x \in N \text{ and } x \text{ is prime}\}$

(ii) $B = \{2, 4, 6, \ldots\}$

(iii) Set of natural number.

(iv) The set of all points on a line.

6. **Equal sets**: Two sets A and B are said to be equal if $n(A) = n(B)$ and very element of A is element of B
    - If set A and B are equal then we write $A = B$, otherwise $A \neq B$ (unequal).

e.g.: -

(i) Let $A = \{1, 2, 3\}$ and $B = \{2, 1, 3\}$ $\Rightarrow A = B$

(ii) Let $A = \{1, 2, 3\}$ and $B = \{2, 1, 3, 1, 3\}$ $\Rightarrow A = B$. ($\because$ we do not repeat any element $\therefore B = \{1, 2, 3\}$)

7. **Equivalent sets**: Two sets A and B are said to be equivalent if $n(A) = n(B)$
    - Equal sets are always equivalent but equivalent set may not be equal.

e.g.: - Let $A = \{1, 2, 3, 4\}$ and $B = \{p, q, r, s\}$ $\because n(A) = n(B) = 4$ $\Rightarrow A \sim B$.

8. **Subsets**: If A is subset of B, then every element of A is also elements of B.
    - It is denoted by $A \subseteq B$ and read as "A is subset of B"
    - If $a \in A$ and $a \in B$ $\Rightarrow$ $A \subseteq B$.
    - If A is not subset of B is written as $A \not\subset B$
    - If $A \subseteq B$ and $B \subseteq A$ $\Rightarrow$ $A = B$.
    - '$\Rightarrow$' read as 'implies'
    - '$\Leftrightarrow$' read as 'double implies' or 'if and only if" or 'iff'.
    - Every set A is subset of itself
    - ϕ is subset of every set.
    - $N \subset W \subset Z \subset Q \subset R \subset C$.
    - Total number of subsets of finite set containing n elements is 2^n.

e.g.: - Let $X =$ set of all students in your school and $Y =$ set of all students in your class, then every students in your class is also students of your school. Therefore Y is subset of X. i.e. $Y \subseteq X$.

9. **Proper subset**: Two sets A and B are such that $A \subset B$ and $A \neq B$, then A is called proper subset of B.
    - Here B is called **superset** of A.
    - Total number of proper subsets of finite set containing n elements is $2^n - 1$.
10. **Power set**: The collection of all subsets of a set A is called power set of A.
    - It is denoted by $P(A)$.
    - Total number of elements in power set it 2^n.
    - Power set of a set is always non empty.
    - If $A = \phi$, then $P(A) = \{\phi\}$.

e.g.: - Let $A = \{1, 2\}$, then $P(A) = \{\phi, \{1\}, \{2\}, \{1, 2\}\}$

11. **Universal set**: the universal set is the superset of all the sets which occurs in the discussion.
    - It is denoted by U.

Example 7. Which of the following is the empty set?

(i) $\{x \mid x \text{ is any real number } x^2 - 1 = 0\}$

(ii) $\{x \mid x^2 - 2 = 0, x \in R\}$

(iii) $\{x \mid x^2+1=0, x \in R\}$ (iv) $\{x \mid x^2-1=x, x \in R\}$

Example 8. Write the power set of set $A=\{\phi,\{\phi\},1\}$.

Example 9. Two finite sets of m and n elements respectively. The total number of elements in power set of first set is 56 more than the total number of elements in power set of the second set. Find the value of m and n respectively.

Intervals as a subset of R

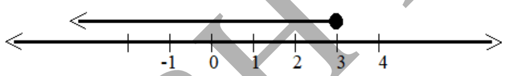
Let a and b be real numbers such that $a<b$.

- The set of real numbers $\{x : x \in R, a<x<b\}$ is called an open interval and is denoted by (a,b). All the values between a and b $\in$ (a,b) but a, b themselves not included.
- The set of real numbers $\{x : x \in R, a \le x \le b\}$ is called closed interval and is denoted by $[a,b]$. This interval contains ends points also. i.e. a, b are included.
- These intervals contain infinitely many points.
- For some particular values of x, we use symbol $\{\ \}$

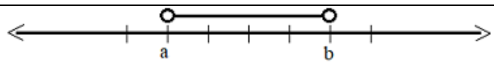
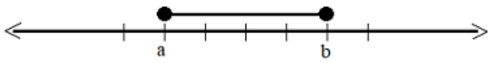
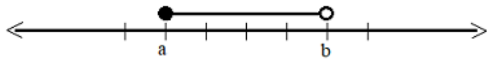
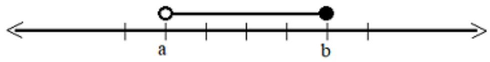
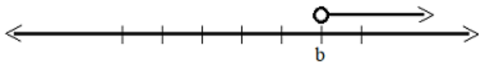
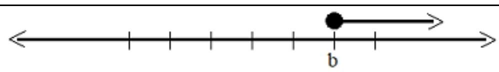
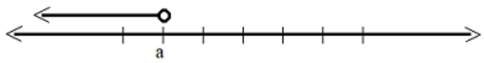
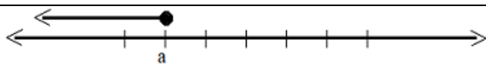
  e.g.: - If $x=1,2$ we can write it as $x=\{1,2\}$ or $x \in \{1,2\}$
- If there are no values of x, then we say $x \in \phi$.
- Intervals are used to described Inequalities.

  e.g.: - The inequalities $x \le 3$ describes all real numbers less than or equal to 3.

  Using interval notation, we write $(-\infty,3]$.

  Using number line 

Some of the standard intervals used to described sets are as follows

Interval Notation	Set-Builder Notation	Graphical Notation
(a,b)	$\{x : x \in R, a<x<b\}$	
$[a,b]$	$\{x : x \in R, a \le x \le b\}$	
$[a,b)$	$\{x : x \in R, a \le x < b\}$	
$(a,b]$	$\{x : x \in R, a < x \le b\}$	
(b,∞)	$\{x : x \in R, x>b\}$	
$[b,\infty)$	$\{x : x \in R, x \ge b\}$	
$(-\infty,\mathrm{a})$	$\{x : x \in R, x<a\}$	
$(-\infty,\mathrm{a}]$	$\{x : x \in R, x \le a\}$	
$(-\infty,\infty)$	$\{x : x \in R\}$	

Example 10. Express the interval in terms of inequalities and graph the interval on number line:

(i) $(-1,4]$

(ii) $[2.5,4]$

(iii) $(-4,\infty)$

Example 11. Rewrite each sentence in both set builder and interval notation.

(i) x is less than 5

(ii) x is greater than or equal to 3.

(iii) x lies between -2 and 5.

(iv) x is at most 4

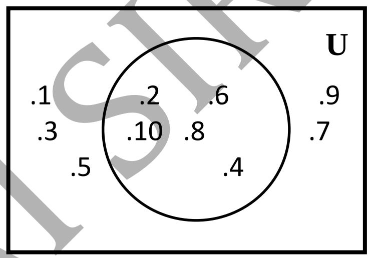
(v) x is positive but not more than 5

Venn diagrams

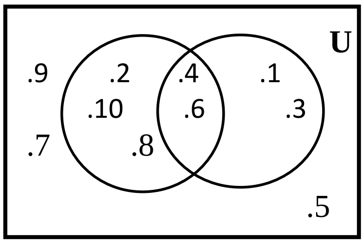
The universal set is represented by rectangle and its subsets are generally represented by circles.

e.g.: - If $U=\{1,2,3,4,5,6,7,8,9,10\}$ and its subset $A=\{2,4,6,8,10\}$,

then it is represented as

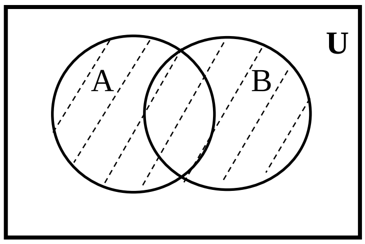


Now, let $B=\{1,3,4,6\}$ another subset of U. Then it can be represented as

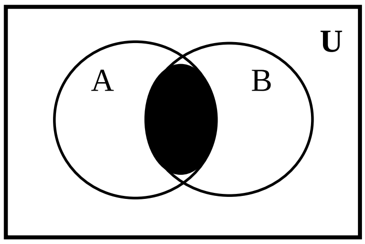


Operations on sets:

1. **Union**: - If A & B are two sets, then their union, $A\cup B$, is the set containing all elements that are in A or in B (or in both A and B).
   - It is denoted by $\cup$.
   - $A\cup B$ is read as "A union B".
   - $A\cup B=\{x : x\in A \text{ or } x\in B\}$.
   - The shaded portion in the figure represent $A\cup B$.



2. **Intersection**: - If A & B are two sets, then their intersection, $A\cap B$, is the set containing all elements that are in both A and B.
   - It is denoted by $\cap$.
   - $A\cap B$ is read as "A intersection B".
   - $A\cap B=\{x : x\in A \text{ and } x\in B\}$.
   - The dark portion in the figure represent $A\cap B$.
   - If $A\cap B=\phi$, then set A & B are called **disjoint set**.



Properties of Union and Intersection

(a) Commutative law: $A\cup B=B\cup A$ & $A\cap B=B\cap A$.

(b) Associative law: $(A\cup B)\cup C=A\cup(B\cup C)$ & $(A\cap B)\cap C=A\cap(B\cap C)$

(c) Distributive law: $A\cup(B\cap C)=(A\cup B)\cap(A\cup C)$ & $A\cap(B\cup C)=(A\cap B)\cup(A\cap C)$.

(d) Laws of ϕ and U : $\begin{matrix} A\cup\phi = A \\ A\cup U = U \end{matrix}$ & $\begin{matrix} A\cap\phi = \phi \\ A\cap U = A \end{matrix}$

(e) Idempotent law: $A\cup A = A$ & $A\cap A = A$.

(f) $A\cap B$ is subset of A as well as B.

(g) A & B are subsets of $A\cup B$.

(h) If $A\subseteq B$, then $A\cup B = B$ and $A\cap B = A$

Example 12. Express each set-in interval notation:

(i) $(1,4)\cap[2,6]$ (iv) $(1,4)\cup[2,6]$

(ii) $(-2,\infty)\cup(3,\infty)$ (v) $(-2,\infty)\cap(3,\infty)$

(iii) $(-2,-1)\cap(2,3)$ (vi) $(-2,-1)\cup(2,3)$

Example 13. Let $A=\{1,2,3\}$, $B=\{3,4\}$ and $C=\{4,5,6\}$ then find the set $A\cup(B\cap C)$.

Example 14. If $N_a=\{an:\forall n\in N\}$, then the set $N_3\cap N_4 = ?$

(i) N_6 (iii) N_{12}

(ii) N_7 (iv) None of these

Example 15. If $A_N=\{ax:\forall n\in N\}$ and $B_N\cap C_N = dN$, where B & $C\in N$ and are relatively prime then

(i) $b=cd$ (iii) $d=cb$

(ii) $c=bd$ (iv) None of these

Example 16. If set A & B are defined as $A=\left\{(x,y): y=\frac{1}{x}, x\neq 0, x\in R\right\}$ &

$B=\{(x,y): y=-x, x\in R\}$, then

(i) $A\cap B = A$ (iii) $A\cap B=\phi$

(ii) $A\cap B = B$ (iv) None of these

Example 17. If set A & B are defined as $A=\{(x,y): y=e^x, x\in R\}$ & $B=\{(x,y): y=x, x\in R\}$, then

(i) $A\cap B = A$ (iii) $A\cap B=\phi$

(ii) $A\cap B = B$ (iv) None of these

Example 18. Let $A=\{x: x\in R, |x|<1\}$; $B=\{x: x\in R, |x-1|\geq 1\}$ and $A\cup B = R-D$, then the set D is –

(i) $\{x:1<x\leq 2\}$ (iii) $\{x:1<x\leq 2\}$

(ii) $\{x:1<x\leq 2\}$ (iv) None of these

Practice Work

Example 19. Express the interval in terms of inequalities and graph the interval on number line:

(i) $(1,6]$ (iii) $[-4,3]$ (v) $[-3,\infty)$

(ii) $[-5,2)$ (iv) $(2,\infty)$ (vi) $(-\infty,3)$

Example 20. Express each English phrase in both set builder and interval notations:

(i) x is less than 6

(ii) x is greater than or equal to -1.

(iii) x lies between 5 and 12, excluding 5 and 12.

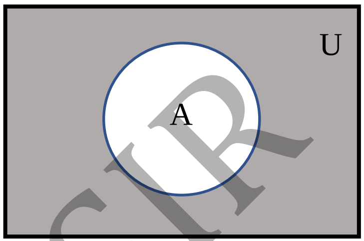
(iv) x lies between 4 and 14, excluding 4 and including 14.

(v) x is at most 7
(vi) x is at least 3 and at most 7

Example 21. Express each set-in interval notation:

(i) $(1,6)\cap[3,8]$
(ii) $(-3,\infty)\cup(4,\infty)$
(iii) $(-5,-3)\cap(-\infty,-4)$
(iv) $(-1,\infty)\cup[5,\infty]$
(v) $(-\infty,2)\cup(-\infty,4)$
(vi) $(-3,-1)\cup[2,4]$

3. **Complement of a set**: Let U be the universal set and A is a subset of U, then the complement of A is the set of all elements of U which are not the elements of A.



- It is denoted by A'
- A' is read as "A complement".
- $A' = \{x : x \in U \text{ and } x \notin A\} = U - A$.
- A' is also subset of U.
- $A \subseteq B \Rightarrow B' \subseteq A'$

Properties

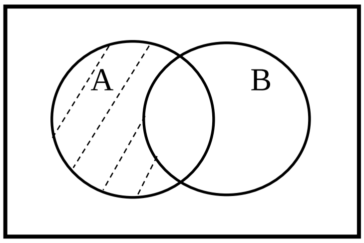
(a) Complement laws: (i) $A \cup A' = U$ (ii) $A \cap A' = \phi$

(b) De Morgan's law: (i) $(A \cup B)' = A' \cap B'$ (ii) $(A \cap B)' = A' \cup B'$

(c) $(A')' = A$

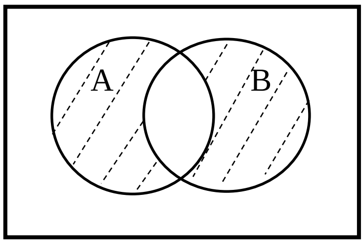
(d) $\phi' = U$ and $U' = \phi$

4. **Difference of sets**: the difference of sets A & B in this order is the set of elements which belong to A but not to B.



- It is denoted by $A - B$
- $A - B$ is read as "A minus B".
- $A - B = \{x : x \in A \text{ and } x \notin B\}$.
- The sets $A - B$, $A \cap B$ and $B - A$ are mutually disjoint sets.

5. **Symmetric difference of two set**: $A \Delta B = (A - B) \cup (B - A)$, it is also equalling to $(A \cup B) - (A \cap B)$.



Properties of Difference and symmetric difference of Two sets

(a) $A - B = A - (A \cap B) = A \cap B'$

(b) $B - A = B - (A \cap B) = B \cap A'$

(c) $A - B = A \Leftrightarrow (A \cap B) = \phi$.

(d) $(A - B) \cup B = (A \cup B)$

(e) $(A - B) \cap B = \phi$

Example 22. If $A = \{1,2,3,4\}$ and $B = \{2,4,6\}$, find $A \Delta B$.

Some Important results on Cardinal number of sets

Let A, B and C are finite sets, and U be the finite universal set, then

(a) $n(A\cup B)=n(A)+n(B)-n(A\cap B)$

(b) If A and B are disjoint sets, then $n(A\cup B)=n(A)+n(B)$.

(c) Number of elements belongs to only A. i.e. $n(A-B)=n(A)-n(A\cap B)$.

(d) Number of elements belongs to only B. i.e. $n(B-A)=n(B)-n(A\cap B)$.

(e) Number of elements belongs to exactly one of A or B

i.e. $n(A\Delta B)=n(A-B)\cup n(B-A)$

$$=n(A\cup B)-n(A\cap B)$$
$$=n(A)-n(A\cap B)+n(B)-n(A\cap B)$$
$$=n(A)+n(B)-2n(A\cap B)$$

(f) $n(A\cup B\cup C)=n(A)+n(B)+n(C)-n(A\cap B)-n(B\cap C)-n(C\cap A)+n(A\cap B\cap C)$

(g) Number of elements in exactly two of the set A, B and C

$=n(A\cap B)+n(B\cap C)+n(C\cap A)-3n(A\cap B\cap C)$

(h) Number of elements belongs to exactly one of A, B, C

$=n(A)+n(B)+n(C)-2n(A\cap B)-2n(B\cap C)-2n(C\cap A)+3n(A\cap B\cap C)$

(i) $n(A^{'}\cup B^{'})=n((A\cap B)^{'})=n(U)-n(A\cap B)$

(j) $n(A^{'}\cap B^{'})=n((A\cup B)^{'})=n(U)-n(A\cup B)$

Example 23. If A and B are two sets such that $n(A)=70$, $n(B)=60$ & $n(A\cup B)=110$, find number of $n(A\cap B)$.

Example 24. If $n(U)=700$, $n(A)=200$, $n(B)=300$ & $n(A\cap B)=100$, then find $n(A^{'}\cap B^{'})$.

Example 25. Set A and B have 3 & 6 element respectively. What can be the minimum number of element in $(A\cup B)$.

Example 26. In a city 20% of the population travels by car, 50% of the population travels by bus and 10% of the population travels by both car and bus. Then find the person travels by car or bus.

Example 27. In a group of 1000 people, there are 750 who can speak Hindi and 400 who can speak Bengali. How many can speak Hindi only? How many can speak Bengali? How many can speak both Hindi and Bengali?