

CS4486: Artificial Intelligence
Homework 1

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Question 1)

a) $UCB = V_i + c \sqrt{\frac{\ln N}{n_i}}$ where V_i : Estimated value of node i
 c : Hyperparameter
 N : Total visitations
 n_i : Visitations of node i

For the three child nodes of root,

$$UCB_{child1} = 1 + 1 \times \sqrt{\frac{\ln 7}{1}} = 1 + \sqrt{\ln 7} = 2.3949$$

$$UCB_{child2} = \frac{21}{4} + 1 \times \sqrt{\frac{\ln 7}{4}} = 5.9474$$

$$UCB_{child3} = \frac{7}{2} + 1 \times \sqrt{\frac{\ln 7}{2}} = 4.4863$$

So the next optimal node is child 2 as it has the highest UCB value. This node again has 3 children so we evaluate UCB for each of them.

$$UCB_{child2.1} = \frac{6}{1} + 1 \times \sqrt{\frac{\ln 4}{1}} = 7.177$$

$$UCB_{child2.2} = \frac{2}{1} + 1 \times \sqrt{\frac{\ln 4}{1}} = 3.177$$

$$UCB_{child2.3} = \frac{7}{1} + 1 \times \sqrt{\frac{\ln 4}{1}} = 8.177$$

Hence, we evaluate child 2.3 as it has the highest UCB value. Therefore, the path selected by MCTS = root \rightarrow 2 \rightarrow 2.3

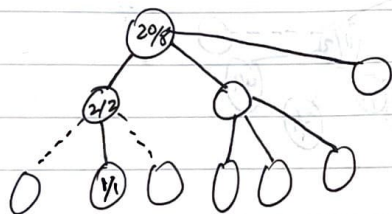
b) Taking $C = 20$

$$UCB_{Node1} = 1 + 20 \sqrt{\ln 7} = 28.8991$$

$$UCB_{Node2} = \frac{21}{4} + 20 \sqrt{\frac{\ln 7}{4}} = 19.1995$$

$$UCB_{Node3} = \frac{7}{2} + 20 \sqrt{\frac{\ln 7}{2}} = 23.2276$$

Node 1 is expanded as it has the highest UCB value. Since all the children of node 1 have not been visited, they may have any value. Arbitrarily expanding node 1.2



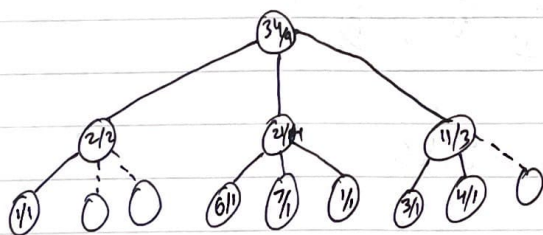
c) Continuing with $C = 20$,

$$UCB_{node1} = 1 + 20 \sqrt{\frac{\ln 8}{2}} = 20.3933$$

$$UCB_{node2} = \frac{21}{4} + 20 \sqrt{\frac{\ln 8}{4}} = 19.42$$

$$UCB_{node3} = \frac{7}{2} + 20 \sqrt{\frac{\ln 8}{2}} = 23.8933$$

Node 3 is chosen. Two out of 3 children of node 3 have not been visited. Arbitrarily choosing node 3.2, exploring it and updating the tree.



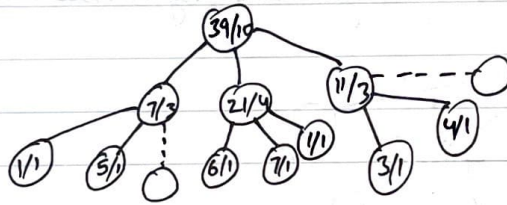
Continuing with $c = 20$

$$UCB_{node1} = 1 + 20 \sqrt{\frac{\ln 9}{2}} = 21.9629$$

$$UCB_{node2} = \frac{21}{4} + 20 \sqrt{\frac{\ln 9}{4}} = 20.0730$$

$$UCB_{node3} = \frac{11}{3} + 20 \sqrt{\frac{\ln 9}{4}} = 18.4897$$

Node 1 is chosen. It has two unexplored nodes, hence arbitrarily choosing 1-2



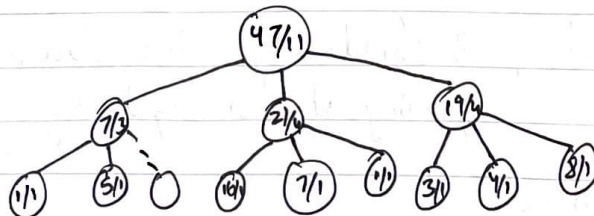
Continuing with $c = 20$

$$UCB_{node1} = \frac{7}{3} + 20 \sqrt{\frac{\ln 10}{3}} = 19.8550$$

$$UCB_{node2} = \frac{21}{4} + 20 \sqrt{\frac{\ln 10}{4}} = 20.4242$$

$$UCB_{node3} = \frac{11}{3} + 20 \sqrt{\frac{\ln 10}{3}} = 21.1884$$

Node 3 is chosen. It has 1 unexplored node, hence expanding it.



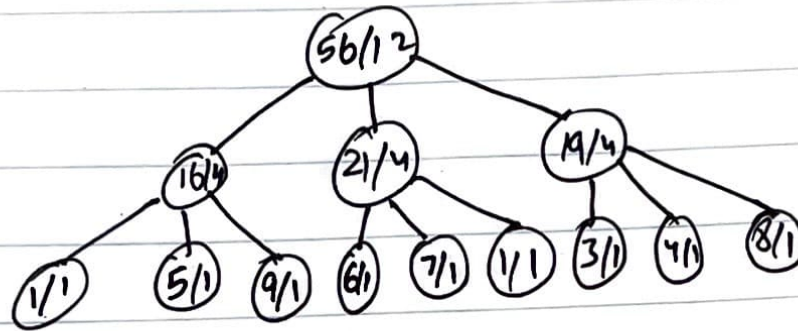
Continuing with $c = 20$

$$UCB_{node1} = \frac{7}{3} + 20 \sqrt{\frac{\ln 11}{3}} = 20.8140$$

$$UCB_{node2} = \frac{21}{4} + 20 \sqrt{\frac{\ln 11}{4}} = 20.7351$$

$$UCB_{Node3} = \frac{19}{4} + 20 \sqrt{\frac{\ln 11}{4}} = 20.2351$$

Node 1 is chosen. It has one unexplored node, hence expanding it.



This ends the MCTS as there are no more nodes to expand. The largest utility leaf node is returned.