Audio Signal Processing For Guitar Pedal Application

Aryan Mosavian Pour

January 6, 2025

1 Equalizer

Equalizers are comprised of various band pass filters. A band pass filter may be represented in the Fourier domain as

$$H(j\omega) = \text{rect}(\frac{1}{W}(\omega - \omega_0)) + \text{rect}(\frac{1}{W}(\omega + \omega_0))$$

$$h[n] = \mathfrak{F}^{-1}_{\text{Discrete}}\{H(j\omega)\}(t) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H(j\omega)e^{-jn\omega}d\omega$$

$$h[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi}(\Theta(\omega + \omega_0 + \frac{W}{2}) - \Theta(\omega - \omega_0 - \frac{W}{2}) + \Theta(\omega - \omega_0 + \frac{W}{2}) - \Theta(\omega + \omega_0 - \frac{W}{2}))e^{-jn\omega}d\omega$$

$$h[n] = \frac{1}{2\pi}(\int_{\omega_0 - \frac{W}{2}}^{\omega_0 + \frac{W}{2}} e^{-jn\omega}d\omega + \int_{-\omega_0 - \frac{W}{2}}^{-\omega_0 + \frac{W}{2}} e^{-jn\omega}d\omega)$$

$$h[n] = \frac{1}{2\pi nj}(e^{-jn(\omega_0 + \frac{W}{2})} - e^{-jn(\omega_0 - \frac{W}{2})} + e^{-jn(-\omega_0 + \frac{W}{2})} - e^{-jn(-\omega_0 - \frac{W}{2})})$$

$$h[n] = \frac{1}{2\pi nj}(e^{jn\frac{W}{2}} - e^{-jn\frac{W}{2}})(e^{jn\omega_0} + e^{-jn\omega_0})$$

$$h[n] = \frac{2}{\pi n}\sin\left(\frac{W}{2}n\right)\cos(\omega_0 n) = \frac{W}{\pi}\frac{\sin\left(\frac{W}{2}n\right)}{\frac{W}{2}n}\cos(\omega_0 n)$$

$$h[n] = \frac{W}{\pi}\text{sinc}(\frac{nW}{2})\cos(\omega_0 n)$$

There are some problems with this solution, though. Most importantly, the system is non-causal. To make it causal, we need to shift our signal and multiply it by some *cropping* function, $c[n]$.

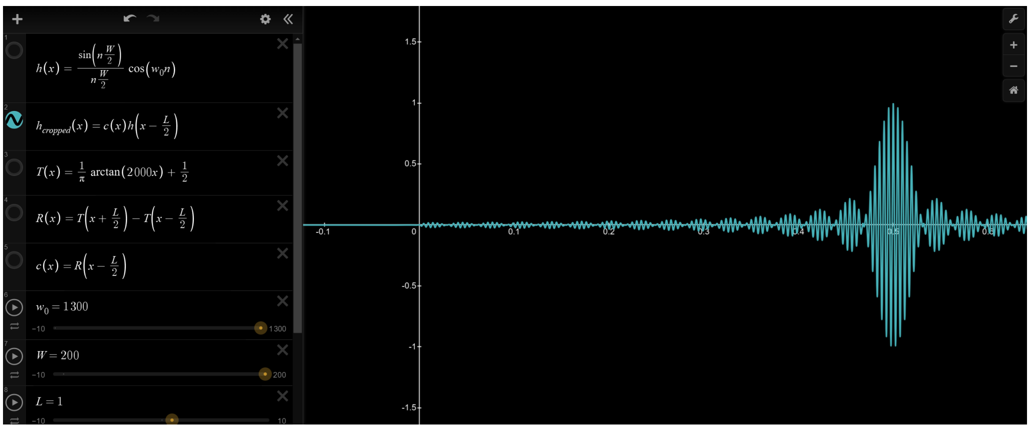
$$c[n] = \Theta(n + \frac{L}{2}) - \Theta(n - \frac{L}{2})$$

$$h_{\text{cropped}}[n] = c[n] \times (h[n] * \delta(L/2))$$

There are various types of windows. I will use a square window as it is the simplest.

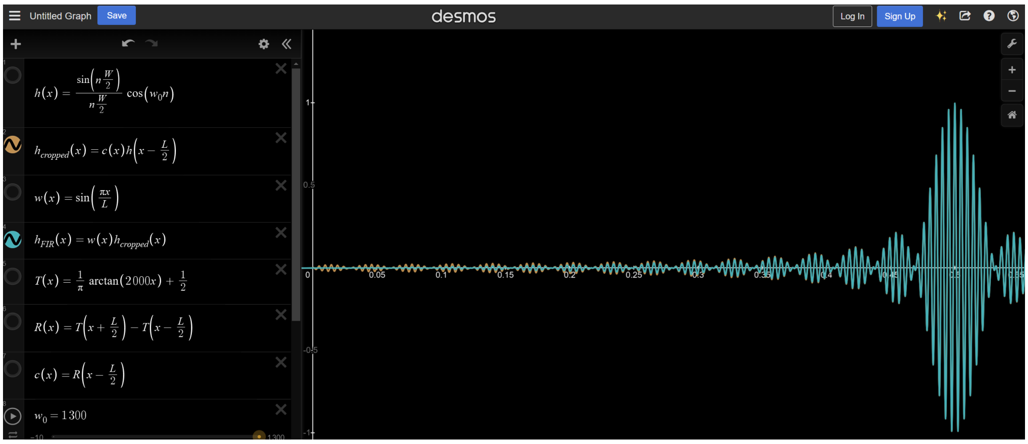
$$h_{\text{cropped}}[n] = (\Theta(n + \frac{L}{2}) - \Theta(n - \frac{L}{2}))\frac{W}{\pi}\text{sinc}(\frac{(n - \frac{L}{2})W}{2})\cos\left(\omega_0(n - \frac{L}{2})\right)$$

For my specifications ($\omega_0 = 1300$, $W = 200$) and using $L = 1$, we get the following windowed sinc:



But this introduces yet ANOTHER problem: discontinuities. These greatly affect the frequency response. As a solution, we multiply by a *window* function which effectively smooths out the function. I will use the sine window, $w[n] = \sin\left(\frac{n\pi}{L}\right)$. We thus get the following function and accompanying graph (with the orange being the cropped function, and the blue being the windowed):

$$h_{\text{FIR}}[n] = \sin\left(\frac{n\pi}{L}\right)(\Theta(n+\frac{L}{2}) - \Theta(n-\frac{L}{2}))\frac{W}{\pi}\text{sinc}(\frac{(n-\frac{L}{2})W}{2})\cos\left(\omega_0(n-\frac{L}{2})\right)$$



We're now at the finish line: we just need to calculate the output,

$$y[n] = (x * h)[n] := \sum_{m=-M}^{M} x[n-m]h[m]$$

The final equation representing our system, which is characterized by ω_0 and W, is:

$$\boxed{y[n] = \sum_{m=-M}^{M} x[m]\sin\left(\frac{n\pi}{L}\right)(\Theta(n+\frac{L}{2}) - \Theta(n-\frac{L}{2}))\frac{W}{\pi}\text{sinc}(\frac{(n-\frac{L}{2})W}{2})\cos\left(\omega_0(n-\frac{L}{2})\right)}$$