Q1

i) number of comparison:

Let n be the number of elements in the array.

- During iteration 1, inner loop makes $(n-1)$ comparisons
- During iteration 2, inner loop makes $(n-2)$ comparisons

In last interation, the patter continued ceases for the outer loop needing only single iteration.

→ Number of comparisons: $(n-1) + (n-2) + \ldots + 2 + 1$

Hence: $$\sum_{i=1}^{k} i = \frac{k(k+1)}{2}$$ (sum of first k integers)

using $k = n-1$,

Number of comparisons: $$\frac{(n-1)(n-1+1)}{2} = \frac{n(n-1)}{2}$$

$$\boxed{\text{so} \longrightarrow \frac{n^2-n}{2} \quad \therefore \text{ In big O notation: } O(n^2)}$$

ii) number of swaps made.

in a randomly shuffled array, assum pair of indices j and k. . where $j < k$.

$\Rightarrow$ hence an inversion $\alpha[j] > \alpha[k]$. where alpha is an array.

As each inversion requires a swap to correct, there are $\frac{n(n-1)}{2}$ such pair (j, k) of indices in an array sized n.

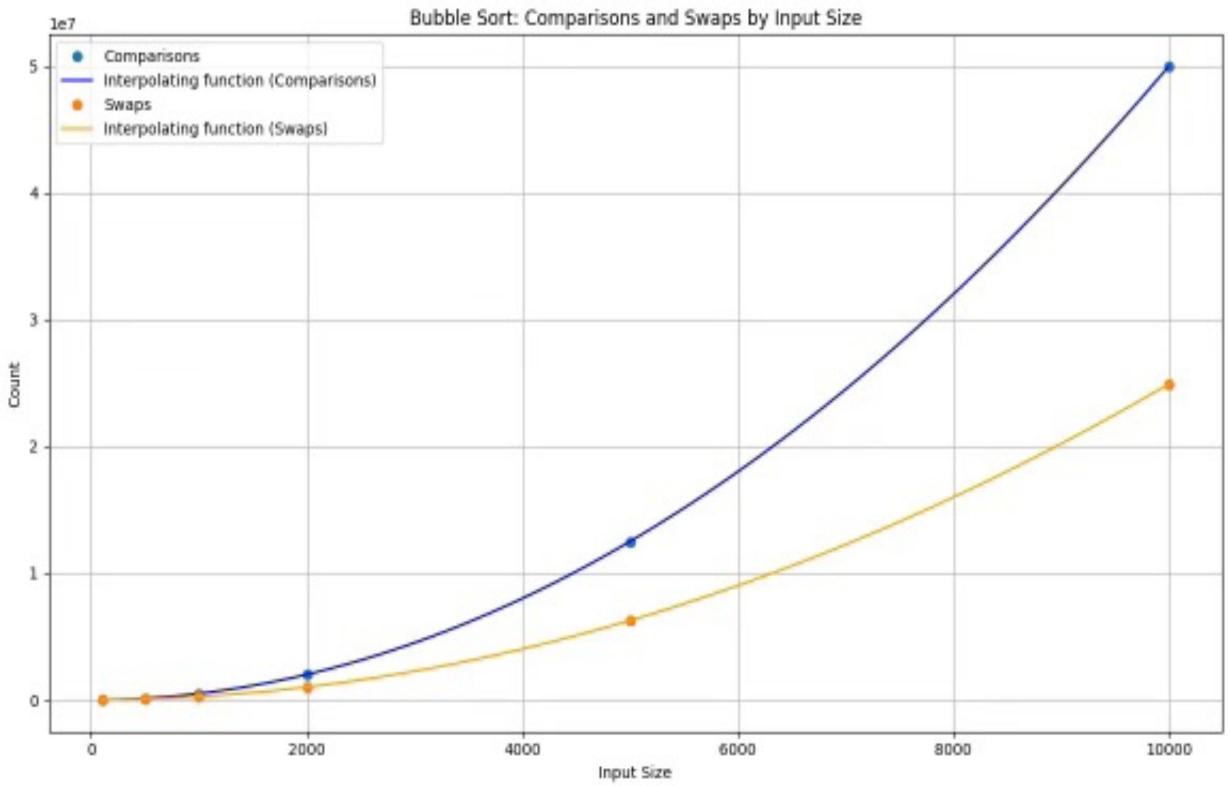
$\Rightarrow$ On averge each inversion appears in $1/2$ of the shuffled array.

Therefore,

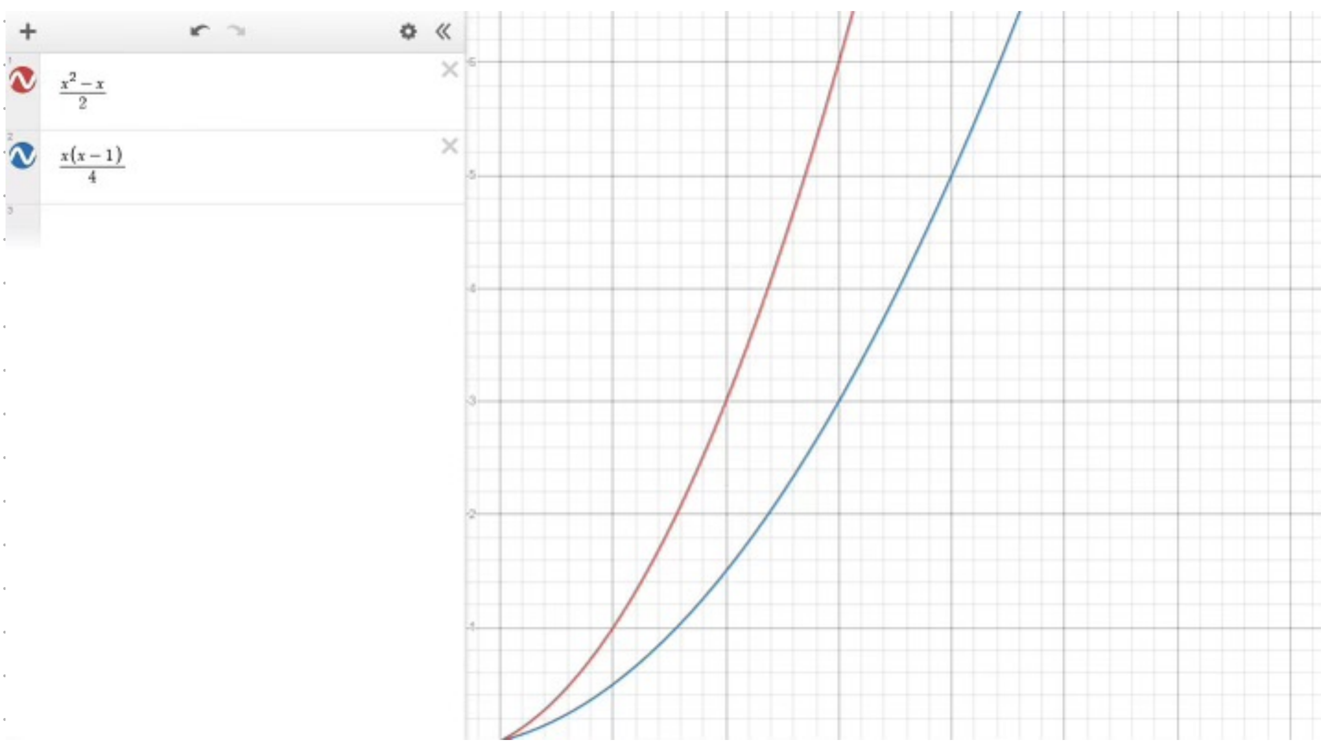
each inversion requires a swap to be corrected,

we can say: number of swaps = number of inversions.

Thus $\Rightarrow$ Average number of swaps in bubble sort $= \frac{1}{2}\left(\frac{n(n-1)}{2}\right) = \boxed{\frac{n(n-1)}{4}}$



Bubble Sort: Comparisons and Swaps by Input Size
1e7
Comparisons
Interpolating function (Comparisons)
Swaps
Interpolating function (Swaps)
Count
Input Size
0
1
2
3
4
5
0
2000
4000
6000
8000
10000



$\frac{x^2-x}{2}$
$\frac{x(x-1)}{4}$

Q4: As we can see the plots for the code and our theoretical plots using desmos. Both are dictated by a quadratic equation with order 2. The results closely match as we can see that for increasing input size, the swap has a lower swap when compared to comparisons. This is indicated in our calculations as well. This differen can be attributed to the different denominator.