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Interval valued inventory model for deterioration, carbon emissions and selling price dependent demand considering buy now and pay later facility

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ABSTRACT

Uncertainty refers to a lack of precise knowledge or information about a particular event, situation, or outcome. It is an inherent characteristic of many real-world phenomena and is often associated with the presence of risk or ambiguity. Uncertainty can arise due to various factors, such as incomplete information, unpredictability, complexity, or variability of a system or process. In many cases, uncertainty can lead to difficulties in decision-making and pose challenges in planning, forecasting, and managing risks. There are different types of uncertainty, including epistemic uncertainty, which arises from the inherent randomness or variability of a system or process. This study encompasses the inclusion of all inventory parameters as uncertainty parameters, which are expressed in the form of intervals. Furthermore, demand for the product is taken into account in interval form. The conversion of the differential equation into an interval differential equation is carried out in order to account for the variability in demand within a certain interval. The utilisation of the centre-radius technique leads to the derivation of the related optimisation problem under a 'buy now, pay later' (BNPL) payment scheme. In order to demonstrate the concept, a numerical case is selected and solved with the MATHEMATICA software and interval order relations. In the end, a comprehensive sensitivity analysis has been conducted to derive insightful conclusions pertaining to this study.

1. Introduction

"Buy now, pay later" (BNPL) is a payment alternative that enables shoppers to buy products or pay for services over time, usually in instalments. BNPL options have become increasingly popular in recent years, particularly for online shopping. With BNPL, customers can typically select the option at checkout and then make a down payment (if required), followed by several equal instalments over a set period of time, usually ranging from a few weeks to several months. Fig. 1 depicts the worldwide transaction value of the BNPL method in the domain of electronic commerce, spanning the years 2019 to 2021, with projected estimates for the years 2022 to 2026.[1] Projections indicate that the volume of BNPL transactions is expected to witness a substantial surge of around 450 billion USD throughout the period spanning from 2021 to 2026. This would entail a heightened rate of progression similar to the observed trend between 2019 and 2021, during which the utilisation of the alternative payment method saw a substantial surge of about 400 percent. For instance, the overall market contribution by BNPL services in local online shopping payments was around 10 times greater in Sweden and Germany than the overall share in international e-commerce payments.

Some BNPL providers may charge interest or fees, while others offer interest-free options. BNPL can be an attractive option for consumers who may not have the funds to pay for a purchase upfront or who prefer to spread out payments over time. However, it is important to keep in mind that missing payments or failing to pay off the balance on time can result in additional fees or damage to one's credit score. Harris [1] created the conventional inventory model in the literature using a cash payment. Because the demand rate for perishable commodities depends on the selling price, the available supplies, and the date of expiration, Feng et al. [2] investigated a cash-on-delivery (COD) model. In the area of multi-retailer and single-manufacturer systems for perishable goods,

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[1] https://www.statista.com/statistics/1311122/global-bnpl-market-value-forecast/.

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Chen [3] examined a production inventory model considering cash payments. The seller in Goyal's [4] economic order quantity (EOQ) model makes a credit payment. The best ordering strategy when there is stock- and price-sensitive demand with an upstream credit payment is looked at by Wu et al. [5]. Zhang [6] developed an efficient method of paying modest invoices in advance to save money and time. Khan et al. [7] extended this advance payment mechanism to include perishable goods with expiration dates. In order to lower the risk of default with credit-risk consumers, Shaikh et al. [8] studied a cash-credit facility on payment (i.e., some amount of the total purchase cost in cash and the rest on credit). Wu et al. [9] extended the inventory model under the downstream partly credit facility for decaying items, considering the longest shelf lives. For evaporating products, Alshanbari et al. [10] discussed an inventory model considering advance-cash payment (i.e., a portion in advance and the remaining in cash). Moreover, Khan et al. [11] established an EOQ model in which the vendor provides a price reduction in exchange for the overall cost of the purchase in advance. To examine a cash payment-advance-credit payment policy, Chang et al. [12], Khan et al. [13], and Li et al. [14] generalised all of the models under the cash payment, advance payment, and credit payment policies.

The deterioration of perishable commodities has a substantial influence on the management of inventories. Due to the perishable nature of these commodities, the probability of spoilage and subsequent waste increases over time. The use of efficient inventory control measures becomes imperative in order to mitigate potential losses. Perishable goods frequently have specific storage conditions, such as temperature regulation, hence introducing intricacies to the management of logistics and inventories. The failure to effectively manage inventory that is in a state of deterioration can lead to a rise in expenses, a decrease in profitability, and consumer discontent stemming from the receipt of rotten or expired items. Therefore, the implementation of efficient inventory management strategies for perishable goods is crucial not only for mitigating financial losses but also for ensuring public health and promoting a sustainable and ethical economic operation. In their study, Panda et al. [15] examined the optimal stocking strategy for a perishable commodity with a consistent degradation rate throughout the storage duration while considering the restricted storage capacity of the warehouse. Afterwards, Das et al. [16] described a production system under a partial credit payment facility for a perishable commodity with a constant deterioration rate. Furthermore, Rahman et al. [17] developed a hybrid inventory model that considers both price and stock dependencies for perishable items. The model also incorporates advance payment-related incentive opportunities and preservation technologies. Duary et al. [18] proposed a mixed payment scheme by combining prepayment and delayed payment for a perishable item with a constant deterioration rate over the storage period. Rukonuzzaman et al. [19] examined a case study for a mango firm in Bangladesh by taking into account an increasing rate of degradation over time. It is evident that a decaying item gradually deteriorates over time, reaching 100 % by the time it has reached its date of expiration (or sell-by date). By taking into account the fact that a perishable product's deterioration rate rises according to the consumption of time, Khan et al. [20] investigated an EOQ model. Viji & Karthikeyan [21] studied an EPQ model for deteriorating items with Weibull distribution and shortage. For new items, Feng and Chan [22] determined the best cycle time and pricing considering downstream and upstream credit payments. Price, lot-sizing, and backordering choices made in response to a seller's request for a cash-advance-credit payment arrangement are investigated by Li et al. [23].

There is a growing consensus that significant global climate change could be caused by carbon emissions from commercial operations. Additionally, reducing carbon emissions has economic benefits, with the most significant one being a decrease in air pollution, which is currently a bigger issue for cities all over the globe. The cost to China's economy of the 1.23 million deaths in 2010 attributed to air pollution, for example, ranged from 9.7 to 13.2 % of China's GDP. US GDP was costed at 3.2–4.6 % by the 103,027 deaths brought on by air pollution. Because of this, a society or nation would profit economically by cutting carbon emissions rather than compensating for them. As a result, several governments have implemented several regulations on industries to reduce carbon emissions, such as the implementation of a tax policy based on emission levels. Based on the study conducted by Benjaafar et al. [24], it has been shown that businesses have the potential to effectively reduce their carbon emissions by adopting operational modifications and collaborating with other actors in the supply chain. In order to reduce the carbon emissions produced by businesses, He et al. [25] calculated the emissions and ideal lot size considering most of the two popular carbon regulations: cap-and-trade (where governments charge businesses a specific amount for each tonne of emissions they create) and carbon tax. These allowances may be put up for auction by the most expensive bidder and subsequently exchanged on secondary markets to determine a carbon price. The effects of environmental legislation and credit periods on inventory management are investigated by Dye and Yang [26] because customers' demand is influenced by credit terms. In accordance with regulations for carbon tax and cap-and-trade, Xu et al. [27] looked at the coordinated pricing and production choices for a

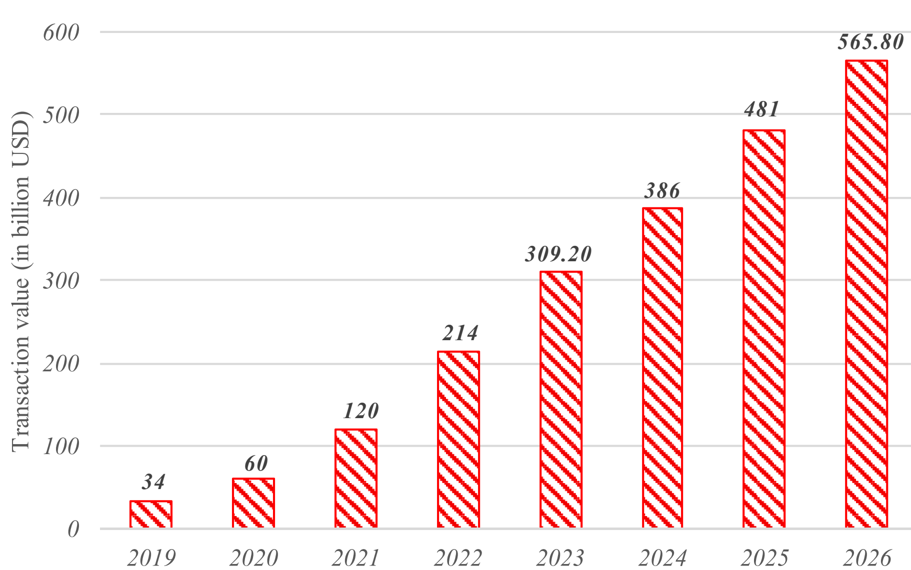

Fig. 1. Worldwide BNPL e-commerce transaction amount from 2019 to 2021, with projections for 2022 to 2026.

Table 1
Recent research work in the inventory literature.

Authors	Demand	Payment policy	Deterioration	Carbon emissions	Interval differential equation
Wu et al. [5]	Stock & price dependent	Trade-credit	Yes	No	No
Manna et al. [39]	Warranty & price dependent	CP	No	Yes	Yes
Rahman et al. [17]	Price & stock dependent	AP	Yes	No	No
Daryanto et al. [31]	Constant	CP	Yes	Yes	No
Alshanbari et al. [10]	Price & frequency of advertisement	AP	Yes	No	No
Zhang et al. [6]	Price dependent	CP	No	No	No
Dye and Yang [26]	Dependent on credit period	Trade-credit	Yes	Yes	No
Khan et al. [7]	Price dependent	AP	Yes	No	No
Taleizadeh et al. [37]	Price sensitive	Trade-credit	No	Yes	No
Ruidas et al. [53]	Selling price & greenness level dependent	CP	No	Yes	No
Shaikh et al. [8]	Ramp type	Trade-credit	Yes	No	No
Rahman et al. [50]	Price dependent	CP	Yes	No	Yes
Duary et al. [18]	Selling price, time and frequency of advertisement dependent	AP and delay in payments	Yes	No	No
Manna & Bhunia [52]	Green level & price dependent	CP	No	No	Yes
Das et al. [54]	Price, green level and time dependent	CP	No	Yes	Yes
This work	Price dependent	BNPL	Yes	Yes	Yes

BNPL: buy now, pay later; AP: advance payment; CP: cash payment.

number of products. Datta [28] investigated a production model in which carbon tax against emissions is taken into account for green investment policy. Darom et al. [29] presented a model based on the supply chain with carbon emissions and safety stock. Goutam and Khanna [30] solve a faulty production inventory problem along with reducing startup costs and carbon emissions. Considering degradation and emissions of carbon, Daryanto et al. [31] solved a three-layer supply chain model. In the same year, Xu et al. [32] investigated a carbon emission inventory model based on multi-resolution emission that takes into account temporal and spatial variations in the Pearl River delta. Taleizadeh et al. [33] investigated carbon emission-related inventory models with planned discounts and partial backordering situations. Lu et al. [34] examined a sustainable manufacturing strategy while considering investment technologies for calculating carbon emissions with the help of the Stackelberg game. In Ruidas et al. [35], an interval-valued carbon emission parameter was used to solve a production inventory model with price-sensitive demand. In their study, Khan et al. [36] investigated the optimal circularity level within a manufacturing system, specifically focusing on a product's carbon emissions within the context of a tax scheme. Recently, Taleizadeh et al. [37], Khan et al. [38], and Manna et al. [39] have formulated different types of inventory models, considering emissions of carbon in their proposed models.

In the year 2020, Nobil et al. [40] investigated the reorder point for an EOQ model in the presence of imperfect quality items. Saif-Eddine et al. [41] developed a genetic algorithm to enhance the optimization of the overall supply chain cost in the inventory location routing problem. Recently, Al-Ashhab [42] proposed a multi-objective optimization model for the design and planning of a resilient closed-loop supply chain network in the face of supply disruptions caused by crises.

To represent the impreciseness of different inventory parameters, several researchers have developed different types of inventory and production inventory models, considering interval-valued demand, different interval-valued cost parameters, interval-valued deterioration rate, etc. Some notable works are mentioned here. In the year 2007, Gupta et al. [43] studied an inventory model with interval-valued inventory cost and a three-component demand rate dependent on the displayed stock level. Gupta et al. [44] solved an inventory problem using a genetic algorithm with advance payment, considering interval-valued inventory costs. Bhunia & Shaikh [45] investigated inventory problems in a two-warehouse system by considering interval-valued parameters via particle swarm optimization. In the year 2017, a production inventory model with interval-valued inventory costs was developed by Bhunia et al. [46]. Ruidas et al. [47] analyzed an EOQ model of two types of defective items in an interval environment. After one year, Shaikh et al. [48] developed an inventory model for non-instantaneous deteriorating items with interval-valued inventory costs. They employed particle swarm optimization to find the best-found solution to their proposed model. Rahman et al. [49] solved a production inventory model via the centre-radius optimisation technique and particle swarm optimization with interval-valued demand. In the same year, an inventory model for deteriorating items where demand is dependent on selling price was solved by Rahman et al. [50] using a parametric approach for an interval differential equation. After that, in 2021, Ruidas et al. [35] solved a production inventory model under price-sensitive demand and interval-valued carbon emission parameters. In the year 2022, an inventory model was solved by Rahman et al. [51] considering interval-valued demand under all units discount facility and deterioration, Manna & Bhunia [52] investigated a green production system with interval-valued demand dependent on price and the green level of the product. Recently, Ruidas et al. [53] investigated a production inventory model of green products under controllable carbon emissions and green subsidies in an interval environment. Manna et al. [39] implemented a production inventory model that includes a warranty policy and carbon emissions-controlled investment via *meta*-heuristic algorithms and Das et al. [54] used interval-valued optimal control theory to analyze the impact of emission reduction and rework policies on green production systems. Moreover, a comprehensive analysis of the relevant literature in relation to the current study is provided, utilising several essential characteristics as outlined in Table 1.

A lack of accurate understanding or information regarding a specific event, circumstance, or result is referred to as uncertainty. It is a common property of real-world phenomena and is frequently linked to uncertainty or risk. A system or process's complexity, unpredictability, variability, or insufficient knowledge are only a few of the many causes of uncertainty. Uncertainty frequently makes it more difficult to make choices and can make it more difficult to plan, make predictions, and manage risks. There are various kinds of uncertainty, such as epistemic uncertainty, which results from a system or process's innate unpredictability or variability. All of the inventory parameters in this study are viewed as uncertainty parameters, and they are all expressed as intervals. This research paper provides a key contribution by examining the comparatively underexplored domain of the BNPL payment method in an environment characterised by uncertainty. Notably, this study introduces the use of interval uncertainty representation in the BNPL payment approach for the first time. The main objective of this study is to develop efficient inventory and pricing methods for perishable goods within the Buy BNPL payment framework. This novel method extends beyond the pursuit of profit maximisation by retailers, encompassing environmental factors, particularly the implications of carbon emissions levies. This study provides vital insights that can aid retailers in developing a balanced and sustainable approach to their operations by integrating financial and environmental considerations. As a result, it makes a substantial contribution to both academic research and industry practices.

The remaining manuscript is organised as follows: The model-building notation and assumptions are described in Section 2. Section 3 presents the mathematical formulation of the inventory system under the BNPL payment scheme in interval uncertainty. A solution approach is given in Section 4, while a numerical example is used in Section 5 to validate the suggested model. Sensitivity studies are performed in Section 6 to provide some managerial insights. In Section 7, the primary findings are summarized, and future research topics are outlined.

2. Notation and assumptions

With the goal of constructing an interval valued EOQ model for decaying items in the presence of pay in later facilities and carbon tax, the subsequent notations are consistently employed in this document, accompanied by the inclusion of certain assumptions.

2.1. Notation

The problem is developed using the ensuing variables and parameters.

Notation	Definition
$\left[\widetilde{x}_L(t),\ \widetilde{x}_U(t)\right]$	Interval-valued level of inventory at time t (units)
$\left[\widetilde{D}_L,\ \widetilde{D}_U\right]$	Market demand (units)
$\left[\widetilde{\alpha}_L,\ \widetilde{\alpha}_U\right]$	Initial fixed demand (units)
$\left[\widetilde{\beta}_L,\ \widetilde{\beta}_U\right]$	Interval-valued demand parameter
$\left[\widetilde{Q}_L,\ \widetilde{Q}_U\right]$	Interval-valued order capacity of the retailer (units)
$\left[\widetilde{\theta}_L,\ \widetilde{\theta}_U\right]$	Interval-valued degrading rate ($0\leqslant\theta_L < \theta_U << 1$)
$[c_{hL},\ c_{hU}]$	Holding cost ($/unit/year)
$[\widehat{c}_{hL},\ \widehat{c}_{hU}]$	Carbon emissions in inventory (tons/unit/year)
$[c_{pL},\ c_{pU}]$	Interval-valued purchased cost ($/unit)
$[\widehat{c}_{pL},\ \widehat{c}_{pU}]$	Interval-valued carbon emissions amount associated per unit purchased (tons/unit)
$\left[\widetilde{\tau}_L,\ \widetilde{\tau}_U\right]$	Interval-valued carbon emission tax ($/ton)
$[CE_L,\ CE_U]$	Interval-valued total amount of carbon emissions per cycle length (tons/year)
$[I_{cL},\ I_{cU}]$	Interval-valued interest charged ($/year)
$[K_L,\ K_U]$	Ordering cost ($/order)
$[\widehat{K}_L,\ \widehat{K}_U]$	Amount of carbon emissions per order (tons/order)
$[SR_L,\ SR_U]$	Sales revenue of the system ($)
$[PC_L,\ PC_U]$	Total purchase cost ($)
$[HC_L,\ HC_U]$	Total holding cost ($)
$[TPL_3,\ TPU_3]$	Total profit of the system ($)
$[APL_3,\ APU_3]$	Interval-valued average profit ($/year)
$\langle APC_3,\ APR_3\rangle$	*c-r* form of average profit ($/year)
Decision variables:	
p	Selling price of the item ($/unit)
T	Cycle length of the purchaser's replenishment in units of time (year)

2.2. Assumptions

i. This inventory model is formulated for a single decaying item, where the demand rate is taken as an interval valued based on the selling price of the item (Rahman et al. [49,50,51], Manna & Bhunia [52], and Das et al. [54]). The interval-valued mathematical form of the demand is given below:

$$\left[\widetilde{D}_L(p),\widetilde{D}_U(p)\right] = \left[\widetilde{\alpha}_L,\ \widetilde{\alpha}_U\right] - \left[\widetilde{\beta}_L,\widetilde{\beta}_U\right]p\ ,\ \text{where } \widetilde{\alpha}_L,\ \widetilde{\beta}_L\ > 0.$$

ii. According to Rahman et al. [49,51], the rate of deterioration of the product is interval-valued, which is given by $\left[\widetilde{\theta}_L,\widetilde{\theta}_U\right]$, where $0\leqslant\widetilde{\theta}_L < \widetilde{\theta}_U << 1$.

iii. During the replenishment cycle, perished products have no repair or replacement or any salvage value.

iv. Inventory management activities and product transportation both contribute to carbon emissions that are taken into consideration. The retailer incurs a tax liability based on the aggregate amount of emissions.

v. An infinite time horizon is considered with negligible lead time.

vi. Due to the pay-later facility, the supplier charges an interval valued interest rate after a certain time period and the rate of interest is $[I_{cL}, I_{cU}]$.

3. Mathematical model

Considering the above-mentioned notation and assumptions, the inventory level $\left[\widetilde{x}_L(t),\ \widetilde{x}_U(t)\right]$ at time t during the time period $[0,T]$ decreases due to deterioration and customers' demand rate (see Fig. 2). Hence, the interval-valued differential equation is given as follows:

$$d[\frac{\widetilde{x}_L(t),\ \widetilde{x}_U(t)\,]}{dt} + \left[\widetilde{\theta}_L,\widetilde{\theta}_U\right]\cdot[\widetilde{x}_L(t),\ \widetilde{x}_U(t)\,] = -\left[\widetilde{D}_L(p),\widetilde{D}_U(p)\right],\ t\in[0,T] \quad (1)$$

with boundary conditions $\left[\widetilde{x}_L(0),\ \widetilde{x}_U(0)\right] = \left[\widetilde{Q}_L,\widetilde{Q}_U\right]$ and $\left[\widetilde{x}_L(T),\ \widetilde{x}_U(T)\right] = [0,0]$.

The parametric form of equation (1) is given by

$$\frac{d\widetilde{x}(r_1,t)}{dt} + \widetilde{\theta}(r_2)\widetilde{x}(r_1,t) = -\widetilde{D}(r_3,p) \quad (2)$$

with boundary conditions $\widetilde{x}(0\ ,\ r_1) = \widetilde{Q}_L + r_1(\widetilde{Q}_U - \widetilde{Q}_L)$ and $\widetilde{x}(T,r_1) = 0$, where

$$\widetilde{x}(t,r_1) = \widetilde{x}_L(t) + r_1\left\{\widetilde{x}_U(t) - \widetilde{x}_L(t)\right\},$$
$$\widetilde{\theta}(r_2) = \widetilde{\theta}_L + r_2\left(\widetilde{\theta}_U - \widetilde{\theta}_L\right),\ \ and$$
$$\widetilde{D}(p,r_3) = \widetilde{D}_L(p) + r_3\left\{\widetilde{D}_U(p) - \widetilde{D}_L(p)\right\}.$$

Using the boundary condition $x(T,r_1) = 0$, the solution of equation (2) is

$$\widetilde{x}(t,r_1) = \frac{\widetilde{D}(p,r_3)}{\widetilde{\theta}(r_2)}\left\{e^{\widetilde{\theta}(r_2)(T-t)} - 1\right\}. \quad (3)$$

Therefore, the lower and upper bounds of the inventory level during the interval of time $[0,T]$ are calculated as:

$$\widetilde{x}_L(t) = \frac{\widetilde{D}_L(p)}{\widetilde{\theta}_U}\left\{e^{\widetilde{\theta}_L(T-t)} - 1\right\} \quad (4)$$

and

$$\widetilde{x}_U(t) = \frac{\widetilde{D}_U(p)}{\widetilde{\theta}_L}\left\{e^{\widetilde{\theta}_U(T-t)} - 1\right\} \quad (5)$$

Employing the condition $\left[\widetilde{x}_L(0),\ \widetilde{x}_U(0)\right] = \left[\widetilde{Q}_L,\widetilde{Q}_U\right]$ in equation (3), one finds

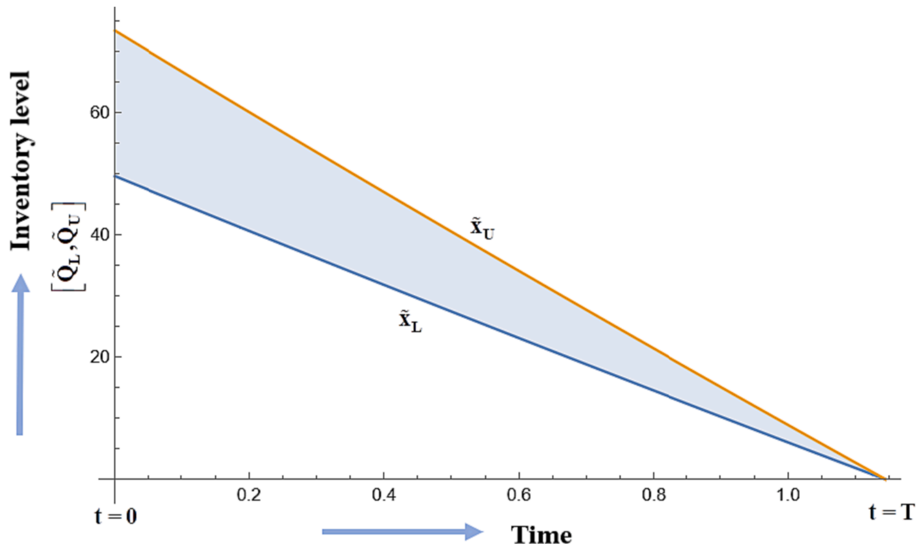

Fig. 2. Pictorial representation of interval-valued inventory level.

$$\widetilde{Q}_L = \frac{\widetilde{D}_L(p)}{\widetilde{\theta}_U}\left\{e^{\tilde{\theta}_L T} - 1\right\} \text{ and } \widetilde{Q}_U = \frac{\widetilde{D}_U(p)}{\widetilde{\theta}_L}\left\{e^{\tilde{\theta}_U T} - 1\right\} \tag{6}$$

Therefore, the interval form of the initial stock level is given by

$$\left[\widetilde{Q}_L, \widetilde{Q}_U\right] = \left[\frac{\widetilde{D}_L(p)}{\widetilde{\theta}_U}\left\{e^{\tilde{\theta}_L T} - 1\right\}, \frac{\widetilde{D}_U(p)}{\widetilde{\theta}_L}\left\{e^{\tilde{\theta}_U T} - 1\right\}\right] \tag{7}$$

The interval form of sales revenue for the system is calculated as follows:

$$[SR_L(p,T), SR_U(p,T)] = \int_0^T \left[\widetilde{D}_L(p), \widetilde{D}_U(p)\right] p\, dt$$

$$= \left[\left(\widetilde{\alpha}_L - \widetilde{\beta}_U p\right)pT, \left(\widetilde{\alpha}_U - \widetilde{\beta}_L p\right)pT\right] \tag{8}$$

The interval valued ordering is

$$[OC_L, OC_U] = [K_L, K_U] \tag{9}$$

Interval valued holding for each replenishment cycle time is as follows:

$$[HC_L(p,T), HC_U(p,T)] = \left[c_{hL}\frac{\widetilde{D}_L}{\widetilde{\theta}_U}\left\{\frac{1}{\widetilde{\theta}_U}\left(e^{\tilde{\theta}_L T} - 1\right) - T\right\}, c_{hU}\frac{\widetilde{D}_U}{\widetilde{\theta}_L}\left\{\frac{1}{\widetilde{\theta}_L}\left(e^{\tilde{\theta}_U T} - 1\right) - T\right\}\right]. \tag{10}$$

The total interval-valued emissions level per business length includes the emissions from procurement activities, the fixed emissions related to order placing activities $[\widehat{K}_L, \widehat{K}_U]$, and the emissions due to holding operations of the stored goods. Hence, the total interval-valued carbon emissions per replenishment are calculated as follows:

$$[CE_L(p,T), CE_U(p,T)] = [\widehat{K}_L, \widehat{K}_U] + [\widehat{c}_{pL}, \widehat{c}_{pU}]\left[\widetilde{Q}_L, \widetilde{Q}_U\right] + [\widehat{c}_{hL}, \widehat{c}_{hU}]\int_0^T\left[\widetilde{x}_L(t), \widetilde{x}_U(t)\right] dt$$

$$= \left[\widehat{K}_L + \widehat{c}_{pL}\widetilde{Q}_L + \widetilde{c}_{hL}\frac{\widetilde{D}_L}{\widetilde{\theta}_U}\left\{\frac{1}{\widetilde{\theta}_U}\left(e^{\tilde{\theta}_L T} - 1\right) - T\right\}, \widehat{K}_U + \widehat{c}_{pU}\widetilde{Q}_U + \widetilde{c}_{hU}\frac{\widetilde{D}_U}{\widetilde{\theta}_L}\left\{\frac{1}{\widetilde{\theta}_L}\left(e^{\tilde{\theta}_U T} - 1\right) - T\right\}\right] \tag{11}$$

The total interval-valued purchasing cost $[PC_L(p,T), PC_U(p,T)]$ is as follows:

$$[PC_L(p,T), PC_U(p,T)] = [c_{pL}, c_{pU}]\left[\widetilde{Q}_L, \widetilde{Q}_U\right]$$

$$= \left[c_{pL}\frac{\widetilde{D}_L(p)}{\widetilde{\theta}_U}\left\{e^{\tilde{\theta}_L T} - 1\right\}, c_{pU}\frac{\widetilde{D}_U(p)}{\widetilde{\theta}_L}\left\{e^{\tilde{\theta}_U T} - 1\right\}\right] \tag{12}$$

In addition, the interval-valued interest charge $[IC_{3L}(T), IC_{3U}(T)]$ for the pay-later facility for the entire cycle is given by

$$[IC_{3L}(T), IC_{3U}(T)] = [I_{cL}, I_{cU}][c_{pL}, c_{pU}]\int_0^T\left[\tilde{x}_L(t),\, \tilde{x}_U(t)\right]dt$$

$$= \left[I_{cL}c_{pL}\left\{\frac{1}{\tilde{\theta}_U}\left(e^{\tilde{\theta}_L T}-1\right)-T\right\},\; I_{cU}c_{pU}\left\{\frac{1}{\tilde{\theta}_L}\left(e^{\tilde{\theta}_U T}-1\right)-T\right\}\right] \quad (13)$$

Utilizing results from (8)-(13), the total profit per replenishment cycle is obtained as follows:

$$[TPL_3(p,T),\, TPU_3(p,T)] = \left[SR_L - OC_U - HC_U - \tilde{\tau}_U CE_U - PC_U - IC_{3U},\; SR_U - OC_L - HC_L - \tilde{\tau}_L CE_L - PC_L - IC_{3L}\right]$$

$$= \begin{bmatrix} p\left(\tilde{\alpha}_L - \tilde{\beta}_U p\right)T - K_U - c_{hU}\dfrac{\tilde{D}_U}{\tilde{\theta}_L}\left\{\dfrac{1}{\tilde{\theta}_L}\left(e^{\tilde{\theta}_U T}-1\right)-T\right\} - \tilde{\tau}_U\left(\widehat{K}_U + \widehat{c}_{pU}Q_U + \widehat{c}_{hU}\dfrac{\tilde{D}_U}{\tilde{\theta}_L}\left\{\dfrac{1}{\tilde{\theta}_L}\left(e^{\tilde{\theta}_U T}-1\right)-T\right\}\right) \\ -c_{pU}\dfrac{\tilde{D}_U(p)}{\tilde{\theta}_L}\left\{e^{\tilde{\theta}_U T}-1\right\} - I_{cU}c_{pU}\left\{\dfrac{1}{\tilde{\theta}_L}\left(e^{\tilde{\theta}_U T}-1\right)-T\right\}, \\ p\left(\tilde{\alpha}_U - \tilde{\beta}_L p\right)T - K_L - c_{hL}\dfrac{\tilde{D}_L}{\tilde{\theta}_U}\left\{\dfrac{1}{\tilde{\theta}_U}\left(e^{\tilde{\theta}_L T}-1\right)-T\right\} - \\ \tilde{\tau}_L\left(\widehat{K}_L + \widehat{c}_{pL}Q_L + \widehat{c}_{hL}\dfrac{\tilde{D}_L}{\tilde{\theta}_U}\left\{\dfrac{1}{\tilde{\theta}_U}\left(e^{\tilde{\theta}_L T}-1\right)-T\right\}\right) - c_{pL}\dfrac{\tilde{D}_L(p)}{\tilde{\theta}_U}\left\{e^{\tilde{\theta}_L T}-1\right\} - I_{cL}c_{pL}\left\{\dfrac{1}{\tilde{\theta}_U}\left(e^{\tilde{\theta}_L T}-1\right)-T\right\} \end{bmatrix} \quad (15)$$

Therefore, the interval form of the average profit of this model per cycle length is presented by $[APL_3(p,T),\, APU_3(p,T)] = \left[\frac{TPL_3(p,T)}{T},\, \frac{TPU_3(p,T)}{T}\right]$.

The centre-radius form of the profit per unit is followed by:

$$APC_3(p,T) = \frac{APL_3(p,T) + APU_3(p,T)}{2} \quad (16)$$

and

$$APR_3(p,T) = \frac{APU_3(p,T) - APL_3(p,T)}{2} \quad (17)$$

4. Solution procedure

The centre-radius $(c-r)$ optimization method (Rahman et al., [50]) and the interval order relation (Bhunia and Samanta, [55]) are used to determine the optimal solution to the maximising problem (16). The following two stages offer the solution to problem (16):

Step 1: Find the following optimization problem's centre value:

Maximize $APC_3(p,T)$

subject to $T > 0,\, p > 0$.

Then, find the solution pair (T^*, p^*). Here $APC_3(p,\ T) = \frac{APL_3(p,T) + APU_3(p,T)}{2}$.

The following symbol is used to signify the set of all maximizers for the aforementioned problem: $U_f = \{(T,p) : APC_3(p,T) = APC_3(p^*, T^*)\}$.

Step 2: The optimization problem's radius is resolved.

Minimize $APR_3(p,T)$
subject to $(p,T) \in U_f$

Here, $APR_3(p,T) = \frac{APU_3(p,T) - APL_3(p,T)}{2}$. (18).

The approach given below addresses the fundamental maximization problem of our presented inventory system.

5. Numerical example

To illustrate the proposed interval-valued inventory model under interval-value price-dependent demand, we considered Example 1 and solved it numerically using MATHEMATICA software.

Example 1. *The various inventory parameter values associated with the suggested model are listed as follows*:

$$\left[\tilde{\alpha}_L,\, \tilde{\alpha}_U\right] = [120, 122],\ [\tilde{\beta}_L, \tilde{\beta}_U] = [1.7, 1.8],\ [\tilde{\theta}_L, \tilde{\theta}_U] = [0.07, 0.08],\ [c_{pL}, c_{pU}] = [8, 10],\ [K_L, K_U] = [50, 52],$$
$$[c_{hL}, c_{hU}] = [0.8, 0.9],\ [\widehat{K}_L, \widehat{K}_U] = [200, 210],\ [\widehat{c}_{pL}, \widehat{c}_{pU}] = [4, 6],\ [\widehat{c}_{hL}, \widehat{c}_{hU}] = [2, 3],\ [I_{cL}, I_{cU}] = [0.09, 0.11],$$
$$[\tilde{\tau}_L, \tilde{\tau}_U] = [0.09, 0.11].$$

The most effective options for Example 1 are presented in Table 2. Furthermore, Fig. 3 is presented to illustrate the interval form of the average profit function in Example 1, specifically in relation to the selling price and the replenishment cycle. The concavity of the centre of

Table 2
Optimal solutions of Example 1.

Variables/unknown parameters	Optimal values
p	\$ 40.223841
T	1.144518 years
$\langle APC_3, APR_3\rangle$	$\langle 1378.805, 411.002\rangle$
$[APL_3, APU_3]$	$[967.804, 1789.81]$
$[\widetilde{Q}_L, \widetilde{Q}_U]$	$[49.6277, 73.4465]$

the average profit in Example 1 is visualized with respect to the selling price and the replenishment cycle in Fig. 4.

6. Sensitivity analyses

By changing the values of several inventory factors from $-20\,\%$ to $+20\,\%$, the sensitivity analyses of Example 1 are conducted to examine the effects of the various inventory parameters on the centre of average profit (APC_3), selling price (p), and total time of replenishment cycle (T). Figs. 5-10 illustrate the graphical representation of the optimal outcomes obtained from these assessments.

The following implications are observed from the preceding assessments:

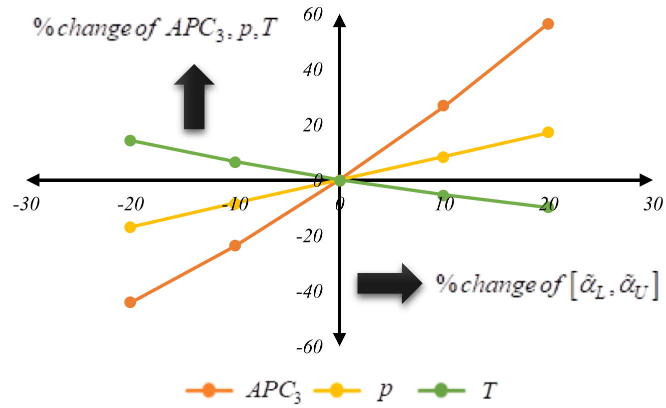

Fig. 5. The effect of changes of $\left[\tilde{\alpha}_L, \tilde{\alpha}_U\right]$ on the centre of average profit (APC_3), length of replenishment cycle (T) and selling price (p).

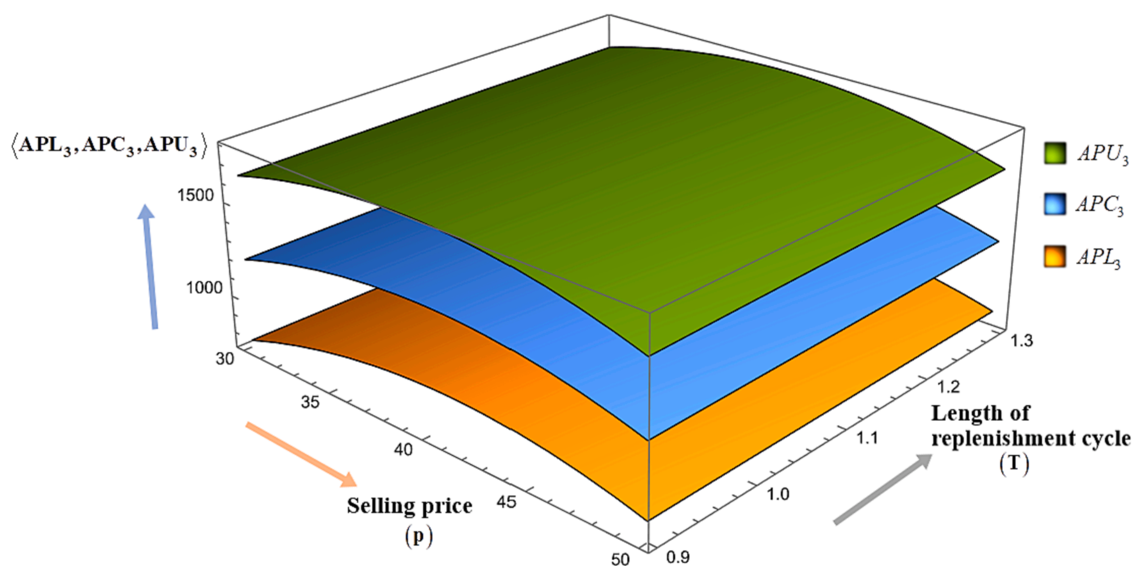

Fig. 3. Average profit of interval forms of Example 1.

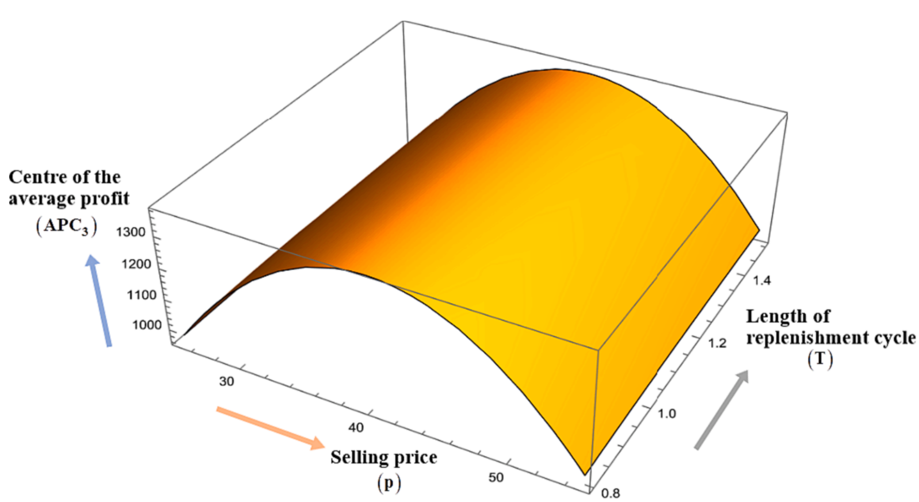

Fig. 4. Concavity of Average profit w.r.t. the length of replenishment cycle (T) and selling price (p) of Example 1.

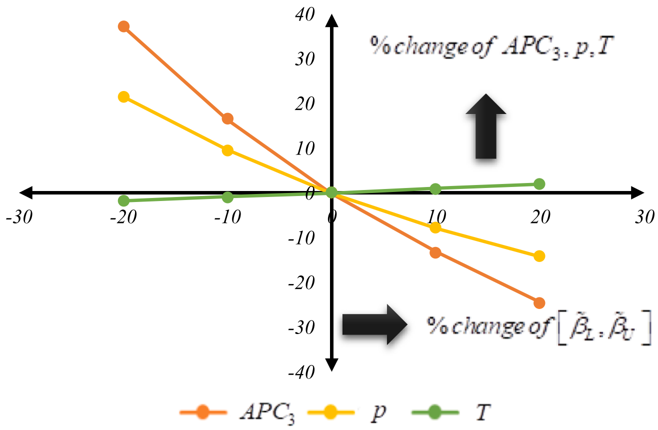

Fig. 6. The effect of changes of $\left[\tilde{\beta}_L, \tilde{\beta}_U\right]$ on the centre of average profit (APC_3), length of replenishment cycle (T) and selling price (p).

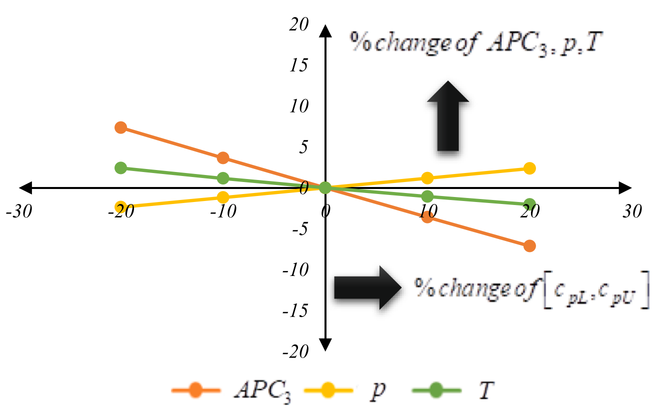

Fig. 7. The effect of changes of $[c_{pL}, c_{pU}]$ on the centre of average profit (APC_3), length of replenishment cycle (T) and selling price (p).

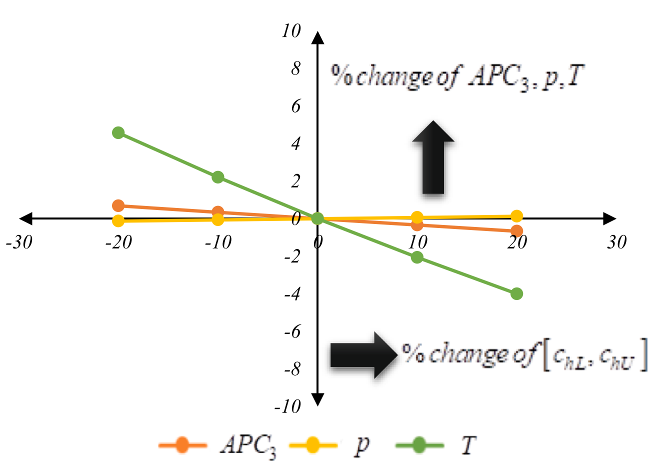

Fig. 8. The effect of changes of $[c_{hL}, c_{hU}]$ on the centre of average profit (APC_3), length of replenishment cycle (T) and selling price (p).

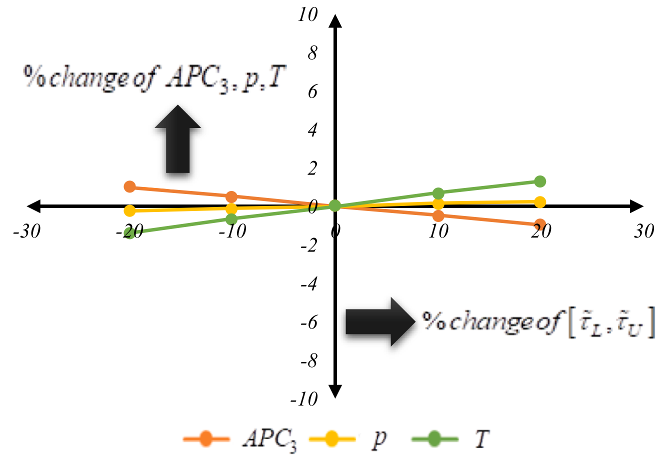

Fig. 9. The effect of changes of $\left[\tilde{\tau}_L, \tilde{\tau}_U\right]$ on the centre of average profit (APC_3), length of replenishment cycle (T) and selling price (p).

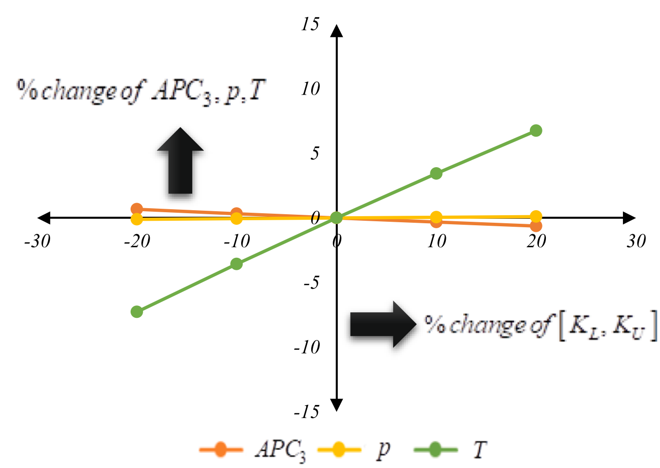

Fig. 10. The effect of changes of $[K_L, K_U]$ on the centre of average profit (APC_3), length of replenishment cycle (T) and selling price (p).

(i) It is seen from Fig. 5 that there appears to be a positive correlation between changes in interval-valued initial fixed demand $\left[\tilde{\alpha}_L, \tilde{\alpha}_U\right]$ and changes in the centre of average profit (APC_3). As $\left[\tilde{\alpha}_L, \tilde{\alpha}_U\right]$ increases (from -20 % to 20 %), the centre of average profit generally increases, indicating that higher initial fixed demand is associated with a significant increase in the centre of average profit. However, there is a negative correlation between changes in $\left[\tilde{\alpha}_L, \tilde{\alpha}_U\right]$ and changes in the selling price (p). As $\left[\tilde{\alpha}_L, \tilde{\alpha}_U\right]$ increases, the selling price tends to decrease (and vice versa), suggesting that higher initial fixed demand may lead to a reduction in the selling price, with the decrease being more pronounced at higher levels of $\left[\tilde{\alpha}_L, \tilde{\alpha}_U\right]$. Likewise, there is a negative correlation between changes in $\left[\tilde{\alpha}_L, \tilde{\alpha}_U\right]$ and changes in the length of the replenishment cycle (T). As $\left[\tilde{\alpha}_L, \tilde{\alpha}_U\right]$ increases, the length of the replenishment cycle (T) tends to decrease (and vice versa).

(ii) Fig. 6 reveals that there is a significant variation in the centre of the average profit (APC_3) when the price-sensitive coefficient in demand ($\left[\widetilde{\beta}_L, \widetilde{\beta}_U\right]$) changes (from −20 % to 20 %). Initially, as $\left[\widetilde{\beta}_L, \widetilde{\beta}_U\right]$ decreases by −20 %, the centre of the average profit increases substantially by 37.11 %, indicating a positive correlation. However, as $\left[\widetilde{\beta}_L, \widetilde{\beta}_U\right]$ continues to increase from 10 % to 20 %, the centre of the average profit decreases significantly, indicating a negative correlation. In addition, there is a negative impact of $\left[\widetilde{\beta}_L, \widetilde{\beta}_U\right]$ on the selling price. This suggests that an increase in the price-sensitive coefficient of demand leads to lower selling prices. Finally, a positive impact of $\left[\widetilde{\beta}_L, \widetilde{\beta}_U\right]$ on the replenishment cycle is observed. As $\left[\widetilde{\beta}_L, \widetilde{\beta}_U\right]$ increases (from −20 % to 20 %), the replenishment cycle (T) generally increases. This indicates that a higher price-sensitive coefficient of demand is associated with longer replenishment cycles.

(iii) Fig. 7 highlights that small changes in the per unit interval-valued purchased cost ($\left[c_{pL}, c_{pU}\right]$) have relatively minor impacts on the centre of average profit (APC_3). This suggests that while cost reduction efforts are valuable, businesses should also focus on other factors, such as pricing and inventory management, to optimize profitability. Moreover, changes in $\left[c_{pL}, c_{pU}\right]$ have minimal effects on selling prices. Businesses may consider adjusting prices in response to cost changes, but these adjustments should be modest to avoid alienating customers. The most significant impact is observed in the replenishment cycle due to changes in $\left[c_{pL}, c_{pU}\right]$. The length of the replenishment cycle (T) falls when $\left[c_{pL}, c_{pU}\right]$ increases.

(iv) Fig. 8 indicates the centre of the average profit (APC_3), the length of the replenishment cycle (T) and the selling price of the product (p) all are not sensitive with the changes of per unit holding cost $[c_{hL}, c_{hU}]$. Thus, this insensitivity in outcomes enables businesses to maintain stable cost structures and pricing strategies, fostering long-term planning and competitive advantage while necessitating prudent risk management.

(v) From Fig. 9, it is also seen that the length of the replenishment cycle (T), the centre of the average profit (APC_3), and the selling price (p) are not sensitive with respect to the changes of emission tax parameter $\left[\widetilde{\tau}_L, \widetilde{\tau}_U\right]$.

(vi) As is shown in Fig. 10, the centre of average profit (APC_3), and the selling price (p) are almost insensitive to changes in interval-valued ordering cost $[K_L, K_U]$. The relatively small impact of changes in interval-valued ordering cost ($[K_L, K_U]$) on the centre of average profit (APC_3) suggests that businesses may have some flexibility in managing ordering costs without significantly affecting their overall profitability. While changes in $[K_L, K_U]$ have a limited impact on selling prices, businesses can consider adjusting prices slightly in response to cost changes to maintain profitability. The most significant impact is observed in the replenishment cycle (T), with changes ranging from −7.27 % to 6.74 % because of changes in $[K_L, K_U]$ from −20 % to 20 %.

6.1. Managerial insights

The utilization of analyses in management yields valuable insights that have significant importance for policymakers. These insights serve as evidence-based guidance, enabling policymakers to develop policies that are successful in achieving their intended objectives. These insights provide a pragmatic comprehension of the relationship between different elements within certain domains, enabling policymakers to formulate policies that effectively tackle real-world difficulties and capitalize on opportunities. Here are some managerial insights about setting the selling price and the replenishment cycle length under a tax regulation on emissions:

- A higher initial fixed demand results in an elevation of the centre of average profit and selling prices. Nevertheless, the duration of replenishment cycles tends to decrease as the initial fixed demand grows. Therefore, in order to enhance profitability, managers ought to implement measures aimed at cultivating the inherent fixed demand and establishing a higher selling price.
- Changes in the price-sensitive coefficient have a large negative influence on the average profit centre. It is important to note that this connection is non-linear. An escalation in price sensitivity results in a decrease in selling prices while simultaneously extending replenishment periods. Therefore, in instances where the price sensitivity coefficient is significantly elevated, it is advisable for the decision manager to establish a reduced selling price in order to augment profitability.
- Negligible effects on average profit are observed when there are slight alterations in the per-unit cost, hence underscoring the necessity of directing attention towards other aspects in order to optimize profit.
- The stability of outcomes persists even when there are variations in per-unit holding costs, enabling firms to uphold their cost structures and pricing strategies. The presence of stability in a given context promotes the cultivation of strategic plans over an extended period of time, hence enhancing the capacity to gain a competitive edge.
- The results demonstrate that variations in the emission tax parameter have minimal impact on profitability, replenishment cycles, and selling prices, suggesting that this parameter has limited influence on the outcomes.
- The influence of modifications in ordering costs on average profit and selling prices is generally insignificant. The replenishment cycle exhibits a substantial variation in response to fluctuations in the ordering cost, therefore yielding a very notable impact. Thus, the manager should employ an investment to reduce the ordering cost.

These insights provide organizations with the necessary information to make well-informed decisions, enhance profitability, effectively manage expenses, develop pricing strategies, and plan for the long term. Additionally, they identify areas where sensitivity to certain parameters may necessitate proactive management and strategic modifications.

7. Conclusions

This work discusses an interval-valued inventory model that takes into account a product's demand as interval-valued with price-dependent demand under the BNPL payment method. The deterioration rate of an item is taken as an interval. In other words, goods with a longer freshness period deteriorate more gradually over time and they fall within an interval. Due to the consideration of all the parameters as interval values, the differential equation of inventory level is presented in interval form and this differential equation is solved using the parametric approach of intervals. The corresponding objective function is transformed into an interval-valued optimisation problem. To solve this interval-valued optimisation problem, interval-order relations (Bhunia and Samanta [55]) and the centre-radius optimisation technique are used, and the transformed problem is solved with the help of MATHEMATICA software. It is noted that the objective function's centre value, which is displayed visually, lies between the objective function's lower bound and upper bound. This model offers instructions for writing business plans for businesses that produce bakeries, pharmacies, cosmetics, cement, chemicals, food products (such as sugar and powder milk), alcoholic beverages, and other products, in addition to the aforementioned industries. The findings indicate that to enhance profitability, managers ought to implement measures aimed at cultivating the inherent fixed demand and establishing a higher selling price. When the price sensitivity coefficient is significantly elevated, the decision manager should establish a reduced selling price in order to augment profitability.

In order to conduct further studies, individuals have the opportunity to include several factors into their analysis. These factors include including nonlinear stock dependent demand, implementing preservation technologies, using carbon cap and trade policies, and employing trade credit, whether at a single or two-level level. Furthermore, the utilisation of the metaheuristic approach is accessible to all individuals seeking to address this intricate situation.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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