Kernel: SageMath 10.2

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ROLL NO : 29

EXPERIMENT NO : 04

AIM : TO FIND LINEAR COMBINATION, SPAN, AND BASIS OF A VECTOR SPACE

QUESTION : 1:- Find The Linear Span of The Set of Vectors

In [1]:
```
V=QQ^4
V1=vector(QQ,[1,1,2,-1])
V2=vector(QQ,[2,3,5,-4])
V3=vector(QQ,[3,4,7,-5])
W=V.span([V1,V2,V3])
W
```

Out[1]:
```
Vector space of degree 4 and dimension 2 over Rational Field
Basis matrix:
[ 1  0  1  1]
[ 0  1  1 -2]
```

QUE 1B

In [2]:
```
V=QQ^4
V1=vector(QQ,[1,3,6,-1])
V2=vector(QQ,[8,3,2,1])
V3=vector(QQ,[9,4,5,-8])
W=V.span([V1,V2,V3])
W
```

Out[2]:
```
Vector space of degree 4 and dimension 3 over Rational Field
Basis matrix:
[      1       0       0  -98/29]
[      0       1       0  395/29]
[      0       0       1 -186/29]
```

QUESTION:-2 - Check If The Following Set Of Vectors in Q^4 are Linearly Dependent

In [3]:
```
V=QQ^4
V1=vector(QQ,[1,-1,2,4])
V2=vector(QQ,[-3,3,2,1])
V3=vector(QQ,[-1,-2,6,9])
L=[V1,V2,V3]
V.linear_dependence(L)==[]
```

Out[3]:
```
True
```

QUE 2B

In [4]:
```
V=QQ^4
V1=vector(QQ,[1,-1,2,4])
V2=vector(QQ,[-3,3,2,1])
V3=vector(QQ,[0,0,0,0])
L=[V1,V2,V3]
V.linear_dependence(L)==[]
```

Out[4]:
```
False
```

QUESTION 3

In [5]:
```
M = Matrix([[1,3,1,3],[0,1,1,0],[-3,0,6,-1],[3,4,-2,1],[2,0,-4,-3]])
M_echelon = M.echelon_form()
basis = M_echelon.rows()
rank = M.rank()
print("Basis of the row space:", basis)
print("Dimension of the row space (Rank):", rank)
```

Out[5]:
```
Basis of the row space: [(1, 0, -2, 0), (0, 1, 1, 0), (0, 0, 0, 1), (0, 0, 0, 0), (0, 0, 0, 0)]
Dimension of the row space (Rank): 3
```

QUE 3B

In [6]:
```
M = Matrix([[2,4,6,8],[0,1,1,0],[3,0,-6,1],[4,-2,3,-1],[2,0,-4,3]])
M_echelon = M.echelon_form()
basis = M_echelon.rows()
rank = M.rank()
print("Basis of the row space:", basis)
print("Dimension of the row space (Rank):", rank)
```

Out[6]:
```
Basis of the row space: [(1, 0, 0, 0), (0, 1, 0, 0), (0, 0, 1, 0), (0, 0, 0, 1), (0, 0, 0, 0)]
Dimension of the row space (Rank): 4
```

QUESTION 4

In [7]:
```
from sage.modules.free_module_element import vector
from sage.matrix.constructor import Matrix
```

In [9]:
```
v1 = vector([1, 2, 3])
v2 = vector([4, 5, 6])
v3 = vector([7, 8, 9])
target_vector = vector([10, 11, 12])
A = Matrix([v1, v2, v3]).transpose()
A = A.augment(target_vector, subdivide=True)
rref = A.rref()
if all(entry == 0 for entry in rref[-1][:-1]):
    print("The target vector is a linear combination of the given vectors.")
else:
    print("The target vector is not a linear combination of the given vectors.")
```

Out[9]:
```
The target vector is a linear combination of the given vectors.
```

QUE 4B

In [11]:
```
v1 = vector([2, 3, 1])
v2 = vector([7, 6, 3])
v3 = vector([4, 9, 3])
target_vector = vector([12, 13, 14])
A = Matrix([v1, v2, v3]).transpose()
A = A.augment(target_vector, subdivide=True)
rref = A.rref()
if all(entry == 0 for entry in rref[-1][:-1]):
    print("The target vector is a linear combination of the given vectors.")
else:
    print("The target vector is not a linear combination of the given vectors.")
```

Out[11]:
```
The target vector is not a linear combination of the given vectors.
```

CONCLUSION:-Problems on linear combination, span and basis are successfully executed .