Airline Pricing: Dynamic Programming

Introduction

The airline industry, a highly competitive market, is constantly balancing maximizing revenue while minimizing customer inconvenience. One strategy that has proven both lucrative and controversial is overbooking flights. Overbooking in the airline industry is a strategic practice where airlines sell more tickets than there are seats available on a flight. This might seem counterintuitive at first, but it's rooted in sound revenue management principles and addresses several operational challenges. Overbooking allows airlines to compensate for customers that don't show for their purchase(no show), where a seat is otherwise occupied by another customer rather than remaining empty. It can help airlines optimize flight occupancy, maximize revenue, and maintain competitive pricing. However, it also carries the risk of over-commitment, resulting in additional costs when passengers are bumped or require upgrades. This report presents a detailed analysis of overbooking policies for coach tickets using a dynamic programming framework. By exploring two distinct policies—the traditional hard cap approach versus a flexible sales control option—we quantify the expected discounted profit, evaluate overbooking frequencies, and assess the associated cost implications.

Problem Overview

In our analysis we handle the challenge of maximizing expected discounted profit by optimizing both the pricing strategy and the ticket sales volume for a specific flight. The primary revenue stems from ticket sales, while the key cost factor is the overbooking penalty incurred when more passengers show up than there are seats available. Fixed costs, although present, are not of focus throughout the analysis.

The flight under consideration features two distinct ticket classes—coach and first-class—with different pricing strategies and demand characteristics. There are two available price points for each class, each associated with its own demand distribution. Importantly, ticket sales occur on a daily basis, with the possibility of selling at most one ticket per class per day. This means that on any given day there is a certain probability (based on the price) a customer will purchase 0/1 first-class tickets and 0/1 coach tickets.

Another thing is that the overbooking policy for this flight is applied selectively:

- **First-Class:** Overbooking is not allowed, maintaining high customer goodwill is critical for this premium segment. Overbooking and then bumping someone due to lack of seats will diminish customer goodwill.
- **Coach:** Overbooking is permitted. However, should more coach passengers show up than there are seats, we have the option to bump some to first-class if space is available. This, in turn, incurs an additional cost due to the higher service cost in first-class. If there is no coach or first-class space available, then overbooked passengers will be bumped off the plane. In this scenario the additional overbook cost is an even higher penalty, which causes an even higher penalty.

Demand dynamics for the tickets are the following:

- **Demand Independence:** The demand for coach and first-class tickets is assumed to be independent.
- **Inter-class Substitution:** When first-class seats are fully booked, potential first-class customers may opt for coach tickets. In such cases, the chance of a sale in coach increases by 3 percentage points, regardless of the coach ticket price.
- **Purchase Behavior:** If there is excessive demand for first-class beyond its seating capacity, customers do not purchase a coach seat, and vice versa if coach demand exceeds the allowable overbooking policy.

Below we have listed the parameters flight that will guide our approach:

- There are 365 days until the flight departs, offering daily opportunities for ticket sales.
- Coach has 100 seats available, with an option to overbook by up to 5 seats. First-Class has 20 seats available, with no overbooking.
- Coach ticket prices can be $300 with a 65% chance of sale per day (increases to 68% when first-class is sold out), or $350 with a 30% chance of sale per day (increases to 33% when first class is sold out).
- First-class ticket prices can be $425 with an 8% chance of sale per day, or $500 with a 4% chance of sale per day.
- Coach ticket holders have a 95% likelihood to show up for the flight, and first-class ticket holders have a 97% likelihood to show up for the flight
- The cost of overbooking is $50 to bump a coach passenger to first-class, or $425 if a coach passenger is overbooked and there is no space in first-class seating.
- The daily discount factor is computed from an annual discount rate of 17%, ensuring that revenue and costs are appropriately weighted over time.

This overview sets the stage for our detailed examination and dynamic programming set up can balance the trade-offs between maximizing revenue from ticket sales and minimizing the costs associated with overbooking, ultimately guiding our strategic decision-making process in the competitive airline industry.

Methodology #1:

Our objective is to maximize the expected discounted profit from ticket sales while accounting for overbooking costs. For this problem we have defined 4 cases with specific outcomes for each case, then we define a dynamic program to deal with the various outcomes, and lastly we run through a backwards induction loop to compute the expected value of each scenario.

Case/Outcomes

Case 1: Neither Coach nor First-class is sold out

In this scenario, we have the option to set prices for both classes. The decision influences the probability of a sale for each ticket class, and all combinations of outcomes are possible…

- No Sale: Neither a coach nor a first-class ticket is sold.
- Coach Sale Only: A coach ticket is sold, but no first-class ticket is sold.
- First-Class Sale Only: A first-class ticket is sold, but no coach ticket is sold.
- Both Sales: Both a coach and a first-class ticket are sold.

Case 2: Coach is sold out, but First-class is not

In this situation, the coach tickets have reached their maximum allowable sales (including the overbooking limit), so no further coach ticket sales can occur. Only first-class tickets remain available for sale, so we only need to handle first-class ticket pricing decisions…

- No Sale: No first-class ticket is sold.
- First-Class Sale: A first-class ticket is sold, leading to an increment in the first-class sales count.

Case 3: First-class is sold out, but Coach is not

In this case, the coach sale probability is manually adjusted by adding 3 percentage points to account for the increased demand from first-class customers who are unable to book there. This time we only have to worry about coach ticket pricing decisions…

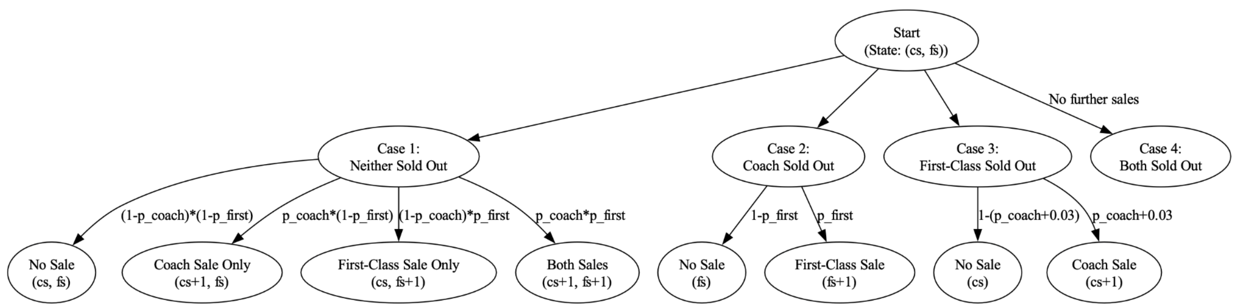
- No Sale: No coach ticket is sold.

- Coach Sale: A coach ticket is sold, updating the coach sales count.

Case 4: Both Coach and First-class tickets are sold out

In the scenario where both coach and first-class tickets have reached their maximum sales limits, no further ticket sales are possible. The value function simply carries the discounted value from the next day. The state remains unchanged, and no additional revenue is generated.

To better illustrate we created the following diagram:



Dynamic Program Definition

State variables:

- **Day:** The current day in the 365-day timeline.
- **Coach Sales (cs):** The number of coach tickets sold so far, which can exceed the coach seating capacity by a defined overbooking limit.
- **First-Class Sales (fs):** The number of first-class tickets sold so far, which is strictly capped by first-class seating capacity.
- The state is implicitly represented by the indices in our DP matrix

Choice/Decision Variables:

- The pricing decisions for coach tickets can be set at \$300 or \$350, and first-class tickets can be set at \$425 or \$500.
- These pricing choices influence the probability of selling a ticket on any given day. The pricing options are iterated over within the DP loop for each option.

Dynamics:

- Depending on whether a ticket is sold in coach or first-class, the state (i.e., the values of cs and fs) is updated accordingly.
- The outcomes are probabilistic, with the immediate revenue being the price of the sold ticket and the future state determined by the new ticket count.

Value function:

$$V(\text{day}, cs, fs)$$

This represents the maximum expected discounted profit from a given state onward. It is updated for each outcome in a specific case. Revenue is the immediate payoff and dp[day + 1, new_cs, new_fs] is the value function for the next state discounted to the present.

Bellman Equation:

$$V(\text{day}, cs, fs) = \max_{\text{decisions}} \{\text{Immediate Revenue} + \text{Daily Discount} \times V(\text{day}+1, cs', fs')\}$$

This is implemented by considering all decision outcomes and selecting the maximum value. This is done for each case, and saves the highest expected value in the DP matrix

Terminal Condition:

This is the value function at the final time step (day 365), where no further ticket sales occur and only overbooking costs are realized. At this point, the value function is the negative of the expected overbooking cost (based on the specific penalties for bumping). It is set by iterating over all possible states on the flight day.

Implementation

To account for the increased demand when first-class tickets are sold out, we defined a helper function that adjusts the sale probability for coach tickets.

```
def get_coach_sale_prob(price, first_sold):
    if first_sold >= first_capacity:
        return coach_options[price] + 0.03
    else:
        return coach_options[price]
```

This function is invoked within the DP loop to ensure the coach sale probability reflects the scenario when first-class is fully booked. The terminal cost is calculated on the flight day (day

365) using the binomial distribution to model the number of passengers who actually show up. This function computes the expected cost incurred from overbooking.

```
def terminal_overbooking_cost(coach_sold, first_sold):
    expected_cost = 0.0
    # Loop over possible coach show-ups (0 to coach_sold)
    for k_coach in range(coach_sold + 1):
        p_coach = binom.pmf(k_coach, coach_sold, coach_show_prob)
        # Loop over possible first-class show-ups (0 to first_sold)
        for k_first in range(first_sold + 1):
            p_first = binom.pmf(k_first, first_sold, first_show_prob)
            # Calculate available first-class seats based on actual first-class show-ups
            available_first = first_capacity - k_first
            extra = max(0, k_coach - coach_capacity)
            bump_to_fc = min(extra, available_first)
            bump_off = extra - bump_to_fc
            cost = bump_to_fc * 50 + bump_off * 425
            expected_cost += p_coach * p_first * cost
    return expected_cost
```

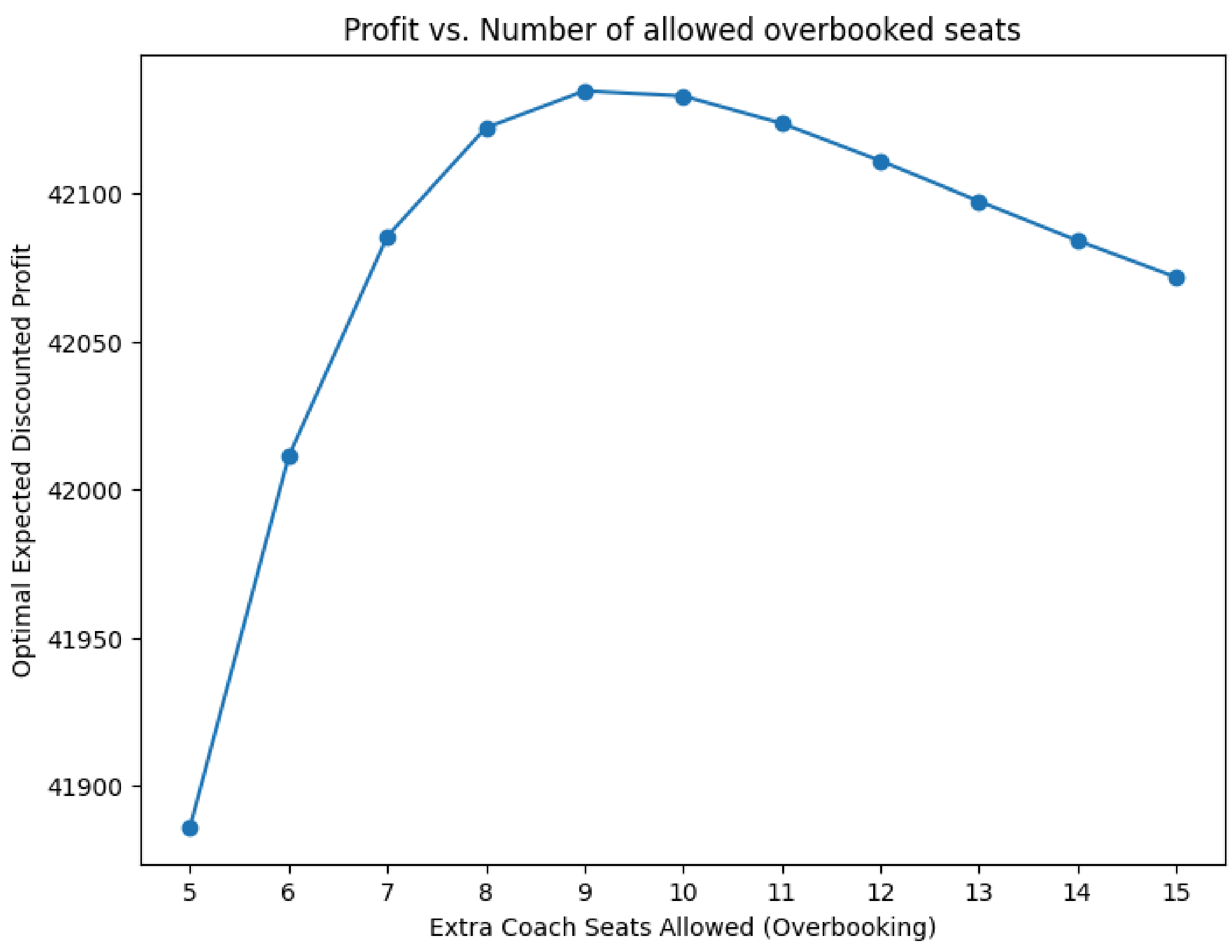
This is used to set the terminal condition that we defined below.

```
for cs in range(allowed_coach_sales + 1):
    for fs in range(first_capacity + 1):
        dp[DAYS, cs, fs] = -terminal_overbooking_cost(cs, fs)
```

On the day of the flight, revenue ceases and only the overbooking cost is incurred. After this setup is when we use a backward induction loop that evaluates each day, state, and pricing decision. For each day from 0 to 364, we update the DP matrix by considering all cases (described previously) and computing the expected value of each outcome.

Results

Our baseline model—without permitting any overbooking—yields an expected profit of $40,654. Introducing a moderate overbooking policy by allowing five extra coach seats increases the expected profit to $41,886, representing an improvement of $1,232. This enhancement demonstrates that carefully calibrated overbooking can effectively capture additional revenue that would otherwise be lost due to no-shows. Moreover, it optimizes seat utilization by balancing the risk of overbooking costs against the revenue benefits. This result underscores the potential of overbooking as a strategic tool to boost profitability, making it an attractive option for our pricing strategy. To explore this further, we evaluated the impact of varying the number of allowed overbooked coach seats from 5 to 15.



From these results, we see that the expected profit steadily increases as overbooking seats move from 5 to 9, peaking at $42,134.62 with 9 overbooked seats. Beyond this point, profit begins to decline slightly. This indicates that while some degree of overbooking enhances revenue, too many overbooked seats eventually erode gains due to rising overbooking penalties. Thus, a moderate overbooking level—around 9 seats in this example—appears to offer the best balance between maximizing revenue and minimize bumping costs.

Methodology #2

In the second approach, we introduce a more flexible overbooking strategy by allowing the airline to dynamically decide not only the ticket prices, but also whether to sell coach tickets on a given day at all. Unlike the first methodology, where coach pricing decisions were limited to high/low pricing, this strategy adds a third option which is to sell no coach tickets at all. This added flexibility enables our airline to better manage overbooking risk by strategically pausing sales when the number of coach tickets sold is nearing the overbooking limit or when there are still many days remaining before departure. By adding this third option into our dynamic programming framework, we give the model the ability to make more nuanced decisions that help us balance revenue with cost control.

Process

To implement this flexible policy, we expanded the dynamic programming logic used in Methodology 1 by introducing an additional action in the coach ticket decision space. Each day, for coach tickets, the model now considers three possible choices: sell at the low price ($300), sell at the high price ($350), or abstain from selling altogether. The "no-sale" option is handled by assigning a zero price and a zero probability of sale, ensuring no state transitions for coach sales occur in that case. First-class ticket pricing remains unchanged, with the model selecting between $425 and $500 as before.

This expanded action space required a nested loop over all combinations of coach and first-class pricing options, including the new "no-sale" scenario for coach. For each combination, we computed the expected revenue and updated the future value in the DP matrix based on probabilistic ticket sales. All four cases from Methodology 1—depending on whether coach and/or first-class are sold out—were retained, with adjustments to ensure that the new no-sale coach option was accounted for in state transitions and value updates.

We also maintained the logic to adjust coach sale probabilities by 3 percentage points when first-class is fully booked, and we preserved the terminal condition by computing the expected overbooking cost on the day of the flight using binomial probabilities. These changes allow the model to make more deliberate sales pacing decisions, especially in scenarios where aggressive selling early on may lead to excessive overbooking penalties.

Dynamic Programming Definitions for Methodology 2

State Variables:

- **Day:** The current day in the 365-day timeline.
- **Coach Sales (cs):** The number of coach tickets sold so far, which can exceed the coach seating capacity by a defined overbooking limit.
- **First-Class Sales (fs):** The number of first-class tickets sold so far, which is strictly capped by first-class seating capacity.

Choice/Decision Variables:

- The pricing decisions for **coach tickets** can now be set at $300, $350, or **"no sale"** (i.e., choose not to sell a coach ticket on that day).
- The pricing decisions for **first-class tickets** remain at $425 or $500.
- These decisions affect the probability of a sale for each ticket class. All combinations of coach and first-class options are iterated over within the DP loop.

Dynamics:

- Depending on the selected coach and first-class pricing options, the probability of a ticket being sold is determined.
- If **"no sale"** is selected for coach, no sale occurs, and the number of coach tickets sold remains the same.
- For all other combinations, the state is updated probabilistically based on whether a ticket is sold or not.
- The immediate reward is the ticket revenue earned that day, and future states are updated accordingly.

Value Function:

$$V(day, cs, fs)$$

This represents the maximum expected discounted profit from a given state onward. It is updated for each possible decision outcome.
The function includes:

- **Immediate revenue** from any tickets sold that day.
- **Discounted future value** from transitioning to the next state: $dp[day + 1, new\_cs, new\_fs]$

Bellman Equation:

$$V(day, cs, fs) = \max_{\text{coach\_option, first\_class\_option}} \{\text{Immediate Revenue} + \text{Daily Discount} \times V(day + 1, cs', fs')\}$$

Where:

- The **maximization** now includes the additional **"no-sale"** coach pricing option.
- The updated cs' and fs′ represent the state after accounting for probabilistic ticket sales based on the chosen pricing strategy

Terminal Condition:

This is the value function at the final time step (**day 365**), when no further ticket sales can occur. At this point:

- Revenue generation stops.
- The value function is set to the **negative expected overbooking cost**, computed using binomial distributions based on the number of tickets sold and passenger show-up probabilities.
- These costs account for bumping passengers either into first-class or off the flight entirely.

Implementation

Due to the third option of not selling a coach ticket on a given day, we had to expand the action space and update our decision logic from methodology #1. Below are the key code changes we made:

We first redefined the coach ticket options:

```
coach_options = [
    ('none', 0, 0.0),        # No sale option
    ('low', 300, 0.65),
    ('high', 350, 0.30)
]
```

Each day, the model now evaluates all combinations of these three coach options and the two first-class options. The no-sale coach option simply results in no revenue and no change in the number of coach tickets sold.

We then loop over all states and decisions using backward induction:

```
# Backward induction over T days
for t in range(1, T + 1):
    for cs in range(coach_max_new + 1):
        for fs in range(fc_capacity + 1):
```

For each combination, we calculate expected revenue and update the value function:

```
immediate_rev = p_c * cp + p_f * fp
```

```
total_value = immediate_rev + beta * exp_future
```

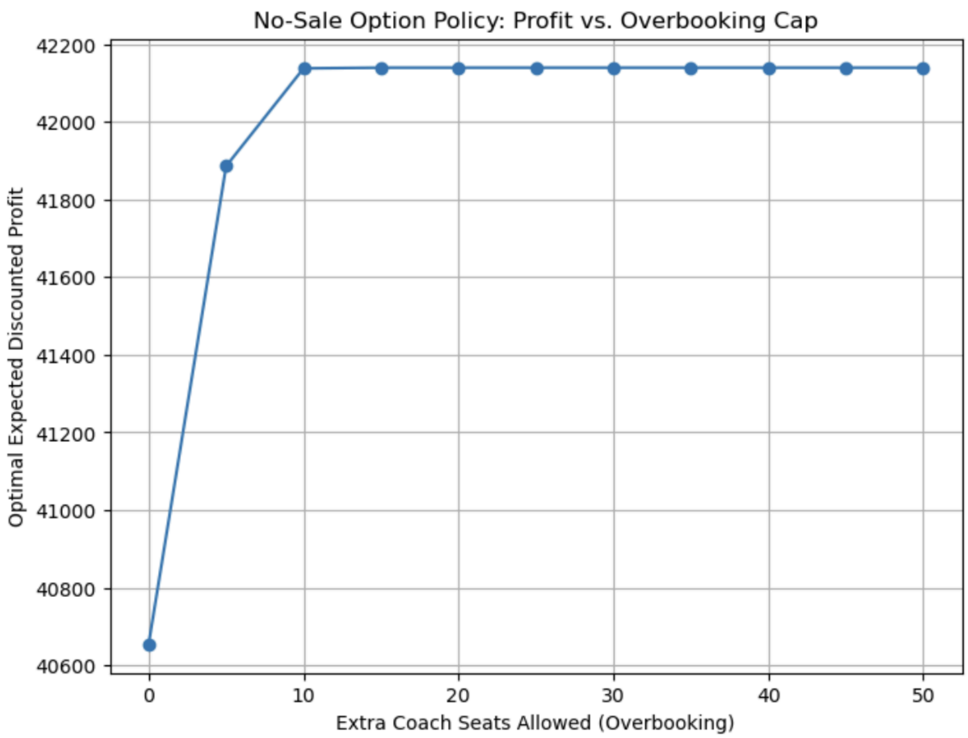
The best value across all actions is stored in the DP matrix for that state. The terminal condition remains the same as in Methodology 1, using binomial distributions to compute expected overbooking costs on the day of the flight.

Key Findings

Using the flexible no-sale policy, we evaluated expected discounted profits across a range of overbooking levels, from 0 to 50 additional coach seats. The model finds an optimal profit when at least 10 extra coach seats are allowed, with stagnating returns beyond that point. The highest expected discounted profit achieved under this policy is approximately $42,140, which remains stable between overbooking levels of 10 to 50.

(Note that the choice of zero overbooked seats combined with no-sale option represents the vanilla, no-sale option asked for in problem 3: which was $40,654 for expected discounted profit)

This performance surpasses the peak profit from Methodology 1 ($42,134.62 at 9 overbooked seats), indicating that the added flexibility of choosing not to sell coach tickets on certain days can marginally improve profitability by limiting excessive overbooking when demand conditions are uncertain.



Methodology Addition: Seasonality Impact

To align results of the analysis closer to the real world scenario, we revise the probability estimates to introduce seasonality component by the multiple of a factor, defined as follows:

$$f = 0.75 + (t / 730)$$

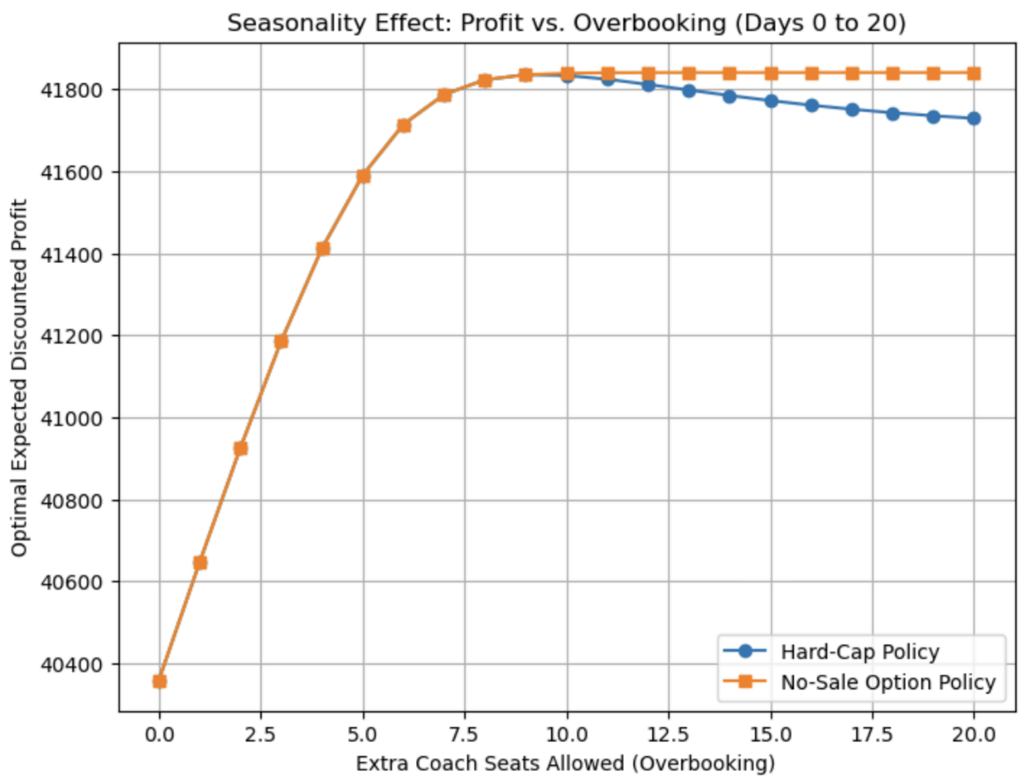
We worked both methodologies with the seasonality factor, and explored different overbooking scenarios. We found it interesting to observe how both methodologies compare when seasonality is incorporated.

Few things changed when seasonality was introduced:

1. Optimal expected profit for the Methodology 1 (referred to as the hard cap policy in the image) reduced marginally after $41,835. Still achieved maximum discounted profit at allowed overbooking of 9 seats.

2. Optimal expected profit for Methodology 2 (referred to as no-sale option in the image) also reduces to $41,841, in comparison to $42,134.62 when seasonality was not accounted for. Moreover, the optimal overbooking policy increased to 16 seats

overbooked. However, even more interesting for the no-sale policy that also had overbooking available and seasonality, was that it plateaued at a maximum discounted profit of $41,841 compared to the decline after the optimal overbooking policy of 9 seats for the hard-cap policy. This demonstrates how the implementation of a no-sale policy can prevent further losses if the estimated allowance for overbooking is incorrect in the actual implementation of an airline pricing model.



Why do estimated profits decline when seasonality is introduced?

Although the average of the seasonality factor over 365 days is around 1, most sales tend to occur earlier in a well-optimized policy—when those probabilities are lower. Since early sales are heavily weighted (because they're less discounted), reducing the chance of an early sale significantly lowers overall expected profits. Meanwhile, even though later days have higher probabilities, the revenue is discounted more heavily. Also, in our problem scope, since we are only allowed to sell one ticket for each class on a given day, we are not able to align with seasonality trends. If there is high demand during a particular time, we will not be able to effectively sell more tickets to capture that demand since we are limited to 1.

Implementation

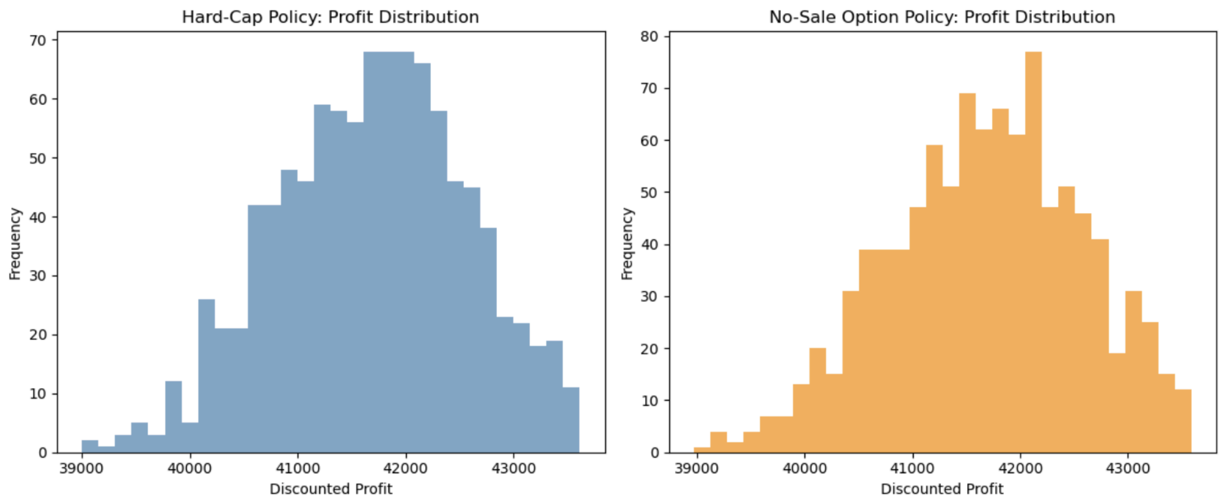
```
# Backward induction over days.
for t in range(1, T + 1):
    # Compute the current day number (1 is earliest, T is last day)
    day = T - t + 1
    season_factor = 0.75 + day / 730.0
```

Simulation Results & Recommendations

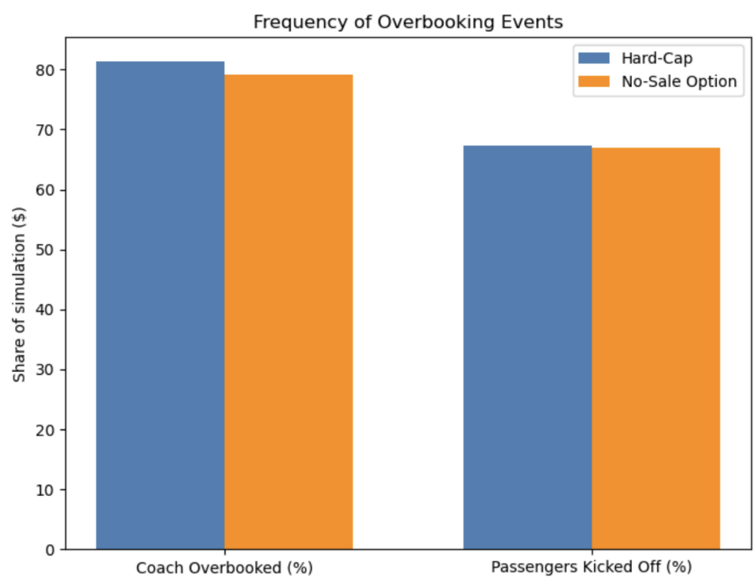
In our study, we conducted 1,000 simulation runs to assess the impact of different overbooking policies on the airline's expected discounted profits. First, we examined scenarios without accounting for seasonality. Under this non-seasonal framework, **Alternative 1** implements a hard cap policy that limits coach ticket sales to a maximum of 109 seats (i.e., 100 physical seats plus an overbooking allowance of 9 seats). In contrast, **Alternative 2** (the policy that provided us the highest expected discounted profit at approximately $42,140) incorporates a no-sale option—providing the flexibility to deliberately withhold sales on a given day—while still imposing the same hard cap of 109 coach tickets.

Next, we incorporated seasonality into the model. With seasonality, **Alternative 3** again applies a hard cap on coach ticket sales at 109 seats. However, in **Alternative 4**, when the no-sale option is available, we relax the hard cap, allowing up to 116 coach tickets (i.e., an overbooking allowance of 16 seats) to reflect the dynamic impact of increased demand as the flight departure approaches. Optimal overbooking values have been identified from the previous sections.

Below, we present visualizations based on the simulation results from the non-seasonality scenarios, which help illustrate the differences in performance between the hard cap and no-sale option methodologies under a constant demand environment.

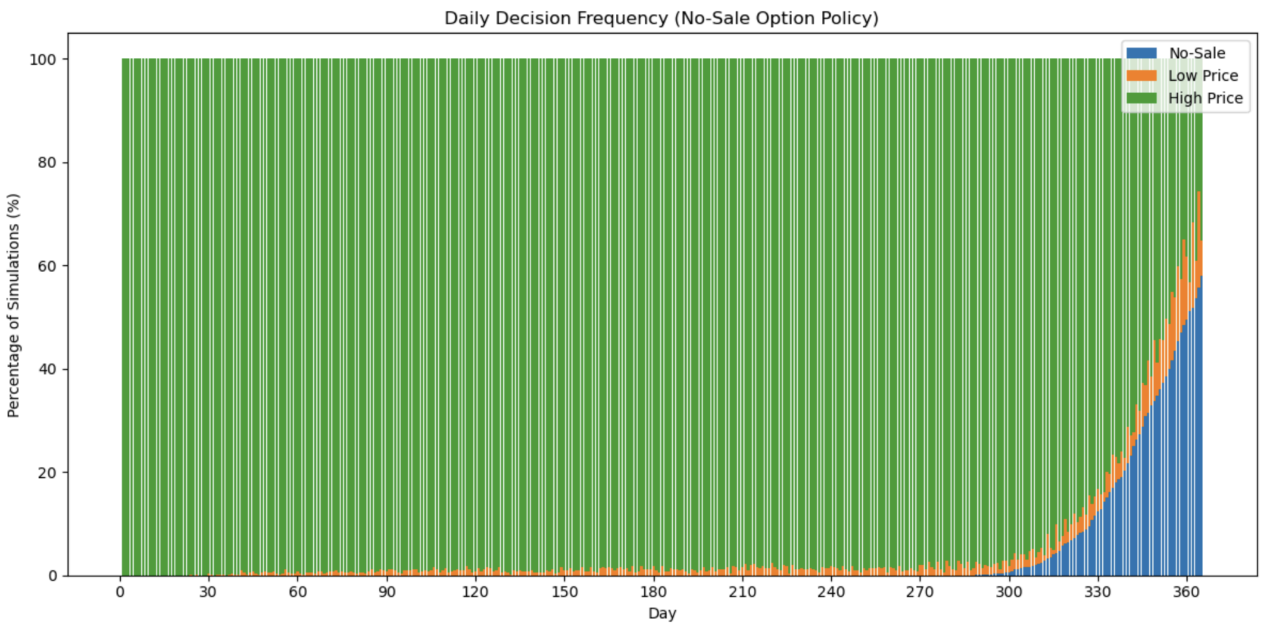


Both methodologies have roughly similar distributions, overall the average discounted profits for Alternative 2 with no-sale are lower than Alternative 1 at roughly $41,681 compared to $41,686. This difference, where Alternative 2 has lower average discounted profits compared to Alternative 1, is likely due to how the profit volatility (standard deviation) for Alternative 2 is higher at roughly $894 compared to a profit volatility of roughly $879.



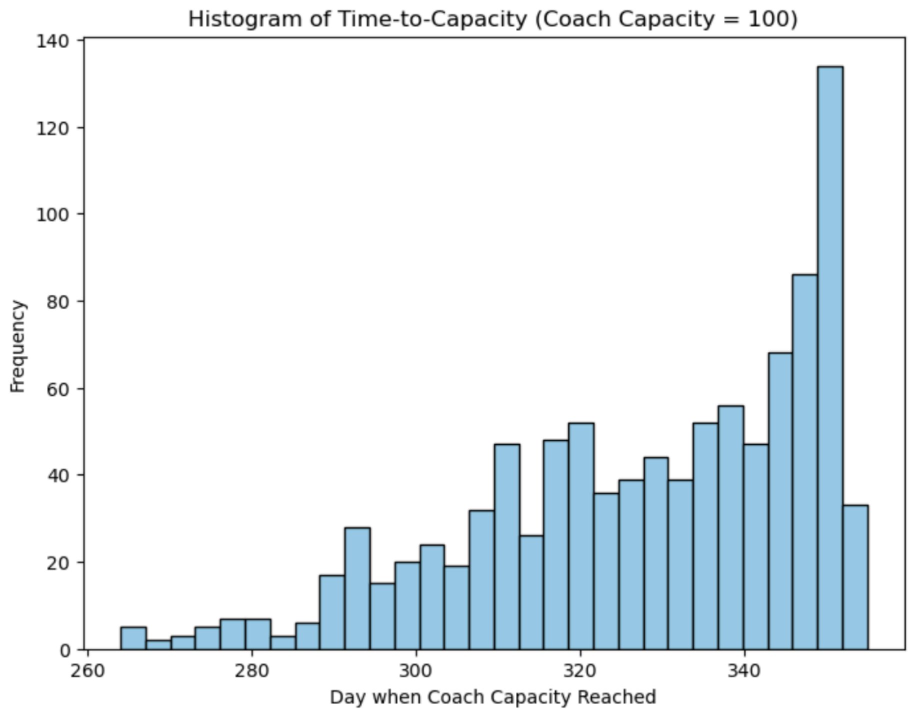
In around 80% of our simulation runs, coach seats were overbooked under both the hard-cap and no-sale option policies (81.3% and 79.2% specifically), and in approximately 67% of those runs (hard-cap 67.3% and no-sale 67% specifically), at least one passenger was denied boarding. The average overbooking costs were also found to be similar with the hard-cap policy having an average cost of $962.67 and the no-sale policy having an average of $954.48. These findings suggest that, while overbooking may be profitable under the assumptions in our model, it also introduces significant potential for customer dissatisfaction and reputational risk.

The following visualization shows how the optimal daily decision (high price / low price / no sell) for coach seats changes across time. Percentages indicate the share of simulations that made that decision. Goal of the visualization to help understand how, on average, the optimal decision changes across time.

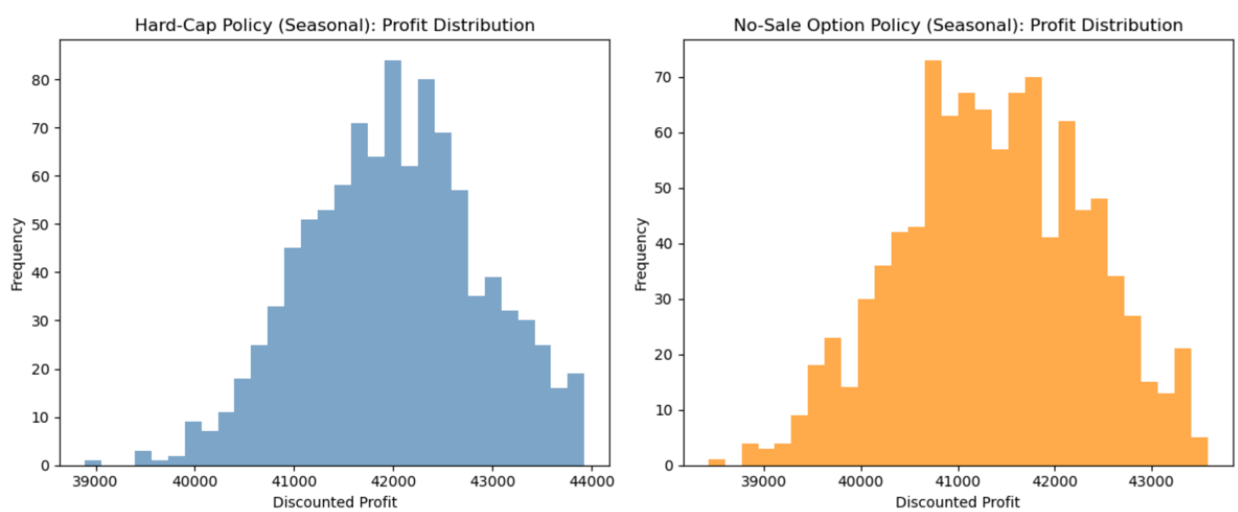


From the chart, we see that for much of the selling horizon, most simulations opt to sell coach seats at the higher price, while toward the very end (roughly the last month), a growing fraction of simulations switch to “no sale.” The presence of the low‑price option is comparatively small throughout, indicating that the model generally finds a high‑price strategy more profitable, unless very late in the sales window when the airline either has already sold enough seats or deems it more optimal to stop selling. Overall, this pattern reflects the interplay of daily discounting, the remaining capacity, and the probability of future demand, highlighting how the airline’s willingness to sell at a premium (or not sell at all) shifts as departure approaches.

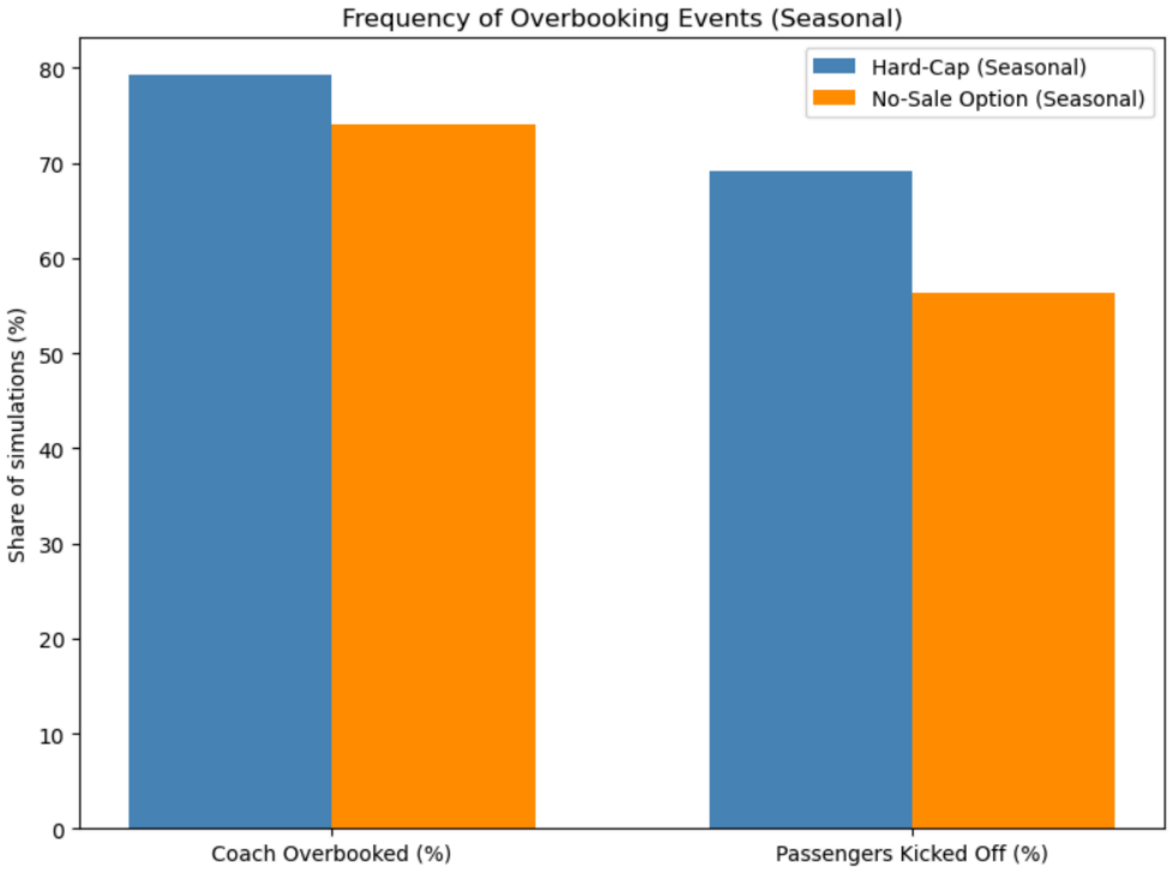
Finally, the visualization below suggests that coaches are sold to capacity much more often towards the close to the last few weeks of the year for the no-sale policy. This is a strong trend even without taking the impact of seasonality into account.



The visualizations based on the simulation results from the seasonality scenarios help illustrate the differences in performance between the hard cap and no-sale option methodologies under a demand environment with seasonality factored in.



The methodologies under seasonal conditions have distributions that differ more, but that still have the same trend of the overall average discounted profits for Alternative 4 with no-sale being lower than Alternative 3 at roughly $41,370 compared to $42,001. This difference, where Alternative 4 has lower average discounted profits compared to Alternative 3, is likely due to how the profit volatility (standard deviation) for Alternative 4 is higher at roughly $961 compared to a profit volatility of roughly $883 for Alternative 3.



Coach seats were overbooked under both the hard‑cap and no‑sale seasonal option policies at 79.2% and 74.1% of the simulations respectively, and in approximately 69.2% of those runs for seasonal hard-cap and 56.4% of those runs for seasonal no-sale option, at least one passenger was denied boarding. The average overbooking costs were also found to differ with the hard-cap policy having an average cost of $958.80 and the no-sale policy having an average of $790.08. These findings continue to suggest that, while overbooking may be profitable under the assumptions in our model, it also introduces significant potential for customer dissatisfaction and reputational risk. However, we can see with the seasonal, no-sale option policy that the occurrence of overbooking and having at least one passenger kicked off was far less compared to the other policies.

The table below compares the average expected profits, and their volatility for the 4 alternatives. Note that we do not consider a vanilla no-sale option (as directed in part 3 of the problem statement) due to its underwhelming results found earlier. We combine the no sale option with overbooking.

Results based on 1000 simulations								
	Daily Choices	Overbooking Allowed	Seasonal?	Expected Discounted Profit	Profit volatility	Avg. Overbooking Cost	% of Coach Overbook	% where at least 1 cust. kicked off
Alt. 1	{High Price, Low Price}	9	Not Included	41,866	879	963	81.3%	67.3%
Alt. 2	{High Price, Low Price, No Sale}	9	Not Included	41,681	894	954	79.2%	67.0%
Alt. 3	{High Price, Low Price}	9	Included	42,001	883	959	79.2%	69.2%
Alt.4	{High Price, Low Price, No Sale}	16	Included	41,370	961	790	74.1%	56.4%

Based on the table, we can see from our forward simulations that Alternative 3 with seasonality but without the no-sale option had the highest generated profits with one of the lowest average overbooking costs.

While overbooking does introduce some reputational risk, our analysis shows that it is certainly a profitable strategy when managed carefully. Simulations and dynamic programming results consistently show increased expected discounted profit when modest overbooking is allowed, with an optimal level around 9–16 coach seats depending on whether seasonality is included. To implement overbooking effectively, airlines should:

1. **Use demand-based dynamic pricing**, adjusting ticket prices based on inventory and time remaining.

2. **Incorporate a no-sale option** in the last weeks before departure to reduce overbooking risk.

3. **Balance overbooking limits with show-up probabilities** and apply tiered penalties for bumping, including soft landing options like first-class upgrades.

A data-driven overbooking policy, combined with flexible sales pacing and probabilistic forecasting, can maximize profits while keeping customer dissatisfaction in check.

Concluding Remarks

This analysis highlights the strategic value of overbooking as a revenue optimization tool in the airline industry. By leveraging dynamic programming and incorporating flexible sales strategies, we demonstrate that overbooking can lead to measurable profit gains. Across both methodologies, profits improved with a moderate level of overbooking, particularly when combined with dynamic pricing and sales pacing decisions.

The key to successful implementation lies in balance. Airlines must weigh the financial upside of higher occupancy against the cost—both tangible and reputational—of denied boarding. Incorporating predictive models, adjusting overbooking thresholds based on real-time demand trends, and providing compensatory mechanisms such as upgrades or rebooking can help mitigate risk. With a data-driven approach, overbooking evolves from a blunt revenue tactic to a refined, high-impact optimization strategy.