# Project 3

**Aryan Soltani Mohammadi  810198558**

$$9u_{xx} = u_t$$ راه دستی ←

$$u_x(0,t) = e^{-3t} \qquad u(x,0) = \sin^2(x)$$

$$u_x(3\pi,t) = e^{-3t}$$

$$w(x,t) = xe^{-3t}$$

$$u(x,t) = v(x,t) + w(x,t)$$

$$v_x(0,t) = 0 \qquad v(x,0) = \sin^2(x) - x$$

$$v_x(3\pi,t) = 0$$

$$w_t = -3xe^{-3t} \longrightarrow v_t - 9v_{xx} = +3xe^{-3t}$$

$$w_{xx} = 0$$

$$\xrightarrow{\text{جواب عمومی}} v(x,t) = \sum_{n=0}^{\infty} T_n(t)\cos\Big(\underbrace{\frac{n\pi}{L}}_{L=3\pi}x\Big)$$

$$\sum_{n=0} \dot{T}_n \cos\left(\frac{n\pi}{L}x\right) + \sum_{n=0}^{\infty} T_n (n\pi)^2 \cos\left(\frac{n}{3}x\right) = 3xe^{-3t}$$

$$\dot{T}_0 + \sum_{n=1}^{\infty}\left(\dot{T}_n + (n\pi)^2 T_n\right)\cos\frac{n\pi}{3} = 3xe^{-3t}$$

$$\longrightarrow T_0 = at + b$$

$$\dot{T}_0 = \frac{1}{6\pi}\int_0^{3\pi} 3x e^{-3t}\,dx =$$

$$= \frac{1}{6\pi}\left(\frac{3x^2}{2}e^{-3t}\right)\Big|_0^{3\pi} = \frac{9\pi}{4}e^{-3t}$$

$$\longrightarrow T_0 = -\frac{9\pi}{6}e^{-3t} + a = -\frac{3\pi}{2}e^{-3t} + c_0$$

$$\dot{T}_n + n^2 T_n = \frac{2}{3\pi}\int_0^{3\pi} 3x e^{-3t}\cos\left(\frac{nx}{3}\right)$$

$$\longrightarrow \dot{T}_n + n^2 T_n = \frac{2}{\pi}e^{-3t} \times \frac{9\left[(-1)^n - 1\right]}{n^2} \quad n \geq 1$$

$$T_n = c_1 e^{-n^2 t} + \frac{18\, e^{-3t}\left[(-1)^n - 1\right]}{\pi n^2 (n^2 - 3)}$$

$$v(x,0) = \frac{-9}{6}\pi + c_0 + \sum_{n=1}^{\infty}\left(\left[c_1 + \frac{18\left[(-1)^n - 1\right]}{\pi n^2 (n^2 - 3)}\right]\right.$$

فرض می‌کنیم که $\sin^2$ وجود ندارد

$$\sin^2 x - x = \underbrace{\frac{-9}{12}\pi + c_0}_{A_0} + \sum_{n=1}^{\infty}\left(\frac{\left[c_1 + 18\left[(-1)^n - 1\right]\right]}{\pi n^2 (n^2 - 3)}\right)$$

$$\frac{1}{6\pi}\int_0^{3\pi}\left(\quad - x\right)dx = \frac{1}{4}(1 - 3\pi)$$

$A_0 = \frac{1}{2}(1-3\pi)$

$$c_1 = \frac{18[(-1)^n - 1]}{\pi n^2(n^2-3)} = \frac{2}{3\pi}\int_0^{3\pi} (\sin^2 x - x)\cos\frac{n x}{3}\,dx$$

$$= \frac{2}{3\pi}\left(\frac{-9[(-1)^n - 1]}{n^2}\right)$$

$$\longrightarrow c_1 = \frac{[(-1)^n - 1][\;-6\;]}{\pi(n^2-3)}$$

$$\rightarrow c_1 = \frac{-6\,[1-(-1)^n]}{\pi(n^2-3)} \quad n \neq 6,\ n \neq 0$$

$$\sin^2 x = \frac{1-\cos 2x}{2}$$

$$c_6 = -\frac{1}{\pi 2}$$

$$\xrightarrow{c_0} u(x,t) = \sum_{n=1}^{\infty} \frac{-6}{\pi(n^2-3)}\left[(-1)^n - 1\right]e^{-n^2 t} + \frac{18e^{-3t}\left[(-1)^n - 1\right]}{\pi n^2(n^2-3)}\Bigg) \cos\frac{n}{3}x + \underbrace{\left(-\frac{1}{2}\right)\cos 2x + \frac{1}{2}}_{\sin^2 \text{ بجای}} x e^{-3t}$$

$$-\frac{3}{2}\pi e^{-3t}$$

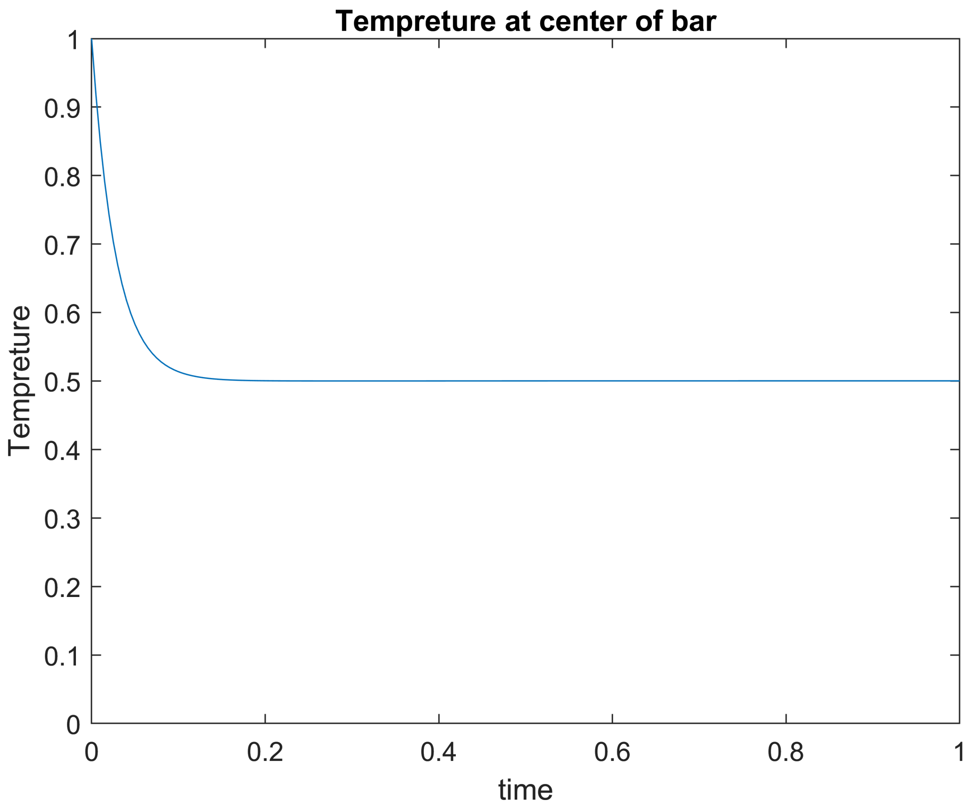
در ابتدا معادله را تفکیک کردیم سپس با روش جواب جزئی معادلات تبدیل به شرایط ODE شدند که با حل آنها دو عدد ثابت مجهول بدست آمد که با قرار دادن شرایط اولیه آن دو را بدست آوردیم همچنین از نظر فیزیکی پاسخ به سمت پایدار رود که (پوسته ها) است و به سمت آن رفته که (پوسته) ها است که سمت ما پایدار رفته به شرایط مرزی وابسته است.

**Question1 Code with using pdepe function**

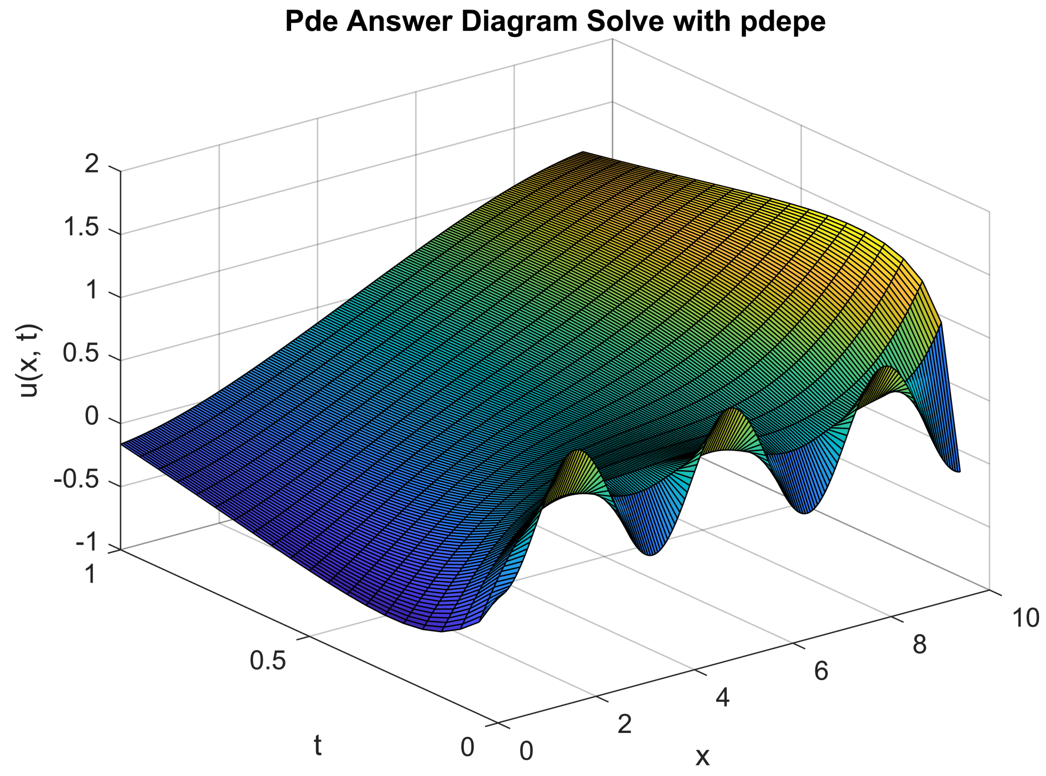
assume that bc conditions are endpoints.

functions are at the end of the report.

```
end_time = 1;
time_step = 0.05;
time_step2 = 0.005;
end_x = 3 * pi;
x = 0:time_step:end_x;
t = 0:time_step:end_time;
x2 = 0:time_step2:end_x;
t2 = 0:time_step2:end_time;
m = 0;
sol = pdepe(m,@pdefun,@icfun,@bcfun,x,t);
sol2 = pdepe(m, @pdefun, @icfun, @bcfun, x2, t2);
u = sol(:,:,1);
u2 = sol2(:, :, 1);
len = length(x2);
if(mod(len, 2)  == 1)
    len = len + 1;
end
pos = len / 2;
plot(t2, u2(:, pos));
ylim([0, 1]);
xlabel('time');
ylabel('Tempreture');
title('Tempreture at center of bar');
```



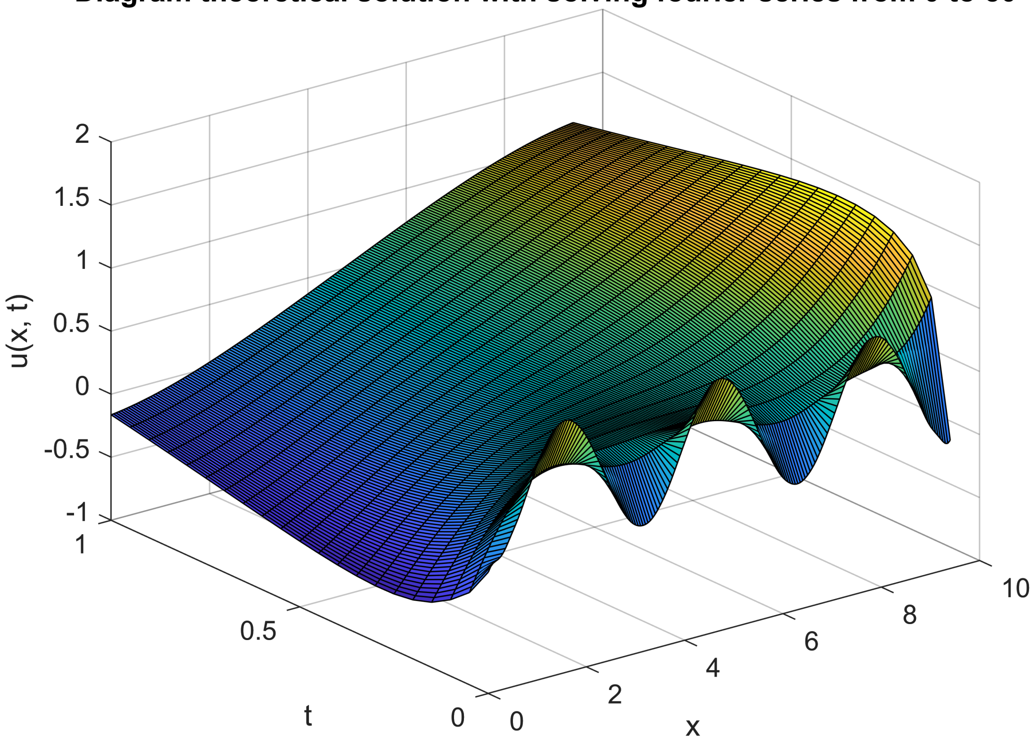

```
surf(x, t, u);
xlabel('x');
ylabel('t');
title('Pde Answer Diagram Solve with pdepe');
zlabel('u(x, t)');
```




**Question 1 code for part theretical diagram by calculating fourier series**

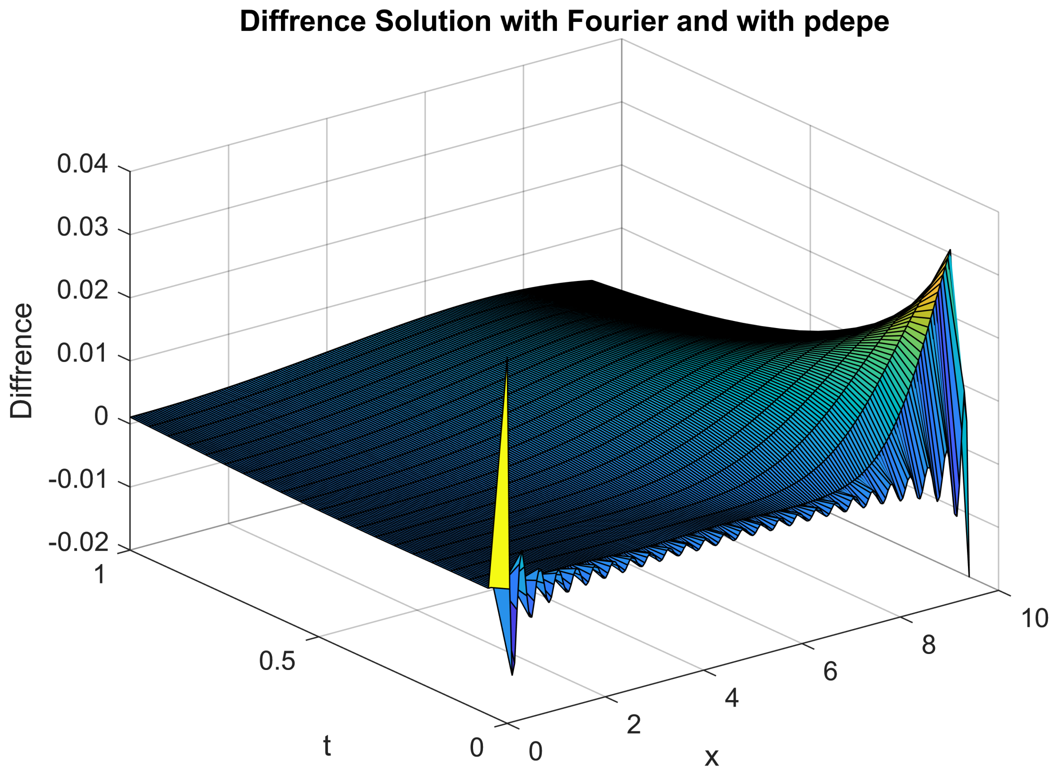
```
pow1 = -1;
v = zeros(length(t), length(x));
for i = 1:length(x)
    for j = 1:length(t)
       pow1 = -1;
       for n = 1:50
         c1 = (6 * (1 - pow1)) /  (pi * (n * n - 3));
         if(n == 6)
             c1 = c1 - 1/2;
         end
         v(j, i) = v(j, i) +  ((c1 * exp(-1 * n * n * t(j))) + ((18 * exp(-1 * 3 * t(j)) * (pow
         pow1 = pow1 * -1;
       end
       v(j, i) = v (j, i)+ x(i) * exp(-3 * t(j)) + 1/2 - 3/2 * pi * exp(-3 * t(j));
    end
end
surf(x, t, v);
xlabel('x');
ylabel('t');
title('Diagram theoretical solution with solving fourier series from 0 to 50')
zlabel('u(x, t)');
```

## Diagram theoretical solution with solving fourier series from 0 to 50




**Question 1 code for diffrence diagrams**

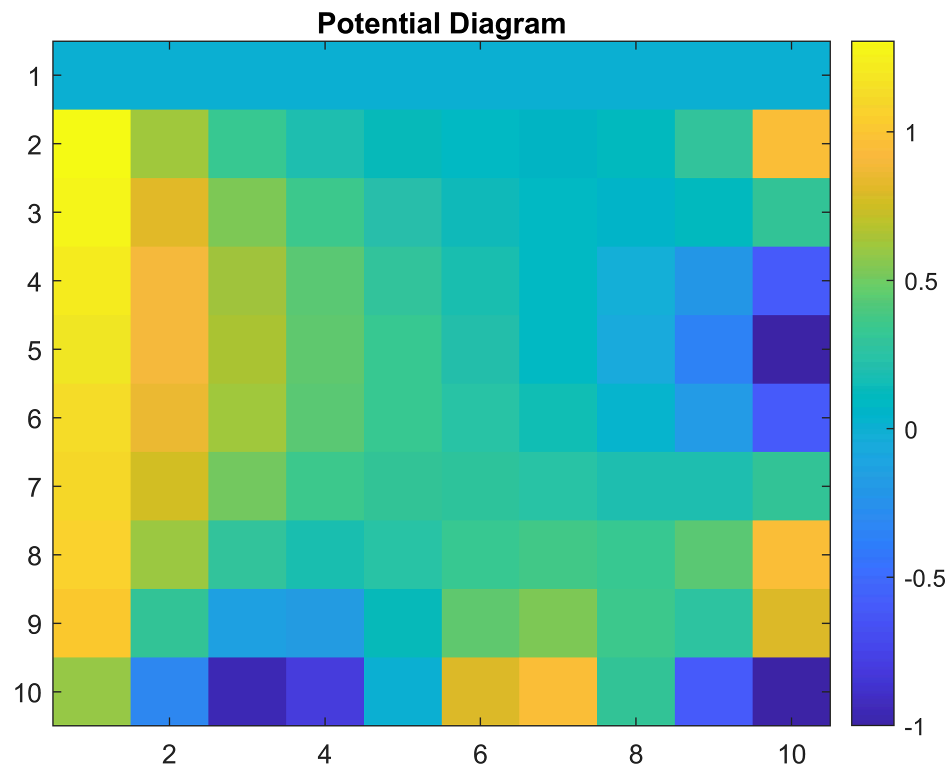
```
surf(x, t, u - v);
xlabel('x');
ylabel('t');
zlabel('Diffrence');
title("Diffrence Solution with Fourier and with pdepe");
```




### Qusetion2 Code

```
mat_div = [];
mat_u = [];
for i = 1: 10;
    for j = 1: 10;
       new_row = zeros(1, 10 * 10);
       if ((i == 1) || (i == 10) || (j == 1) || (j == 10))
            new_row((i - 1) * 10 + (j - 1) + 1) = 1;
       else
            new_row((i - 1) * 10 + (j - 1) + 1) =-4;
            new_row((i - 2) * 10 + (j - 1) + 1) = 1;
            new_row((i - 1) * 10 + (j - 2) + 1) = 1;
            new_row((i) * 10 + (j - 1) + 1) = 1;
            new_row((i - 1) * 10 + (j) + 1) = 1;
       end
       if(i == 1)
            ans = 0;;
       elseif(i == 10)
            ans = cos((3 * pi * j)/ 10);
       elseif(j == 1)
            ans = exp((1 - i/10)/3);
       elseif(j == 10)
            ans = sin(3 * pi * i / 10);
       else
            ans = 0;
       end
```

```
        mat_u = [mat_u; ans];
        mat_div = [mat_div; new_row];
    end
end
[row, col] = size(mat_div);
fi = inv(mat_div) * mat_u;
fi2 = [];
for i = 1:10
    index = (i - 1) * 10;
    row_fi = [];
    for j = 1:10
        row_fi = [row_fi, fi(index + j)];
    end
    fi2 = [fi2; row_fi];
end
imagesc(fi2);
title('Potential Diagram')
colorbar;
```




### Functions Question1

```
function [c, f, s] = pdefun(x, t, u, dudx)
    c = 1 / 9;
    f = dudx;
    s = 0;
end
```

```
function [pl, ql, pr, qr] = bcfun(xl, ul, xr, ur, t)
    ql = 1;
    pl = -1 * exp(-3 * t);
    qr = 1;
    pr = -1 * exp(-3 * t);
end

function u = icfun(x)
    u = sin(x) * sin(x);
end
```