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210205

CGS 698C

Assignment - 1

Part-1

1. (a) Sample space $\Omega = \{HH, HT, TH, TT\}$

(b) Event space $\Sigma = \{ \phi, \{HH\}, \{HT\}, \{TH\}, \{TT\},$
$\{HH, TH\}, \{HH, HT\}, \{HH, TT\}, \{HT, TH\}, \{HT, TT\}$
$\{TH, TT\}, \{HH, HT, TH\}, \{HT, TH, TT\}, \{HH, HT, TT\}$
$\{HH, TH, TT\}, \{HH, HT, TH, TT\}. \}$

(c) (i) $P(\{HH\}) = P(\{HT\}) = P(\{TH\}) = P(\{TT\}) = P$
also $P(\{HH\}) + P(\{HT\}) + P(\{TH\}) + P(\{TT\}) = 1$

$$P+P+P+P=1 \Rightarrow \boxed{P = \frac{1}{4}}$$

(ii) Probability of at least 1 Head
$P(\{HH, TH, HT\}) = 3/4$

(iii) Probability of exact 1 head =
$P(\{HT, TH\}) = 1/2$

Part-2

2.1 p = probability of recognizing correct word = 0.9.
K = number of correctly recognized words
n = total number of words.

$$f(K, n, p) = \frac{n!}{K!\,(n-K)!} p^K (1-p)^{(n-K)}.$$

$p = 0.9 \qquad n = 50 \qquad K = 45$

$$f(45, 50, 0.9) = \frac{50!}{45!\,5!} (0.9)^{45} (0.1)^{5}$$

$$f(45, 50, 0.9) = 0.184$$
$$= 0.18$$

(2.2) $f(k, \lambda) = \dfrac{\lambda^k e^{-\lambda}}{k!}$ $\lambda = 10$

k road accidents in a day.

(a) Probability of zero accidents.

$$f(0,10) = \frac{(10)^0 e^{-10}}{0!} = e^{-10}$$
$$= 4.5 \times 10^{-5}$$

(b) Probability of more than 7 & less than 10.

$$f(8,10) + f(9,10) = \frac{(10)^8 e^{-10}}{8!} + \frac{(10)^9 e^{-10}}{9!}$$
$$= 0.237$$
$$= 0.24$$

(c) Graph

Part-3

(3.1) $f(x) = \dfrac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$

X random variable, normally distributed with above probability density function.

PAGE NO.:

(a) probability density of obtaining $x=0$ given that $\mu=1$; $\sigma=1$

$$f(0) = \frac{1}{\sqrt{2\pi}} e^{-\frac{(0-1)^2}{2}} = \frac{e^{-1/2}}{\sqrt{2\pi}}$$

$$= \frac{1}{\sqrt{2\pi e}} = 0.24$$

(b) probability density $x=\cancel{0}$, $\mu=0$, $\sigma=1$.

$$f(1) = \frac{1}{\sqrt{2\pi}} e^{-\frac{(1-0)}{2}} = \frac{1}{\sqrt{2\pi e}} = 0.24$$

(c) $P(x_1 \leq X \leq x_2) = 0.3$

$P(x_1 \leq X \leq x_3) = 0.45$

$$P(x_2 \leq X \leq x_3) = P(x_1 \leq X \leq x_3) - P(x_1 \leq X \leq x_2)$$

$$= 0.45 - 0.3$$

$$= 0.15$$

Part-4

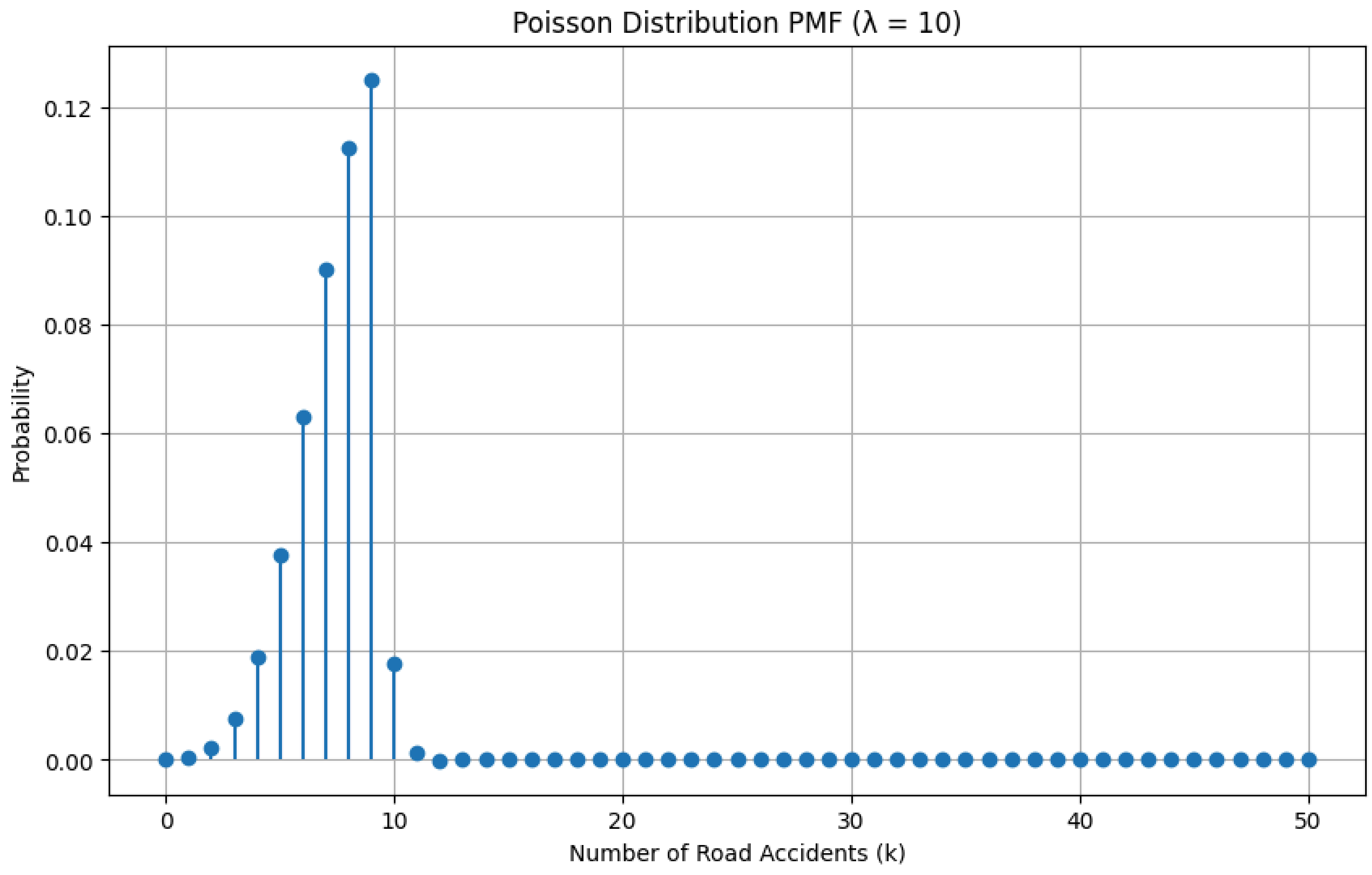
(4.1) Graph.

(c) ~~$\mu = 6.04$~~ $\mu \approx 6$

In [ ]:

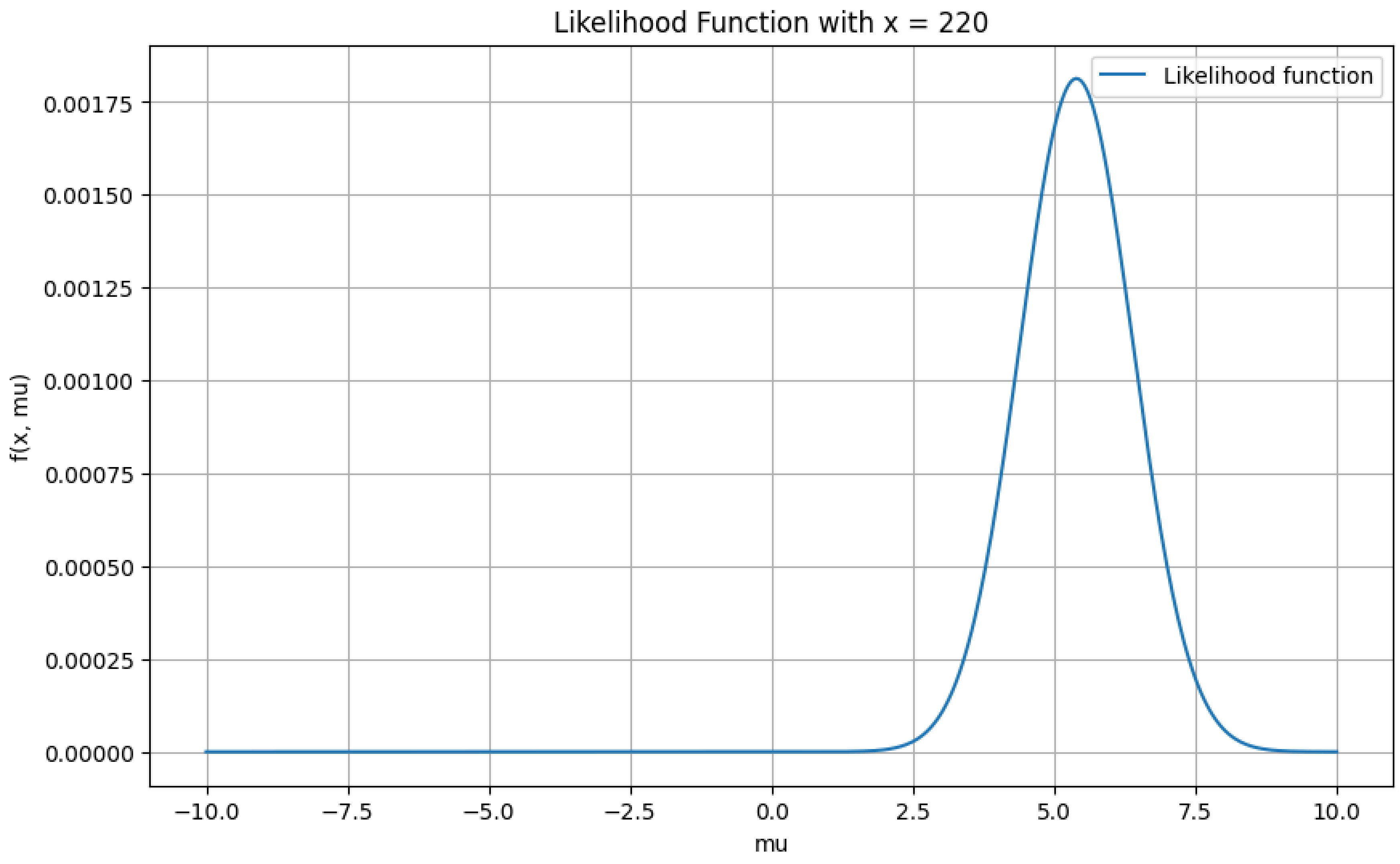
```
import numpy as np
import matplotlib.pyplot as plt
from math import factorial, exp


def poisson_pmf(k, lambd):
    return (lambd**k * exp(-lambd)) / factorial(k)


lambd = 10
k_values = np.arange(0, 51)
probabilities = [poisson_pmf(k, lambd) for k in k_values]


plt.figure(figsize=(10, 6))
plt.stem(k_values, probabilities, basefmt=" ", use_line_collection=True)

plt.xlabel('Number of Road Accidents (k)')
plt.ylabel('Probability')
plt.title('Poisson Distribution PMF (λ = 10)')
plt.grid(True)
plt.show()
```

```
C:\Users\aryan\AppData\Local\Temp\ipykernel_18896\4063919027.py:16: MatplotlibDeprecationWarning: The 'use_line_collection' parameter of stem() was deprecated in Matplotl
ib 3.6 and will be removed two minor releases later. If any parameter follows 'use_line_collection', they should be passed as keyword, not positionally.
  plt.stem(k_values, probabilities, basefmt=" ", use_line_collection=True)
```



In [ ]:

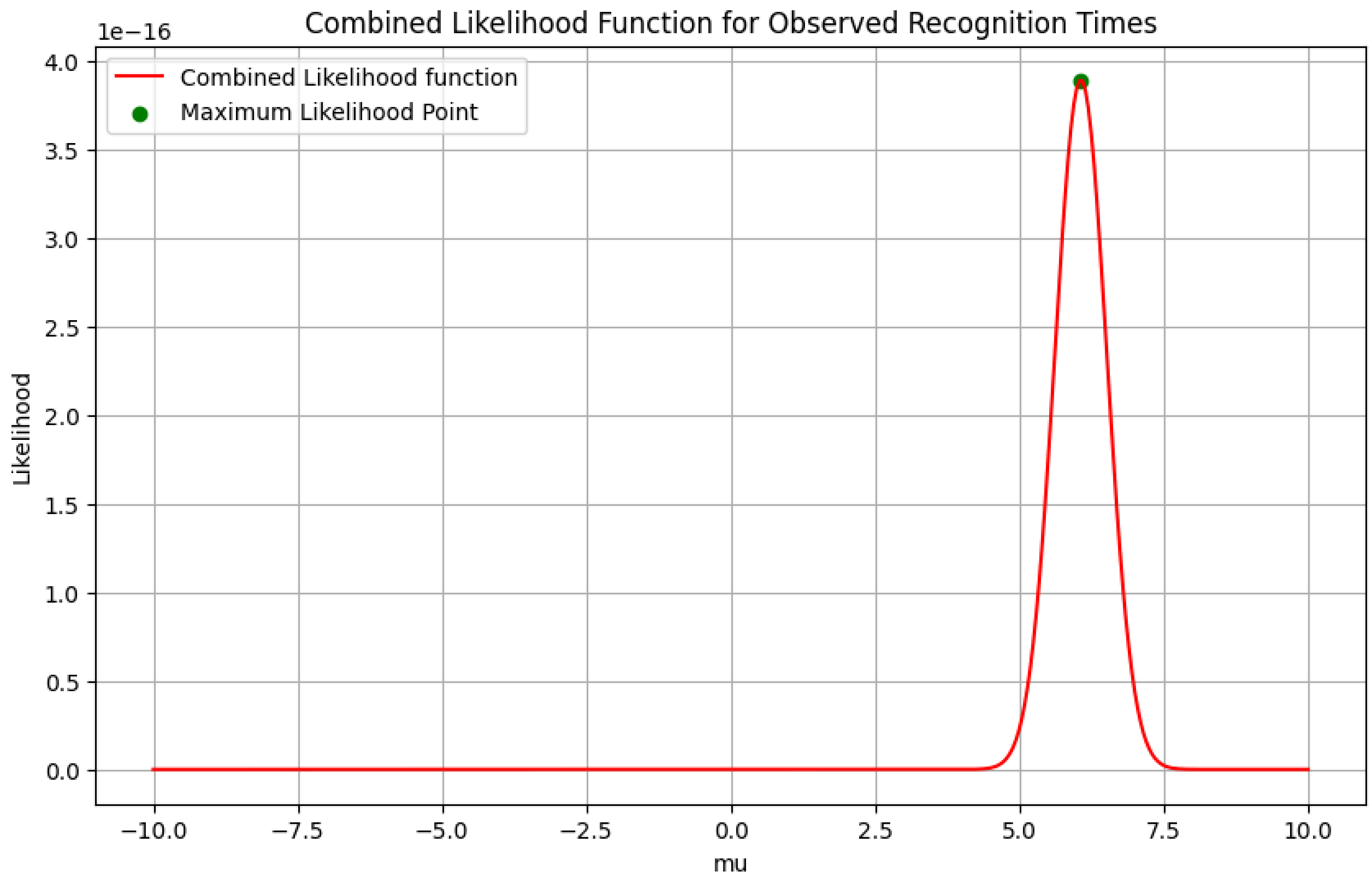
```
import numpy as np
import matplotlib.pyplot as plt


def f(x, mu):
    return (1 / (x * np.sqrt(2 * np.pi))) * np.exp(-((np.log(x) - mu) ** 2) / 2)


x = 220


mu_values = np.linspace(-10, 10, 400)
f_values = f(x, mu_values)


plt.figure(figsize=(10, 6))
plt.plot(mu_values, f_values, label='Likelihood function')
plt.xlabel('mu')
plt.ylabel('f(x, mu)')
plt.title('Likelihood Function with x = 220')
plt.legend()
plt.grid(True)
plt.show()
```



In [ ]:

```
x_values = np.array([303, 443, 220, 560, 880])

def combined_likelihood(mu, x_values):
    likelihood = 1
    for x in x_values:
        likelihood *= f(x, mu)
    return likelihood


combined_f_values = [combined_likelihood(mu, x_values) for mu in mu_values]

plt.figure(figsize=(10, 6))
plt.plot(mu_values, combined_f_values, label='Combined Likelihood function',color = 'red')
plt.xlabel('mu')
plt.ylabel('Likelihood')
plt.title('Combined Likelihood Function for Observed Recognition Times')
plt.legend()
plt.grid(True)
# plt.show()

max_likelihood_mu = mu_values[np.argmax(combined_f_values)]
print(f"The value of μ that maximizes the likelihood is approximately: {max_likelihood_mu}")

plt.scatter(max_likelihood_mu, np.max(combined_f_values), color='green', label='Maximum Likelihood Point')

plt.legend()
plt.show()
```

```
The value of μ that maximizes the likelihood is approximately: 6.0401002506265655
```


Combined Likelihood Function for Observed Recognition Times
1e−16
4.0
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
Likelihood
Combined Likelihood function
Maximum Likelihood Point
−10.0
−7.5
−5.0
−2.5
0.0
2.5
5.0
7.5
10.0
mu