Data structure & algorithm

Data structure

- particular way of storing and organizing data in a computer so that it can be used efficiently

Ex:- arrays, linked lists, stacks, queue, trees, graphs etc.

DS → Linear, Non-Linear

Linear	Non-Linear
⇝ linear order	⇝ non-linear order
⇝ linked lists, stack etc.	⇝ Trees and graphs

Algorithm

Step by step fixed unambiguous instruction to solve a problem

properties ⇝

1) Each step must be a basic operator like we can't write directly in algo that sort the given numbers
2) Should terminate in finite amount of time
3) Should produce atleast 1 output
4) Should take 0 or more input
5) Proper mapping
6) Platform independent
7) Every statement must be unambigous

Steps required

- Problem Definition
- Development model
- Design
- Testing ⇝ To find error
- Analysis ⇝ Measuring S.C & T.C
- Optimize ⇝ minimize T.C & S.C
- implementation

Analysis of algorithm

it means not only just measuring T.C & S.C but also analysing the performance of algo.

⇝ How to analyze performance of algorithm

2 ways

Apriori analysis	Aposteriori analysis
⇝ Before implementation	⇝ After implementation
⇝ Can calculate approx S.C & T.C	⇝ Used to calculate exact S.C & T.C
⇝ We use this while analysing	⇝ Not used
	⇝ Bcz it gives credit to machine & not developer of algo

⇝ How to analyze algo before implementation

S-1 Hypothetical language

S-2 Hypothetical CPU with ∞ RAM (Each fundamental operation need 1 unit of time)

S-3 Select a matric for analyzing algo

S-4 Approach for categorizing the running of algo (Asymptotic notation)

Fundamental operations

Assignment	Arithmatic	Logical	Relational	I/O
=	+, -, *, /, %	AND, OR, NOT	>, <, ==, !=	read, write

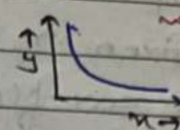
Ways of apriori analysis

- Step count method
- Order of magnitude
  - ↳ we use this
  - ⇝ Calculate only major operator
  - Ex:- $f(n) = n^2 + 3n + 2$
    - $\rightarrow (n^2)$ Ans

Time complexity
quantifies the amount of time (CPU time) taken by an algorithm to run as a funcⁿ of I/P size

Space complexity
Quantifies the space (RAM size) taken by an algorithm to run as a funcⁿ of I/P size

Tangents to the curve at ∞



→ x & y axis are asymptotes to this curve at ∞

→ curve is bounded by x & y axis

Asymptotic notation
- → O (Big-oh)
- → Ω (Big-omega)
- → Θ (theta)
- → o (Small-oh)
- → ω (Small-omega)

Asymptotic analysis of algorithms
- Analysing time & space needed by algorithm when $n \to \infty$
- Analysis of algorithm is generally done for very large input size
- Apriori analysis gives time needed by algorithm as a funcⁿ of Input size
- Algo 1: $f(n) = n^2 + n + 1$
  Algo 2: $f(n) = n^2$

  Mathematically A1 need more time but asymptotically ($n \to \infty$) both have same time complexity
- A1 : n^2 , A2 : $10000n$

→ Mathematically we will say till 9999 inputs, A2 need more time but beyond 10000 A1 need more time

Asymptotically A1 needs more time than A2.

Conclusion : 1) constants don't matter
2) Take dominating term

reasons : • $n \to \infty$
• machine independent

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→ We should have ways to represent time and space complexity of algorithm which is independent of machine specification & these ways are asymptotic notations.

① Big-oh notation:
Given 2 f^n $f(n)$ & $g(n)$
we say $f(n) = O(g(n))$ if we were able to find atleast 1 +ve constant c & a value of n called n_0 [$n_0 \geq 1$] such that $f(n) \leq c \cdot g(n)$ $\forall$ $n \geq n_0$

$O \to \leq$

• $f(n) = O\,g(n)$ means $g(n)$ is greater than or equal to $f(n)$ by taking help of c (constant) i.e
$f(n) \leq c \cdot g(n)$

• if $g(n)$ is already bigger then no need to take help of c ($c=1$) but if $g(n)$ is smaller than $f(n)$ then we need to find c such that $c \cdot g(n)$ is always bigger or equal to $f(n)$ but this relationship holds starting from which value of n?? this value is n_0

n → no. of inputs always greater than or equal to 1

Q- $f(n) = n$, $g(n) = 3n+2$

(i) $f(n) = O(g(n))$

Ans- Clearly
$g(n) \geq f(n)$
so take $c = 1$
and yes
$f(n) = O(g(n))$

(ii) $g(n) = O(f(n))$

Ans- here for any constant c greater than 3
let $c = 3.1$
$3n+2 \leq 3.1(n)$
$2 \leq 0.1n$
$n_0 = 20$ $\boxed{20 \leq n}$ ✓
$3n+2 \leq 3.1n$ $\forall n \geq 20$

n_0 should be an integer

Note

Q- $f(...$
(i...

No

(1)
(2)
(3)

②

Note: Big-oh represents upper bound
Big-omega represents lower bound

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and
independent
asymptotic
find
called
$n \geq n_0$
equal
take
n
$c \cdot g(n)$
of
stant
s
o

Q- $f(n) = 1000n$, $g(n) = n^2$

(i) $f(n) = O(g(n))$

$1000n = O(n^2)$

$1000n \leq c \cdot n^2$

let $c = 1$

$1000n \leq n^2$

$\boxed{n_0 = 1000}$

if $c = 2$

$n_0 = 500$

(ii) $g(n) = O(f(n))$

$n^2 = O(1000n)$

$n^2 \leq c \cdot 1000n$

analyse and you'll get to know that we can't find any value greater for which it is always

→ Not possible

Note: if time taken by an algorithm is independent of input size then its time complexity is $O(1)$

conclusion:

(1) always take dominating term for big-oh
(2) if $f(n)$ is $O(n)$ then $f(n)$ is $O(n^2)$, $O(n^3)$ & so on
(3) $O(n)$ represents ∞ func^n i.e. its not a single func^n → whose dominating term is $\leq n$

② Big-omega notation:

Given 2 func^n $f(n)$ & $g(n)$
we say $f(n) = \Omega\, g(n)$ if we are able to find atleast 1 +ve constant c & a value of n called n_0 s.t

$$f(n) \geq c \cdot g(n) \quad \forall n \geq n_0$$

Ex:- $f(n) = 3n+2$, $g(n) = n$

$f(n) = \Omega(g(n))$

$3n+2 \geq c \cdot n$

let $c = 1$

$\forall n \geq 1$

$\boxed{n_0 = 1}$

$g(n) = \Omega(f(n))$

$n \geq c \cdot (3n+2)$

let $c = 1/4$

$n \geq \frac{3}{4}n + \frac{1}{2}$

$\forall n \geq 2$

$\boxed{n_0 = 2}$

Conclusion:

(1) Always take dominating term for Ω

(2) If $f(n)$ is $\Omega(n^3)$ then $f(n)$ is $\Omega(n^2)$, $\Omega(n)$ & so on.

(3) $\Omega(n)$ represents ∞ functions i.e not a single func^n^

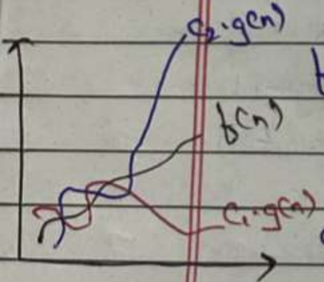
(3) Theta notation:

Given 2 f^n $f(n)$ & $g(n)$

we say $f(n) = \Theta(g(n))$ if we are able to find atleast 2 +ve constant c_1 & c_2 & a value of n called n_0 $(n_0 \geq 1)$ such that

$$c_1 \cdot g(n) \leq f(n) \leq c_2 \cdot g(n) \quad \forall n \geq n_0$$

$f(n) \leq c_2 \cdot g(n) \quad \forall n \geq n_0$ - (i) big-oh



$f(n) \geq c_1 \cdot g(n) \quad \forall n \geq n_0'$ - (ii) big-omega

from (i) & (ii)

$$c_1 \cdot g(n) \leq f(n) \leq c_2 \cdot g(n) \quad \forall n \geq \max(n_0, n_0')$$

Q- $f(n) = 3n+2$, $g(n) = n$

(i) $f(n) = \Theta(g(n))$

$c_1 \cdot g(n) \leq f(n) \leq c_2 \cdot g(n)$

$c_1 \cdot n \leq 3n+2 \leq c_2 \cdot n$

let $c_1 = 1$ and $c_2 = 4$

$n \leq 3n+2 \leq 4n$

$n_0 = 2$

(ii) $g(n) = \Theta(f(n))$

$c_1 \cdot f(n) \leq g(n) \leq c_2 \cdot f(n)$

$c_1 \cdot (3n+2) \leq n \leq c_2 \cdot (3n+2)$

let $c_1 = \frac{1}{4}$ let $c_2 = 1$

$\frac{(3n+2)}{4} \leq n \leq 3n+2$

$n = 2$

$n_0 = 2$

Q- $f(n) = 1000n$, $g(n) = n^2$

(i) $f(n) = \Theta(g(n))$

$c_1 \cdot n^2 \le f(n) \le c_2 \cdot n^2$

$c_1 \cdot n^2 \le 1000n \le c_2 \cdot n^2$

not possible bcz of dominating term (omega fails)

let $c_2 = 1$ (Big-oh pass)

ANS ⇝ not correct eqn

(ii) $g(n) = \Theta(f(n))$

$c_1 \cdot 1000n \le n^2 \le c_2 \cdot 1000n$

let $c_1 = 1$ (omega pass)

not possible bcz of dominating term (Big-oh fails)

ANS ⇝ Not a correct reln

Note : If time taken by algorithm is independent of input size, then its time complexity is $\Theta(1)$

Conclusion :

(1) Always take dominating term for Θ

(2) If $f(n)$ is $\Theta(n^3)$ then $f(n)$ is $\Theta(n^2)$, $\Theta(n)$ & so on — wrong statement only valid for equal

(3) $\Theta(n)$ represents ∞ funcn i.e. not a single funcn

(4) Ω is lower bound, O is upper bound, Θ is tight bound

Note : Theta notation is the most suitable and accurate among all because

if we say $\Theta(n)$ ⇝ it is guranteed that 'n' is dominating term

but

if we say $O(n)$ ⇝ not guranteed bcz 'n' se niche wali power bhi dominating term hoge

and

if we say $\Omega(n)$ ⇝ not guranteed bcz n^2, n^3 & so on ye sab dominating term hoge

④ Small-oh notation

Given 2 funcⁿ $f(n)$ & $g(n)$
we say $f(n) = o(g(n))$ if for every +ve constant c
we are able to find a value of n called n_0 s.t.

$$f(n) < c \cdot g(n) \quad \forall n \geq n_0$$

agar $f(n) = n^2$ toh $g(n)$ ko hamesha bdi power lena like $g(n) = n^3, n^4, \ldots$

$f(n) = 3n+2$, $g(n) = n^2$

~~f(n) = o(~~ $f(n) = o(g(n))$ ✓ $\qquad g(n) = o(f(n))$ ✗

secondry definition

$f(n) = o(g(n))$ iff

$$\lim_{n\to\infty} \frac{f(n)}{g(n)} = 0$$

simply we can say that when $n \to \infty$

$g(n) >>>>> f(n)$

⑤ Small omega notation

Given 2 funcⁿ $f(n)$ & $g(n)$
we say $f(n) = \omega(g(n))$ if for every +ve constant c
we are able to find a value of n called n_0 s.t.

$$f(n) > c \cdot g(n) \quad \forall n \geq n_0$$

agar $f(n) = n^3$ toh $g(n)$ ko hamesa choti power lena like $g(n) = n^2, n, n\log n$ so on

another definition

$f(n) = \omega(g(n))$

$$\lim_{n\to\infty} \frac{g(n)}{f(n)} = 0$$

when $n \to \infty$

$f(n) >>>> g(n)$

Q- why small Θ never exists

Ans- Theta notation is comprised of big-oh and big-omega

$$C_1 \cdot g(n) \le f(n) \le C_2 \cdot g(n)$$

here this equal sign
is the reason for
existing of theta notation
and not existing for small Θ

bcz according to small Θ

$$C_1 \cdot g(n) < f(n) < C_2 \cdot g(n)$$

Yaha pe dominating term
equal ho hi nhi skte bcz
ek mai hme power bde chahiye
or ek mai power choti chahiye

㊟ Some important topics to build background for properties

① $f(n) = 3n+2$
$g(n) = 6n-4$

we can say $f(n) = O(n)$ -(i)
& also say $g(n) = O(n)$ -(ii)

Now we can't conclude $f(n) = g(n)$
bcz big-oh represents set of ∞ funcⁿ

technically we need to write
$f(n) \in O(n)$
$g(n) \in O(n)$

but for better we write
$f(n) = O(n)$

that's why we cant equate $f(n) = g(n)$

$\Theta(n) \rightarrow$ set of ∞ asymptotically +ve func^n whose dominating term is $\leq n$

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② Asymptotically -ve func^n

$f(n) = -n^2 + 4n + 1$

curve reaches -ve after some value and then remains -ve !

⤳ we don't use big-oh for asymptotically -ve func^n bcz time can't be negative

③ $f(n) \rightarrow a$, $g(n) \rightarrow b$

$f(n) = O(g(n)) \rightsquigarrow a \leq b$

$f(n) = \Omega(g(n)) \rightsquigarrow a \geq b$

$f(n) = \Theta(g(n)) \rightsquigarrow a = b$

$f(n) = o(g(n)) \rightsquigarrow a < b$

$f(n) = \omega(g(n)) \rightsquigarrow a > b$

⤳ ignore constants

⤳ Take only dominating term

④ All func^n are asymptotically comparable ?

NO

$f(n) = n$

$g(n) = n^{1+\sin n}$ ⤳ lies b/w (n^0 to n^2)

$f(n) = O(g(n))$ ✗

$f(n) = \Omega(g(n))$ ✗

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Properties of Asymptotic notation

① Reflexivity

$R \subseteq A \times A$

$x R x \ \forall x \in A$

$f(n) = O(f(n))$ ✓
$f(n) = \Omega(f(n))$ ✓
$f(n) = \Theta(f(n))$ ✓
$f(n) = o(f(n))$ ✗
$f(n) = \omega(f(n))$ ✗

$f(n) = n$

$n = O(n)$ ✓
$n = \Omega(n)$ ✓
$n = \Theta(n)$ ✓
$n = o(n)$ ✗
$n = \omega(n)$ ✗

but if $n = o(n^2)$ ✓
$n^2 = \omega(n)$ ✓

func

constants
dominatin

② Symmetric

$R \subseteq A \times A$

$\forall a, b \in A \quad a R b \rightarrow b R a$

$f(n) = O(g(n))$ ✗ $\rightarrow (a \le b) \neq (b \le a)$
$f(n) = \Omega(g(n))$ ✗ $(a \ge b) \neq (b \ge a)$
$f(n) = \Theta(g(n))$ ✓ $(a = b) = (a = b)$
$f(n) = o(g(n))$ ✗ $(a < b) \neq (b < a)$
$f(n) = \omega(g(n))$ ✗ $(a > b) \neq (b > a)$

③ Transitive

$R \subseteq A \times A$

$\forall a, b, c \in A : a R b R c \rightarrow a R c$

$\rightarrow a \le b \ \& \ b \le c \Rightarrow a \le c$

$f(n) = O(g(n)) \ \& \ g(n) = O(h(n)) \rightarrow f(n) = O(h(n))$ ✓
$f(n) = \Omega(g(n)) \ \& \ g(n) = \Omega(h(n)) \rightarrow f(n) = \Omega(h(n))$ ✓
$f(n) = \Theta(g(n)) \ \& \ g(n) = \Theta(h(n)) \rightarrow f(n) = \Theta(h(n))$ ✓
$f(n) = o(g(n)) \ \& \ g(n) = o(h(n)) \rightarrow f(n) = o(h(n))$ ✓
$f(n) = \omega(g(n)) \ \& \ g(n) = \omega(h(n)) \rightarrow f(n) = \omega(h(n))$ ✓

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Summary

		R	S	T
$\le$	O	✓	✗	✓
$\ge$	Ω	✓	✗	✓
$=$	Θ	✓	✓	✓
$<$	o	✗	✗	✓
$>$	ω	✗	✗	✓

(4) Transpose symmetry

$$f(n) = O(g(n)) \text{ iff } g(n) = \Omega(f(n))$$

$a \le b$ & $b \ge a$ ⇝ yes, its true

(5)

$$\overset{a}{f(n)} = O(\overset{b}{g(n)})$$
$$\underset{c}{d(n)} = O(\underset{d}{h(n)})$$

$$f(n) + d(n) = O(\max(g(n), h(n)))$$

yes, its true

(6)

$$f(n) = O(g(n))$$
$$d(n) = O(h(n))$$

$$f(n) * d(n) = O(g(n) * h(n))$$

(7) $n^2 + \Theta(n) = \Theta(n^2)$

$n^2 + C_1 * n + C_2$ ⇝ we don't know C_1 & C_2 ⇝ also complicated to find

that's why

we ANS ⇝ $\Theta(n^2)$ by property

(8) $n^2 + O(n) = O(n^2)$

also a true identity

$n^2 + O(n) = O(n^2)$

$\{3n+2,\ 5n-6,\ \vdots,\ \log n,\ \sqrt{n},\ \vdots\}$

(9) $O(n) + o(n) = O(n)$

$\{3n+2,\ 5n-9,\ \vdots,\ \log n,\ \sqrt{n},\ \vdots\}$ $\{\log n,\ \sqrt{n},\ \vdots\}$ $= \times$

not a true identity

(10) $\sum_{i=1}^{n} O(i) = O(1) + O(2) + O(3) + \cdots + O(n) = O(n)$ ✗

bcz its a set of infinite funcⁿ that why we can't expand it like that.

$1+2+3+\cdots+n = \frac{n(n+1)}{2} = O(n^2)$ ✓

$\sum_{i=1}^{n} O(i) = O(n)$ ⇝ wrong statement

Q- Let $f(n)$, $g(n)$ & $h(n)$ be 3 +ve funcⁿ defined as follows

$\overset{a}{f(n)} = O(g(n))$ but $\overset{b}{g(n)} \neq O(f(n))$ $a \le b$ & $b \not\le a$ ⇓ $\boxed{a < b}$ —(I)

$g(n) = O(h(n))^{c}$ and $h(n) = O(g(n))$ $b \le c$ & $c \le b$ ⇓ $\boxed{b = c}$ (II)

which are True

a) $f(n) + g(n) = O(h(n))$ ✓

b) $f(n) = o(h(n))$ ✓

c) $h(n) * g(n) = O(h(n) * h(n))$ ✓

d) $f(n) * g(n) = O(h(n) * g(n))$ ✗

a) $a + b \le c$ (yes)

b) $a < c$ (yes)

c) $c * b = c * c$ (yes)

d) $a * \cancel{b} = c * \cancel{b}$ (No)

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a) $O(n)$ ~~b) $\Omega(n)$~~
c) ~~$\Theta(n)$~~ d) ~~$\omega(n)$~~

Q- $f(n) = \Omega(n)$, $g(n) = O(n)$, $h(n) = \Theta(n)$
then $(f(n) * g(n)) + h(n)$

Ans-

$f(n) \geq n$ $\quad$ $g(n) \leq n$ $\quad$ $h(n) = n$

$a \geq n$ $\quad$ $b \leq n$ $\quad$ $c = n$

$(a * b) + c$

$\geq n * \leq n + n \Rightarrow n^2 * \sqrt{n} + n$

n^2 $\quad$ $\sqrt{n}$ $\quad$ $n * c + n = n$

ANS ⇝ b

Q-

a) $O(f(n)) + O(g(n)) = O(f(n) + g(n))$ ✓
b) $O(f(n)) * O(g(n)) = O(f(n) * g(n))$ ✓
c) $n^2 + 6n + n^3 = O(n^4)$ ✓
d) $n^2 + 6n + n^3 = \Theta(n^4)$ ✓

a) $O(n) + O(n^2)$ $\quad$ $O(n + n^2)$ $O(n^2)$

$O(n^2)$ $\quad$ Yes, True

b) $O(n) * O(n^2)$ $O(n * n^2)$

$O(n^3) \equiv O(n^3)$

imp. log properties

(1) $\log_2 10 = \dfrac{\log_{10} 10}{\log_{10} 2}$

(2) $\log_x 4 = \dfrac{\log_e 4}{\log_e x}$

$$\boxed{\log_{10} n = \frac{\log_2 n}{\log_2 10} = \frac{\log_e n}{\log_e 10}}$$

$\log_{10} n = \Theta(\log_2 n) = \Theta(\log_e n) = \Theta(\log_{50} n) \ldots$

a)
b)
c)
d)
e)
f)
g

Asymptotic notations ke andr hume log ka base lagane ki koye jrurat nhi pdte bcz hum ek base se dusre base mai change krdete hai constant se divide krke

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a) $O(n)$ b) $\Omega(n)$
c) $\Theta(n)$ d) $\omega(n)$

Growth of func^n

	f(n)	g(n)
a)	n	n^2
b)	$\sqrt{n}$	n
c)	$\log n$	n
d)	$\log(\log N)$	$\log N$
e)	n	$n\log N$
f)	n	2^n
g)	n^{1000}	2^n

we can do this changement

$< \Rightarrow \leq$

$> \Rightarrow \geq$

$o \Rightarrow O$

$\omega \Rightarrow \Omega$

but not

$\leq \not\Rightarrow <$

$\geq \not\Rightarrow >$

$O \not\Rightarrow o$

ANSWERS

a) $f(n) = o(g(n))$
$f(n) = O\, g(n)$
$g(n) = \omega f(n) \Rightarrow g(n) = \Omega f(n)$

b) $f(n) = o(g(n))$
$f(n) = O(g(n))$
$g(n) = \omega f(n) \Rightarrow g(n) = \Omega f(n)$

c) Same like option a & b

d) Same like option a & b

e) Same like option a & b

f) Same like option a & b

g) Same like option a & b

n^{1000} 2^n

let $n = 2^{15}$

$(2^{15})^{1000}$ $2^{2^{15}}$

2^{15000} 2^{32768} ✓

★★★ $n^{finite} <<<< 2^n$

$\frac{\log_e n}{\log_e 10}$

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$n >>> \log n$
$n^2 >>> \log n$
$n^3 >>> \log n$
$n^{1/2} >> \log n$

Positive no:
(1) $n > \log n$
(2) $(\log n)^{\text{finite}} < n$

① $f(n)$ $g(n)$
$\sqrt{n}$ $\log n$

$\rightsquigarrow$ $f(n) = \omega(g(n)) \Rightarrow f(n) = \Omega(g(n))$
$g(n) = o(f(n)) \Rightarrow g(n) = O(f(n))$

② $f(n)$ $g(n)$
$\sqrt{\log n}$ $\log(\log n)$

$\log n >> (\log(\log n))^{\text{finite}}$

$\rightsquigarrow$ $f(n) = \omega(g(n)) \Rightarrow f(n) = \Omega(g(n))$
$g(n) = o(f(n)) \Rightarrow g(n) = O(f(n))$

③ $f(n)$ $g(n)$
2^n $<<$ $n!$

$\rightsquigarrow$ $f(n) = O(g(n)) \Leftarrow f(n) = o(g(n))$
$g(n) = \omega(f(n)) \Rightarrow g(n) = \Omega(f(n))$

④ $f(n)$ $g(n)$
$n!$ $<<$ n^n

$n * (n-1) * (n-2) \ldots$
$n * n * n * n$

Same result like 3rd

⑤ $f(n)$ $g(n)$
2^{n+1} 2^n

$2^n \cdot \cancel{2}$ $\cancel{2}$

$f(n) = \Theta(g(n))$
$\hookrightarrow$ $f(n) = O(g(n))$
$f(n) = \Omega(g(n))$

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$f(n)$ $g(n)$

⑥ $2^{2n} >> 2^n$

2^{2*n} 2^n

$(2^2)^n$ 2^n

$4^n >>> 2^n$

$f(n) = \omega(g(n)) \Rightarrow f(n) = \Omega(g(n))$

$g(n) = o(f(n)) \Rightarrow g(n) = O(f(n))$

⑦ $f(n)$ $g(n)$

$(n+k)^m$ n^m where k, m are constants

$k=2, m=3$

$(n+2)^3$ n^3

$\cancel{n^3} + \ldots$ $\cancel{n^3}$

$f(n) = \Theta(g(n))$

$\hookrightarrow f(n) = O(g(n))$

$f(n) = \Omega(g(n))$

⑧ $f(n)$ $g(n)$

$2^{n^2} >> n!$

Same like result 6

Q. $f(n) = 10n\log n$

$g(n) = 0.0001n^2$

let $f(n) < g(n)$ $\forall$ $n \geq 10^x$. Find smallest value of x

Ans- $f(n) = o(g(n)) \Rightarrow f(n) < \underset{1}{c} \cdot g(n)$ $\forall$ $n \geq 10^x$

$10 * 10^x \log 10^x < 0.0001(10^x)^2$

$x * 10^{x+1} < 10^{-4}(10^{2x})$

$x * 10^{x+1} < 10^{2x-4}$

$x < \frac{10^{2x-4}}{10^{x+1}}$

$x < 10^{(2x-4)-(x+1)}$

$x < 10^{x-5}$

using hit and trial

$\boxed{x=6}$

Tip: Jab dono taraf log mai comparison ho toh 2 ki power lagake try kro

$(1.001)^n >>> n^{1000000}$

Q- $f(n) << g(n)$

$n \qquad (\log n)^{\log n}$

$<$ let $n = 2^{10}$ $\quad 10^{\log_2 2^{10}}$

$2^{10} \qquad (\log_2 2^{10})^{\log_2 2^{10}}$

$2^{10} \qquad 10^{10}$

$f(n) = o(g(n)) \Rightarrow f(n) = O(g(n))$

$g(n) = \omega(f(n)) \Rightarrow g(n) = \Omega(f(n))$

Q- $f(n) \qquad g(n)$

$n^{\sqrt{n}} \qquad (\log n)^n$

let $n = 2^{10}$

Same result as above

$(2^{10})^{(2^{10})^{1/2}} \qquad (\log_2 2^{10})^{2^{10}}$

$(1024)^{32} << (10)^{1024}$

Q- $f_1(n) = n^2$

$f_2(n) = n\log n$

$f_3(n) = n\sqrt{n}$

$f_4(n) = e^n$

$f_5(n) = n$

$f_6(n) = 2^n$

$f_7(n) = 1/n$

$e^n > 2^n > n^2 > n\sqrt{n} > n\log n > n > \frac{1}{n}$ Ans

arrange them

Q- $f_1(n) = 2^n$

$f_2(n) = n\log n$

$f_3(n) = n^{3/2}$

$f_4(n) = n^{\log n}$

$2^n > n^{\log n} > n^{3/2} > n\log n$

$2^n \qquad n^{\log n}$

take log both side

$n\log_2 2 > \log n \log n$

$\{ \log(n!) = \Theta(n\log n)$

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ake try KTO

$) \Rightarrow f(n) = O(g(n))$

$\Rightarrow g(n) = \Omega(f(n))$

Q- $f_1(n) = \log n$

$f_2(n) = (\log n)^{\log n}$

$f_3(n) = \sqrt{n}$

$f_4(n) = n$

arrange

$\log n < \sqrt{n} < n < (\log n)^{\log n}$

$n \qquad (\log n)^{\log n} \sim \text{let } n = 2^{10}$

$2^{10} << 10^{10}$

Q- $f_1(n) = n^n$

$f_2(n) = n^{\log n}$

$f_3(n) = n^{\sqrt{n}}$

$f_4(n) = (\log n)^n$

$f_5(n) = n!$

$f_6(n) = 2^n$

$f_7(n) = \log(n!)$

$f_8(n) = n\log n$

as above

$n^n > n! > (\log n)^n > 2^n > n^{\sqrt{n}} > n^{\log n} > \log(n!)$

★★★★

Q- if $a<b$ then $f(n^a) = O f(n^b)$ where $a, b \geq 1$

Ans- let $f(n) = \frac{1}{n}$

$gn > n > \frac{1}{n}$

$f(n) = 3n+2$

$f(n^2) = 3n^2+2$

$f(n^a) = 3n^a+2$

$f(n^a) = \frac{1}{n^a}$, $f(n^b) = \frac{1}{n^b}$

$a < b$

$\frac{1}{n^a} > \frac{1}{n^b}$

~~but~~

so $f(n^a) \neq O f(n^b)$

$n! = O(n^n)$

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(2) Polynomial bounded function

Given any funcⁿ $f(n)$. if we have any polynomial funcⁿ greater than or equal to $f(n)$ then we say $f(n)$ is polynomial bounded.

every polynomial funcⁿ is polynomial bounded funcⁿ

n is bounded by n^2

Polynomial Fⁿ → P.B.F

P F → P.B.F (one way)

~PF → ~PBF ⇝ False

PBF → PF ⇝ False

~PBF → ~PF ⇝ True

funcⁿ	Polynomial Bounded funcⁿ
n	n^2
n^2	n^3
n^5+n^2+1	n^6
$6n^3+4n+5$	n^4
$n\log n$	n^2
$\log n$	n
$n\log(\log n)$	n^2
$n^2\log n$	n^3
2^n	×
$n^{\log n}$	×
n^n	×

not a polynomial funcⁿ but still bounded

$f(n)$ is polynomially bounded iff

$$\log(f(n)) = O(\log n)$$

Q- which of the funcⁿ are polynomially bounded

a) $n!$ → $O(n\log n)$ × No

b) $\log(n!)$ → $O(n\log n) \approx n\log n \Rightarrow \log n + \log\log = O(\log n)$ yes

c) $(\log n)!$ → ×

d) $\log(\log n)!$ → yes lee agar $\log(n!)$ aarha toh ye bhi ayega

e) $(\log(\log n))!$ → yes

use above formula

Note : $(n!)$ is the perfect example of a func^n which is not polynomially bounded but $\log(f(n))$ is polynomially bounded

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Q- which of the following are True

(1) $f(n) = O\, f(n/2)$

let $f(n) = 4^n$ — False

$f(n) = 4^n$, $f(n/2) = 4^{n/2} = (4^n)^{1/2} = (2^{2n})^{1/2} = 2^n$

$f(n) \neq O(f(n/2))$

(2) $f(n) = O\, f(n^2)$

let $f(n) = \frac{1}{n}$ — False

$f(n^2) = \frac{1}{n^2}$ $\quad \frac{1}{n} > \frac{1}{n^2}$

$f(n) \neq O(f(n^2))$

(3) $f(n) = O(f(n)^2)$

let $f(n) = \frac{1}{n}$ — False

$(f(n))^2 = \frac{1}{n^2}$

$f(n) \neq O((f(n))^2)$

(4) if $f(n) = O(g(n))$ then $2^{f(n)} = O(2^{g(n)})$

Ans- Let $f(n) = n$ $g(n) = n/2$ — False

$2^n \neq O(2^{n/2})$

(5) $1000^* n\log n = O\left(\frac{n\log n}{1000}\right)$ — True

ignore constant

Q-

$$f(n) = \begin{cases} n^4 & 0 < n < 1000 \\ n^2 & n \geq 1000 \end{cases}$$

$$g(n) = \begin{cases} n' & 0 < n < 100 \\ n^3 & n \geq 100 \end{cases}$$

as we know, we calculate for big terms

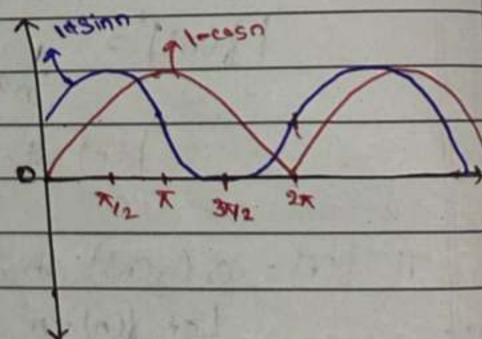
$f(n) = n^2$
$g(n) = n^3$

$f(n) = o(g(n)) \Rightarrow f(n) = O(g(n))$
$g(n) = \omega(f(n)) \Rightarrow g(n) = \Omega(f(n))$

Q-

$f(n) = n^{\overset{-1 \text{ to } +1}{1+\sin n}} \rightsquigarrow n^0$ to n^2

$g(n) = n^{\underset{-1 \text{ to } 1}{1-\cos n}} \rightsquigarrow n^0$ to n^2



we can't conclude any notation here bcz it is changing

Q-

$f(n) = n^{\sin n}$ ↗ -1 to $+1$

$g(n) = n^{2+\cos n}$ ↗ -1 to $+1$.

$f(n) \rightsquigarrow n^{-1}$ to n'

$g(n) \rightsquigarrow n'$ to n^3

$f(0)=0$	$g(0)=3$
$f(\pi/2)=1$	$g(\pi/2)=2$
$f(\pi)=0$	$g(\pi)=1$
$f(3\pi/2)=-1$	$g(3\pi/2)=2$
$f(2\pi)=0$	$g(2\pi)=3$

$f(n) = O(g(n)) \leftarrow f(n) = o(g(n))$
$g(n) = \omega(f(n)) \rightsquigarrow g(n) = \Omega(f(n))$

as we can see g(n) is always greater than f(n) but when we look at quesn it seems they are equal at some point

Note: Jab bhi compare krte time thoda bhi doubt hai toh value leke compare kro kro.

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Q- $f(n) = n!$
$g(n) = (n-1)!$
$h(n) = (n-1)! + n$

$n! = n * (n-1)!$

$g(n) = o f(n) \Rightarrow g(n) = O(f(n))$

$f(n) = \omega(g(n)) \Rightarrow f(n) = \Omega(g(n))$

$g(n) = \Theta(h(n))$

become equal to g(n) bcz of negotiating constant

$h(n) = g(n) < f(n)$

Q- $f(n) = n!$
$f(n) = (\log n)^{\log n}$
$f(n) = n^{\log n}$ $\rightarrow n^{\log n} > (\log n)^{\log n}$
$f(n) = \log(n!)$
$f(n) = \sqrt{n}$
$f(n) = (\log(\log n))!$
$f(n) = (\log n)!$ $\rightarrow (\log n)! > (\log(\log n))!$
$f(n) = 2^n$
$f(n) = n^{\sqrt{n}}$ $\rightarrow 2^n > n^{\sqrt{n}}$

$\rightsquigarrow$ n! is biggest

$n! > 2^n > n^{\sqrt{n}}$

ANS $\rightsquigarrow$ $n! > 2^n > n^{\sqrt{n}} > n^{\log n} > (\log n)^{\log n} > (\log n)! > \log(n!) > \sqrt{n} > (\log(\log n))!$

Q- $f(n) = \sqrt{n}$ $\quad g(n) = n \log n$

Same part ko uda do

$\sqrt{n} \times 1$ $\quad \sqrt{n} \times \sqrt{n} \times \log N$

$\sqrt{n} \log n >> 1$

$g(n) >> f(n)$

$f(n) = o(g(n)) \Rightarrow f(n) = O(g(n))$
$g(n) = \omega(f(n)) \Rightarrow g(n) = \Omega(f(n))$

★★★ Note: first remove common part then take log