Volatility Modeling and Forecasting using Time Series Models

Presented

In Partial Fulfillment of the Requirements
for the Degree of Bachelor of Statistics

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Abstract

This project presents an application of time series modeling techniques to forecast financial market volatility on various stocks across different sectors. Recognizing the empirical regularities in financial return series—such as volatility clustering and time-varying variance—we employ the Generalized Autoregressive Conditional Heteroskedasticity (GARCH) framework to model and forecast conditional volatility. The analysis begins with the transformation of price data into log returns, followed by standard time series diagnostics, including stationarity checks and tests for conditional heteroskedasticity. A GARCH(1,1) model is then estimated to capture the dynamic structure of volatility. Model selection and validation are carried out using information criteria and residual diagnostics. Forecasting performance is evaluated using statistical accuracy measures such as Mean Squared Error (MSE) and visual inspection of predicted volatility patterns. This project demonstrates the effectiveness of time series methods, particularly GARCH models, in capturing key features of financial data and highlights their applicability in real-world risk modeling and forecasting contexts.

Acknowledgements

I would like to express my heartfelt gratitude to Dr. Ayanendranath Basu for his invaluable guidance and support throughout this project. His expertise and insights into the field have greatly enriched my understanding and have been instrumental in shaping the direction of my research. I am particularly thankful for his encouragement and constructive feedback, which motivated me to strive for excellence.

I would also like to extend my appreciation to my friends, who have been a constant source of support and encouragement. Their collaboration and willingness to share ideas and resources made this project a more enjoyable and enriching experience. I am grateful for their companionship and the many discussions that helped refine my thoughts and approaches.

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Chapter 1

Introduction

Volatility is a fundamental concept in financial markets, reflecting the degree of uncertainty or risk associated with the return of an asset. In the context of financial time series, volatility is rarely constant over time — instead, it tends to cluster, with periods of high volatility followed by periods of relative calm. This stylized fact renders constant-variance models inadequate for modeling financial returns. To address this, time series models such as the Generalized Autoregressive Conditional Heteroskedasticity (GARCH) family have been developed to capture time-varying volatility through conditional heteroskedastic structures.

This project focuses on the application of time series methods — specifically GARCH modeling — to forecast volatility in the Indian stock market. Instead of analyzing a single index, this study uses daily return data from a diverse set of stocks across different sectors, allowing for a broader examination of volatility dynamics in the market. The aim is to assess whether GARCH models can effectively capture the conditional variance patterns present in individual stock returns and to compare volatility behavior across sectors.

The analysis involves transforming price series into log returns, testing for stationarity and ARCH effects, and fitting GARCH(1,1) models to each stock. Residual diagnostics and information criteria are employed to validate model adequacy. The project adopts a hands-on, data-driven approach rooted in statistical theory, illustrating how time series models with non-constant variance assumptions can be practically applied to real-world financial datasets. The findings provide insights into sectoral differences in volatility structure and highlight the strengths and limitations of univariate GARCH models in capturing financial risk.

Chapter 2

Report

2.1 Theory

Financial return series often display characteristics such as volatility clustering, where large shocks are followed by large shocks and small shocks by small ones. Traditional time series models with constant variance assumptions fail to capture such behavior. To address this, Engle (1982) introduced the *Autoregressive Conditional Heteroskedasticity* (ARCH) model, which models the variance of the current error term as a function of past squared errors.

2.1.1 ARCH Model

Let $\{r_t\}$ denote a return series, modeled as:

$$r_t = \mu + \epsilon_t, \quad \epsilon_t = \sigma_t z_t, \quad z_t \sim \text{i.i.d. } N(0,1)$$

So $r_t \sim N(\mu, \sigma_t^2)$ which means that the model assumes the variance of the returns of the stocks as time varying. In an **ARCH**(q) model, the conditional variance σ_t^2 is defined as:

$$\sigma_t^2 = \omega + \sum_{i=1}^{q} \alpha_i \epsilon_{t-i}^2$$

where:

- $\omega > 0$ ensures a strictly positive baseline variance, which is also referred as long-run variance,
- $\alpha_i \geq 0$ are coefficients capturing the influence of past squared shocks. (In the context of return modeling, shock represents new information or random disturbances that cause a deviation from the expected return.)

The ARCH model captures time-varying volatility but may require a high order q to adequately describe persistent volatility in financial data.

2.1.2 GARCH Model

To improve parsimony while capturing long memory in volatility, Bollerslev (1986) generalized the ARCH model to the *Generalized ARCH* or GARCH model, by allowing past variances to influence the current variance.
The GARCH(p, q) model is given by:

$$\sigma_t^2 = \omega + \sum_{i=1}^{q} \alpha_i \epsilon_{t-i}^2 + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2$$

where:

- $\omega > 0$, $\alpha_i \geq 0$, and $\beta_j \geq 0$ ensure positive variance,

- α_i represent ARCH terms (effects of past squared shocks),
- β_j represent GARCH terms (effects of past variances).

A widely used specification is the GARCH$(1,1)$ model:

$$\sigma_t^2 = \omega + \alpha_1 \epsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2$$

This model is capable of capturing volatility clustering and persistence in a parsimonious way. For the GARCH$(1,1)$ model to be covariance stationary, it is required that $\alpha_1 + \beta_1 < 1$.

Key Features of ARCH/GARCH Models

- Capture time-varying conditional variance.
- Model volatility clustering, a key characteristic of financial returns.
- Provide better forecasts of risk and volatility than constant-variance models.

Extensions to the basic GARCH framework, such as EGARCH and GJR-GARCH, allow for asymmetries in volatility responses to positive and negative shocks.

2.2 Data Preparation

2.2.1 Train-Test Split and Data Period

To evaluate the forecasting performance of time series models like GARCH, it is essential to divide the data into a *training set* and a *testing set*. The training set is used to estimate the model parameters, while the testing set allows us to assess the model's ability to predict out-of-sample volatility.

In this project, historical daily closing price data was collected for multiple stocks across different sectors of the Indian stock market. The full dataset contains daily Open, Low, High, Close, Volume for the trading days from 2019/01/01 to 2024/12/31. From this:

- The **training set** consists of the first T days (e.g., 80% of the data), used to estimate GARCH model parameters.
- The **testing set** consists of the remaining $N - T$ days (e.g., 20%), used for out-of-sample forecasting and performance evaluation.

This split enables a more realistic assessment of how well the model generalizes to new, unseen data — a crucial requirement for any forecasting model.

Price	Close	High	Low	Open	Volume	Log_Return
Ticker	^NSEI	^NSEI	^NSEI	^NSEI	^NSEI	
Date						
2015-01-05	8378.400391	8445.599609	8363.900391	8407.950195	118200	-0.002033
2015-01-06	8127.350098	8327.849609	8111.350098	8325.299805	172800	-0.030422
2015-01-07	8102.100098	8151.200195	8065.450195	8118.649902	164100	-0.003112
2015-01-08	8234.599609	8243.500000	8167.299805	8191.399902	143800	0.016221
2015-01-09	8284.500000	8303.299805	8190.799805	8285.450195	148000	0.006042

Figure 2.1: Sample of the Data

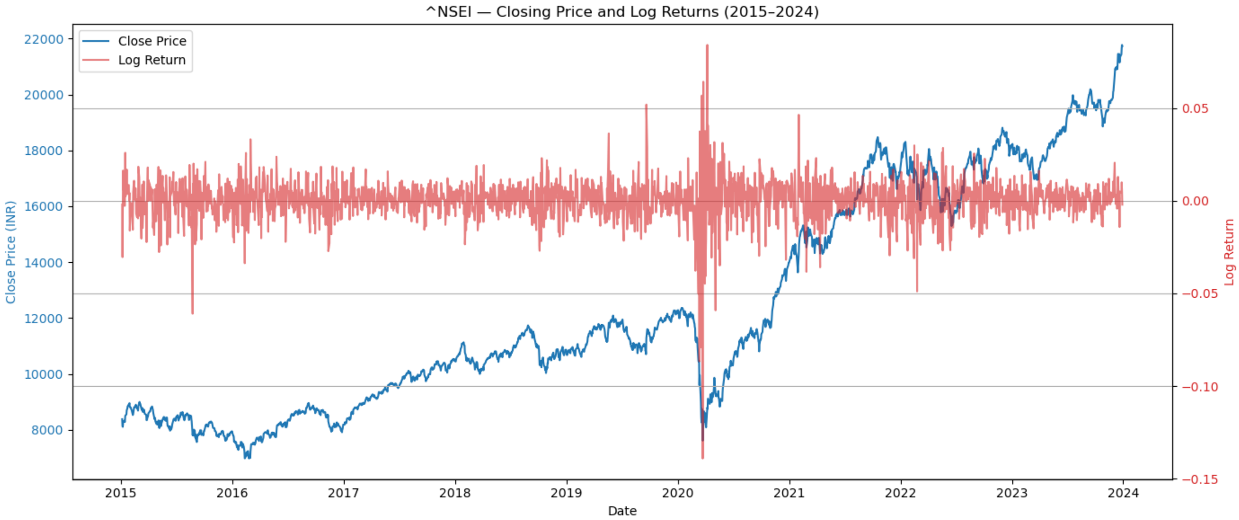

Figure 2.2: Plot of Raw Prices and Daily Log Returns

2.2.2 Log Returns and Motivation

Instead of working with raw price levels, the analysis is performed on the series of *log returns*. Given a time series of closing prices P_t, the log return r_t is defined as:

$$r_t = \log\left(\frac{P_t}{P_{t-1}}\right) = \log(P_t) - \log(P_{t-1})$$

Log returns are preferred over simple arithmetic returns for the following reasons:

- **Time Additivity:** Log returns are additive over time, i.e., the log return over multiple periods is the sum of the log returns of the individual periods.
- **Stationarity:** Price series are typically non-stationary, but return series (especially log returns) are more likely to be stationary, a key assumption in time series modeling.
- **Normality Approximation:** Log returns tend to approximate normality better than raw returns, especially over shorter time intervals.
- **Scale Invariance:** Log returns are dimensionless and can be compared across assets of different price levels.

As a result, modeling volatility on log returns provides more robust and interpretable results in the context of time series forecasting.

2.3 Methodology

- After collecting and prepossessing the data, we conduct the *Augmented Dickey-Fuller (ADF)* test to check if the log returns series is stationary or not . Then for each stock a $\mathrm{GARCH}(1,1)$ model was estimated. To predict the volatility for day t, we have used the $\mathrm{GARCH}(1,1)$ model with the training data as all the previous trading days available into the training dataset.
- After modeling we have plotted the realised volatility vs forecasted volatility to visualize if our prediction is accurate or not. Volatility of Day t is estimated by volatility of log-returns of days $t-k$ to t where k is the number of lookback days,it depends on stock and the sector. Also the residuals of the model has been plotted. To check if it is a white noise or not we have also plotted the autocorrelation function (ACF) of standardized residuals and of squared standardized residuals.

2.4 Results

The model has been fitted to the index $NSEI$ and various stocks from different sectors- ’SBI’,’TCS’,’ONGC’.’RELI

2.4.1 NSEI

Using data from 2019/01/01 to 2023/01/01 we have forecasted the volatility from year 2023 to 2024. For each day all the previous trading days have been used for training the model so that it can capture the recent volatility trends.

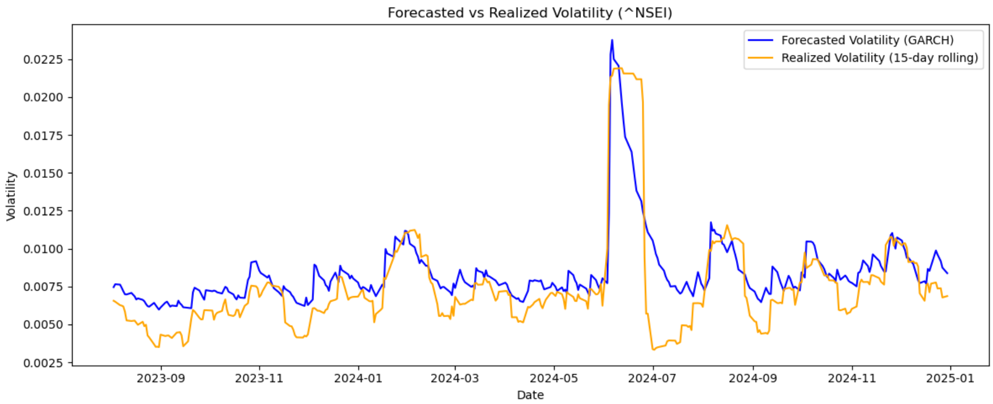

Figure 2.3: Forecasted vs Actual Volatility of NSEI

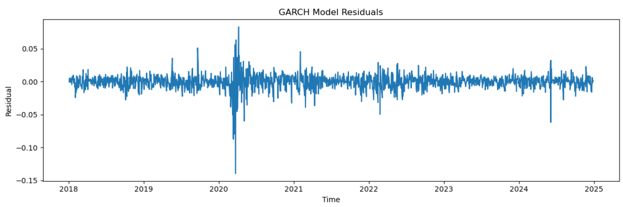

Figure 2.4: GARCH Model Residuals of NSEI

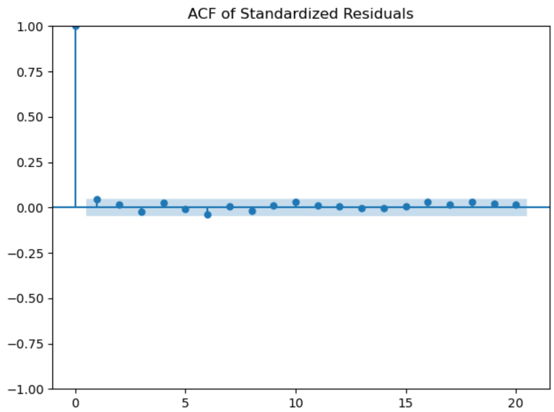

Figure 2.5: ACF of Standardized Residuals

Considering the plots above and the model estimates we can make the following observations:

- As from table 3.1 we can see $\alpha_1 + \beta_1 < 1$, proving the fact that the model is covariance stationary.
- Figure 2.4 highlights the fact that the residuals are white noise. ACF of Standardized Residuals in figure 2.5 shows that there is no autocorrelation between the Standardized Residuals (residuals divided by conditional variance) since there are no spikes beyond the confidence interval.
- Also from figure 2.3 the pattern for both realised volatility and forecasted volatility are same: high volatility follows high volatility and low volatility follows low volatility.

2.4.2 TCS

Using data from 2019/01/01 to 2023/01/01 we have forecasted the volatility from year 2023 to 2024. For each day all the previous trading days have been used for training the model so that it can capture the recent volatility trends. 15 day rolling window has been used for realised volatility.

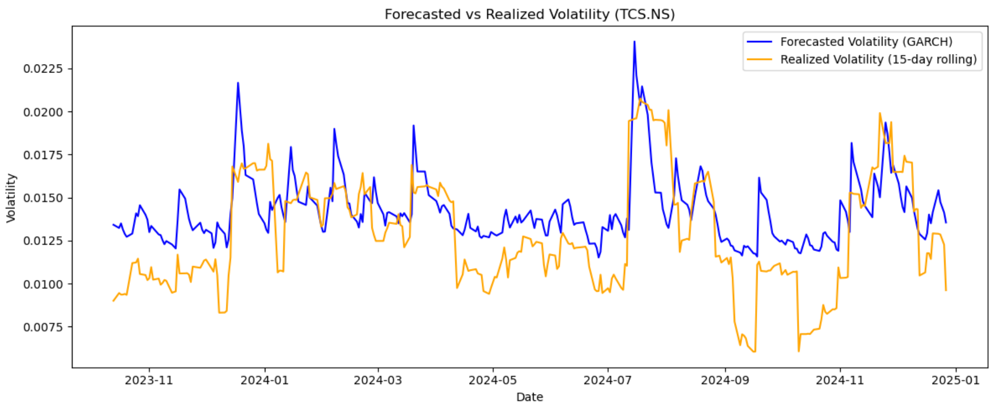

Figure 2.6: Forecasted vs Actual Volatility of TCS

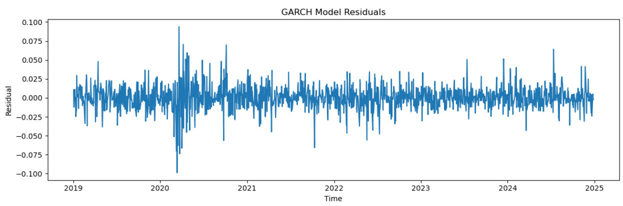

Figure 2.7: GARCH Model Residuals of TCS

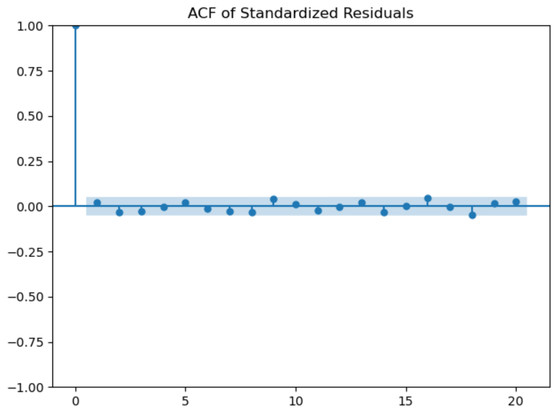

Figure 2.8: ACF of Standardized Residuals

Considering the plots above and the model estimates we can make the following observations:

- As from table 3.2 we can see $\alpha_1 + \beta_1 < 1$, proving the fact that the model is covariance stationary.
- In figure 2.7 a high spike can be seen around the year $2020 - 2021$ which is because of the market fluctuations during corona virus.
- Figure 2.7 highlights the fact that the residuals are white noise. ACF of Standardized Residuals in figure 2.8 shows that there is no autocorrelation between the Standardized Residuals (residuals divided by conditional variance) since there are no spikes beyond the confidence interval.
- Also from figure 2.6 the pattern for both realised volatility and forecasted volatility are same: high volatility follows high volatility and low volatility follows low volatility.
- Despite of all the above, there is a shortcoming in this forecast.In figure 2.6 we can see the high volatility areas are predicted well by the model, but the model overestimates the low volatility area. Although we can use this model market regime detection which basically differentiates between high and low volatility regions.

2.4.3 ONGC

Using data from 2019/01/01 to 2023/01/01 we have forecasted the volatility from year 2023 to 2024. For each day all the previous trading days have been used for training the model so that it can capture the recent volatility trends. 15 day rolling window has been used for realised volatility.

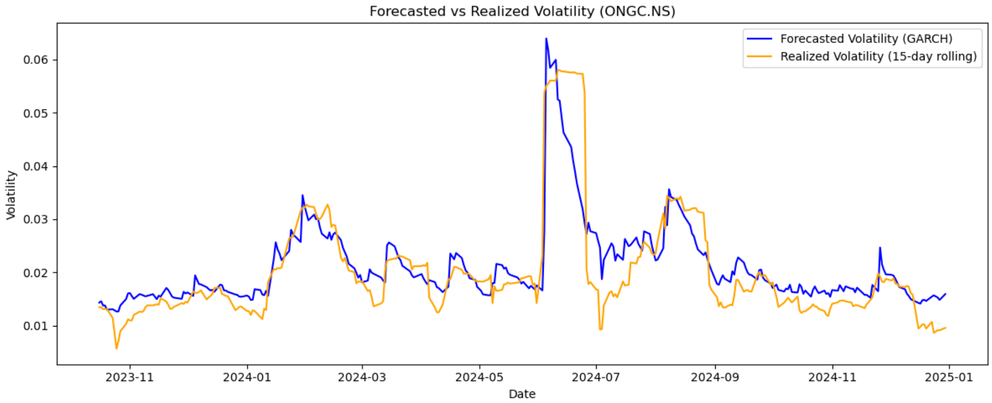

Figure 2.9: Forecasted vs Actual Volatility of ONGC

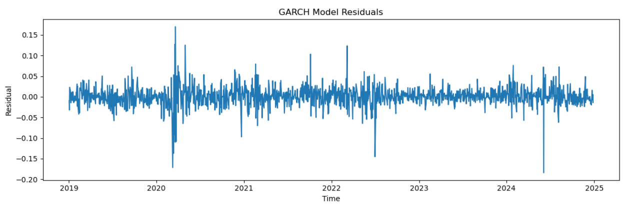

Figure 2.10: GARCH Model Residuals of ONGC

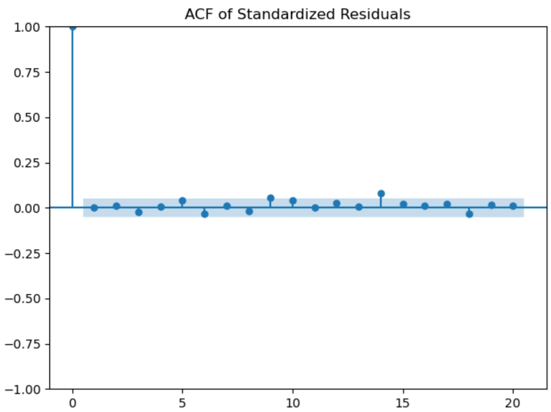

Figure 2.11: ACF of Standardized Residuals

Considering the plots above and the model estimates we can make the following observations:

- As from table 3.3 we can see $\alpha_1 + \beta_1 < 1$, proving the fact that the model is covariance stationary.
- $p-$values of the coefficients and intercepts obtained from the model (see table 3.3 are all less than 0.005. This implies the lagged shock term and variance term are highly significant for this model.
- Figure 2.10 highlights the fact that the residuals are white noise. ACF of Standardized Residuals in figure 2.11 shows that there is no autocorrelation between the Standardized Residuals (residuals divided by conditional variance) since there are no spikes beyond the confidence interval.
- Also from figure 2.9 the pattern for both realised volatility and forecasted volatility are same: high volatility follows high volatility and low volatility follows low volatility.
- Unlike the other stocks, we have been successful to predict all the high and low volatility regions and all the upward and downward direction of market movement has been well captured by GARCH$(1,1)$. This accuracy is also visible from the error metrics in table 2.1. MAPE is less than 20% which is great for a GARCH $(1,1)$ model.

2.4.4 SBI

Using data from 2018/01/01 to 2023/01/01 we have forecasted the volatility from year 2023 to 2024. For each day all the previous trading days have been used for training the model so that it can capture the recent volatility trends. 15 day rolling window has been used for realised volatility.

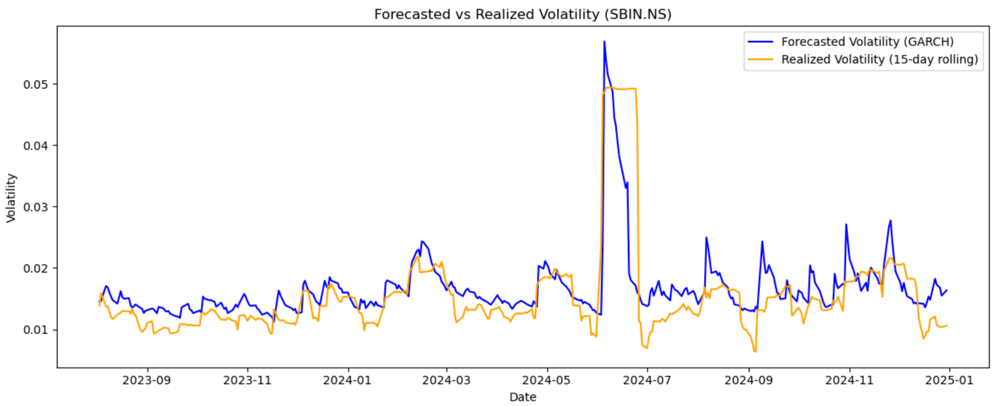

Figure 2.12: Forecasted vs Actual Volatility of SBI

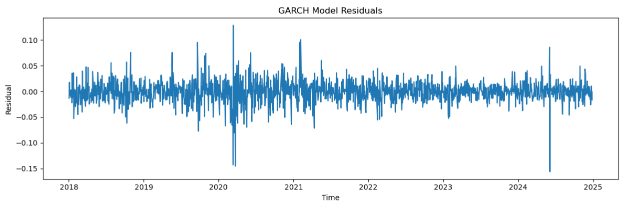

Figure 2.13: GARCH Model Residuals of SBI

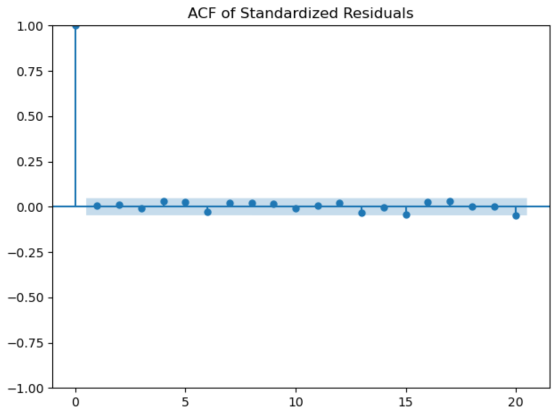

Figure 2.14: ACF of Standardized Residuals

Considering the plots above and the model estimates we can make the following observations:

- All the p-values for intercept and coefficients of the model are both < 0.0001. So all the variables used in the model are statistically significant.
- In figure 2.13 a high spike can be seen around the year $2020 - 2021$ which is because of the market fluctuations during corona virus.
- Figure 2.13 highlights the fact that the residuals are white noise. ACF of Standardized Residuals in figure 2.14 shows that there is no autocorrelation between the Standardized Residuals (residuals divided by conditional variance) since there are no spikes beyond the confidence interval.
- Also from figure 2.12 the pattern for both realised volatility and forecasted volatility are same: high volatility follows high volatility and low volatility follows low volatility.
- Despite of all the above, there is a shortcoming in this forecast.In figure 2.12 we can see the high volatility areas are predicted well by the model, but the model overestimates the low volatility area. Although we can use this model market regime detection which basically differentiates between high and low volatility regions.

2.4.5 RELIANCE

Using data from 2019/01/01 to 2023/01/01 we have forecasted the volatility from year 2023 to 2024. For each day all the previous trading days have been used for training the model so that it can capture the recent volatility trends. 15 day rolling window has been used for realised volatility.

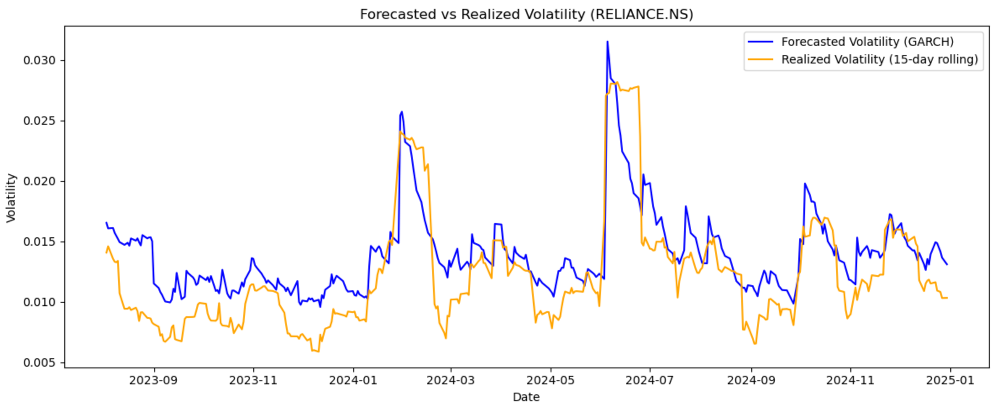

Figure 2.15: Forecasted vs Actual Volatility of ONGC

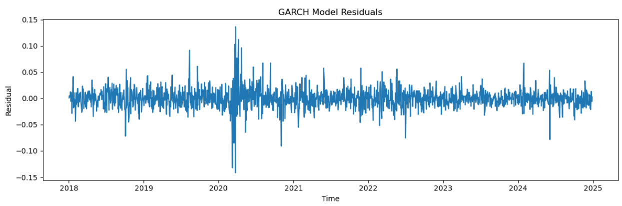

Figure 2.16: GARCH Model Residuals of RELIANCE

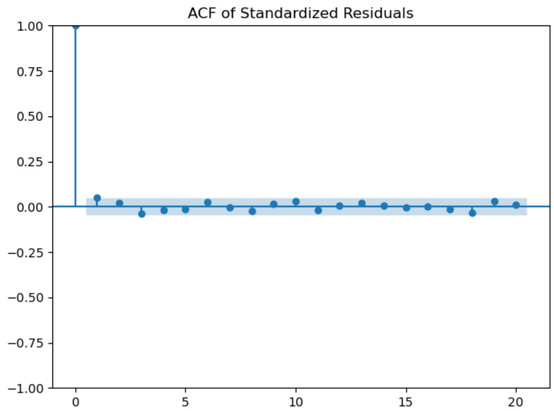

Figure 2.17: ACF of Standardized Residuals

Considering the plots above and the model estimates we can make the following observations:

- As from table 3.5 we can see $\alpha_1 + \beta_1 < 1$, proving the fact that the model is covariance stationary.
- $p-$values of the coefficients and intercepts obtained from the model (see table 3.5 are all less than 0.005. This implies the lagged shock term and variance term are highly significant for this model.
- Figure 2.16 highlights the fact that the residuals are white noise. ACF of Standardized Residuals in figure 2.17 shows that there is no autocorrelation between the Standardized Residuals (residuals divided by conditional variance) since there are no spikes beyond the confidence interval.
- Also from figure 2.15 the pattern for both realised volatility and forecasted volatility are same: high volatility follows high volatility and low volatility follows low volatility.
- MAPE for this stock is 24.21% which is considered a decent level of error metric for GARCH(1, 1) model.

2.5 Error Metrics

To assess the forecasting performance of the GARCH(1,1) model, we compare the *forecasted volatility* ($\hat{\sigma}_t$) with the *realized volatility* (σ_t), which is estimated using a rolling standard deviation of returns. The following error metrics are used for evaluation:

1. **Mean Squared Error(MSE):**

$$\mathbf{MSE} = \frac{1}{n}\sum_{i=1}^{n}(\sigma_i - \hat{\sigma}_t)^2$$

   This measures the average squared difference between the forecasted and realized volatilities. It penalizes larger errors heavily.

2. **Mean Absolute Error (MAE):** It is the average of the absolute difference between realized volatility and predicted volatility. Mathematically we can write it as

$$\mathbf{MAE} = \frac{1}{n}\sum_{i=1}^{n}|\sigma_i - \hat{\sigma}_t|$$

   MAE is more robust to outliers compared to MSE.

3. **Root Mean Squared Error(RMSE):**

$$\mathbf{RMSE} = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(\sigma_i - \hat{\sigma}_t)^2}$$

   Square root of MSE is named as RMSE. RMSE provides an interpretable measure in the same units as volatility, offering a sense of the typical error magnitude.

4. **Mean Absolute Percentage Error (MAPE):**

$$\mathbf{MAE} = \frac{100}{n}\sum_{i=1}^{n}\frac{|\sigma_i - \hat{\sigma}_t|}{\sigma_t}$$

   This expresses forecast error as a percentage, making it easier to interpret across scales. However, it can be unstable when $\sigma_t$is close to zero.

Galloway (2018) in *"A Review of Forecasting Volatility Models"* highlights that MAPE is often not suitable for financial time series models, especially in volatility forecasting, where the data is rarely symmetric, and extreme small values can lead to very large percentage errors. So we will mainly focus on MAE and RMSE to check the accuracy of the model. Below is the table consisting the errors calculated for different stocks over a span of nearly 1 year. Except MAPE rest two are calculated as decimals.

Stock	MAE	RMSE	MAPE(%)
NSEI	0015	0.0020	25.52
TCS	0.0023	0.0027	21.74
ONGC	0.0035	0.0050	19.86
RELIANCE	0.0025	0.0030	24.21
SBI	0.0031	0.0049	21.77

Table 2.1: Error Calculation over 1 year

2.6 Conclusion and Further Work

This project successfully implemented a GARCH(1,1) model to forecast the volatility of stock market returns using historical data from the NSEI index. The use of log returns, rolling forecasts, and comparison with realized volatility provided a realistic framework for evaluating the model's predictive performance. The residual analysis showed that the model effectively captured the key characteristics of financial time series, especially volatility clustering. Most importantly, the ACF plots of the standardized residuals indicated that the GARCH model reduced autocorrelation and conditional heteroskedasticity, validating its use for volatility modeling. This project demonstrates that GARCH models are valuable tools in financial analytics, particularly in risk management and portfolio monitoring, where volatility forecasting plays a critical role.

Going forward, this work can be extended by integrating ARIMA models for the mean return process, exploring more advanced GARCH variants like EGARCH or GJR-GARCH to model asymmetric effects, and applying the forecasts to risk management tasks such as Value at Risk (VaR). Additionally, incorporating macroeconomic variables or comparing GARCH forecasts with machine learning methods could provide deeper insights, particularly for portfolio optimization and volatility-based trading strategies.

Chapter 3

Appendix

3.1 Model Coefficients and Statistical Significance

Table 3.1: Model Estimates for NSEI

Parameter	Coefficient	Std. Error	p-value
omega	0.000003	0.000000	0.000000
alpha[1]	0.100000	0.024477	0.000044
beta[1]	0.880000	0.017588	0.000000

Table 3.2: Model Estimates for TCS

Parameter	Coefficient	Std. Error	p-value
omega	0.000023	0.000000	0.000000
alpha[1]	0.100000	0.028728	0.000500
beta[1]	0.800000	0.021210	0.000000

Table 3.3: Model Estimates for ONGC

Parameter	Coefficient	Std. Error	p-value
omega	0.000022	0.000002	0.000000
alpha[1]	0.136073	0.045653	0.002877
beta[1]	0.830004	0.032840	0.000000

Table 3.4: Model Estimates for SBIN

Parameter	Coefficient	Std. Error	p-value
omega	0.000044	0.000006	0.000000
alpha[1]	0.200000	0.048786	0.000041
beta[1]	0.700000	0.047558	0.000000

Table 3.5: Model Estimates for RELIANCE

Parameter	Coefficient	Std. Error	p-value
omega	0.000007	0.000000	0.000000
alpha[1]	0.100000	0.027722	0.000310
beta[1]	0.880000	0.023498	0.000000

3.2 Python Code

The python code needed for the model fitting and error calculation has been uploaded here. In the 'ticker' variable enter the ticker of the stock of which you want to forecast volatility. the default 'ticker' is set to the ticker of "NSEI".