(ii) Using heat pump. The power consumption is

$$\dot{W} = \frac{\dot{Q}_k}{\mathcal{E}_h} \cong \frac{1}{3}\dot{Q}_k$$

if $\mathcal{E}_h = 3$. Thus, power consumption of the heat pump is very much lower.

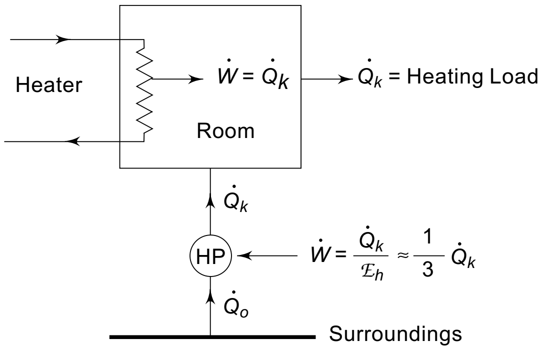

Fig. 2.8 Heating of a room by electric heater and heat pump

2.4 BEST REFRIGERATION CYCLE: THE CARNOT PRINCIPLE

It is possible to show that the cooling energy ratio of a refrigeration cycle working between two temperatures will be maximum when the cycle is a reversible one. For the purpose, consider a reversible refrigerating machine R and another irreversible refrigerating machine I, both working between two heat reservoirs at temperatures T_k and T_o, and absorbing the same quantity of heat Q_o from the cold reservoir at T_o as shown in Fig. 2.9 (a).

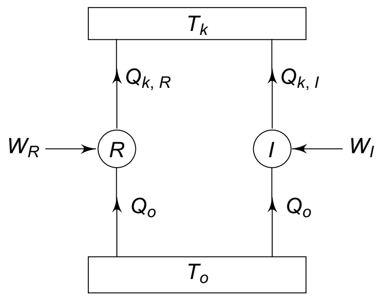

Fig. 2.9 (a) Reversible and irreversible refrigerating machines

Now, to prove the contrary, let us assume that the COP of the irreversible machine is higher than that of the reversible machine, viz., $\mathcal{E}_I > \mathcal{E}_R$. Hence

$$\frac{\dot{Q}_o}{W_I} > \frac{\dot{Q}_o}{W_R}$$

$$W_R > W_1$$

And since

$$Q_{k,R} = Q_o + W_R$$

$$Q_{k,I} = Q_o + W_I$$

we have

$$Q_{k,R} > Q_{k,I}$$

and

$$Q_{k,R} - Q_{k,I} = W_R - W_I = W_{\text{net}}$$

If now, the reversible refrigerating machine is made to work as a heat engine and the irreversible refrigerating machine continues to work as a refrigerating machine, as shown in Fig. 2.9(b), the resultant combined system will work as a perpetual motion machine of the second kind taking heat equal to $Q_{k,R} - Q_{k,I}$ from the hot reservoir and converting it completely into work, thus violating the Kelvin-Planck statement of the Second Law applicable to heat engines as shown in Fig. 2.10.

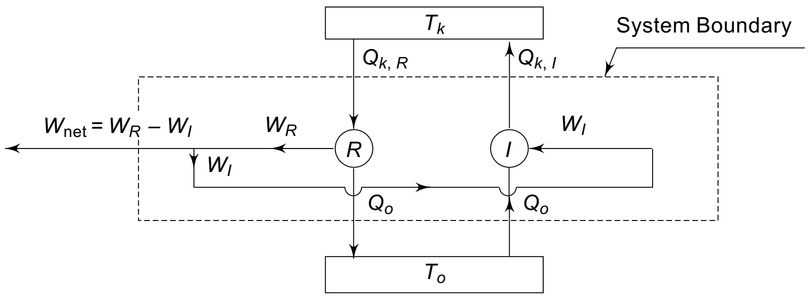

Fig. 2.9(b) Reversible refrigerating machine working as a heat engine in combination with on irreversible refrigerating machine

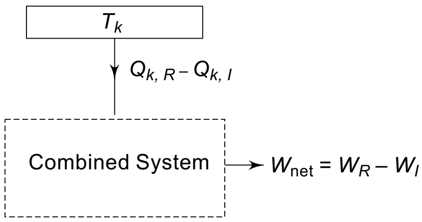

Fig. 2.10 Combined system resulting in a perpetual motion machine of the second kind thus violating the Second Law

It is, therefore, concluded that a refrigeration cycle operating reversibly between two heat reservoirs has the highest coefficient of performance. Likewise, it can also be shown that all reversible refrigeration cycles have the same COP. These are two corollaries of Second Law comprising the *Carnot Principle*.

2.4.1 Reversed Carnot Cycle

We now know that a reversible refrigeration cycle has the maximum COP. We know further that a reversible heat engine can be reversed in operation to work as a refrigerating machine.

Sadi Carnot, in 1824, proposed a reversible heat-engine cycle as a measure of maximum possible conversion of heat into work. A reversed Carnot cycle can, therefore, be employed as a reversible refrigeration cycle, which would be a measure of maximum possi-

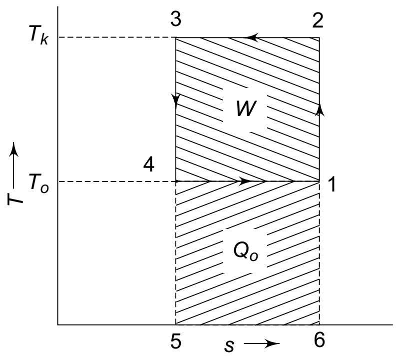

Fig. 2.11 Reversed Carnot Cycle

ble COP of a refrigerating machine operating between two temperatures T_k of heat rejection and T_o of refrigeration.

A reversed Carnot cycle, for a unit mass of the working substance, is shown in Fig. 2.11 on a T–s diagram. The cycle consists of two isothermals and two isentropics as follows:

Process 1—2	Isentropic compression, $s_1 = s_2$
Process 2—3	Isothermal heat rejection to the hot reservoir at T_k = const.
Process 3—4	Isentropic expansion, $s_3 = s_4$
Process 4—1	Isothermal heat absorption from the cold reservoir at T_o = const.

The areas on the T-s diagram, representing $\int Tds$ give the heat transfers, and work done in the cycle as follows:

Heat absorbed from cold body,	$Q_o = T_o \, \Delta s$	= area 1-4-5-6
Heat rejected to hot body,	$Q_k = T_k \, \Delta s$	= area 2-3-5-6
Work done,	$W = Q_k - Q_o = (T_k - T_o) \, \Delta s$	= area 1-2-3-4

Hence we obtain Carnot values of COP for cooling and heating as

$$\mathcal{E}_{c,\text{ Carnot}} = \frac{\dot{Q}_o}{W} = \frac{T_o}{T_k - T_o} = \frac{1}{\dfrac{T_k}{T_o} - 1}$$

$$\mathcal{E}_{h,\text{ Carnot}} = \frac{\dot{Q}_k}{W} = \frac{T_k}{T_k - T_o} = \frac{1}{1 - T_o/T_k}$$

Effect of Operating Temperatures We, thus, see that the Carnot COP depends on the operating temperatures T_k and T_o only. It does not depend on the working substance (refrigerant) used.

For cooling, T_o is the refrigeration temperature and T_k is the temperature of heat rejection to the surroundings. The lowest possible refrigeration temperature is $T_o = 0$ (absolute zero) at which $\mathcal{E}_c = 0$. The highest possible refrigeration temperature is $T_o = T_k$, i.e., when the refrigeration temperature is equal to the temperature of the surroundings (ambient) at which $\mathcal{E}_c = \infty$. Thus, Carnot COP for cooling varies between 0 and ∞.

For heating, T_o is the temperature of heat absorption from the surroundings and T_k is the heating temperature. Theoretically, the COP for heating varies between 1 and ∞.

It may, therefore, be noted that to obtain maximum possible COP in any application,

(i) the cold body temperature T_o should be as high as possible, and
(ii) the hot body temperature T_k should be as low as possible.

The lower the refrigeration temperature required, and higher the temperature of heat rejection to the surroundings, the larger is the power consumption of the refrigerating machine. Also, the lower is the refrigeration temperature required, the lower is the refrigerating capacity obtained.

Consider now, for example, a domestic refrigerator which produces refrigeration at –25°C (248 K). Let heat be rejected to the ambient at 60°C (333 K).

The maximum possible COP of this refrigerator would be

$$(COP_c)_{max} = \frac{248}{333 - 248} = \frac{248}{85} = 2.9$$

Thus, the refrigerator would produce a maximum of 290 W of refrigeration per 100 W of power consumption. The most popular size, 165 L internal volume refrigerators of most manufacturers produce 89 W of refrigeration, and the power of the electric motors running these refrigerators is around 110 W. Considering that the refrigerators have a running time of 75% only, the average power consumption would come to 82.5 W. Thus, the actual COP of these machines would be only 1.08.

Compare this with the COP of a room air conditioner which produces refrigeration at a higher temperature of about 5°C (278 K). Assuming the temperature of heat rejection the same as 60°C (333 K), the maximum possible COP would be

$$(COP_c)_{max} = \frac{278}{333 - 278} = \frac{278}{55} = 5$$

It would, thus, produce 5 kW of refrigeration per kW of power consumption. A 1.5 TR (5.3 kW) air conditioner will have a minimum power consumption of 1.05 kW. The actual air conditioner will, however, have power consumption of the order of 2.0 kW or more considering 75% running time. The actual COP is thus 2.6 or even less.

The above two examples show that the COP of a refrigeration system decreases, and power consumption increases as we go to lower and lower refrigeration temperatures.

2.4.2 Selection of Operating Temperatures

The selection of temperature T_0 depends on the particular application of refrigeration. Consider, for example, the simple summer air-conditioning system shown in Fig. 2.12(a). The room is maintained at temperature t_i equal to 25°C.

To offset the heat entering the room, the air must be supplied at a temperature lower than 25°C, at t_s equal to 15°C.

The air conditioner will, therefore, cool the room air from 25°C to 15°C and then supply it back to the room.

Accordingly, the refrigerant temperature t_o must be less than 15°C to absorb heat Q_o from the air maintaining a finite temperature difference Δt across the heat exchanger as shown in Fig. 2.12(b). If the temperature difference is zero, the *area requirement* of the heat exchanger will be infinite. Thus for air conditioning in summer, the temperature t_o is of the order of 0 to 10°C usually about 5°C. In a similar manner, the approximate refrigeration temperature requirements can be found out for various other applications as given in Table 2.1.

Table 2.1 Refrigeration temperature requirements of common applications

No.	*Application*	*Refrigeration Temperature, t_o, °C*
1.	Air conditioning in summer	0 to 10
2.	Cold storages	–10 to 2
3.	Domestic refrigerators	–25
4.	Frozen foods	–35
5.	Freeze drying and IQF (Instant Quick Freezing)	–35 to –45

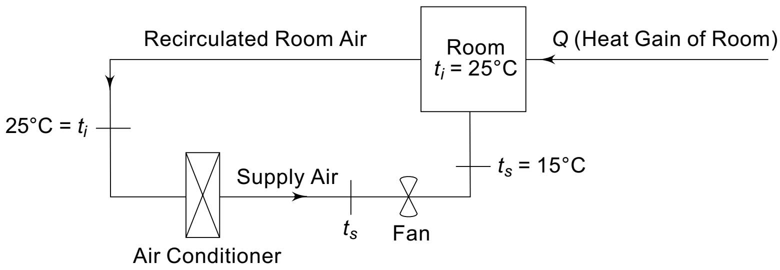

Fig. 2.12(a) Simple summer air-conditioning system

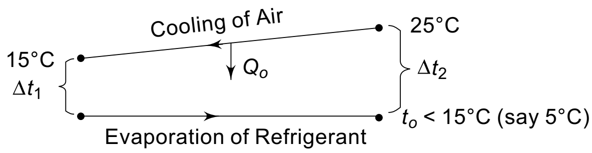

Fig. 2.12(b) Temperatures in a cooling coil

The selection of temperature T_k depends on the surrounding medium used for heat rejection. There are three possible media in the surroundings to which heat Q_k may be rejected, viz., air, water and ground. The units that use air as a cooling medium are called the *air-cooled units* and those using water are called the *water-cooled units*.

Consider an air-cooled unit. Let the surrounding air temperature (in summer) be 45°C. Also, let the rise in temperature of air, after absorbing heat Q_k, be 10°C, as shown in Fig. 2.13(a). Hence the temperature of heat rejection t_k has to be greater than 55°C, say 65°C, so that the temperature differentials Δt_1 and Δt_2, across the heat exchanger (HE) are 20°C and 10°C respectively, and the arithmetic mean temperature difference is (20 + 10)/2 = 15°C.

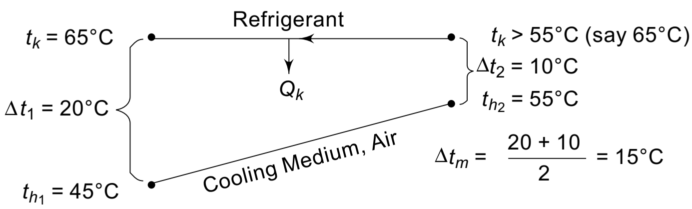

Fig. 2.13(a) Temperatures during heat rejection in an air-cooled refrigerating machine

Air is widely used as a cooling medium in small refrigerating machines such as refrigerators, water coolers, window-type air conditioners and small-package units.

Water as a cooling medium is preferable to air as it affords a lower value of T_k because of the following reasons:

(i) It is available at a temperature lower than that of air. Its temperature approaches the wet bulb temperature of the surrounding air. This is the limiting temperature to which heated water can be cooled in a cooling tower or a spray pond.

(ii) The specific heat of water is about four times that of air. Thus, for the same heat rejection Q_k and the same mass flow, the temperature rise of water is one-fourth that of air and correspondingly T_k is lower.

(iii) Water has a higher heat transfer coefficient than air mainly because of its high thermal conductivity, say, 5000 as against 100 in forced convection and 10 in free convection for air in $Wm^{-2}\ K^{-1}$. Thus, for the same heat rejected Q_k and the same area of the heat exchanger, the temperature difference Δt required across the heat exchanger is less which also results in a lower value of T_k.

Note *The lower value of heat transfer coefficient of air is made up by providing extended surface on air-side in actual equipment.*

Consider, for example, a water-cooled unit, in place of the air-cooled unit of the preceding example. Let the wet bulb temperature of the surrounding air be 28°C (even though the dry bulb temperature may be 45°C). Then the temperature of water from the cooling tower may be taken to be about 30°C. With the same mass flow for air and water, the temperature rise of water as a result of its higher specific heat will be 10/4 = 2.5°C only, though taken as 5°C in design as shown in Fig. 2.13 (b). The temperature of water leaving the heat exchanger is then 35°C.

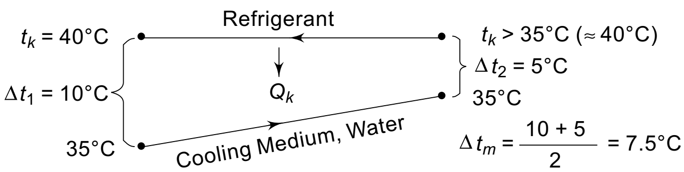

Fig. 2.13(b) Temperatures during heat rejection in a water-cooled refrigerating machine

Let the mean temperature difference required with water be 7.5°C instead of 15°C for air as a consequence of the higher heat transfer coefficient. Then the temperature of heat rejection required need only be 40°C for water as compared to 65°C for air. Normal design temperature T_k in Delhi is 43°C .

Thus, the use of water as a cooling medium results in a lower T_k, higher COP and lower power consumption in a refrigeration plant. At the same time, we have much more compact and smaller condenser even with a smaller value of Δt_m (7.5°C with water as against 15°C with air in the above illustration).

Large refrigeration plants, including central air-conditioning plants, therefore, are invariably water-cooled as the saving in the costs of power consumption exceeds the added cost of a water-cooling plant (cooling tower, pump, piping, etc.). Examples 2.1 and 2.2 amply illustrate the advantages of water over air, as a cooling medium. Although, the calculations are based on the reversed Carnot cycle for the working substance, yet finite temperature differentials have been assumed in the heat exchangers as external irreversibilities.

The ground is also sometimes used as a heat sink, specially in locations where extreme high and low temperatures are reached and the ground temperatures are lower in summer and higher in winter as compared to the surrounding temperatures.

Example 2.1

(a) The ambient air temperatures during summer and winter in a particular locality are 45°C and 15°C respectively. Find the values of Carnot COP for an air conditioner for cooling and heating, corresponding to refrigeration temperatures of 5°C for summer and heating temperature of 55°C for winter. Assume suitable temperature differences in the exchanger that exchanges heat with the surroundings.

(b) If water from the cooling tower at 30°C is used as a cooling medium with 3°C temperature differential for air-conditioning in summer, what will be the Carnot COP for cooling?

(c) Also, find the theoretical power consumption per ton of refrigeration in each case. Assume no increase in the temperature of the surrounding air or water.

Solution

(a) *Air as a cooling medium*

For summer: $t_k = 45 + 15 = 60°C$ (Assuming 15°C temperature difference for air-cooled)

$t_o = 5°C$

$$\mathcal{E}_c, \text{Carnot} = \frac{273 + 5}{60 - 5} = \frac{278}{55} = 5.05$$

For winter: $t_k = 55°C$

$t_o = 15 - 5 = 10°C$ (Assuming 5°C temp. difference)

$$\mathcal{E}_h, \text{Carnot} = \frac{273 + 55}{55 - 10} = \frac{328}{45} = 7.3$$

(b) *Water as a cooling medium*

$t_k = 30 + 10 = 40°C$ (Assuming 10°C temp. difference for water-cooled)

$t_o = 5°C$

$$\mathcal{E}_c, \text{Carnot} = \frac{273 + 5}{40 - 5} = \frac{278}{35} = 7.94$$

(c) *Power consumption*

$$\dot{W} = \dot{Q}_o / \mathcal{E}_c$$

For the air-cooled unit, $\dot{W} = 211/5.05 = 41.8$ kJ/min = 0.7 kW

For the water-cooled unit, $\dot{W} = 211/7.94 = 26.6$ kJ/min = 0.44 kW

Note *Saving in power with water-cooled is 37%. In actual practice, it is much more.*

Example 2.2

(*a*) *A reversed Carnot cycle air conditioner of 1 TR capacity operates with cooling coil temperature* $t_o = 5°C$. *The surrounding air at 43°C is used as a cooling medium rising to a temperature of 53°C. The temperature of heat rejection is* $t_k = 55°C$. *The overall heat transfer coefficient of the heat exchanger between the working substance and the surrounding air is* $U = 250$ *W.* m^{-2} K^{-1}. *Determine the mass flow rate of the surrounding air entering the heat exchanger, area of the heat exchanger, COP and power consumption of the air conditioner.*

(*b*) *If water at 30°C is used as a cooling medium in the same heat exchanger, will its area be adequate or inadequate for necessary heat rejection? Assume that the mass flow rate of water remains the same as that of air. Also for water,* $U = 2500$ *W.* m^{-2} K^{-1}.

(*c*) *If the area is reduced to 1* m^2, *what will be the temperature of heat rejection* t_k, *temperature rise of water, COP, power consumption, percentage saving in power consumption and percentage reduction in the heat-exchanger area as compared with air as a cooling medium.*

Solution

(a) *Air as a cooling medium* Referring to Fig. 2.13 (a), the various heat exchanger temperatures in this case are as follows:

$$t_{h_1} = 43°C,\ t_{h_2} = 53°C,\ t_k = 55°C$$

$$\text{COP} = \frac{273 + 5}{55 - 5} = \frac{278}{50} = 5.56$$

Power consumption

$$\dot{W} = \frac{\dot{Q}_o}{\mathcal{E}_c} = \frac{3.5167}{5.56} = 0.63 \text{ kW}$$

Heat rejected

$$\dot{Q}_k = \dot{Q}_o + \dot{W} = 3.5167 + 0.63 = 4.15 \text{ kW}$$

Mass flow rate of cooling air

$$\dot{m}_a = \frac{\dot{Q}_k}{C_p\,\Delta t_h} = \frac{4.15}{1.005(53 - 43)} = 0.413 \text{ kg/s}$$

LMTD in the heat exchanger

$$\Delta t_m = \frac{\Delta t_1 - \Delta t_2}{\ln \dfrac{\Delta t_1}{\Delta t_2}} = \frac{(55 - 43) - (55 - 53)}{\ln\left(\dfrac{55 - 43}{55 - 53}\right)} = \frac{10}{\ln 6} = 5.58°C$$

Area of the heat exchanger

$$A = \frac{\dot{Q}_k}{U\,\Delta t_m} = \frac{4.15 \times 10^3}{250(5.58)} = 2.975 \text{ m}^2$$

(b) *Water as a cooling medium*

First approximation: Let $t_k = 35°C$

$$\text{Then COP} = \frac{273 + 5}{35 - 5} = 9.27$$

$$\dot{Q}_k = \dot{Q}_o + \dot{W} = \dot{Q}_o + \frac{\dot{Q}_o}{\mathcal{E}_c} = \dot{Q}_o\left(1 + \frac{1}{\mathcal{E}_c}\right)$$

$$= 3.5167\left(1 + \frac{1}{9.27}\right) = 3.9 \text{ kW}$$

$$\Delta t_h = \frac{\dot{Q}_k}{\dot{m}_w C_p} = \frac{3.9}{0.413\ (4.1868)} = 2.26°C$$

$$t_{h_2} = 30 + 2.26 = 32.26°C$$

LMTD in the heat exchanger

$$\Delta t_m = \frac{(35 - 30) - (35 - 32.26)}{\ln \dfrac{35 - 30}{35 - 32.26}} = 3.76°C$$

$$A = \frac{\dot{Q}_k}{U\Delta t_m} = \frac{3.9}{2.5\ (3.76)} = 0.41 \text{ m}^2$$

Checking with the actual area of 2.975 m^2, it is seen that temperature t_k could be much lower.

Second approximation of $t_k = 33°C$ gives $A = 0.95$ m^2.

The area required is still one-fourth of the available area of 2.975 m^2. Thus, even if t_k is taken equal to $t_{h_2} = 32.24$ so that $\Delta t_2 = 0$, then $\Delta t_1 = 32.24 - 30 = 2.24°C$, and $\Delta t_m = 1.12°C$, it gives an area requirement of only (1.63/1/12) (0.95) = 1.38 m^2 which is still less than 2.975 m^2.

Hence the area of the heat exchanger is more than adequate with water as a cooling medium and can be reduced to at least one-third.

(c) *Water as a cooling medium* ($A = 1$ m^2)

By iteration, we find the solution, viz.,

$$t_k = 32.94°C$$

$$\text{COP} = \frac{273 + 5}{32.94 - 5} = 9.95$$

$$\dot{Q}_k = 3.5167\left(1 + \frac{1}{9.76}\right) = 3.87 \text{ kW}$$

$$\Delta t_h = \frac{3.87}{0.413\,(4.1868)} = 2.24°C$$

$$t_{h_2} = 32.24°C$$

$$\Delta t_m = \frac{(32.9 - 30) - (32.9 - 32.24)}{\ln \dfrac{32.9 - 30}{32.9 - 32.24}} = 1.5°\text{C}$$

$$A = \frac{3.87}{2.5\,(1.5)} = 1.0\ \text{m}^2$$

$$\text{Power consumption, } \dot{W} = \frac{3.5167}{9.95} = 0.35\ \text{kW}$$

$$\text{Saving in power consumption} = \frac{0.63 - 0.35}{0.63} \times 100 = 44.4\%$$

Saving in area of the heat exchanger

$$= \frac{2.975 - 1}{2.975} \times 100 = 66\%$$

Note *Indiscriminate use of air-cooled window-type air conditioners is wasteful of energy and equipment. Insted of using large number of window units in a big building, it is much more desirable to install a* ***central air conditioning*** *plant which is always water-cooled. This will save power and reduce cost. It will also minimise thermal pollution of the environment by diminishing* $\dot{W}$ *and* $\dot{Q}_k$.

2.5 VAPOUR AS A REFRIGERANT IN REVERSED CARNOT CYCLE

The reversed Carnot cycle can be made almost completely practical by operating in the liquid-vapour region of a pure substance as shown in Fig. 2.14.

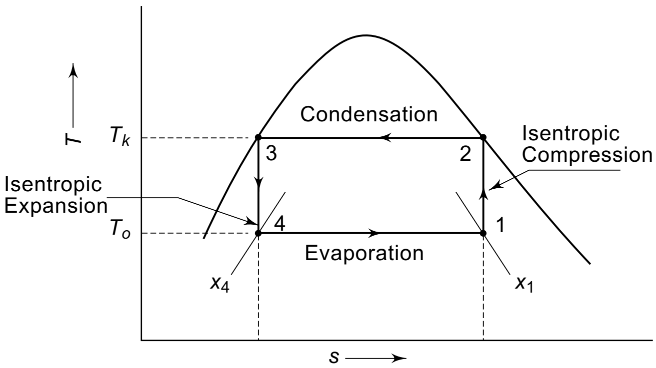

Fig. 2.14 Reversed Carnot cycle with vapour as a refrigeran

The isothermal processes of heat rejection (2-3) and heat absorption (4-1) of the Carnot cycle are achieved by making use of the phenomena of condensation and evaporation of a pure substance at constant pressure and temperature. This alternate condensation and evaporation of a working substance is accompanied by alternate isentropic compression (1-2) and expansion (3-4) processes.