$$\text{Work done, } w_{4-1} = p_4 v_4 \ln \frac{v_1}{v_4} = RT_o \ln \frac{v_1}{v_4} \tag{2.9}$$

$$\text{Refrigerating effect, } q_o = q_{4-1} = w_{4-1} \text{ (for a perfect gas)}$$

Net work of the cycle.

$$w = |w_{2-3}| - w_{4-1} = RT_k \ln \frac{v_2}{v_3} - RT_o \ln \frac{v_1}{v_4} \tag{2.10}$$

Refrigerating effect, $\quad q_o = q_{4-1} = RT_o \ln \dfrac{v_1}{v_4}$ (2.11)

Now for the isentropic processes 1-2 and 3-4

$$\frac{T_k}{T_o} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{p_3}{p_4}\right)^{\frac{\gamma-1}{\gamma}}$$

and

$$\frac{T_k}{T_o} = \left(\frac{v_1}{v_2}\right)^{\gamma-1} = \left(\frac{v_4}{v_3}\right)^{\gamma-1} = r^{\gamma-1}$$

where r is the compression ratio for the isentropic processes.

Hence $\qquad \dfrac{p_2}{p_1} = \dfrac{p_3}{p_4}$, and $\quad r = \dfrac{v_1}{v_2} = \dfrac{v_4}{v_3}$

or $\qquad \dfrac{v_2}{v_3} = \dfrac{v_1}{v_4} = \dfrac{p_3}{p_2} = \dfrac{p_4}{p_1}$

since $\qquad p_2 v_2 = p_3 v_3$ and $p_4 v_4 = p_1 v_1$

We have, for the COP for cooling

$$\mathcal{E}_c = \frac{q_o}{w} = \frac{RT_o \ln \dfrac{v_1}{v_4}}{RT_k \ln \dfrac{v_2}{v_3} - RT_o \ln \dfrac{v_1}{v_4}}$$

$$= \frac{T_o}{T_k - T_o}$$

$$= \frac{1}{\dfrac{T_k}{T_o} - 1} = \frac{1}{r^{\gamma-1} - 1} \tag{2.12}$$

Thus COP is a function of compression ratio only.

2.7 LIMITATIONS OF REVERSED CARNOT CYCLE

It is found that serious practical difficulties are encountered in the application of Carnot cycle.

In the reversed Carnot cycle with vapour as refrigerant, the isothermal processes of condensation and evaporation are internally reversible processes, and they are easily achievable in practice although there may be some problem in having only partial evaporation. However, isentropic compression and expansion processes have some limitations which are discussed in Chap. 3. In brief, it is difficult to design an expander to handle a mixture of largely liquid and partly vapour for the process 3-4. Also, because of the internal irreversibilities in the compressor and the expander, the actual COP of the Carnot cycle is very low, though the ideal cycle COP is the maximum. A cycle which is closest to the reversed Carnot vapour cycle is the vapour compression cycle described in Chap. 3.

There are two drawbacks of reversed Carnot cycle with gas as a refrigerant:

(i) Firstly, it is not possible to devise, in practice, isothermal processes of heat absorption and rejection, 4-1 and 2-3 in Fig. 2.15 with gas as the working substance. These are impractical as these will be infinitely slow.

(ii) Secondly, the cycle on *p-v* diagram is very narrow since the volume is changing both during the reversible isothermal and reversible adiabatic processes. Drawn correctly to scale, the Carnot *p-v* diagram is much thinner than the diagram illustrated in Fig. 2.15. As a result, the stroke volume of the cylinder is very large. The cycle, therefore, suffers from poor actual COP as a result of irreversibilities of the compressor and expander.

A gas refrigeration cycle, which is closest to reversed Carnot cycle with gas as a refrigerant, is described in Chap. 11.

2.8 ACTUAL REFRIGERATION SYSTEMS

Although the Carnot cycle is theoretically the most efficient cycle between given temperatures T_k and T_o, it has limitations for practical use. It is, therefore, found useful only as a criterion of perfection of cycle. In an actual cycle, the COPs, $\mathcal{E}_c$ and $\mathcal{E}_h$, will be less than their Carnot values. For the purpose of comparison between the actual and Carnot values, we define the *second law efficiency* or *exergetic efficiency* for cooling and heating, $(\eta_{\text{II}})_c$ and $(\eta_{\text{II}})_h$ as below:

$$(\eta_{\text{II}})_c = \frac{\mathcal{E}_c}{\mathcal{E}_{c,\,\text{Carnot}}}$$

$$(\eta_{\text{II}})_h = \frac{\mathcal{E}_h}{\mathcal{E}_{h,\,\text{Carnot}}}$$

Note that $\mathcal{E}_c$ and $\mathcal{E}_h$ are the first law COPs.

The conventional refrigeration systems work on the *vapour compression cycle* which is closest to the Carnot vapour cycle and has a high COP. *Gas cycle refrigeration* is used in aircraft refrigeration. Among the less conventional ones are the heat-operated refrigerating machines working on the *vapour absorption cycle* and *steam ejector cycle*.

There are also the low temperature refrigeration or cryogenic cycles, e.g., Linde cycle, Claude cycle, etc., used for the liquefaction of gases. Also, we have Philips liquefier which employs a cycle approaching the reversible Stirling cycle.