Enthalpies at 1 and 4

$$h_1 = 8.9 + 0.951\ (174.2 - 8.9) = 166 \text{ kJ/kg}$$

$$h_4 = 8.9 + 0.322\ (174.2 - 8.9) = 62.1 \text{ kJ/kg}$$

Work of compression, $w_C = h_2 - h_1 = 201.5 - 166 = 35.5$ kJ/kg

Work of expansion, $w_E = h_3 - h_4 = 69.5 - 62.1 = 7.4$ kJ/kg

Refrigerating effect, $q_o = h_1 - h_4 = 166 - 62.1 = 103.9$ kJ/kg

Heat rejected, $q_k = h_2 - h_3 = 201.5 - 69.5 = 132$ kJ/kg

Net work, $w = w_C - w_E = 35.5 - 7.4 = 28.1$ kJ/kg

$= q_k - q_o = 132 - 103.9 = 28.1$ kJ/kg

COP of the cycle, $\mathcal{E}_c = \dfrac{q_o}{w} = \dfrac{103.9}{28.1} = 3.74$

Alternatively, we have for Carnot COP

$$(\mathcal{E}_c)\ \text{Carnot} = \frac{T_o}{T_k - T_o} = \frac{273 - 30}{35 - (-30)} = 3.74$$

(b) Actual COP, $\mathcal{E}_c = 0.75 \times 3.74 = 2.8$

Power consumption per ton, $\dot{W} = \dfrac{\dot{Q}_o}{\mathcal{E}_c} = \dfrac{3.5167}{2.8} = 1.256$ kW

Heat rejected per ton, $\dot{Q}_k = \dot{Q}_o + \dot{W} = 3.516 + 1.256 = 4.722$ kW

2.6 GAS AS A REFRIGERANT IN REVERSED CARNOT CYCLE

Figure 2.15 shows the Carnot cycle 1-2-3-4 with gas as a refrigerant, illustrated on *T-s* and *p-v* diagrams.

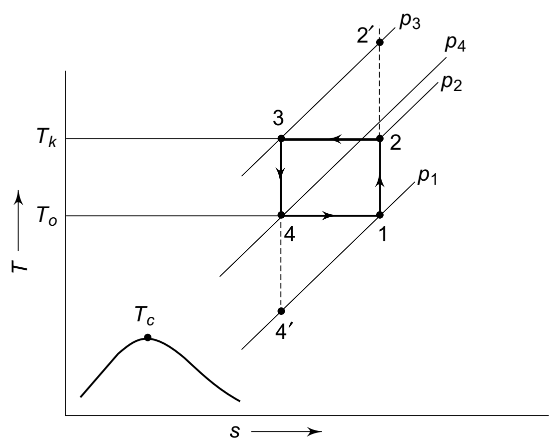

Fig. 2.15(a) Reversed Carnot cycle with gas as a refrigerant on *T-s* diagram

The four processes of the cycle are analysed as non-flow processes as follows in Eqs. (2.6), (2.7), (2.8) and (2.9).

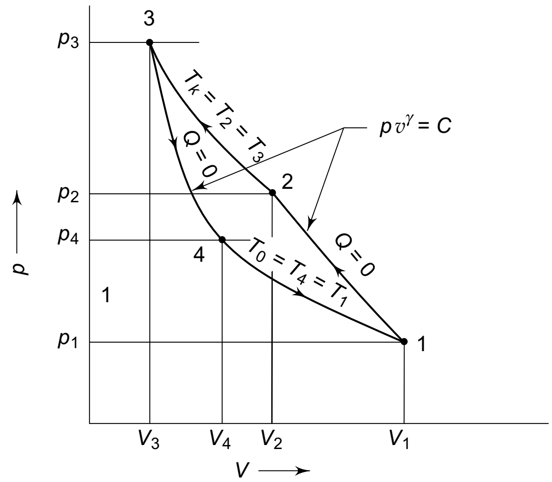

Fig. 2.15(b) Reversed Carnot cycle with gas as a refrigerant on *p-υ* diagram

1–2 Isentropic compression: $Q = 0$
Pressure increases from p_1 to p_2
Specific volume reduces from v_1 to v_2
Temperature increases from $T_o = T_1$ to $T_k = T_2$

$$\text{Work done, } |w_{1-2}| = \frac{p_2 v_2 - p_1 v_1}{\gamma - 1} = \frac{R(T_k - T_o)}{\gamma - 1} \tag{2.6}$$

2–3 Isothermal compression and heat rejection: $T_2 = T_3 = T_k$
Pressure increases from p_2 to p_3
Specific volume reduces from v_2 to v_3

$$\text{Work done, } |w_{2-3}| = p_2 v_2 \ln \frac{v_2}{v_3} = RT_k \ln \frac{v_2}{v_3} \tag{2.7}$$

Heat rejected, $q_k = q_{2-3} = w_{2-3}$ (for a perfect gas in an isothermal process)

3–4 Isentropic expansion: $Q = 0$
Pressure falls from p_3 to p_4
Specific volume increases from v_3 to v_4
Temperature decreases from $T_k = T_3$ to $T_o = T_4$

$$\text{Work done, } w_{3-4} = \frac{p_3 v_3 - p_4 v_4}{\gamma - 1} = \frac{R(T_k - T_o)}{\gamma - 1} \tag{2.8}$$

4–1 Isothermal expansion and heat absorption: $T_4 = T_1 = T_o$
Pressure falls from p_4 to p_1
Specific volume increases from v_4 to v_1

$$\text{Work done, } w_{4-1} = p_4 v_4 \ln \frac{v_1}{v_4} = RT_o \ln \frac{v_1}{v_4} \tag{2.9}$$

$$\text{Refrigerating effect, } q_o = q_{4-1} = w_{4-1} \text{ (for a perfect gas)}$$

Net work of the cycle.

$$w = |w_{2-3}| - w_{4-1} = RT_k \ln \frac{v_2}{v_3} - RT_o \ln \frac{v_1}{v_4} \tag{2.10}$$

Refrigerating effect, $\quad q_o = q_{4-1} = RT_o \ln \dfrac{v_1}{v_4}$ (2.11)

Now for the isentropic processes 1-2 and 3-4

$$\frac{T_k}{T_o} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{p_3}{p_4}\right)^{\frac{\gamma-1}{\gamma}}$$

and

$$\frac{T_k}{T_o} = \left(\frac{v_1}{v_2}\right)^{\gamma-1} = \left(\frac{v_4}{v_3}\right)^{\gamma-1} = r^{\gamma-1}$$

where r is the compression ratio for the isentropic processes.

Hence $\qquad \dfrac{p_2}{p_1} = \dfrac{p_3}{p_4}$, and $r = \dfrac{v_1}{v_2} = \dfrac{v_4}{v_3}$

or $\qquad \dfrac{v_2}{v_3} = \dfrac{v_1}{v_4} = \dfrac{p_3}{p_2} = \dfrac{p_4}{p_1}$

since $\qquad p_2 v_2 = p_3 v_3$ and $p_4 v_4 = p_1 v_1$

We have, for the COP for cooling

$$\mathcal{E}_c = \frac{q_o}{w} = \frac{RT_o \ln \dfrac{v_1}{v_4}}{RT_k \ln \dfrac{v_2}{v_3} - RT_o \ln \dfrac{v_1}{v_4}}$$

$$= \frac{T_o}{T_k - T_o}$$

$$= \frac{1}{\dfrac{T_k}{T_o} - 1} = \frac{1}{r^{\gamma-1} - 1} \tag{2.12}$$

Thus COP is a function of compression ratio only.

2.7 LIMITATIONS OF REVERSED CARNOT CYCLE

It is found that serious practical difficulties are encountered in the application of Carnot cycle.