CS6462

*Probabilistic and Explainable AI*

# Lesson 9

# *First-Order Probabilistic Programming Language (FOPPL)*

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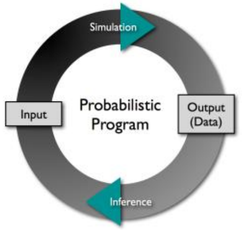
# FOPPL Features



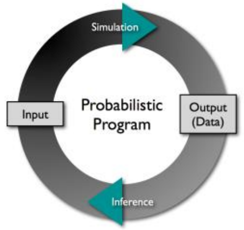

*First-Order Probabilistic Programming Language**:

- includes most common features of programming languages;
- conditional statements, e.g., ***if***
- primitive operations, e.g.,+,-,*,/
- user-defined functions:
  - must be first order, i.e., functions cannot accept other functions as arguments
  - cannot be recursive

- FOPPL programs are models describing distributions over a finite number of random variables
- compile any program written in FOPPL to a data structure that represents a graphical model

** "An Introduction to Probabilistic Programming", book by J.W. van de Meent, B. Paige, H. Yang, F. Wood]*

# FOPPL Syntax



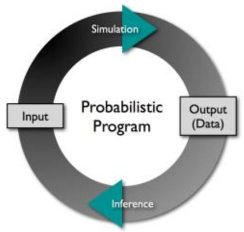

*Grammar*:

$$
\begin{aligned}
v &::= \text{variable} \\
c &::= \text{constant value or primitive operation} \\
f &::= \text{procedure} \\
e &::= c \mid v \mid (\texttt{let}\ [v\ e_1]\ e_2) \mid (\texttt{if}\ e_1\ e_2\ e_3) \\
&\quad \mid (f\ e_1\ \ldots\ e_n) \mid (c\ e_1\ \ldots\ e_n) \\
&\quad \mid (\texttt{sample}\ e) \mid (\texttt{observe}\ e_1\ e_2) \\
q &::= e \mid (\texttt{defn}\ f\ [v_1\ \ldots\ v_n]\ e)\ q
\end{aligned}
$$

- FOPPL is a Lisp variant that is based on Clojure*
- two sets of production rules: for expressions ***e*** and for programs ***q***
- rules for ***q****:* a program can either be a single ***e***, or ***(defn f . . .)*** followed by ***q***

Hickey, R. (2008), 'The Clojure Programming Language'. In: *Proceedings of the 2008 Symposium on Dynamic Languages. New York, NY, USA, pp. 1:1–1:1,* ACM.

# FOPPL Expressions



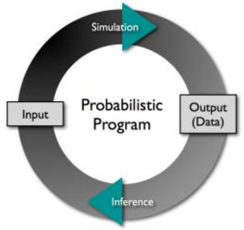

$$
\begin{aligned}
e ::= \; & c \mid v \mid (\texttt{let}\ [v\ e_1]\ e_2) \mid (\texttt{if}\ e_1\ e_2\ e_3) \\
\mid \; & (f\ e_1\ \ldots\ e_n) \mid (c\ e_1\ \ldots\ e_n) \\
\mid \; & (\texttt{sample}\ e) \mid (\texttt{observe}\ e_1\ e_2)
\end{aligned}
$$

- constant ***c****:* a value of a primitive data type such as a *number*, *string*, or *Boolean*, a built-in primitive function (e.g., ***+***), or a value of any other data type that can be constructed using primitive procedures (lists, vectors, hash-maps, and distributions)

  *primitive operation example*: **(+ a b)** → *a + b (Python equivalent)*

- variable ***v***: a symbol that references the value of another expression in the program
- ***let*** form (***let [v*** $e_1$***]*** $e_2$) binds the value of the expression $e_1$ to the variable ***v,*** which can then be referenced in the expression $e_2$ (the body of the let expression)
- ***if*** form (***if*** $e_1$ $e_2$ $e_3$) takes the value of $e_2$ when the value of $e_1$ is logically true and the value of $e_3$ when $e_1$ is logically false

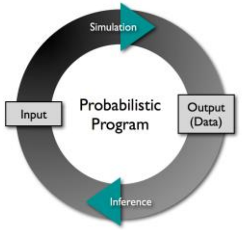
# FOPPL Expressions (cont.)




$$
\begin{aligned}
e ::= \; & c \mid v \mid (\texttt{let}\ [v\ e_1]\ e_2) \mid (\texttt{if}\ e_1\ e_2\ e_3) \\
& \mid (f\ e_1\ \ldots\ e_n) \mid (c\ e_1\ \ldots\ e_n) \\
& \mid (\texttt{sample}\ e) \mid (\texttt{observe}\ e_1\ e_2)
\end{aligned}
$$

- function application ***(f*** $\boldsymbol{e_1 \ldots e_n}$***)*** calls the user-defined function ***f*** with arguments $\boldsymbol{e_1}$ through $\boldsymbol{e_n}$
- primitive procedure applications ***(c*** $\boldsymbol{e_1 \ldots e_n}$***)*** calls a built-in function ***c,*** such as ***+***
- sample form ***(sample e)*** represents an unobserved random variable; it accepts a single expression ***e***, which must evaluate to a distribution object, and returns a value that is a sample from this distribution
- observe form ***(observe*** $\boldsymbol{e_1\ e_2}$***)*** represents an observed random variable; it accepts an argument $\boldsymbol{e_1}$, which must evaluate to a distribution that is a condition for the next argument $\boldsymbol{e_2}$ ,i.e., $\boldsymbol{P(e_2 \mid e_1)}$

# FOPPL Data Structures




*Vector and map data structures:*

- vectors: constructed by the expression ***(vector*** $\boldsymbol{e_1 \ldots e_n}$***)***

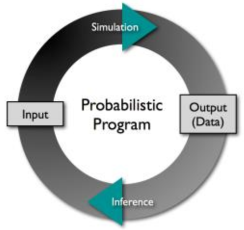
  Example: ***(vector 1 2 3 4 5 6)*** – vector defining the die sample space

- hash maps: constructed by the expression ***(hash-map*** $\boldsymbol{e_1\ e'_1 \ldots e_n\ e'_n}$***)*** - constructed from a sequence of key-value pairs $\boldsymbol{e_1\ e'_1}$

***Functions operating over data structures:***

- ***(first e) -*** retrieves the first element of a list or vector ***e***
- ***(rest e)****,* ***(last e)****,* ***(append*** $\boldsymbol{e_1\ e_2}$***)***
- ***(get*** $\boldsymbol{e_1\ e_2}$***) -*** retrieves an element at index $\boldsymbol{e_2}$ from a list or vector $\boldsymbol{e_1}$ , or the element at key $\boldsymbol{e_2}$ from a hash map $\boldsymbol{e_1}$
- ***(put e1 e2 e3)****,* ***(remove e1 e2)***

# FOPPL Loops



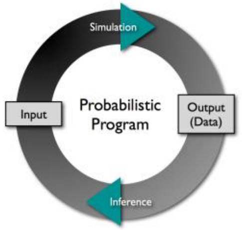

*For loop syntax:*

$$
\begin{array}{l}
\texttt{(foreach}\ c \\
\quad [v_1\ e_1\ \ldots\ v_n\ e_n] \\
\quad e'_1\ \ldots\ e'_k\texttt{)}
\end{array}
\Rightarrow
\begin{array}{l}
\texttt{(vector} \\
\quad \texttt{(let}\ [v_1\ \texttt{(get}\ e_1\ \texttt{0)}\ \ldots\ v_n\ \texttt{(get}\ e_n\ \texttt{0)}] \\
\qquad e'_1\ \ldots\ e'_k\texttt{)} \\
\quad \vdots \\
\quad \texttt{(let}\ [v_1\ \texttt{(get}\ e_1\ \texttt{(-}\ c\ \texttt{1))}\ \ldots\ v_n\ \texttt{(get}\ e_n\ \texttt{(-}\ c\ \texttt{1))}] \\
\qquad e'_1\ \ldots\ e'_k\texttt{))}
\end{array}
$$

*Interpretation:*

- ***c*** is a non-negative integer constant
- ***foreach*** form iterates incrementally c-times over the structure $[v_1\ e_1 \ldots v_n\ e_n]$ where it associates each variable $v_i$ with a vector $e_i$ and returns the values $v_i$ assigned with the c-index elements of vectors $e_i$ ; consecutively these values $v_i$ are passed to the $e_1$´ … $e_k$´ structure and can be referenced from there

# FOPPL Loops (cont.)




*For loop example:*

```
foreach 5
 [x (range 1 6)
  y y-values]
```

```
(let [y-values [2.1 3.9 5.3 7.7 10.2]
      slope (sample (normal 0.0 10.0))
      intercept (sample (normal 0.0 10.0))]
  (foreach 5
    [x (range 1 6)
     y y-values]
    (let [fx (+ (* slope x) intercept)]
      (observe (normal fx 1.0) y))))
```

*Interpretation:*

- ***c** = 5 (# of iterations)*
- iterations and vectors produced:

  ***c=0,  x=1, y=2.1***

  ***c=1,  x=2, y=3.9***

  ***c=2,  x=3, y=5.3***

  ***c=3,  x=4, y=7.7***

  ***c=4,  x=5, y=10.2***

# FOPPL Sampling

*Definition:*

```
(let [xi sample(d, e1, ... , en)])
```

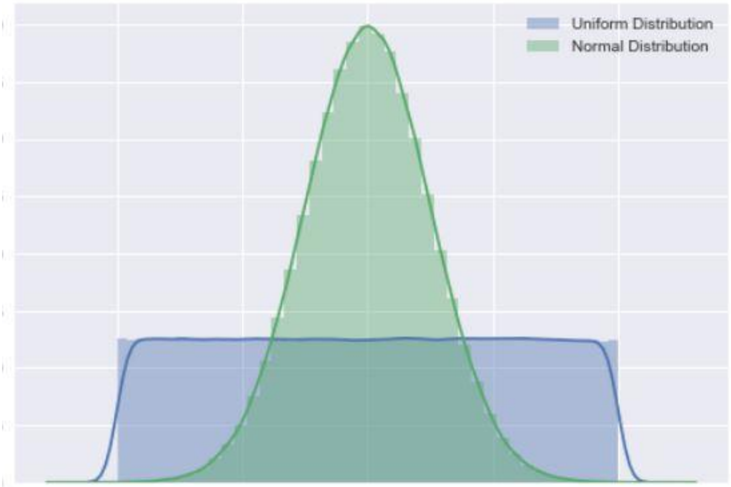
*Interpretation:*

- sample from the distribution ***d*** is taken and stored under the name ***xi***
- ***sample*** construct accepts a distribution object ***d***, which must evaluate to a distribution object and a set of expressions ***ei*** representing the distribution inputs
- distributions are constructed using primitives provided by the FOPPL

*Example*:

- ***x  Uniform(0, 1)*** is represented as ***(let [x sample(uniform 0 1)])***

  and returns a value that is a sample from this distribution object
- ***(let [x sample (normal 0.0 1.0)])***
- ***(let [x sample (poisson 10.0)])***



# FOPPL Observe Conditioning Statement

*Definition:*

```
observe( (d e1, ... , en)c)
```

*Interpretation:*

- 1) factors the density according to the distribution ***d***, with ***e1,..., en*** and the observed data ***c*** , and 2) represents an observed random variable
- accepts an argument ***d,*** which must evaluate to a distribution that provides the conditions on the next argument ***c***
- output: ***P(c|d)***

*Example*:

- **(observe (normal 0 1) 2) → P(2 | Normal(0,1))**
- **(observe (beta 1 5) 2) → P(2 | Beta(1,5))**

# FOPPL If Statement

*Definition:*

If statement***: if ( boolean expr.) expr1 expr2***

*Examples:*

```
(if (> mean 0)
        (observe (normal mean 1) y1)
        (observe (normal mean -1) y2))
```

*If (mean > 0)*
*(observe (normal mean 1) y1)*
*else*
*(observe (normal mean -1) y2)*

***(if (< x-0.5 0)***
***(observe (normal x 1.0) 2)***
***(observe (beta x 5) 2 )***
***)***

*If ((x-0.5) > 0)*
*(observe (normal x 1.0) 2)*
*else*
*(observe (Beta x 5) 2)*

# FOPPL Function Declaration

*Definition:*

- ***defn*** statement***: ( defn*** *name* ***[****args****] (****body****) )***
- takes a set of arguments and a set of expressions in the body to evaluate during the forward execution

*Example:*

```
(defn observe-data [slope intercept x y]
  (let [fx (+ (* slope x) intercept)]
    (observe (normal fx 1.0) y)))
```

```
(let [y1 (observe-data slope intercept 1.0 2.1)]
```

# FOPPL Example

*Example:* reasoning about the bias of a coin

```
(let [prior (beta a b)
      x (sample prior)
      likelihood (bernoulli x)
      y 1]
  (observe likelihood y)
  x)
```

*Interpretation:*

***prior (beta a b)*** → ***beta*** distribution over ***a*** and ***b*** is assigned to ***prior*** variable

***x (sample prior)*** → sample from the distribution ***prior*** is taken and stored in variable ***x***

***Likelihood (bernoulli x)*** → ***bernoulli*** distribution over ***x*** is assigned to ***likelihood*** variable

***y 1*** → ***1*** (heads) is assigned to ***y*** variable

***(observe likelihood y)*** → computes the posterior conditional probability ***p(y|likelihood)***

***x*** → x stores the value of ***y***

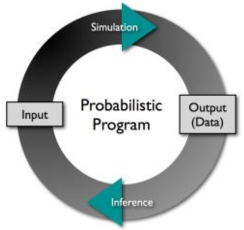
# FOPPL Example (cont.)

*Example:* Bayesian linear regression

```
(let [y-values [2.1 3.9 5.3 7.7 10.2]
      slope (sample (normal 0.0 10.0))
      intercept (sample (normal 0.0 10.0))]
  (foreach 5
    [x (range 1 6)
     y y-values]
    (let [fx (+ (* slope x) intercept)]
      (observe (normal fx 1.0) y)))

[slope intercept])
```

# Summary

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- primitive operations, e.g.,**+,-,*,/**
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  - must be first order, i.e., functions cannot accept other functions as arguments
  - cannot be recursive

- FOPPL programs are models describing distributions over a finite number of random variables

*Expectation:*

- you are expected to be able to <u>read</u>/<u>interpret</u> FOPPL examples

*Next Lesson* – Lesson: Graph-Based Inference

# Thank You!

Questions?