CS6462

*Probabilistic and Explainable AI*

# Lesson 3

# *Probability Multiplication*

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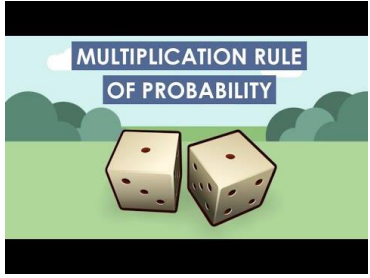
# Probability and Multiplication




*Definition:*

- a way to find the probability of two or more events happening at the same time

*Recall Conditional Probability*:

- the probability of ***E*** happening given that ***F*** has occurred ***(E|F)***

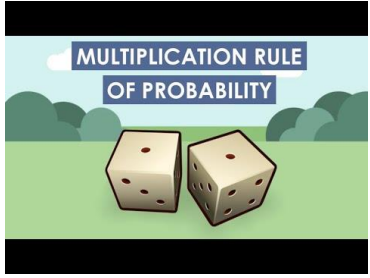
*Multiplication Rules:*

- general multiplication rule
  ***P(E ∩ F) = P(E|F) * P(F) = P(F|E) * P(E)***

$$P(E|F) = \frac{P(E \cap F)}{P(F)}, \text{ iff } P(F) > 0$$

- specific multiplication rule
  ***P(E ∩ F) = P(E) * P(F)*** – iff ***E*** and ***F*** are independent

# General Multiplication – Multiple Events




*General Multiplication Rule for Three and More Events:*

**The intersection of events determines when events occur together.**

- the probability of three dependent events ***F, E, D*** occurring together is:

  ***P(F and E and D) = P(F)*P(E given F)*P(D given E and F)***

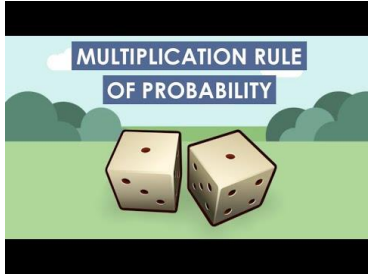
  ***P(F ∩ E ∩ D) = P(F) * P(E|F) * P(D|E|F)***

- the probability of four dependent events ***F, E, D, C*** occurring together is:

  ***P(F and E and D and C) = P(F)*P(E given F)*P(D given E and F) *P(C given D and E and F)***

  ***P(F ∩ E ∩ D ∩ C) = P(F) * P(E|F) * P(D|E|F) * P(C|D|E|F)***

# General Multiplication – Example




*Example:*

***X***: A bag contains 6 black marbles and 4 white marbles. Two marbles are drawn from the bag, without replacement. What is the probability that both marbles are white?

$S = \{b_1, b_2, b_3, b_4, b_5, b_6, w_1, w_2, w_3, w_4\}$

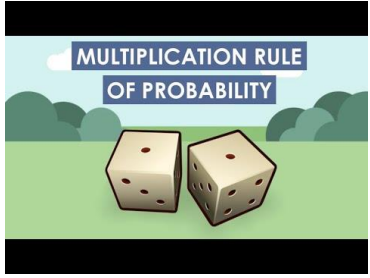
$P(S) = 1, P(b_1) = \ldots = P(b_6) = P(w_1) = \ldots = P(w_4) = 1/10$

$F = \{w\}$ - an event "the ball is white", $P(F) = 4/10$ (there are 10 marbles in the bag)

$E = \{w\}$ - an event "the ball is white", $P(E|F) = 3/9$ (there are 9 marbles in the bag)

$P(F \cap E) = P(E|F) * P(F) = 3/9 * 4/10 = 2/15 = 0.1333 =$ **13.33 %**

# General Multiplication – Example (cont.)




*Example:*

***X***: We have 52 cards. We want to find out the probability of getting *three jacks* consecutively if we don't return the drawn card each draw.

***S = {4*j, 48*c}*** – 4 jacks + 48 other cards

***P(S) = 1, P(j) = P(c) = 1/52***

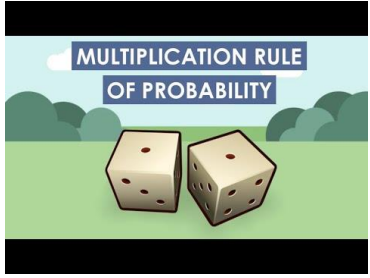
$E_1 = \{j\}$ - an event "the card is a jack", $P(E_1) = 4/52$ (there are 52 cards)

$E_2 = \{j\}$ - an event " the card is a jack", $P(E_2|E_1) = 3/51$ (there are 51 cards and 3 jacks)

$E_3 = \{j\}$ - an event "the card is a jack ", $P(E_3|E_2|E_1) = 2/50$ (there are 50 cards and 2 jacks)

$P(E_1 \cap E_2 \cap E_3) = P(E_1)*P(E_2|E_1)*P(E_3|E_2|E_1) = 4/52 * 3/51 * 2/50 = 1/13 * 1/17 * 1/25 = 1/5525$

# Specific Multiplication – Example




*Example:*

***X***: Calculate the probability of obtaining “heads” during two consecutive coin flips.

***S = {h, t}***

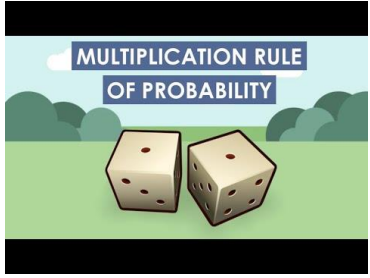
***P(S) = 1, P(h) = P(t) = 1/2***

***F = {h}*** - an event "the coin shows heads", ***P(F) = 1/2*** (the number of samples does not change)

***E = {h}*** - an event "the coin shows heads", ***P(E) = 1/2*** (the number of samples does not change)

$$P(F \cap E) = P(F) * P(E) = 1/2 * 1/2 = 1/4 = 0.25 = 25\ \%$$

# Specific Multiplication – Example (cont.)




*Example:*

***X***: You have 10 pairs of pants and 3 are black. You have 16 shirts and 4 are white. You grab your pants and shirt randomly from the closet. What is the probability that you grab a black pair of pants and a white shirt?

$S_1 = \{3*b, 7*p\}$ – 3 black pants and 7 other pants

$S_2 = \{4*w, 12*s\}$ – 4 white shirts and 12 other shirts

$P(S_1) = 1, P(b) = P(p) = 1/10$

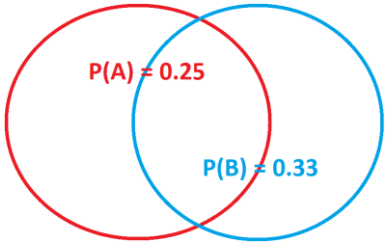
$P(S_2) = 1, P(w) = P(s) = 1/16$

$F = \{b\}$ - an event “grabbed a black pair of paints", $P(F) = 3/10$

$E = \{h\}$ - an event "grabbed a white shirt", $P(E) = 4/16$

$P(F \cap E) = P(F) * P(E) = 3/10 * 4/16 = 3/10*1/4 = 3/40=0.075 =$ **7.5 %**

# Summary




*Probability multiplication:*

a way to find the probability of two or more events happening at the same time

*Multiplication Rules:*

- general multiplication rule

  $P(E \cap F) = P(E|F) * P(F)$

  $P(E \cap F \cap D) = P(F) * P(E|F) * P(D|E|F)$

- specific multiplication rule

  $P(E \cap F) = P(E) * P(F)$ – iff ***E*** and ***F*** are independent

- *Next Lesson* – Counting

# Thank You!

Questions?