Assigment - 0

Q-1 Find out the Z-transform and ROC of the following function

1) $x(n) = \{\underset{\uparrow}{1}, 2, 1, 2, 3\}$

$\rightarrow$ $$X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$$

$$= 1 \cdot z^{-0} + 2z^{-1} + 1z^{-2} + 2z^{-3} + 3z^{-4}$$

$$= 1 + 2z^{-1} + z^{-2} + 2z^{-3} + 3z^{-4}$$

ROC :- $z - \{0, \infty\}$

Stability :- system is stable because ROC includes unit circle

Causal :- System is causal because it is defined for $n \geq 0$

2) $x[n] = (n+1)^2 u(n)$

$\rightarrow$ $$X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$$

$$= \sum_{n=0}^{\infty} (n+1)^2 z^{-n}$$

ROC :- $|z| > 1$

Stability :- System is not stable because ROC does not include the unit circle.

Causality :- The system is causal because it is defines only for $n \geq 0$.

3) $x(n) = \log n$

$\rightarrow$ $X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$

$= \sum_{n=0}^{\infty} \log n \, z^{-n}$

The function log(n) is not typically defined for $n \leq 0$ and is not causal function. It's also not well defined for $n < 1$, therefore z-transform isn't usually considered for this function.

ROC :- NA because standard z-transform is not exist.

Stability :- NA

Causality :- The system is not causal since log(n) is not defined for $n < 1$.

4) $x(n) = 2^n u(n) - 0.5^n u(-n-1)$

$\rightarrow$ $X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$

$= \sum_{n=-\infty}^{\infty} 2^n u(n) z^{-n} - 0.5^n u(-n-1) z^{-n}$

$= \sum_{n=0}^{-\infty} 2^n z^{-n} - \sum_{n=-\infty}^{-1} 0.5^n z^{-n}$

$= \frac{1}{1-2z^{-1}} - \frac{1}{1-0.5z}$

$= \frac{z}{z-2} - \frac{1}{1-0.5z}$

$\rightarrow$ ROC : i) for $2^n u(n) \rightarrow |z| > 2$
ii) for $-0.5^n u(n) \rightarrow |z| < 0.5$

The final ROC will be the intersection of both sequence which is empty. So the z-transform does not exist in this combined form.

$\rightarrow$ Stability :- NA. because no overlapping ROC.

$\rightarrow$ causality :- The first part is causal but the second part is anti-causal.

5) $x[n] = u(-n-6)$

→ $X(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n}$

$= \sum_{n=-\infty}^{\infty} u(-n-6) z^{-n}$

$= \sum_{n=-\infty}^{-6} z^{-n} = z^{6} \sum_{n=-\infty}^{-6} z^{n}$

$= \frac{z^{6}}{1-z}$

→ ROC: The ROC is $|z| < 1$ where series is converges.

→ Stability :- NA, because ROC does not include the unit circle.

→ Causality :- The system is non-causal because $x[n]$ is defined $n \le -6$.

6) $x(n) = \delta(n+5)$

→ $X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$

$= z^{-(-5)} = z^{5}$ (∵ because $x(n) = 1$ for $n = -5$)

ROC :- $z - \{0\}$

→ Stability :- The system is stable as the ROC includes the unit circle.

→ Causality :- The system is non-causal because the impulse is at $n = -5$

Q-2 Find out the inverse z-transform of the following $H(z)$ if the system is causal,

1) $H(z) = 1 + z + z^{-1}$

→ 1 is for $\delta[n]$
z is for $\delta[n+1]$
z^{-1} is for $\delta[n-1]$

Here the $\delta[n+1]$ is defined for $n < 0$, so the system is non-causal.

2) $H(z) = \log(1 + z^{-1})$

Power series of $\log(1+z^{-1})$ is

$$\sum_{K=1}^{\infty} (-1)^{K+1} \frac{z^{-K}}{K}$$

So, the Inverse z transform is -

$$h[n] = \begin{cases} -1/K \, (-1)^{K+1} & \text{for } n = K \, (K \geq 1) \\ 0 & \text{, otherwise} \end{cases}$$

or $h[n] = \sum_{k=1}^{\infty} (-1)^{k+1} \frac{1}{k} \delta[n-k]$

3) $H(z) = \frac{z^2+2z-3}{z^3+z^2-4z+3}$

→ Here, $z^3+z^2-4z+3 = (z-1)^2(z+3)$

now, $H(z) = \frac{A}{z-1} + \frac{B}{(z-1)^2} + \frac{C}{z+3}$

now Multiply by demominator,

$A(z-1)(z+3) + B(z+3) + C(z-1)^2 = z^2+2z-3$

$\therefore A(z^2+2z-3) + B(z+3) + C(z^2-2z+1) = z^2+2z-3$

$\therefore Az^2+2Az-3A + Bz+3B+Cz^2-2Cz+C = z^2+2z-3$

$\therefore z^2(A+C) + z(2A+B-2C) + (C+3B-3A) = z^2+2z-3$

So, $A+C = 1$

$2A+B-2C = 2$

$-3A+3B+C = -3$

So, $A = 1,\ B = 2,\ C = -1$

Thus, $H(z) = \frac{1}{z-1} + \frac{2}{(z-1)^2} - \frac{1}{z+3}$

So, the inverse z-transforms are

$H[n] = u[n] + 2(n+1)u[n] - (-3)^n u[n]$

4) $H(z) = \frac{(z^2+1)(2z-3)}{z^2-4z+3}$

→ Here, $(z^2+1)(2z-3) = 2z^3 - 3z^2 + 2z - 3$

So, $H(z) = \frac{2z^3 - 3z^2 + 2z - 3}{z^2 - 4z + 3}$

and, $z^2 - 4z + 3 = (z-1)(z-3)$

So, $H(z) = \frac{2z^3 - 3z^2 + 2z - 3}{(z-1)(z-3)}$

now, $H(z) = \frac{A}{z-1} + \frac{B}{z-3}$

So, $2z^3 - 3z^2 + 2z - 3 = A(z-3) + B(z-1)$

now $z=3$ > $B(3-1) = 2(27) - 3(9) + 2(3) - 3$

$2B = 54 - 27 + \cancel{18}\ 6 - 3$

$2B = 27 + 3$

$B = 15$

now $z=1$ > $2(1) - 3(1) + 2(1) - 3 = -2A$

$2 - 3 + 2 - 3 = -2A$

$-2 = -2A$

$A = 1$

so, $H(z) = \frac{1}{z-1} + \frac{15}{z-3}$

now, Inverse z-transform,

$H(n) = u[n] + 15 \cdot 3^n u[n]$

$= (1 + 15 \cdot 3^n) u[n]$

5> $H(z) = \frac{5}{(z-3)^2 (z-1)}$

→ Here $H(z) = \frac{A}{z-1} + \frac{B}{z-3} + \frac{C}{(z-3)^2}$

so, $5 = A(z-3)^2 + B(z-1)(z-3) + C(z-1)$

$5 = A[z^2 - 3z + 9] + B(z^2 - 4z + 3) +$
$C[z-1]$

$5 = Az^2 - 3Az + 9A + Bz^2 - 4Bz + 3B +$
$cz - C$

∴ $5 = z^3 [A] + z^2 (A + B) + z[C - 4B - 3A]$
$+ (9A + 3B - C)$

so, $A = 5/4$, $C = 5/2$

So, $H(z) = \frac{5/4}{z-1} + \frac{B}{z-3} + \frac{5/2}{(z-3)^2}$

So, Inverse z-transform is -

$h[n] = \frac{5}{4} u[n] + B \cdot 3^n u[n] +$

$\frac{5}{2} n \cdot 3^n \cdot u[n]$

Q-3 LCCD equation of LTI system is as following :-

$y(n) = y(n-1) + 5x(n) + 2x(n-1)$

Find the impulse response, frequency response, magnitude response and phase response of system. If input is $2^n u(n)$, what will be the output.

→ For impulse response, $x(n) = \delta(n)$

So, $h(n) = h(n-1) + 5\delta(n) + 2\delta(n-1)$

now $n = 0$ > $h(0) = h(-1) + 5\delta(0) + 2\delta(-1)$

assuming system is causal -

$h(0) = 0 + 5 + 2(0) = 5$

and $n=1$ } $h(1) = h(0) + 5\delta(1) + 2\delta(0)$
$= 5 + 5(0) + 2(1)$
$= 7$

For $n=2$ } $h(2) = h(1) + 5\delta(2) + 2\delta(1)$
$= 7 + 2(0) + 5(0)$
$= 7$

So $h(n) = \begin{cases} 5, & n=0 \\ 7, & n>0 \end{cases}$

→ now Frequency response $H(e^{j\omega})$

$$H(e^{j\omega}) = 5 + 7\sum_{n=1}^{\infty} e^{-j\omega n}$$

$$H(e^{j\omega}) = 5 + \frac{7}{e^{j\omega}-1}$$

→ Magnitude Response -

$$|H(e^{j\omega})| = \left| 5 + \frac{7}{e^{j\omega}-1} \right|$$

→ Phase Response -

$$\angle H(e^{j\omega}) = \arg\left(5 + \frac{7}{e^{j\omega}-1} \right)$$

→ Output for input $x(n) = 2^n u(n)$

$$y(n) = \sum_{K=0}^{n} h(K)\, x(n-K)$$

$$y(n) = 5(2^n) + 7\sum_{K=1}^{n} 2^{n-K}$$

$$= 5(2^n) + 7(2^n - 1)$$
$$= 12 \cdot 2^n - 7$$

Q-4 If transfer function of LTI system i $z^2 + 2z - 3 / z^3 + z^2 - 4z + 3$. Find out relatio ship between input and output. Also find output unit step respones of the following system.

→ $$H(z) = \frac{z^2 + 2z - 3}{z^3 + z^2 - 4z + 3}$$

$$\frac{Y(z)}{X(z)} = \frac{z^2 + 2z - 3}{z^3 + z^2 - 4z + 3}$$

$$\therefore (z^3 + z^2 - 4z + 3)\, Y(z) = (z^2 + 2z - 3)\, X(z)$$

$$\therefore z^3 Y(z) + z^2 Y(z) - 4z Y(z) + 3Y(z) = z^2 X(z) + 2z X(z) - 3X(z)$$

Using difference equation -

$$\therefore y(n+3) + y(n+2) - 4y(n+1) + 3y(n) = x(n+2) + 2x(n+1) - 3x(n)$$

→ now, unit step response :-

$$X(z) = \frac{1}{1-z^{-1}}, \text{ for } |z| > 1$$

The output $Y(z)$ is given by :-

$$Y(z) = H(z) \cdot X(z)$$

$$= \frac{z^2 + 2z - 3}{z^3 + z^2 - 4z + 3} \cdot \frac{1}{1 - 1/z}$$

Q-5 Find out convolution of $2^n u(n)$ and $5^n u(-n-1)$ using z transform.

→ Let $x_1[n] = 2^n u(n)$
$x_2[n] = 5^n u(-n-1)$

so, $X_1(z) = \sum_{n=0}^{\infty} 2^n z^{-n}$

$$= \sum_{n=0}^{\infty} (2/z)^n$$

$$= \frac{1}{1 - 2/z} = \frac{z}{z-2}$$

so, $X_2(z) = \sum_{n=-\infty}^{-1} 5^n z^{-n}$

putting $m = -n \Rightarrow X_2(z) = \sum_{m=1}^{m=\infty} 5^{-m} z^m$

so, $X_2(z) = \dfrac{(z/5)^2}{1 - z/5}$

$$= \frac{z^2}{25(1 - z/5)}$$

$$= \frac{z^2}{25(z-5)}$$

now $X(z) = X_1(z) \cdot X_2(z)$

$$= \frac{z}{z-2} \cdot \frac{z^2}{25(z-5)}$$

$$= \frac{z^3}{25(z-5)(z-2)}$$

now Inverse z-transform is -

$$\frac{z^3}{25(z-5)(z-2)} = \frac{A}{z-5} + \frac{B}{z-2}$$

$$z^3 = A(z-2) + B(z-5)$$

Apor $(z=2) \Rightarrow$ $8 = A(0) + B(-3)$

$8 = -3B$

$B = -8/3$

$z^3 = Az - 2A + Bz - 5B$

$z^3 = z(A+B) - (2A+5B)$

$A + B = 0$ & $5B + 2A = 0$

Solving $A = 0$, $B = 0$

so, $y(n) = x_1(n) * x_2(n)$

$$= \frac{1}{25} \left[2^n u(n) - 5^n u(n) \right]$$

Q-6 Find DTFT of the following sequence

a) $(n+1) a^n u(n)$

$\rightarrow$ $X_1(n) \xrightarrow{F} X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} (n+1) a^n u(n) e^{-j\omega n}$

$$= \sum_{n=0}^{\infty} (n+1) a^n e^{-j\omega n}$$

$$= \frac{1}{(1 - a e^{-j\omega})^2}$$

b) $x(n) = \frac{\sin(\omega_0 n)}{\pi n}$

$\rightarrow$ $X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} \sin(\omega_0 n) / \pi n \cdot e^{-j\omega n}$

$$= \begin{cases} 1, & |\omega - \omega_0| < \omega_0 \\ 0, & \text{otherwise} \end{cases}$$

c) $x(n) = \cos(\omega_0 n + \beta)$

$\rightarrow$ $X(e^{j\omega}) = \frac{1}{2}\left[e^{j\beta}\delta(\omega-\omega_0) + e^{-j\beta}\delta(\omega+\omega_0)\right]$

d) $x(n) = e^{j\omega_0 n} u(n)$

$\rightarrow$ $X(e^{j\omega}) = \sum_{n=0}^{\infty} e^{j\omega_0 n} e^{-j\omega n}$

$$= \frac{1}{1-e^{j(\omega_0-\omega)}}, \quad |\omega_0-\omega|<1$$

e) $x(n) = 1$ for $-M < n < M$

$\rightarrow$ $X(e^{j\omega}) = \sum_{n=-M}^{M} e^{-j\omega m}$

$$= \frac{\sin((2M+1)\omega/2)}{\sin \omega/2} e^{-jM\omega}$$

Q.7   Determine IDTFT of the following sequence

$\rightarrow$ $x(n) = \frac{1}{2\pi} \operatorname{sinc}(n/4)$

$$H_{bs}(e^{j\omega}) = \begin{cases} 1 & \text{if } |\omega| < \omega_0 \\ 0 & \text{if } |\omega| > \omega_0 \end{cases}$$

→ The IDTFT of the above sequences :-

$$x(n) = \frac{1}{2\pi} \sum_{k=-\infty}^{\infty} H_{bs}(e^{j\omega_k})\, e^{j\omega_k n}$$

$$= \frac{1}{2\pi} \sum_{k=-\infty}^{\infty} e^{j\omega_k n}$$

$$= \frac{1}{2\pi} \times \frac{1 - e^{-j\omega_k n}}{1 - e^{-j\omega_a n}}$$

$$= \frac{\sin \pi n/4}{\cancel{8}\, \pi n}$$

Q-8 Find out linear and circular convolution of the following sequences.

→ $y(n) = x(n) * h(n)$

$$= \sum_{k=-\infty}^{\infty} x[k]\, h[n-k]$$

$= \cancel{10}$ {0, 0, 1, 2, 3, 4, 5, 6, 5, 4, 3, 2, 1, 0, 0}

circular convolution

$$y(n) = \sum_{k=0}^{N-1} x[k]\, h[(n-k) \bmod N]$$

$= \cancel{a}$ {5, 4, 4, 4, 4, 4, 5, 6}

Q-9 Consider an LTI system with $|H(e^{j\omega})|$ and $\arg|H(e^{j\omega})|$ is shown in figure. Find impulse response of the system

→ $H(e^{j\omega}) = e^{j\theta(\omega)}$

$$h(t) = \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{j\omega t} e^{j\theta(\omega)}\, d\omega$$

$$= \frac{1}{2\pi j}\left(e^{j5\pi t/6} - e^{j\pi t/2}\right)$$

$$+ \frac{1}{2\pi j}\left(e^{j\pi t/2} - e^{-j5\pi t/6}\right)$$

$$= \frac{2}{\pi}\sin\frac{5\pi t}{6}$$

For $x(t) = \cos(\pi n)u(n)$

$$y(t) = x(t) * h(t)$$

$$= \frac{2}{\pi}\int_{-\infty}^{\infty} \cos(\pi n)\sin 5\pi t/6\, u(t)\,dt$$

$$= \frac{2}{\pi}\int_{0}^{\infty} \cos(\pi n)\sin 5\pi t/6\, dt$$

$$= \frac{1}{\pi}\left[\frac{\sin(5\pi t/6 - \pi n)}{5\pi/6} - \frac{\sin(5\pi/6 + \pi n)}{5\pi/6}\right]$$

For unit step input $x(t) = u(t)$

$$y(t) = \frac{2}{\pi} \int_0^t \sin 5\pi\tau/6 \cdot d\tau$$

$$= \frac{2}{\pi} \left[\frac{-6}{5\pi} \cos \frac{5\pi\tau}{6} \right]_0^t$$

$$= \frac{-12}{\pi} \cos \frac{5\pi t}{6} + \frac{12}{\pi}$$

Q-10 Prove the following properties of z-transform.

1) Differentiation of $X(z)$

→ if $X(z) = Z\{x[n]\}$, then the z-transform of $n \cdot x[n]$ is given by:

$$Z\{n.x[n]\} = -z \frac{dX(z)}{dz}$$

→ Proof :-

$$Z\{n.x[n]\} = \sum_{n=-\infty}^{\infty} n \cdot x[n] z^{-n}$$

so, $$\frac{dX(z)}{dz} = \frac{d}{dz}\left(\sum_{n=-\infty}^{\infty} x[n] z^{-n} \right)$$

$$\frac{dX(z)}{dz} = \sum_{n=-\infty}^{\infty} x[n] \frac{d}{dz} z^{-n}$$

$$= \sum_{n=-\infty}^{\infty} x[n] \cdot (-n) z^{-n-1}$$

now multiplying $-z$ both side

$$-z \frac{dX(z)}{dz} = \sum_{n=-\infty}^{\infty} n \cdot x[n] z^{-n}$$

hence, $Z\{n \cdot x[n]\} = -z \frac{dX(z)}{dz}$

2) Time shifting

→ if $X(z) = Z\{x[n]\}$, then the z-transform of $x[n-k]$ is given by:

$$Z\{x[n-k]\} = z^{-k} X(z)$$

Proof :-

→ Consider the z-transform of $x[n-k]$:

$$Z\{x[n-k]\} = \sum_{n=-\infty}^{\infty} x[n-k] z^{-n}$$

now putting $m = n-k$, hence $n = m+k$

$$Z\{x[n-k]\} = \sum_{m=-\infty}^{\infty} x[m] z^{-(m+k)}$$

$$= z^{-k} \sum_{m=-\infty}^{\infty} x[m] z^{-m}$$

this simplifies to:-

$$Z\{x[n-k]\} = z^{-k} X(z)$$

3) Convolution of sequences

→ if $y[n] = x_1[n] * x_2[n]$ then the z-transform of $y[n]$ is the product of the individual z-transforms:-

$$Z\{y[n]\} = X_1(z) \cdot X_2(z)$$

→ Proof:-

The convolution of two sequences $x_1[n]$ and $x_2[n]$ is defined as:-

$$y[n] = \sum_{k=-\infty}^{\infty} x_1[k] x_2[n-k]$$

Taking the z-transform of $y[n]$:

$$Z\{y[n]\} = \sum_{n=-\infty}^{\infty} \left(\sum_{k=-\infty}^{\infty} x_1[k] x_2[n-k] \right) z^{-n}$$

$$= \sum_{k=-\infty}^{\infty} x_1[k] \sum_{n=-\infty}^{\infty} x_2[n-k] z^{-n}$$

make substitution $m = n-k$, hence $n = m+k$

$$Z\{y[n]\} = \sum_{k=-\infty}^{\infty} x_1[k] z^{-k} \sum_{m=-\infty}^{\infty} x_2[m] z^{-m}$$

This simplifies to:

$$Z\{y[n]\} = X_1[z] \cdot X_2[z]$$

4) Time Reversal :-

→ Property :-

if $X(z) = Z\{x[n]\}$ then the z-transform of $x[-n]$ is given by:

$$Z\{x[-n]\} = X(1/z)$$

→ Proof :-

consider the z-transform of $x[-n]$

$$Z\{x[-n]\} = \sum_{n=-\infty}^{\infty} x[-n]\, z^{-n}$$

make a substitution $m = -n$, hence $n = -m$

$$Z\{x[-n]\} = \sum_{m=-\infty}^{\infty} x[m]\, z^{m}$$

$$Z\{x[-n]\} = X(1/z)$$

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Q-11 State and Prove symmetry property of DTFT.

→ Statement :- The DTFT of a real-valued sequence is Hermitian symmetric, meaning its magnitude is even and its phase is odd. mathematically,

$$X^*(e^{-j\omega}) = X(e^{j\omega})$$

where :-

- $X(e^{j\omega})$ is the dtft of the real-valued sequence $x(n)$.
- $X(-e^{j\omega})$ is the complex conjugate of the DTFT evaluated at $e^{j\omega}$

→ Proof :-

Let $x(n)$ be a real-valued sequence, then its DTFT is:

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x(n) e^{-j\omega n}$$

$$X^*(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x^*(n) e^{j\omega n}$$

Since $x(n)$ is real, $x^*(n) = x(n)$.
therefore.

$$X^*(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x(n) e^{j\omega n} = X(e^{j\omega})$$

now, let's evaluate $X^*(e^{j\omega})$ at $-e^{j\omega}$

$$X^*(-e^{j\omega}) = \sum_{n=-\infty}^{\infty} x(n) e^{j\omega n}$$

substituting $k=-n$ and using the fact that $x(n)$ is real

$$X^*(-e^{j\omega}) = \sum_{k=\infty}^{-\infty} x(-k) e^{-j\omega k}$$

$$= \sum_{k=-\infty}^{\infty} x(k) e^{-j\omega k} = X(e^{j\omega})$$

Q-12 $x(n) = \{1, 1, 2, 2\}$, $h(n) = \{1, 2, 3, 4\}$

a) Find linear convolution using circular convolution

→

	1	2	3	4
1	1	2	3	4
1	1	2	3	4
2	2	4	6	8
2	2	4	6	8

linear = $\{1, 3, 7, 13, 14, 14, 8\}$

circular = $\{15, 17, 15, 13\}$

Q-13 Find 8 point DFT of $x(n) = \cos(n\pi/4)$ then find amplitude and phase spectra.

→ Here $x[n] = \cos n\pi/4$

$x[n] = \{1, 1/\sqrt{2}, 0, -1/\sqrt{2}, -1, -1/\sqrt{2}, 0, 1/\sqrt{2}\}$

now $X(K) = \sum_{n=0}^{7} x(n) \cdot e^{-j\frac{2\pi kn}{8}}$

$X(0) = \sum_{n=0}^{7} x(n) = 0$

$X(1) = \sum_{n=0}^{7} x(n) \cdot e^{-j\frac{\pi}{4}n}$

$= 1 + 1/\sqrt{2}\, e^{-j\pi/4} + 0 - 1/\sqrt{2}\, e^{-j3\pi/4}$
$- 1\, e^{-j\pi/2} + -1/\sqrt{2}\, e^{-j5\pi/4} + 0$
$+ 1/\sqrt{2}\, e$

$= 0$

similarly for $X(2), X(3), \ldots X(7)$ non-zero values will occur at the $X(2)$ and $X(6)$

$X(0) = 0$, $X(1) = 0$, $X(2) = 4$, $X(3) = 0$,
$X(4) = 0$, $X(5) = 0$, $X(6) = 4$
$X(7) = 0$

$|X(K)| = [0, 0, 4, 0, 0, 0, 4, 0]$
$\arg(X(K)) = [0, 0, 0, 0, 0, 0, \pi, 0]$

Q-14 For each of the following system, determine whether the system is i) stable ii) causal iii) Linear iv) time Invariant v) memoryless

1) $y[n] = x[n-n_0]$

→ Here output is shifted version of input. so it is stable.

→ Here if $n_0 \geq 0$ it is causal
else non-causal

→ Here the system is line shifted so it is linear

→ Here the time shift in input will be same as time shift in o/p. so it is time inveriat

→ Here the o/p is depends on the past i/p. so it is not memoryless.

2) $y[n] = x[-n]$

→ the system is time reversal. so it is stable

→ the system is defined for $x<0$. so it is non-causal.

→ the system follows additivity and homogeneity. so it is linear

→ Here the timeshift of i/p and o/p is not same, so it is non time-invariant.

The system depends on a different time index. so it is not memoryless

3) $y(n) = x^2(n)$

→ the o/p and i/p both are bounded. so it is stable

→ the system is causal.

→ the system is non-linear

→ the time-shift in i/p and o/p will be same so it is time invariant

→ the output at time t is depends only on the current input. so it is memoryless.

4) $y(n) = \log(x(n))$

→ if $x>0$ then it is stable.

→ the system is defined for $n>0$ only so it is causal

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→ the system is non-linear

→ if the input is time-shifted by same amount so it is time-invariant.

→ the output at time n depends only on the input at time n. so it is memoryless

5) $y(t) = \frac{d}{dt} x(t)$

→ the system is not stable.

→ the system is causal

→ the system follows the principle of linearity and homoginity. so it is linear

→ the system is time invariant

. the system is not memoryless

[illegible]

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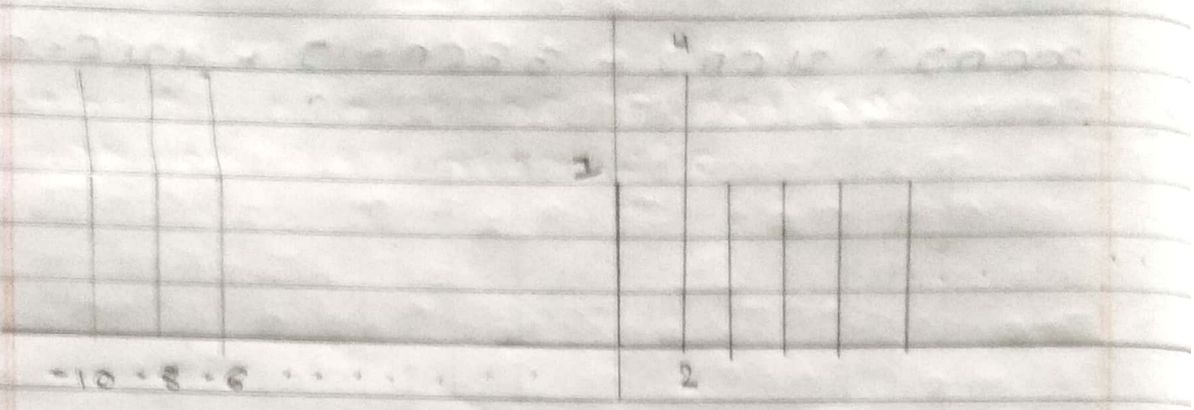
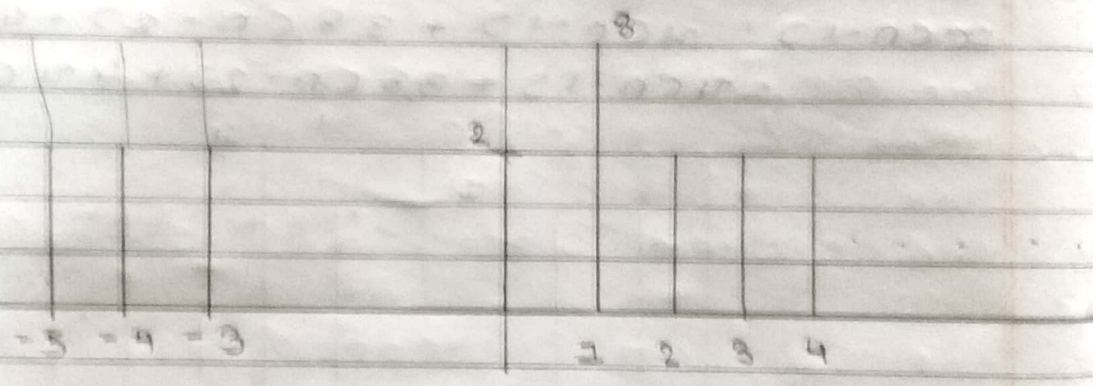
[illegible]

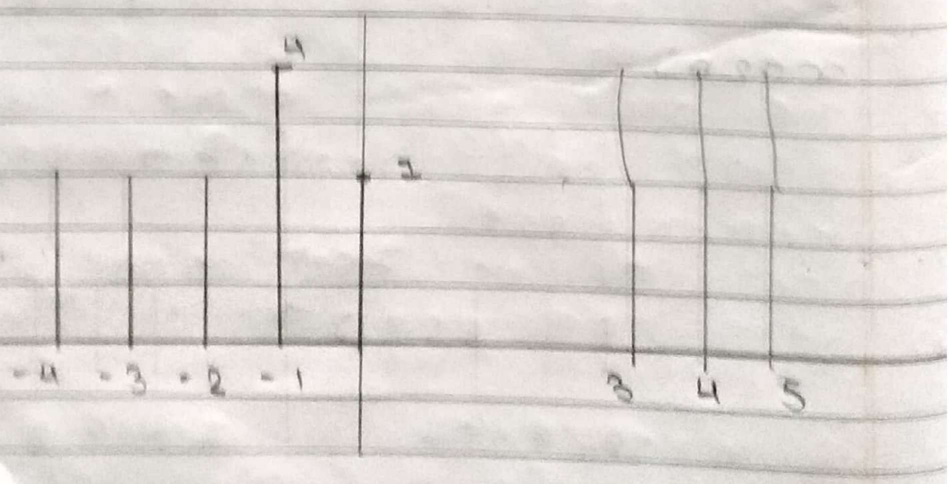
$x(n/2)$



$2x(n)$



$x(-n)$



Q-16 Find out whether the signal is periodic aperiodic. Find the period of following signals :

1) $x(n) = e^{j\pi n/6}$

$e^{j\omega}$ period $\Rightarrow \frac{\omega_0}{2\pi} = \frac{\pi}{6 \times 2\pi} = \frac{1}{12}$

~~Period = P2~~

2) $x(n) = \sin(3n)$

→ Period $= \frac{3}{2\pi} \Rightarrow \frac{\omega_0}{2\pi}$

$3/2\pi$ is not rational number so the signal is aperiodic

3) $x(n) = e^{jn/1.41}$

Period $= \frac{\omega_0}{2\pi} = \frac{1}{1.41 \times 2\pi} \neq$ rational number

signal is aperiodic

4) $x(n) = \sin(3\pi n) + \cos(6\pi N)$

$N_1 = \frac{2\pi}{3\pi} = \frac{2}{3}$, $N_2 = \frac{2\pi}{6\pi} = \frac{1}{3}$

Here LCM is not founded, so it is aperodic

5) $x(n) = \frac{\sin \pi n}{\pi n}$

this is a sinc function which is aperodic