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Q-1 To determine the exponential form of the discrete fourier series :- we know that

$$W_N^K = e^{-j(2\pi/N)K}$$

Given $N=4$ hence

$$W_4^1 = e^{-j2\pi/4} = \cos\pi/2 - j\sin\pi/2 = -j$$
$$W_4^2 = W_4^1 \cdot W_4^2 = -1$$
$$W_4^3 = W_4^2 \cdot W_4^1 = -1 \cdot -j = j$$
$$W_4^4 = W_4^2 \cdot W_4^2 = -1 \cdot -1 = 1$$

For $K=0$, $X(0) = \sum_{n=0}^{3} x(n) W_4^{0\cdot n} = x(0) + x(1) + x(2) + x(3)$

$$= 0 + 1 + 2 + 3 = 6$$

For $K=1$, $X(1) = \sum_{n=0}^{3} x(n) W_4^{n}$

$$= x(0)W_4^0 + x(1)W_4^1 + x(2)W_4^2 + x(3)W_4^3$$

$$= (0)(1) + (1)(-j) + (2)(-1) + (3)(j)$$
$$= 0 - j - 2 + 3j$$
$$= -2 + 2j$$

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For $k=2$ $\quad X(2) = \sum_{n=0}^{3} x(n) W_4^{2\cdot n}$

$x(0) W_4^0 + x(1) W_4^2 + x(2) W_4^4 + x(3) W_4^6$

$(0)(1) + (1)(-1) + (2)(1) + (3)(-1)$

$0 - 1 + 2 - 3$

-2

For $k=3$ $\quad X(3) = \sum_{n=0}^{3} x(n) W_4^{3n}$

$x(0) W_4^0 + x(1) W_4^3 + x(2) W_4^6 + x(3) W_4^9$

$(0)(1) + (1)(j) + (2)(-1) + (3)(-j)$

$0 + j - 2 - 3j$

$-2 - 2j$

so the expotential fourier series

$$x(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(k) W_N^{-kn}$$

$$= \frac{1}{4} \sum_{k=0}^{3} X(k) W_4^{-kn}$$

$1/4 \left[X(0) W_4^{\cdot 0} + X(1) W_4^{-n} + X(2) W_4^{-2n} + X(3) W_4^{-3n} \right]$

$1/4 \left[(0)(1) + (-2 + 2j) e^{j\pi n/2} - 2 e^{j\pi n} + (-2 - 2j) e^{j 3\pi n/2} \right]$

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$$\therefore \quad x(n) = \frac{1}{2}\left[3 + (-1+j)e^{j\pi n/2} - e^{j\pi n} - (1+j)e^{j3\pi n/2}\right]$$

→ To determine the trignometric form of fourier series

$$A(0) = X(0)/N = 6/4 = 3/2$$

$$A(1) = [X(1) + X(3)]/4$$

$$= [(-2+2j) + (-2-2j)]/4$$

$$= -1$$

$$\cancel{A}\ B(1) = j[X(1) - X(3)]$$

$$= j[(-2+2j) - (-2-2j)]/4$$

$$= -1$$

$$A(4/2) = X(2)/4 = -1/2$$

→ Therefore the trignometric fourier series :-

$$x(n) = \frac{3}{2} - \cos\left[\frac{\pi n}{2}\right] - \sin\left[\frac{\pi n}{2}\right] - \frac{1}{2}\cos \pi n$$

Q-2 Linearity :-

→ Consider two periodic sequences $x_1(n)$ and $x_2(n)$ both with period N, such that,

$DFS[x_1(n)] = X_1(K)$ and
$DFS[x_2(n)] = X_2(K)$

then, $DFS[a_1 x_1(n) + a_2 x_2(n)] = a_1 X_1(K) + a_2 X_2(K)$

Time Shipting :-

→ If $x(n)$ is a periodic sequence with N samples and

$DFS[x(n)] = X(K)$

then, $DFS[x(n-m)] = e^{-j(2\pi/N)mK} X(K)$

Periodic Convolution :-

→ Let $x_1(n)$ and $x_2(n)$ be two periodic sequences with ~~seq~~ period N with

$DFS[x_1(n)] = X_1(K)$
$DFS[x_2(n)] = X_2(K)$

$$\begin{bmatrix} 1 & 1 & 0 & -1 & 0 \\ 0 & 1 & 1 & 0 & -1 \\ -1 & 0 & 1 & 1 & 0 \\ 0 & -1 & 0 & 1 & 1 \\ 1 & 0 & -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ -1 \\ -1 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \\ 0 \\ -3 \\ -2 \\ 2 \end{bmatrix}$$

~~He~~ Hence $x_3(n) = \{3, 0, -3, -2, 2\}$

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→ Hence,

$$x_3(n) = 1/4 \sum_{k=0}^{3} X_3(k) e^{j2\pi nk/4}$$

$$= 1/4 [60 + (-4j) e^{j\pi n/2} + 4j e^{j3\pi n/2}]$$

$$x_3(0) = 1/4 [60 + (-4j) + 4j] = 15$$

$$x_3(1) = 1/4 [60 - 4j e^{j\pi/2} + 4j e^{j3\pi/2}]$$
$$= 1/4 [60 - 4j(+j) + 4j(-j)]$$
$$= 1/4 [60 + 4 + 4] = 17$$

$$x_3(2) = 1/4 [60 - 4j e^{j\pi} + 4j e^{j3\pi}]$$
$$= 1/4 [60 - 4j(-1) + 4j(-1)]$$
$$= 1/4 [60 + 4j - 4j] = 15$$

$$x_3(3) = 1/4 [60 + (-4j)(-j) + 4j(j)]$$
$$= 1/4 [60 - 4 - 4]$$
$$= 13$$

$$x_3(n) = [15, 17, 15, 13]$$

Q-11 Here $x_1(n) = \{1, 1, -1, -1\}$
and $x_2(n) = \{1, 0, -1, 0, +1\}$
and $N = 5$

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For $K=0$, $X_2(0) = \sum_{n=0}^{3} x_2(n) e^{-j2\pi n(0)/4}$

$= 10$

For $K=1$, $X_2(1) = \sum_{n=0}^{3} x_2(n) e^{-j\pi n/2}$

$= 1 + 2e^{-j\pi/2} + 3e^{-j\pi} + 4e^{-j3\pi/2}$

$= 1 + 2(-j) + 3(-1) + 4(j) = -2 + 2j$

For $K=2$, $X_2(2) = \sum_{n=0}^{3} x_2(n) e^{-j\pi n}$

$= 1 + 2e^{-j\pi} + 3e^{-j\pi} + 4e^{-j3\pi}$

$= 1 + 2(-1) + 3(1) + 4(-1) = -2$

For $K=3$, $X_2(3) = \sum_{n=0}^{3} x_2(n) e^{-j3\pi n/2}$

$= 1 + 2e^{-j3\pi/2} + 3e^{-j3\pi} + 4e^{-j9\pi/2}$

$= 1 + 2j + 3(-1) + 4(-j)$

$= -2 - 2j$

$X_2(K) = \{10, -2+2j, -2, -2-2j\}$

$X_3(K) = X_1(K) \cdot X_2(K)$

$= \{60, -4j, 0, 4j\}$

now, $x_3(n) = IDFT\{X_3(K)\}$

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for $k=0$ $X_1(0) = \sum_{n=0}^{3} x_1(n) e^{-j2\pi n/4(0)}$

$= 6$

for $k=1$ $X_1(1) = \sum_{n=0}^{3} x_1(n) e^{-j\pi n/2}$

$= 1 + e^{-j\pi/2} + 2e^{-j\pi} + 2e^{-j3\pi/2}$

$= 1 - j + 2(-1) + 2j$

$= -1 + j$

for $k=2$ $X_1(2) = \sum_{n=0}^{3} x_1(n) e^{-j\pi n}$

$= 1 + e^{-j\pi} + 2e^{-j2\pi} + 2e^{-j3\pi}$

$= 0$

for $k=3$ $X_1(3) = \sum_{n=0}^{3} x_1(n) e^{-j3n\pi/2}$

$= 1 + e^{-j3\pi/2} + 2e^{-j3\pi} + 2e^{-j9\pi/2}$

$= 1 + j + 2(-1) - j(2)$

$= -1 - j$

$X_1(k) = \{6, -1+j, 0, -1-j\}$

ii) when $x_2(n) = \{1, 2, 3, 4\}$

so, $X_2(k) = \sum_{n=0}^{3} x_2(n) e^{-j2\pi nk/4}$

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For $m=2$

$$x_3(2) = \sum_{n=0}^{3} x_1(n)\, x_2(2-n,(\text{mod } 4))$$

$$= 15$$

for $m=3$

$$x_3(3) = \sum_{n=0}^{3} x_1(n)\, x_2(3-n,(\text{mod } 4))$$

$$= 13$$

$$x_3(n) = \{15, 17, 15, 13\}$$

c) Here the DFT and IDFT are used for finding circular convolution

$$X_3(k) = X_1(k)\, X_2(k)$$

and ~~$X_3(k)$~~ $x_3(n) = \frac{1}{N} \sum_{k=0}^{N-1} X_3(k) e^{j2\pi nk/N}$

i) When $x_1(n) = [1, 1, 2, 2]$

$$X_1(k) = \sum_{n=0}^{N-1} x_1(n) e^{-j2\pi nk/N}$$

for $N=4$ $\sum_{n=0}^{3} x_1(n) e^{-j2\pi nk/4}$

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b) circular convolution

$$x_3(m) = \sum_{n=0}^{N-1} x_1(n)\, x_2(m-n, (\text{mod } N)),$$
$$m = 0, 1, \ldots, N-1$$

For $m = 0$

$$x_3(0) = \sum_{n=0}^{3} x_1(n)\, x_2(-n, (\text{mod } 4))$$

$x_2(0, \text{mod } 4) = x_2(0)$
$x_2(-1, \text{mod } 4) = x_2(3)$
$x_2(-2, \text{mod } 4) = x_2(2)$
$x_2(-3, \text{mod } 4) = x_2(1)$

$$x_3(0) = x_1(0)\,x_2(0) + x_1(1)\,x_2(3) + x_1(2)\,x_2(2) + x_1(3)\,x_2(1)$$

$= (1)(1) + (1)(4) + (2)(3) + (2)(2)$

$= 1 + 4 + 6 + 4$

$= 15$

For $m = 1$

$$x_3(1) = \sum_{n=0}^{3} x_1(n)\, x_2(1-n, (\text{mod } 4))$$

$= 17$

For $n = 1$, $x(1) = 1/4 \sum_{K=0}^{3} X(K) e^{j\pi K/2}$

$= 1/4 [3 + (2+j) e^{j\pi/2} + e^{j\pi} + (2-j) e^{j3\pi/2}]$

$= 1/4 [3 + (2+j)j - 1 + 2(-j)(-j)] = 0$

For $n = 2$ $x(2) = 1/4 \sum_{K=0}^{3} X(K) e^{j\pi K}$

$= 1/4 [3 + (2+j) e^{j\pi} + e^{j2\pi} + (2-j) e^{j3\pi}]$

$= 1/4 [3 + (2+j)(-1) + 1 + (2-j)(-1)]$

$= 0$

For $n = 3$ $x(3) = 1/4 \sum_{K=0}^{3} X(K) e^{j3\pi K/2}$

$= 1/4 [3 + (2+j) e^{j3\pi/2} + e^{j3\pi} + (2-j) e^{j9\pi/2}]$

$= 1/4 [3 + (2+j)(-j) - 1 + (2-j)(j)] = 1$

$x(n) = \{2, 0, 0, 1\}$

Q-10 a) linear convolution

$x_2(n)$ \ $x_1(n)$	1	1	2	2
1	1	1	2	2
2	2	2	4	4
3	3	3	6	6
4	4	4	8	8

$x_1(n) * x_2(n) = \{1, 3, 7, 13, 14, 14, 8\}$

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For $n=3$ $\quad x(3) = 1/4 \sum_{K=0}^{3} X(K) e^{-j3\pi K/2}$

$= 1/4 (1 + 1 e^{j3\pi/2} + 2 e^{j3\pi} + 3 e^{j9\pi/2})$]
$= 1/4 (1 + 1(-j) + 2(-1) + 3j)$
$= 1/4 (1 - j - 2 + 3j)$
$= 1/4 (-1 + 2j)$
$= -1/4 + 1/2 j$

$x(n) = \{ 7/4 , -1/2 - 1/2 j , -1/4 , -1/4 + 1/2 j \}$

Q-9 $\quad X(K) = \{ 3 , 2+j , 1 , 2-j \}$

The IDFT is defined as

$$x(n) = \frac{1}{N} \sum_{K=0}^{N-1} X(K) e^{j2\pi Kn/N}$$

Given $N=4$, $x(n) = 1/4 \sum_{K=0}^{3} X(K) e^{j\pi nK/2}$

For $n=0$ $\quad x(0) = 1/4 \sum_{K=0}^{3} X(K) e^{0}$

$= 1/4 [3 + (2+j) + 1 + (2-j)]$
$= 2$

Q-8 $x(n) = ?$ $X(K) = \{1, 1, 2, 3\}$

The inverse DFT is defined as

$$x(n) = \frac{1}{N} \sum_{K=0}^{N-1} X(K)\, e^{j2\pi nK/N}$$

$$= \frac{1}{4} \sum_{K=0}^{3} X(K)\, e^{j2\pi nK/4}$$

For $n=0$, $x(0) = 1/4 \{1 + 1 + 2 + 3\}$
$= 7/4$

For $n=1$ $x(1) = 1/4 \sum_{K=0}^{3} X(K)\, e^{j\pi K/2}$

$= 1/4\,(1 + 1e^{j\pi/2} + 2e^{j\pi} + 3e^{j3\pi/2})$
$= 1/4\,(1 + 2j + 3(-1) + 4(-j))$
$= 1/4\,(-2 - 2j) = -1/2 - 1/2\,j$

For $n=2$ $x(2) = 1/4 \sum_{K=0}^{3} X(K)\, e^{j\pi K}$

$= 1/4\,(1 + e^{j\pi} + 2e^{j2\pi} + 3e^{j3\pi})$
$= 1/4\,(1 + 2(-1) + 2(1) + 3(-1))$
$= 1/4\,(1 - 1 + 2 - 3) = -1/4$

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For $k=1$ $X(1) = \sum_{n=0}^{3} x(n)\, e^{-j\pi n/2}$

$= 1 + 0.707\, e^{-j\pi/2} + 0 + (-0.707)\, e^{-j3\pi/2}$

$= 1 + 0.707(-j) + 0 - (0.707)j$

$= 1 - 1.414j$

For $k=2$ $X(2) = \sum_{n=0}^{3} x(n)\, e^{-j\pi n}$

$= 1 + (0.707)\, e^{-j\pi} + 0 + (-0.707)\, e^{-j3\pi}$

$= 1 + (0.707)(-1) + 0 + (-0.707)(-1)$

$= 1$

For $k=3$ $X(3) = \sum_{n=0}^{3} x(n)\, e^{j3\pi n/2}$

$= 1 + (0.707)\, e^{-j3\pi/2} + 0 + (-0.707)\, e^{-j9\pi/2}$

$= 1 + (0.707)j + 0 + (-0.707)(-j)$

$= 1 + 1.414j$

$X(k) = \{1,\ 1-1.414j,\ 1,\ 1+1.414j\}$

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Q-6 a) Given $x(n) = \delta(n)$

$$X(e^{j\omega}) = \sum_{n=0}^{N-1} x(n) e^{-j\omega n}$$

$$X(e^{j\omega}) = \sum_{n=0}^{N-1} \delta(n) e^{-j\omega n} = 1$$

b) Given $x(n) = \delta(n-n_0)$

$$X(e^{j\omega}) = \sum_{n=0}^{N-1} \delta(n-n_0) e^{-j\omega n}$$

$$= e^{-j\omega n_0}$$

Q-7 Given $N = 4$

$$x(n) = [\cos(0), \cos \pi/4, \cos \pi/2, \cos 3\pi/4]$$
$$= [1, 0.707, 0, -0.707]$$

The DFT is

$$X(K) = \sum_{n=0}^{3} x(n) e^{-j\pi n K/2} \quad K = 0, 1, 2, 3$$

For $K = 0$ $\quad X(0) = \sum_{n=0}^{3} x(n) = 1$

For $K=4$ $X(4) = \sum_{n=0}^{5} x(n) e^{-j4\pi n/3}$

$= 1 + e^{-j4\pi/3} + 2e^{-j8\pi/3} + 2e^{-j4\pi} + 3e^{-j16\pi/3} + 3e^{-j20\pi/3}$

$= 1 + (-0.5 + j0.866) + 2(-0.5 - j0.866) + 2(1) + 3(-0.5 + j0.866) + 3(-0.5 - j0.866)$

$= -1.5 - j0.866$

For $K=5$ $X(5) = \sum_{n=0}^{5} x(n) e^{-j5\pi n/3}$

$= 1 + e^{-j5\pi/3} + 2e^{-j10\pi/3} + 2e^{-j5\pi} + 3e^{-j20\pi/3} + 3e^{-j25\pi/3}$

$= 1 + (-0.5 + 0.866j) + 2(-0.5 + j0.866) + 2(-1) + 3(-0.5 - j0.866) + 3(0.5 - 0.866j)$

$= -1.5 - 2.598j$

→ The corresponding amplitude response -
{12, 2.999, 1.732, 0, 1.732, 2.999}

→ Phase response - {0, $-\pi/3$, $-\pi/6$, 0, $\pi/6$, $\pi/3$}

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For $k=1$ $X(1) = \sum_{n=0}^{5} x(n) e^{-j\pi n/3}$

$= 1 + e^{-j\pi/3} + 2e^{-j2\pi/3} + 2e^{-j\pi} + 3e^{-j4\pi/3} + 3e^{-j5\pi/3}$

$= 1 + 0.5 - j0.866 + 2(-0.5 - j0.866) + 2(-1) + 3(-0.5 + j0.866) + 3(0.5 + j0.866)$

$= -1.5 + j2.598$

For $k=2$ $X(2) = \sum_{n=0}^{5} x(n) e^{-2j\pi n/3}$

$= 1 + e^{-j2\pi/3} + 2e^{-j4\pi/3} + 2e^{-j2\pi} + 3e^{-j8\pi/3} + 3e^{-j10\pi/3}$

$= 1 + (-0.5) - j0.866 + 2(-0.5 + j0.866) + 2(1) + 3(-0.5 - j0.866) + 3(-0.5 + j0.866)$

$= -1.5 + j0.866$

For $k=3$ $X(3) = \sum_{n=0}^{5} x(n) e^{-j\pi n}$

$= 1 + e^{-j\pi} + 2e^{-j2\pi} + 2e^{-j3\pi} + 3e^{-j4\pi} + 3e^{-j5\pi} = 0$

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Q-4 $x(n) = \{0.2, \underset{\uparrow}{0.2}, 0.2\}$

$\rightarrow$ $$X(K) = \sum_{n=-\infty}^{\infty} x(n) e^{-j2\pi K n / N}$$

$$= \frac{1}{5}\left[e^{j\omega} + 1 + e^{-j\omega}\right] \text{ at } \omega = 2\pi K/N$$

$$= \frac{1}{5}\left[1 + 2\cos\omega\right] \text{ at } \omega = 2\pi K/N \text{ and } N = 3$$

$$= \frac{1}{5}\left[1 + 2\cos(2\pi K/3)\right]$$

where $K = 0, 1, N-1$

Q-5 The N-Point DFT of a finite duration sequence $x(n)$ is defined as

$$X(K) = \sum_{n=0}^{N-1} x(n) e^{-j2\pi n K / N}$$

For $K=0$ $X(0) = \sum_{n=0}^{5} x(n) = 1 + 1 + 2 + 2 + 3 + 3$

$= 12$

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Q - 3 Here $x(n) = \{0, 2, 2, 0, 0, 0, 3, 3, 0\}$

$$X(k) = \sum_{n=0}^{N-1} x(n) e^{-j2\pi kn/N}$$

$$= \sum_{n=6}^{7} x(n) e^{-j2\pi kn/N}$$

For $k=1 \Rightarrow X(1) = \sum_{n=0}^{7} x(n) e^{-j2\pi n/7}$

$= x(0) e^{-j2\pi(0)/7} + x(1) e^{-j2\pi/7} + x(2) e^{-j4\pi/7}$
$x(3) e^{-j6\pi/7} + x(4) e^{-j8\pi/7} + x(5) e^{-j10\pi/7} +$
$x(6) e^{-j12\pi/7} + x(7) e^{-j14\pi/7}$

$= 2e^{-j2\pi/7} + 2e^{-j4\pi/7} + 3e^{-j12\pi/7} +$
$3e^{-j14\pi/7}$

$= 2(0.1428 - j(\cancel{0}^{0.78183})) + 2(0.9673 - j(0.25350)) +$
$+ 3(0.6234 - j(-0.78180)) +$
$3(1 - 0)$

$= 0.2856 - 1.5636j + 1.9346 - j0.507 +$
$1.8702 + 2.3454j + 3$

$= 7.0904 + 0.2748j$

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→ If $X_3(k) = X_1(k)X_2(k)$, then the periodic sequence $x_3(n)$ with Fourier series coefficients $X_3(k)$ is $x_3(n) = \sum_{m=0}^{N-1} x_1(m)x_2(n-m)$

Hence, DFS $\left|\sum_{m=0}^{N-1} x_1(m)x_2(n-m)\right| = X_1(k)X_2(k)$

Q-3 Here, $x(n) = \{0, 2, 2, 0, 0, 0, 3, 3, 0\}$

$$X(k) = \sum_{n=0}^{N-1} x(n)e^{-j2\pi kn/N}$$

$$\text{for } k = 1 \Rightarrow X(1) = \sum_{n=0}^{7} x(n)e^{-j2\pi n/7}$$

$= x(0)e^{-j2\pi(0)/6} + x(1)e^{-j2\pi/6} + x(2)e^{-j4\pi/6}$
$+ x(3)e^{-j6\pi/6} + x(4)e^{-j8\pi/6} +$
$x(5)e^{-j10\pi/6} + x(6)e^{-j12\pi/6}$

$= 2[\cos\pi/3 - j\sin\pi/3] + 2[\cos 2\pi/3 - j\sin 2\pi/3]$
$+$