Q $Z = 3x_1 + 5x_2 + 4x_3$

Constrain
$$\begin{cases} 2x_1 + 3x_2 \le 8 \\ 2x_2 + 5x_3 \le 10 \\ 3x_1 + 2x_2 + 4x_3 \le 15 \\ x_1, x_2, x_3 \ge 0 \end{cases}$$

Change Inequality to Equality.

$$2x_1 + 3x_2 + S_1 = 8$$
$$2x_2 + 5x_3 + S_2 = 10$$
$$3x_1 + 2x_2 + 4x_3 + S_3 = 15$$

$$\boxed{Z = 3x_1 + 5x_2 + 4x_3 + 0S_1 + 0S_2 + 0S_3}$$

Matrix $Ax = b$

b = Solution

Put the value of x_1 & x_2 & $x_3 = 0$

Eqⁿ 1 $\Rightarrow$ $S_1 = 8$

2 $\Rightarrow$ $S_2 = 10$

3 $\Rightarrow$ $S_3 = 15$

$$\begin{array}{c}
\begin{bmatrix} x_1 & x_2 & x_3 & s_1 & s_2 & s_3 \end{bmatrix} \; 6*1 \\
\begin{bmatrix} 2 & 3 & 0 & 1 & 0 & 0 \\ 0 & 2 & 5 & 0 & 1 & 0 \\ 3 & 2 & 4 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 8 \\ 10 \\ 15 \end{bmatrix} \\
3 \times 6 \qquad \text{Solution}
\end{array}$$

$$z_j = \sum \text{Coeff} * x_j$$

c_j = Coefficent of decision variable

$$z_1 = [0 * 2] + [0 * 0] + [0 * 3] = 0$$

$$Z_2 = [0 * a_{21}] + [0 * a_{22}] + [0 * a_{23}] = 0$$

Coeff	Solution	C_j	3	5	4	0	0	0	Ratio
			x_1	x_2	x_3	S_1	S_2	S_3	
0	8	S_1	2	3	0	1	0	0	8/3 (Min)
0	10	S_2	0	2	5	0	1	0	10/2
0	15	S_3	3	2	4	0	0	1	15/2
		Z_j	0	0	0	0	0	0	
		$C_j - Z_j$	3	5	4	0	0	0	

↑

Coeff	Solution	C_j	3	5	4	0	0	0	Ratio
			x_1	x_2	x_3	S_1	S_2	S_3	
5	8/3	x_2	2/3	1	0	1/3	0	0	—
0	14/3	S_2	-4/3	0	5	-2/3	1	0	14/15
0	29/3	S_3	5/3	0	4	-2/3	0	1	29/12
		Z_j	10/3	5	0	5/3	0	0	
		$C_j - Z_j$	-1/3	0	4	-5/3	0	0	

If Row is key Row then

$$\boxed{a_{ij} = \frac{\text{Old element}}{\text{Key element}}}$$

If Row is not key Row

$$a_{ij} = \text{Old element} - \frac{\text{Corr key Row element} * \text{Corr key Coln elem}}{\text{key element}}$$

$$\boxed{a_{ij} = \text{Old element} - \text{New Row element} * \text{Corr key Colum element}}$$

Coef	Sol^n	C_j	3	5	4	0	0	0	Ratio
			x_1	x_2	x_3	S_1	S_2	S_3	
5		x_2	$2/3$	1	0	$1/3$	0	0	
4	$14/15$	x_3	$-4/15$	0	1	$-2/15$	$1/5$	0	
0		S_1							
		Z_j							
		$C_j - Z_j$							

Q 4.2

Maximize $Z = 4x_1 + 3x_2 + 0s_1 + 0s_2 + 0s_3 + 0s_4$

subject to the constraints

(i) $2x_1 + x_2 + s_1 = 1{,}000$, (ii) $x_1 + x_2 + s_2 = 800$

(iii) $x_1 + s_3 = 400$, (iv) $x_2 + s_4 = 700$

and $x_1, x_2, s_1, s_2, s_3, s_4 \geq 0$

		$c_j \rightarrow$	4	3	0	0	0	0	
Basic Variables Coefficient c_B	*Basic Variables* B	*Basic Variables Value* $b\ (= x_B)$	x_1	x_2	s_1	s_2	s_3	s_4	*Min Ratio* x_B/x_1
0	s_1	1,000	2	1	1	0	0	0	1,000/2 = 500
0	s_2	800	1	1	0	1	0	0	800/1 = 800
0	s_3	400	1	0	0	0	1	0	400/1 = 400 $\rightarrow$
0	s_4	700	0	1	0	0	0	1	not defined
$Z = 0$		z_j	0	0	0	0	0	0	
		$c_j - z_j$	4 $\uparrow$	3	0	0	0	0	

In Table 4.7, since $c_j - z_j = 4$ is the largest positive number, we apply the following row operations

C_B	B	b	4	3	0	0	0	0	Ratio
			x_1	x_2	S_1	S_2	S_3	S_4	
0	S_1	200	0	1	1	0	-2	0	200
0	S_2	400	0	1	0	1	-1	0	400
4	x_1	400	1	0	0	0	1	0	—
0	S_4	700	0	1	0	0	0	1	700
		z_j	4	0	0	0	4	0	
		$C_j - z_j$	0	3	0	0	-4	0	

C_B	B	b	4	3	0	0	0	0	Ratio
			x_1	x_2	S_1	S_2	S_3	S_4	
3	x_2	200	0	1	1	0	-2	0	200
0	S_2	200	0	0	-1	1	1	0	200
4	x_1	400	1	0	0	0	1	0	400
0	S_4	500	0	0	-1	0	2	1	250
		z_j	4	3	3	0	-2	0	
		$C_j - z_j$	0	0	-3	0	2	0	

$$z = 4x_1 + 3x_2$$

$$= 4 \times 200 + 3 \times 600$$

$$= 800 + 1800 = 2600$$

C_B	B	b	4	3	0	0	0	0	Ratio
			x_1	x_2	S_1	S_2	S_3	S_4	
3	x_2	600	0	1	-1	2	0	0	
0	S_3	200	0	0	-1	1	1	0	
4	x_1	200	1	0	1	-1	0	0	
0	S_4	100	0	0	1	-2	0	1	
		z_j	4	3	1	2	0	0	
		$C_j - z_j$	0	0	-1	-2	0	0	