Two Phase Method

Only minimization problems will be Solved by Two phase method

⇒ Convert to minimization

⇒ Constraints

For '≥' or '=' type Constraint Substract Slack Variables and add artificial Variable.

For '≤' type Constraints add Slack Variable.

→ For Phase 1, But Objective function is revised

at the end of Phase 1 check whether the objective function value is zero in the optimal table

If yes ? ↓ Go to Phase 2 : No → Conclude No optimal Solution

Example $\quad \text{Min } z = 10x_1 + 6x_2 + 2x_3$

Subject to
$$-x_1 + x_2 + x_3 \geq 1$$
$$3x_1 + x_2 - x_3 \geq 2$$
$$x_1, x_2 \text{ and } x_3 \geq 0$$

$$\text{min } z = 10x_1 + 6x_2 + 2x_3 + 0S_1 + 0S_2 + A_1 + A_2$$

Sub to
$$-x_1 + x_2 + x_3 - S_1 + A_1 = 1$$
$$3x_1 + x_2 - x_3 - S_2 + A_2 = 2$$
$$x_1, x_2, x_3, S_1, S_2, A_1, \text{ and } A_2 \geq 0$$

$$\boxed{\text{Min } z = A_1 + A_2}$$

Subject to
$$-x_1 + x_2 + x_3 - S_1 + A_1 = 1$$
$$3x_1 + x_2 - x_3 - S_2 + A_2 = 2$$
$$x_1, x_2, x_3, S_1, S_2, A_1 \;\&\; A_2 \geq 0$$

C_{Bi}	C_j	0	0	0	0	0	1	1	Solution
	B.V	x_1	x_2	x_3	S_1	S_2	A_1	A_2	
1	A_1	-1	1	1	-1	0	1	0	1
1	A_2	3	1	-1	0	-1	0	1	2
	z_j	2	2	0	-1	-1	1	1	3
	$C_j - z_j$	-2	-2	0	1	1	0	0	

$C_j - z_j \geq 0$, $\leftarrow$ Should be non-negative.

Select the Smallest Value as key column

&

Select the Largest Value From the Solution as key Row.

Iteration 1

C_{B_i}	C_j / B.V	x_1 (0)	x_2 (0)	x_3 (0)	S_1 (0)	S_2 (0)	A_1 (1)	Solution
1	A_1	0	$4/3$	$2/3$	-1	$-1/3$	0	1
0	X_1	1	$1/3$	$-1/3$	0	$-1/3$	0	$2/3$
	Z_j	0	$4/3$	$2/3$	-1	$-1/3$	1	
	$C_j - Z_j$	0	$-4/3$	$-2/3$	1	$1/3$	0	

Iteration – II

C_{B_i}	C_j / B.V	x_1 (0)	x_2 (0)	x_3 (0)	S_1 (0)	S_2 (0)	Solution
0	X_2	0	1	$1/2$	$-3/4$	$-1/4$	$5/4$
0	X_1	1	0	$-1/2$	$1/4$	$-1/4$	$1/4$
	Z_j	0	0	0	0	0	0
	$C_j - Z_j$	0	0	0	0	0	

All the $C_j - Z_j \geq 0$.

at the End of the Phase – I, the Value of objective function must be 0.

Phase II

Do not include artificial Variable.

C_{B_i}	C_j / B.V	10 (x_1)	6 (x_2)	2 (x_3)	0 (S_1)	0 (S_2)	Solution
6	X_2	0	1	$1/2$	$-3/4$	$-1/4$	$5/4$
10	X_1	1	0	$-1/2$	$1/4$	$-1/4$	$1/4$
	z_j	10	6	-2	-2	-4	10
	$c_j - z_j$	0	0	4	2	4	

In linear programming problems sometimes we see that the constraints may have $\geq$, $\leq$ or = signs. In such problems, basis matrix is not obtained as an identity matrix in the first simplex table; therefore, we introduce a new type of variable called, the artificial variable. These variables are fictitious and cannot have any physical meaning. The introduction of artificial variable is merely to get starting basic feasible solution, so that simplex procedure may be used as usual until the optimal solution is obtained. Artificial variable can be eliminated from the simplex table as and when they become zero *i.e*, non–basic. This process of eliminating artificial variable is performed in **PHASE I of the solution. PHASE II** is then used for getting optimal solution. Here the solution of the linear programming problem is completed in two phases, this method is known as **TWO PHASE SIMPLEX METHOD**. Hence, the two–phase method deals with removal of artificial variable in the fist phase and work for optimal solution in the second phase. If at the end of the first stage, there still remains artificial variable in the basic at a positive value, it means there is no feasible solution for the problem given. In that case, it is not necessary to work on phase II. If a feasible solution exists for the given problem, the value of objective function at the end of phase I will be zero and artificial variable will be non-basic. In phase II original objective coefficients are introduced in the final tableau of phase I and the objective function is optimized.

Comparison between maximisaton case and minimisation case

S.No.	*Maximisation case*	*Minimisation case*
	Similarities:	
1.	It has an objective function.	This too has an objective function.
2.	It has structural constraints.	This too has structural constraints.
3.	The relationship between variables and constraintsis linear.	Here too the relationship between and variables constraints is linear.
4.	It has non-negativity constraint.	This too has non-negativity constraints.
5.	The coefficients of variables may be positive or negative or zero.	The coefficient of variables may be positive, Negative or zero.
6.	For selecting out going variable (key row) lowest replacement ratio is selected.	For selecting out going variable (key row) lowest replacement ratio is selected.
	Differences:	
1.	The objective function is of maximisation type.	The objective function is of minimisation type.
2.	The inequalities are of ≤ type.	The inequalities are of ≥ type.
3.	To convert inequalities into equations, **slack variables** are added.	To convert inequalities into equations, **surplus Variables are subtracted and artificial surplus variables are added.**
4.	While selecting incoming variable, highest positive Opportunity cost is selected from net evaluation Row.	While slecting, incoming variable, lowest element in the net evaluation row is selected (highest number with negative sign).
5.	When the elements of net evaluation row are either Negative or zeros, the solution is optimal	When the element of net evaluation row are either positive or zeros the solution is optimal.

Solve the Minimization LPP.

$$\text{Min } z = 1x - 3y + 2z$$

$$3x - 1y + 3z \le 7$$

$$-4x + 3y + 8z \le 10$$

$$\text{Max } z = -x + 3y - 2z$$

Q

$$\text{Max } z = 4a + 5b$$

$$2a + 4b \le 8$$

$$1a + 3b \ge 9$$

Solⁿ $z = 0a + 0b + 0S_1 + 0S_2 - A_1$

$$2a + 4b + S_1 = 8$$

$$a + 3b + 0S_1 - S_2 + A_1 = 9$$

$a = 0 \quad b = 0 \quad S_1 = 8 \quad S_2 = 0 \quad A_1 = 9 \quad z = -9$

C_B	B	Solⁿ	0 a	0 b	0 S_1	0 S_2	-1 A_1	Ratio
0	S_1	8	2	4	1	0	0	2
-1	A_1	9	1	3	0	-1	1	3
	z_j		-1	-3	0	1	-1	
	$C_j - z_j$		1	3	0	-1	0	

$a = 0 \quad b = 2 \quad S_1 = 0 \quad S_2 = 0 \quad A_1 = 3 \quad z = -3$

C_B	B	Solⁿ	0 a	0 b	0 S_1	0 S_2	-1 A_1	Ratio
0	b	2	$1/2$	1	$1/4$	0	0	
-1	A_1	3	$-1/2$	0	$-3/4$	-1	1	
	z_j		$1/2$	0	$3/4$	1	-1	
	$C_j - z_j$		$-1/2$	0	$-3/4$	-1	0	