Simplex Method

$$Z = 3x_1 + 2x_2 + 5x_3$$

$$x_1 + 2x_2 + x_3 \leq 430$$

$$3x_1 + 2x_3 \geq 260$$

$$x_1 + 4x_2 + x_3 \leq 420$$

$$x_1, x_2, x_3 \geq 0$$

Convert these from general form to Standard

1) Check wheater the objective function of LPP is maximized or minimized.

If it is to be minimized then we convert it into maximization

$$\text{Max } Z' = -\text{Min } Z$$

1) Max $Z = 3x_1 + 2x_2$

Subject to = $x_1 + x_2 \leq 4$

$x_1 - x_2 \leq 2$

$x_1, x_2 \geq 0$

This Remains as it is.

(2) Min $z = 3x_1 + 2x_2 + 5x_3$

Subject to
$$x_1 + 2x_2 + x_3 \le 430$$
$$3x_1 + 2x_3 \le 260$$
$$x_1 + 4x_2 + x_3 \le 420$$
$$x_1, x_2, x_3 \ge 0$$

SLPP

$$\text{Max } z' = -3x_1 - 2x_2 - 5x_3$$

Note → if $x_1 > 0$, & x_2 is unrestricted

$$2x_1 + x_2 \le 4 \qquad x_1 \ge 0$$

& x_2 is unrestricted.

$$\Rightarrow 2x_1 + (x_2' - x_2'') \le 4$$
$$x_1, x_2', x_2'' \ge 0$$

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1) Max $z = 3x_1 + 2x_2$

Subject to = $x_1 + x_2 \le 4$
$$x_1 - x_2 \le 2$$
$$x_1, x_2 \ge 0$$

This Remains as it is.

$$z = 3x_1 + 2x_2 + 0S_1 + 0S_2$$
$$x_1 + x_2 + S_1 = 4$$
$$x_1 - x_2 + S_2 = 2$$

S_1 and S_2 are slack variables with cost 0.

SLPP

$$\text{Max } z' = -3x_1 - 2x_2 - 5x_3$$

St

$$x_1 + 2x_2 + x_3 + S_1 \leq 430$$

$$3x_1 + 2x_3 \leq 260$$

$$x_1 + 4x_2 + x_3 \geq 420$$

$$x_1, x_2, x_3 \geq 0$$

$$\text{Max } z' = -3x_1 - 2x_2 - 5x_3$$

St

$$x_1 + 2x_2 + x_3 + S_1 = 430$$

$$3x_1 + 2x_3 + S_2 = 260$$

$$x_1 + 4x_2 + x_3 - S_3 = 420$$

S_1 & S_2 are Slack Variable and S_3 is surplus Variable with Cost 0.

$\leq$ add Slack var
$\geq$ Subs Surplus Var

Check whether all the b_i $(i=1,2,\dots n)$ are positive or not. If any b_i is −ve then multiply the inequation of Constraints by −1.

Q Max $z = 3x_1 + 2x_2 + 5x_3$

Subject to

$$x_1 + 2x_2 + x_3 \leq 430$$

$$3x_1 + 2x_3 \leq 460$$

$$x_1 + 4x_2 \leq 420$$

$$x_1, x_2, x_3 \geq 0$$

Maximize $Z = 12x_1 + 16x_2$

$$10x_1 + 20x_2 \le 120$$
$$8x_1 + 8x_2 \le 80$$
$$x_1, x_2 \ge 0$$

Sol^n Max $Z = 12x_1 + 16x_2 + 0S_1 + 0S_2$

$$10x_1 + 20x_2 + S_1 = 120$$
$$8x_1 + 8x_2 + S_2 = 80$$
$$x_1, x_2, S_1 \& S_2 \ge 0$$

Key element

Initial Simplex table

CBi	C_j	12	16	0	0	Solution	Ratio
	Basic Variable	X_1	X_2	S_1	S_2		
0	S_1	10	20	1	0	120	$\frac{120}{20} = 6$
0	S_2	8	8	0	1	80	$\frac{80}{8} = 10$
	Z_j	0	0	0	0	0	
$C_j - Z_j$		12	16	0	0		

C_j = Coefficient of the objective function.

$$Z_j = \sum_{i=1}^{2} (CB_i)(a_{ij})$$

We have two basic variables.

$$z_j = (0*10) + (0*8) = 0$$
$$(0*20) + (0*8) = 0$$
$$(0*1) + (0*0) = 0$$
$$(0*0) + (0*1) = 0$$

Now Compare $c_j - z_j$

$$12 - 0 = 12$$
$$16 - 0 = 16$$
$$0 - 0 = 0$$
$$0 - 0 = 0$$

Optimality Condition:

For Max: all $c_j - z_j \leq 0$

For Min: all $c_j - z_j \geq 0$

But $c_j - z_j \geq 0$ so, we did'nt reach optimality. So, we need to Solve further

Select the Max $(c_j - z_j) = 16$

The Max $c_j - z_j$ column will be key Column.

Now find the Ratio between Solutn and key Column.

$\Rightarrow \quad \cancel{120/20} = 6 \,, \quad \cancel{80/8} = 10$

Select the Minimum Ratio.

The Row with minimum Ratio is key Row. x_1 = Entering, S_2 = leaving

Iteration - I

CB_i	C_j	12	16	0	0	Solution	Ratio
	Basic Variable	x_1	x_2	S_1	S_2		
16	x_2	$1/2$	1	$1/20$	$0/20$	6	
0	S_2	4	0	$-2/5$	1	32	
	Z_j	8	16	$4/5$	0	0	
	$C_j - Z_j$	4	0	$-4/5$	0		

Divide the Constraint Coefficient with key element

$$\boxed{\text{New Value} = \text{old value} - \frac{\text{Corr. key Column value} \times \text{corr. key row value}}{\text{key element}}}$$

$$= 8 - \frac{8 \times 10}{20}$$

$$= \boxed{4}$$

$$\Rightarrow \quad 8 - \frac{8 * 20}{20} = \boxed{0}$$

$$\Rightarrow \quad 0 - \frac{8 * 1}{20} = \boxed{-\frac{2}{5}}$$

$$\Rightarrow \quad 1 - \frac{8 * 0}{20} = \boxed{1}$$

$$\Rightarrow \quad 80 - \frac{8 \times 120}{20} = 32$$

$$z_1 = 16 * \frac{1}{2} + 0 * 4 = 8$$

$$z_2 = \qquad = 16$$

$$z_3 = 16 * \frac{1}{20} + 0 * \frac{-2}{5} = \frac{4}{5}$$

$$z_4 = 0$$

Still we have 1 positive value

So we will proceed further

Iteration - I

CB_i	C_j / Basic Variable	12 X_1	16 X_2	0 S_1	0 S_2	Solution	Ratio
16	X_2	$1/2$	1	$1/20$	$0/20$	6	$\frac{6}{1/2} = 12$
0	S_2	4	0	$-2/5$	1	32	$\frac{32}{4} = 8$
	Z_j	8	16	$4/5$	0	0	
	$C_j - Z_j$	4	0	$-4/5$	0		

S_2 is leaving variable, X_1 is entering variable

Iteration - II

CB_i	C_j / Basic Variable	12 X_1	16 X_2	0 S_1	0 S_2	Solution	Ratio
16	X_2	0	1	$1/10$	$-1/8$	2	
12	X_1	1	0	$-1/10$	$1/4$	8	
	Z_j	12	16	$\frac{4}{10}$	1	128	
	$C_j - Z_j$	0	0	$\frac{-4}{10}$	-1		

$$x_1 x_1 = \frac{1}{2} - \frac{\frac{1}{2} * 4}{4} = 0$$

$$x_2 x_2 = 1 - \frac{\frac{1}{2} * 0}{4} = 1$$

$$x_2 s_1 = \frac{1}{20} - \frac{\frac{1}{2} * \frac{-2}{5}}{4} = 0$$

$$x_2 s_2 = 0 - \frac{\frac{1}{2} * 1}{4} = \frac{-1}{8}$$

$$= 6 - \frac{\frac{1}{2} * 32}{4} = 2$$

All the $C_j - Z_j$ values should be less than 0.

$x_1 = 12$ & $Z(\text{opt}) = 128$.

$x_2 = 16$

LPP using Simple Method

1. Solve the following LPP using Simplex Method.

Minimize $Z = 2x_1 - 3x_2 + 6x_3$

Subject to
$$3x_1 - x_2 + 2x_3 \leq 7$$
$$2x_1 + 4x_2 \geq -12$$
$$-4x_1 + 3x_2 + 8x_3 \leq 10$$
$$x_1, x_2, x_3 \geq 0$$

Sol^n This is a minimization method.

$\rightarrow$ $3x_1 - x_2 + 2x_3 + S_1 \cancel{\leq} 7$

$\rightarrow$ $-2x_1 - 4x_2 + S_2 \cancel{\leq} 12$

$-4x_1 + 3x_2 + 8x_3 + S_3 \cancel{\leq} 10$

$$Z = 2x_1 - 3x_2 + 6x_3 + 0S_1 + 0S_2 + 0S_3$$

CB:	C_j	2	−3	6	0	0	0	Sol^n	Ratio
	Basic Variable	x_1	x_2	x_3	S_1	S_2	S_3		
0	S_1	3	−1	2	1	0	0	7	
0	S_2	−2	−4	0	0	1	0	12	
0	S_3	−4	3	8	0	0	1	10	
	Z_j	0	0	0	0	0	0	0	
	$C_j - Z_j$	2	−3	6	0	0	0		

Optimality Condition :

For minimization Problem :

All the $C_j - Z_j \geq 0$ / $Z_j - C_j \leq 0$

For maximization Problem :

All the $C_j - Z_j \leq 0$ / $Z_j - C_j \geq 0$

So, we will select the Smallest Value.

CB:	C_j / Basic Variable	2 x_1	-3 x_2	6 x_3	0 S_1	0 S_2	0 S_3	Sol^n	Ratio
0	S_1	3	-1	2	1	0	0	7	-7
0	S_2	-2	-4	0	0	1	0	12	$-12/4 = -3$
0	S_3	-4	3	8	0	0	1	10	$10/3$
	Z_j	0	0	0	0	0	0	0	
	$C_j - Z_j$	2	-3	6	0	0	0		

After finding key Column, now find key Row.

Select the Most Negative, from $\boxed{C_j - Z_j}$

Intersection = 3 is called key element.

Note :- If all the Ratio are negative, the solut^n is unbounded.

Select the smallest positive value from Ratio.

Iteration – I

CB:	C_j / Basic Variable	2 x_1	−3 x_2	6 x_3	0 S_1	0 S_2	0 S_3	Solⁿ	Ratio
0	S_1								
0	S_2								
−3	x_2	$-4/3$	$3/3$	$8/3$	0	0	$1/3$	$10/3$	
	Z_j								
	$C_j - Z_j$								

CB:	C_j / Basic Variable	2 x_1	−3 x_2	6 x_3	0 S_1	0 S_2	0 S_3	Solⁿ	Ratio
0	S_1	3	−1	2	1	0	0	7	−7
0	S_2	−2	−4	0	0	1	0	12	$-12/4 = -3$
0	S_3	−4	3	8	0	0	1	10	$10/3$
	Z_j	0	0	0	0	0	0	0	
	$C_j - Z_j$	2	−3	6	0	0	0		

Old Value − [Corr. key Col. value] x [Corr. new row value]