# Alpha-beta Pruning

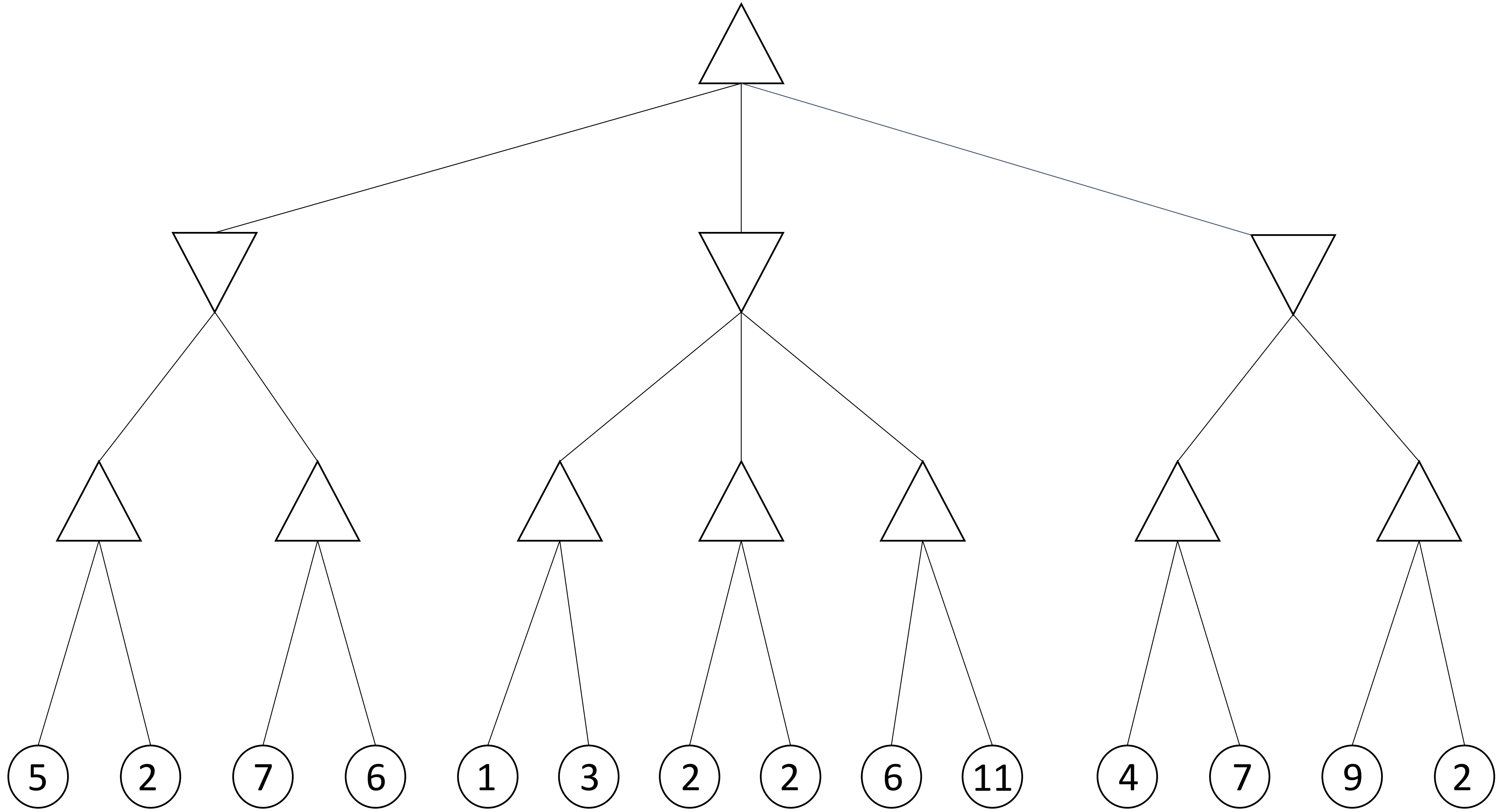
## The trouble with Minimax

- The full Minimax search of all possible ways a game could be played takes time.
- If there is any time constraint – and there always is! – then it may not be possible to explore the whole game tree.
- A **lookahead** limit solves this, but fast **heuristics** tend to be imperfect – a game state might be evaluated as better or worse than it turns out to be when actually playing.
- Another solution is to **prune** some of the branches –never look at some possible moves.

## Alpha-Beta Pruning

Alpha-beta pruning is an extension of Minimax which allows you to avoid searching subtrees of moves which do not lead to the optimal minimax solution.

It will always return the same solution as Minimax, but will usually do so faster.



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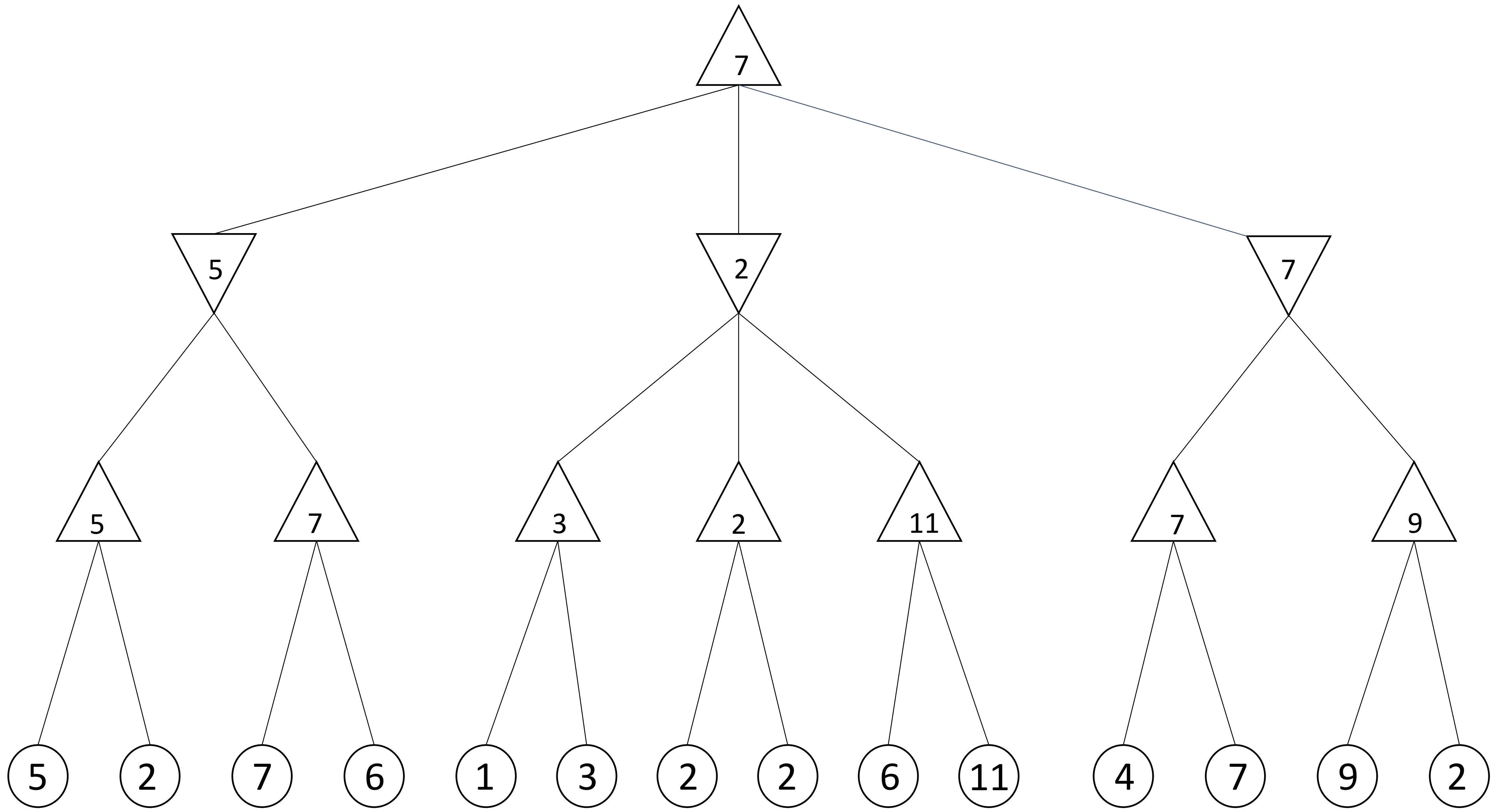
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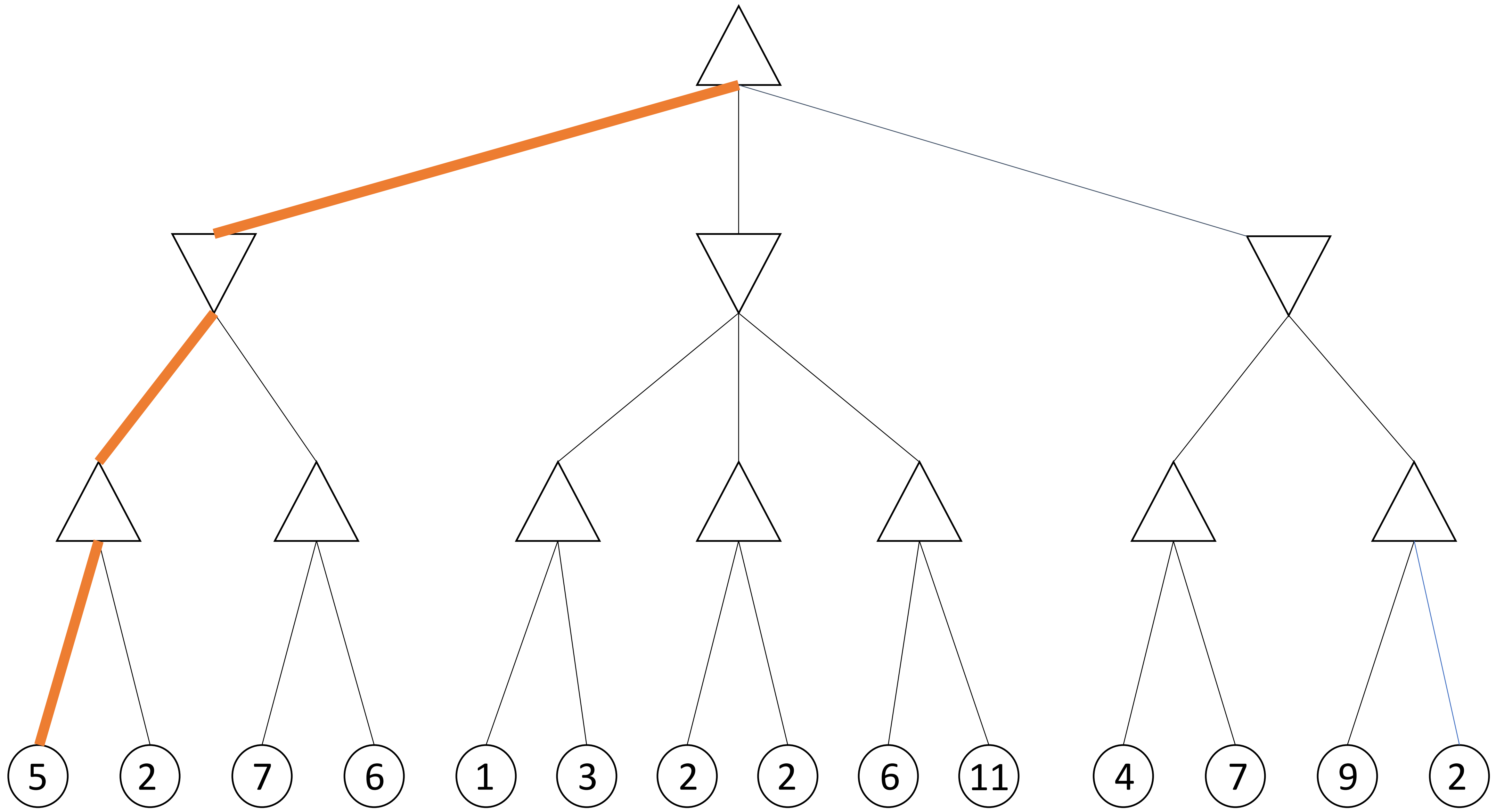
path we are currently exploring ━━

path to currently best-known solution ━━

explored but not the best path ─

pruned (not explored) branches ╌╌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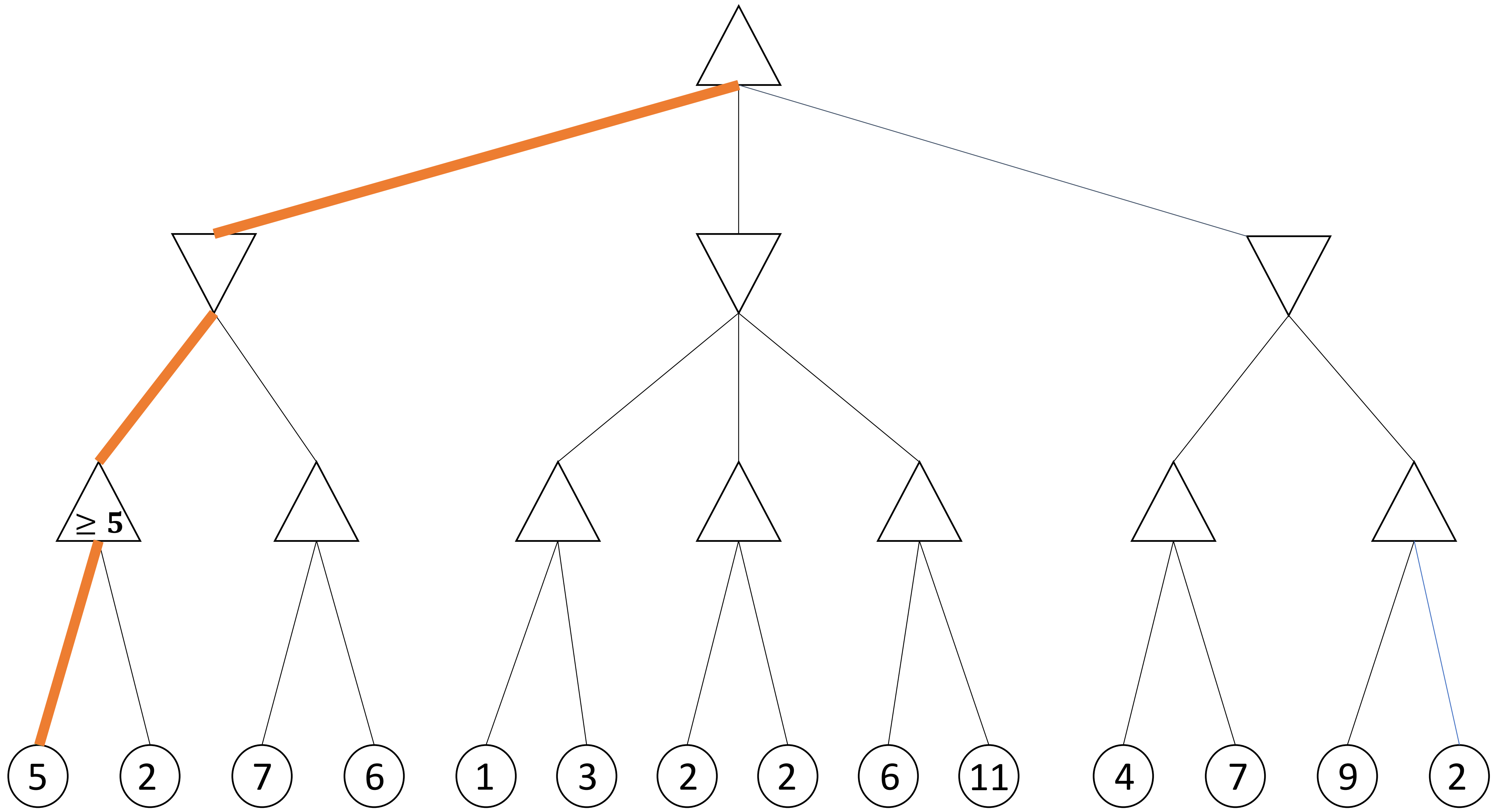
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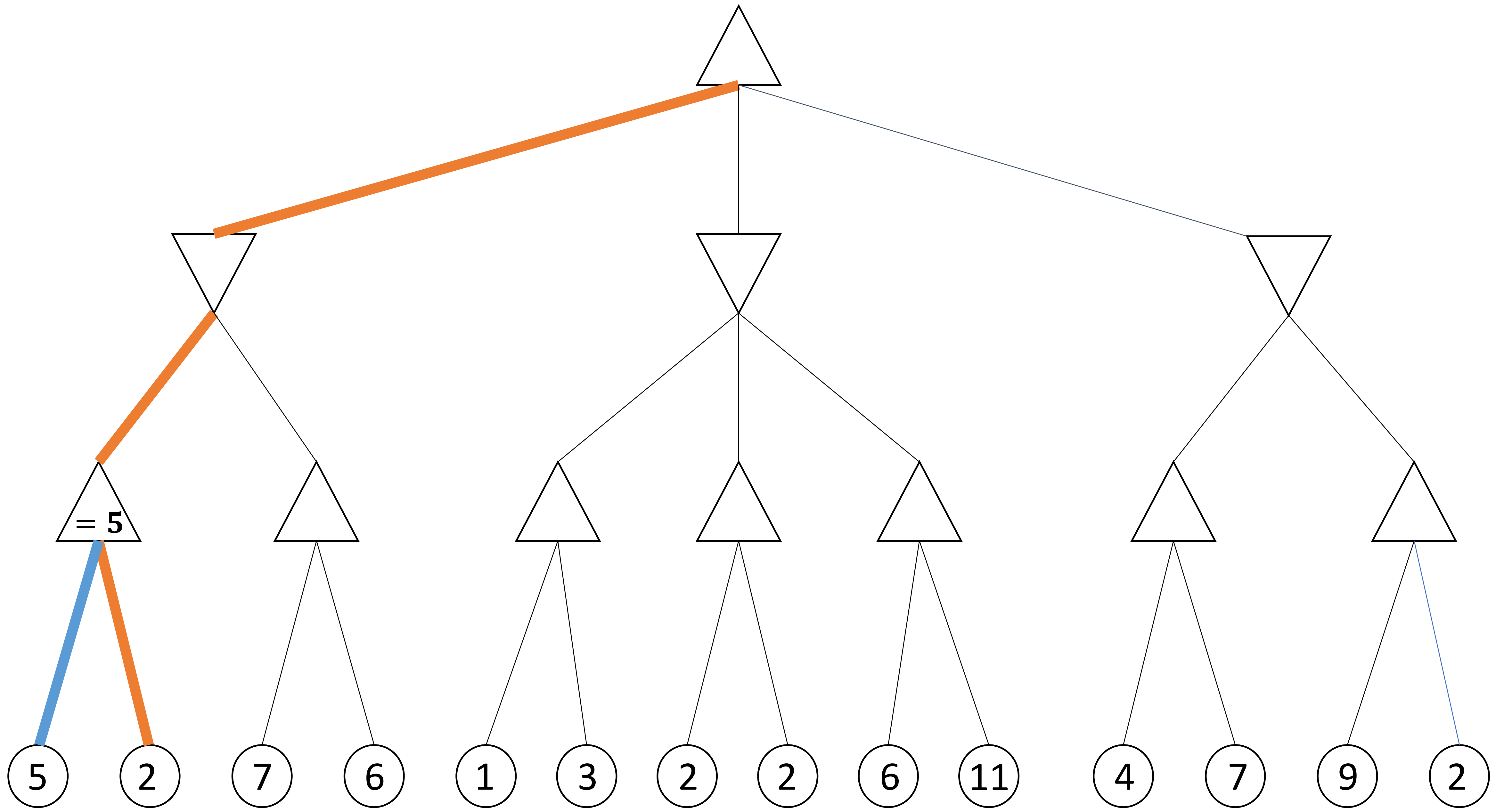
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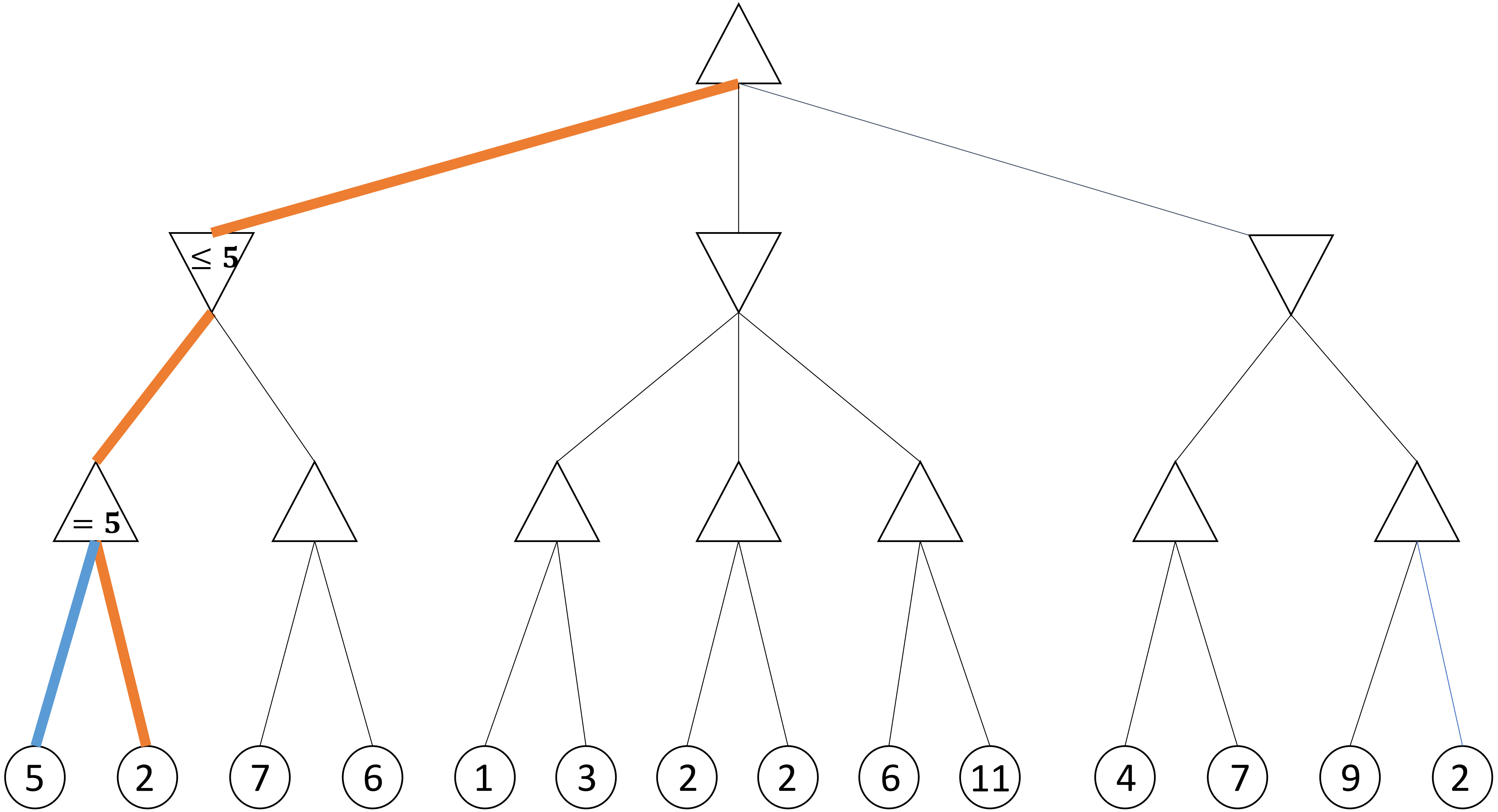
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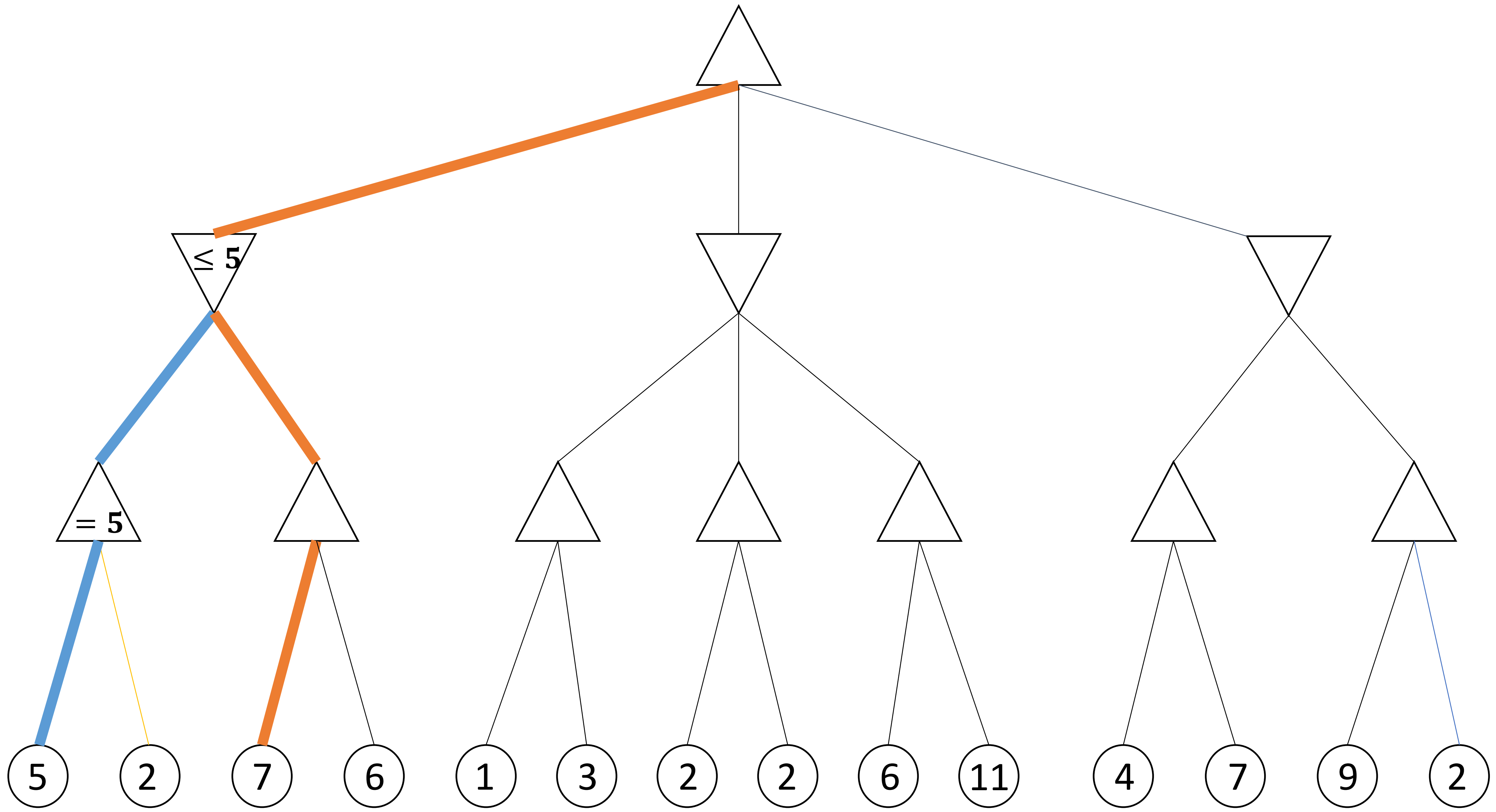
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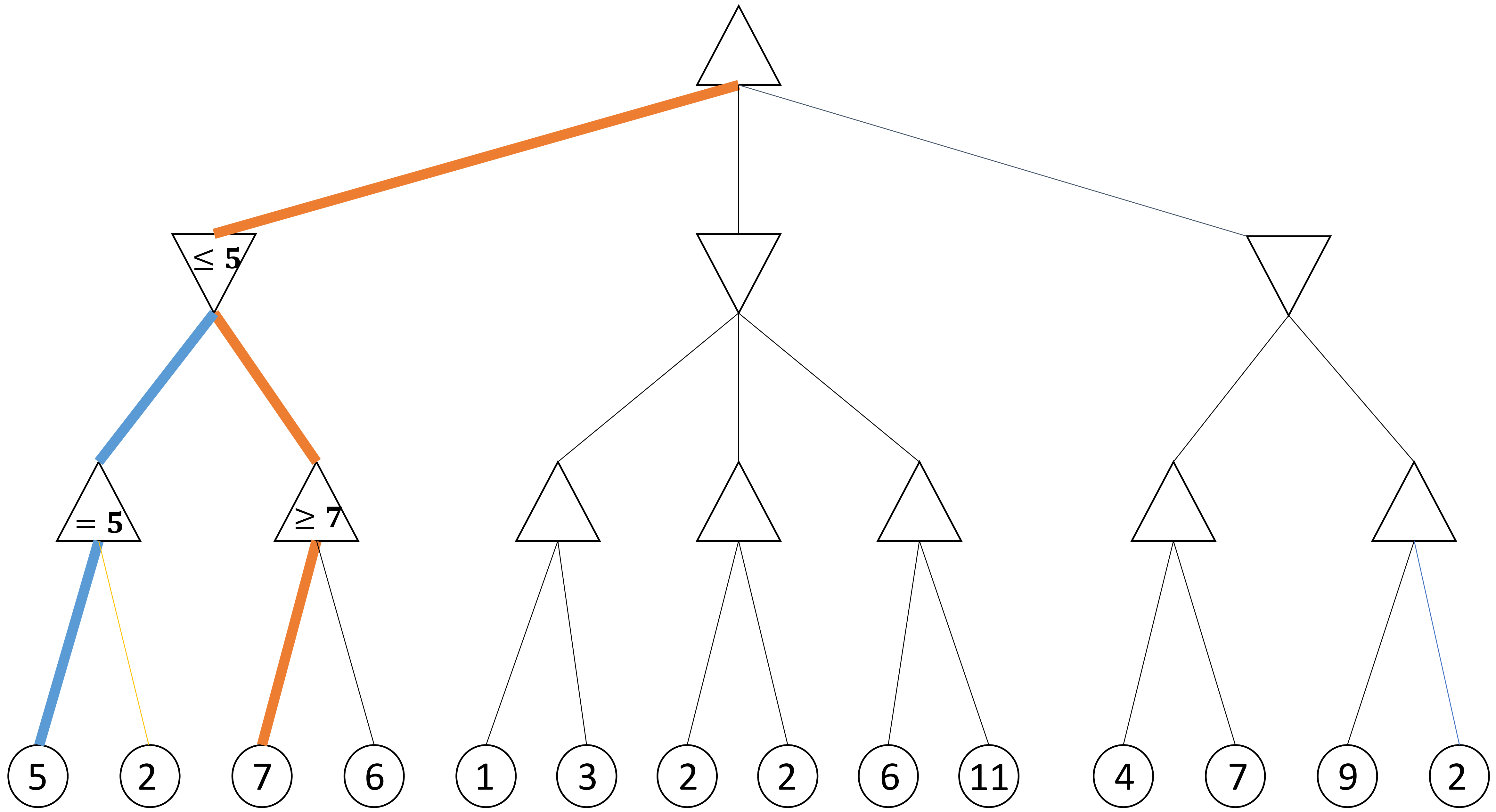

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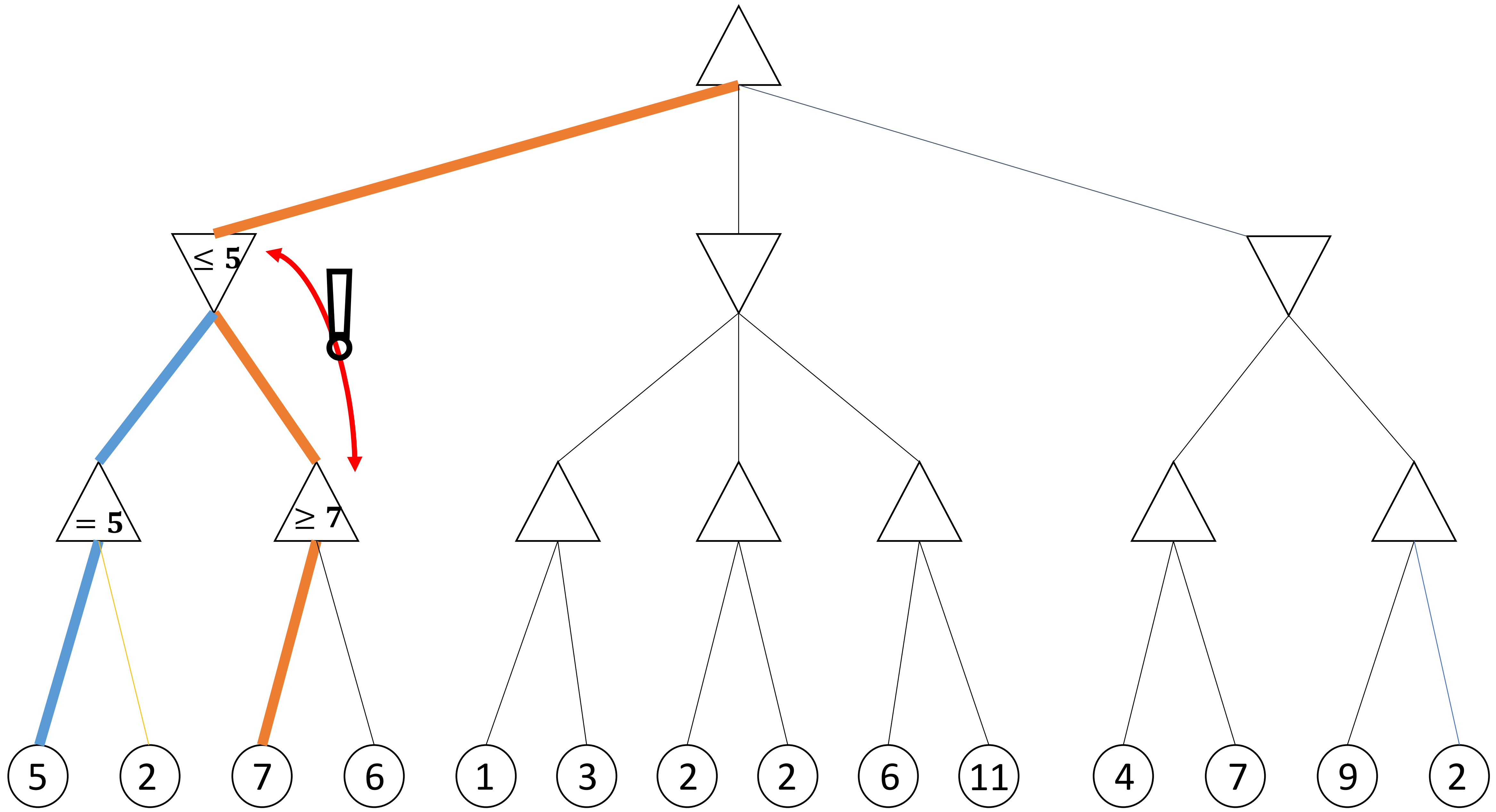
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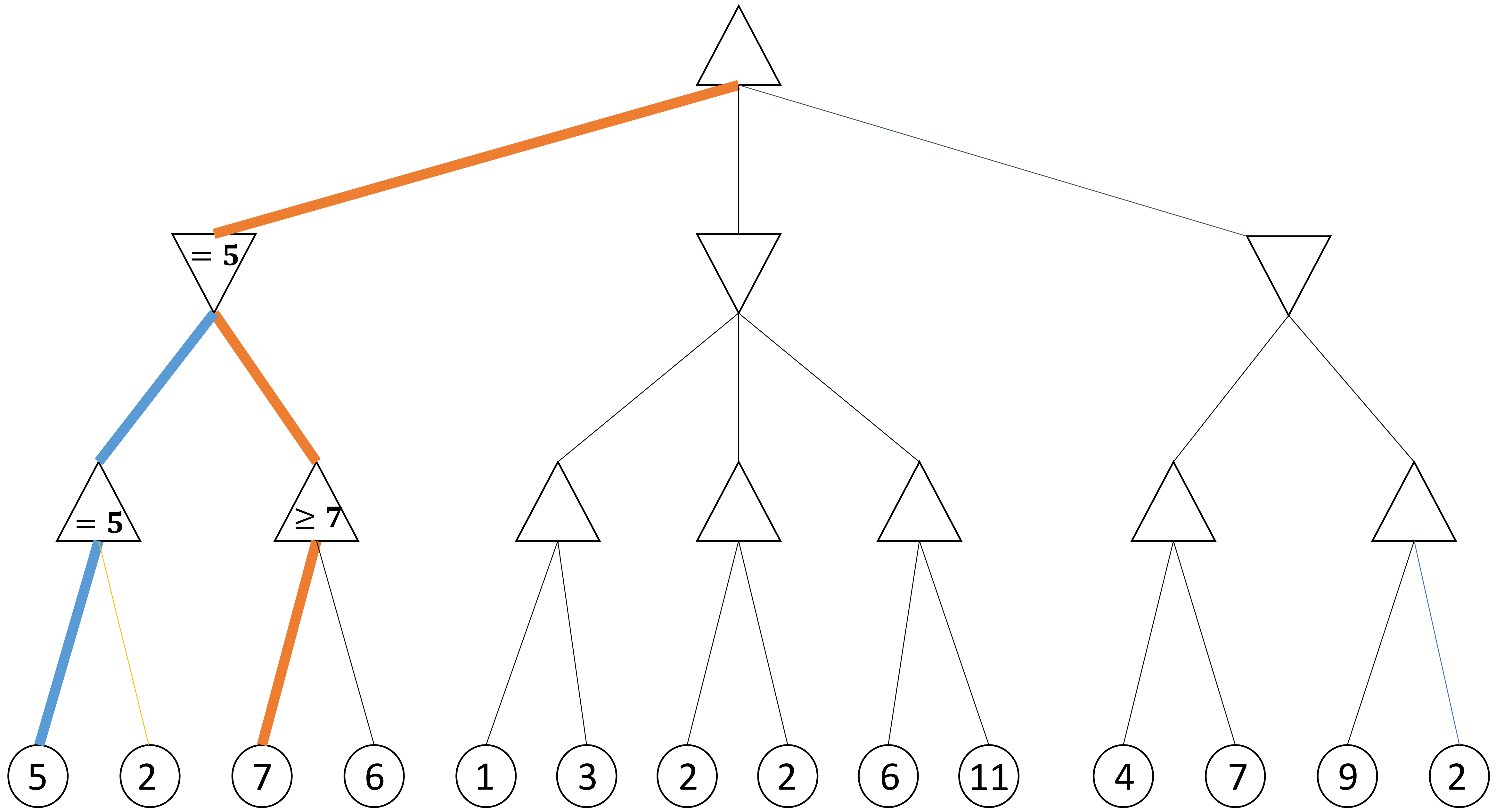
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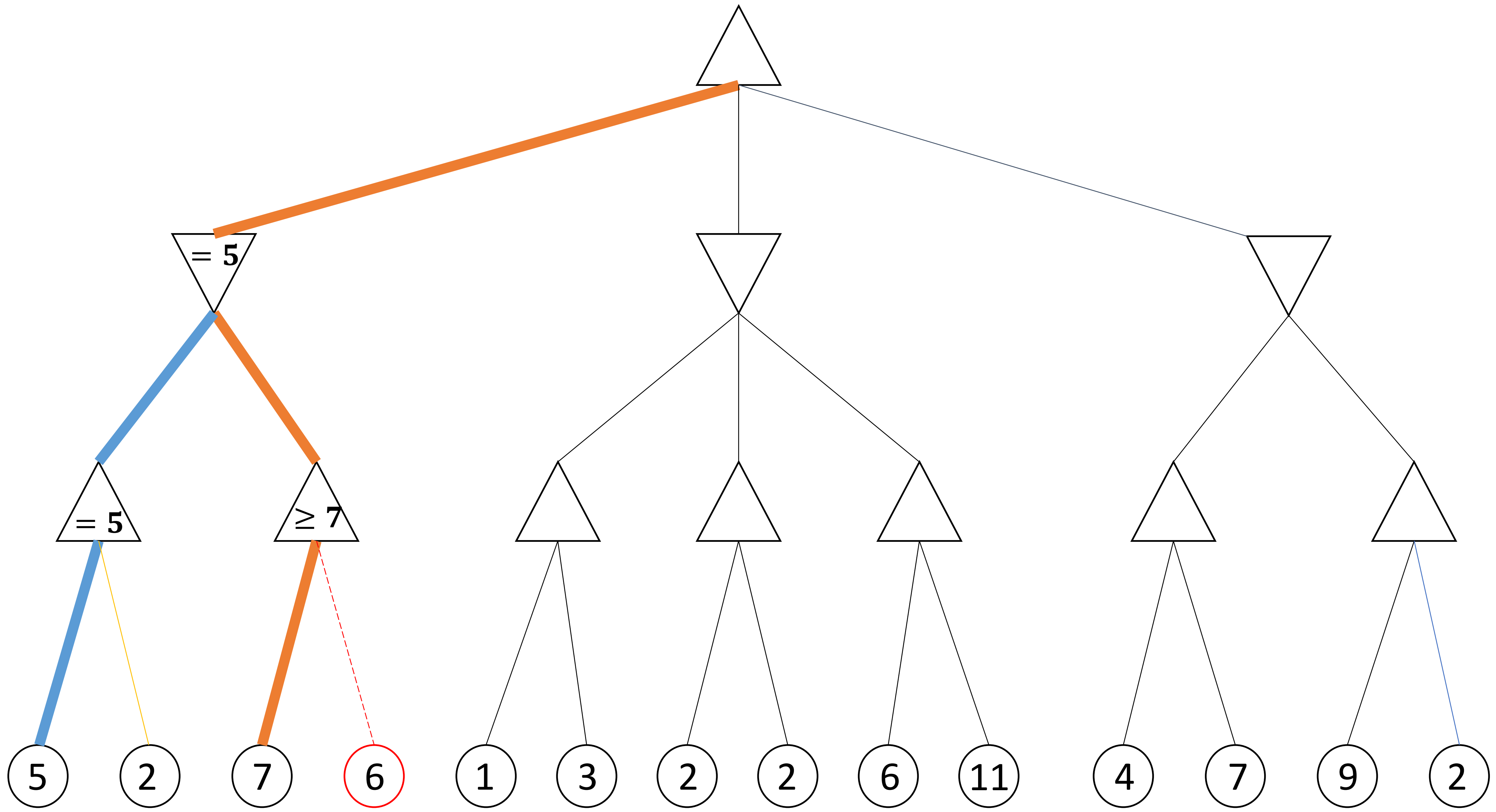

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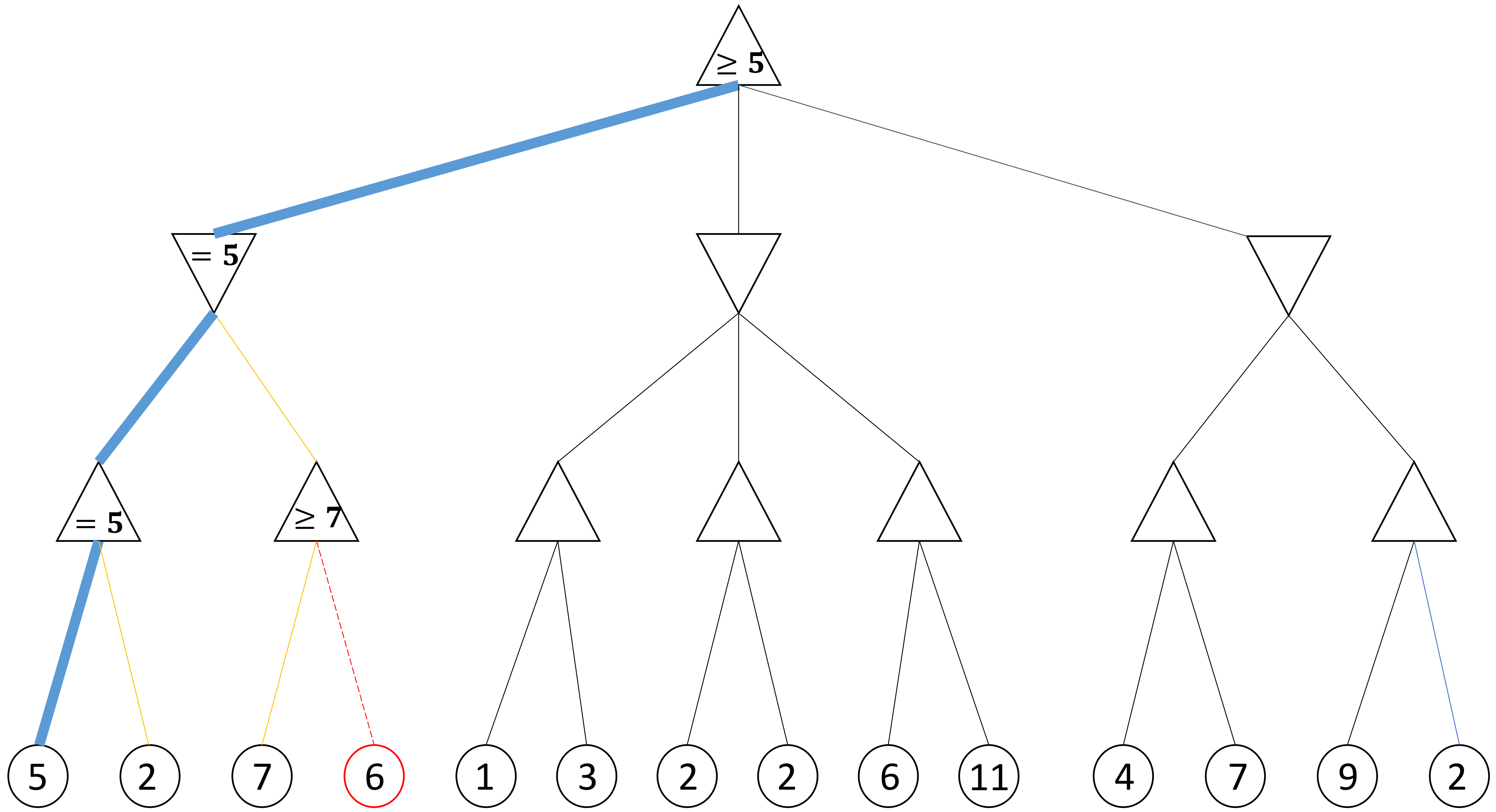
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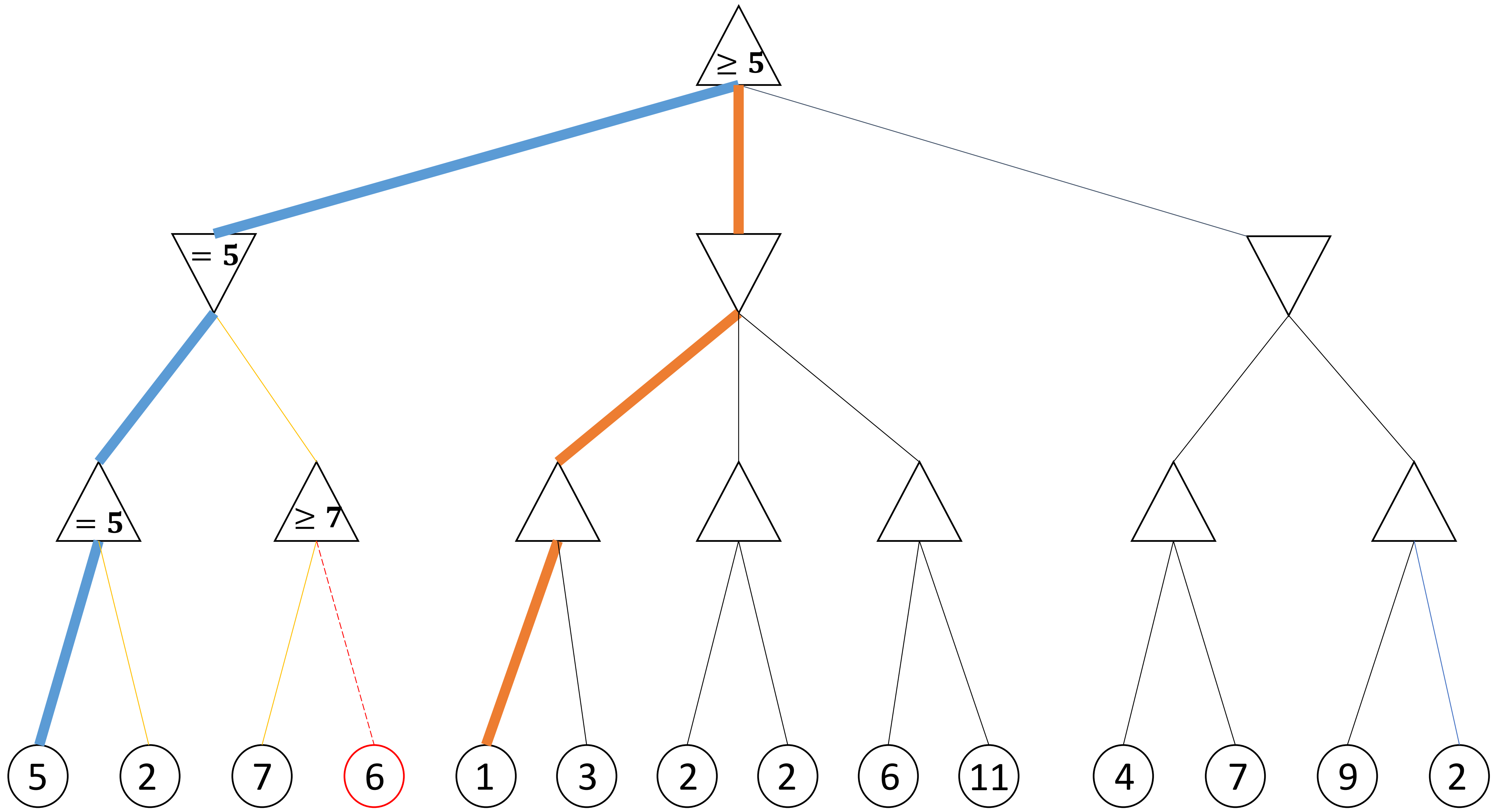
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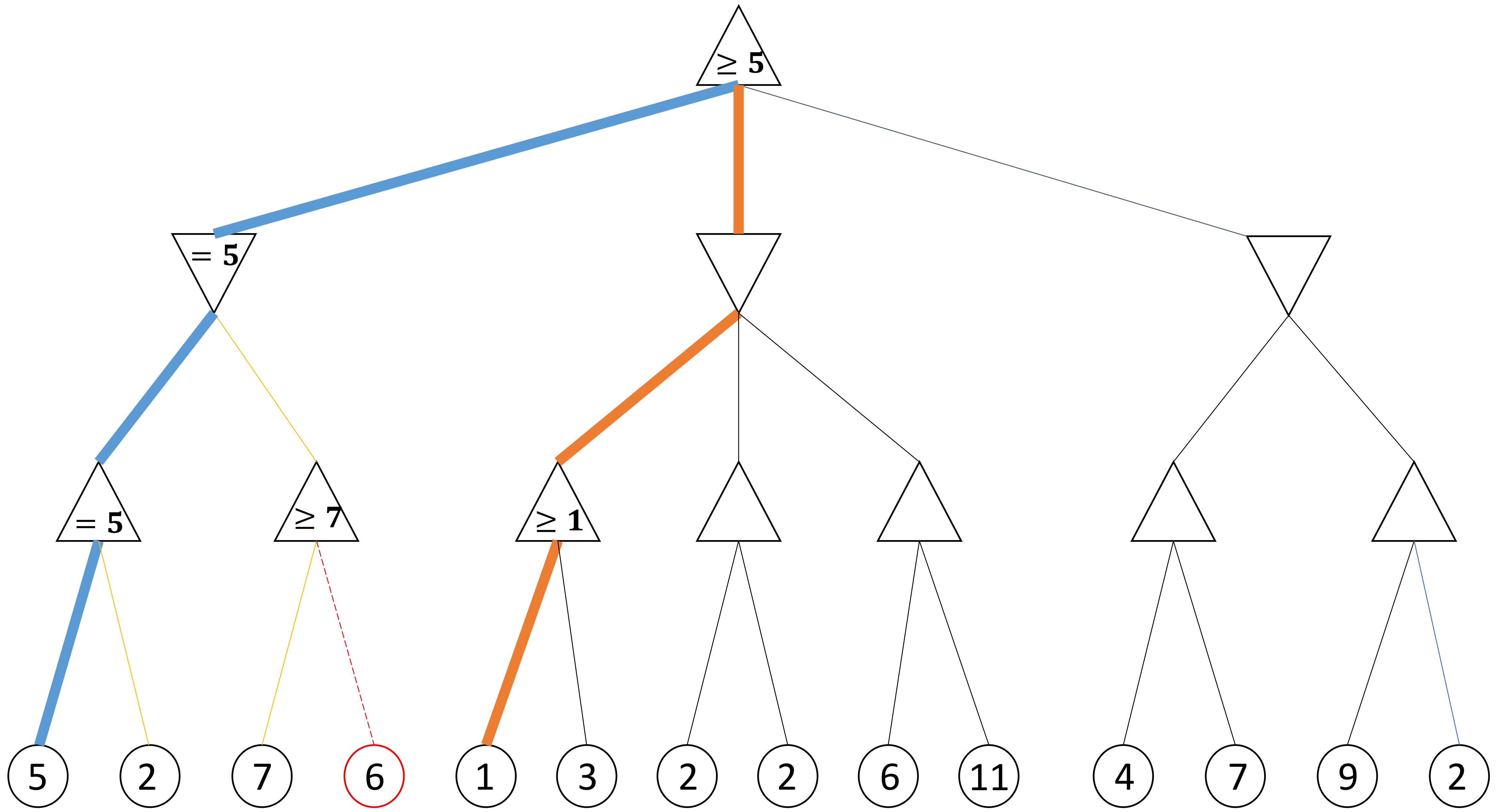
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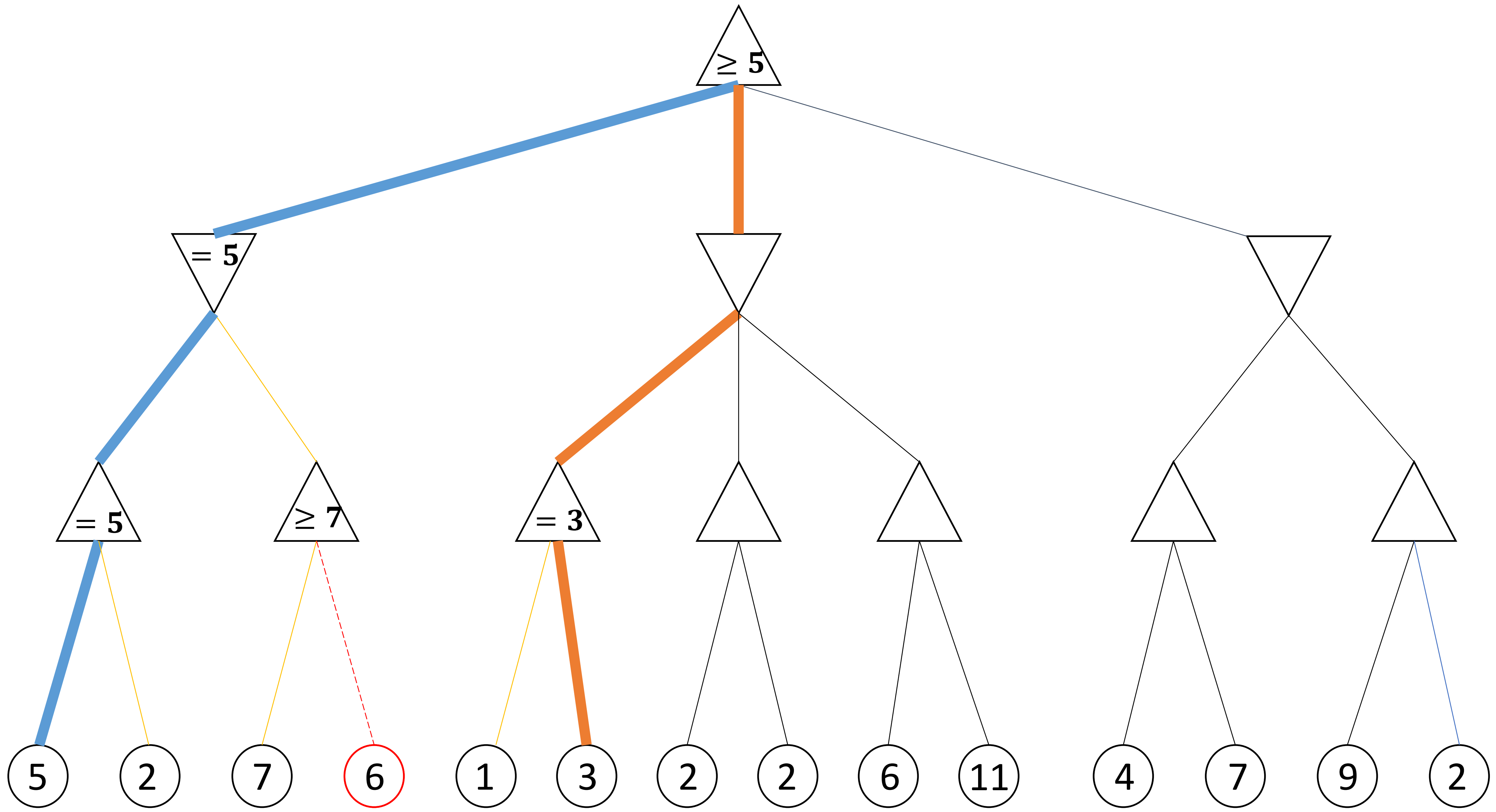
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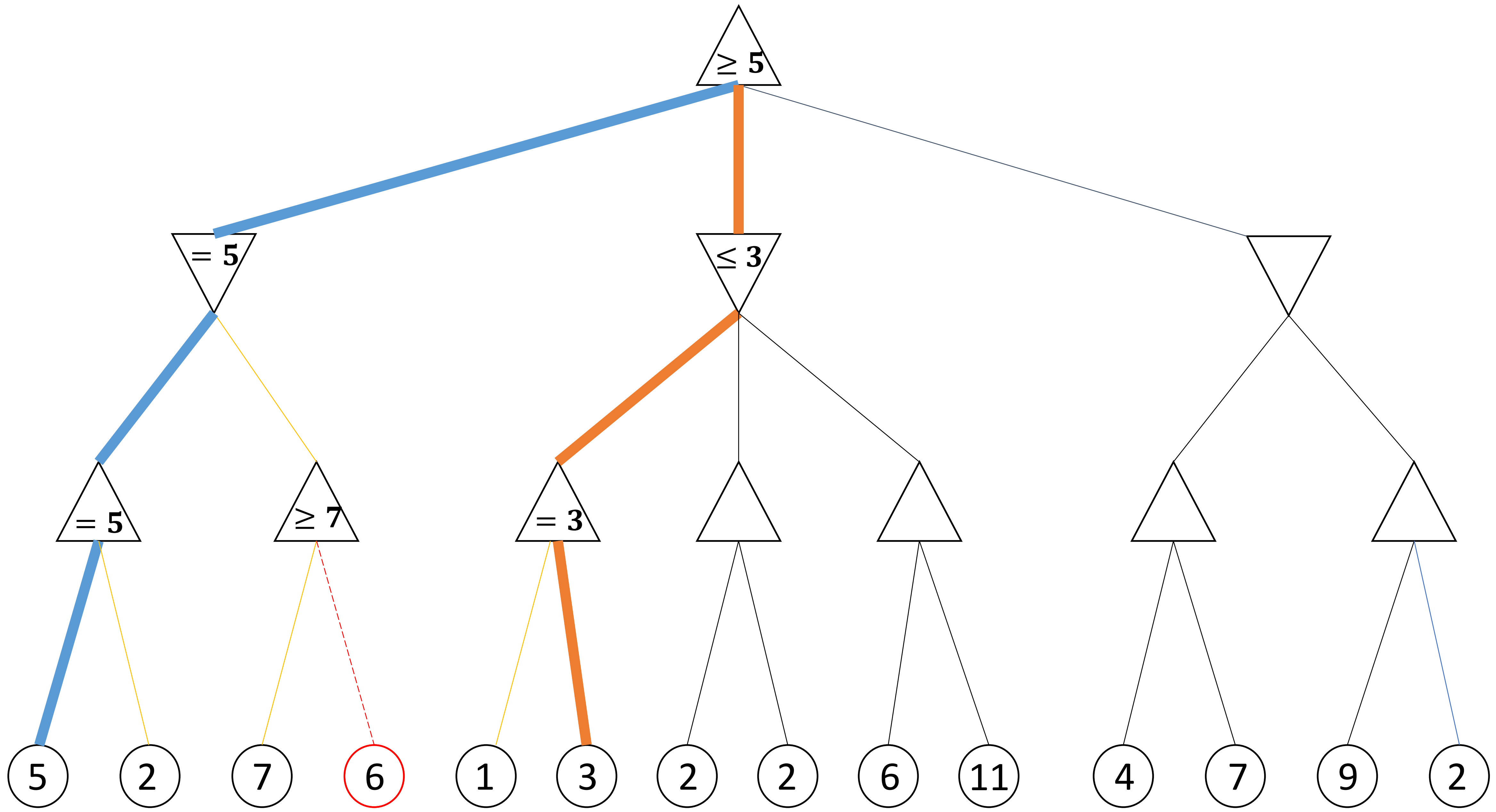
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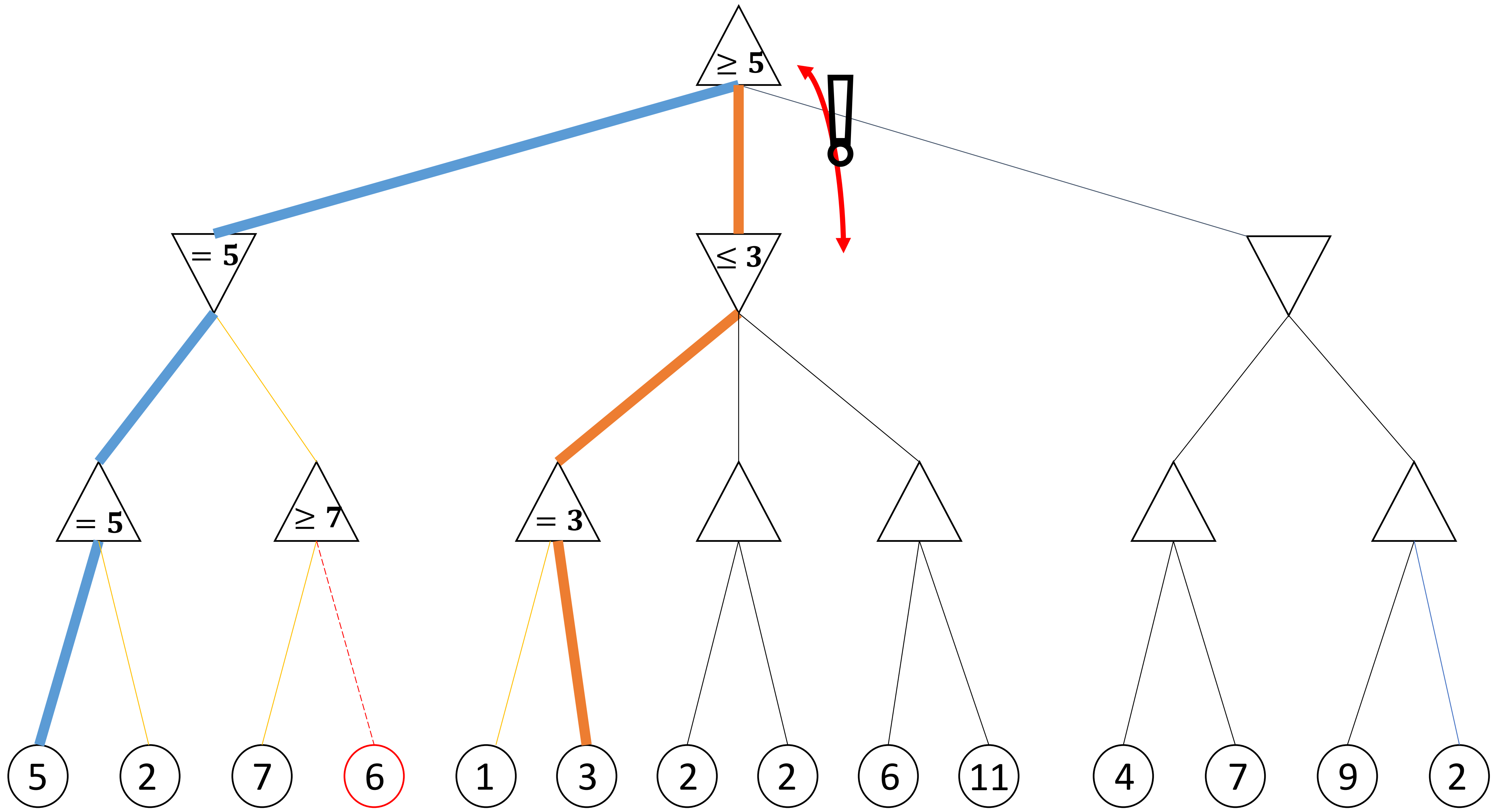

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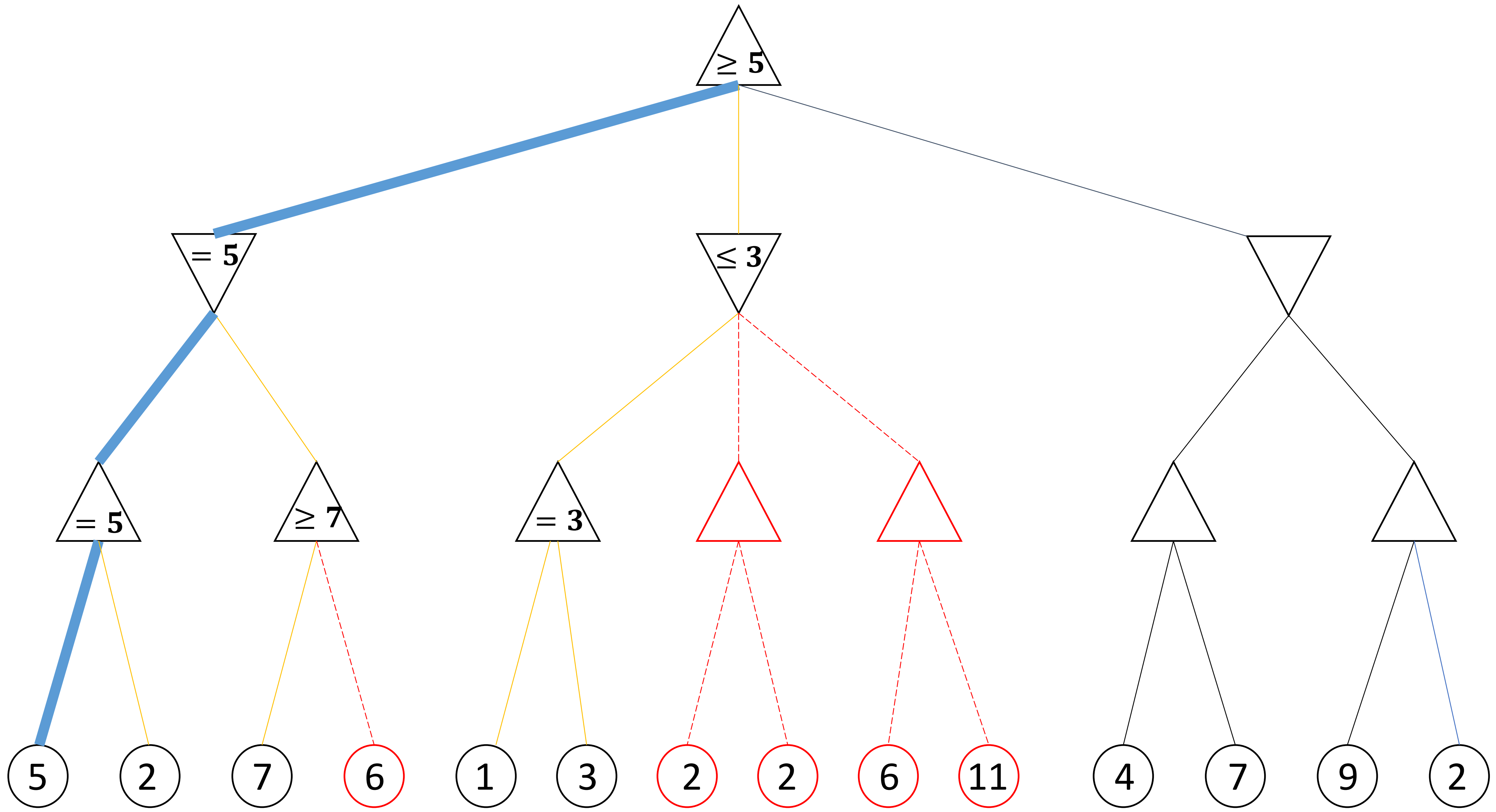
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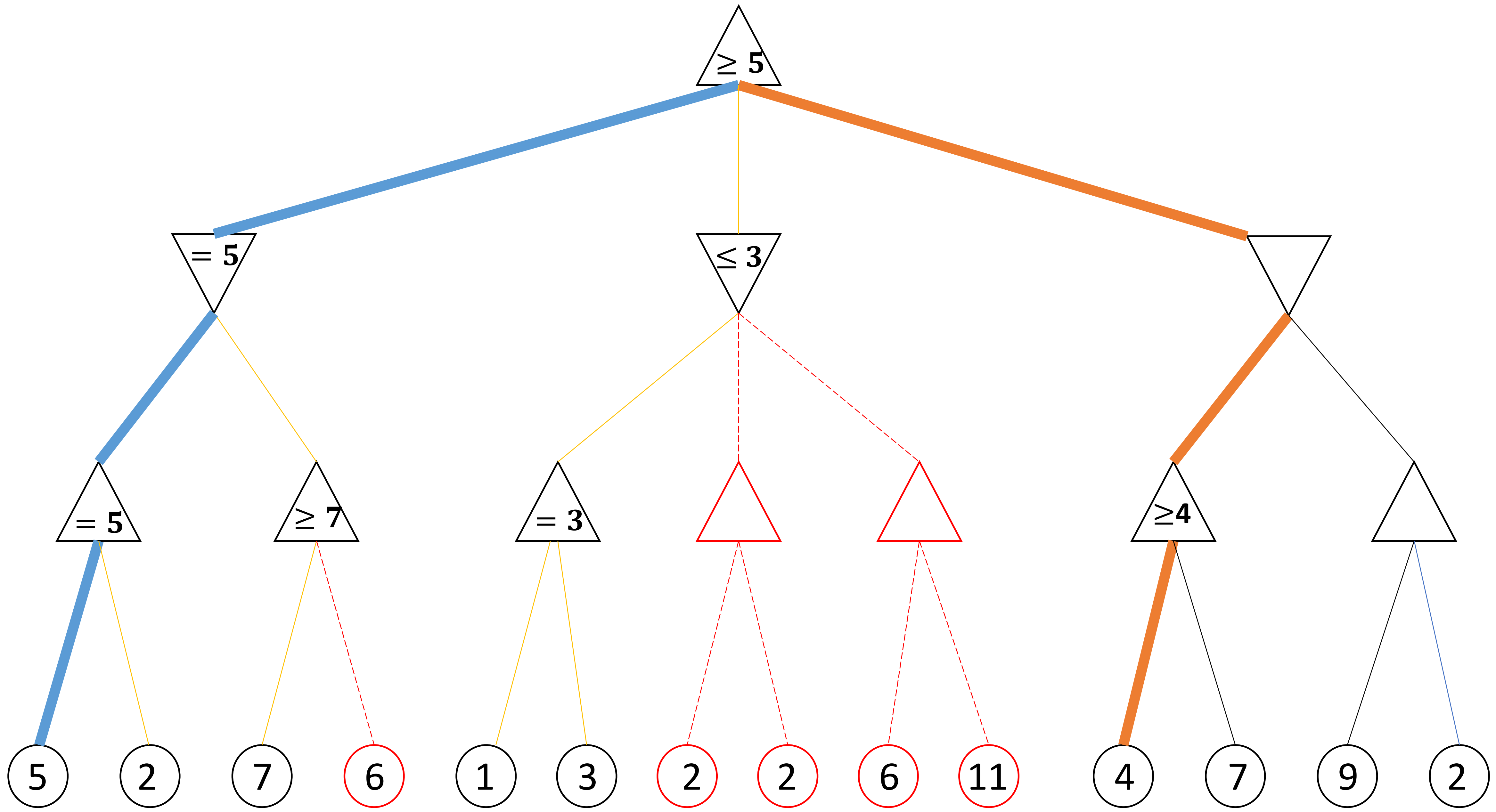

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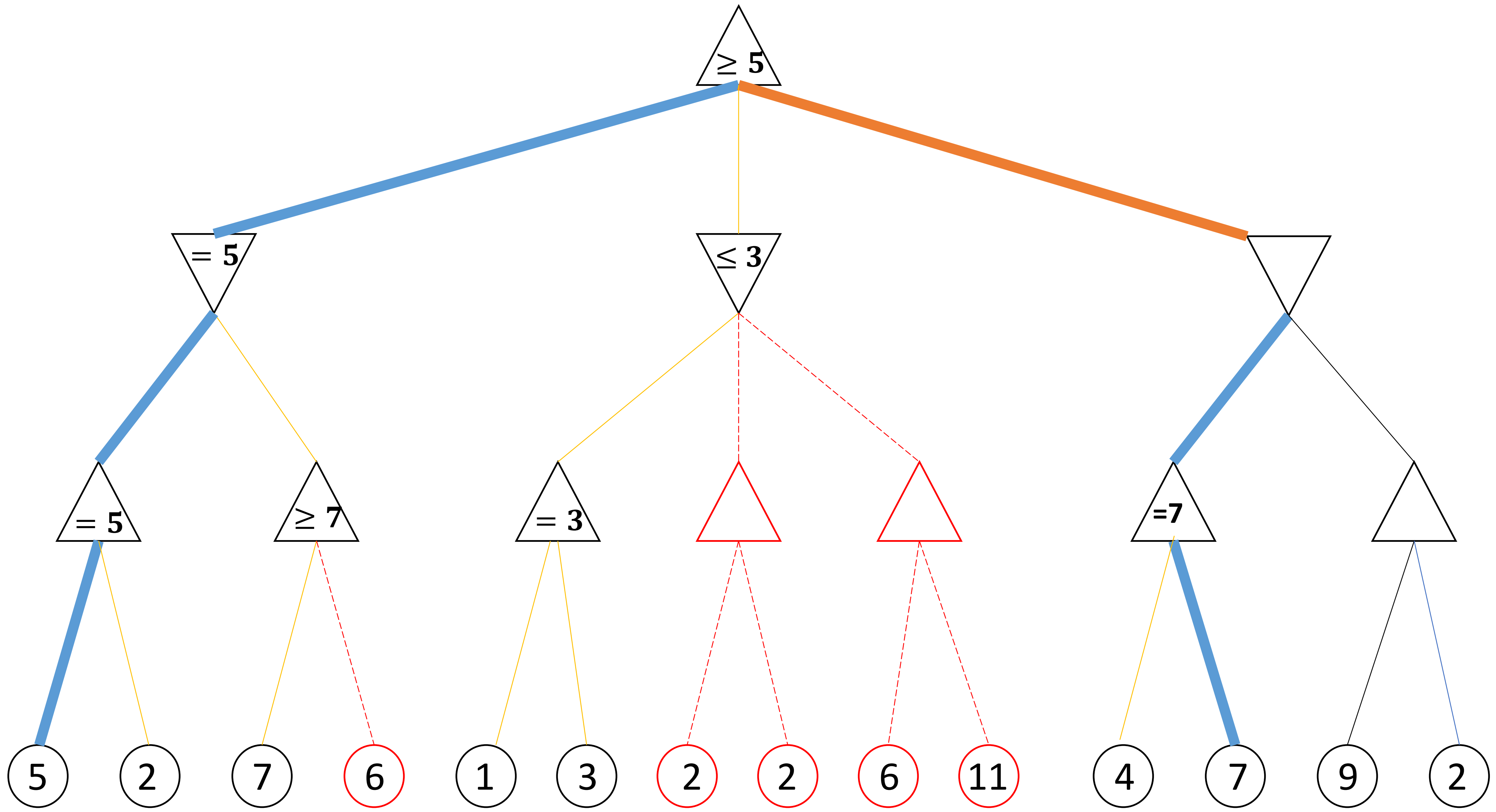
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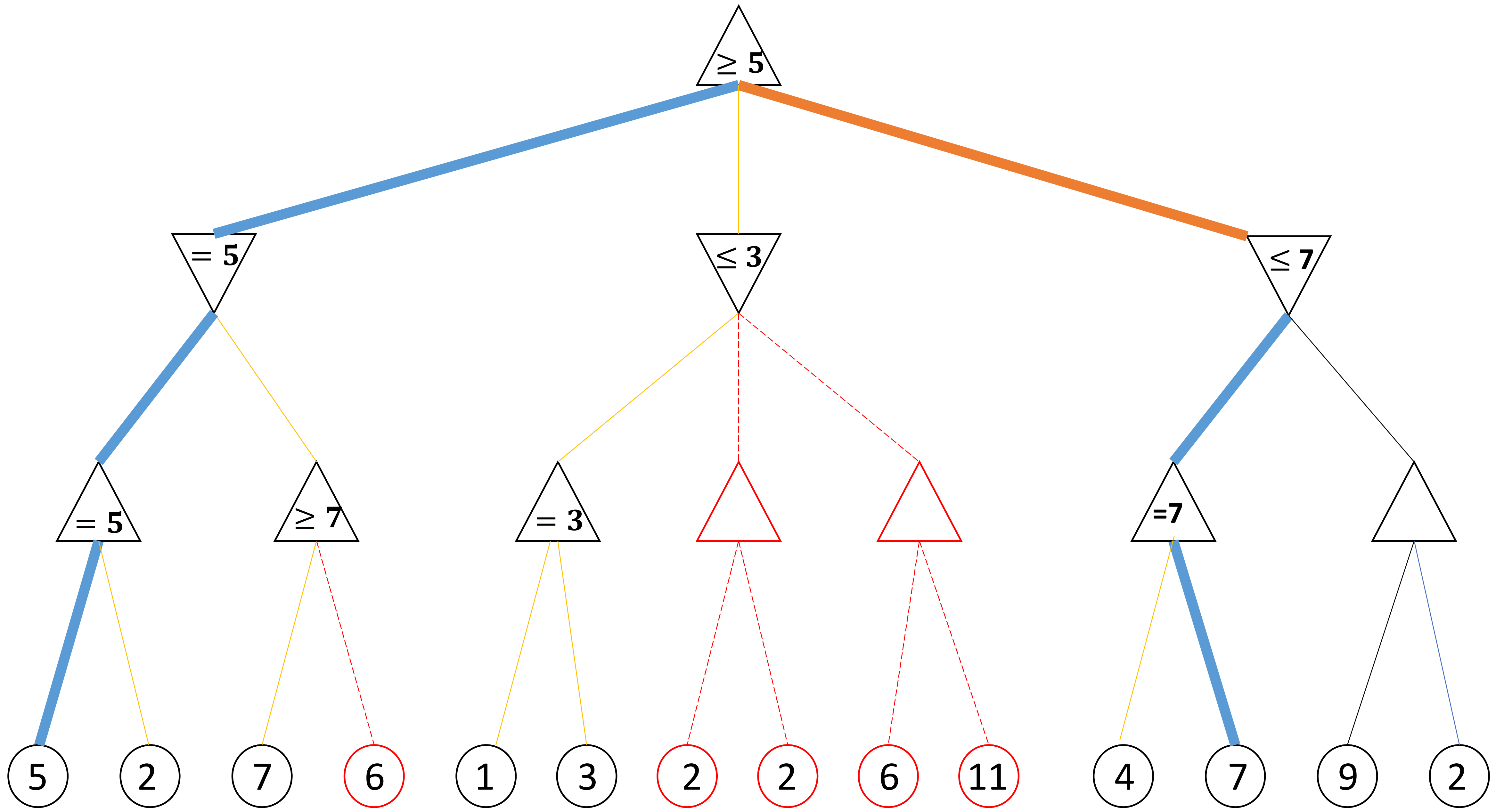
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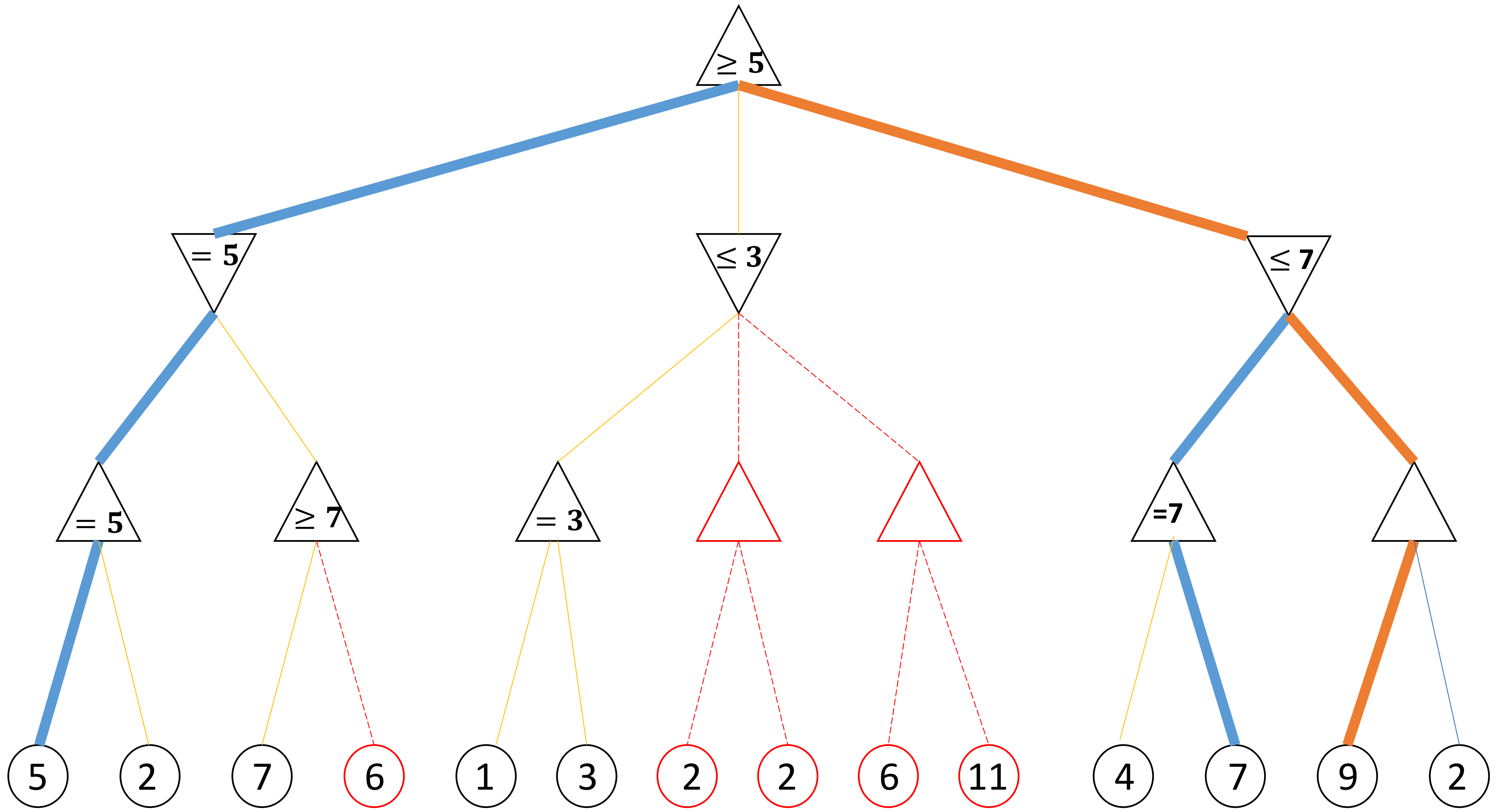
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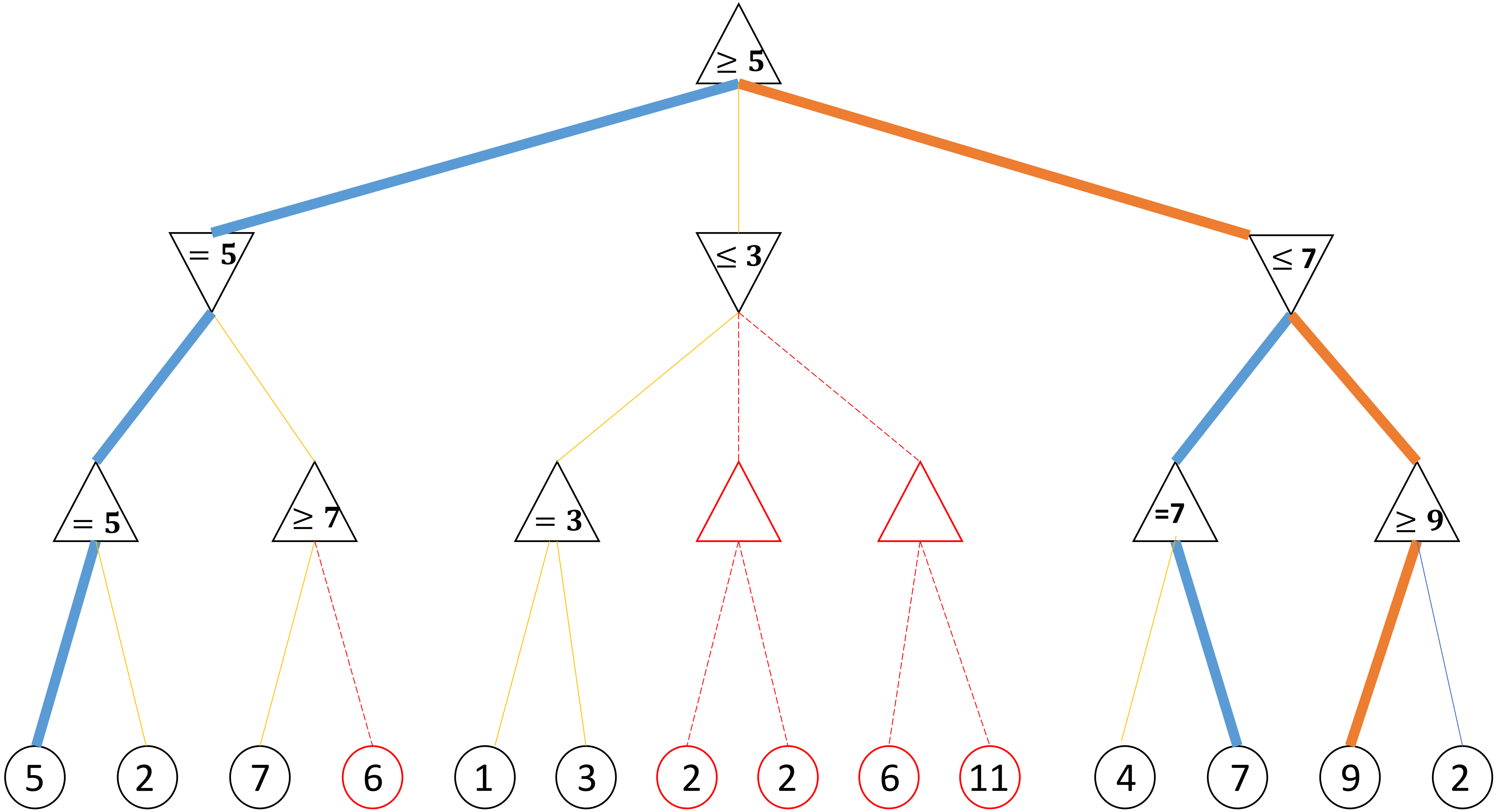
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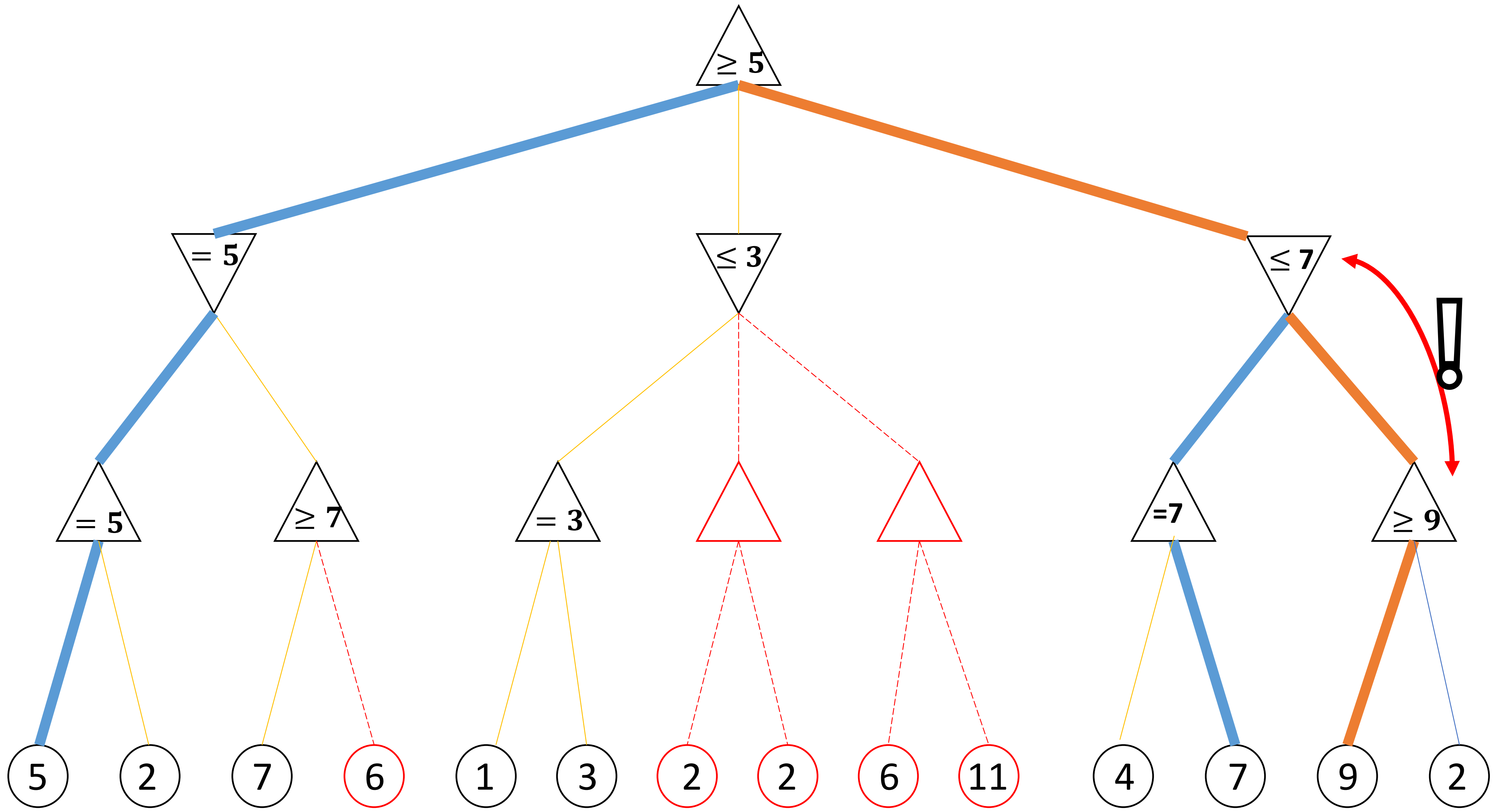
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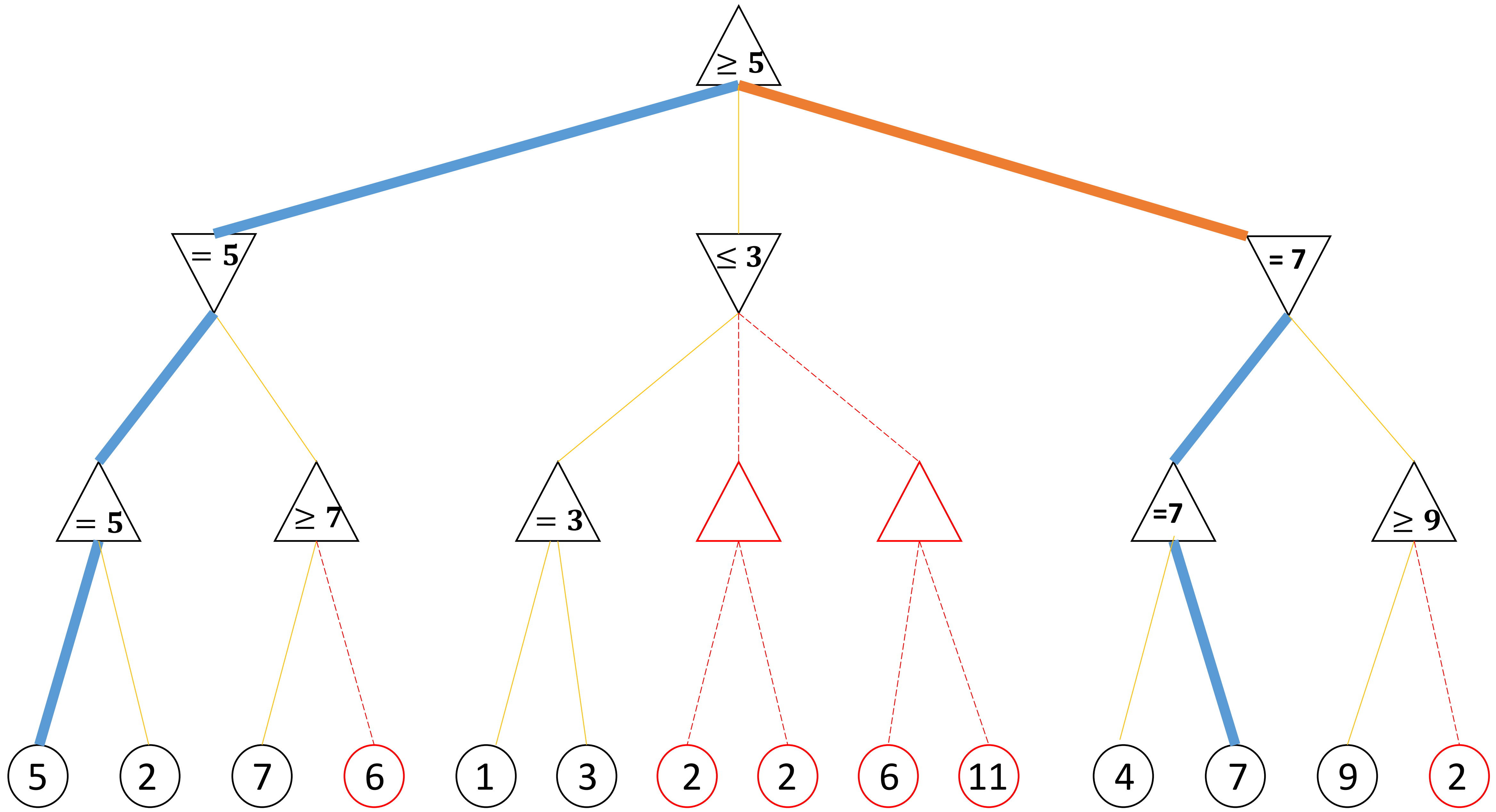
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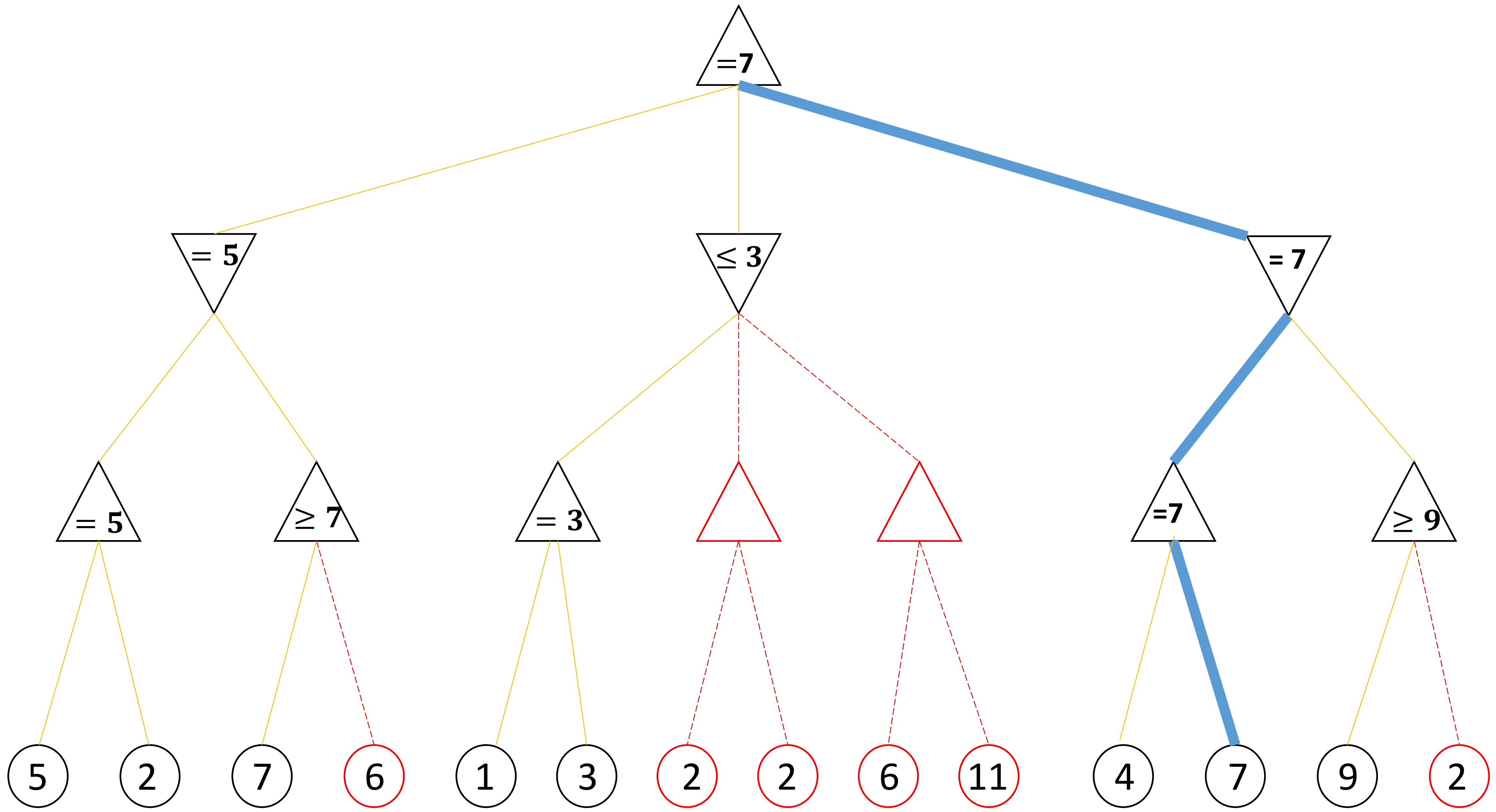
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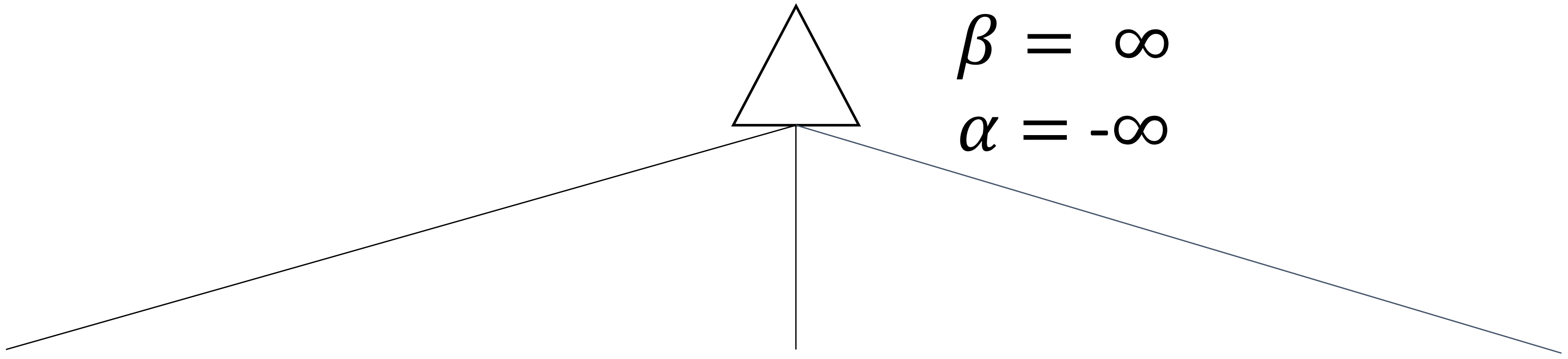
## Why “Alpha-beta” ?

Any new node is considered as a game-state of a potential solution only if the following equation holds

$$\alpha \leq v \leq \beta$$

where $v$ is the current (estimate of the) value of the node.

Therefore $$\alpha \leq \beta$$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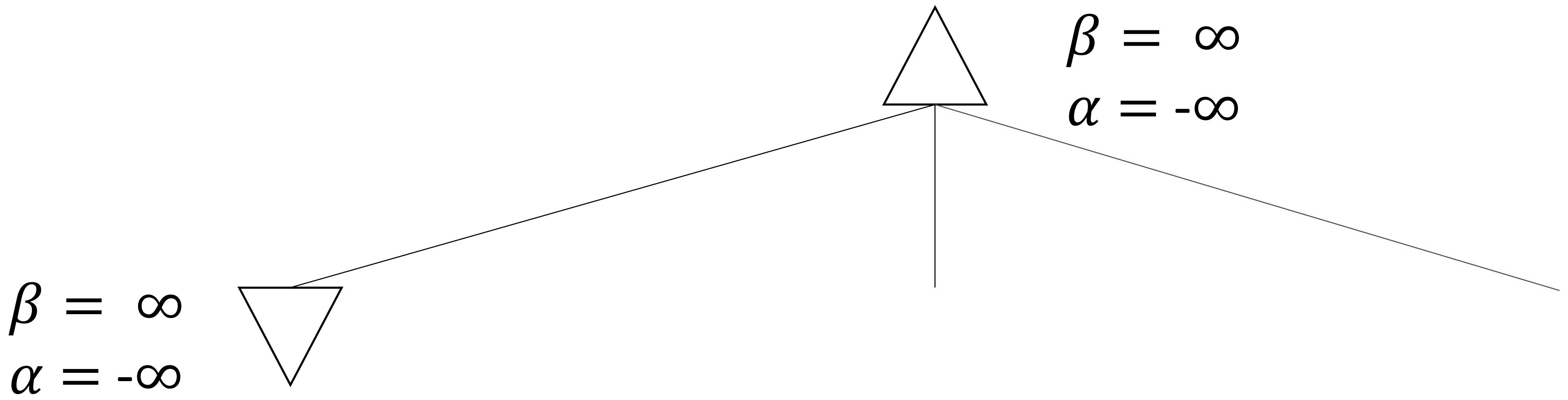
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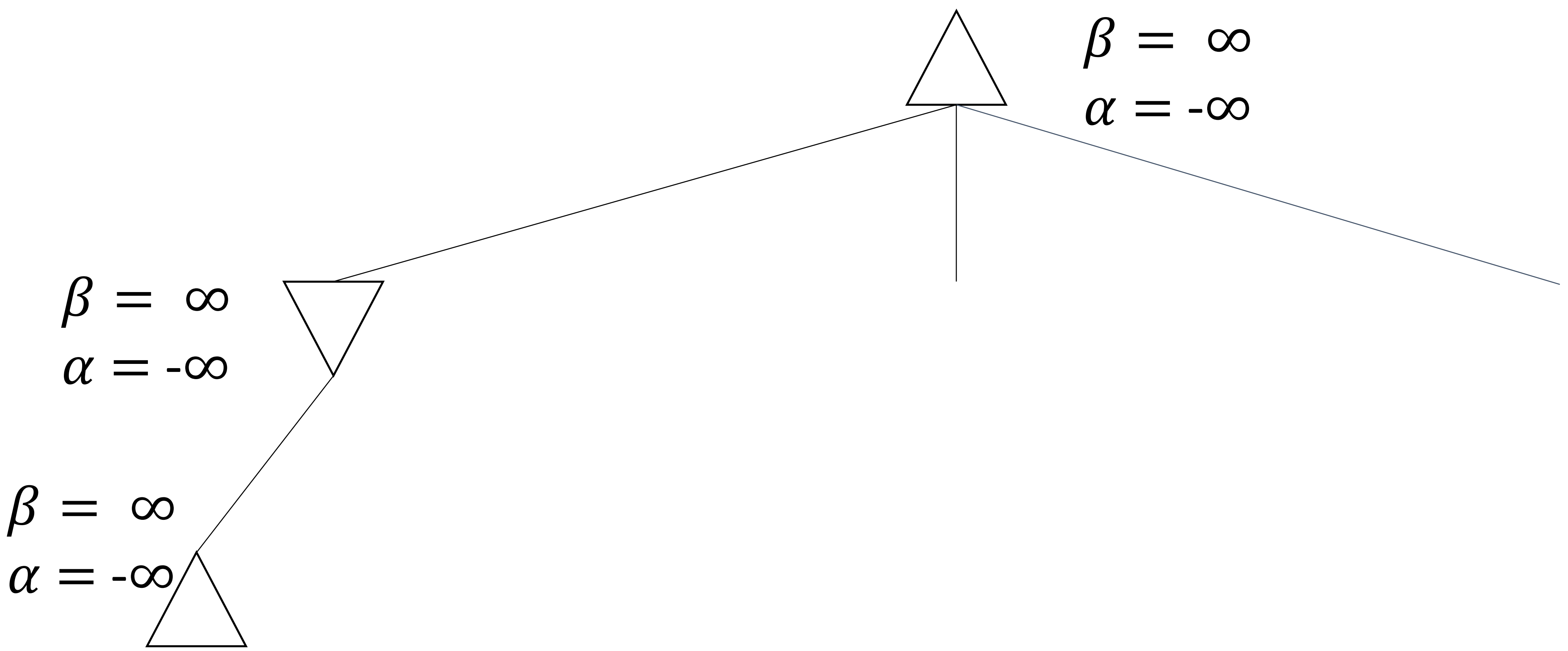
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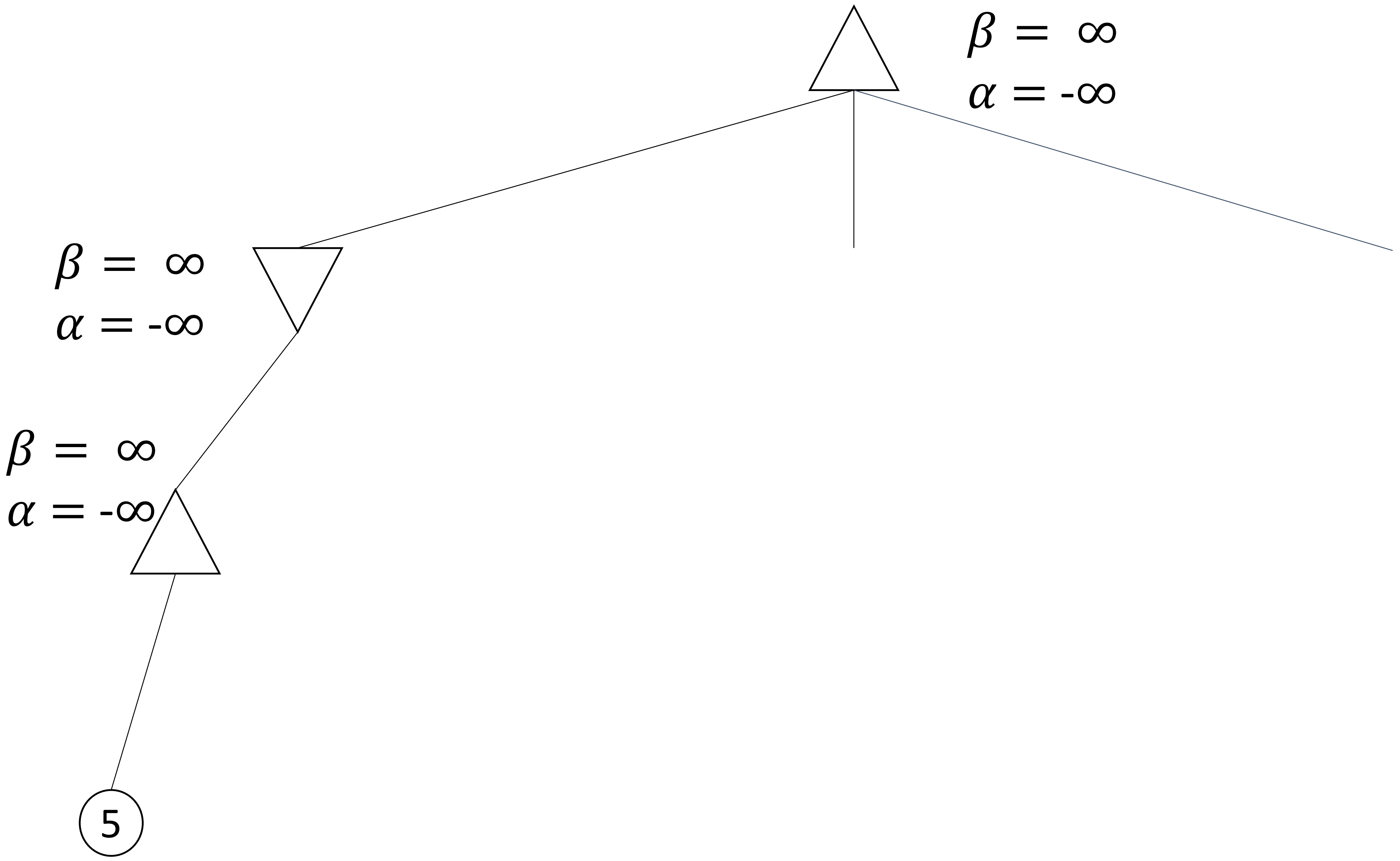
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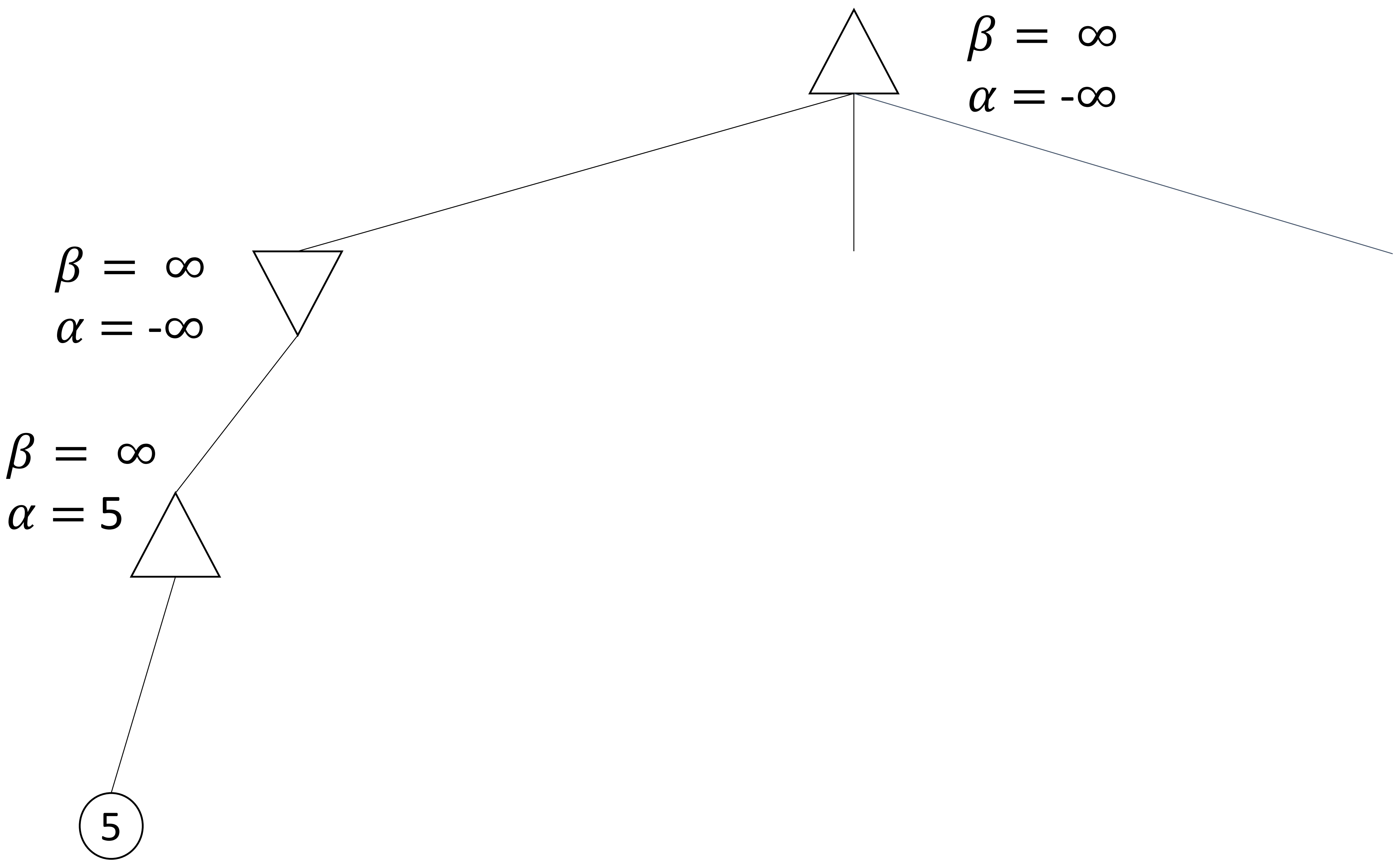
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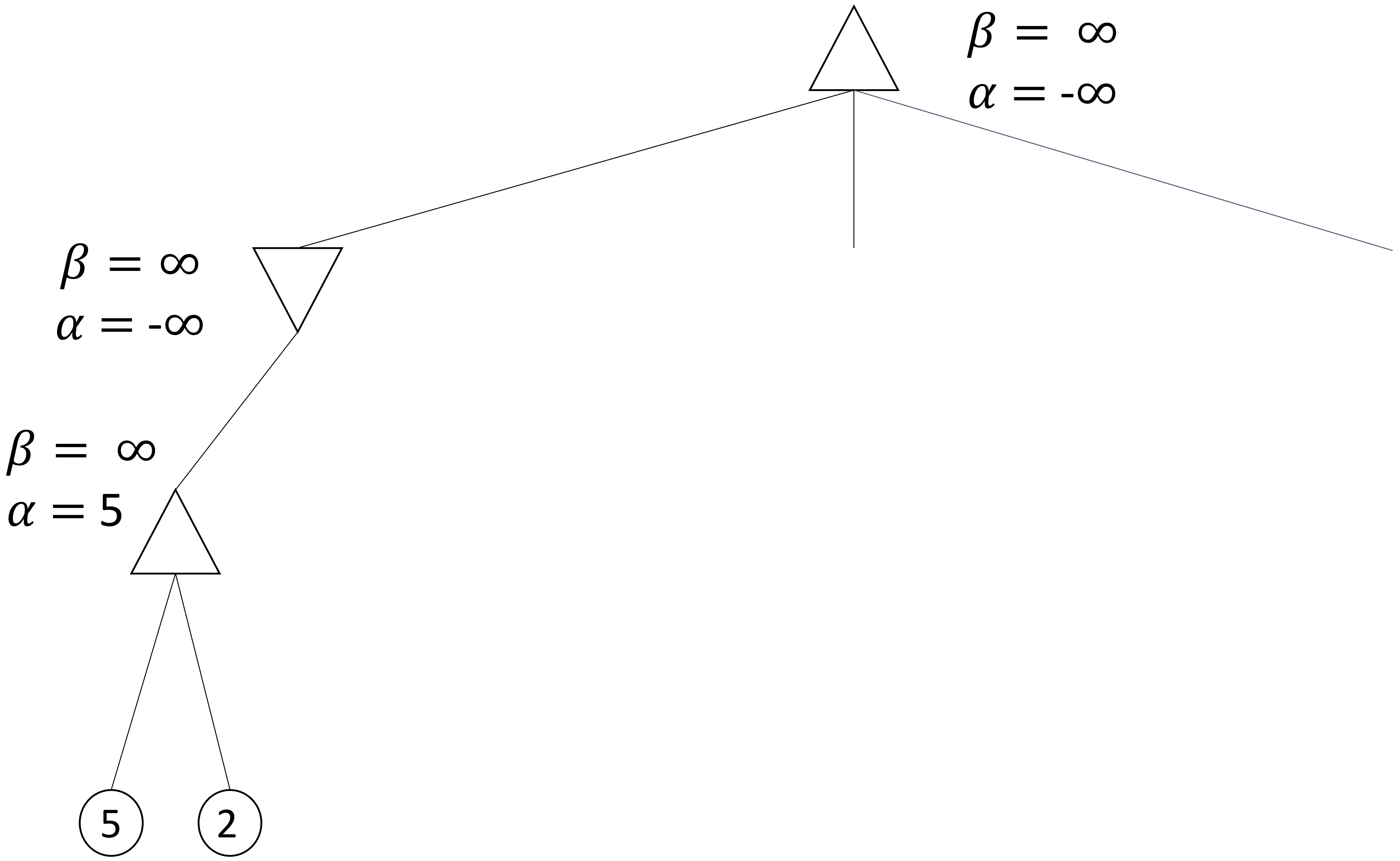
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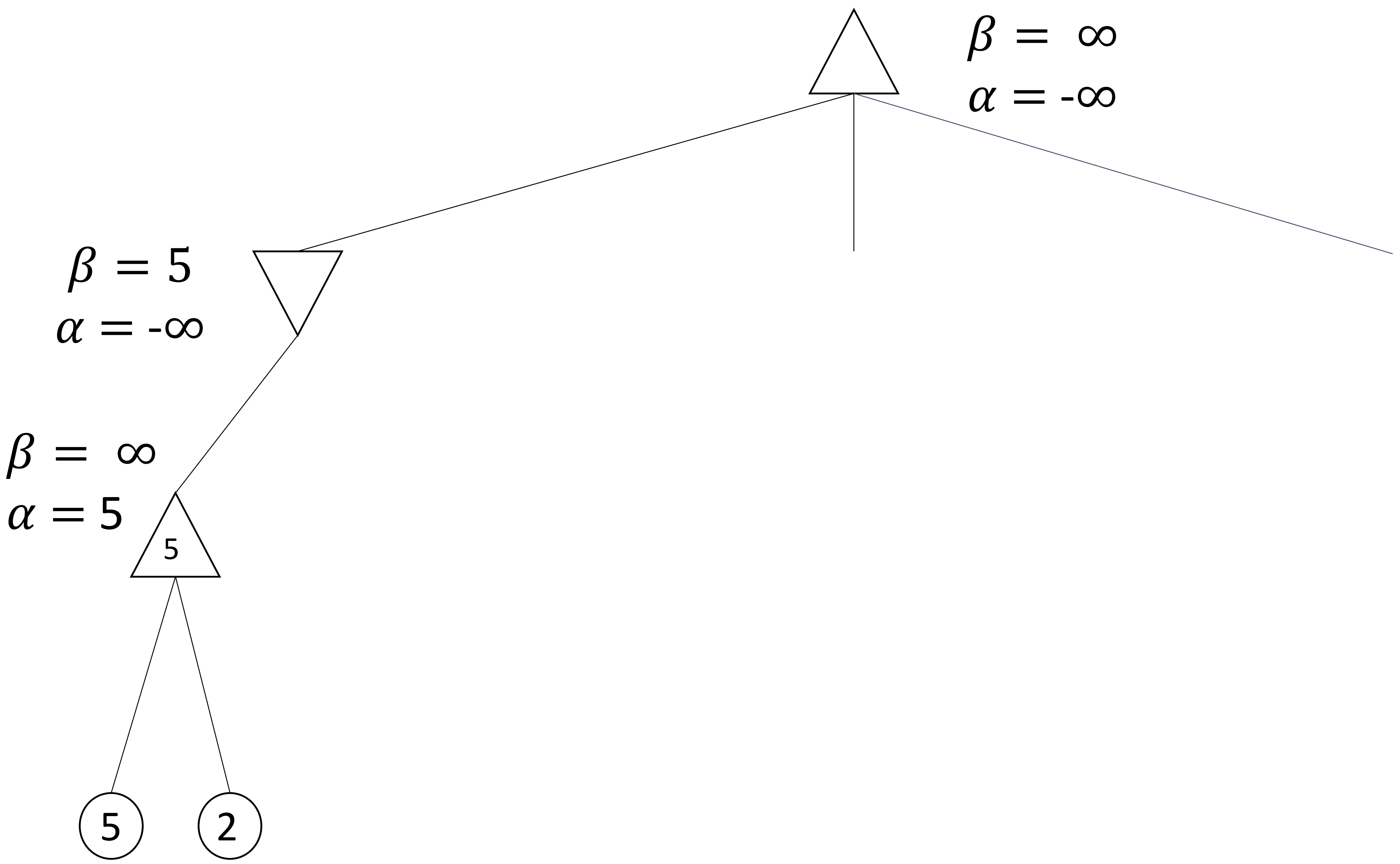
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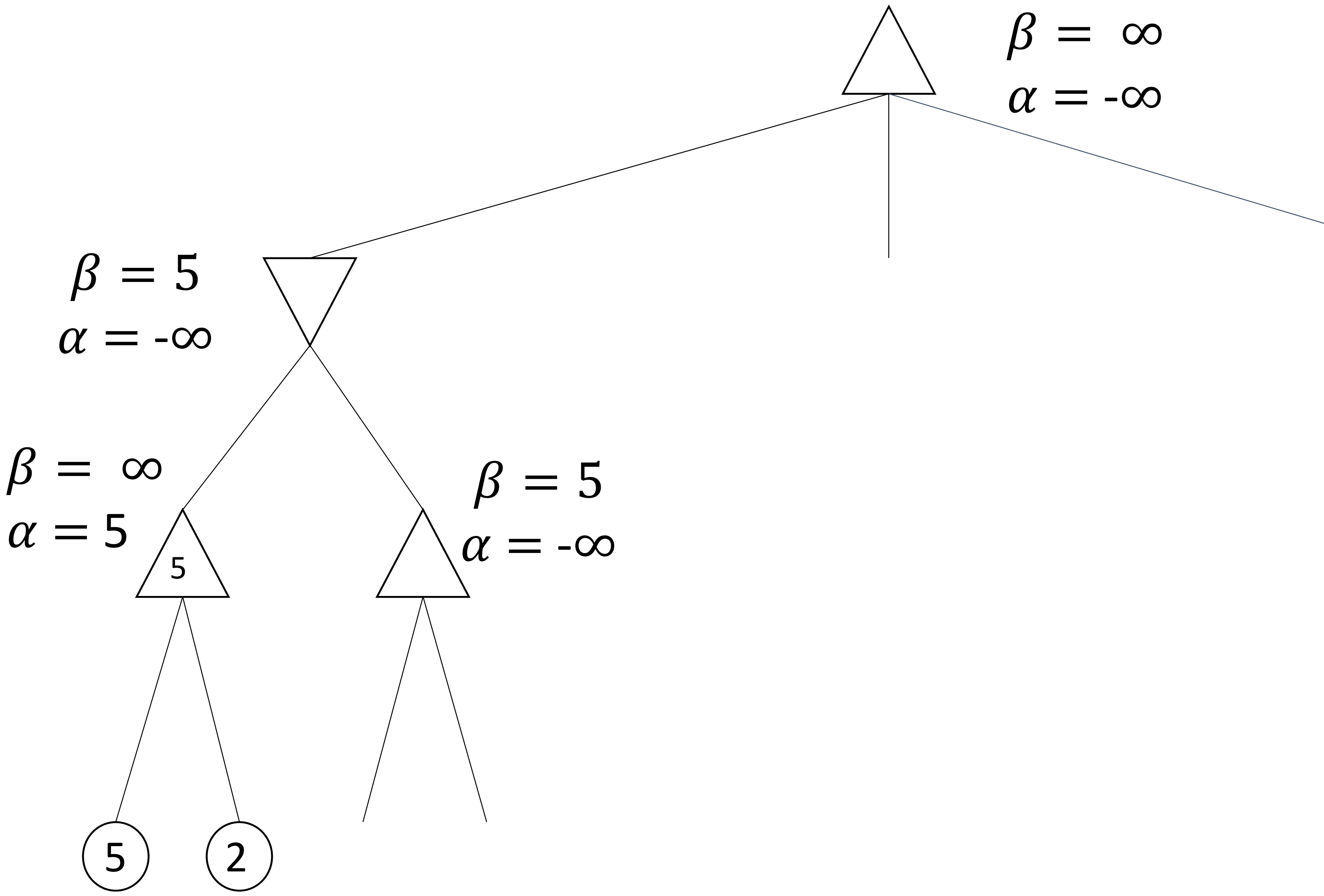

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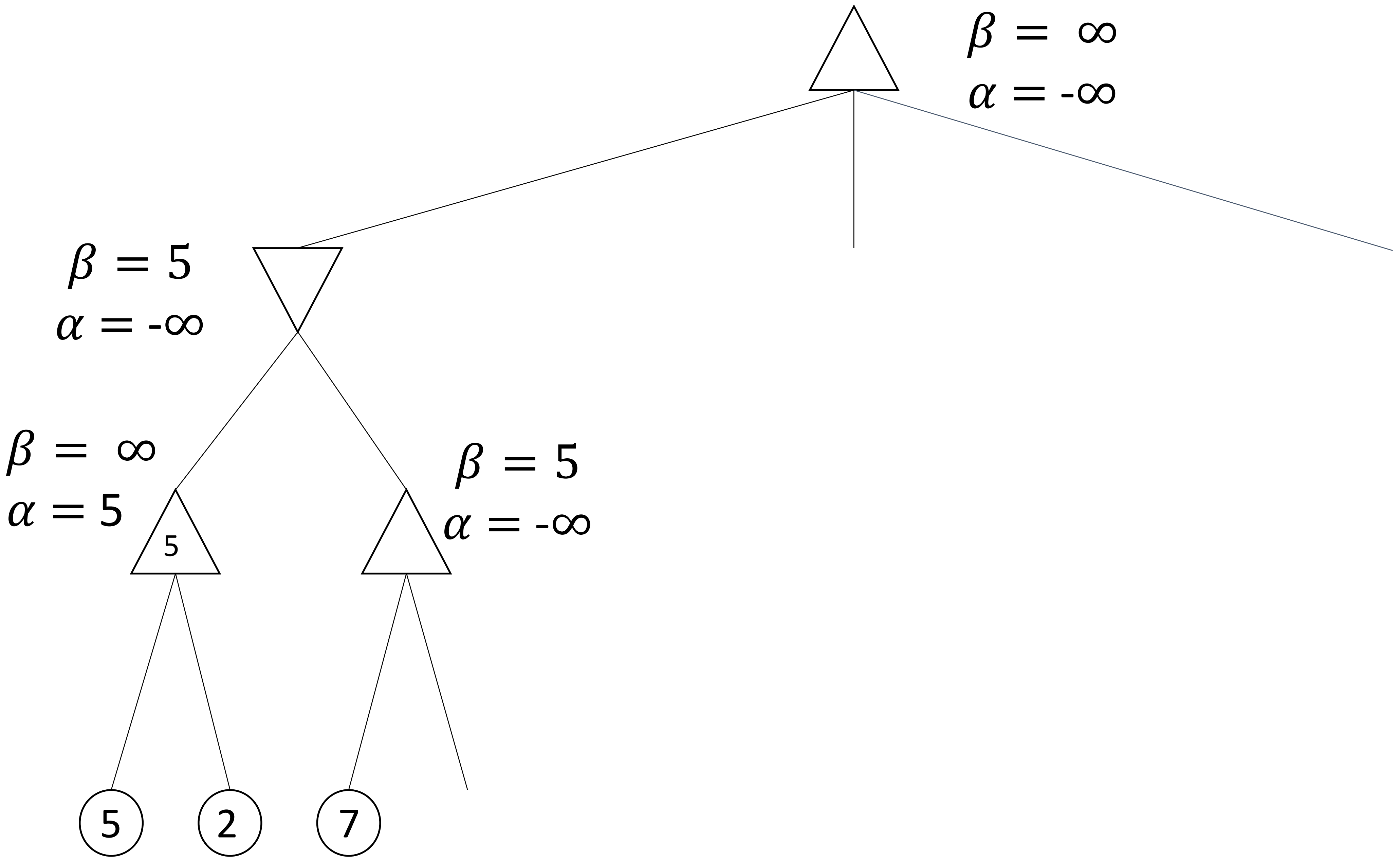
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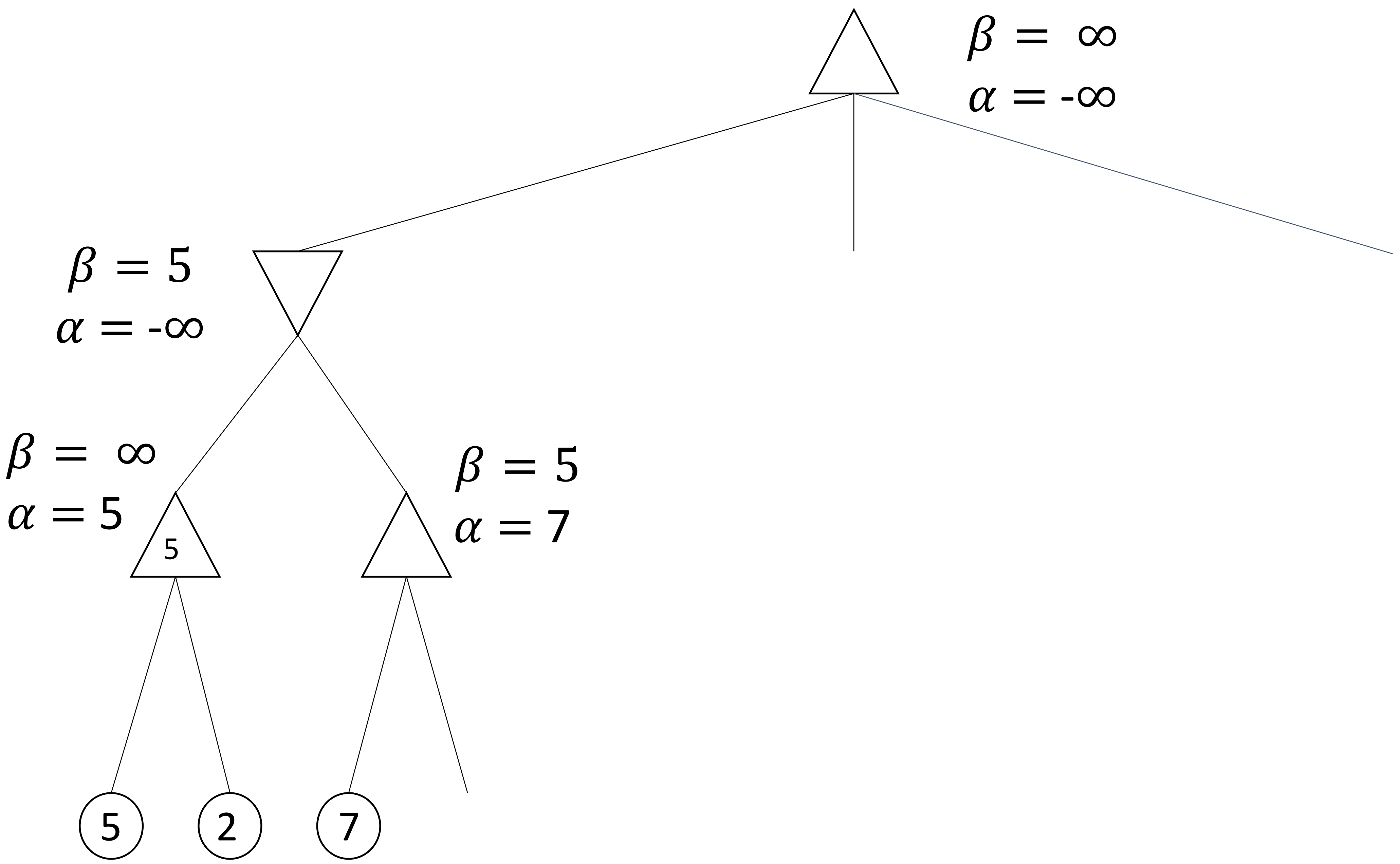
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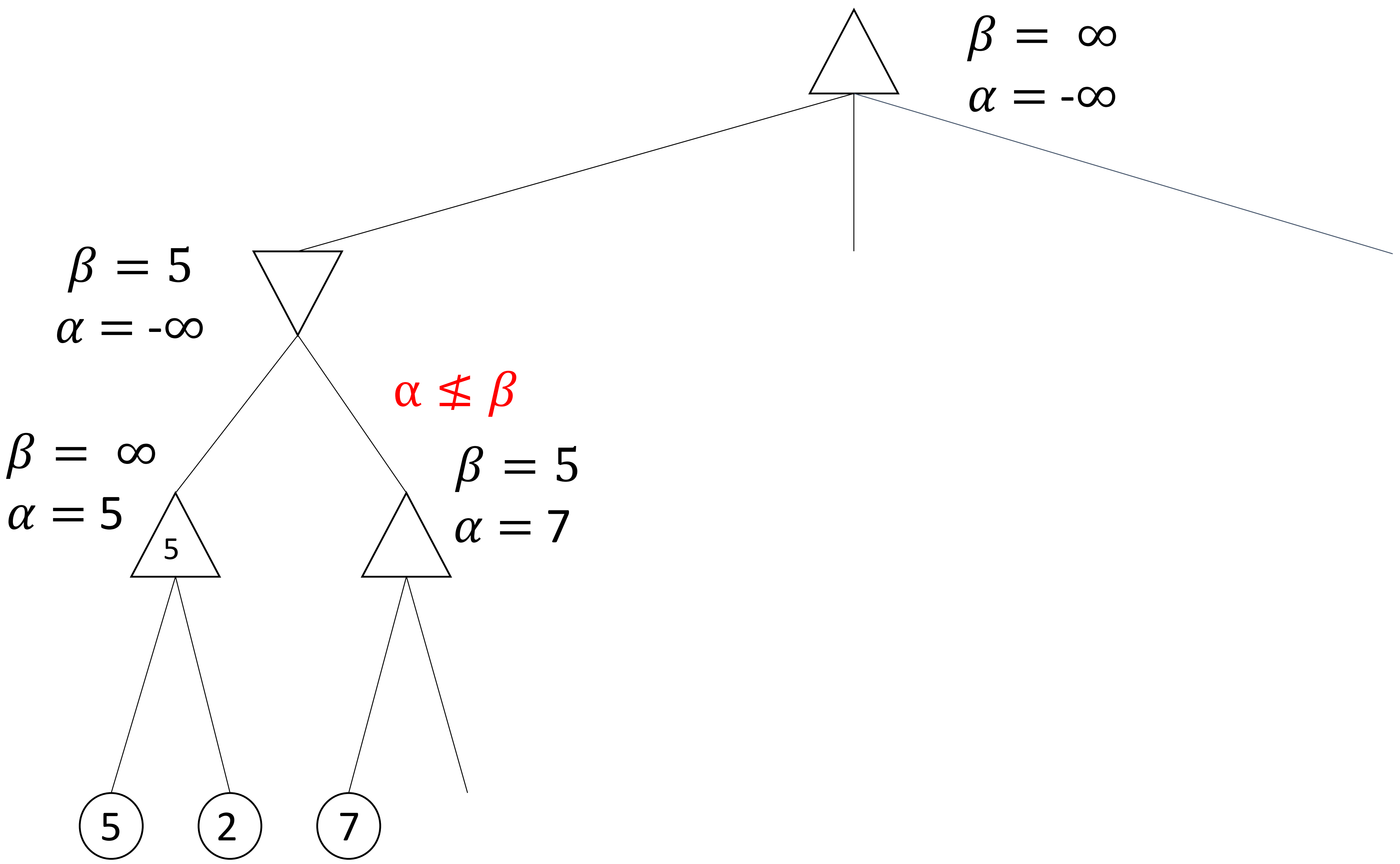
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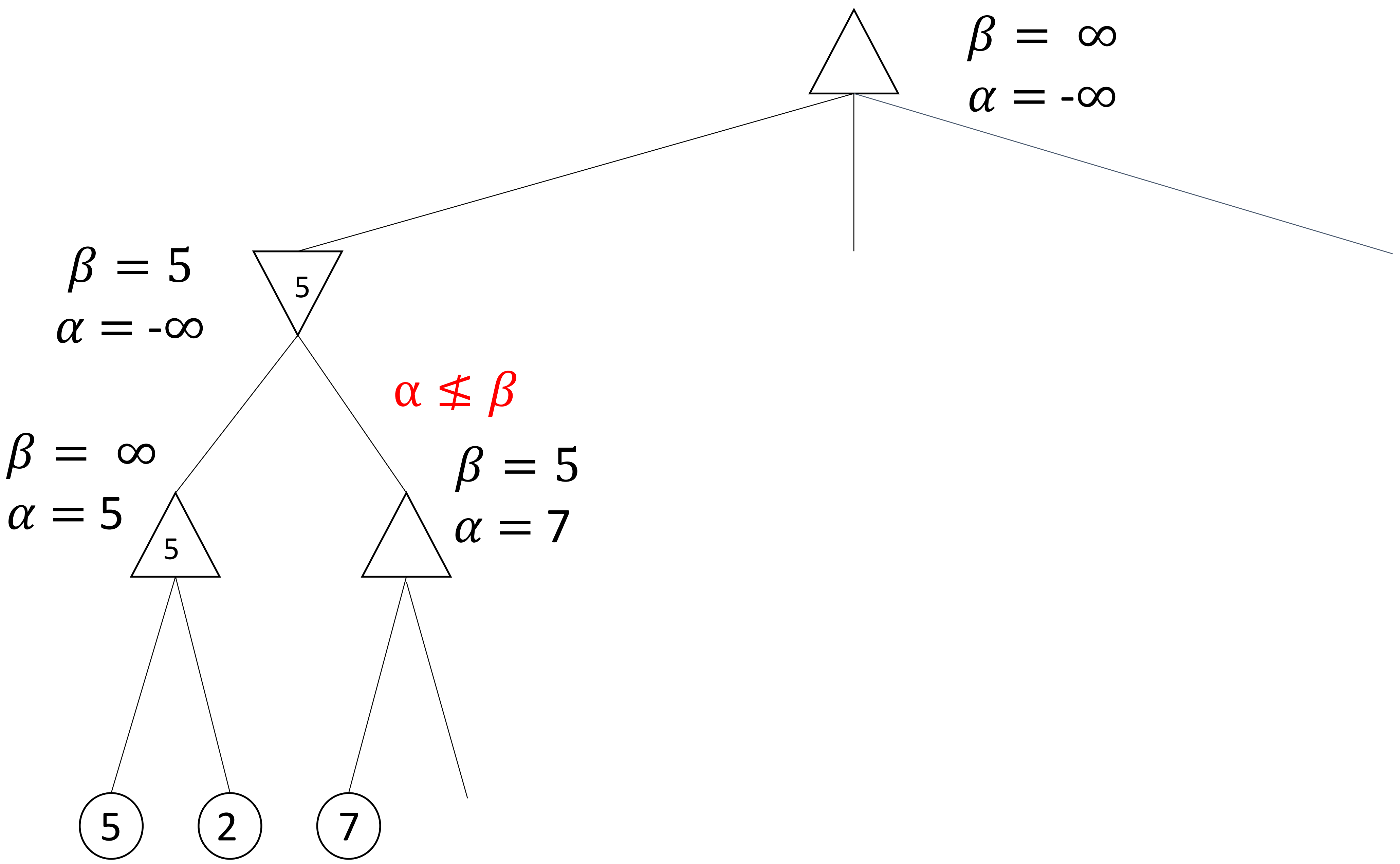
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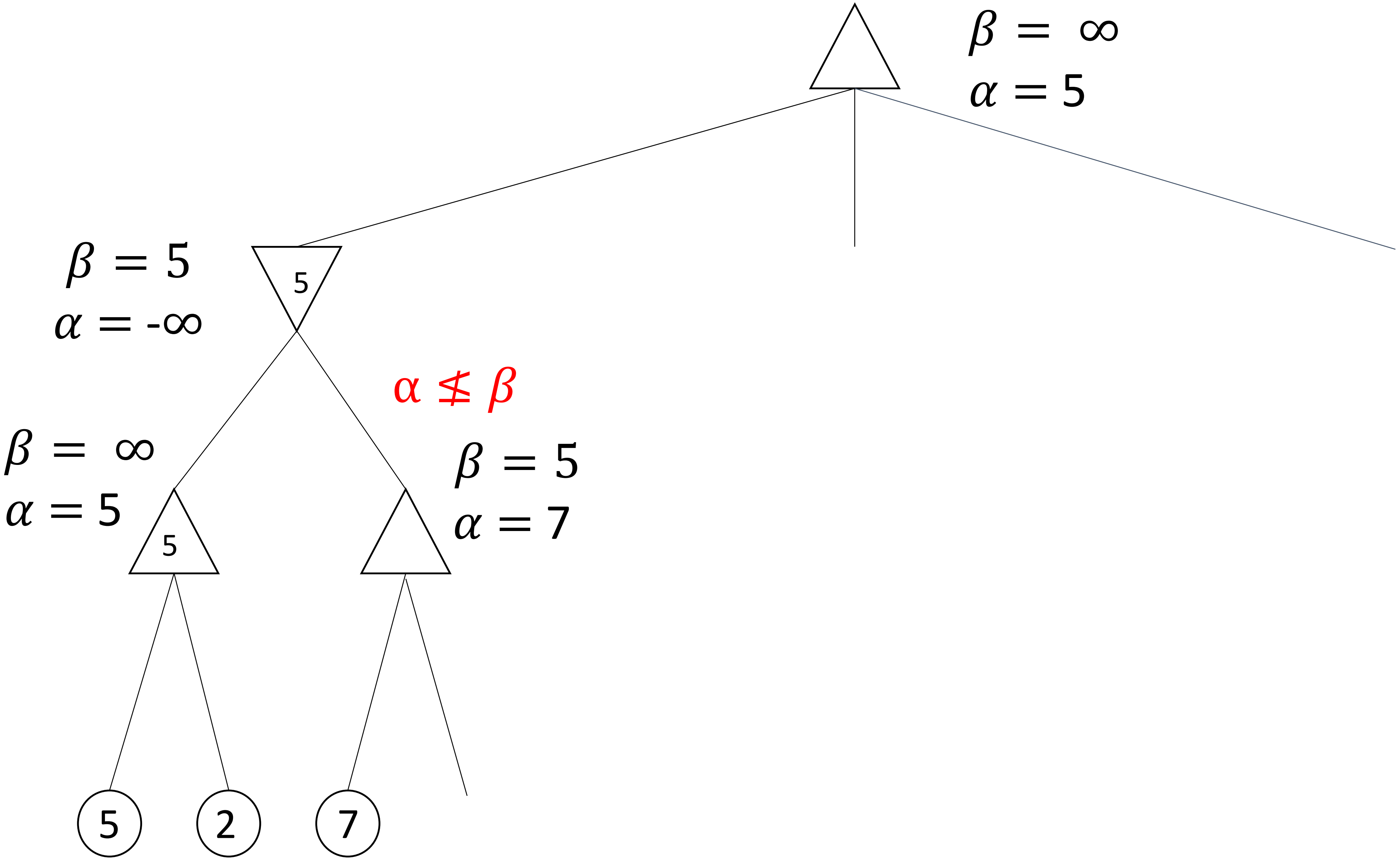
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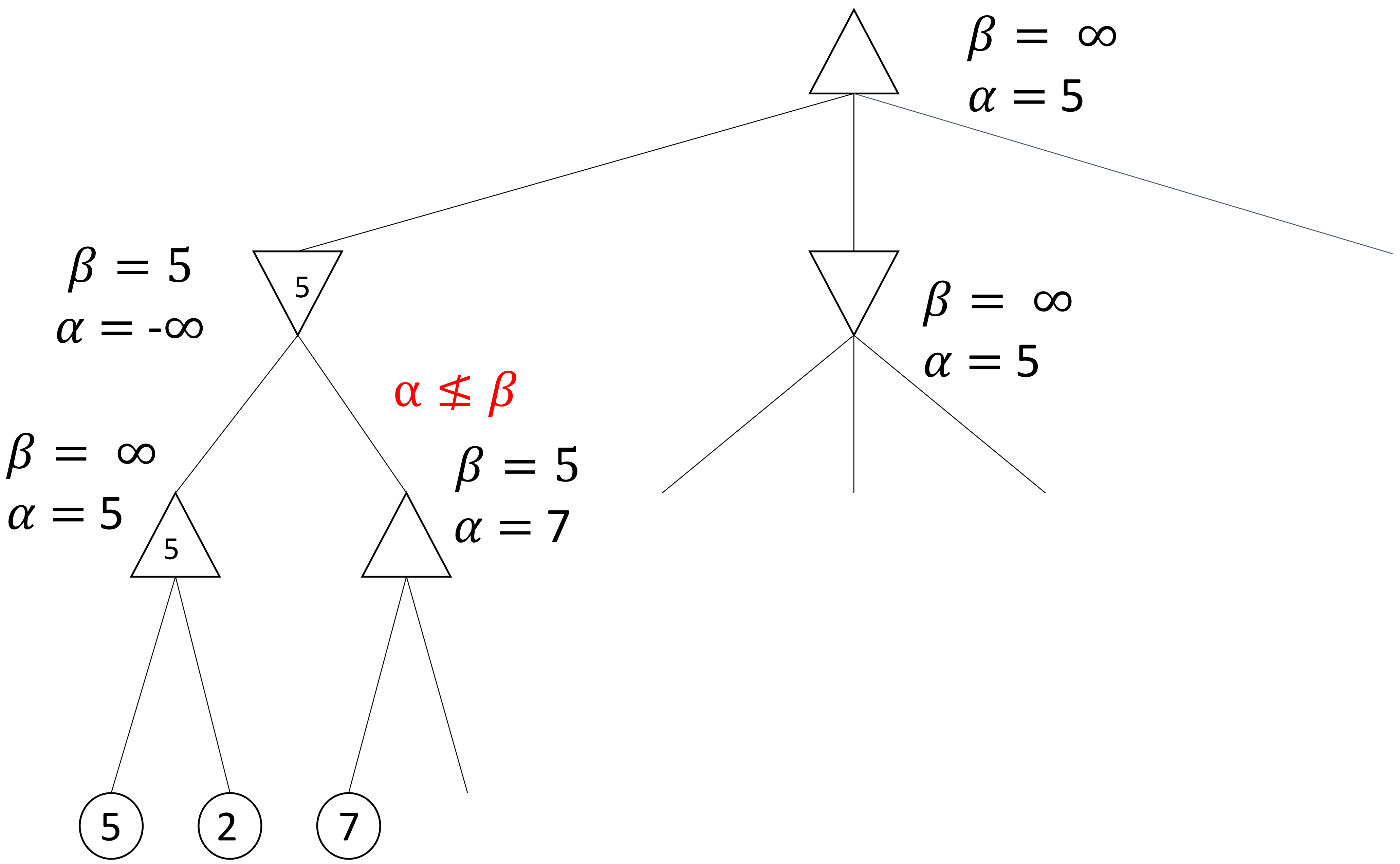

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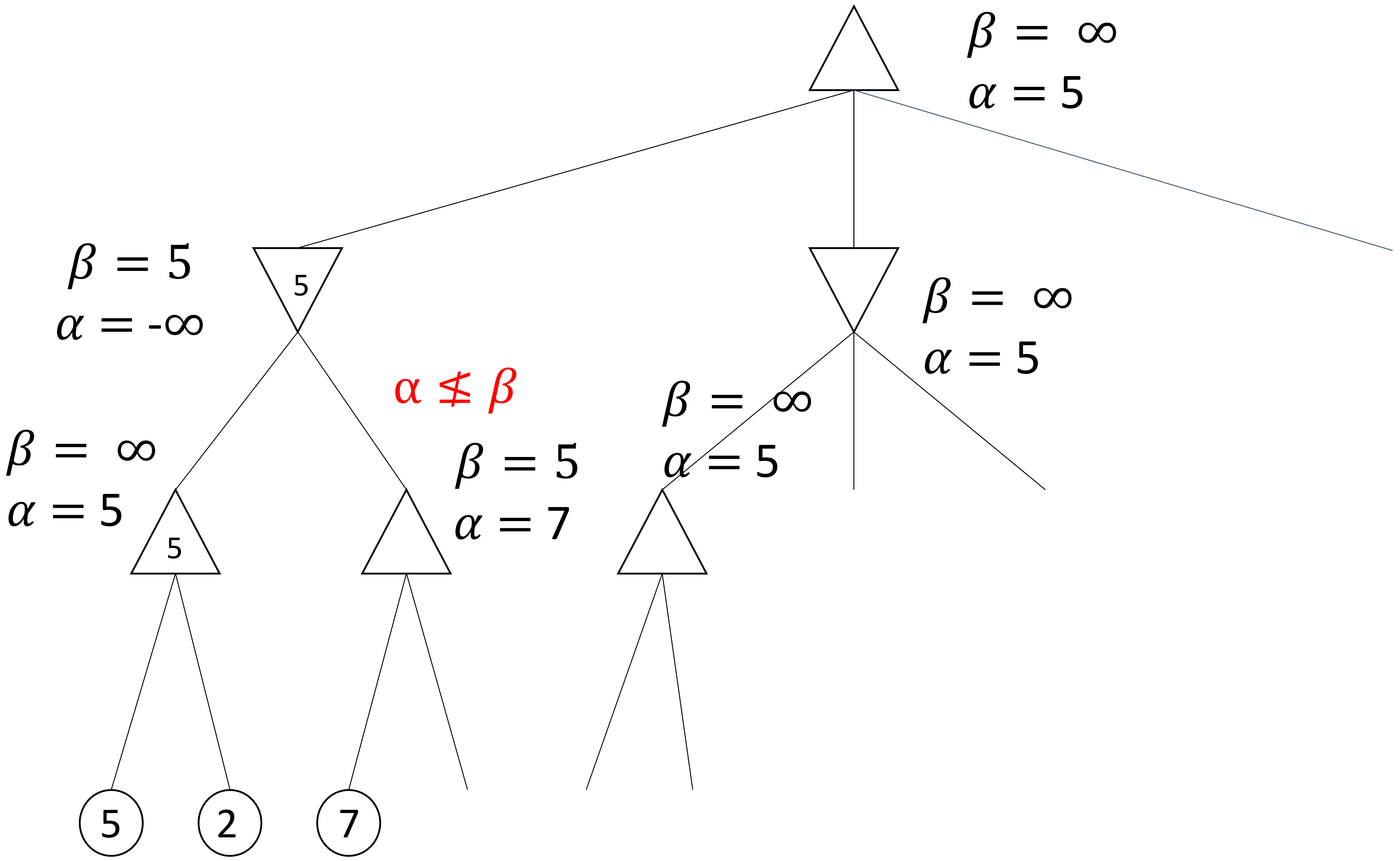

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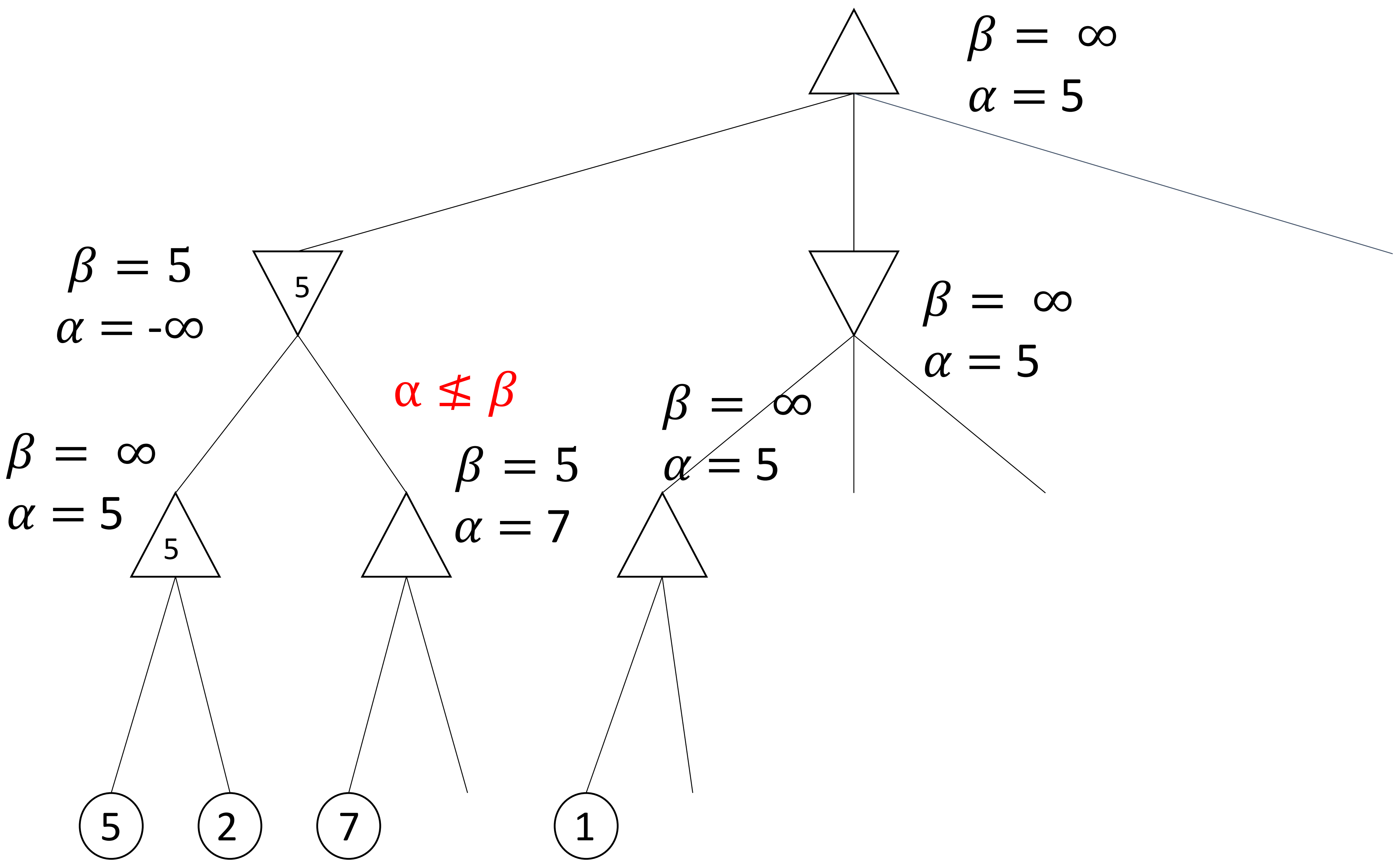
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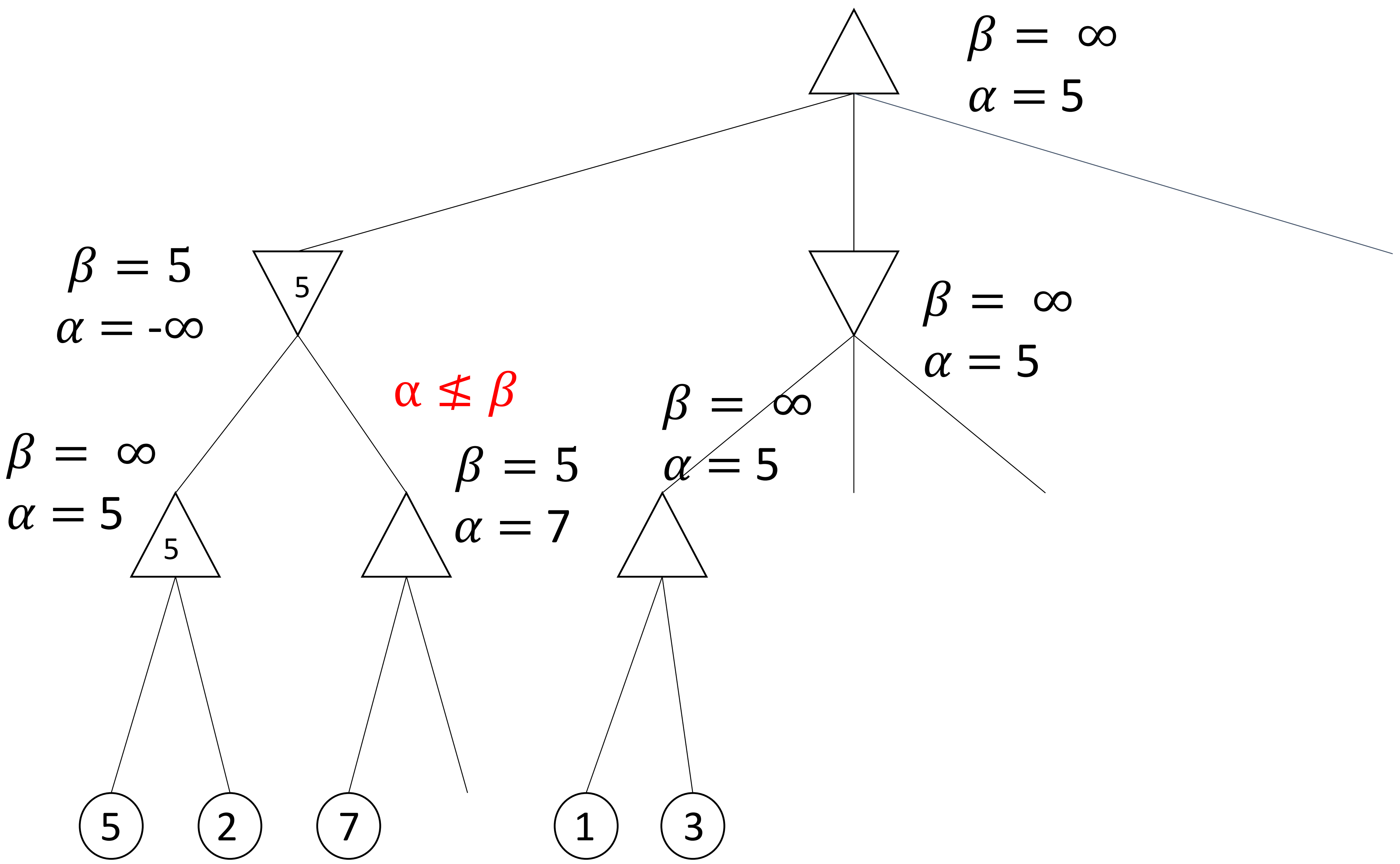

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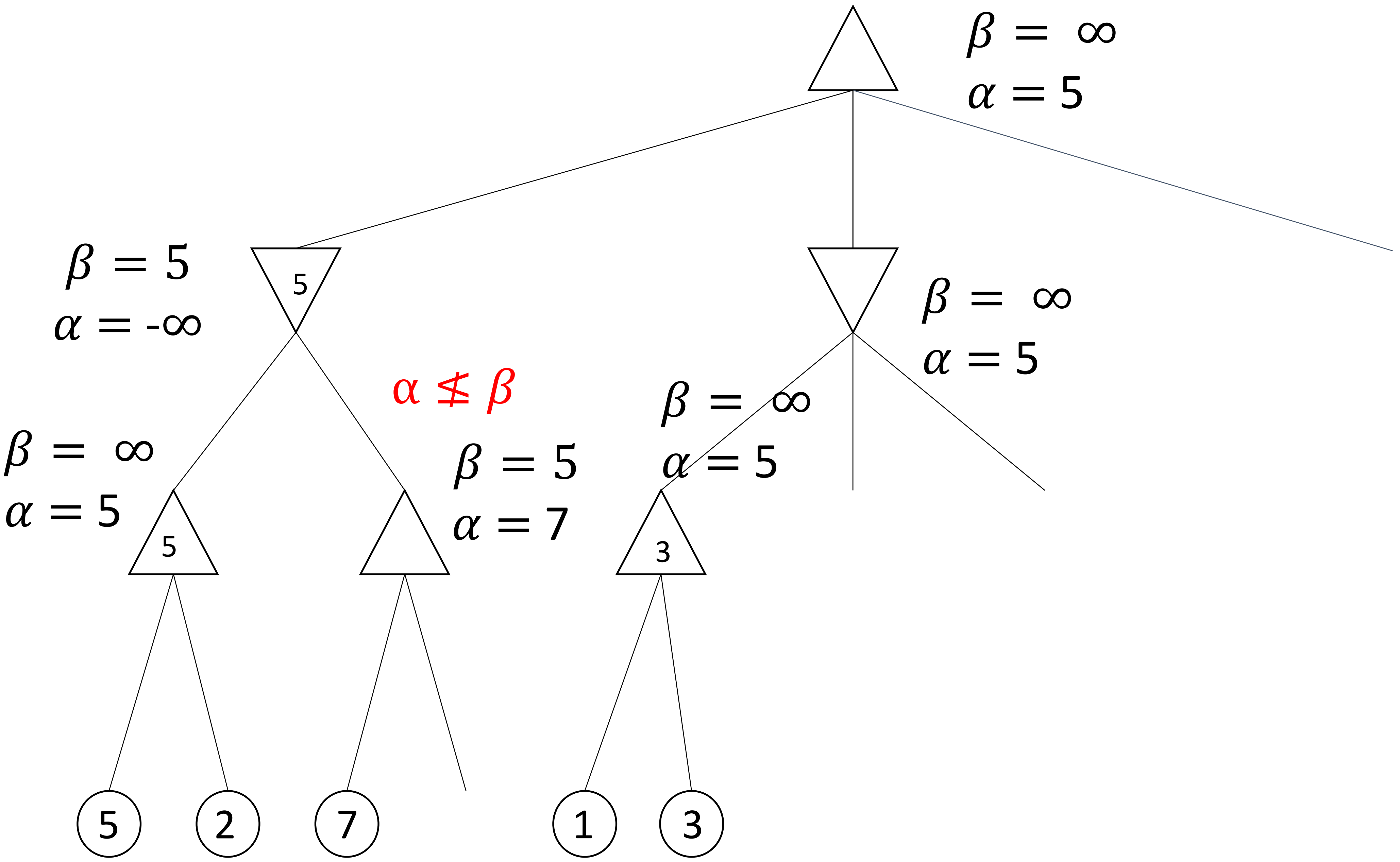

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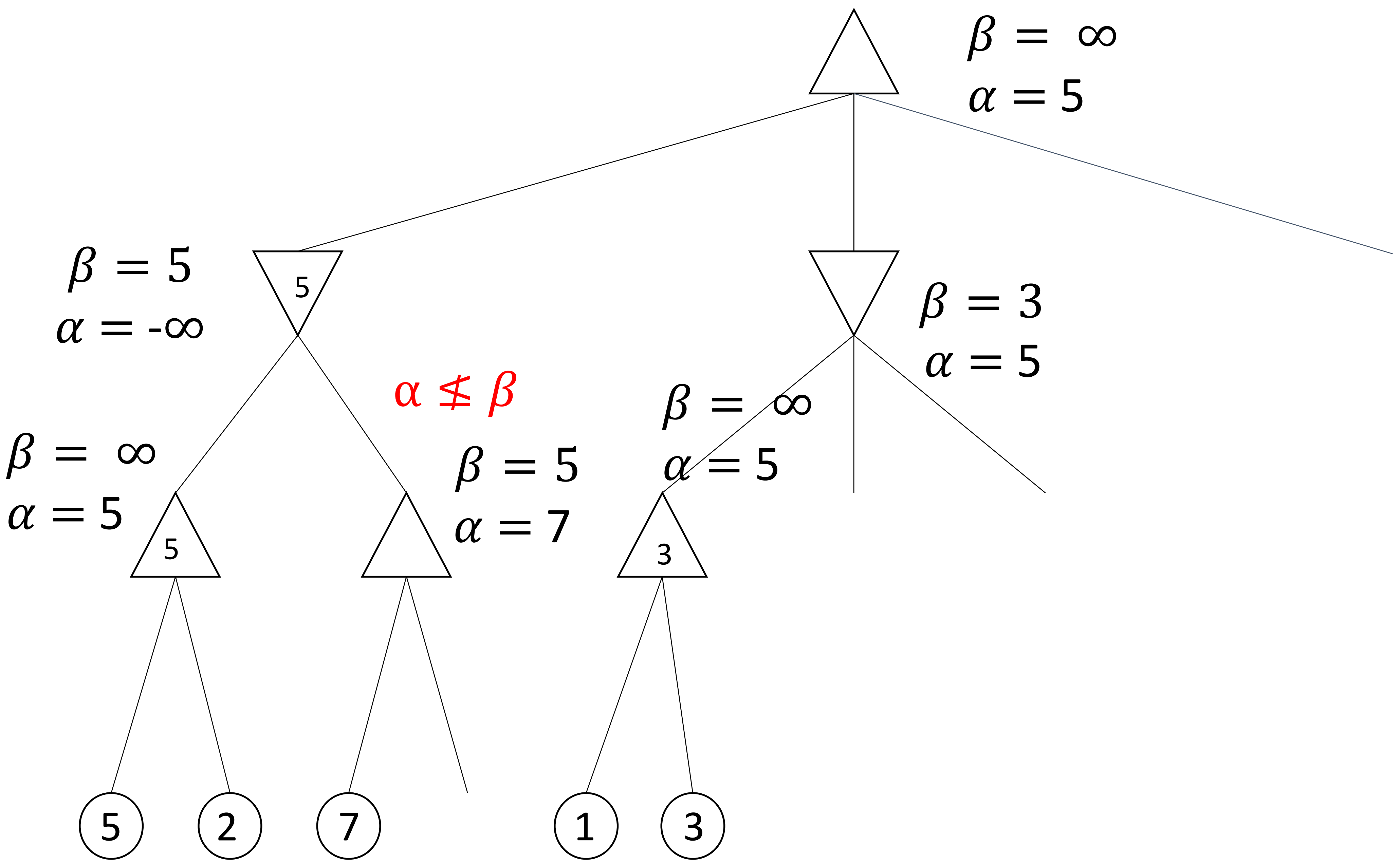

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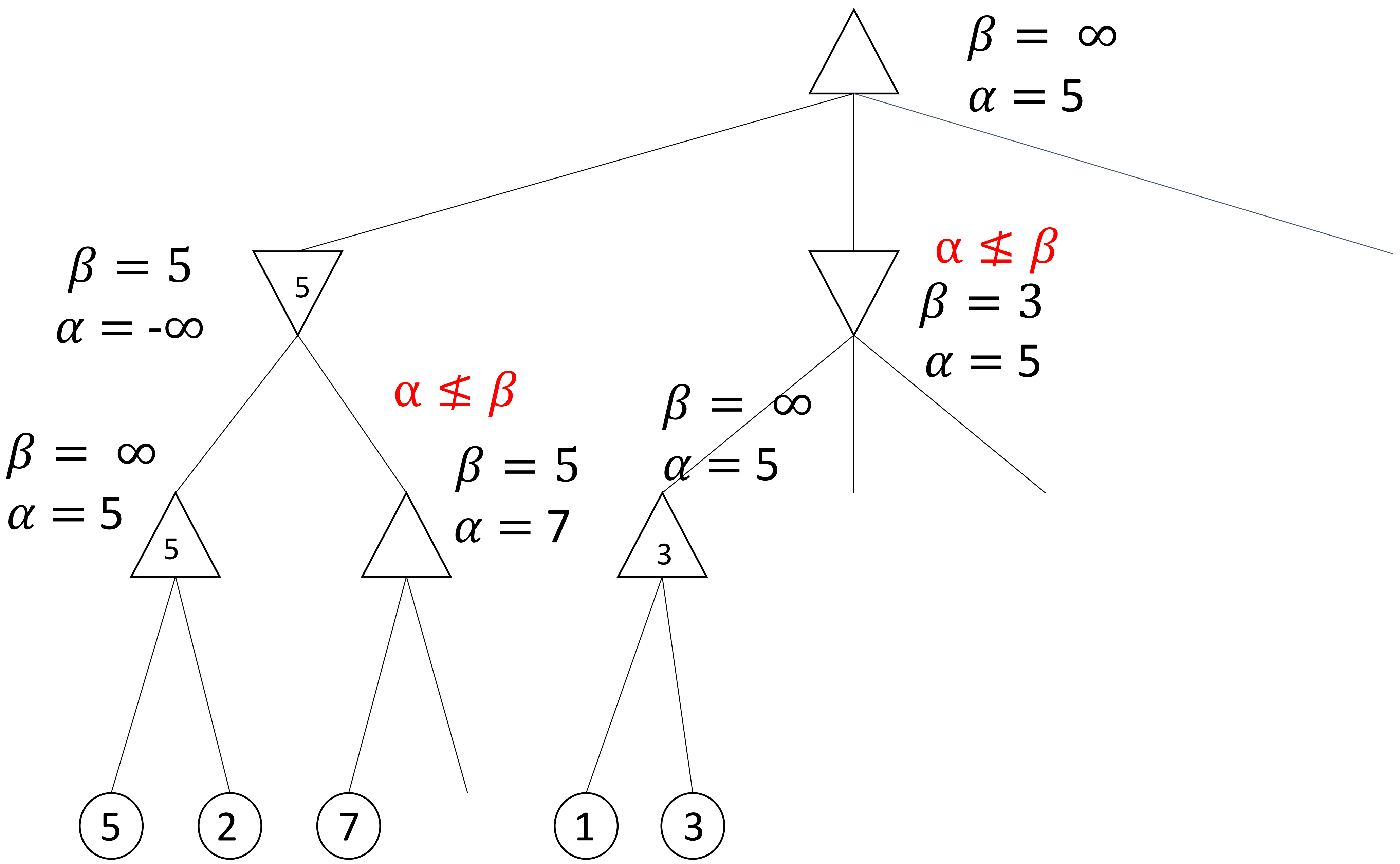

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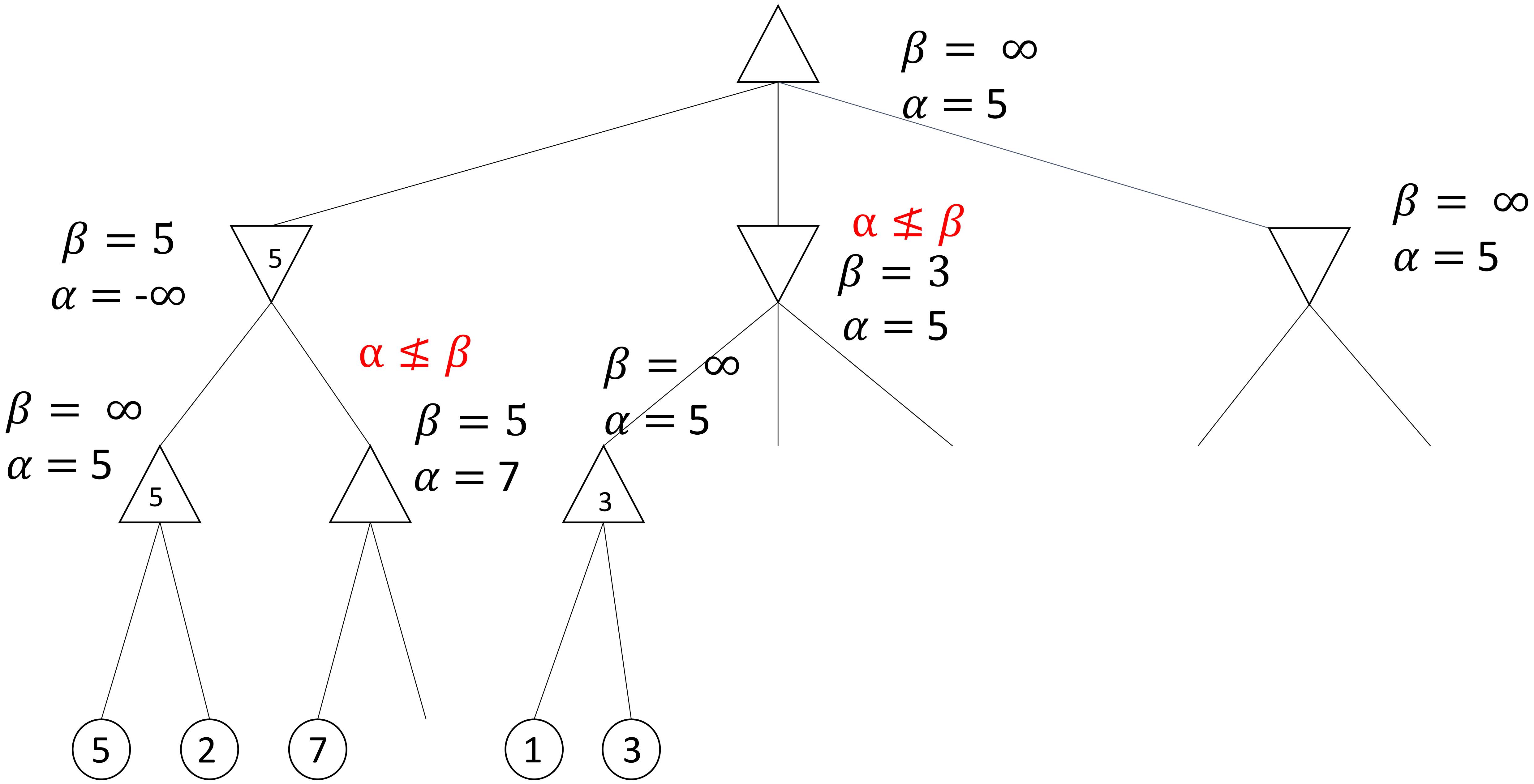

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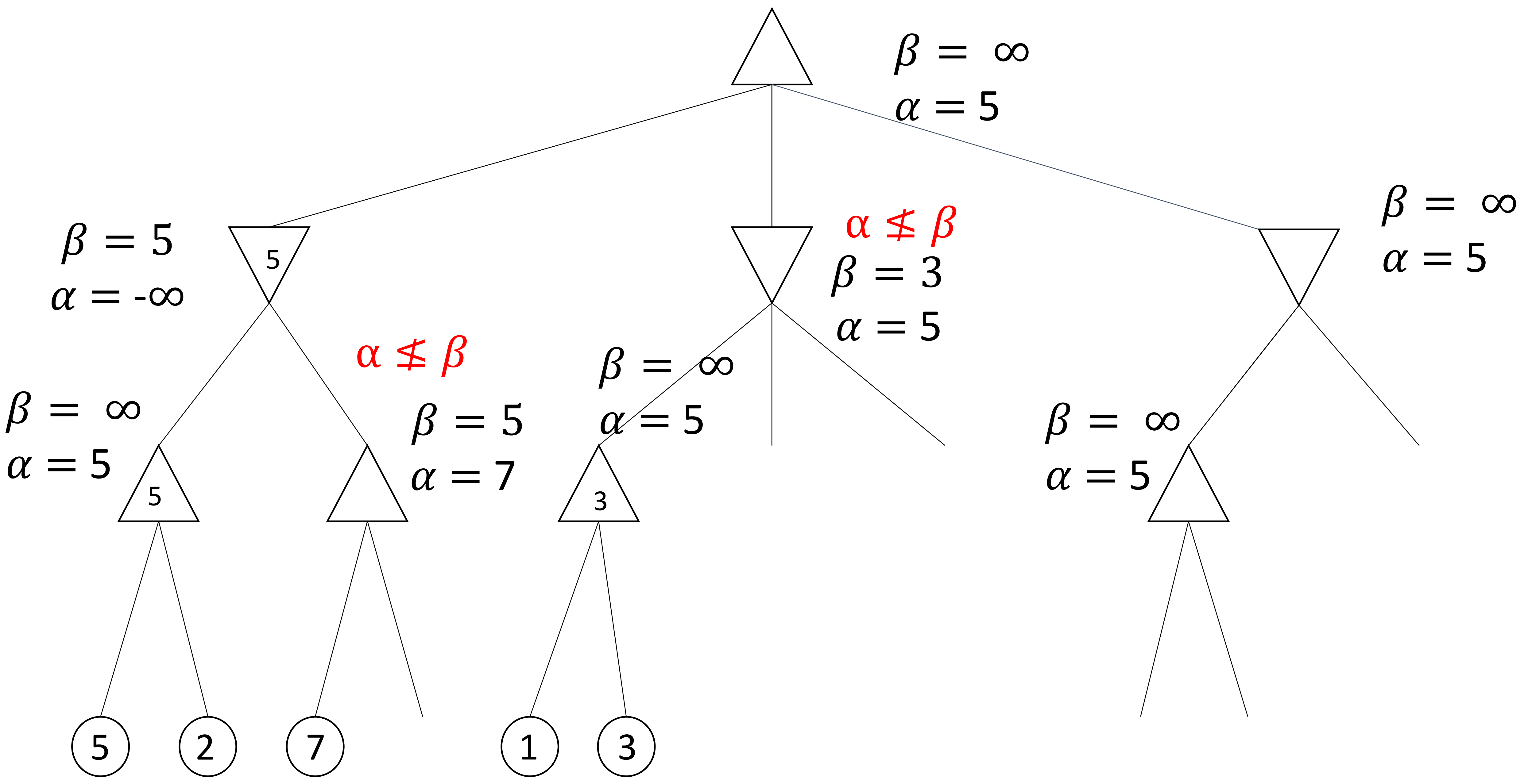
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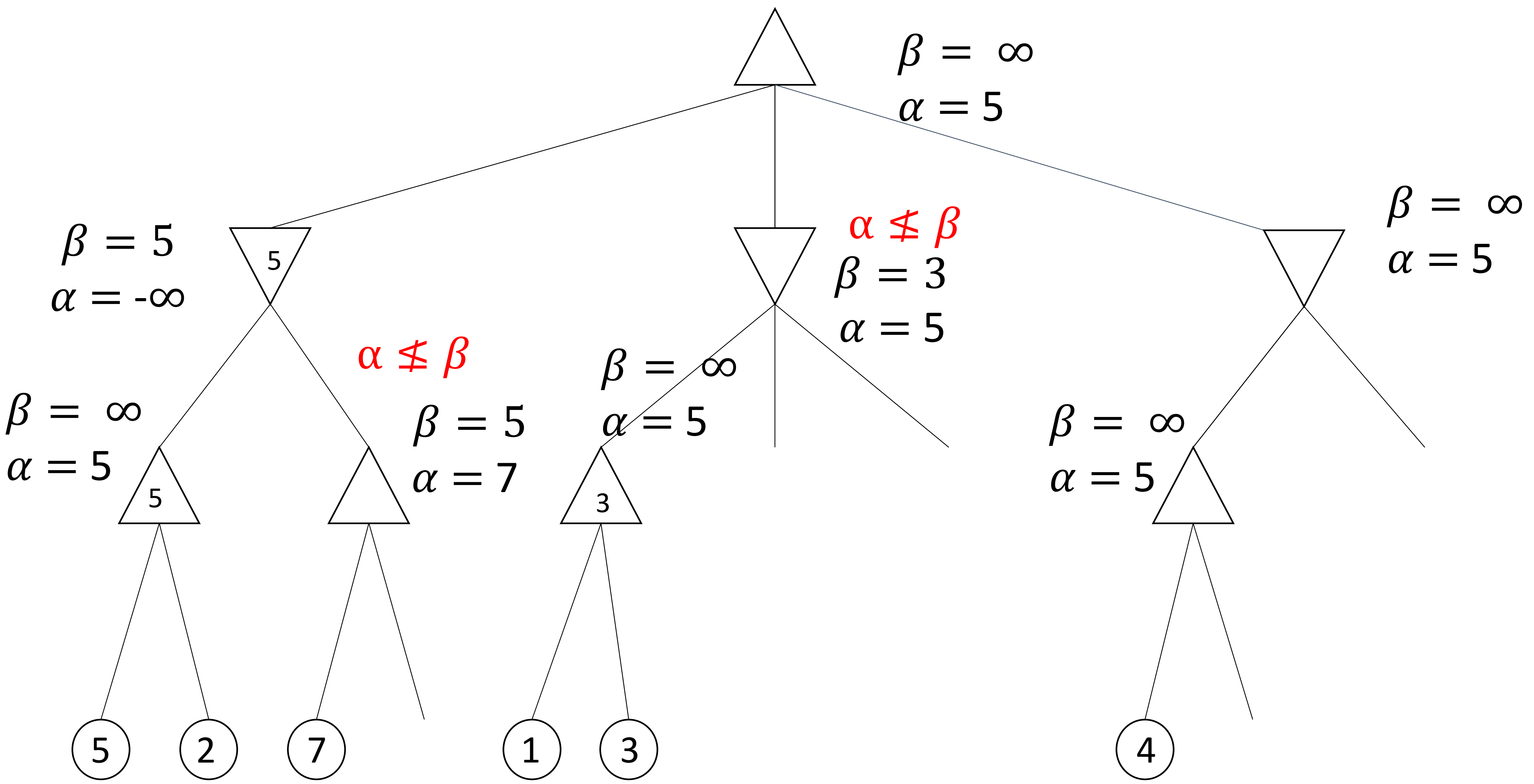

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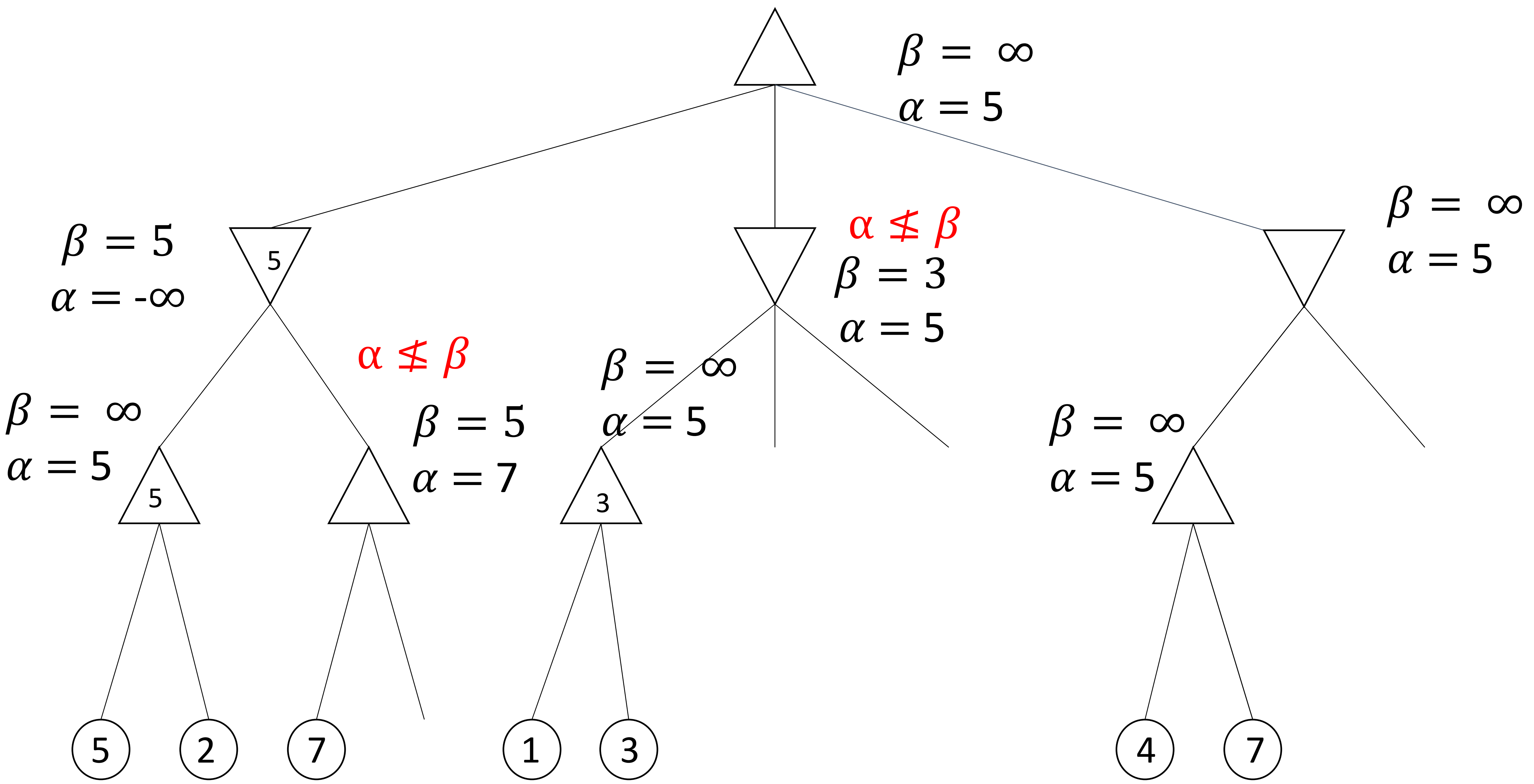
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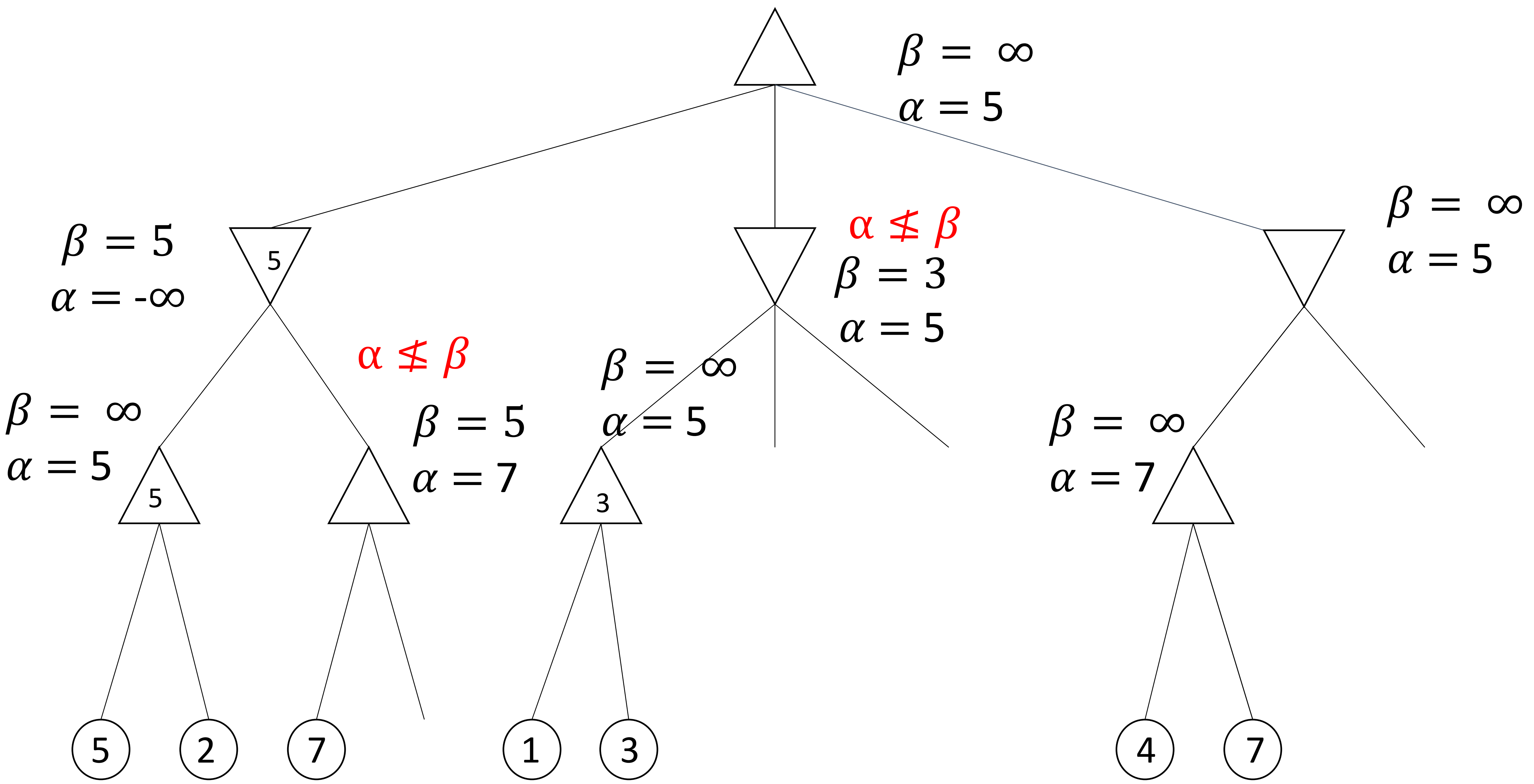
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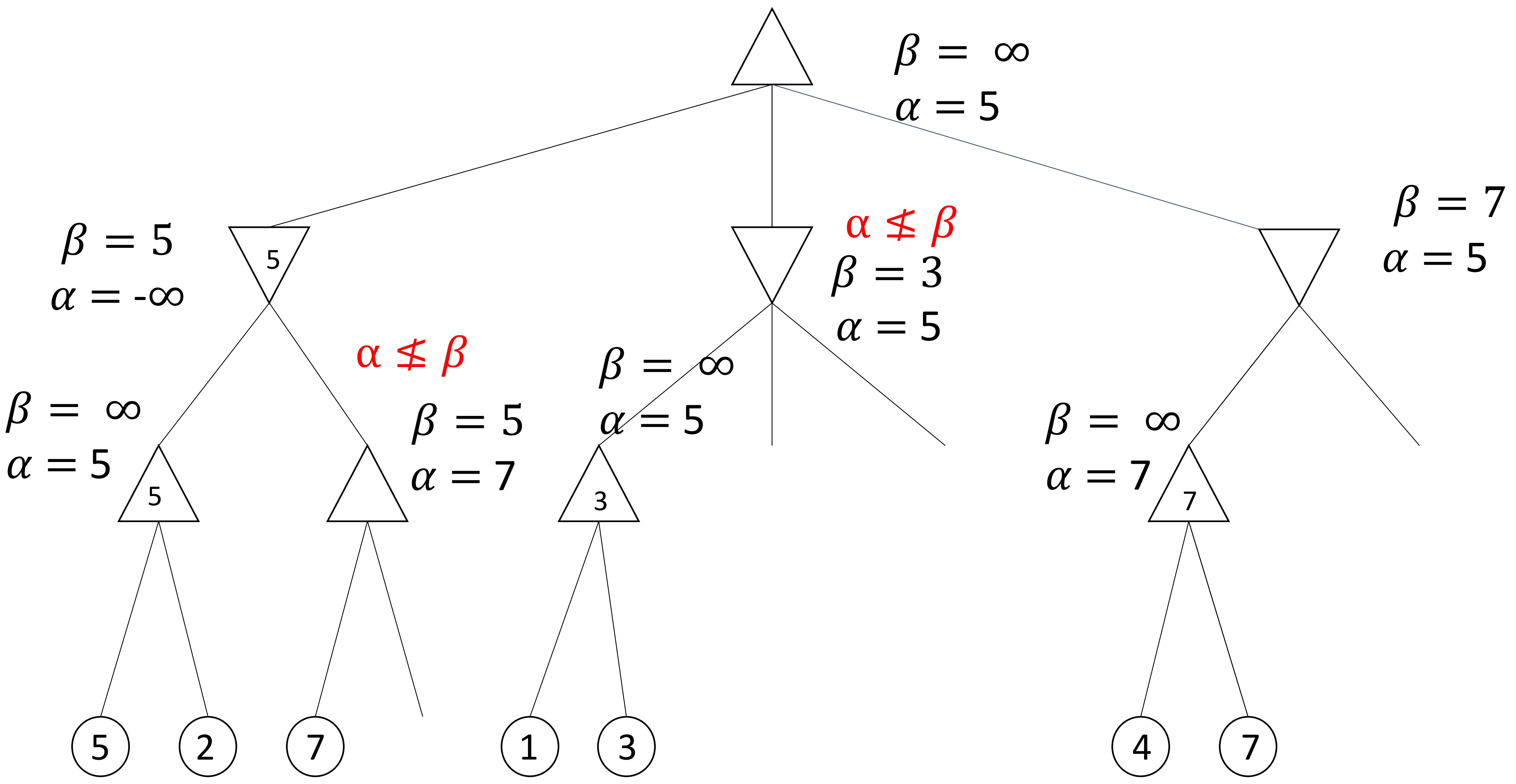
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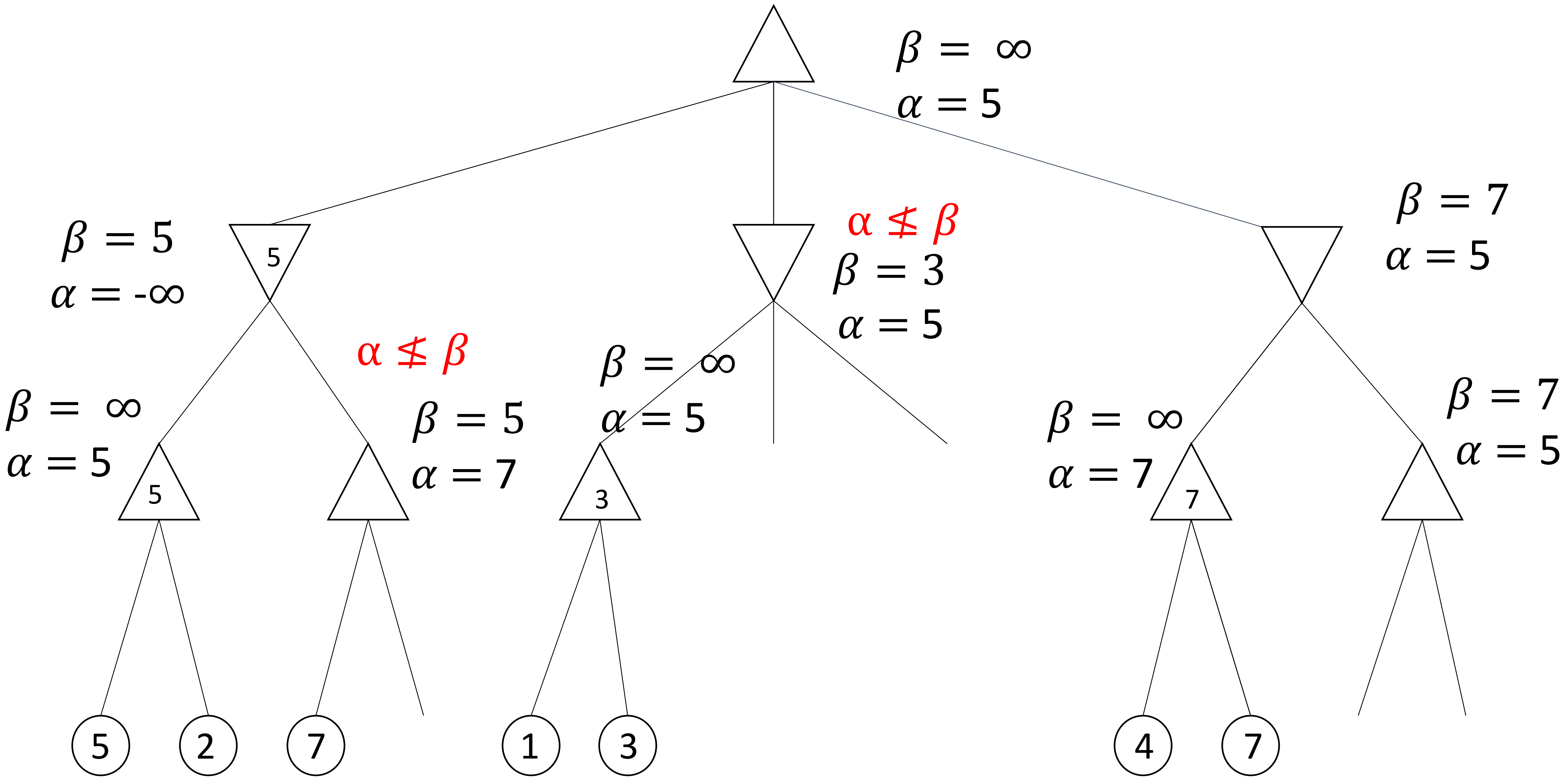
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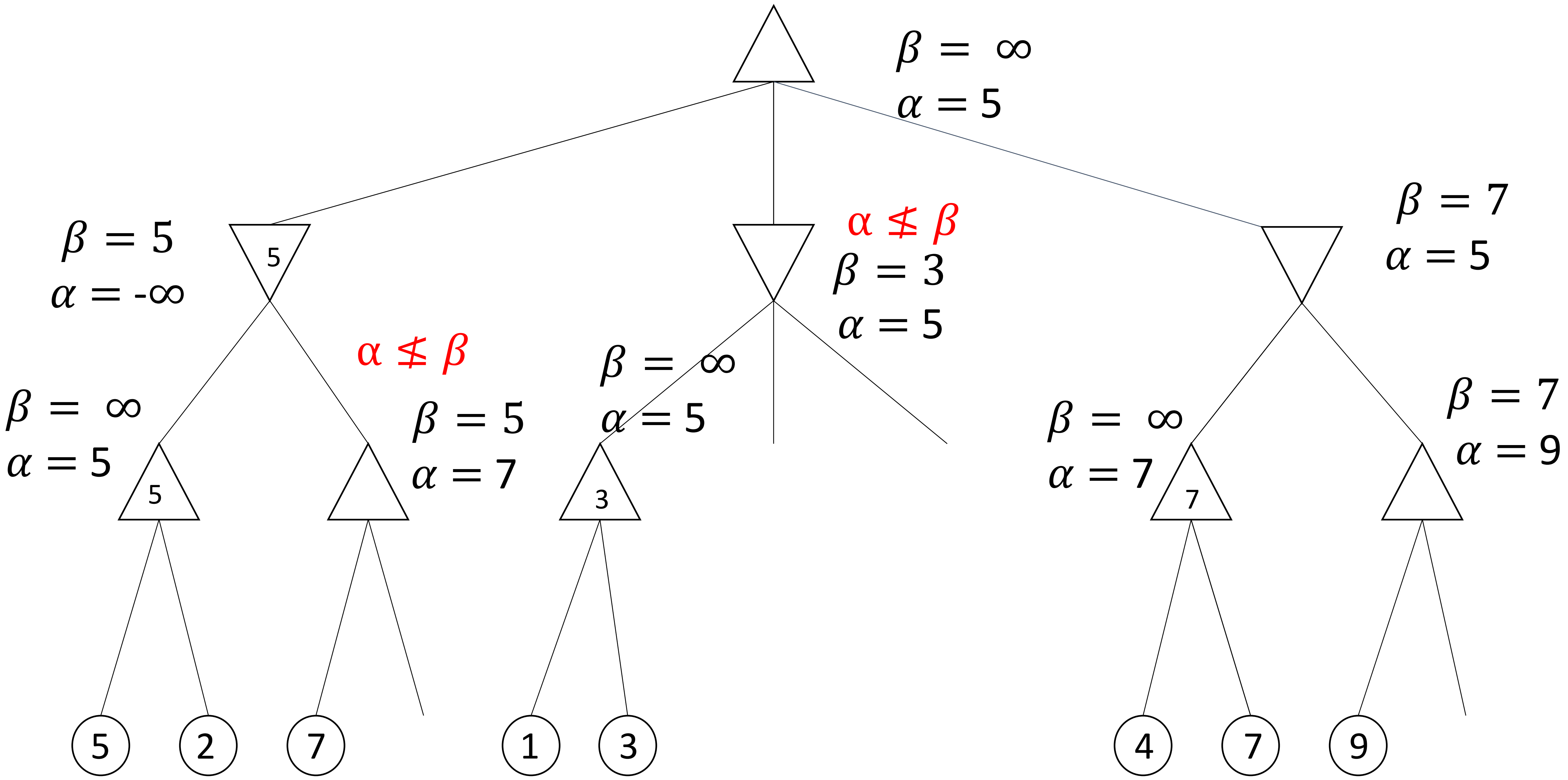

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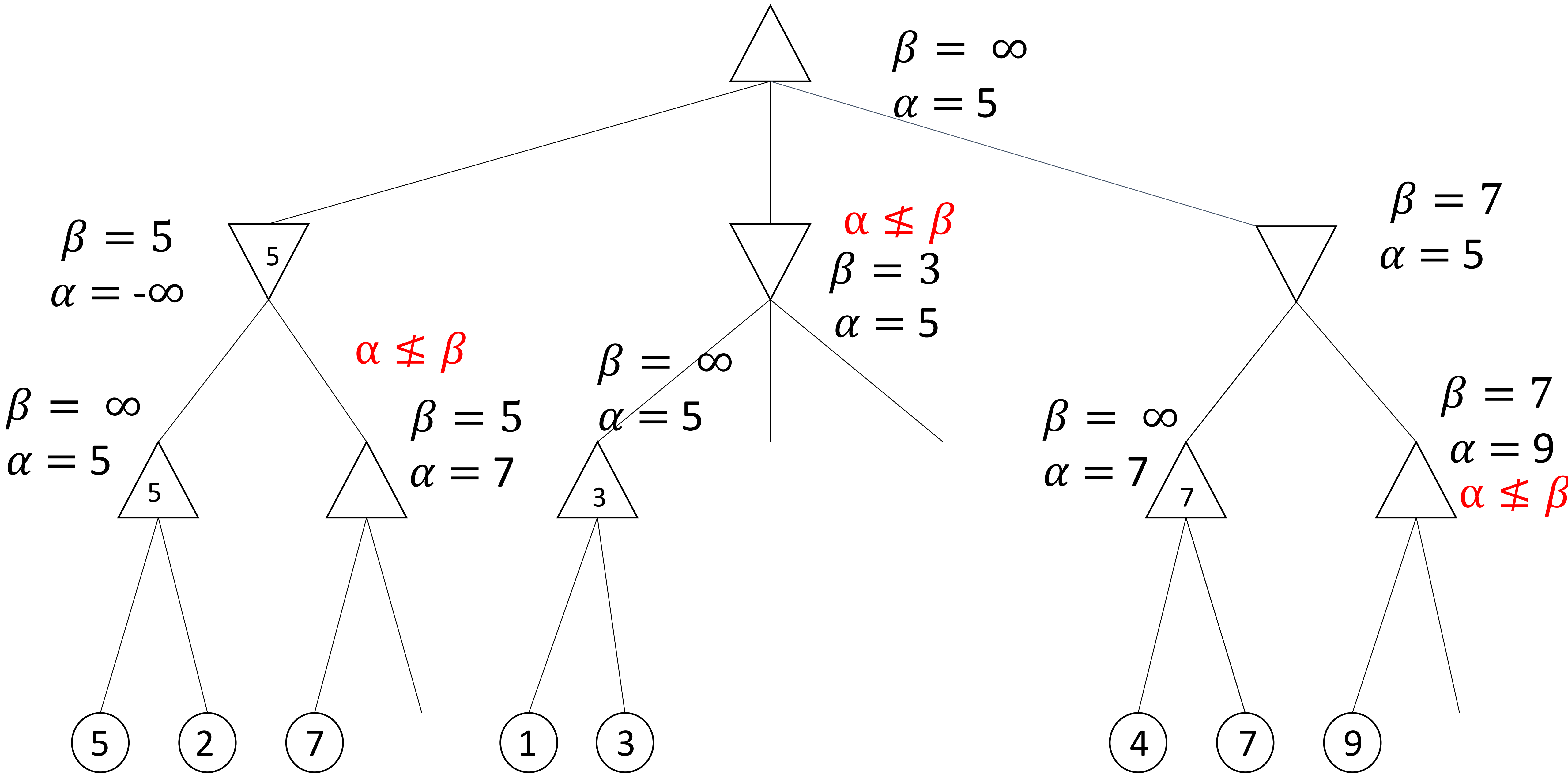

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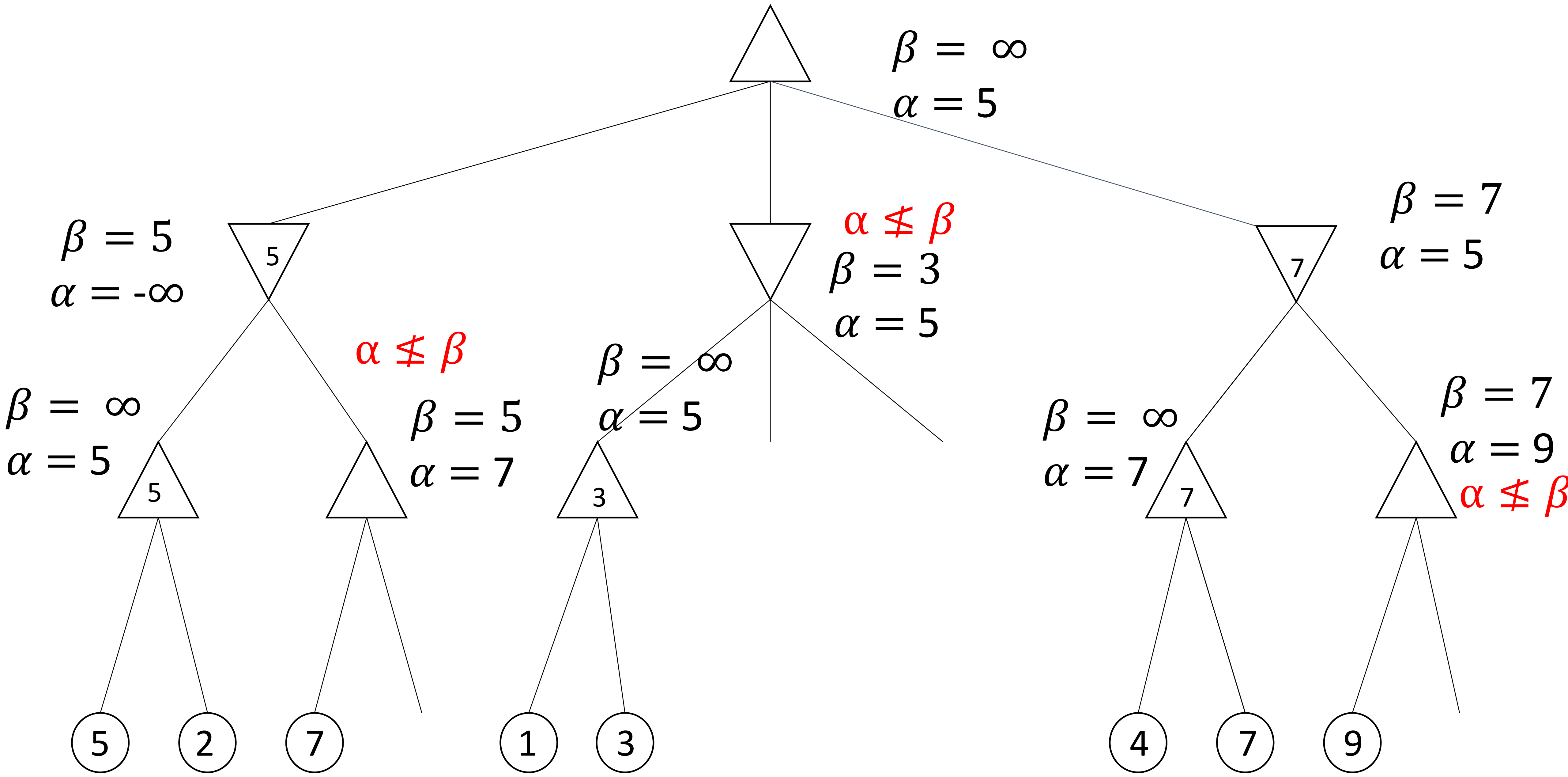

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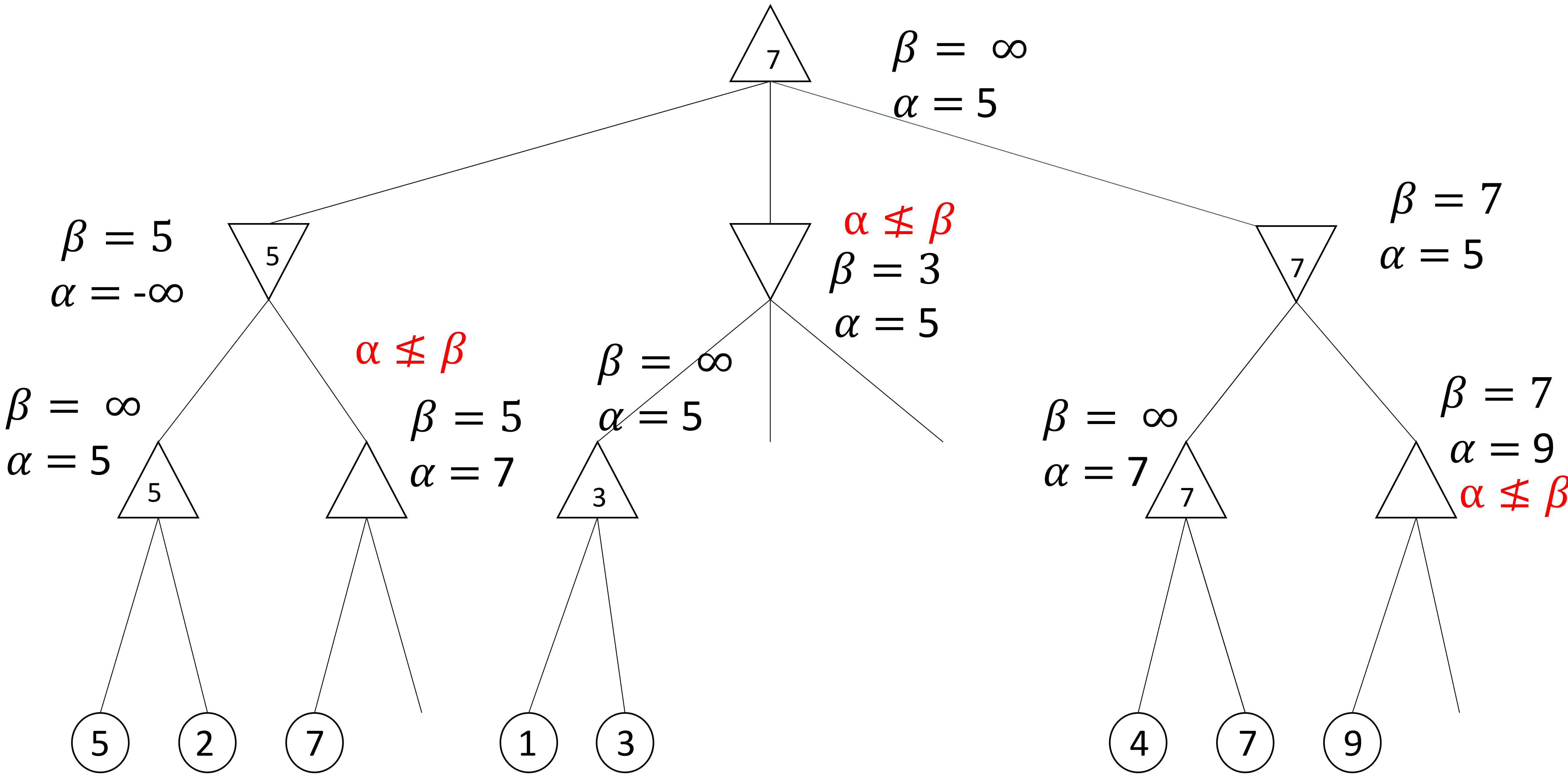
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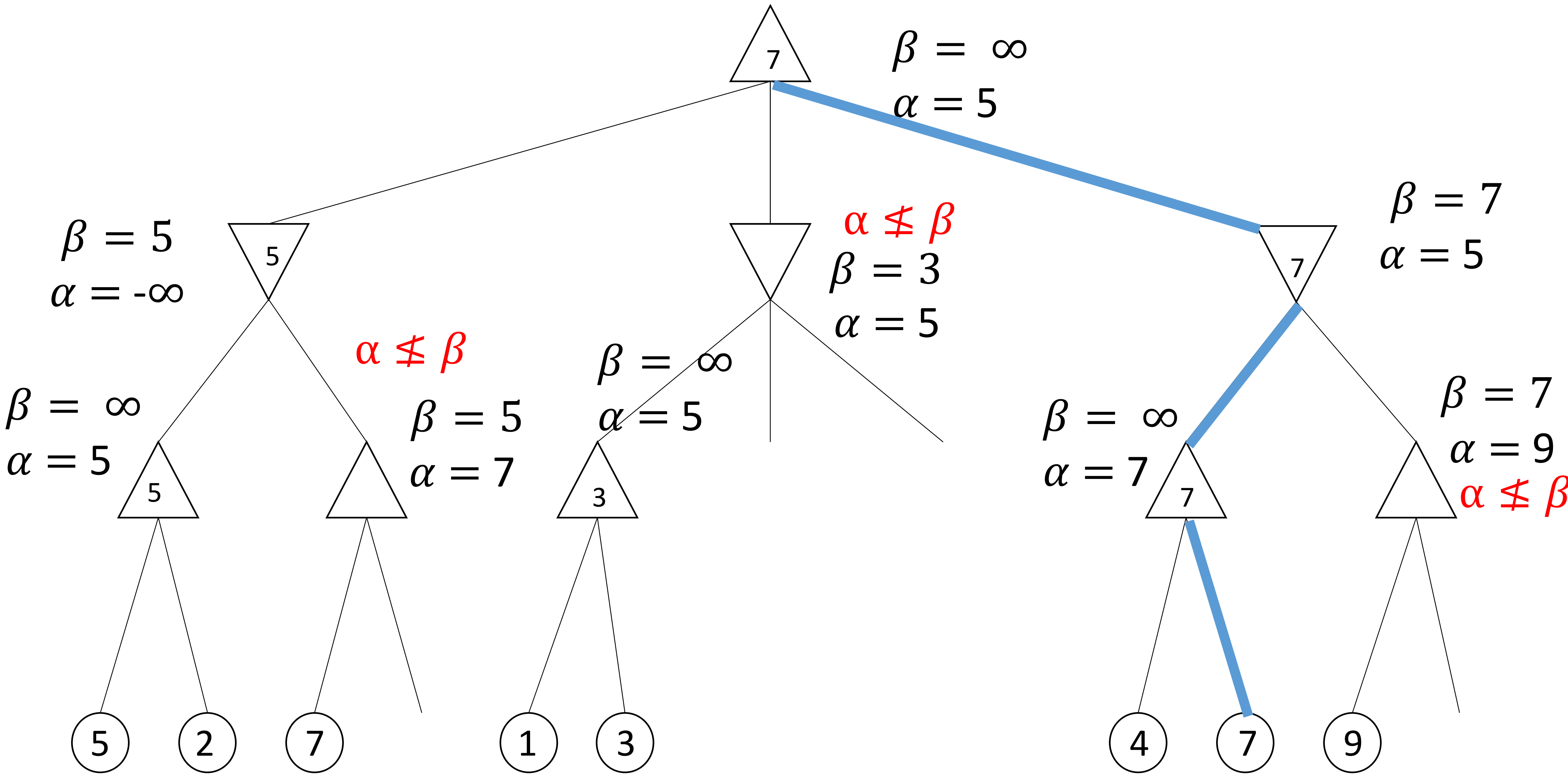
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## Alpha-Beta Pruning Practice

http://inst.eecs.berkeley.edu/~cs61b/fa14/ta-materials/apps/ab_tree_practice/

## Optimizing even more...

- Make the heuristic better

- Explore children in a different order – most ‘promising’ first

- Avoid exploring some children altogether?

And remember – make the rest of the code as fast as possible!

- Hint – if you find yourself calculating the same thing multiple times in the same definition, use a variable defined via a `where` clause: Haskell will then calculate it at most once.