$$L(\theta) = -y \log g_\theta(x) - (1-y) \log(1 - g_\theta(x))$$

$$g_\theta(x) = \sigma(f_\theta(x)) \qquad \sigma(z) = \frac{\exp(z)}{1+\exp(z)}$$

$$f_\theta(x) = \theta^T . x \qquad = \frac{1}{1+\exp(-z)}$$

$$\frac{dL}{d\theta} = \frac{dL}{dg} \cdot \frac{dg}{df} \cdot \frac{df}{d\theta}$$

$$\frac{dL}{dg} = -\frac{y}{g_\theta(x)} + \frac{(1-y)}{1-g_\theta(x)}$$

$$= \frac{-y(1-g_\theta(x)) + (1-y)\, g_\theta(x)}{g_\theta(x) \cdot (1-g_\theta(x))}$$

$$= \frac{g_\theta(x) - y}{g_\theta(x) \cdot (1-g_\theta(x))} \qquad 1$$

$$\frac{dg}{df} = + \frac{\exp(-f)}{(1+\exp(-f))^2}$$

$$g = \frac{1}{1+\exp(-f)}$$

$$\exp(-f) = \frac{1}{g} - 1$$

$$= g_\theta(x) \cdot (1 - g_\theta(x))$$

2

$$\frac{df}{d\theta} = x^T$$

3

$$1 \times 2 \times 3 = [g_\theta(x) - y] \, x^T$$

✓

$$P(y=1|\theta,x) = g_\theta(x) = \frac{1}{1+\exp(-f_\theta(x))}$$

$$P(y=0|\theta,x) = 1 - g_\theta(x)$$

$$= \frac{\exp(-f_\theta(x))}{1+\exp(-f_\theta(x))}$$

$$\frac{P(y=1|\theta,x)}{P(y=0|\theta,x)} = \exp(f_\theta(x))$$

$$\log \quad = f_\theta(x) = \theta^T x$$

$$X = \begin{bmatrix} \text{how many minutes you exercise} \\ \text{— apples you eat/day} \\ \text{— cigarettes you smoke/day} \end{bmatrix}$$

$y = 1 \rightarrow$ cancer

$\phantom{y = {}} 0 \rightarrow$ not cancer

$$\theta = \begin{bmatrix} -1.4 \\ -1.2 \\ 1.3 \end{bmatrix}$$

if we eat one more apple/day

risk of cancer reduced

by $\exp(+1.2)$

if we smoke one more cig/day

— increases by

$\exp(1.3)$

True

Prediction	$y=-1$	$y=1$
$y=-1$	TN	FN
→ $y=1$	FP	TP

① Perfect $FP = FN = 0$

$$\text{Accuracy} = 1 = \frac{TN + TP}{N}$$

$$\text{Error} = 1 - \text{Acc} = 0$$

$$\text{Precision} = \frac{TP}{TP + FP} = \frac{TP}{TP + 0} = 1$$

$$\text{Recall} = \frac{TP}{TP + \underbrace{FN}_{0}} = 1$$

$$TPR = 1$$

$$FPR = 0$$

② Random guess

	$y=-1$	$y=1$	
$y=-1$	$N_n/2$	$N_p/2$	
$y=1$	$N_n/2$	$N_p/2$	
	N_n	N_p	N

$$\text{Accuracy} = \frac{N_n/2 + N_p/2}{N} = \frac{1}{2}$$

$$\text{Error} = 1/2$$

$$\text{Precision} = \frac{N_p/2}{N_p/2 + N_n/2} = \frac{N_p}{N}$$

$$\text{Recall} = \frac{N_p/2}{N_p} = 1/2$$