MATH1005/MATH6005 Final Exam Semester 1 2022, Marking Criteria

| Problem 1a | |
|---|---|
| Marks | Description |
| 2 | Demonstrates mastery of how to use a truth table to determine whether or not a logical equivalence holds by correctly using a truth table, or truth tables, to show that this logical equivalence holds. |
| 1 | A minor error but demonstrates knowledge of how a truth table may be used to show that a logical equivalence holds |
| 0 | Otherwise |

| Problem 1b | |
|---|---|
| Marks | Description |
| 2 | Demonstrates mastery of writing the structural part of an argument that proceeds via the contrapositive. Introduces the variable G. Makes the correct hypothesis. Makes the correct conclusion (may give a statement that is logically equivalent to the one listed in the solutions). |
| 1 | A minor error (like forgetting to introduce G, or supposing that the chromatic number of G is greater than or equal to 4), but demonstrates knowledge of how to structure an argument via the contrapositive. At an absolute minimum the response contains p->q and ~q->~p. |
| 0 | Otherwise |

| Problem 1c | |
|---|---|
| Marks | Descriptions |
| 1 | Demonstrates mastery of the notation of sets and cartesian products by correctly answering the question and using notation accurately throughout |
| 0.5 | A minor error such as forgetting the empty set but must use notation correctly |
| 0 | Otherwise |

| Problem 1d | |
|---|---|
| Marks | Descriptions |
| 3 | An excellent element proof. Introduces variables, uses notation correctly (uses ⇔ between statements, never = between statements, etc), justifies each step with definitions and laws, and does not skip steps. |
| 2.5 | Almost an excellent element proof. As for 3 but doesn't introduce at least some of the variables or does not justify every step or skips an important step. |
| 2 | Pretty good element proof. Perhaps makes at least two of the errors described in 2.5. |
| 1 | Demonstrates some understanding of what to do, but misuses notation in a significant way (like writing = between statements, writing ⇔ between sets, proving only one containment) OR only draws the Venn diagrams |
| 0 | Otherwise |

| Problem 1e | |
|---|---|
| Marks | Descriptions |
| 2 | Describes an explicit counterexample and demonstrates that it is a counterexample |
| 1.5 | Correctly identifies that the statement is false, describes an explicit counterexample but does not demonstrate that it is a counterexample. |
| 1 | Correctly identifies that the statement is false, but does not provide an explicit counterexample |
| 0 | Otherwise |

| Problem 2a | |
|---|---|
| Marks | Descriptions |
| 3 | An excellent induction. The induction structure is explicit (does not have to be exactly like the solutions, but a clear base step and inductive step, explicit inductive hypothesis and invokes the principle of induction) and explicitly states when the inductive hypothesis is used. The inductive step is effective. |
| 2.5 | Almost an excellent induction. As for 3 but a minor error or omission. |
| 2 | A pretty good induction. Perhaps cannot make the inductive step work, but structures the argument well. |
| 1.5 | Cannot make the inductive step work, and the structure has a minor error. |
| 1 | Demonstrates some understanding of mathematical induction, but does not rise to the level of 2 |
| 0 | Otherwise |

| Problem 2b | |
|---|---|
| Marks | Descriptions |
| 1 | Effectively invokes generalised pigeon hole principle, or the pigeon hole principle, to explain why there is at least one PIN shared by four or more customers. |
| 0.5 | Mentions the pigeon hole principle, but the proof is not quite effective, or makes mention of pigeons and pigeon holes without making explicit reference to the pigeon hole principle. |
| 0 | Otherwise |

| Problem 2c | |
|---|---|
| Marks | Descriptions |
| 3 | An excellent computation that demonstrates mastery of probability when outcomes are equally likely. Explicitly justifies the use of formula $P(E) = \|E\|/\|S\|$, and explains how to correctly count the sets to be counted. |
| 2.5 | Almost an excellent computation. As for 3 but omits the justification that counting can be used to compute the probability, or part of the explanation needs expanding. |
| 2 | A pretty good computation. Perhaps a minor error in one part of the computation, or correct computation but almost no explanation and no justification as to why counting can be used |
| 1 | Demonstrates some understanding of how to use counting to compute a probability. |
| 0 | Otherwise |

| Problem 2d(i) | |
|---|---|
| Marks | Descriptions |
| 1 | Demonstrates mastery of the ideas of how to think through a probability experiment by explaining how each ordered pair represents what happened in the experiment and correctly describing the outcomes as a set of 16 ordered pairs |
| 0.5 | Demonstrates some understanding of the ideas of how to think through a probability experiment, perhaps by correctly describing the outcomes as a set of 16 ordered pairs. |
| 0 | Otherwise |

| Problem 2d(ii) | |
|---|---|
| Marks | Descriptions |
| 2 | Demonstrates knowledge of what it means for events to be independent by correctly computing the three probabilities required and determining that the events are not independent. |
| 1 | Demonstrates some understanding of what it means for events to be independent, perhaps by giving a correct definition or by computing (perhaps incorrectly) the probabilities of E1, E2 and their intersection |
| 0.5 | Does not demonstrate understanding of what it means for events to be independent but correctly computes the probabilities of E1 and E2. |
| 0 | Otherwise |

NOTE: GRADES FOR 2d(i) and 2d(ii) are combined for entry in spreadsheet

| Problem 3a | |
|---|---|
| Marks | Descriptions |
| 3 | Demonstrates mastery of the vocabulary of walks, paths and circuits by giving three correct answers (numbers and lists) and walks are described as alternating lists of vertices and edges (it is necessary to list the edges because there are parallel edges in the graph). |
| 2 | Demonstrates a pretty good understanding of the vocabulary of walks, paths and circuits, but for a minor error (omitting the trivial circuits in (iii) or not having the same circuit starting at different points in (iii) or lists the correct number of walks IN EACH CASE but writes walks as a list of vertices without the edges in between |
| 1.5 | Demonstrates a good understanding of the vocabulary of walks, paths and circuits but writes walks as a list of vertices without the edges in between (it is necessary to list the edges because there are parallel edges in the graph) and therefore does not distinguish between some walks that are different (would lead to answers infinitely many, 1, 8)<br>Answers to only two parts from three parts for instance (i) and (ii) are correct. |
| 1 | Demonstrates some understanding of walks, paths and circuits, but does not rise to the level of 2 or 1.5. Answers to only one part of the three parts are correct. |
| 0 | Otherwise |

| Problem 3b | |
|---|---|
| Marks | Descriptions |
| 1 | Mentions something about arranging CPUs for the purpose of parallel computing |
| 0 | Otherwise |

| Problem 3c(i) | |
|---|---|
| Marks | Descriptions |
| 1 | Demonstrates knowledge of the vocabulary of graph isomorphisms by stating a correct definition (may leave out “for all u_1, u_2 \in V(G)” as we left this out in the lecture slides) |
| 0 | Otherwise |

| Problem 3c(ii) | |
|---|---|
| Marks | Descriptions |

| | |
|---|---|
| 2 | Demonstrates mastery of enumerative combinatorics and graph isomorphisms and knowledge of the complete bipartite graph by correctly counting isomorphisms and explaining the reasoning. |
| 1 | Demonstrates understanding of what the complete bipartite graph is, and some idea that an isomorphism must map vertices of degree m to vertices of degree m. May have the incorrect answer. |
| 0.5 | Demonstrates understanding of what the complete bipartite graph is |
| 0 | Otherwise |

Note: Score for 3c(i) and 3c(ii) combined for entry into the spreadsheet.

| Problem 3d | |
|---|---|
| Marks | Descriptions |
| 3 | Demonstrates mastery of Hamilton circuits and proofs by providing a well-structured and effective argument that the graph has no Hamilton circuit. |
| | |
| 2 | Pretty good argument. As for 3 but omits one or more than one necessary justification. No false statements are made. |
| 1 | Demonstrates knowledge of what a Hamilton circuit is. |
| 0 | Otherwise |

| Problem 4a | |
|---|---|
| Marks | Descriptions |
| 1 | Demonstrates knowledge of the Nearest Neighbour Algorithm by “correctly” listing the input and output.  To be correct, the input must list “weighted complete graph” and the output must list “Hamilton circuit for G”  and must NOT list other things |
| 0.5 | If missing “complete” for input and otherwise satisfying for 1 |
| 0 | Otherwise (including if the response claims that the algorithm produces a minimal weight Hamilton circuit) |

| Problem 4b | |
|---|---|
| Marks | Descriptions |
| 1 | Demonstrates knowledge of Fleury’s algorithm and Euler circuits by correctly identifying the condition.<br><br>OR<br>If they mentioned at most 2 vertices are odd and must start at an odd degree vertex |
| 0.5 | If they mentioned at most 2 vertices are odd (but not starting at odd) |
| 0 | Otherwise |

| Problem 4c | |
|---|---|
| Marks | Descriptions |
| 2 | Demonstrates understanding of Kruskal’s algorithm by correctly naming it and drawing the correct minimal spanning tree. |
| 1 | Meets 1 of the 2 criteria for a 2 |
| 0 | Otherwise |

| Problem 4d | |
|---|---|
| Marks | Descriptions |
| 3 | Demonstrates understanding of Dijkstra’s algorithm by producing lists that are correct and complete. |
| 2.5 | 3 except no A in path |

| | |
|---|---|
| 2 | Demonstrates a pretty good understanding of Dijkstra's algorithm but at least one of the lists is incorrect or incomplete. There is evidence that the misunderstanding is a minor error or minor misunderstanding. |
| 1 | Demonstrates some understanding of Dijkstra's algorithm but at least one of the lists is incorrect or incomplete |
| 0 | Otherwise |

| Problem 4e | |
|---|---|
| Marks | Descriptions |
| 3 | Demonstrates understanding of the vertex labelling algorithm by producing lists that are correct and complete. |
| 2 | Demonstrates pretty good understanding of the vertex labelling algorithm but at least one of the lists is incorrect or incomplete. There is evidence that the misunderstanding is a minor error or minor misunderstanding. |
| 1.5 | Demonstrates some understanding of the vertex labelling algorithm but at least one of the lists is incorrect or incomplete AND with understanding of virtual flow. |
| 1 | Demonstrates some understanding of the vertex labelling algorithm but at least one of the lists is incorrect or incomplete<br><br>(if the solution does not demonstrate understanding of virtual flow then it cannot score more than 1) |
| 0 | Otherwise |

| Problem 5a | |
|---|---|
| Marks | Descriptions |
| 2 | Demonstrates understanding of transition diagrams for Markov process by drawing a correct directed graph with correct labels, including correct deductions about the labels on edges for which explicit information is not given in the problem. |
| 1 | Demonstrates some understanding of transition diagrams for Markov process by drawing a directed graph with labels on edges. May have made an error in transcribing information from the problem or may omit edges (and/or their labels) not explicitly mentioned in the problem.<br><br>OR<br><br>Makes a correct transition matrix instead of a transition diagram. |
| 0.5 | Draws a directed graph<br><br>OR<br><br>Makes a transition diagram |
| 0 | Otherwise |

| Problem 5b(i) | |
|---|---|
| Marks | Descriptions |
| 1 | Demonstrates an understanding of webgraphs, and the ability to model a similar situation, by correctly describing the vertices and edges (including the correct direction). |
| 0.5 | Describes the vertices correctly |
| 0 | Otherwise |

| Problem 5b(ii) | |
|---|---|
| Marks | Descriptions |
| 2 | Demonstrates understanding of the two key hypotheses of the PageRank model by making analogous statements about the employee graph (language may be rough, but there is enough evidence to suggest an understanding of both) |
| 1 | Makes at least one of the two statements listed in the solution (language may be rough, but there is enough evidence to suggest an understanding of at least one hypothesis) |
| 0 | Otherwise |

Note: Scores for 5b(i) and 5b(ii) and combined for entry in spreadsheet.

| Problem 5c(i) | |
|---|---|
| Marks | Descriptions |
| 1 | A correct webgraph |
| 0.5 | If added transition probability and dotted arrow for D |
| 0 | Otherwise |

| Problem 5c(ii) | |
|---|---|
| Marks | Descriptions |
| 2 | A correct basic transition matrix (also accept the same matrix but with ¼, ¼, ¼ , ¼ along the bottom row, even though this is technically incorrect). |
| 1 | As for 2 but row 4 is not one of the two versions accepted<br><br>OR<br><br>Minor mistakes with transition probability<br><br>OR<br><br>Not basic. e.g added damping. |
| 0.5 | Makes two errors from 1 |
| 0 | Otherwise |

| Problem 5c(iii) | |
|---|---|
| Marks | Descriptions |
| 2 | Demonstrates understanding that the page rank vector is the steady-state vector for the system and this can be checked by checking that M’v = v. |
| 1 | Demonstrates understanding that the page rank vector is the steady-state vector for the system but suggests doing the computation of a steady-state vector instead of just using the v as given<br><br>OR<br><br>As for 2 but uses M when should use M’<br><br>OR<br><br>As for 2 but did vM’ (as matrix multiplication do not commute) |

| | |
|---|---|
| | OR<br>Only give M’v=v without interpretation |
| 0.5 | Makes both errors from 1 |
| 0 | Otherwise |