Question 1 In rôle-playing games, such as the original, *Dungeons and Dragons*, gamers use a variety of dice shapes, not just the standard cube. For example a 'd12' has the shape of a regular dodecahedron. A (fair) d12 has twelve faces numbered 1,2,. . . ,12, and each face is equally likely to become the top face (the face that is read) when the die is 'thrown'.

Suppose gamer Alice does not have a d12 but does have a d6 (*i.e.* a regular cubic die with six faces labelled 1,2,. . . ,6.) To simulate throwing a d12, Alice throws her d6 twice, resulting in a pair of values $(a, b) \in \{1, \ldots, 6\}^2$. She then combines the two values in some way to come up with a value in $\{1, \ldots, 12\}$.

(a) Write out the sample space S for this 'experiment'.

$$\left\{ \begin{array}{cccccc} (1,1), & (1,2), & (1,3), & (1,4), & (1,5), & (1,6), \\ (2,1), & (2,2), & (2,3), & (2,4), & (2,5), & (2,6), \\ (3,1), & (3,2), & (3,3), & (3,4), & (3,5), & (3,6), \\ (4,1), & (4,2), & (4,3) & ,(4,4), & (4,5), & (4,6), \\ (5,1), & (5,2), & (5,3), & (5,4), & (5,5), & (5,6), \\ (6,1), & (6,2), & (6,3), & (6,4), & (6,5), & (6,6) \end{array} \right\}$$

(b) One way to combine a and b would be to add them. But this would not accurately simulate a d12 throw because it would be impossible to score 1.
To avoid this problem Alice decides to use the formula: $v = \left\lceil \frac{ab}{3} \right\rceil$.
Write out the event $E \subseteq S$ coresponding to $v = 1$.

We need $ab \leq 3$ so

$$E = \{(1,1),\ ((1,2),\ (1,3),\ (2,1),\ (3,1)\}.$$

(c) What is the probability of event E above and in what way does it demonstrate that Alice's method also does not accurately simulate a d12 throw?

$$\mathbb{P}(E) = \frac{|E|}{|S|} = \frac{5}{36}.$$

On a d12 every outcome has equal probability $\frac{1}{12}$. But $\frac{5}{36} \neq \frac{1}{12}$.

(d) How *could* Alice combine a and b to accurately simulate a d12 throw?
(Gamers actually do this, so you can Google an answer if you're desperate!)

One possibility is:

$$v = \begin{cases} a & \textit{if } b \textit{ is even} \\ a+6 & \textit{if } b \textit{ is odd} \end{cases}$$

For example, $\mathbb{P}(v{=}5) = \frac{1}{36}|\{(5,2),(5,4),(5,6)\}| = \frac{3}{36} = \frac{1}{12}$.

Question 2 To evaluate the quality of a micro-finance program, some participants are selected, and their economic situation measured. In a given evaluation a sample of 100 participants, out of a total 10 000, is randomly selected. Every participant has the same probability of being selected.

Assume that 40 out of the 10 000 participants have actually seen their economic situation deteriorate. We seek the probability that at least one of these is in the sample.

(a) Describe a sample space S for this problem, and give a formula for $|S|$.

The sample space S is the set of all possible samples of size 100 drawn from the population of size 10,000. So

$$|S| = C(10\,000, 100) = \boxed{\binom{10\,000}{100}}.$$

(b) Let E be the event "at least one of the participants in the random selection has seen her/his economic situation deteriorate". Give a formula for $|E^c|$.

The set E^c comprises all samples from S that contain noone whose economic situation has deteriorated. Removing those 40 people from the population leaves 9960 to choose from. So

$$|E^c| = C(9960, 100) = \boxed{\binom{9960}{100}}.$$

(c) With the help of WolframAlpha, or some other electronic aid, calculate $\mathbb{P}(E)$.

$$\mathbb{P}(E) = 1 - \frac{|E^c|}{|S|} = 1 - \frac{\binom{9960}{100}}{\binom{10\,000}{100}} \approx \boxed{0.3316 \approx 33\%}.$$

Question 3 A *Binomial experiment* comprises a fixed number n of 'trials' where each trial has the same probability p of 'success'. The probability that a binomial experiment results in k successes is given by

$$\mathbb{P}(k \text{ successes}) = \binom{n}{k} p^k (1-p)^{n-k}.$$

For example, what is the probability of scoring two 6s from twelve throws of a standard die? Here $n = 12$, $p = \frac{1}{6}$ and $k = 2$, so the the probability is $\binom{12}{2}\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^{10}$.

(a) Use your calculator to find the probabilities of

(i) two 6s from twelve throws, $k = 2,\ n = 12,\ p = \frac{1}{6}$.

$$\mathbb{P}(\textit{two } 6s) = \binom{12}{2}\left(\frac{1}{6}\right)^2\left(\frac{5}{6}\right)^{10} = \frac{12 \cdot 11 \cdot 5^{10}}{2 \cdot 1 \cdot 6^{12}} \approx \boxed{0.296}.$$

(ii) one 6 from twelve throws and $k = 1,\ n = 12,\ p = \frac{1}{6}$.

$$\mathbb{P}(\textit{one } 6) = \binom{12}{1}\left(\frac{1}{6}\right)^1\left(\frac{5}{6}\right)^{11} = \frac{12 \cdot 5^{11}}{1 \cdot 6^{12}} \approx \boxed{0.269}.$$

(iii) no sixes from twelve throws $k = 1,\ n = 12,\ p = \frac{1}{6}$.

$$\mathbb{P}(\textit{no } 6s) = \binom{12}{0}\left(\frac{1}{6}\right)^0\left(\frac{5}{6}\right)^{12} = \left(\frac{5}{6}\right)^{12} \approx \boxed{0.112}.$$

(b) Calculate the probability of scoring *at least two 6s* from twelve throws.

$$\begin{aligned}\mathbb{P}(\textit{at least two } 6s) &= 1 - \mathbb{P}(\textit{one } 6) - \mathbb{P}(\textit{no } 6)\\ &\approx 1 - 0.269 - 0.112 \approx \boxed{0.619}.\end{aligned}$$

(c) In practice Binomial probabilities are usually found from a book of tables or from on-line tables or calculators. Both density and (cumulative) distribution values are available. With the help of an on-line statistical calculator such as *Stat Trek* (`http://stattrek.com/online-calculator/binomial.aspx`) find the probability of scoring more than two but less than eight 6s from 30 throws of a standard die. Give all the values you obtained, what they represented, and how you used them to obtain your answer.

Setting "Probability of success on a single trial" to 0.16667*, "Number of trials" to* 30 *and "Number of successes (X)" to* 7*, we get, as the third line of the output, " Cumulative Probability:* $P(X \leq 7)$ 0.886303*.....".*

Changing "Number of successes (X)" to 3*, we get, as the second line of the output, " Cumulative Probability:* $P(X < 3)$ 0.102782*.....".*

So $\mathbb{P}(3 \leq X \leq 7) = \mathbb{P}(X \leq 7) - \mathbb{P}(X < 3) \approx 0.8863 - 0.1028 = \boxed{0.7835}.$

Question 4 The income and education level of each person on the electoral roll for Queanberra is recorded as a pair $(x, y) \in \{1, 2, 3\}^2$, where 1 stands for low, 2 for average, and 3 for high, *e.g.* $(2, 3)$ represents a highly educated person with average income.

Let S denote the set of all people on the Queanberra electoral roll, and define random variables $X, Y : S \to \{1, 2, 3\}$ by $X(s), Y(s)$ are the income and educational levels of person s. Let $p_{i,j} = \mathbb{P}(\{(X(s) = i) \wedge (Y(s) = j)\})$ for $1 \leq i, j \leq 3$. Assume that

$$(p_{i,j})_{1 \leq i,j \leq 3} = \begin{pmatrix} 0.05 & 0.10 & 0.05 \\ 0.10 & 0.20 & 0.10 \\ 0.15 & 0.20 & 0.05 \end{pmatrix}.$$

(a) Explain what $p_{1,2}$ represents.

$\boxed{\textit{Probability that } s \textit{ has low income and average education level.}}$

(b) Find the probability of the event AI: "the person has an average income"?

$p_{2,1} + p_{2,2} + p_{2,3} = 0.10 + 0.20 + 0.10 = \boxed{0.40}$.

(c) Find the probability of the event AE: "the person has an average education level"?

$p_{1,2} + p_{2,2} + p_{3,2} = 0.10 + 0.20 + 0.20 = \boxed{0.50}$.

(d) Describe the event $AI \cup AE$ and find its probability.

$\boxed{s \textit{ has average income or average education level}}$

$$\begin{aligned} \mathbb{P}(AI \cup AE) &= \mathbb{P}(AI) + \mathbb{P}(AE) - \mathbb{P}(AI \cap AE) \\ &= 0.40 + 0.50 - p_{2,2} \;=\; 0.90 - 0.20 \;=\; \boxed{0.70}. \end{aligned}$$

(e) Are the events AI and AE independent?

$\mathbb{P}(AI) \times \mathbb{P}(AE) = 0.40 \times 0.50 = 0.20 = p_{2,2} = \mathbb{P}(AI \cap AE)$.

So $\boxed{\textit{Yes}}$ *they are independent.*

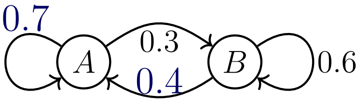
(f) Are the random variables X and Y independent?

$\boxed{\textit{No}}$. *E.g. let* LI = *'Low Income' and let* LE = *'Low Ed. level'. Then*

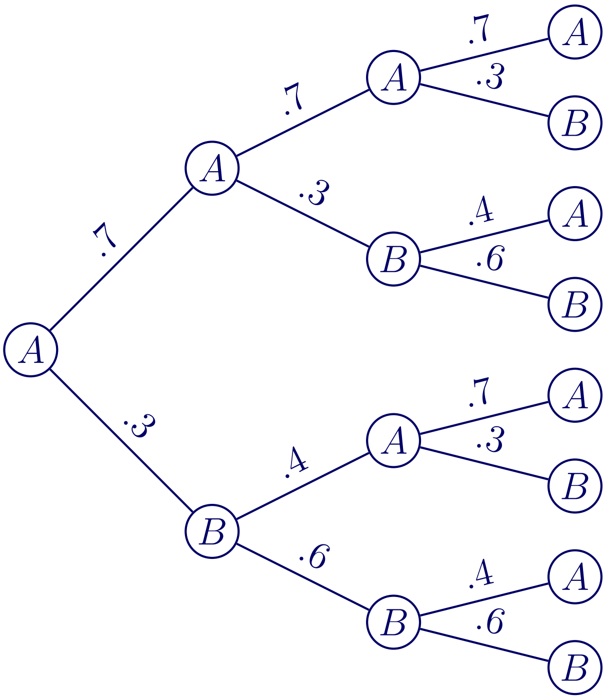
$$\begin{aligned} \mathbb{P}(X(1)) \times \mathbb{P}(Y(1)) &= \mathbb{P}(LI) \times \mathbb{P}(LE) \\ &= (0.05{+}0.10{+}0.05{+}) \times (0.05{+}0.10{+}0.15) \\ &= 0.20 \times 0.30 = 0.06 \neq 0.05 = p_{1,1} = \mathbb{P}(X(1) \cap Y(1)). \end{aligned}$$

Question 5 A Markov process has two states A and B with transition graph below.

(a) Write in the two missing probabilities.



(b) Suppose the system is in state A initially. Use a tree diagram to find the probability that the system will be in state B after three steps.



$.7\times.7\times.7=0.343$

$\mathbf{.7\times.7\times.3=0.147}$

$.7\times.3\times.4=0.084$

$\mathbf{.7\times.3\times.6=0.126}$

$.3\times.4\times.7=0.084$

$\mathbf{.3\times.4\times.3=0.036}$

$.3\times.6\times.4=0.072$

$\mathbf{.3\times.6\times.6=0.108}$

Probability system in state B after 3 steps = $\boxed{\mathbf{0.417}}$

(c) The transition matrix for this process is $T = \boxed{\begin{bmatrix} .7 & .3 \\ .4 & .6 \end{bmatrix}}$.

(d) Use T to recalculate the probability found in (b).

$$\begin{aligned}
\mathbf{x}_3 = (T')^3\mathbf{x}_0 &= \begin{bmatrix} .7 & .4 \\ .3 & .6 \end{bmatrix}\begin{bmatrix} .7 & .4 \\ .3 & .6 \end{bmatrix}\begin{bmatrix} .7 & .4 \\ .3 & .6 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} \\
&= \begin{bmatrix} .7 & .4 \\ .3 & .6 \end{bmatrix}\begin{bmatrix} .7 & .4 \\ .3 & .6 \end{bmatrix}\begin{bmatrix} .7 \\ .3 \end{bmatrix} \\
&= \begin{bmatrix} .7 & .4 \\ .3 & .6 \end{bmatrix}\begin{bmatrix} .61 \\ .39 \end{bmatrix} \\
&= \begin{bmatrix} .583 \\ .417 \end{bmatrix}
\end{aligned}$$

So we again get: Probability system in state B after 3 steps = $\boxed{0.417}$.

Question 6 Let $T = \begin{bmatrix} 9/10 & 1/10 \\ 7/10 & 3/10 \end{bmatrix} = \begin{bmatrix} .9 & .1 \\ .7 & .3 \end{bmatrix}$.

(a) Use the 'Matrix Calculator' computer application `http://matrixcalc.org/en/` to calculate T^2, T^4, T^8 and T^{16} to 3dp accuracy. (You can progressively insert the results back into an input matrix, so there is no need to physically enter anything more than the four entries of T.)

$$T^2 = \begin{bmatrix} 22/25 & 3/25 \\ 21/25 & 4/25 \end{bmatrix} = \begin{bmatrix} 0.880 & 0.120 \\ 0.840 & 0.160 \end{bmatrix}$$

$$T^4 = (T^2)^2 = \begin{bmatrix} 547/625 & 78/625 \\ 546/625 & 79/625 \end{bmatrix} \approx \begin{bmatrix} 0.875 & 0.125 \\ 0.874 & 0.126 \end{bmatrix}$$

$$T^8 = (T^4)^2 = \begin{bmatrix} 341797/390625 & 48828/390625 \\ 341796/390625 & 48829/390625 \end{bmatrix} \approx \begin{bmatrix} 0.875 & 0.125 \\ 0.875 & 0.125 \end{bmatrix}$$

$$T^{16} = (T^8)^2 = \begin{bmatrix} 133514404297/152587890625 & 19073486328/152587890625 \\ 133514404296/152587890625 & 19073486329/152587890625 \end{bmatrix} \approx \begin{bmatrix} 0.875 & 0.125 \\ 0.875 & 0.125 \end{bmatrix}$$

(b) Based on (a) guess a steady state vector for the Markov process with transition matrix T.

To 3 decimal place accuracy, the rows of T^{16} are the same, so these rows are the steady state vector.

$$S = \boxed{\begin{bmatrix} 0.875 \\ 0.125 \end{bmatrix}} = \begin{bmatrix} 7/8 \\ 1/8 \end{bmatrix} = \frac{1}{8}\begin{bmatrix} 7 \\ 1 \end{bmatrix}.$$

(c) Use the transpose matrix T' to verify that your guess from (b) is correct. Do the calculation by hand.

$$T'S = \begin{bmatrix} .9 & .7 \\ .1 & .3 \end{bmatrix} \frac{1}{8} \begin{bmatrix} 7 \\ 1 \end{bmatrix} = \frac{1}{8} \begin{bmatrix} 6.3+0.7 \\ 0.7+0.3 \end{bmatrix} = \frac{1}{8} \begin{bmatrix} 7 \\ 1 \end{bmatrix} = S.$$

Question 7

(a) By solving the relevant system of equations find the steady state vector for the Markov process with transition matrix $T = \begin{bmatrix} .7 & .3 \\ .2 & .8 \end{bmatrix}$. Do this by hand calculation, using matrix inverse.

We need to solve $(T' - I)S = 0$ *subject to* S *being a probability vector.*

Using $T' - I = \begin{bmatrix} .7 & .2 \\ .3 & .8 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} -.3 & .2 \\ .3 & -.2 \end{bmatrix}$ *and* $\begin{bmatrix} 1 & 1 \end{bmatrix} S = 1,$

the shortcut method says we must solve the linear system given by

$$\begin{bmatrix} -.3 & .2 \\ 1 & 1 \end{bmatrix} S = \begin{bmatrix} 0 \\ 1 \end{bmatrix}.$$

So

$$\begin{aligned} S &= \begin{bmatrix} -.3 & .2 \\ 1 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 1 \end{bmatrix} \\ &= \frac{1}{-.3 - .2} \begin{bmatrix} 1 & -.2 \\ -1 & -.3 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \frac{1}{-0.5} \begin{bmatrix} -.2 \\ -.3 \end{bmatrix} = \boxed{\begin{bmatrix} .4 \\ .6 \end{bmatrix}} \quad \left(= \begin{bmatrix} 2/5 \\ 3/5 \end{bmatrix} \right) \end{aligned}$$

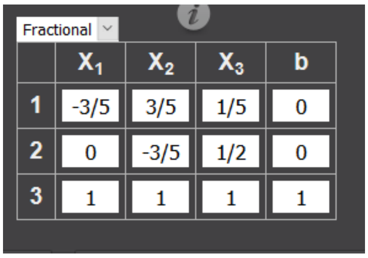
Check: $T'S = \begin{bmatrix} .7 & .2 \\ .3 & .8 \end{bmatrix} \begin{bmatrix} .4 \\ .6 \end{bmatrix} = \begin{bmatrix} .28 + .12 \\ .12 + .48 \end{bmatrix} = \begin{bmatrix} .4 \\ .6 \end{bmatrix} = S \checkmark.$

(b) Using Matrix Reshish[1] to solve the relevant equations by Gauss-Jordan Elimination, find the steady state vector for the Markov process with transition matrix

$T = \begin{bmatrix} \nicefrac{2}{5} & 0 & \nicefrac{3}{5} \\ \nicefrac{3}{5} & \nicefrac{2}{5} & 0 \\ \nicefrac{1}{5} & \nicefrac{1}{2} & \nicefrac{3}{10} \end{bmatrix}$. Use the 'Fractional' input style.

Using the shortcut method as in (a) we need to solve the system of equations with augmented matrix:

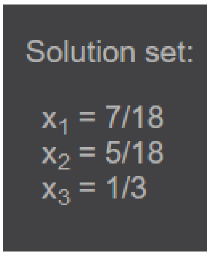
$$\begin{bmatrix} -3/5 & 3/5 & 1/5 & 0 \\ 0 & -3/5 & 1/2 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}.$$



Fractional

	X_1	X_2	X_3	b
1	-3/5	3/5	1/5	0
2	0	-3/5	1/2	0
3	1	1	1	1

Input to, and output from, Reshish is shown at right.



Solution set:

x_1 = 7/18
x_2 = 5/18
x_3 = 1/3

So $S = \begin{bmatrix} \nicefrac{7}{18} \\ \nicefrac{5}{18} \\ \nicefrac{1}{3} \end{bmatrix} = \boxed{\frac{1}{18} \begin{bmatrix} 7 \\ 5 \\ 6 \end{bmatrix}}$ *Check:*

$$T'S = \begin{bmatrix} \nicefrac{2}{5} & \nicefrac{3}{5} & \nicefrac{1}{5} \\ 0 & \nicefrac{2}{5} & \nicefrac{1}{2} \\ \nicefrac{3}{5} & 0 & \nicefrac{3}{10} \end{bmatrix} \frac{1}{18} \begin{bmatrix} 7 \\ 5 \\ 6 \end{bmatrix} = \frac{1}{18} \begin{bmatrix} \nicefrac{14}{5} + 3 + \nicefrac{6}{5} \\ 0 + 2 + 3 \\ \nicefrac{21}{5} + 0 + \nicefrac{9}{5} \end{bmatrix} = \frac{1}{18} \begin{bmatrix} 7 \\ 5 \\ 6 \end{bmatrix} = S. \checkmark$$

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[1] `https://matrix.reshish.com/`

Question 8

(a) Carefully prove that the steady state vector for the Markov process with transition matrix $T = \begin{bmatrix} t_{11} & t_{12} \\ t_{21} & t_{22} \end{bmatrix}$ is $S = \dfrac{1}{t_{12}+t_{21}} \begin{bmatrix} t_{21} \\ t_{12} \end{bmatrix}$ (providing $t_{12}+t_{21} \neq 0$).

We need to show (i) that S is a probability vector and (ii) that $T'S = S$.

(i) Since T is a transition matrix $t_{ij} \geq 0 \forall i, j$.

Hence $\dfrac{t_{21}}{t_{12}+t_{21}} \geq 0$ and $\dfrac{t_{12}}{t_{12}+t_{21}} \geq 0$ (given that $t_{12}+t_{21} \neq 0$).

Also $\dfrac{t_{21}}{t_{12}+t_{21}} + \dfrac{t_{12}}{t_{12}+t_{21}} = \dfrac{t_{21}+t_{12}}{t_{12}+t_{21}} = 1.$

So S satisfies both requirements of a probability vector.

(ii)
$$T'S = \begin{bmatrix} t_{11} & t_{21} \\ t_{12} & t_{22} \end{bmatrix} \frac{1}{t_{12}+t_{21}} \begin{bmatrix} t_{21} \\ t_{12} \end{bmatrix} = \frac{1}{t_{12}+t_{21}} \begin{bmatrix} t_{11}t_{21} + t_{21}t_{12} \\ t_{12}t_{21} + t_{22}t_{12} \end{bmatrix}$$
$$= \frac{1}{t_{12}+t_{21}} \begin{bmatrix} t_{21}(t_{11}+t_{12}) \\ t_{12}(t_{21}+t_{22}) \end{bmatrix} = \frac{1}{t_{12}+t_{21}} \begin{bmatrix} t_{21} \\ t_{12} \end{bmatrix} = S,$$

since $t_{11}+t_{12} = 1$ and $t_{21}+t_{22} = 1$ as T is a transition matrix.

(b) What happens if $t_{12}+t_{21} = 0$?

Since $t_{12}, t_{21} \geq 0$ (they are probabilities) this is only possible if $t_{12}=t_{21}=0$. This requires $t_{11}=t_{22}=1$, so that $T=\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and hence $T'S=S$ for any vector S at all. So in this case all probability vectors are steady state vectors.

(c) Check that the formula proved for (a) gives the correct result for Q2c and/or Q3a.

Q3a: $T = \begin{bmatrix} .7 & .3 \\ .2 & .8 \end{bmatrix}$. *Formula gives* $S = \dfrac{1}{.3+.2} \begin{bmatrix} .2 \\ .3 \end{bmatrix} = \begin{bmatrix} 2/5 \\ 3/5 \end{bmatrix}$ ✓

Q2c: $T = \begin{bmatrix} .9 & .1 \\ .7 & .3 \end{bmatrix}$. *Formula gives* $S = \dfrac{1}{.1+.7} \begin{bmatrix} .7 \\ .1 \end{bmatrix} = \begin{bmatrix} 7/8 \\ 1/8 \end{bmatrix} = \begin{bmatrix} .875 \\ .125 \end{bmatrix}$ ✓

NB: Student's answer does not need to be as fulsome as this.

Question 9 The *Ehrenfest urns model* is used as part of an explanation of how gas diffusion works. Two urns A and B contain between them a fixed number of balls. At each time step one ball is selected at random (equal probabilities), removed from its current urn and placed into the other one. For the case of four balls the model has five states corresponding to A containing 0, 1, 2, 3 or 4 balls.

(a) Explain why the transition matrix for this Markov process is $T = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 1/4 & 0 & 3/4 & 0 & 0 \\ 0 & 1/2 & 0 & 1/2 & 0 \\ 0 & 0 & 3/4 & 0 & 1/4 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$.

State	*balls in*		*Chance ball is picked from* A	*state change*	*Chance ball is picked from* B	*state change*	*matrix entries*				
i	A	B					t_{i1}	t_{i2}	t_{i3}	t_{i4}	t_{i5}
1	0	4	*no chance*	–	*certainty*	$1 \to 2$	0	1	0	0	0
2	1	3	1 *in* 4	$2 \to 1$	3 *in* 4	$2 \to 3$	1/4	0	3/4	0	0
3	2	2	2 *in* 4	$3 \to 2$	2 *in* 4	$3 \to 4$	0	1/2	0	1/2	0
4	3	1	3 *in* 4	$4 \to 3$	1 *in* 4	$4 \to 5$	0	0	3/4	0	1/4
5	4	0	*certainty*	$5 \to 4$	*no chance*	–	0	0	0	1	0

(b) Explain why, for any n, T^n will always contain some zeroes.
(Hint: Can the number of balls in A change by 1 in an even number of steps?)

At each time step the number of balls in A changes by exactly 1 (increase or decrease) and so the parity (odd or even) of the state number alternates at each time step.
In particular, in an even number of steps the parity does not change, so it is impossible to go from state 1 to state 2 in any even number of steps. Thus for even n the row1, col2 entry of T^n is always 0.
By similar reasoning, for odd n the row1, col1 entry of T^n is always 0.

(c) The property of T stated in (b) makes this Markov process *non-regular.* Non-regular Markov processes do not necessarily converge towards a steady state. Find the steady state vector for this process (use the computer as in 3(b)) and explain why this will never be reached starting from any *known* (*i.e.* probability 1) starting state.

Use Guass-Jordan Elimination in Matrix Reshish to solve $(T'-I)S=0$ with shortcut:

Augmented matrix:

$$\begin{bmatrix} -1 & 1/4 & 0 & 0 & 0 & 0 \\ 1 & -1 & 1/2 & 0 & 0 & 0 \\ 0 & 3/4 & -1 & 3/4 & 0 & 0 \\ 0 & 0 & 1/2 & -1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} \longrightarrow S = \begin{bmatrix} 1/16 \\ 1/4 \\ 3/8 \\ 1/4 \\ 1/16 \end{bmatrix}$$

Generalising from the argument in (b), for even n the i,j-th entry in T^n will be 0 whenever $i+j$ is odd, and when n is odd the i,j-th entry in T^n will be 0 whenever $i+j$ is even.
Starting from a known state i means that the initial state vector $\mathbf{x}_0$ is all 0s except for a single 1 in the i-th position. This in turn means that $T^n\mathbf{x}_0$ will be the i-th column of T^n and so contain 0s. Hence $T^n\mathbf{x}_0$ cannot converge to S.