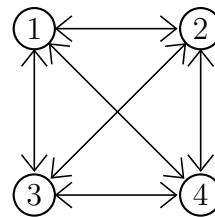


1. Jeremy is taking random walks on the digraph shown at right.

At any vertex Jeremy has three choices as to where to go next:

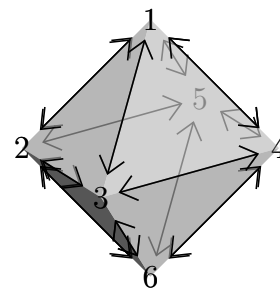
- go to the vertex diagonally across the square.
- go to the adjacent vertex around the square in either a clockwise or an anticlockwise direction;



He takes the first option with probability one half but his probability for the other options is equal to one tenth of the destination vertex number. For example, if he is at vertex 1, his probabilities of next visiting vertices 2, 3 and 4 are 0.2, 0.3 and 0.5 respectively.

- List all the walks of length 2 from vertex 1 to vertex 2 and the probabilities associated with each of them. Hence find the probability that a walk of length 2 that starts at vertex 1 finishes at vertex 2.
- List all the walks of length 3 from vertex 1 to vertex 2 (there are a lot!) and the probabilities associated with each of them. Hence find the probability that a walk of length 3 that starts at vertex 1 finishes at vertex 2.
- Compile Jeremy's transition matrix T . Check that it is stochastic.
- Calculate T^2 (by hand or computer) and use it to check your answer to (a).
- Calculate T^3 (by hand or computer) and use it to check your answer to (b). If the answers don't agree, you probably missed some of the walks.
- Use T to verify that, if Jeremy keeps walking, then in the long run his visits to any vertex v will constitute a fraction $(v + 5)/30$ of his visits to all vertices. (Thus, for example, 20% of his visits will be to vertex 1 and 30% to vertex 4.)

2. Anton the ant is wandering around the structure shown at right, made from twelve length 1 struts joined at six vertices 1,...,6, making a regular octahedron with the struts as edges. He walks at a steady pace along the edges and at each vertex makes a random (equal-probability) choice as to which edge to walk next (this includes the possibility of retracing the previous edge).



- Anton starts at vertex 1. Explain why, after a walk of length two, he is twice as likely to be at vertex 6 than at vertex 5.
- Again starting from vertex 1, what is the probability that after a walk of length 3 (not necessarily using distinct edges), Anton is back at vertex 1?
- Write out Anton's transition matrix T . Write it in the form $T = fM$ where f is a fraction and M is a symmetric matrix of zeros and ones.
- Making good use of the simple nature of M to save work, calculate T^2 , writing it in similar simple form. Use T^2 to check your answer to (a).
- Calculate $T^4 = (T^2)^2$. Use a computer if you like, but it is not really necessary.
- Suppose Anton has been wandering for a long time. By comparing T , T^2 and T^4 , and by considering the symmetry inherent in the situation, estimate the proportion of Anton's vertex visits that have been to vertex 1.
- Support your answer to (f) by demonstrating a steady state vector for T .

