This is Assignment 8 for MATH1005 students in a Friday workshop. It is due at 6 pm on the Thursday after Workshop 8 (6 days after was released).

There are four problems. The numbering of the problems is strange because the numbering is taken from a much larger document that has many problems from which I can select. As long as you can see four different problems, then you have the complete assignment.

You should write your best solutions to the problems here, and then upload your solutions before the due time. Here are three ways you may complete the assignment:

1. Print the assignment sheet. Write your solutions in pen or pencil on the print out. Scan your completed assignment, turn the file into a single .pdf file, then upload your solution file to Wattle.

2. Write your solutions in pen or pencil on blank paper. You should clearly label your solutions and you should write them in the order in which the problems appear in your assignment. Scan your completed assignment, turn the file into a single .pdf file, then upload your solution file to Wattle.

3. Download the assignment sheet to a tablet. Annotate the file using your favourite annotation software. **Flatten the file**–this makes your annotations a permanent part of the file, and if you do not do this then we see only a blank assignment in our grading software. Upload your flattened solution file to Wattle.

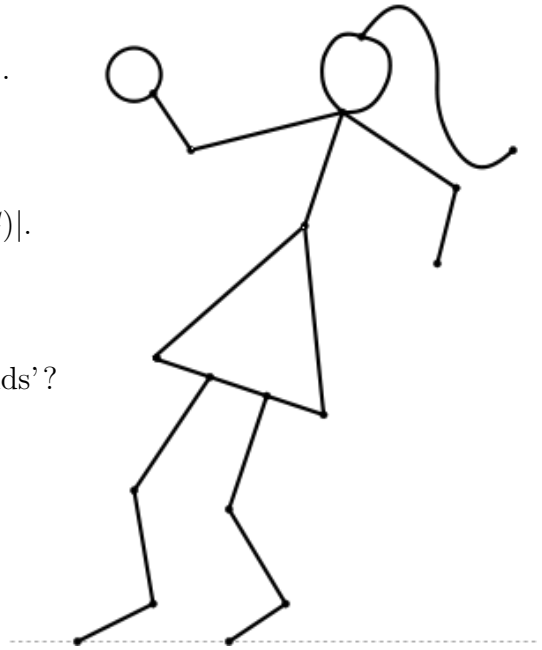
In all cases, the file you upload must be a .pdf file.

Please remember to plan your time carefully so you are not trying to submit your assignment at the last minute. No late work is accepted.

Please enjoy,

AP

Question 1* For the 'girl'[1] graph G:



(a) Number all the vertices and hence find $|V(G)|$.

(b) Mark the edges $a, b, c, \ldots$ and hence find $|E(G)|$.

(c) What are the degrees of each of the girl's 'hands'?

(d) List all vertices adjacent to the girl's 'neck'.

(e) List all edges incident on the girl's 'waist'

(f) Does the graph have a vertex of degree 4?
If so, specify one.

(g) Does the graph have an isolated vertex?
If so, specify one.

(h) Does the graph have any parallel edges?
If so, specify a pair.

(i) Does the graph have any circuits?
If so, specify one.

(j) Is the graph simple? Justify your answer.

[1]This is the 'Hackenbush girl' see `https://en.wikipedia.org/wiki/Hackenbush`.
The game of Hackenbush is discussed in MATH2301.

Question 3A$^+$ In a *food web matrix* there is one **c**olumn for each **c***onsumer* under study and one **r**ow for each studied **r***esource* (food item) for the these consumers. Some consumers are food for other consumers. A 1 in column c row r means c consumes r. The food web matrix at right relates to the following species in Lake Michigan, USA:

A: Walleye B: Bloater C: Chironomids F: Waterflea
L: Lamprey U: Burbot W: Whitefish.

	A	B	L	U	W
A	0	0	1	0	0
B	1	0	0	1	0
C	0	1	0	1	1
F	1	1	0	0	1
U	0	0	1	0	0
W	0	0	1	0	0

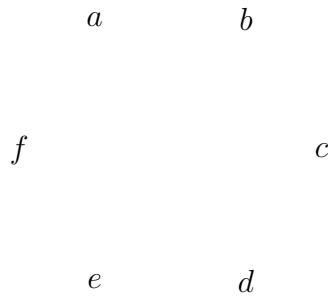
(a) Draw a digraph representing this food web.
[Draw it in rough on scrap paper first; it may take a few attempts to make it look presentable.]

(b) Draw a niche overlap graph for this food web.
[For this graph, two consumers are adjacent iff they share a resource.
Again, draw in rough first.]

Question 3B$^+$ A relation R on $\{a, b, c, d, e, f\}$ is specified by the table at right.

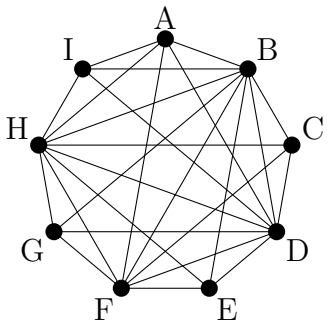
item	is related to
a	b, d, e
b	a, f
c	a, b
d	a, d, e, f
e	a, c, d
f	b, d, f

(a) Draw a digraph representing R:
[When there is an arc in both directions between two items, use a single two-way arrow.]



(b) Write out the adjacency matrix for R.

Question 4A* For the graph G at right:



(a) State the degree of each vertex.

Vertex:	A	B	C	D	E	F	G	H	I
Degree:									

(b) Exhibit subgraphs of G isomorphic to K_{18}, K_{36}, K_{44} and K_5.

K_{18}	K_{36}	K_{44}	K_5

(c) Prove that G does not have a subgraph isomorphic to K_{45}.

Question 4B* Draw a graph with five vertices of degrees 0, 1, 2, 3 and 4, or say why you believe this to be impossible.

Question 6A [This problem is Problem 14 in Excercise set 9.9 of Epp. (2019). Discrete Mathematics with Applications, metric Edition. Cengage. (our optional text)] A drug-screening test is used in a large population of people of whom 1.5% actually use drugs. Suppose that the false positive rate is 2% and the false negative rate is 1%. Thus a person who uses drugs tests positive for them 99% of the time, and a person who does not use drugs tests negative for them 98% of the time

(i) What is the probability that a randomly chosen person who tests positive for drugs actually uses drugs?

(ii) What is the probability that a randomly chosen person who tests negative for drugs does not use drugs?