This is Assignment 2 for MATH1005 students in a Friday workshop. It is due at 6 pm on the Thursday after Workshop 2 (6 days after was released).

There are four problems. The numbering of the problems, and the page numbering, is strange because the numbering is taken from a much larger document that has many problems from which I can select. As long as you can see four different problems, then you have the complete assignment.

You should write your best solutions to the problems here, and then upload your solutions before the due time. Here are three ways you may complete the assignment:

1. Print the assignment sheet. Write your solutions in pen or pencil on the print out. Scan your completed assignment, turn the file into a single .pdf file, then upload your solution file to Wattle.

2. Write your solutions in pen or pencil on blank paper. You should clearly label your solutions and you should write them in the order in which the problems appear in your assignment. Scan your completed assignment, turn the file into a single .pdf file, then upload your solution file to Wattle.

3. Download the assignment sheet to a tablet. Annotate the file using your favourite annotation software. **Flatten the file**–this makes your annotations a permanent part of the file, and if you do not do this then we see only a blank assignment in our grading software. Upload your flattened solution file to Wattle.

In all cases, the file you upload must be a .pdf file.

Please remember to plan your time carefully so you are not trying to submit your assignment at the last minute. No late work is accepted.

Please enjoy,

AP

Question 1* Let $U = \{$**S,M,B,P,A,GC,N,C,SC,W,G,H,T,Cs,D,Ta**$\}$ be the set of the 16 largest cities in Australia[1], and let E and L denote the subsets of the universal set U comprising respectively cities in the Eastern States (Queensland, NSW&ACT, Victoria & Tasmania) and cities with populations over a million.
How many members have each of the following sets? Show your enumeration/calculation.

(a) E^c (b) $E \cap L$ (c) $E \cup L$

(d) $E \backslash L$ (e) $E \triangle L$ (f) $\mathcal{P}(E)$ [the power set of E]

Question 2* In computing jargon a string of four binary digits ('bits'), such as `0101` and `1000`, is called a 'nibble'. (= half a byte!) Let universal set U be the set of all sixteen different nibbles and for $i = 0, 1, \ldots, 4$ let U_i denote the set of all those nibbles that have exactly i ones; e.g. $U_3 = \{$`1110,1101,1011,0111`$\}$.

Decide the truth or falsity of each of the following. Beware of deliberate traps! Briefly justify your answers.

(a) $\{$`1100,0011`$\} \in U_2$ (b) $|\mathcal{P}(U_2)| = 64$ (c) $\{$`0100,0110`$\} \in U_1 \times U_2$

(d) $U_2 \triangle U_3 = U_2 \cup U_3$ (e) $U_1 \subset (U_3)^c$ (f) $\{U_1, U_2, U_3, U_4\}$ is a partition of U

Question 6$^+$ Functions $a, b, c, d : \mathbb{N} \to \mathbb{N}$ are defined by the rules below. In each case decide whether the function is injective (one-to-one), surjective (onto), neither or both (bijective). Justify your answers.

$$a(n) = \lfloor (n+1)/2 \rfloor \quad b(n) = \frac{n(n+1)}{2} \quad c(n) = 1 + (n-2)^2 \quad d(n) = \begin{cases} 10 - n & \text{if } n < 10 \\ n & \text{if } n \geq 10 \end{cases}$$

For function a, $\lfloor x \rfloor$ denotes the 'floor' of x, the greatest integer not greater than x.
E.g. $\lfloor 5.5 \rfloor = 5$, $\lfloor 6 \rfloor = 6$.

Question 7$^+$
Let $S = \{z \in \mathbb{Z} \mid 1 \leq z \leq 10\}$. Let

E denote the set of even integers,
O denote the set of odd integers.

Determine whether or not each of the following sets is a partition of S. Justify your answer in each case.

(a) $\mathcal{P}_1 = \{\{1, 5, 7\}, \{2\}, \{3, 4, 6\}, \{8, 9, 10\}\}$

(b) $\mathcal{P}_2 = \{E \cap S, O \cap S, \{1\}\}$

(c) $\mathcal{P}_3 = \{\mathbb{N} \cap S, S \setminus \mathbb{N}\}$

[1] `https://en.wikipedia.org/wiki/List_of_cities_in_Australia_by_population`