This is Assignment 3 for MATH1005 students in a Friday workshop. It is due at 6 pm on the Thursday after Workshop 3 (6 days after was released).

There are four problems. The numbering of the problems, and the page numbering, is strange because the numbering is taken from a much larger document that has many problems from which I can select. As long as you can see four different problems, then you have the complete assignment.

You should write your best solutions to the problems here, and then upload your solutions before the due time. Here are three ways you may complete the assignment:

1. Print the assignment sheet. Write your solutions in pen or pencil on the print out. Scan your completed assignment, turn the file into a single .pdf file, then upload your solution file to Wattle.

2. Write your solutions in pen or pencil on blank paper. You should clearly label your solutions and you should write them in the order in which the problems appear in your assignment. Scan your completed assignment, turn the file into a single .pdf file, then upload your solution file to Wattle.

3. Download the assignment sheet to a tablet. Annotate the file using your favourite annotation software. **Flatten the file**–this makes your annotations a permanent part of the file, and if you do not do this then we see only a blank assignment in our grading software. Upload your flattened solution file to Wattle.

In all cases, the file you upload must be a .pdf file.

Please remember to plan your time carefully so you are not trying to submit your assignment at the last minute. No late work is accepted.

Please enjoy,

AP

Question 1#

Teams P,Q,R,S,T and U each play every other team once at home and once away. The results for the home team in each match are displayed in the table at right. For example S defeats Q at home but loses to Q away. There are no drawn matches. Relations α, β and γ on the set of six teams are defined below.
Draw a (directed) graph for each relation and say whether or not the relation is a function. If it is not a function, say why not.

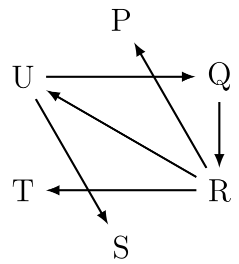
		Away					
		P	Q	R	S	T	U
Home	P		L	W	W	W	W
	Q	L		L	W	W	W
	R	L	W		W	L	L
	S	W	W	W		L	W
	T	W	W	W	L		W
	U	W	L	W	L	W	

(a) $x \alpha y \Leftrightarrow x$ defeats y at home and loses to y away

(b) $x \beta y \Leftrightarrow x$ defeats y away and loses to y at home

(c) $x \gamma y \Leftrightarrow x$ defeats y both at home and away

Draw your graphs in a similar way to the example at right, which is for the relation δ on the set of six teams, where $x \delta y$ means that x loses both matches against y.



Question 2#

Functions $a, b, c, d \ : \mathbb{N} \times \mathbb{N} \to \mathbb{N}$ are defined by the rules below. In each case decide whether the function is injective (one-to-one), surjective (onto), neither or both (bijective). Justify your answers.

$a(x, y) = xy \qquad b(x, y) = x + y \qquad c(x, y) = (x + y)^2 + x$

$d(x, y) = (m - 1)^2 + m + x - y$, where $m = \max(x, y)$.

For functions c and d it may help to draw up a 4×4 grid and write the image of (x, y) in the cell (x, y).

Question 5+

Let $x = 10110_2,\ y = 10011_2,\ s = x + y,\ d = x - y,\ p = xy$.

(a) Calculate s, d and p directly in binary. Keep the answers in binary.

(b) Convert x, y to hexadecimal and recalculate s, d and p directly in hexadecimal.

(c) Finally convert x, y to decimal, recalculate s, d and p (in decimal) and convert the answers to hexadecimal and thence to binary.
Use the results to check your answers to (a) and (b).

Question 6$^+$ This question is about the toggle-plus-one method as it relates to the storage of integers in computer words. It also shows how this method avoids the need for separate subtraction circuits.

As demonstrated in lectures, for a binary word W, toggle-plus-one means:

toggle:	replace every 1 by 0, every 0 by 1, then
add one:	treating W as a binary number, add 1.
	Ignore any carry beyond the length of the word.

For $l \in \mathbb{N}$ let $S_l = \{n \in \mathbb{Z} : -2^{l-1} \leq n < 2^{l-1}\}$. Then S_l is the set of all integers that can be stored in words of length l bits, using the standard computer representation. The rules governing the storage of an integer n in a word W may be summarised as:

Rule 1:	n is negative if and only the left-most bit of W is 1
Rule 2:	$-n$ is stored as the word obtained from W by toggle-plus-one.
	This is true even when n is negative.
Rule 3:	If the left-most bit of W is 0 than $n = W_2$.
	i.e. n is retrieved by treating W as a binary number.

In computer arithmetic, subtraction uses negation and addition: $x - y = x + (-y)$.
In the addition, any carry beyond the length of the word is ignored.
As an example, take $x = 10110_2$, $y = 10011_2$ and use 8-bit computer arithmetic on:

(a) $x - y$ (b) $y - x$ (c) $-x - y$

Check your results by expressing x, y and your answers in decimal.