This is Assignment 7 for MATH1005 students in a Friday workshop. It is due at 6 pm on the Thursday after Workshop 7 (6 days after was released).

There are four problems. The numbering of the problems is strange because the numbering is taken from a much larger document that has many problems from which I can select. As long as you can see four different problems, then you have the complete assignment.

You should write your best solutions to the problems here, and then upload your solutions before the due time. Here are three ways you may complete the assignment:

1. Print the assignment sheet. Write your solutions in pen or pencil on the print out. Scan your completed assignment, turn the file into a single .pdf file, then upload your solution file to Wattle.

2. Write your solutions in pen or pencil on blank paper. You should clearly label your solutions and you should write them in the order in which the problems appear in your assignment. Scan your completed assignment, turn the file into a single .pdf file, then upload your solution file to Wattle.

3. Download the assignment sheet to a tablet. Annotate the file using your favourite annotation software. **Flatten the file**–this makes your annotations a permanent part of the file, and if you do not do this then we see only a blank assignment in our grading software. Upload your flattened solution file to Wattle.

In all cases, the file you upload must be a .pdf file.

Please remember to plan your time carefully so you are not trying to submit your assignment at the last minute. No late work is accepted.

Please enjoy,

AP

Question 1^+ In rôle-playing games, such as the original, *Dungeons and Dragons*, gamers use a variety of dice shapes, not just the standard cube. For example a 'd8' has the shape of a regular octahedron. A (fair) d8 has eight faces numbered 1,2,. . . ,8, and each face is equally likely to become the top face (the face that is read) when the die is 'thrown'.
Suppose gamer Alice does not have a d8 but does have a d6 (*i.e.* a regular cubic die with six faces labelled 1,2,. . . ,6.) To simulate throwing a d8, Alice throws her d6 twice, resulting in a pair of values $(a, b) \in S = \{1, \ldots, 6\}^2$. She then adds the two values and subtracts 8 if the sum is more than 8. The result is a value v in $\{1, \ldots, 8\}$, as required.

(a) Write out the partition of the sample space S comprising the eight events $\{v = 1\}, \{v = 2\}, \ldots, \{v = 8\}$.

(b) Find the probability of each of the eight events listed for part (a).

(c) Alice's simulation of a d8 throw is not perfect, but it's pretty good. What are its best and worst features in your opinion?

(d) How *could* Alice use a d6 to accurately simulate a d8 throw?
Hint: 6^2 is not divisible by 8, but 6^3 is.

Question 3^+ A *Binomial experiment* comprises a fixed number n of 'trials' where each trial has the same probability p of 'success'. The probability that a binomial experiment results in k successes is given by

$$\mathbb{P}(k \text{ successes}) = \binom{n}{k} p^k (1-p)^{n-k}.$$

The probability of scoring a 4 on one throw of a d4 die is 0.25.

(a) Use your calculator to find the probabilities of scoring, with a d4 die,

(i) four 4s from six throws,

(ii) five 4s from six throws and

(iii) six 4s from six throws

(b) Calculate the probability of scoring *at most three 4s* from six throws of a d4.

(c) In practice Binomial probabilities are usually found from a book of tables or from on-line tables or calculators. Both density and (cumulative) distribution values are available. With the help of an on-line statistical calculator such as *Stat Trek* (`http://stattrek.com/online-calculator/binomial.aspx`) find the probability of scoring more than four but no more than ten 4s from 25 throws of a d4. Give all the values you obtained, what they represented, and how you used them to obtain your answer.

Question 6^+ Let $T = \begin{bmatrix} 2/5 & 3/5 \\ 1/5 & 4/5 \end{bmatrix} = \begin{bmatrix} .4 & .6 \\ .2 & .8 \end{bmatrix}$.

(a) Use the 'Matrix Calculator' computer application `http://matrixcalc.org/en/` to calculate T^2, T^4, T^8 and T^{16} to 3dp accuracy. (You can progressively insert the results back into an input matrix, so there is no need to physically enter anything more than the four entries of T.)

(b) Based on (a) guess a steady state vector for the Markov process with transition matrix T.

(c) Use the transpose matrix T' to verify that your guess from (b) is correct. Do the calculation by hand.

Question 7^+

(a) By solving the relevant system of equations find the steady state vector for the Markov process with transition matrix $T = \begin{bmatrix} .6 & .4 \\ .1 & .9 \end{bmatrix}$. Do this by hand calculation, using matrix inverse.

(b) Using Matrix Reshish[1] to solve the relevant equations by Gauss-Jordan Elimination, find the steady state vector for the Markov process with transition matrix

$$T = \begin{bmatrix} 2/5 & 1/5 & 2/5 \\ 0 & 4/5 & 1/5 \\ 1/2 & 3/10 & 1/5 \end{bmatrix}.$$ Use the 'Fractional' input style.

[1] `https://matrix.reshish.com/`