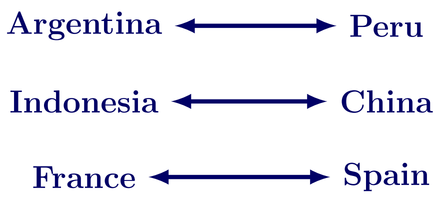
These questions are for practice, in preparation for Workshop 2.

1. Let $A = \{Argentina, Indonesia, China, Peru, France, Spain\}$.
A relation $R \subseteq A \times A$ is defined by

$aRb \iff a$ and b are part of the same continent.

(The continents are Africa, Antartica, Asia, Australia, Europe, N.America, S.America)

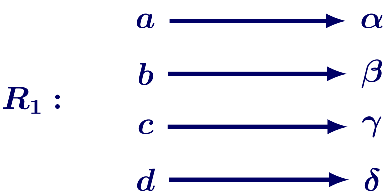
Draw a graph representing this relation.



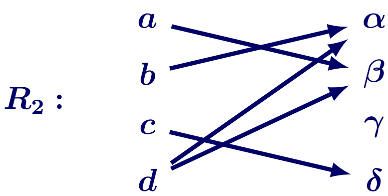
(Note two-way arrows)

2. Let $A = \{a, b, c, d\}$, $B = \{\alpha, \beta, \gamma, \delta\}$. Draw graphs representing the following relations.

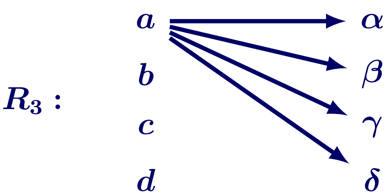
(a) $R_1 = \{(a, \alpha), (b, \beta), (c, \gamma), (d, \delta)\}$



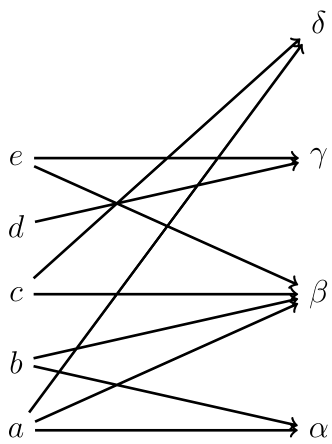
(b) $R_2 = \{(a, \beta), (b, \alpha), (c, \delta), (d, \alpha), (d, \beta)\}$.



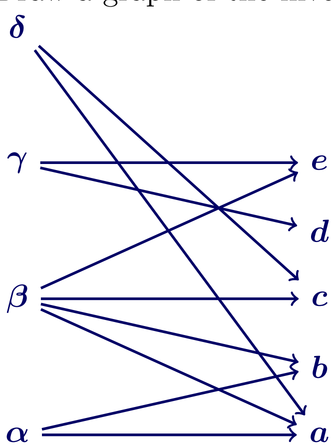
(c) $R_3 = \{(a, \alpha), (a, \beta), (a, \gamma), (a, \delta)\}$.



3. Let $S = \{a, b, c, d, e\}$ be a set of senators and $P = \{\alpha, \beta, \gamma, \delta\}$ be a set of policies.
Let $R \subseteq S \times P$ be the relation defined by sRp if s supports p, using the graph below.



(a) Draw a graph of the inverse relation R^{-1}.



(b) What does $pR^{-1}s$ mean? Answer in words, referring to senators and policies.

Policy p is supported by senator s.

(c) What is $\{s \in S \ ; \ pR^{-1}s\}$? Answer in words, referring to senators and policies.

The set of senators that support policy p.

4. For each of the following relations $R \subseteq \{a, b, c\} \times \{\alpha, \beta, \gamma\}$ decide whether R is a function. If not, say why.

(a) $R_1 = \{(a, \alpha), (b, \beta), (c, \gamma)\}$. **Yes. There is exactly one pair containing a, exactly one pair containing b and exactly one pair containing c.**

(b) $R_2 = \{(a, \alpha), (a, \beta), (a, \gamma)\}$. **No. There is more than one pair containing a. (Also no pair containing b, nor c.)**

(c) $R_3 = \{(a, \alpha), (b, \alpha), (c, \alpha)\}$. **Yes. Same reason as for (a).**

(d) $R_4 = \{(a, \alpha), (b, \alpha), (c, \gamma)\}$. **Yes. Same reason as for (a).**

5. Let A, B, C be sets, each with at least two members. Define a function F as shown at right:

$$F : A \times B \times C \to B$$
$$(a, b, c) \mapsto b.$$

(a) Is F injective (one-to-one)? Why or why not?
No. Suppose $a_1, a_2 \in A$, $a_1 \neq a_2$, $b_1 \in B$, $c_1 \in C$. Then

$$F(a_1, b_1, c_1) = b_1 = F(a_2, b_1, c_1).$$

So two different members of the domain have the same value.

(b) Is F surjective (onto)? Why or why not?
Yes. $\forall b \in B, F(a_1, b, c_1) = b$.
So every member of the codomain is the image of some member of the domain.

6. Let U be a set, and $p \in U$. Let $F \subseteq \mathcal{P}(U) \times \mathcal{P}(U)$ be defined by SFT if and only if $S \cup \{p\} = T$. **Note that F is a function, with rule $F(S) = S \cup \{p\}$.**

(a) State the domain and codomain of F. **Domain = Codomain = $\mathcal{P}(U)$.**

(b) Determine the range of F. **Range = $\{T \subseteq U : p \in T\}$. (Since $\forall T \in \mathcal{P}(U)$ $p \in T \implies T = F(T)$.)**

(c) Is F injective (one-to-one)? Why or why not? **No. Counterexample: $F(\emptyset) = F(\{p\}) = \{p\}$ (and $\emptyset \neq \{p\}$).**

(d) Is F surjective (onto)? Why or why not? **No. Counterexample: $\emptyset \in$ Codomain but $\emptyset \notin$ Range.**

7. Let $\mathbb{Z}$ denote the set of integers; $\mathbb{Z} = \mathbb{N} \cup \{0\} \cup \{-n \; ; \; n \in \mathbb{N}\}$

Define functions F and G by
$$F : \mathbb{Z} \to \mathbb{Z}, \quad z \mapsto z^2. \qquad G : \mathbb{Z} \to \mathbb{Z}, \quad z \mapsto z + 1.$$

(a) Explain why G is bijective (a one-to-one correspondence).
G is injective because if $z_1 \neq z_2$ then $z_1 + 1 \neq z_2 + 1$; *i.e.* $G(z_1) \neq G(z_2)$
G is surjective because $\forall y \in \mathbb{Z}\ \exists z \in \mathbb{Z}\ G(z) = y$; viz $z = y - 1$.

(b) Complete each of the following by providing signature and rule:

$$G^{-1} : \mathbb{Z} \to \mathbb{Z}, \quad z \mapsto z - 1. \qquad FoG : \mathbb{Z} \to \mathbb{Z}, \quad z \mapsto (z+1)^2. \qquad GoF : \mathbb{Z} \to \mathbb{Z}, \quad z \mapsto z^2 + 1.$$