Question 1 Recall that a natural number n is *prime* provided that $n > 1$ and n has no positive divisors other than 1 and itself (e.g. 17 is prime but 18 is not). For universal set $U = \{n \in \mathbb{N} : n \leq 15\}$ define sets P and H as follows:

$$P = \{n \in U : n \text{ is prime}\} \qquad H = \{n \in U : n > 10\}.$$

How many members have each of the following sets? Show your enumeration/calculation.

(a) P^c (b) $P \cap H$ (c) $P \cup H$

(d) $P \backslash H$ (e) $P \triangle H$ (f) $\mathcal{P}(P)$ [the power set of P].

Question 2 Recall that a natural number n is *prime* provided that $n > 1$ and n has no positive divisors other than 1 and itself (e.g. 17 is prime but 18 is not) and that n is *composite* if it is product of two smaller natural numbers (e.g. 18 is composite because $18 = 6 \times 3$). Define sets U, P and C as follows:

$$U = \{n \in \mathbb{N} : n \leq 15\} \quad P = \{n \in U : n \text{ is prime}\} \quad C = \{n \in U : n \text{ is composite}\}$$

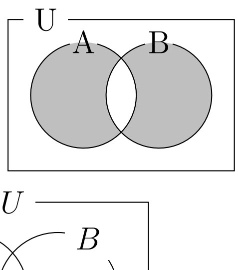
Decide the truth or falsity of each of the following. Beware of deliberate traps! Briefly justify your answers.

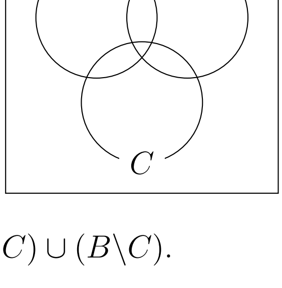
(a) $\{1, 2, 3\} \subseteq P$ (b) $\{8, 9, 10\} \in C$ (c) $P \cap C = \emptyset$

(d) $\{11, 12\} \in P \times C$ (e) $P \subset P \times C$ (f) $\{P, C\}$ is a partition of U

Question 3 A **Venn diagram** is a graphical reprsentation of sets, subsets and elements. Inside a rectangle for the universal set are one or more circles (or shapes) representing sets. Dots inside the circles (or shapes) represent elements. A subset of the universe, and its relationship to the sets represented by circles, can be indicated by shading various regions on the Venn diagram.

For example, this Venn diagram represents the symmetric difference of A and B.





(a) Using copies of the skeleton Venn diagram at right, draw five diagrams, one for each of the following:

i) $A \cup B$ ii) $(A \cup B) \backslash C$

iii) $A \backslash C$ iv) $B \backslash C$ v) $(A \backslash C) \cup (B \backslash C)$

(b) Based on your answers to (a) decide whether $(A \cup B) \backslash C = (A \backslash C) \cup (B \backslash C)$.

(c) Use an element proof (and a logical equivalence) to prove your answer to (b).

Question 4

(a) Draw a Venn diagram showing four sets A, B, C, D in the most general configuration for which $A \subseteq C$ and $B \subseteq D$.

(b) By referring to your answer to (a), decide on the truth or falsity of the claim that, for all sets A, B, C, D,

$$[(A \subseteq C) \wedge (B \subseteq D)] \Rightarrow [(A \triangle B) \subseteq (C \triangle D)].$$

(c) [Challenge] Prove your answer to (b).

Question 5

For any integers x and y, we say that x **divides** y if there exists an integer k such that $y = kx$.

For each integer x, we define $M_x = \{z \in \mathbb{Z} \mid x \text{ divides } z\}$.

(a) Describe M_5 using set-roster notation.

(b) What is another name for M_2?

(c) What is another name for $\mathbb{Z} \setminus M_2$?

(d) Prove the following statement: $M_3 \neq M_5$.

(e) Prove or disprove the following statement: $M_7 \cap M_5 = \emptyset$

Question 6

In this problem we use the definitions and notation of Question 5. Recall that the notation $A \subsetneq B$ reads "A is a subset of B and A is not equal to B."

Determine which of the following statements, if any, are true. Give a brief reason in each case.

(a) $M_7 \subseteq M_{14}$

(b) $M_{14} \subseteq M_7$

(c) $M_7 \subsetneq M_{14}$

(d) $M_{14} \subsetneq M_7$

Question 7

Let $S = \{z \in \mathbb{Z} \mid -5 \leq z \leq 5\}$. Let

$$\begin{aligned} &E \text{ denote the set of even integers,} \\ &O \text{ denote the set of odd integers.} \end{aligned}$$

Determine whether or not each of the following sets is a partition of S. Justify your answer in each case.

(a) $\mathcal{P}_1 = \{\{-5, 1, 4, 5\}, \{-4, -3\}, \{-2, 0\}, \{-1, 2, 3\}\}$

(b) $\mathcal{P}_2 = \{\mathbb{N} \cap S, \mathbb{N} \setminus S\}$

(c) $\mathcal{P}_3 = \{E \cap S, O \cap S, \{0\}\}$

(d) $\mathcal{P}_4 = \{\mathbb{N} \cap S, S \setminus \mathbb{N}\}$

Question 8

Consider the following statement: No integers x and y exist for which $6x + 24y = 13$.

(a) Clearly identify the logical structure, including appropriate quantification, of the statement under consideration.

(b) Use a proof by contradiction to prove the statement under consideration.