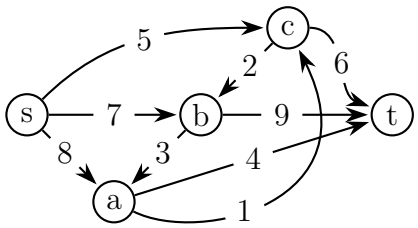
Instructions: *See Worksheets 1 and 2*

The first three questions relate to flows and matchings, topics covered in section D2 on Weighted Graphs. The remaining questions relate to the final section of material, D3 on Random Walks. Unfortunately neither topic is covered in Epp's textbook, but full notes on these topics are on the Wattle site and contain a number of worked examples. However, it is very difficult to fully understand the application of the max flow and matching algorithms from the static lecture slide notes. If you have not done so already ***you are strongly advised to (re)view the recordings of the relevant lectures****. In the lectures the application of each algorithm is revealed dynamically so that, crucially, you can see the precise order in which everything is done.*

Question 1 The diagram at right shows the capacities and directions of all links in a network with source s, target t and intermediate nodes a, b and c. Use the labelling algorithm to find the maximum flow through the network and how it can be achieved. Prove that your flow is maximum by finding a cut of equal value.



Question 2 The matrix at right shows the capacities for a network with source at vertex A and target at vertex H.

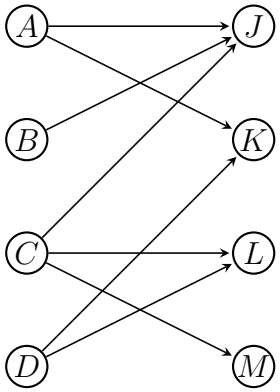
$$\begin{bmatrix} 0 & 3 & 4 & 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 3 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 4 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 5 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(a) Find a minimum cut. Specify the partition of the vertices, the edges making up the cut, and the value of the cut.

(b) Use the minimum cut to find a maximum flow, using only integer flows. There is no need to use the labelling algorithm. Draw a diagram showing capacity and flow value for every directed edge.

Question 3 Four children at a daycare are selecting snuggle toys for nap time. Avril likes to snuggle the Jaguar and the Koala; Bai only likes to snuggle the Jaguar; Carletta like to snuggle the Jaguar, the Llama and the Moo-moo Cow; and Dan likes to snuggle the Koala and the Llama. Use the labelling algorithm to find a perfect match-up.

[With such small sets it is easy to solve this without the help of any algorithm. The point of the question is to demonstrate how the labelling algorithm works on a matching problem.]



Question 4 Let G be the digraph with adjacency matrix A shown at right. A random walk on G has all transition probabilities equal to $1/4$ except that all transition to vertex 1 have probability $1/2$.

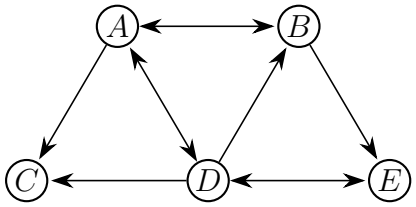
$$A = \begin{bmatrix} 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 \end{bmatrix}$$

(a) Draw the graph and mark on it the transition probabilities.

(b) Compile the transition matrix T and verify that it is stochastic.

(c) On average over the long term, what proportion of time will the walker spend at vertex 1? Hint: There are really only two unknowns.

Question 5 For the webgraph shown at right, find the PageRanks of each page, assuming no damping. You will need to use the computer.



Question 6 Let G be the graph with adjacency matrix A shown at right. Find the PageRank vector for this webgraph using a damping factor of 90%. You will need to use the computer.

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \end{bmatrix}$$