Question 1 Define the logical variables a, d and f as follows:

$$\begin{aligned} a &= \text{“The ASX is at least 4000”} \\ d &= \text{“The Dow is at least 1200”} \\ f &= \text{“The FTSE is at least 5000”} \end{aligned}$$

Express each of the following in symbols, as succinctly as you can.

(a) The ASX is at least 4000, and either the Dow is at least 1200 or the FTSE is at least 5000, but not both.

(b) When the FTSE is below 5000 the Dow is below 1200.

(c) Either the ASX is at least 4000 and the Dow is at least 1200, or the ASX is below 4000 and the Dow is below 1200. [*Try to express this with just one logical connective.*]

Question 2 Negate each of the statements below.
Use as natural sounding English as you can manage, and try to avoid using the word ‘not’. Do not use symbols.

(a) She will win silver or gold.

(b) She will win silver if she fails to win gold.

(c) She will win gold in the 100m event and in the 200m event.

Question 3 When solving the equation $2x - 6 = 0$ we might write out the solution method in ‘shorthand’ something like this:

$$2x - 6 = 0 \quad \Rightarrow \quad 2x = 6 \quad \Rightarrow \quad x = 3.$$

In our model of logic, this appears to have the form $p \Rightarrow q \Rightarrow r$. However, this statement form is ambiguous because the $\Rightarrow$ connective is not associative; *i.e.* $(p \Rightarrow q) \Rightarrow r$ and $p \Rightarrow (q \Rightarrow r)$ mean different things.

(a) Use truth table(s) to show that $[(p \Rightarrow q) \Rightarrow r] \not\equiv [p \Rightarrow (q \Rightarrow r)]$.

(b) Is the shorthand solution scheme notation $p \Rightarrow q \Rightarrow r$ correctly represented by either $(p \Rightarrow q) \Rightarrow r$ or $p \Rightarrow (q \Rightarrow r)$? If so, which one; if not, what should it be?

Question 4 Let p : "If the new drug succeeds, diabetes rates will fall".

(a) Write out the converse of p. Is this equivalent to p?

(b) Write out the contrapositive of p. Is this equivalent to p?

(c) Express p using the phrase "necessary condition".

Question 5 For each of the following sentences, say whether the sentence is a true statement, a false statement or a predicate. Also give the negation of each sentence.

(a) If $x^2 > 0$ then $x > 0$.

(b) $\forall x \in \mathbb{N}\ \exists y \in \mathbb{N}\ \ x = y^2$.

(c) $\exists! x \in \mathbb{N}\ \ 3x - x^2 = 2$. [*This one is tricky!*]

Question 6

(a) Construct a circuit diagram corresponding to the input-output (truth) table at right. Do this by employing the standard method of first writing out a logical expression, in disjunctive normal form (*i.e.* the disjunction of several conjunctions), that has the given truth table, and then converting this to a circuit using only AND, NOT and OR gates.

inputs		output
X	Y	
1	1	0
1	0	1
0	1	1
0	0	0

(b) (*Challenge*) Construct a circuit diagram corresponding to the same input-output table but this time using only NAND gates. Try to use as few gates as you can.

Question 7 Determine whether or not the following argument is valid.

> If Adam's kids stay up past their bedtime, then they are grumpy in the morning. If the kids are grumpy in the morning, then Adam is late for work. Adam was late for work, therefore the kids stayed up past their bedtime.