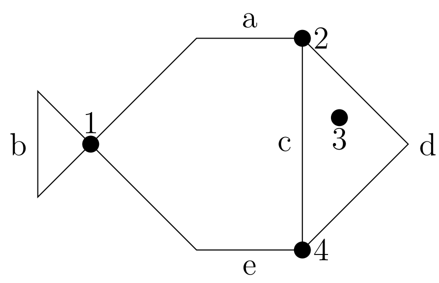
Question 1 For the 'fish' graph G at right find:



(a) $V(G)$.

(b) $|E(G)|$.

(c) All edges incident on vertex 2.

(d) All vertices adjacent to vertex 2.

(e) All edges adjacent to edge c.

(f) A loop.

(g) An isolated vertex.

(h) A pair of parallel edges.

(i) The adjacency matrix of G.

Question 2

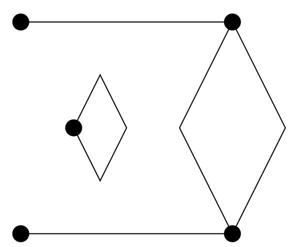
(a) Draw diagrams for the graphs with adjacency matrices below:

(i) $$\begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 2 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

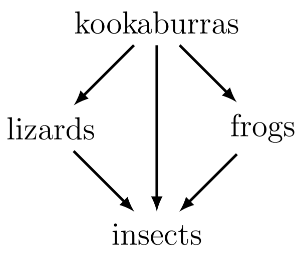
(ii) $$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 \\ 0 & 2 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

(b) Are the graphs of (a) isomorphic? Justify your answer.

(c) Is the graph (a)(i) isomorphic to the graph with diagram at right? Justify your answer.



Question 3A The digraph W at right represents a foodweb.
An edge from a to b indicates that a eats b.
Let k, l, f, i denote kookaburras, lizards, frogs and insects.



(a) Which of these are edges of W: $(k, l), (l, f), (f, i), (i, k)$?

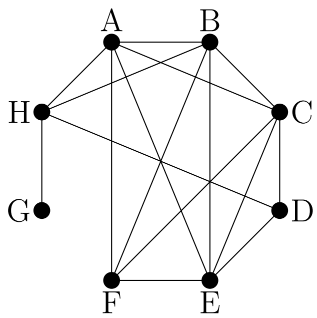
(b) Draw the niche overlap graph for W.

Question 3B A relation $R \subseteq \{1, 2, 3, 4, 5\}^2$ is defined by $xRy \Leftrightarrow 0 < x - y < 3$.

(a) Draw a digraph representing R.

(b) Write out the adjacency matrix for R.

Question 4A For the graph G at right:



(a) State the degree of each vertex.

Vertex:	A	B	C	D	E	F	G	H
Degree:								

(b) Exhibit subgraphs of G isomorphic to $K_{1,5}$, $K_{2,4}$, $K_{3,3}$ and K_5.

$K_{1,5}$	$K_{2,4}$	$K_{3,3}$	K_5

(c) Prove that G does not have a subgraph isomorphic to $K_{3\,4}$.
Hint: Concentrate on vertices G and H.

Question 4B Draw a graph with five vertices, each of degree 3, or say why you believe this to be impossible.

Question 5A A pair of fair 6-sided dice, one red and one blue, are rolled. What is the probability that the sum of the numbers showing face up is 6, given that both of the numbers are odd?
An excellent response will identify: the experiment; how an outcome is recorded; the sample space; how probabilities are assigned to events; and any particular events of interest.

Question 6A [This problem is Problem 14 in Excercise set 9.9 of Epp. (2019). Discrete Mathematics with Applications, metric Edition. Cengage. (our optional text)]
A drug-screening test is used in a large population of people of whom 4% actually use drugs. Suppose that the false positive rate is 3% and the false negative rate is 2%. Thus a person who uses drugs test positive for them 98% of the time, and a person who does not use drugs tests negative for them 97% of the time

(i) What is the probability that a randomly chosen person who tests positive for drugs actually uses drugs?

(ii) What is the probability that a randomly chosen person who tests negative for drugs does not use drugs?

Question 7A Recall the following from our lecture notes.

Lemma:
For any probability experiment with sample space S, and for any events $A, B \subseteq S$, if $\mathbb{P}(A) \neq 0$ then

$$\mathbb{P}(A \cap B) = \mathbb{P}(B|A)\mathbb{P}(A).$$

Theorem:[Bayes' Theorem]
For any probability experiment with sample space S, for any $n \in \mathbb{N}$, for any partition $\{B_1, B_2, \ldots, B_n\}$ of S and for any event $A \subseteq S$, if $\mathbb{P}(A) \neq 0$ and for all $i \in \{1, 2, \ldots, n\}$ we have $\mathbb{P}(B_i) \neq 0$, then for all $k \in \{1, 2, \ldots, n\}$ we have

$$\mathbb{P}(B_k|A) = \frac{\mathbb{P}(A|B_k)\mathbb{P}(B_k)}{\sum_{i=1}^{n} \mathbb{P}(A|B_i)\mathbb{P}(B_i)}$$

(i) Write out the structural part of a proof of the lemma that proceeds directly.

(ii) Complete the proof of the lemma you started in part (i).
HINT: Use the definition of conditional probability.

(iii) Write out the structural part of a proof of Bayes' Theorem that proceeds directly.

(iv) In our lecture proof of Bayes' Theorem, we wrote a sequence of 8 equalities to establish that, under the hypotheses made, the conclusion of Bayes' Theorem holds. Each equality was accompanied by a justification. Below is a table of the 8 justifications in the order in which they appeared in our lecture proof of Bayes' Theorem. Use these to finish the proof of Bayes' Theorem you started in part(ii).

#	Justification for algebraic manipulation
1	(By defn of $\mathbb{P}(B_k|A)$)
2	(Applying the lemma, which is OK because $\mathbb{P}(B_k) \neq 0$)
3	(Because $A \cap S = A$)
4	(Because $\{B_1, \ldots, B_n\}$ is a partition of S, we have $S = B_1 \cup B_2 \cup \cdots \cup B_n$)
5	($\cap$ distributes over $\cup$)
6	(Applying the sum rule, which is OK because $B_1, \ldots, B_n$ are mutually disjoint)
7	(Applying the lemma n times, which is OK because $\mathbb{P}(B_i) \neq 0$ for $i \in \{1, 2, \ldots, n\}$)
8	(Using Σ notation)