
These questions are for practice, in preparation for Workshop 1.

1. Which of the following sentences are statements?

- (a) Australia is a country.
- (b) Australia is the greatest country.
- (c) Is Australia a country?
- (d) Tell me if Australia is a country.
- (e) Australia is in the Northern hemisphere.
- (f) I always lie.

If (f) is a statement, is it true or false?

2. Find the negation of the following statements.

- (a) I am young.
- (b) You are rich.
- (c) New Zealand always wins.
- (d) This is not fair.
- (e) This is not what you think it is not.

3. Rewrite the following statements symbolically as in the example.

Example: I am a mathematician, but I am not crazy.

p : I am a mathematician. q : I am crazy. r : I am a mathematician, but I am not crazy.

$r \equiv p \wedge \neg q$.

- (a) Either I get it, or I don't.
- (b) I live in Australia or in France.
- (c) There will be an election soon, and we will win.
- (d) Either you have earned enough points and have been a member since 2002, or you have earned enough points and paid \$100.

4. Construct truth tables for the following statements.

- (a) $(p \vee q) \wedge \neg r$.
- (b) $(p \oplus q) \vee r$.

5. Give an example of a tautology and an example of a contradiction.

6. Are these two statements logically equivalent?

(a) $p \vee (q \wedge r)$.

(b) $(p \vee q) \wedge (p \vee r)$.

7. Are these two statements logically equivalent?

(a) $\neg(p \vee q \vee r)$.

(b) $p \wedge q \wedge \neg r$.

8. Formalise the following statements as in the example.

Example: People who do not give up always succeed.

p : you do not give up. q : you succeed. $p \implies q$.

(a) You will get a discount if you apply early.

(b) Musicians are cool.

(c) No machine running Microsoft's Windows runs well.

9. Negate the following expressions.

(a) If GDP grows, people are happier.

(b) If I have a coffee, I feel energetic.

(c) You can get it if you really want.

10. Reasoning by contrapositive, what can be concluded from the following statements?

Guns don't kill people. I kill people.

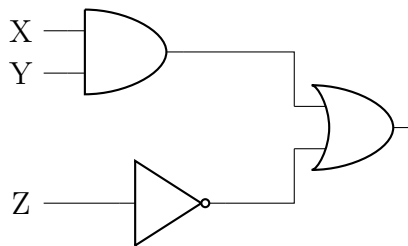
11. Find the condition in the following statements, and determine if it is necessary, sufficient, or both.

- (a) To get a table you need to have a reservation.
- (b) Only people who arrive early might get a ticket.
- (c) If you were a member of our earlier program, you are automatically a member of the new program.

12. Are the following arguments valid or not?

- (a) If the user has been inactive for five minutes, then turn the display off. The display is on. Therefore the user has been active in the last five minutes.
- (b) If we are given more time to prepare a plan and have a representative on the committee, then we will reach a consensus. No consensus has been reached. This means that we were not given enough time to prepare our plan and did not have a representative on the committee.
- (c) People who have done a lot of mathematics are logical. Therefore logical people have done a lot of mathematics.

13. Find the logic expression that corresponds to this circuit, and give its truth table.



14. Draw a circuit corresponding to the following truth table.

X	Y	Z	$output$
1	1	1	0
1	1	0	0
1	0	1	0
1	0	0	1
0	1	1	1
0	1	0	1
0	0	1	0
0	0	0	0

15. The NOR gate \Downarrow has the truth table at right. Construct the following gates using only NOR gates.

X	Y	$NOR(X, Y) = X \downarrow Y$
1	1	0
1	0	0
0	1	0
0	0	1

1. NOT

2. AND

3. NAND

16. Which of these sentences could be predicates?

1. User x is not allowed to view this page.
2. For every user x , the cache has been cleared.
3. This lecture is boring.
4. For every user x , page y has been deleted.

17. Negate the following statements.

- (a) Snakes can't swim.
- (b) Fast growing countries are all in Asia.
- (c) All sheep are black.

18. Negate the following statements.

- (a) $\exists x \quad p(x) \implies q(x)$.
- (b) $\exists x \quad \forall y \quad p(y)$.
- (c) $\forall x \quad \exists z \quad \exists w \quad \forall t \quad p(x, z) \vee q(w, t) \implies \neg r(x, z, w, t)$

19. Is the following argument valid?

It is false that snakes can't swim. Indeed, sea snakes can. So snakes can swim.