These questions are for practice, in preparation for Workshop 2.

1. Let $A = \{Argentina, Indonesia, China, Peru, France, Spain\}$.
A relation $R \subseteq A \times A$ is defined by

$aRb \iff a$ and b are part of the same continent.
(The continents are Africa, Antartica, Asia, Australia,
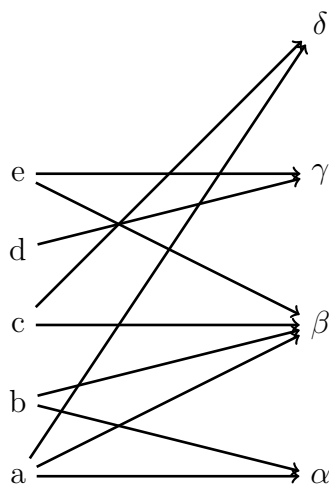Europe, N.America, S.America)

Draw a graph representing this relation.

2. Let $A = \{a, b, c, d\}$, $B = \{\alpha, \beta, \gamma, \delta\}$. Draw graphs representing the following relations.

(a) $R_1 = \{(a, \alpha), (b, \beta), (c, \gamma), (d, \delta)\}$ (b) $R_2 = \{(a, \beta), (b, \alpha), (c, \delta), (d, \alpha), (d, \beta)\}$.

(c) $R_3 = \{(a, \alpha), (a, \beta), (a, \gamma), (a, \delta)\}$.

3. Let $S = \{a, b, c, d, e\}$ be a set of senators and $P = \{\alpha, \beta, \gamma, \delta\}$ be a set of policies.
Let $R \subseteq S \times P$ be the relation defined by sRp if s supports p, using the graph below.

(a) Draw a graph of the inverse relation R^{-1}.

(b) What does $pR^{-1}s$ mean? Answer in words, referring to senators and policies.

(c) What is $\{s \in S \; ; \; pR^{-1}s\}$? Answer in words, referring to senators and policies.

4. For each of the following relations $R \subseteq \{a, b, c\} \times \{\alpha, \beta, \gamma\}$ decide whether R is a function. If not, say why.

(a) $R_1 = \{(a, \alpha), (b, \beta), (c, \gamma)\}$.

(b) $R_2 = \{(a, \alpha), (a, \beta), (a, \gamma)\}$.

(c) $R_3 = \{(a, \alpha), (b, \alpha), (c, \alpha)\}$.

(d) $R_4 = \{(a, \alpha), (b, \alpha), (c, \gamma)\}$.

5. Let A, B, C be sets, each with at least two members. Define a function F as shown at right:

$$F : A \times B \times C \to B$$
$$(a, b, c) \mapsto b.$$

(a) Is F injective (one-to-one)? Why or why not?

(b) Is F surjective (onto)? Why or why not?

6. Let U be a set, and $p \in U$. Let $F \subseteq P(U) \times P(U)$ be defined by SFT if and only if $S \cup \{p\} = T$.

(a) State the domain and codomain of F.

(b) Determine the range of F.

(c) Is F injective (one-to-one)? Why or why not?

(d) Is F surjective (onto)? Why or why not?

7. Let $\mathbb{Z}$ denote the set of integers; $\mathbb{Z} = \mathbb{N} \cup \{0\} \cup \{-n \; ; \; n \in \mathbb{N}\}$

Define functions F and G by
$$F : \quad \mathbb{Z} \to \mathbb{Z} \qquad\qquad G : \quad \mathbb{Z} \to \mathbb{Z}$$
$$z \mapsto z^2. \qquad\qquad\qquad z \mapsto z+1.$$

(a) Explain why G is bijective (a one-to-one correspondence).

(b) Complete each of the following by providing signature and rule:

G^{-1} : FoG : GoF :