This is Assignment 4 for MATH1005 students in a Friday workshop. It is due at 6 pm on the Thursday after Workshop 4 (6 days after was released).

There are four problems. The numbering of the problems is strange because the numbering is taken from a much larger document that has many problems from which I can select. As long as you can see four different problems, then you have the complete assignment.

You should write your best solutions to the problems here, and then upload your solutions before the due time. Here are three ways you may complete the assignment:

1. Print the assignment sheet. Write your solutions in pen or pencil on the print out. Scan your completed assignment, turn the file into a single .pdf file, then upload your solution file to Wattle.

2. Write your solutions in pen or pencil on blank paper. You should clearly label your solutions and you should write them in the order in which the problems appear in your assignment. Scan your completed assignment, turn the file into a single .pdf file, then upload your solution file to Wattle.

3. Download the assignment sheet to a tablet. Annotate the file using your favourite annotation software. **Flatten the file**–this makes your annotations a permanent part of the file, and if you do not do this then we see only a blank assignment in our grading software. Upload your flattened solution file to Wattle.

In all cases, the file you upload must be a .pdf file.

Please remember to plan your time carefully so you are not trying to submit your assignment at the last minute. No late work is accepted.

Please enjoy,

AP

Question 1* True or false? Justify.

(a) $51 \bmod 13 = -1$

(b) $13 \bmod 51 = 12$

(c) $51\,000 \bmod 13 = 1$

(d) $51 \equiv 13 \pmod{12}$

(e) $12 \equiv 51 \pmod{13}$

(f) $51 \equiv -1 \pmod{13}$

Question 4⋆

Read the following algorithm:

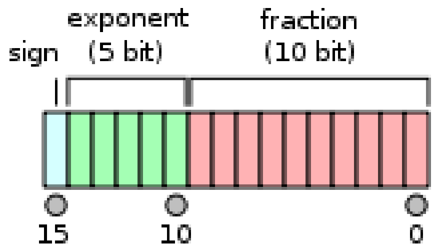
Algorithm for converting a fraction x into binary with p binary places.

Inputs: fraction $x \in \mathbb{Q}$, $0 < x < 1$ and number of places $p \in \mathbb{N}$.
Outputs: bits $b_1, b_2, \ldots, b_p$ such that $(0.b_1 b_2 \ldots b_p)_2$ is the best approximation to x using p binary places.
Method:
Initialise: $j \leftarrow 1, \quad b_1, b_2, \ldots, b_p \leftarrow 0$
Loop: if $j = p + 1$ stop.
  $x \leftarrow 2x$
  If $x \geq 1$ $[\ b_j \leftarrow 1, \quad x \leftarrow x - 1\ ]$
  $j \leftarrow j + 1$
Repeat loop

(a) Verify that the algorithm converts $\frac{3}{5}$ to 0.10011_2 when $p = 5$. Can you spot how to express $\frac{3}{5}$ exactly in repeating binary?

(b) Express $\frac{3}{7}$ as accurately as possible with 8 binary places.

Question 5* The shortest IEEE standard for representing rational numbers is called *half-precision floating point*. It uses a 16-bit word partitioned as in the diagram at right. (This diagram is taken from the Wikipedia article on the subject, where more details can be found.)



As described in lectures, to store a rational number x it is first represented as $(-1)^s \times m \times 2^n$ with $1 \le m < 2$. The sign bit s is stored as the left-most bit (bit 15), the mantissa m (called "significand" in IEEE parlance) is stored in the right-most 10 bits (bits 9 to 0), and the exponent n is stored in the 5 bits in between (bits 14 to 10). However:

- *Only the fractional part of* m *is stored.* Because $1 \le m < 2$, the binary representation of m **always** has the form $1 \cdot \star\star\star\ldots$ where the stars stand for binary digits representing the fractional part of m. Hence there is no need to store the $1\cdot$ part.

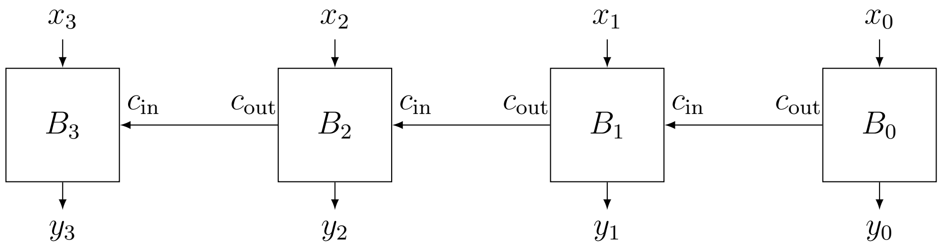
- *the exponent* n *is stored with an "offset".* In order to allow for both positive and negative exponents, but to avoid another sign bit, the value stored is $n + 15$. In principle this means that the five exponent bits can store exponents in the range $-15 \le n \le 16$, but 00000 and 11111 are reserved for special purposes so in fact n is restricted to the range $-14 \le n \le 15$.

(a) A rational number x is stored in half-precision floating point as the word DEAF_{16}. (That's hex shorthand for the 16-bit binary word.) Write x in ordinary decimal notation.

(b) Given that $\frac{1}{5} = 0.\overline{0011}_2$, find the word representing -43.2 as accurately as possible in half-precision floating point. Give your answer using hex shorthand, like the word you were given for (a).

Question 6* A '4-bit incrementer' has input x, $0 \le x \le 15$, expressed in binary as $x_3x_2x_1x_0$, and output $y = x + 1$, expressed in binary in binary as $y_3y_2y_1y_0$. So for example if $x = 1011_2$, $(x_3 = x_1 = x_0 = 1,\ x_2 = 0)$ then $y = 1100_2$ $(y_3 = y_2 = 1,\ y_1 = y_0 = 0)$. In the special case where $x = 1111_2$, y is specified to be 0000_2, *i.e* carry to a y_4 is suppressed.

A circuit for a 4-bit incrementer can be drawn in block form as:



Using any suitable gates, draw circuit diagrams for each of the blocks B_0, B_1, B_2 and B_3. Hint: Two will be suitably re-labelled half adders, and the other two will be adaptations of half adders.