ADS-5

S.1 (a) All four methods are present in "rib.cpp" file.

(b) I have put the time limit to 1.0 sec.
Increased the value by 10.
Table is present by the name "Table.pdf"

(c) All the three methods return the same value rather then closed form. In closed form rounding error can be generated.

(d) Graph is present in the file "plot.pdf" file

Scanned with CamScanner

S.2

Page No.
Date: / /20

(a) Addition, substraction and bit shifting can be done in linear time that is $\theta(n)$ so according to the brute force implementation of multiplication a number A with n bits when it gets multiplied with another number B with n bits then we have n bit shift that occurs. After every bit shift there is the multiplication of the bit for n times with n bit in A.

So

$T(n) = n\theta(n) + n\theta(n)$
$= 2n\theta(n)$
$= \theta(n^2)$

(b) Taking a number 7549

$K = 7549 \quad (A = a_1 \times 10^{n/2} + a_2)$

$a_1 = 75, \; a_2 = 49$

$75 \times 10^{n/2} + 49$
$75 \times 100 + 49$
$7500 + 49$
7549

$x = a \cdot 10^{n/2} + b$
$y = c \cdot 10^{n/2} + d$

$x \cdot y = (a 10^{n/2} + b)(c 10^{n/2} + d)$

$= ac\, 10^n + ad\, 10^{n/2} + bc\, 10^{n/2} + bd$
$= (a \cdot c\, 10^n) + (a \cdot d\, 10^{n/2}) + (b \cdot c\, 10^{n/2}) + bd$

We have

$a \cdot c \; 10^n$

$$ac \cdot 10^n + 10^{n/2}\left[(a+b)\cdot(c+d) - ac - bd\right] + bd$$

Now

$$(a \cdot c\, 10^n) + (a \cdot d\, 10^{n/2}) + (b \cdot c\, 10^{n/2}) + bd$$

$$(a \cdot c\, 10^n) + 10^{n/2}(a \cdot d + b \cdot c) + bd$$

Here we have 4 multiplication altogether. To decrease the number we can write the term

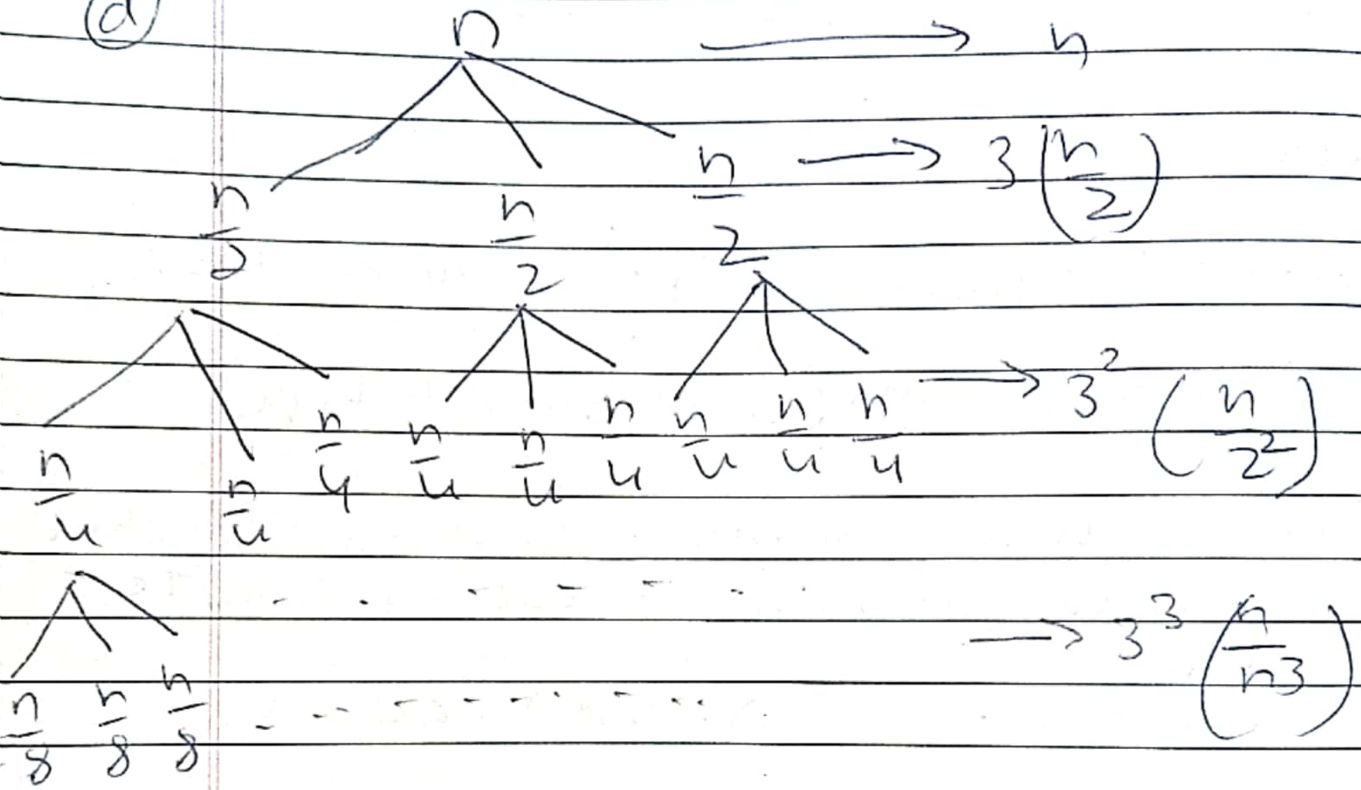
$$a \cdot d + bc = (a+b)\cdot(c+d) - ac - bd.$$

$$a \cdot c\, 10^n + 10^{n/2}\left[(a+b)\cdot(c+d) - ac - bd\right) + bd$$

(c) So the algorithm requires three distinct multiplication of size $n/2$

$$T(n) = 3T\left(\frac{n}{2}\right) + \Theta(n)$$

(d)



$$3^a\left(\frac{n}{2^a}\right)$$

$$\frac{n}{2^n} = 1$$

$$n = 2^n$$

$$n = \log_2 n$$

~~log n = n log~~

Time complexity is the sum of all the levels.

$$n + \frac{3n}{2} + 3^2\left(\frac{n}{2^n}\right) + \cdots + 3^a\left(\frac{n}{2^a}\right)$$

As the ratio is $\frac{3}{2}$ so it is a geometric series with

$a = 1$

$$S_\infty = \frac{a(r^n - 1)}{r - 1} = \frac{\left(\frac{3}{2}\right)^n - 1}{\frac{3}{2} - 1}$$

$$= \frac{\left(\frac{3}{2}\right)^n - 1}{\frac{1}{2}}$$

$$T(n) = 2n\left(\left(\frac{3}{2}\right)^n - 1\right)$$

$$= 2n\left(\left(\frac{3}{2}\right)^{\log_2 n} - 1\right)$$

$$= 2n\left[n^{\log_2 \frac{3}{2}} - 1\right]$$

$$= \cancel{2n}\{ 2n^{1 + \log_2 \frac{3}{2}} - 2n$$

$$= 2n^{\log_2 2 + \log_2 \frac{3}{2}} - 2n$$

$$= 2n^{\log_2 3} - 2n$$

$$= 2\left[n^{\log_2 3} - n\right]$$

$n^{1.58} = n^{\log_2 3}$ is greater than n

so

$\theta(n) = n^{\log_2 3} \cong n^{1.58}$.

(e) $T(n) = 3T\left(\frac{n}{2}\right) + \theta(n)$

$a = 3 \quad b = 2$

$= n^{\log_b a}$

$= n^{\log_2 3} = n^{1.58}$

$f(n)$ in term of $n^{\log_b (a)}$?

$f(n) < n^{\log_b a}$

$f(n) = \theta(n^{1.55 - \epsilon})$ for $\epsilon > 0.58$

$T(n) = \theta(n^{1.58})$

$T(n) = \theta\, n^{\log_2 3}$