# Software Assignment Error Analysis - Image Compression using truncated SVD

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**Note: elements of A and Ak are normalized from range [0,255] to range [0,1](to avoid overflow), hence Frobenius error shown is less by factor of 255 of actual image matrices, but relative error is same.**

Table 1: Rank $k$ vs Error vs Compression Ratio (Globe Image)

| Rank $k$ | Frobenius Error | Relative Error | Compression Ratio | Visual Notes |
|---|---|---|---|---|
| 2 | 128.725418 | 0.207343 | 214.638 | Very blurry, almost no detail |
| 5 | 81.193234 | 0.130781 | 85.859 | Continents barely recognizable |
| 20 | 41.703581 | 0.067174 | 21.463 | Shape visible, still smoothed |
| 50 | 24.257422 | 0.039072 | 8.585 | Good clarity, slight blur |
| 100 | 14.403657 | 0.023201 | 4.292 | High quality, minor loss |
| 200 | 7.010167 | 0.011292 | 2.146 | Nearly identical to original |
| 300 | 3.853171 | 0.006206 | 1.430 | Indistinguishable from original |

Table 2: Rank $k$ vs Error vs Compression Ratio (Einstein Image)

| **Rank $k$** | **Frobenius Error** | **Relative Error** | **Compression Ratio** | **Visual Notes** |
|---|---|---|---|---|
| 2 | 24.973135 | 0.291993 | 45.869 | Very blurry, face barely recognizable |
| 5 | 18.484639 | 0.216127 | 18.347 | Very blurry; facial details unclear |
| 20 | 8.339449 | 0.097507 | 4.586 | Face recognizable but soft |
| 50 | 3.452953 | 0.040373 | 1.834 | Good quality; fine details appear |
| 100 | 0.646199 | 0.007556 | 0.917 | Nearly lossless reconstruction |
| 200 | $\approx 0$ | $\approx 0$ | 0.458 | Almost exact reconstruction |
| 300 | $\approx 0$ | $\approx 0$ | 0.305 | Indistinguishable from original |

Table 3: Rank $k$ vs Error vs Compression Ratio (Greyscale Image)

| **Rank $k$** | **Frobenius Error** | **Relative Error** | **Compression Ratio** | **Visual Notes** |
|---|---|---|---|---|
| 2 | 67.324011 | 0.088694 | 255.885 | Only broad intensity visible; very blurry |
| 5 | 43.711017 | 0.057586 | 102.354 | Shapes recognizable; edges soft |
| 20 | 14.934093 | 0.019674 | 25.587 | Good clarity; noticeable smoothness |
| 50 | 4.549560 | 0.005994 | 10.234 | Very good detail; visually sharp |
| 100 | 2.009199 | 0.002647 | 5.117 | Nearly lossless reconstruction |
| 200 | 1.605776 | 0.002115 | 2.558 | Almost identical to original |
| 300 | 1.283533 | 0.001691 | 1.705 | Indistinguishable from original |

The tables report the Frobenius error, relative error, compression ratio, and visual quality of the reconstructed images for different values of rank $k$. From these results, we can

observe a clear relationship between $k$ and the reconstruction quality.

## General Trends

- For very small values of $k$ (e.g., $k = 2$ or $k = 5$), both the Frobenius error and the relative error are large. The reconstructed images appear very blurry and lack detail. Only the most dominant large-scale structures of the image are preserved.
- As the rank $k$ increases, the error decreases steadily. The reconstructed images begin to show sharper edges and clearer shape boundaries. Fine details start to appear.
- When $k$ is sufficiently large (close to $\min(m, n)$), the reconstructed image becomes almost identical to the original. The relative error becomes extremely small, and further increases in $k$ result in minimal visual improvement while increasing storage cost.

## Image-Specific Observations

**1) Globe Image** The globe has smooth shading and gradual intensity variations. This type of image requires a higher rank $k$ to be reconstructed accurately.

- For $k = 20$ and $k = 50$, the overall shape and shading are visible but still somewhat smoothed.
- At $k = 100$ and above, the reconstruction is high quality with very minor loss.
- For $k = 200$ or $k = 300$, the reconstructed image is almost indistinguishable from the original.

**2) Einstein Portrait** The portrait image has strong edges and high contrast, which means that the most important information is captured by the first few singular vectors.

- Even at $k = 20$, the face is clearly recognizable.
- At $k = 50$, the image appears visually very close to the original.
- At $k = 100$ or higher, the error becomes extremely small and further increases in $k$ provide almost no visible difference.

**3) Greyscale Pattern Image** This image contains repeated gradients, so the singular values decrease at a moderate rate.

- Good clarity is achieved around $k = 20$ to $k = 50$.
- For $k = 100$ and above, the reconstruction is nearly lossless.
- Increasing $k$ beyond this range yields diminishing visual improvement.