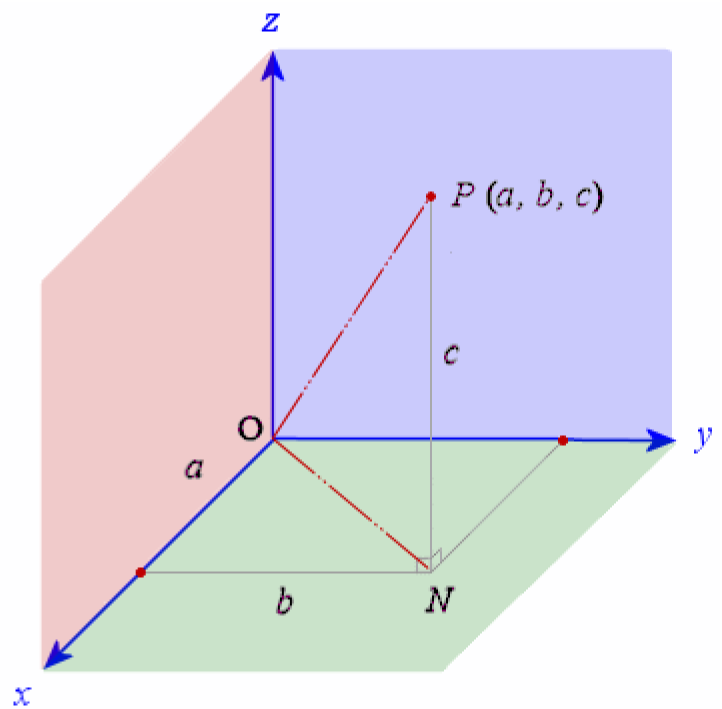
3D Transformations

3D Transformation

- Generalize from of 2D by including **z** coordinate
- Straightforward for translation and scale ☺
- Rotation little more difficult ☹
- Homogeneous coordinates: 4 components
- Order of transformation matrices: 4×4

3D Point

- We will consider points as column vectors. Thus, a typical point **P** with coordinates (x, y, z) is represented as:



$$P = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

3D Point Homogenous Coordinate

- A 3D point $\mathbf{P}$ is represented in homogeneous coordinates by a 4-dimensional vector

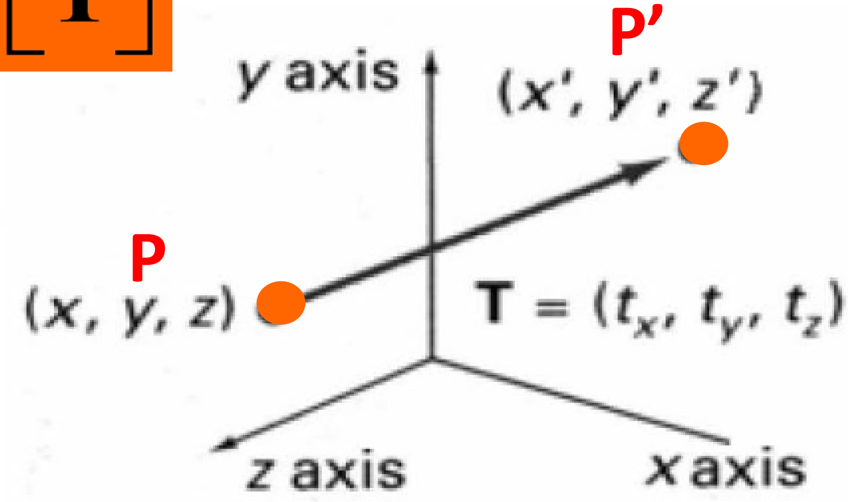
$$\mathbf{P} = \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

3D Translation

- P is translated to P' by:

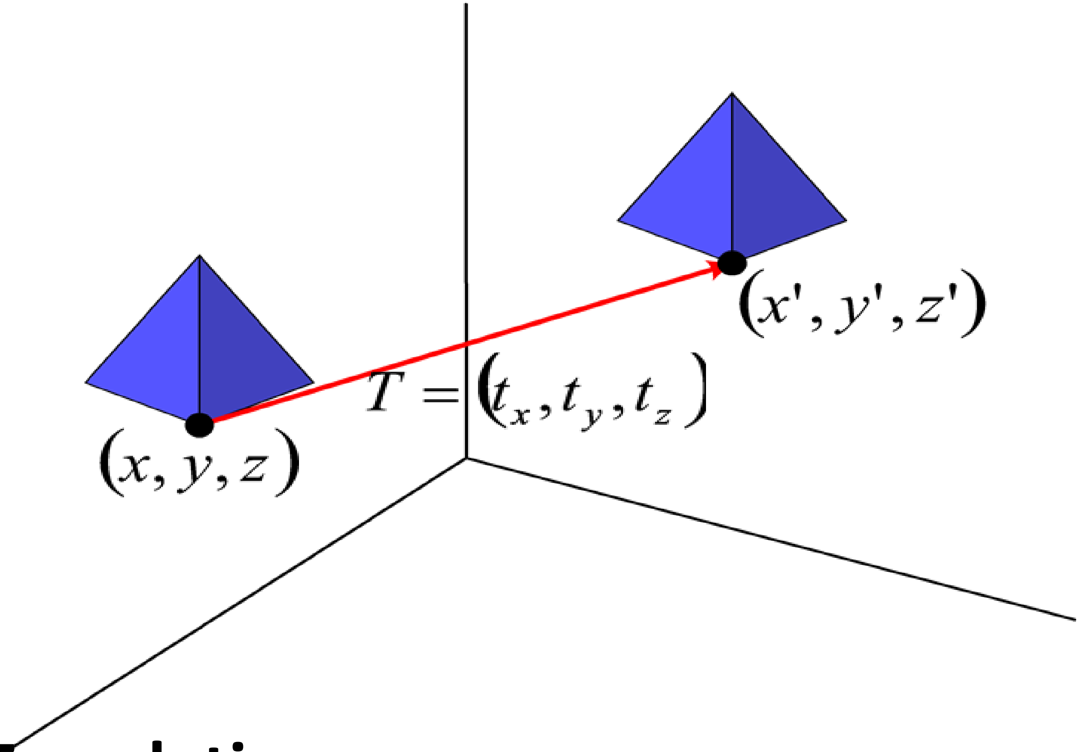
$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & t_x \\ 0 & 1 & 0 & t_y \\ 0 & 0 & 1 & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$$\mathbf{P}' = \mathbf{T} \cdot \mathbf{P}$$



3D Translation

- An Object represented as a set of polygon surfaces
  - Translated by translating each vertex of each surface and redraw the polygon facets in the new position.



- **Inverse Translation:**

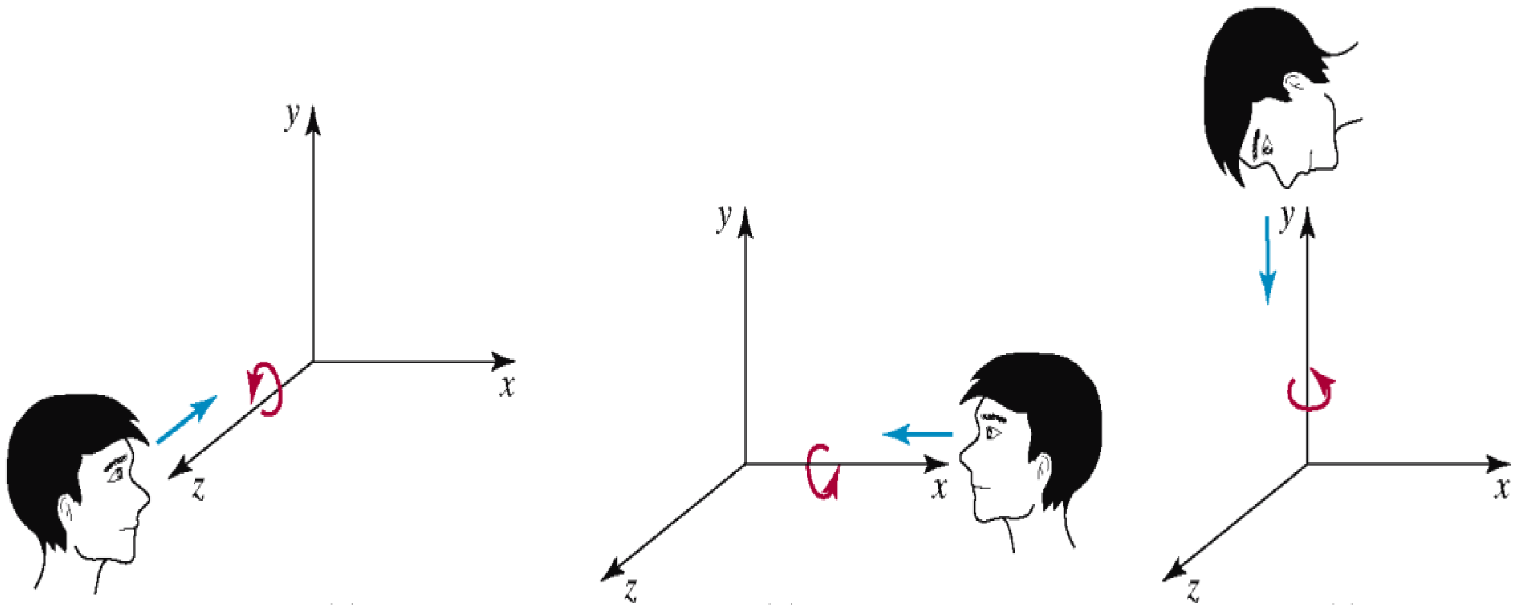
$$\boxed{T^{-1}(t_x, t_y, t_z) = T(-t_x, -t_y, -t_z)}$$

3D Rotation

- In general, rotations are specified by
  - *Rotation axis* and
  - *Angle*

3D Rotation

- The easiest **rotation axes** are those that parallel to the coordinate axis.
- Positive rotation **angles** produce counterclockwise rotations about a coordinate axix, if we are looking along the positive half of the axis toward the coordinate origin.

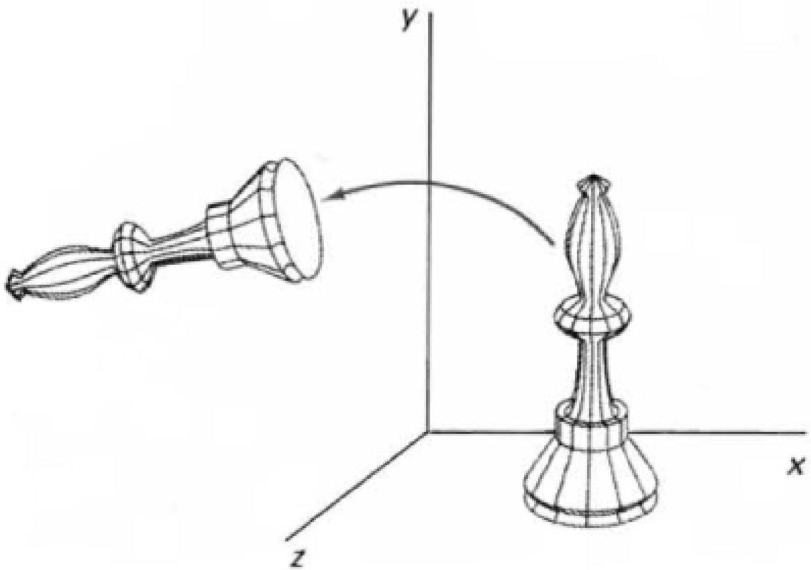


Coordinate Axis Rotations

- **Z-axis rotation:** For z axis same as 2D rotation:

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$$\mathbf{P}' = \mathbf{R}_z(\theta) \cdot \mathbf{P}$$

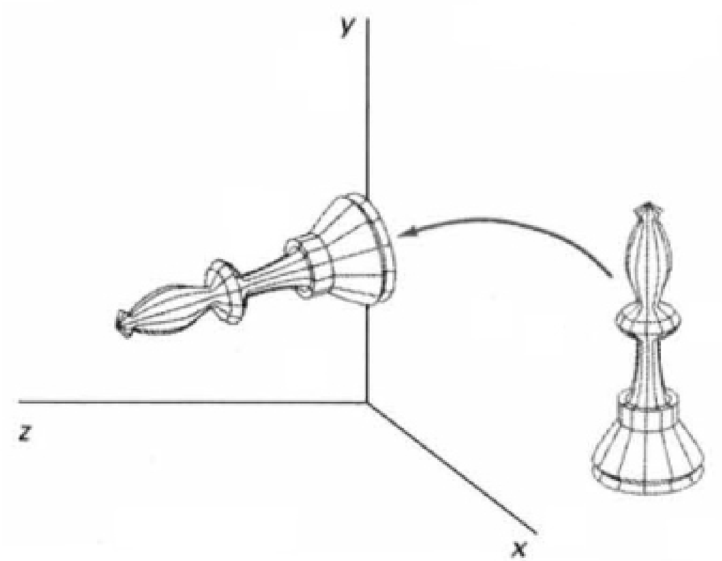


Coordinate Axis Rotations

- **X-axis rotation:**

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta & 0 \\ 0 & \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$$\mathbf{P'} = \mathbf{R}_x(\theta) \cdot \mathbf{P}$$

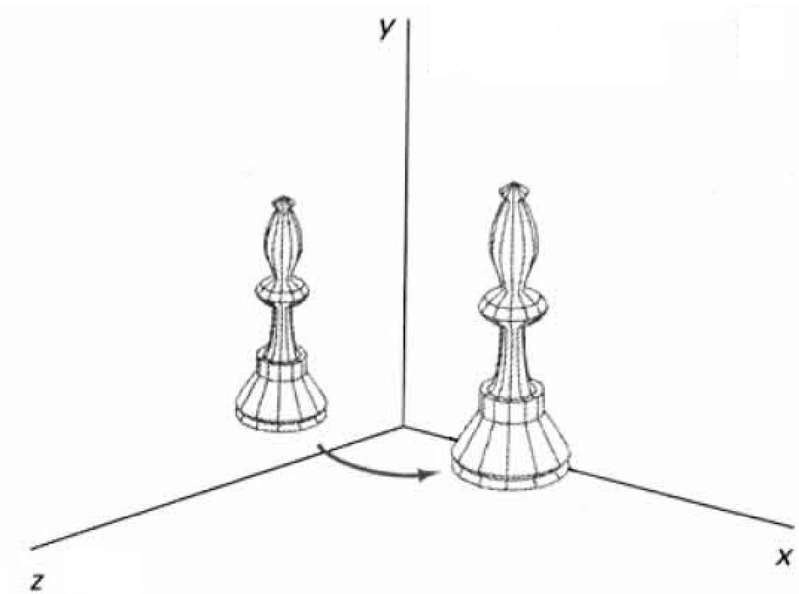


Coordinate Axis Rotations

- **Y-axis rotation:**

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & 0 & \sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$$\mathbf{P'} = \mathbf{R}_y(\theta) \cdot \mathbf{P}$$



Rotations for an arbitrary axis

(1)

(2)

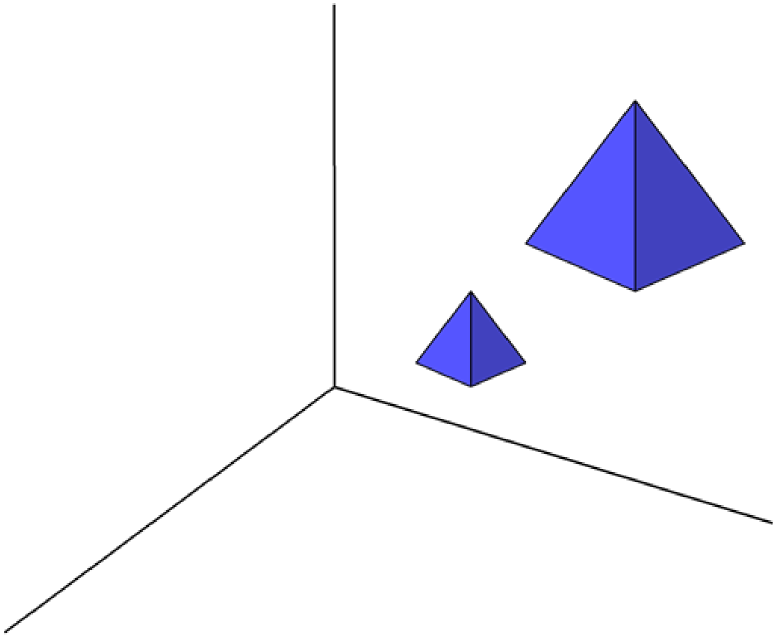
(3)

3D Scaling

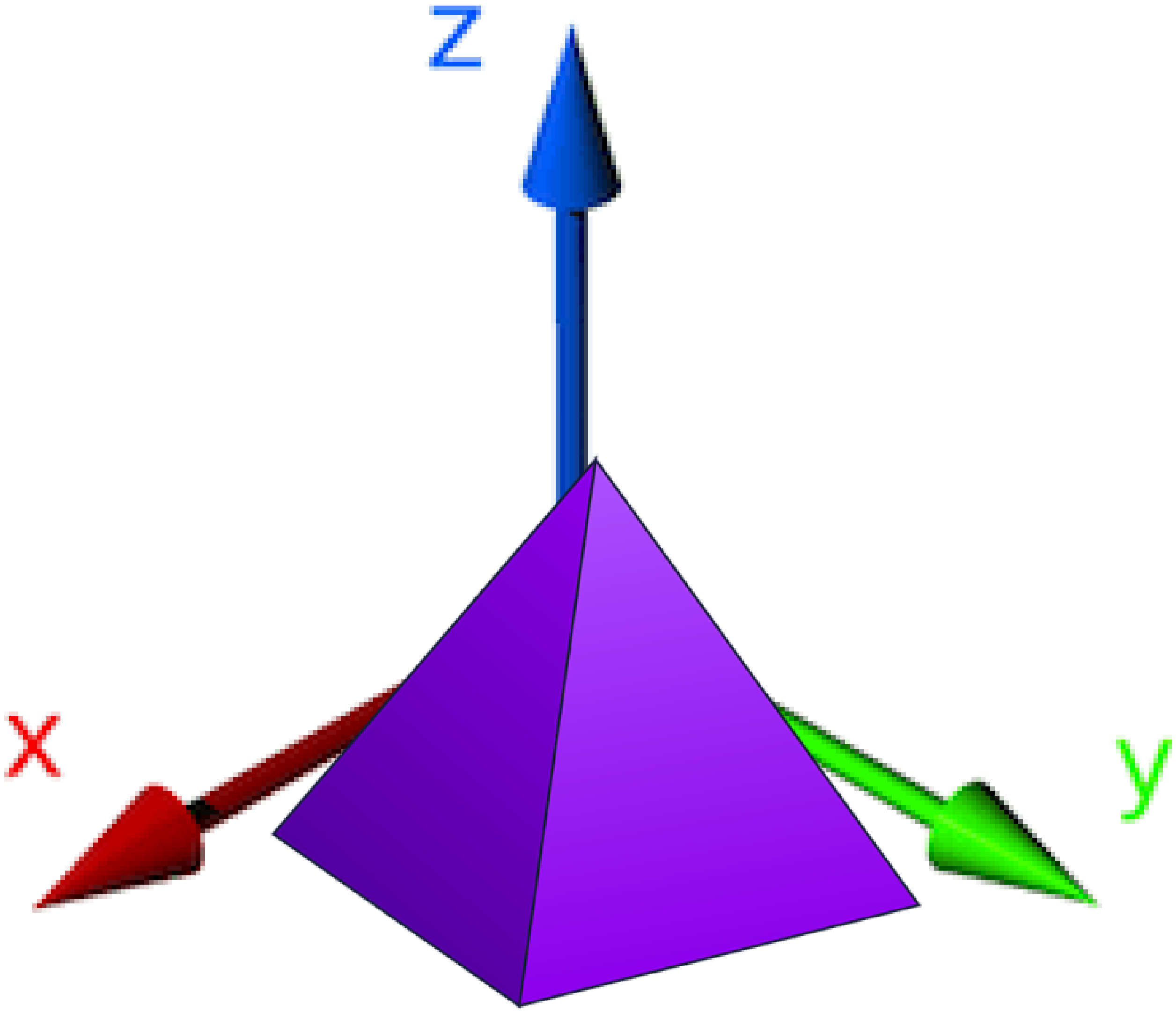
- **About origin:** Changes the size of the object and repositions the object relative to the coordinate origin.

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} s_x & 0 & 0 & 0 \\ 0 & s_y & 0 & 0 \\ 0 & 0 & s_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

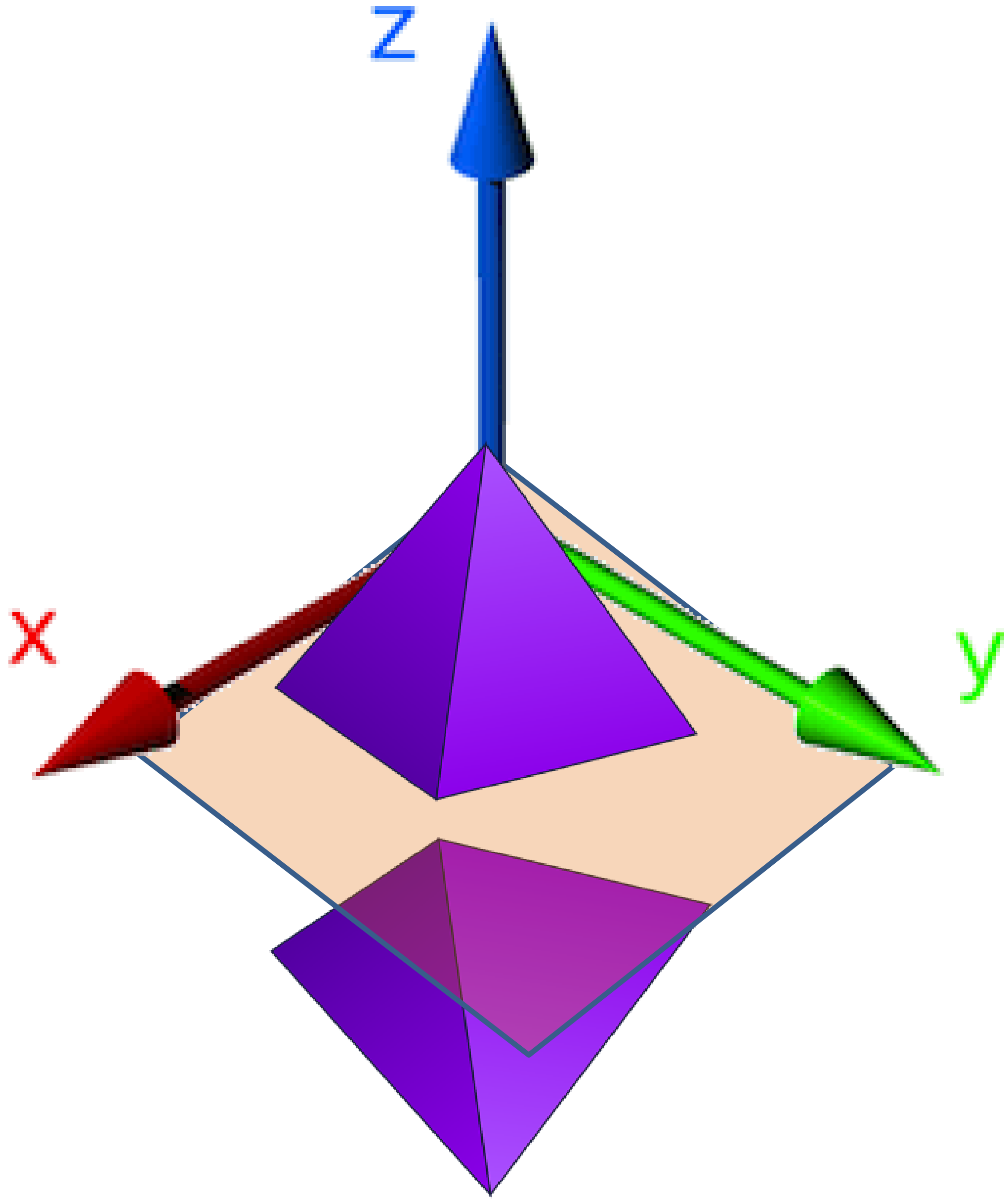
$$\mathbf{P}' = \mathbf{S} \cdot \mathbf{P}$$



3D Reflection



3D Reflection



3D Reflections

About a plane:

- A reflection through the **xy** plane:

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

- A reflections through the **xz** and the **yz** planes are defined similarly.

Thank you!!