# Categorical Data (2 of 2)

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## Overview of Bivariate and Multivariate Categorical Variables

- 1. Bivariate and multivariate categorical variables allow us to analyze and comprehend relationships between two or more categorical variables respectively.
- 2. Bivariate analysis involves examining the relationship between two variables. For instance, we might examine the relationship between a person's gender (male, female, or non-binary) and whether they own a car (yes or no). By exploring these two categorical variables together, we can discern potential correlations or associations.
- 3. On the other hand, multivariate analysis involves the simultaneous observation and analysis of more than two variables. This form of analysis can help reveal complex interactions and dependencies between multiple variables that cannot be detected in bivariate analyses. For example, we might want to explore the relationship between a person's gender, car ownership status, and their level of education (high school, bachelor's, master's, etc.). Both bivariate and multivariate analyses are essential in statistical and data analysis as they allow us to uncover relationships and patterns in data

## **Bivariate Categorical Variables**

- 1. Bivariate categorical data can be understood as a type of data where we are examining two categorical variables simultaneously, where we need to explore relationships or differences between the two factors, using contingency tables.
- 2. Contingency Table: A contingency table, also known as a cross-tabulation or crosstab, is a type of table in a matrix format that displays the (multivariate) frequency distribution of the variables. In the case of a univariate factor variable, a contingency table is essentially the same as a frequency table, as there's only one variable involved. In more complex analyses involving two or more variables, contingency tables provide a way to examine the interactions between the variables [(Agresti, 2007). [1]

3. Data: Let us work with the same mtcars data from the previous chapter. Suppose we have run the following code:

```
# Load the required libraries, suppressing annoying startup messages
library(tibble)
suppressPackageStartupMessages(library(dplyr))
# Read the mtcars dataset into a tibble called tb
data(mtcars)
tb <- as_tibble(mtcars)
attach(tb)
# Convert several numeric columns into factor variables
tb$cyl <- as.factor(tb$cyl)
tb$vs <- as.factor(tb$vs)
tb$am <- as.factor(tb$gear)</pre>
```

#### Frequency Table for Bivariate Categorical Variables

- 1. As an illustration, let us investigate the bivariate relationship between the number of cylinders (cyl) and whether the car has an automatic (am) or manual (1 = manual, 0 = automatic) transmission.
- 2. table(): We can use the table() function to generate a contingency table of these two variables.

```
0 1
4 3 8
6 4 3
8 12 2
```

In this example, a two-way frequency table of am and cyl is created using the table() function. The frequency of each grouping of categories is displayed in the table that results. As an illustration, there are 8 cars with a manual gearbox and 4 cylinders.

• addmargins(): The addmargins() function is used to add row and/or column totals to a table.

```
t0 <- table(tb$cyl, tb$am)
addmargins(t0)
        1 Sum
4
     3
        8
            11
6
     4
        3
             7
8
    12
        2
            14
Sum 19 13
```

The addmargins() function is used to add row and column totals to a table

addmargins() adds both row and column totals to the table. Hence, this command adds both the row and column totals to the contingency table t0.

```
t0 <- table(tb$cyl, tb$am)
addmargins(t0,1)

0 1
4 3 8
6 4 3
8 12 2
Sum 19 13
```

Here, the 1 in the function call indicates that we want to add row totals. So, this command adds the totals (sum) of each row (representing cyl) to the contingency table t0.

```
t0 <- table(tb$cyl, tb$am)
addmargins(t0,2)

0 1 Sum
4 3 8 11
6 4 3 7
8 12 2 14
```

In this command, the 2 specifies that we want to add column totals. Thus, it adds the totals (sum) of each column (representing am) to the contingency table t0.

3. **xtabs()**: This function in R provides a more versatile way to generate cross tabulations or contingency tables. It differs from the **table()** function by allowing the use of weights and formulas. Here's how we can use it:

In the command above, we've used xtabs() to construct a cross-tabulation of am and cyl. The syntax ~ cyl + am is interpreted as a formula, signifying that we aim to cross-tabulate these variables. The output is a table akin to what we obtain with table(), but with the added advantage of accommodating more intricate analyses.

An important advantage of xtabs() over table() is its superior handling of missing values or NAs; it doesn't automatically exclude them, which is beneficial when dealing with real-world data that often includes missing values. [1]

4. **ftable()**: The **ftable()** function in R is a powerful tool that offers an advanced way to create and display contingency tables. Here's an example of its use:

```
ftable(tb$cyl, tb$am)

0 1

4 3 8
6 4 3
8 12 2
```

- In this scenario, we've employed the ftable() function to create a contingency table of am and cyl. The output of this function is similar to what we get using table(), but it presents the information in a flat, compact layout, which can be particularly helpful especially when dealing with more than two variables.
- One key advantage of ftable() is that it creates contingency tables in a more readable format when dealing with more than two categorical variables, making it easier to visualize and understand complex multivariate relationships.
- Do note, like xtabs(), ftable() also handles missing values or NAs effectively, making it a reliable choice for real-world data that might contain missing values.
- 5. We can also use package dplyr to generate contingency tables.

```
library(dplyr)
tb %>%
  group_by(cyl, am) %>%
  summarise(Frequency = n())
```

`summarise()` has grouped output by 'cyl'. You can override using the `.groups` argument.

```
# A tibble: 6 x 3
# Groups:
             cyl [3]
               Frequency
  cyl
        am
  <fct> <fct>
                   <int>
1 4
        0
                        3
2 4
                        8
        1
3 6
        0
                        4
4 6
        1
                        3
5 8
                       12
        0
                        2
6 8
        1
```

## **Proportions Table for Bivariate Categorical Variables**

6. **prop.table()**: The **prop.table()** function is an advantageous tool in R, particularly when it's necessary to understand the relative proportions rather than raw frequencies. This function converts a contingency table into a table of proportions. Here is how we could utilize this function:

8 0.375 0.062

• In the example above, we first generate a frequency table with the table() function, using cyl and am as our variables. Then, we employ the prop.table() function to convert this frequency table (freq\_table) into a proportions table (prop\_table).

- This resulting prop\_table reveals the proportion of each combination of cyl and am categories relative to the total number of observations. This can provide insightful context, allowing us to see how each combination fits into the overall distribution. For instance, we could learn what proportion of cars in our dataset have 4 cylinders and a manual transmission.
- A noteworthy benefit of prop.table() is its ability to normalize the data, providing a perspective based on relative proportions instead of absolute numbers, which can often provide a clearer view of the underlying patterns in the data.
- 7. We can alternately use package dplyr to achieve this as follows.

```
library(dplyr)
tb %>%
  group_by(cyl, am) %>%
  summarise(Frequency = n()) %>%
  mutate(Proportion = Frequency / sum(Frequency))
```

`summarise()` has grouped output by 'cyl'. You can override using the `.groups` argument.

```
# A tibble: 6 x 4
# Groups:
             cyl [3]
  cyl
               Frequency Proportion
        am
  <fct> <fct>
                   <int>
                               <dbl>
1 4
        0
                       3
                               0.273
2 4
                       8
                               0.727
        1
3 6
                       4
        0
                               0.571
4 6
        1
                       3
                               0.429
5 8
                      12
                               0.857
        0
6 8
                       2
                               0.143
```

In the code above, group\_by(cyl, am) groups the data by cyl and am, summarise(Frequency = n()) calculates the frequency for each group, and mutate(Proportion = Frequency / sum(Frequency)) calculates the proportions by dividing each frequency by the total sum of frequencies. The mutate() function adds a new column to the dataframe, keeping the original data intact.

• **Rounding**: If we wanted to round-off the Proportion up to 4 decimal places, we could write the following code.

```
library(dplyr)
tb %>%
  group_by(cyl, am) %>%
  summarise(Frequency = n()) %>%
  mutate(Proportion = round(Frequency / sum(Frequency), 4))
```

`summarise()` has grouped output by 'cyl'. You can override using the `.groups` argument.

```
# A tibble: 6 x 4
# Groups:
           cyl [3]
 cyl
             Frequency Proportion
        am
  <fct> <fct>
                 <int>
                             <dbl>
1 4
        0
                      3
                             0.273
2 4
        1
                      8
                             0.727
3 6
        0
                      4
                             0.571
4 6
                      3
        1
                             0.429
                     12
5 8
        0
                             0.857
6 8
        1
                      2
                             0.143
```

#### **Adding Margins to Proportions Tables**

- 1. Different proportions provide various perspectives on the relationship between categorical variables in our dataset. We can calculate i) Proportions for Each Cell; (ii) Row-Wise Proportions; (iii) Column-Wise Proportions. This forms a crucial part of exploratory data analysis.
- 2. Proportions for Each Cell: This calculates the ratio of each cell to the overall total.

```
freq <- table(tb$cyl, tb$am)
# Compute cell-wise proportions
cellprop <- prop.table(freq)
# Add totals for each row and column
cellprop <- addmargins(cellprop)
# Round off the results to three decimal places and display as percentages
100*round(cellprop, 3)</pre>
```

```
0 1 Sum
4 9.4 25.0 34.4
```

```
6 12.5 9.4 21.9
8 37.5 6.2 43.8
Sum 59.4 40.6 100.0
```

3. Row-Wise Proportions: Here, we compute the proportion of each cell relative to the total of its row.

```
# Compute row-wise proportions
rowprop <- prop.table(freq, margin = 1)
# Append totals for each row and column
rowprop <- addmargins(rowprop,2)
# Round off the results to three decimal places and display as percentages
100*round(rowprop, 3)</pre>
```

```
0 1 Sum
4 27.3 72.7 100.0
6 57.1 42.9 100.0
8 85.7 14.3 100.0
```

4. **Column-Wise Proportions**: In this step, we determine the proportion of each cell relative to the total of its column.

```
# Compute column-wise proportions
colprop <- prop.table(freq, margin = 2)
# Append totals for each row and column
colprop <- addmargins(colprop,1)
# Round off the results to three decimal places and display as percentages
100*round(colprop, 3)</pre>
```

```
0 1
4 15.8 61.5
6 21.1 23.1
8 63.2 15.4
Sum 100.0 100.0
```

In the examples provided, the prop.table() function is initially applied with the margin parameter to calculate cell, row, and column proportions from the contingency table. We then invoke addmargins() to sum up and include the marginal totals for each row and column. Lastly, the round() function aids in displaying the proportions with one decimal place.

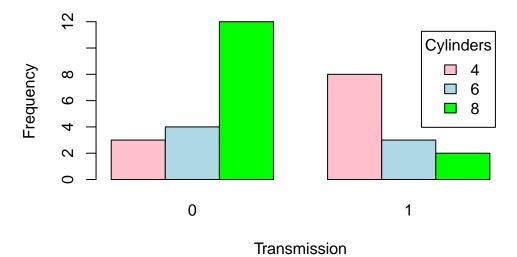
#### Visualizing Bivariate Categorical Variables

- 1. Grouped Barplots and Stacked Barplots serve as powerful tools for representing and understanding bivariate categorical data, where both variables are categorical in nature.
- 2. Grouped Barplots, often referred to as side-by-side bar plots, illustrate the relationship between two categorical variables by placing bars corresponding to one category of a variable next to each other, differentiated by color or pattern. This layout facilitates a direct comparison between categories of the second variable. Grouped bar plots are particularly effective when we are interested in comparing the distribution of a categorical variable across different groups
- 3. On the other hand, stacked bar plots present a similar relationship between two categorical variables, but rather than aligning bars side by side, they stack bars on top of one another. This results in a single bar for each category of one variable, with the length of different segments in each bar corresponding to the counts or proportions of the categories of the other variable. Stacked bar plots are advantageous when we're interested in the total size of groups as well as the distribution of a variable across groups. [2]

#### 4. Grouped Barplot

```
# Create a table with count by transmission and number of cylinders
freq <- table(tb$cyl, tb$am)</pre>
freq
   0
      1
  3
      8
6 4
      3
8 12
     2
# Create a Grouped bar plot
barplot(freq,
        beside = TRUE,
        col = c("pink", "lightblue", "green"),
        xlab = "Transmission", ylab = "Frequency",
        main = "Grouped Barplot of Frequency by transmission and cylinders",
        legend.text = rownames(freq),
        args.legend = list(title = "Cylinders"))
```

## Grouped Barplot of Frequency by transmission and cylind-



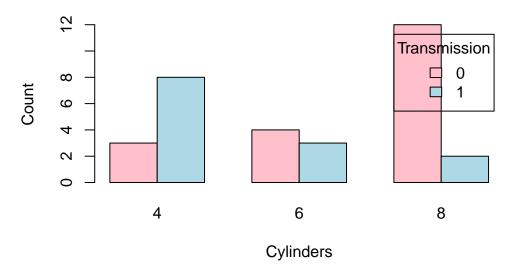
- 5. Here's a detailed explanation of each argument passed to the barplot() function:
- freq: This is the dataset being visualized, which we anticipate to be a contingency table of am and cyl variables.
- beside = TRUE: This argument is specifying that the bars should be positioned next to each other, which means that for each level of am, there will be a distinct bar for each level of cyl.
- col = c("pink", "lightblue", "green"): Here, we are setting the colors of the bars to pink, light blue, and green.
- xlab = "Transmission" and ylab = "Frequency": These arguments set the labels for the x and y-axes, respectively.
- main = "Grouped Barplot of Frequency by transmission and cylinders": This argument assigns a title to the plot.
- legend.text = rownames(freq): This creates a legend for the plot, using the row names of freq as the legend text.
- args.legend = list(title = "Cylinders"): This sets the title of the legend to "Cylinders".

Consider this alternate barplot.

```
# Create a table with count by transmission and number of cylinders
freqInverted <- table(tb$am, tb$cyl)
freqInverted</pre>
```

```
4 6 8
0 3 4 12
1 8 3 2
```

## **Grouped Barplot of Frequency by cylinders and transmiss**



The most significant differences from the previous Grouped Barplot and this one are as follows:

freqInverted: The contingency table's axes have been swapped or inverted. Hence, the table's rows now correspond to the am variable (transmission), and its columns correspond to the cyl variable (cylinders).

xlab = "Cylinders" and ylab = "Count": These arguments set the labels for the x and y-axes, respectively. This is a departure from the previous plot where the x-axis represented 'Transmission'. In this case, the x-axis corresponds to 'Cylinders'.

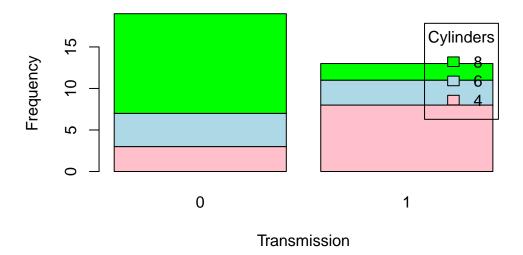
legend.text = rownames(freqInverted) and args.legend = list(title = "Transmission"):
In the legend, the roles of 'Transmission' and 'Cylinders' are reversed compared to the previous
plot.

To put it succinctly, the main distinction between the two plots is the swapping of the roles of the cyl and am variables. In the second plot, 'Cylinders' is on the x-axis, which was occupied by 'Transmission' in the first plot. This perspective shift helps to understand the data in a different light, adding another dimension to our exploratory data analysis. [2]

#### 5. Stacked Barplot

```
# Create a table with count by transmission and number of cylinders
freq <- table(tb$cyl, tb$am)</pre>
freq
   0
     1
4 3 8
6 4 3
8 12 2
# Create a Stacked bar plot
barplot(freq,
        beside = FALSE,
        col = c("pink", "lightblue", "green"),
        xlab = "Transmission", ylab = "Frequency",
        main = "Stacked Barplot of Frequency by transmission and cylinders",
        legend.text = rownames(freq),
        args.legend = list(title = "Cylinders"))
```

## Stacked Barplot of Frequency by transmission and cylinde



There are a few key differences between this Stacked Barplot and the original Grouped Barplot:

beside = FALSE: In the original code, beside = TRUE was used to generate a grouped bar plot, where each set of bars corresponding to each transmission type (automatic or manual) were displayed side by side. However, with beside = FALSE, we obtain a stacked bar plot. In this plot, the bars corresponding to each cylinder category (4, 6, or 8 cylinders) are stacked on top of one another for each transmission type.

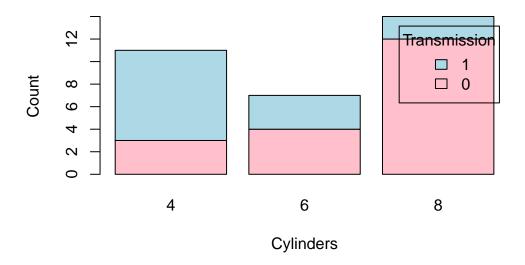
main = "Stacked Barplot of Frequency by transmission and cylinders": The title of the plot also reflects this change, mentioning now that it's a stacked bar plot instead of a grouped bar plot.

Finally, here is a second Stacked Barplot, corresponding to the second Grouped Barplot discussed above.

```
# Create a table with count by transmission and number of cylinders
freqInverted <- table(tb$am, tb$cyl)
freqInverted</pre>
```

4 6 8 0 3 4 12 1 8 3 2

## Stacked Barplot of Frequency by cylinders and transmissi



6. The grouped bar plot helps in comparing the number of cylinders across transmission types side by side, while the stacked bar plot gives an overall comparison in terms of total number of cars, with the frequency of each cylinder type stacked on top of the other. The choice between a stacked and a grouped bar plot would depend on the specific aspects of the data one would want to highlight. Taken together, grouped and stacked bar plots offer visually appealing and intuitive methods for presenting bivariate categorical data, allowing us to understand and analyze relationships between categorical variables in a meaningful way. [3]

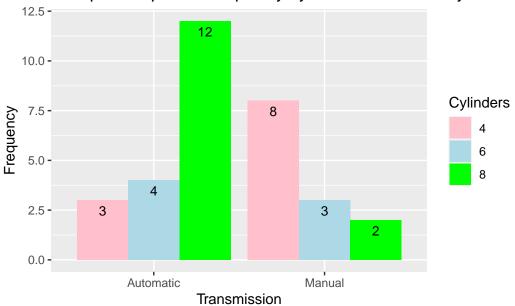
## Visualizing Bivariate Categorical Variables using ggplot2

#### Grouped Barplot using ggplot2

```
freq <- table(tb$cyl, tb$am)</pre>
```

```
# Load required library
  library(ggplot2)
Attaching package: 'ggplot2'
The following object is masked from 'tb':
    mpg
  # Convert the table to a data frame and reshape to 'long' format
  df <- as.data.frame.table(freq)</pre>
  names(df) <- c("Cylinders", "Transmission", "Frequency")</pre>
  df$Cylinders <- factor(df$Cylinders)</pre>
  df$Transmission <- factor(df$Transmission, labels = c("Automatic", "Manual"))</pre>
  # Create the Grouped Barplot using ggplot2
  ggplot(df,
         aes(x = Transmission, y = Frequency, fill = Cylinders)) +
    geom_bar(stat = "identity",
             position = position_dodge()) +
    geom_text(aes(label=Frequency),
               vjust=1.6, color="black", position = position_dodge(0.9), size=3.5) +
    labs(x = "Transmission", y = "Frequency", fill = "Cylinders",
         title = "Grouped Barplot of Frequency by transmission and cylinders") +
    scale_fill_manual(values = c("pink", "lightblue", "green"))
```





In this code:

We first convert the frequency table into a data frame that ggplot2 can use. This involves converting the table to a data frame and then reshaping it to long format.

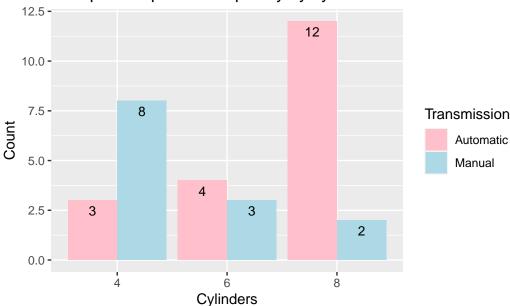
We then use the ggplot() function to initiate the plot and specify the aesthetic mappings. Here, we map 'Transmission' to the x-axis, 'Frequency' to the y-axis, and 'Cylinders' to the fill aesthetic which controls the color of the bars.

geom\_bar() with stat = "identity" is used to add the bar geometry to the plot. position = position\_dodge() ensures the bars are placed side by side, which creates the grouped effect.

labs() is used to add labels to the plot.

scale\_fill\_manual() allows us to manually specify the colors for the bars.

## Grouped Barplot of Frequency by cylinders and transmission



The geom\_text() function is used in the same way as in the previous example to add frequency labels to the bars. The scale\_fill\_manual() function is used to specify the colors for the different transmission types.

#### Stacked Barplots using ggplot2

To replicate the stacked bar plot with ggplot2, we can utilize the same dataframe transformation as before. Then, we apply the geom\_bar() function but without the dodge positioning this time, since we want the bars stacked. Here is how you can accomplish this:

```
# Load required library
library(ggplot2)

# Convert the table to a data frame and reshape to 'long' format
df <- as.data.frame.table(freq)
names(df) <- c("Cylinders", "Transmission", "Frequency")</pre>
```

### Stacked Barplot of Frequency by transmission and cylinders

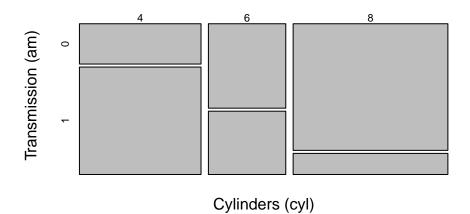


In this code, we used geom\_bar(stat = "identity") to create a stacked bar plot. The scale\_fill\_manual() function is used to specify the colors for the different cylinder categories. Again, we changed the order of the Transmission and Cylinders columns to suit this specific plot. The geom\_text() function is used to add the labels to the stacked bars. We use position\_stack() to position the labels in the middle of the stacked sections. The vjust argument inside position\_stack() controls the vertical positioning of the labels, and 0.5 puts them in the middle. As before, scale\_fill\_manual() allows us to specify the colors for the different cylinder categories.

- Mosaic plots for Bivariate Categorical Variables
- 1. A mosaic plot is a graphical method for visualizing data from two or more qualitative

- variables / categorical data. It is a form of area plot that can provide a visual representation of the frequency or proportion of the different categories within the variables.
- 2. In a mosaic plot, we interpret two categorical variables on two axes. The width of each section on one axis signifies the proportion of that particular category in our dataset. Conversely, the height of a section on the other axis illustrates the proportion of that category, contingent on the specific category from the first variable. Therefore, the area of each rectangle directly corresponds to the frequency or proportion of observations falling within that specific combination of categories (Hartigan & Kleiner, 1981).
- 3. They can be used to identify breaks in independence or test hypotheses regarding the connections between the variables. They are especially helpful for examining interactions between two or more categorical variables.
- 4. The following code generates a mosaic plot from a contingency table of two variables: cyl (cylinders) and am (transmission).

## Mosaic of Cylinder count and Transmission type



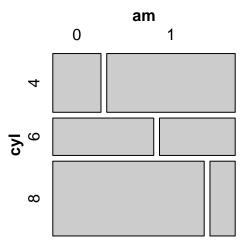
The output of this line of code would be a mosaic plot that visualizes the distribution of the number of cylinders by the type of transmission. Each block's width in the plot would be proportional to the number of cylinders, and the height would be proportional to the transmission type.

The vcd package, short for Visualizing Categorical Data, provides various visual and analytical methods for categorical data (Meyer, Zeileis, & Hornik, 2020). Following this, a mosaic plot is created from the variables cyl (cylinders) and am (transmission type) using the mosaic() function.

```
# Load the vcd package
library(vcd)
```

Loading required package: grid

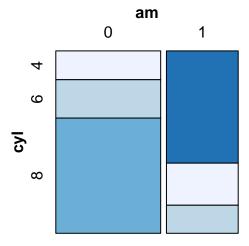
# losaic of Cylinder count and Transmission typ



We can customize the shading color of the mosaic plot by setting the color using the col argument inside the mosaic() function. To specify shades of blue, we could use the RColorBrewer package and its brewer.pal() function to generate a color palette:

```
# Load necessary packages
library(vcd)
library(RColorBrewer)
# Define the color palette
```

# losaic of Cylinder count and Transmission typ



#### In this code:

- brewer.pal(4, "Blues") creates a color palette of 4 different shades of blue.
- highlighting = "cyl" means that the shading will differentiate the categories in the cyl variable.
- highlighting\_fill = cols applies the cols color palette to the shading.

## **Multivariate Categorical Variables**

#### Three Way Relationship

• table()

```
table(cyl
         , gear
          , am)
, , am = 0
   gear
cyl 3
         4
            5
     1
         2
            0
     2
         2
  6
            0
  8 12
         0
            0
    am = 1
   gear
cyl
     3
         4
            5
         6
            2
  4
     0
  6
     0
         2
            1
  8
     0
         0
            2
```

In this example, a three-way contingency table of cyl, gear, and am is created using the table() function. The frequency of each grouping of categories is displayed in the table that results. One vehicle has four cylinders, three gears, and an automatic transmission, whereas eight vehicles have four cylinders, four gears, and manual transmissions. The resulting table, which has a two-dimensional table for each level of the am variable, is three-dimensional.

```
• xtabs()
  xtabs(~ cyl + gear + am
         , data = mtcars)
, , am = 0
   gear
           5
     3
        4
cyl
  4
     1
        2
           0
     2
        2
  6
            0
        0
  8 12
, , am = 1
```

```
gear

cyl 3 4 5

4 0 6 2

6 0 2 1

8 0 0 2
```

In this example, a three-way contingency table of cyl, gear, and am is created using the xtabs() function. The frequency of each grouping of categories is displayed in the table that results. One vehicle has four cylinders, three gears, and an automatic transmission, whereas eight vehicles have four cylinders, four gears, and manual transmissions. The resulting table, which has a two-dimensional table for each level of the am variable, is three-dimensional. The output table matches the one created by the table() function used in the preceding example exactly.

```
• ftable()
  ftable(gear + cyl ~ am
           , data = mtcars)
   cyl
                     4
                        6
am
0
              2 12
                     2
                        2
                            0
                               0
                                   0
                                      0
                     6
                        2
             0
                 0
                            0
                               2
1
                                   1
```

In this example, a three-way contingency table of gear, cyl, and am is created using the ftable() function. The frequency of each grouping of categories is displayed in the table that results. One car has four cylinders, three gears, and an automatic gearbox, whereas there are eight cars with four cylinders, four speeds, and manual transmissions. One table exists for each level of the am variable, resulting in a two-dimensional table. Similar to the table created by the xtabs() function used in the preceding example, the output table is produced.

#### Four Way Relationship

```
am
         cyl
                  6
                             6 8
gear vs
     0
3
               0
                  0 12
                         0
                                 0
               1
                  2
                          0
      1
                             0
4
      0
               0
                  0
                      0
                         0
                             2
                                 0
                  2
               2
                                 0
5
               0
                      0
                         1
      1
                  0
                      0
                             0
```

In this example, we establish a four-way contingency table containing am, cyl, gear, and vs using the ftable() function. The frequency of each grouping of categories is displayed in the table that results. There are two vehicles with a 6-cylinder, 3-gear, automatic transmission, and inline engine, for instance, and three vehicles with four cylinders. The resulting table, which has two two-dimensional tables for each level of the am variable, is four-dimensional.

#### Visualization of MULTIVARIATE Categorical Variables

```
# Load the mtcars dataset
data(mtcars)

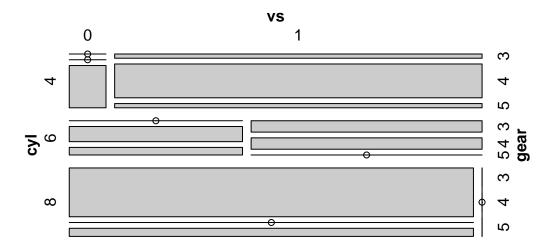
# Install and load the vcd package (if it's not already installed)
install.packages("vcd")

Installing package into '/cloud/lib/x86_64-pc-linux-gnu-library/4.3'
(as 'lib' is unspecified)

library(vcd)

# Create a mosaic plot of mpg (miles per gallon) vs. vs (engine shape)
mosaic(~ cyl + vs + gear, data = mtcars, main = "Mosaic Plot of MPG vs. VS")
```

## Mosaic Plot of MPG vs. VS



## References

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