Aryo Karel M   AFL 2 - Techniques of Differentiation

1.) a.) $y_0 > x_0^2 \rightarrow$ titik $x_0\ y_0$ berada di atas kurva $y = x^2$ karena garis singgung cuma menyentuh kurva di satu titik, tidak akan ada garis singgung yang melalui $x_0\ y_0$

b.) $y_0 = x_0^2 \rightarrow$ titik $x_0 y_0$ terletak tepat pada kurva $y = x^2$ karena garis singgung menyentuh kurva pada titik tersebut, maka akan ada satu garis singgung yang melalui $x_0 y_0$. untuk mencari persamaan garis singgung ini, kita perlu menentukan kemiringannya.

↳ turunan $y = x^2$ terhadap x adalah $\frac{dy}{dx} = 2x \rightarrow x = x_0 \rightarrow m = 2x_0$

↳ $y - y_0 = m(x - x_0)$

$y - y_0 = 2x_0(x - x_0)$

$y - y_0 = 2x_0^2 - 2x_0^3$ ← satu garis singgung

c.) $y_0 < x_0^2 \rightarrow$ titik $x_0 y_0$ terletak di bawah kurva $y = x^2$ Tidak ada garis singgung yang melalui $x_0 y_0$ karena titik tersebut terletak di bawah kurva

3.) $f'(c) = \frac{f(5) - f(1)}{5-1} = \frac{f(5) - 2}{4}$ | $f'(c) \geq \frac{3}{2} \rightarrow \frac{f(5) - 2}{4} \geq \frac{3}{2} \rightarrow f(5) - 2 \geq 6$

$f(5) \geq 8$

4.) $f'(c) = \frac{f(3) - f(0)}{3-0} \rightarrow \frac{f(3) + 4}{3}$ | $f'(c) \geq 3 \rightarrow \frac{f(3) + 4}{3} \geq 3 \rightarrow f(3) + 4 \geq 9$

$f(3) \geq 5$

5.) a.) $f'(c) = \frac{f(4) - f(1)}{4-1} \rightarrow \frac{f(4) - 6}{3}$ | $f'(c) \leq -2 \rightarrow \frac{f(4) - 6}{3} \leq -2$

$f(4) - 6 \leq -6$

b.) $f'(x) = \frac{f(1) - f(0)}{1-0} \rightarrow \frac{6 - f(0)}{1}$ | $f'(x) \leq -2 \rightarrow \frac{6 - f(0)}{1} \leq -2$

$6 - f(0) \leq -2$

$f(0) \geq 8$

6.) $g'(x) = 1 - \cos(x)$ | $g'(x) \geq 0$

↳ $g(0) = 0 - \sin(0) = 0$ dan $g(x)$ monoton meningkat $\rightarrow g(x) \geq 0$

($\sin(x) \leq x$ untuk semua $x \geq 0$)

7.) $f(x)' = x - (1 - \cos x) \rightarrow f'(x) = 1 + \sin x$

$-1 \leq \sin x \leq 1$ untuk semua $x \rightarrow 0 \leq 1 + \sin x \leq 2$

$h(x) \geq h(0) = 0$

$x - (1 - \cos x) \geq 0 \rightarrow 1 - \cos x \leq x$

8.) $\frac{d}{dx}(x^2) = \frac{d}{dy}\left(\frac{1-y^2}{1+y^2}\right)$

$2x\frac{dx}{dy} = \frac{(1-(y^2)(-2y)-(1-y^2)(2y)}{(1+y^2)^2}$

$2x\frac{dx}{dy} = \frac{-2y-2y^3+2y+2y^2}{(1+y^2)^2}$

$2x\frac{dx}{dy} = \frac{-4y}{(1+y^2)^2}$

$\frac{dx}{dy} = -\frac{2y}{x(1+y^2)}$

$\left(\frac{dx}{dy}\right)^2 = \left(\frac{-2y}{x(1-y)y^2}\right)^2 = \frac{4y^2}{x^2(1+y^2)^2}$

$1-x^4 = 1-\left(\frac{1-y^2}{1+y^2}\right)^2$

$= \frac{(1+y^2)^2-(1-y^2)^2}{(1+y^2)^2}$

$= \frac{4y^2}{(1+y^2)^2}$

$\frac{\frac{1-x^4}{1-y^4} = \frac{4y^2}{(1+y^2)^2}}{\frac{1}{2}-y^4} \rightarrow \frac{4y^2}{(1+y^2)^2(1-y^4)}$

$= \frac{4y^2}{(1+y^2)^2(1-y^2)(1+y^2)}$

$= \frac{4y^2}{(1+y^2)^4}$

9.) $x^2+y^2=1$

$\frac{d}{dx}(x^2+y^2) = \frac{d}{dx}$

$2x+2y\frac{dy}{dx}=0$

$2x+2yy'=0$

$y' = \frac{-x}{y} \rightarrow \frac{d}{dx}\left(-\frac{x}{y}\right) = -\frac{d}{dx}\left(\frac{x}{y}\right)$

$= -\left(\frac{y'-x\frac{dy}{dx}}{y^2}\right)$

$= -\left(\frac{y+x\cdot\frac{x}{y}}{y^2}\right) = -\left(\frac{y+\frac{x^2}{y}}{y^2}\right) = -\left(\frac{y^2+x^2}{y^3}\right)$

$= y'' = -\left(\frac{1}{y^3}\right)$

$\frac{d}{dx}\left(-\frac{1}{y^3}\right) = -\frac{d}{dx}\left(\frac{1}{y^3}\right) = \left(\frac{-3\cdot y'\cdot y^{-4}}{1}\right) = -(3(-\frac{x}{y})\cdot y^{-4})$

$= -(-3\cdot\frac{-x}{y}\cdot\frac{1}{y^4}) = y''' = -\left(\frac{3x}{y^5}\right)$

10.) a.) $3x^2+3y^2\frac{dx}{dy} = 9y+9x\frac{dy}{dx}$

$3x^2+3y^2y' = 9y+9xy'$

$3xy^2y'-9xy' = 9y-3x^2$

$y'(3y^2-9x) = 9y-3x^2$

$y' = \frac{9y-3x^2}{3y^2-9x} = \frac{3(3y-x^2)}{3(y^2-3x)} = \frac{3y-x^2}{y^2-3x}$

b.) $y' = \frac{3y-x^2}{y^2-3x}$

$y'' = \frac{d}{dx}\left(\frac{3y-x^2}{y^2-3x}\right)$

11.) $y-y_0 = f'(x_0)(x-x_0)$

$y=0$

$0-y_0 = f'(x_0)(x-x_0)$] x

$x = x_0 - \frac{y_0}{f'(x_0)}$

$x=0$

$y-y_0 = f'(x_0)(0-x_0)$] y

$y = y_0 - x_0 f'(x_0)(0-x_0)$

$y = y_0 - x_0 f'(x_0)$

$x_0 = 2$

$y_0 = 3$

$x^2-4 = k(y-3)$