Multivatiate Calculus, Quiz 5 worksheet (section-13.2)

1. If $r(t) \neq 0$, show that $\frac{d}{dt}|r(t)| = \frac{1}{|r'(t)|} r(t) \cdot r'(t)$.

2. If $u =< \sin(t), \cos(t), t >$ and $v(t) =< t, \cos(t), \sin(t) >$, then find (a)

$$\frac{d}{dt}[u(t) \cdot v(t)]$$

(b)

$$\frac{d}{dt}[u(t) \times v(t)]$$

3. Find the unit tangent vector $\mathbf{T(t)}$ at the given point:

$$r(t) = \sin(t)\mathbf{i} + 5t\mathbf{j} + \cos(t)\mathbf{k}, (0, 0, 1)$$

4. Find the derivative of the following vector valued function:

(a)

$$r(t) =< t^2, \cos(t^2), \sin^2(t) >$$

(b)

$$r(t) = \left\langle \sqrt{t-2}, 3, \frac{1}{t^2} \right\rangle$$

5. Evaluate the integral:

(a)

$$\int_0^1 \left(\frac{1}{t+1}\mathbf{i} + \frac{1}{t^2+1}\mathbf{j} + \frac{t}{t^2+1}\mathbf{k} \right) \, dt$$

(b)

$$\int_0^{\frac{\pi}{4}} \left(\sec(t)\tan(t)\mathbf{i} + t\cos(2t)\mathbf{j} + \sin^2(2t)\cos(2t)\mathbf{k} \right) \, dt$$

(c)

$$\int \left(te^{2t}\mathbf{i} + \frac{t}{1-t}\mathbf{j} + \frac{1}{\sqrt{1-t^2}}\mathbf{k} \right) \, dt$$

(d)

$$\int \left(\frac{1}{1+t^2}\mathbf{i} + te^{t^2}\mathbf{j} + \sqrt{t}\mathbf{k} \right) \, dt$$

6. Find $r(t)$ if $r'(t) = t\mathbf{i} + e^t\mathbf{j} + te^t\mathbf{k}$ and $r(0) = \mathbf{i} + \mathbf{j} + \mathbf{k}$.