Creating a model to optimize the routes of public transport

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Introduction I

- What is Vehicle Routing Problem (VRP)?
- What is No-Depot Vehicle Routing Problem with Multiple Visits (NDVRPMV)?
- Novelty:
  - mathematically model of NDVRPMV,
  - apply real work data.
- Practical Relevance: economic improvements based on logistic optimization in transportation.
- Research Relevance:
  - formulation of NDVRPMV,
  - comparison of accuracy and performance of different solving methods,
  - this topic studied in [1] and [2]
- Research object: public transportation and its optimization.
- Hypothesizes:

Introduction II

- Number of nodes in a graph influences more on the running time of the algorithm compared to the number of routes.
- The integration of a visualization application for all routes will enhance the decision-making process and improve the understanding of the VRP solutions.
- The mathematical model proposed for the NDVRPMV will effectively capture the key constraints and objectives of the problem, leading to accurate and practical solutions.
- The proposed solution methodology, visualization, and use of real-world data will showcase the research's scalability and applicability to real-world scenarios, providing insights for public transportation industries.

- Goal: Create a mathematical model for NDVRPMV and compare the performance and the accuracy of different solving methods for this problem.
- Methodology: Exploratory Research and Computer Programming.

Introduction III

- Tasks:
  - Research different types of VRP
  - Research different solution methods for VRP
  - Research Linear Programming
  - Research Genetic Algorithm
  - Research Graph Neural Networks
  - Collect real-world data of bus stations in Astana
  - Create a mathematical model for NDVRPMV
  - Solve NDVRPMV using Linear Programming
  - Solve NDVRPMV using Genetic Algorithms
  - Compare the accuracy and performance of those methods

Literature Review: Variants of VRP

1. Classical VRP [3],
2. Capacitated VRP (CVRP) [3],
3. VRP with Time Windows (VRPTW) [3],
4. VRP with Pick-Up and Delivery (VRPPD) [3],
5. VRP with Nodal demand [4],
6. Consistent VRP (ConVRP) [5],
7. Generalized consistent VRP (GenConVRP) [5],
8. Reach VRP (rVRP) [6].

Literature Review: Solution Methods for VRP

1. Linear Programming (LP) or Mixed Integer Programming (MIP) [7],
2. Large Neighborhood Search (LNS) [5],
3. Ant Colony Optimization Algorithm (ACO) [8], [4],
4. Genetic Algorithm (GA) [9],
5. Graph Neural Networks (GNN) [10].

Data Acquisition

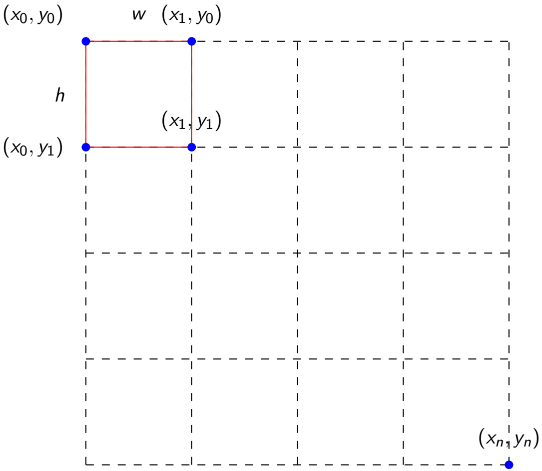

Figure: Rectangle partitioning

Methodology: Linear Programming I

$$\min f = z \tag{1}$$

Subject to:

$$\forall (i,j,k) \in V^2 \times R : x_{ijk} \in \{0,1\}, \tag{2}$$

$$\forall (i,j) \in V \times (V \setminus \{i\}) : \sum_{k=1}^{m} x_{ijk} \geq 1, \tag{3}$$

$$\forall (k,i) \in R \times V : x_{iik} = 0, \tag{4}$$

$$\forall k \in R : \sum_{i \in V} x'_{ki} \geq 1, \tag{5}$$

$$\forall (k,i) \in R \times V : x'_{ki} \leq 1, \tag{6}$$

$$\forall (i,j,k) \in V^2 \times R : x_{ijk} - x_{jik} = 0, \tag{7}$$

$$\forall k \in R : z - \sum_{i=1}^{n} \sum_{j=1}^{n} x_{ijk} \cdot d_{ij} \geq 0, \tag{8}$$

Methodology: Linear Programming II

$$\min f = \sum_{i \in V} \sum_{j \in V} x_{ij} \cdot d_{ij} = X \odot D, \tag{9}$$

Subject to:

$$\forall i \in V : x_{ii} = 0, \tag{10}$$

$$\forall (i, j) \in V^2 : x_{ij} - x_{ji} = 0, \tag{11}$$

$$\forall i \in V : \sum_{j \in V} x_{ij} = 2, \tag{12}$$

where $V = \{1, 2, \ldots, n\}$ and $R = \{1, 2, \ldots, m\}$.

Methodology: Linear Programming III

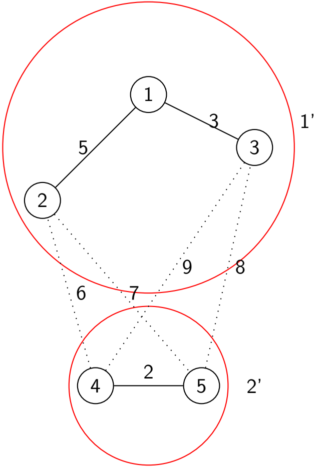

Figure: Component connection example

$$\forall (i,j) \in N^2 : d'_{ij} = min\{d_{i_s j_s}, d_{i_s j_e}, d_{i_e j_s}, d_{i_e j_e}\}, \qquad (13)$$

Methodology: Genetic Algorithms

Fitness function:

$$f = \frac{100}{\max_{k \in R}\{\sum_{i=1}^{n_k} d_{(x_{ki})(x_{k(i+1)})}\}} \quad (14)$$

Selection:

- Tournament selection.
- Biased random selection.

Crossover:

- Crossover over the individual.
- Crossover over the particular route.

Mutation:

- Swapping mutation.
- Rotation mutation.
- Removal mutation.
- Addition mutation.

Visualization I

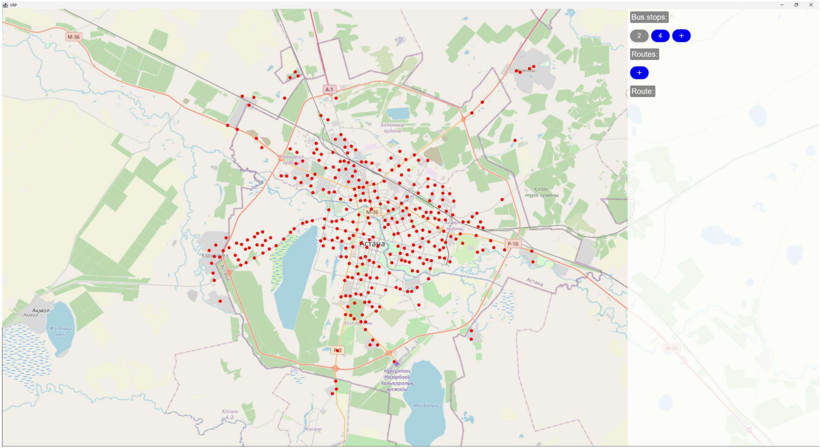

Figure: Map of Astana

Visualization II

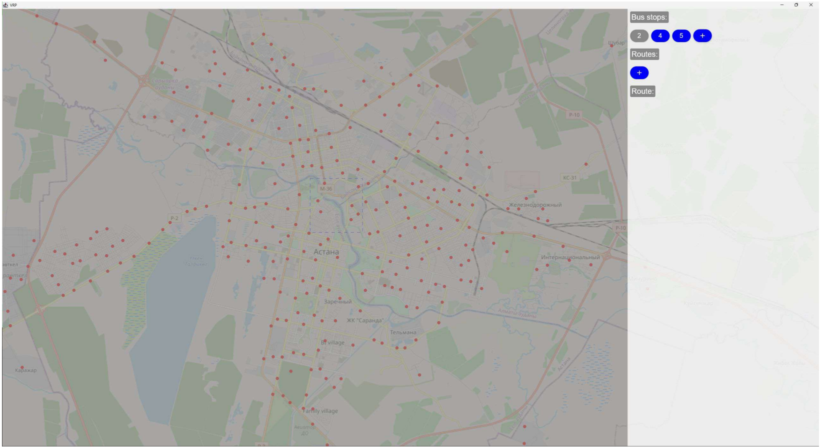

Figure: Selection of bus stops on a map

Visualization III

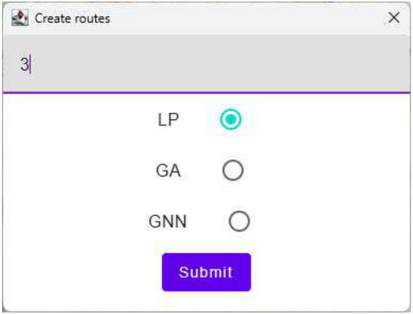

Figure: Route generation dialog

Visualization IV

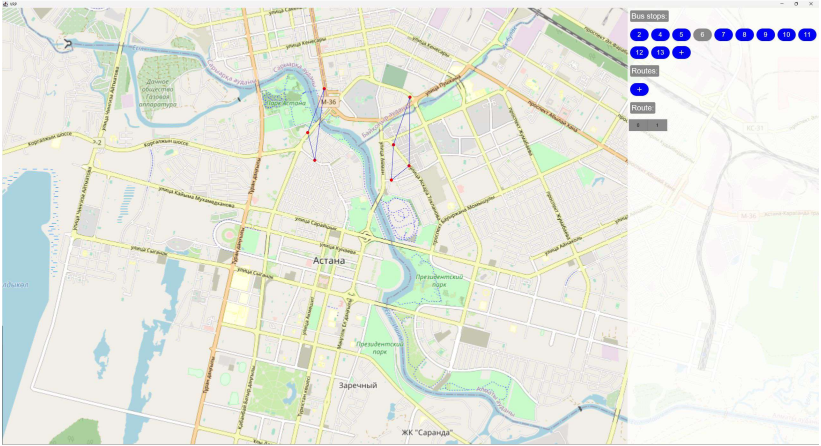

Figure: Applying LP for VRP with 2 routes

Visualization V

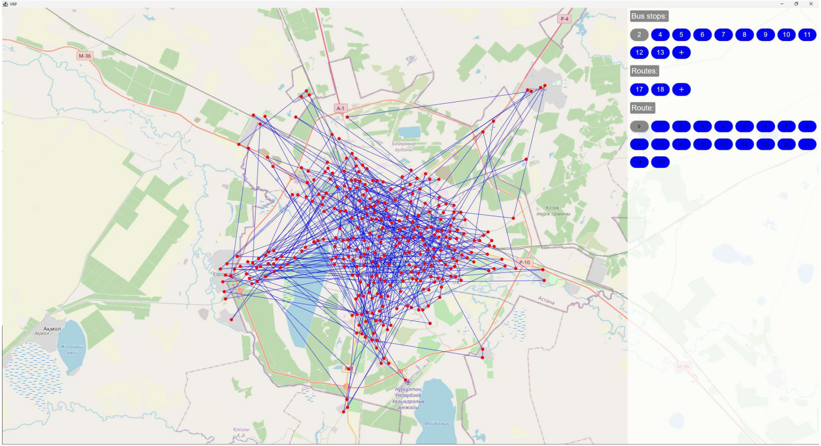

Figure: Applying genetic algorithm with 100 generations for whole Astana

Visualization VI

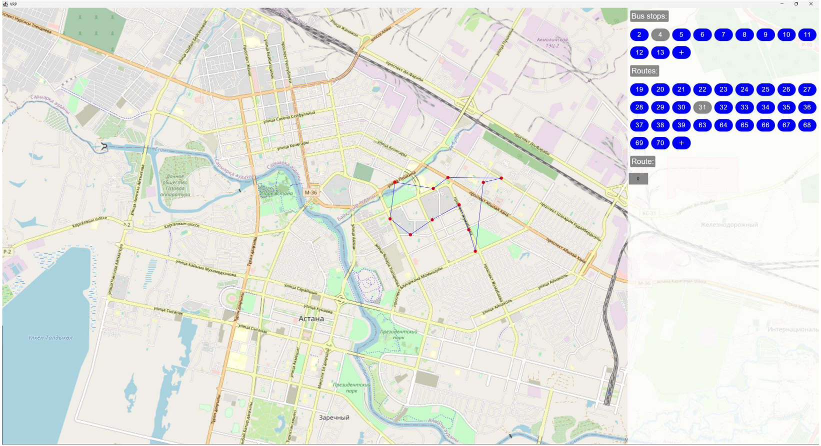

Figure: Applying LP for VRP with 1 route

Visualization VII

Figure: Applying GA for VRP with 1 route

Results I

The time complexity of the LP approach can be approximated because the time complexity of the simplex method is exponential and the time complexity of constraints generation is cubic:

$$R(n, m) = a_1^{n^2 \cdot m} \cdot a_2^{n^2} \cdot a_3^{nm} \cdot a_4^{m} + a_5 n^2 \cdot m + a_6 n^2 + a_7 nm + a_8 m \quad (15)$$

$$R(n, m) = a_1^{n^2 \cdot m} \cdot a_2^{n^2} \cdot a_3^{nm} \cdot a_4^{m} \quad (16)$$

$$3600 = 0.997^{n^2 \cdot m} \cdot 1.037^{n^2} \cdot 1.019^{nm} \cdot 0.878^{m} \quad (17)$$

Maximizing this equation we will get (83.0684916980356, 12.5663037920871). However, this conclusion is flawed because it considers execution on a single machine with a small sample.

Results II

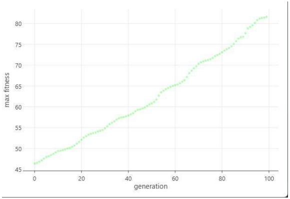

Figure: Max fitness with respect to the number of generations

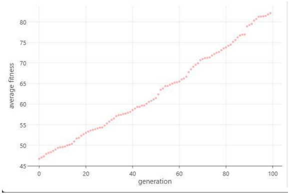

Figure: Average fitness with respect to the number of generations

Results III

The performance of the genetic algorithm is significantly better because it has polynomial time complexity $O(g \cdot p \cdot (p + m \cdot n))$, where g is the number of generations, p is the population size, m is the number of routes and n is the number of nodes. The accuracy of the genetic algorithm is satisfactory, but it will be preferable to use the LP solution for small graphs. The fitness function grows linearly for medium graphs that show that each generation is important in the optimization process.

Future works

- Apply GNN [11].
- Apply Reinforcement Learning [12, 13].
- Apply Covariance Matrix Adaptation Evolution Strategy [14].
- Apply ACO.
- Compare the accuracies of methods discussed above.
- Find more precise different ways to compute the distance between bus stops.
- Improve the UI of the application.

Conclusion I

1. Various types of VRP had been studied.
2. Multiple solution methods were discussed.
3. Real-world data was collected.
4. Mathematical model for NDVRPMV had been formulated.
5. LP and GA had been researched.
6. NDVRPMV had been solved using LP and GA.
7. Performance and accuracy were analyzed of LP and GA in the context of NBVRPMV.
8. Software was created to conveniently show the results of algorithms in use.
9. The performance analysis confirmed that the number of nodes significantly influences the performance of LP and GA algorithms in solving optimization problems.

Conclusion II

10. The choice between GA and LP depends on the specific use case, with GA being more suitable for fast solutions and processing large graphs, while LP is a better choice for other scenarios.
11. The developed mathematical model of NDVRPMV effectively represents the constraints and objective function, providing a valuable tool for analyzing and solving instances of the problem.
12. Real-world data and applications derived from the research can be applied to practical situations, as they accurately represent real-world scenarios and can be extended beyond the original scope of the study.
13. The developed software improves understanding of VRP solutions by providing a visual representation and enhancing usability, allowing researchers and practitioners to analyze and explore different VRP solution methods easily.

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