MMDM Lab №1

Our names

March 29, 2024

1 Problem description

The aim of problem is to find a set of minimum total cost routes for a capacitated vehicles (couriers). Each courier starts and ends the route at the flower market and has to serve a set of cities with following constraints:

1. Each city needs only certain number of flowers
2. Couriers are paid for the distance travelled
3. The total demand of route does not exceed the capacity of the courier

Let's try to formalize our problem.

- N is the set of cities with $N = \{1, 2, 3, ..., n\}$
- M is the set of courier with $M = \{1, 2, 3, ..., m\}$
- q_v is the capacity of the courier v
- d_i is the demand of the city i
- c_{ij} is the distance between city i and city j
- y_{iv} is the amount of cargo delivered to client i by courier v

Our goal is to minimize the total cost (distance):

$$\min_x \sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{v=1}^{m} c_{ij} x_{ijv} \tag{1}$$

$$x_{ijv} \in \{0, 1\}, \tag{2}$$

1 - courier v visited city i, 0 - otherwise

$$\sum_{i=1}^{n}\sum_{v=1}^{m} x_{ijv} = 1, \qquad \forall j \in \{1, 2, ..., n\} \tag{3}$$

$$\sum_{i=0}^{n} x_{izv} = \sum_{j=0}^{n} x_{zjv}, \qquad \forall z \in \{0, ..., n\}, \forall v \in \{1, ..., m\} \tag{4}$$

$$y_{iv} \leq d_i \sum_{j=1}^{n} x_{ijv}, \qquad \forall i \in \{1, ..., n\}, \forall v \in \{1, ..., m\} \tag{5}$$

$$\sum_{v=1}^{m} y_{iv} = d_i, \qquad \forall i \in \{1, ..., n\} \tag{6}$$

$$\sum_{i=1}^{n} y_{iv} \leq q_v, \qquad \forall v \in \{1, ..., n\} \tag{7}$$

The function (1) minimizes the total delivery cost (distance). The claim (3) ensures that each city is visited by exactly one courier and the limitation (4) requires that every courier can leave the flower store only once, and the number of couriers visited every city and returning to the store is equal to the number of the couriers leaving. Constraints (5) and (6) guarantee that the amount of cargo delivered by courier is equal to city demand. And the last requirement (7) ensures that the courier can carry only certain number of cargo (flowers).

2 Algorithm

1. First of all, we generate initial population $P = \{s_0, s_1, ..., s_k\}$. Here we have k solutions that includes set of the couriers with sequence of the cities they serve.

2. Then we find the best solution $F^* = \min_i f(s_i)$. In our case f is the cost (distance) function with all constraints mentioned above

3. Randomly chose 2 parents s_1 and s_2 from population and apply our crossover operator: $child = C(s_1, s_2)$

4. Then we apply mutation operator in order to expand the search area and preserve diversity in the population: $child' = M(child)$

5. If $f(child') < F^*$, we update F^*.

6. Finally, we append *child* to our population and remove the worst solution from it.

7. We repeat step 3 to step 6 t times to reach a sufficient number of iterations to find the best solution.