Mandatory Assignment 2

Asbjørn Fyhn & Emil Beckett Kolko

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Introduction

Exercise 1

The CRSP Monthly dataset contains both observations before 1962 and after 2020. We remove those observations such that the dataset only contains data from 1962-2020. Thereafter, we only keep stocks that have exactly 708 observations of excess return. This ensures that there are no stocks with interrupted observations in our dataset, as there is exactly 708 months between January 1962 and December 2020. Our investment universe now consists of 119 different stocks with an average monthly excess return of 0.77%

Exercise 2 The portfolio choice problem for a transactions-cost adjusted certainty equivalent maximization with risk aversion parameter γ is given by

$\omega^*_{t+1} := \arg\max \left(\hat{\omega}'\mu - \nu_t(\omega, \omega_{t^+}, \beta) - \frac{\gamma}{2}\omega'\hat{\Sigma}\omega\right)$

Where $\omega \in \mathbb{R}^N$, $\iota'\omega = 1$

In the mandatory assignment the proposed transaction costs are specified as

$TC(\omega, \omega_{t^+}) = \lambda(\omega - \omega_{t^+})'\Sigma(\omega - \omega_{t^+})$

To follow the proofs presented in Hautsch & Voigt (2019) we define $\lambda \equiv \frac{\beta}{2}$ where $\beta > 0$ is just a cost parameter like λ.

The optimal portfolio thus takes the form

$\omega^*_{t+1} := \arg\max \left(\hat{\omega}'\mu - \frac{\beta}{2}(\omega - \omega_{t^+})'\Sigma(\omega - \omega_{t^+}) - \frac{\gamma}{2}\omega'\hat{\Sigma}\omega\right) = \arg\max\ \omega'\mu^* - \frac{\gamma}{2}\omega'\Sigma^*\omega$

Where

$\Sigma^* = \left(1 + \frac{\beta}{\gamma}\right)\Sigma$

And

$\mu^* = \mu + \beta\Sigma\omega_{t^+}$

With these new return parameters, we can derive a closed-form solution for the mean-variance efficient portfolio. We compute the mean-variance efficient portfolio by solving for γ:

$\omega^*_{t+1} = \frac{1}{\gamma}\left(\Sigma^{*-1} - \frac{1}{\iota'\Sigma^{*-1}\iota}\Sigma^{*-1}\iota\iota'\Sigma^{*-1}\right)\mu^* + \frac{1}{\iota'\Sigma^{*-1}\iota}\Sigma^{*-1}\iota$

$= \frac{1}{\gamma+\beta}\left(\Sigma^{-1} - \frac{1}{\iota'\Sigma^{-1}\iota}\Sigma^{-1}\iota\iota'\Sigma^{-1}\right)(\mu + \beta\Sigma\omega_{t^+}) + \omega^{mvp}$

$= \omega_{t+1} + \frac{\beta}{\gamma+\beta}(\omega_{t^+} - \omega^{mvp})$

Where ω_{t+1} is the efficient portfolio without transaction costs and risk aversion parameter $\gamma + \beta$.

$\omega^{mvp} = \frac{1}{\iota'\Sigma^{-1}\iota}\Sigma^{-1}\iota$ is the minimum variance allocation.

We see that the optimal weights are a linear combination of the efficient portfolio without transaction costs and the difference between the minimum variance portfolio and the portfolio before reallocation weights. The weight $\frac{\beta}{\beta+\gamma}$ only depends on the risk aversion and the cost parameter (which is not nescesarily linked to volatility). Thus, the weight $\frac{\beta}{\beta+\gamma}$ is not affected by Σ directly. Only ω_{t+1} ω^{mvp} is affected by Σ directly. Therefore, the effect of making transaction costs proportional to volatility is ambigous and depends on how β and Σ affect each other.