

Mandatory Assignment 2

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Exercise 1 and 2:

When computing the momentum we use the market capitalization as opposed to the stock price. The benefit of doing so, is that our computations are not affected by events that increase or decrease the number of outstanding stocks such as stock issuances or buybacks. If we had used the stock price in our computations as opposed to the market capitalization, such events would have artificially increased or decreased our calculated momentum.

Exercise 3:

The equal-weighted average values of the 12-month momentum (Average_Mom_12) and market capitalization (Average_mktcap) is shown in the table 1 below:

Table 1

Momentum decile	Average momentum	Average market cap
1	-61.82	376.38
2	-35.18	971.25
3	-20.36	1650.68
4	-9.18	2348.25
5	0.44	2972.50
6	10.00	3354.10
7	21.01	3457.84
8	35.78	3092.28
9	61.34	2257.36
10	208.71	1370.94

Furthermore, we present the average excess return and the CAPM alpha for the ten momentum-sorted portfolios in table 2. The excess returns are reported in the “factors_ff3_monthly” dataset and the CAPM alphas are estimated with OLS from the CAPM equation:

$$r_{i,t} - r_{f,t} = \alpha_i + \beta_i(r_{m,t} - r_{f,t}) + \epsilon_{i,t}$$

The null-hypothesis is that alpha is zero and the alternative hypothesis is that alpha is different from zero:

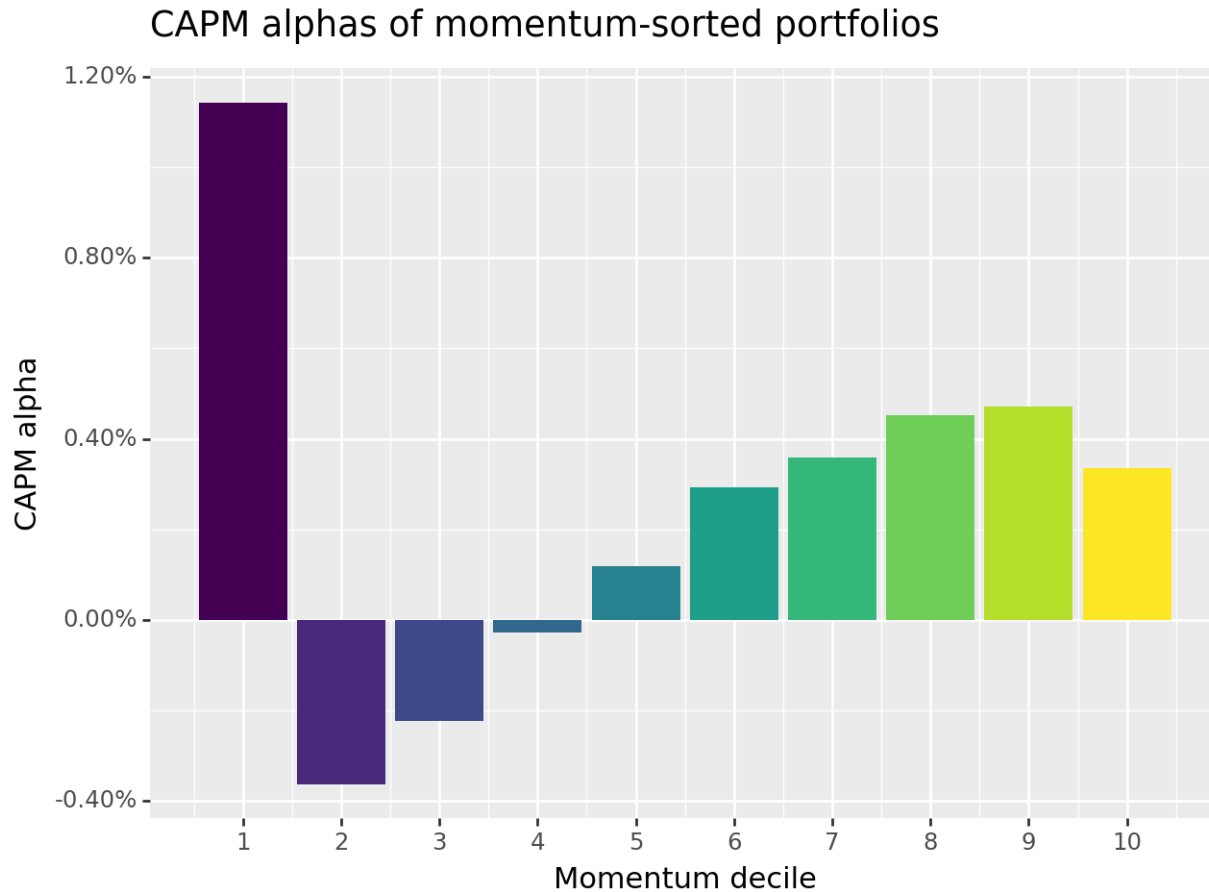
$$H_0 : \alpha_i = 0$$

$$H_A : \alpha_i \neq 0$$

Table 2

Momentum decile	Excess return	Alpha	t-statistic	p-value
1	0.0477	0.0114	18.98	0.00
2	0.0215	-0.0036	-10.57	0.00
3	0.0133	-0.0022	-8.08	0.00
4	0.0126	-0.0003	-1.09	0.27
5	0.0114	0.0012	5.35	0.00
6	0.0096	0.0029	13.57	0.00
7	0.0129	0.0036	16.43	0.00
8	0.0141	0.0045	19.34	0.00
9	0.0157	0.0047	17.72	0.00
10	0.0309	0.0034	9.19	0.00

We see from the t-statistics and p-values, that we can reject the null-hypothesis in all cases except for the portfolio of 4th decile momentum stocks. As such, alpha is significantly different from zero in all other cases. To get a better sense of the distribution of alpha accross the ten portfolios, we present the data in a graph:



We see that the lowest decile stocks offer high alphas compared to the rest of the stocks. We expected the lowest performing stocks to have a negative alpha in accordance with most of the literature. The 2nd and 3rd decile stocks are more aligned with the literature showing negative alphas and the stocks in the 5th decile and up offer positive alphas. These findings represent an imperfection in the market. In the following, we test if we can exploit this imperfection with a momentum strategy that goes long past winners and short past losers.

Specifically, we examine the alpha and beta of a portfolio that shorts the 1st decile portfolio and goes long the 10th decile portfolio. We note that this entails shorting stocks with high alpha to buy stocks with lower alpha, which seems counter-intuitive. We analyze the alpha and beta of this portfolio with t-statistics. Here our hypotheses are:

$$H_0 : \alpha = 0$$

$$H_A : \beta = 0$$

To compute the Newey-West standard errors we must choose a bandwidth based on lags for the estimation. The choice here is rather arbitrary and not data-driven, but we choose a lag length of 12 months, since that is also the lag length of the momentum stocks. Our results are presented below:

OLS Model:

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long_short ~ 1 + mkt_excess
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Coefficients:

	Estimate	Std. Error	t-Statistic	p-Value
Intercept	-0.002	0.003	-0.907	0.364
mkt_excess	-0.217	0.077	-2.805	0.005

Summary statistics:

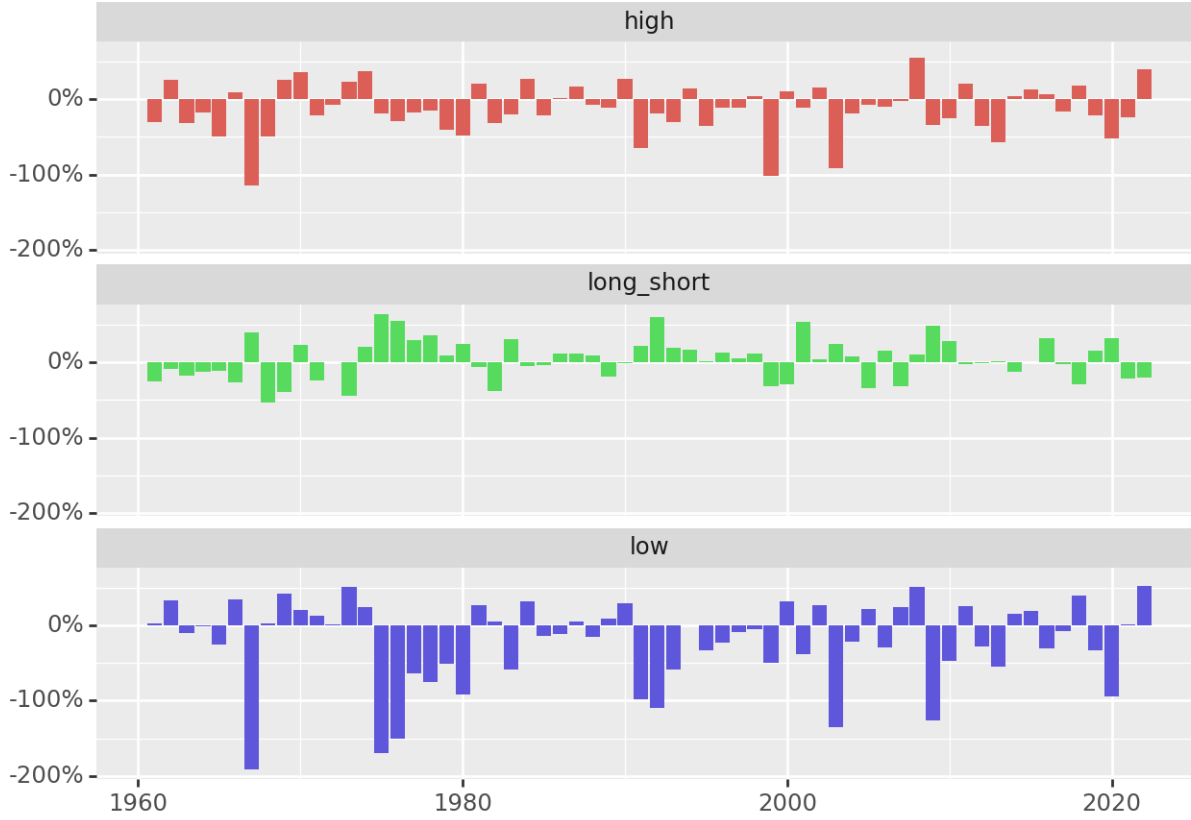
- Number of observations: 736
- R-squared: 0.018, Adjusted R-squared: 0.017
- F-statistic: 7.866 on 1 and 734 DF, p-value: 0.005

We cannot reject the null hypothesis that the returns, α , of the momentum strategy is 0. This finding is in accordance with the CAPM model, that arbitrage strategies are not possible. However, our results deviate from most literature which suggests that it is possible to obtain a positive α with the momentum strategy. The deviation in our results can be explained by the high alphas of portfolio 1, which is not usually observed.

Our results show a significant negative β , which means the momentum (long/short) portfolio is negatively correlated with the market. This type of portfolio is rare and can be used to reduce the market risk of other investments with positive betas. This type of return mimics the return of gold, which historically have a negative β .

To further our understanding of the returns of the portfolio we plot the graph below:

Annual returns of momentum portfolios



The graphs generally show an expected pattern with both positive and negative returns. All portfolios exhibit autocorrelation in the returns, which further the argument for a momentum portfolio. Interestingly, the momentum long/short portfolio seems to deliver the highest returns even though the two other portfolios have positive alphas.

Exercise 4 In this exercise, we want to estimate a model where the excess return of a stock is linear dependent on its own 60 lags. The regression model looks as follows:

$$r_{i,t+1} = \sum_{k=1}^{60} b_k r_{i,t-k} + \sum_{k=1}^{60} c_k r_{i,t-k}^2$$

First, we start by cleaning the dataset meaning that we remove the smallest stocks. More precisely, we remove the stocks with the 5% lowest market capitalization.

Next, we demean the dependent variable $r_{i,t+1}$ by month by month subtracting the mean from series. Thus by construction, the mean of the excess return in each month is 0. At the same time, we standardize the lagged excess return by month by month dividing with the standard deviation. In each month, the standard deviation across the firms is one.

The entire dataset has now been cleaning and transformed in such a way that we are ready to take the analysis further. We will estimate regression coefficient using ridge regression. Thus, we will have to choose the hyperparameter, α .

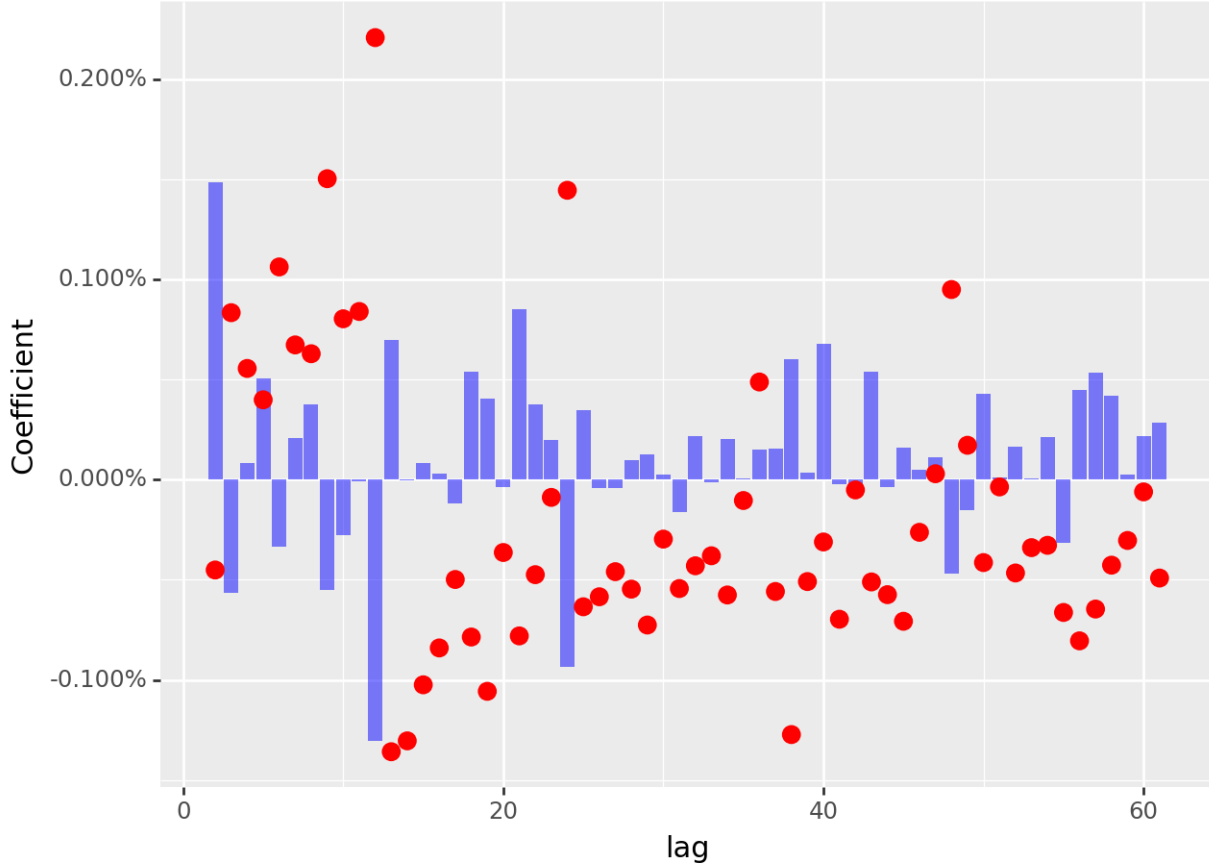
The hyperparameter is chosen by minimizing the estimated mean squared prediction error, MSPE.

$$MSPE = E \left(\frac{1}{T} \sum_{t=1}^T (\hat{y}_t - y_t)^2 \right)$$

Before fine tuning the model, we divide our dataset into a training set for hyperparameter selection. Since MSPE can only be estimated, we employ cross-validation. This involves averaging the MSPE across K data folds over a range of hyperparameters. The hyperparameter that yields the lowest MSPE is then selected.

We tune the model over a grid of alphas that goes from 0.0 to 10000.0 with 100 datafolds.

After tuning the model, we set alpha to 0.007



We notice that the coefficient in front of $r_{i,t-k}^2$ seem to be large for $k = 12, 24, 36, 48$. The corresponding lagged not squared excess return are negative. Meaning that the excess returns are predicted to be lower when the excess return same month in the previous years were positive. Based on this analysis, one might suggest to short those stocks that in a certain month previous years experienced positive excess returns.

Table 3

lag	r	r^2	lag	r	r^2	lag	r	r^2
2	0.001484	-0.000451	22	0.000375	-0.000474	42	-0.000030	-0.000051
3	-0.000565	0.000834	23	0.000197	-0.000089	43	0.000540	-0.000511
4	0.000084	0.000555	24	-0.000937	0.001445	44	-0.000037	-0.000574
5	0.000505	0.000399	25	0.000349	-0.000635	45	0.000159	-0.000707
6	-0.000335	0.001062	26	-0.000041	-0.000585	46	0.000047	-0.000263
7	0.000206	0.000673	27	-0.000043	-0.000459	47	0.000112	0.000029
8	0.000375	0.000629	28	0.000096	-0.000547	48	-0.000468	0.000949
9	-0.000552	0.001502	29	0.000124	-0.000726	49	-0.000153	0.000172
10	-0.000275	0.000803	30	0.000023	-0.000297	50	0.000427	-0.000414
11	-0.000010	0.000839	31	-0.000162	-0.000544	51	0.000013	-0.000036
12	-0.001304	0.002207	32	0.000219	-0.000430	52	0.000163	-0.000466
13	0.000700	-0.001358	33	-0.000014	-0.000380	53	0.000005	-0.000340
14	-0.000003	-0.001304	34	0.000204	-0.000576	54	0.000212	-0.000328
15	0.000081	-0.001024	35	0.000006	-0.000104	55	-0.000317	-0.000663
16	0.000032	-0.000840	36	0.000148	0.000487	56	0.000449	-0.000805
17	-0.000118	-0.000498	37	0.000153	-0.000558	57	0.000535	-0.000646
18	0.000537	-0.000786	38	0.000604	-0.001273	58	0.000417	-0.000427
19	0.000406	-0.001057	39	0.000035	-0.000509	59	0.000026	-0.000304
20	-0.000038	-0.000364	40	0.000679	-0.000311	60	0.000218	-0.000061
21	0.000850	-0.000780	41	-0.000021	-0.000697	61	0.000285	-0.000491