Mandatory Assignment 1

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1 Efficient portfolios and estimation uncertainty

In this analysis, we evaluate the statistical error in portfolio weights by contrasting the plug-in estimation method against the theoretically optimal portfolio weights. Utilizing historical sample moments as a benchmark, we simulate returns for DOW Jones constituents, estimating mean and covariance matrices, and subsequently deriving portfolio weights. Comparing these plug-in estimates with the true optimal weights, we uncover substantial deviations, particularly in scenarios with limited data. The study, in line with Brandt 2010, highlights the challenges of relying on simulation techniques, emphasizing their susceptibility to parameter uncertainty. Through a comprehensive assessment of minimum variance and efficient tangent portfolios, alongside Sharpe ratios, we underscore the significance of acknowledging statistical limitations in real-world portfolio optimization.

2 Exercises

The Capital Asset Pricing Model (CAPM) is a cornerstone in finance, linking expected asset returns to systematic risk. Our focus lies in testing the theoretical CAPM against plug-in estimation methods, emphasizing mean-variance analysis. The efficient frontier, a vital aspect of this analysis, optimally balances risk and return in portfolio construction, offering insights into achieving optimal investment outcomes, based on the true estimates. In the first part of this analysis we wish to calculate the minimum variance- and efficient tangent portfolio and the Sharpe Ratios of the constituents of the DOW Jones index, such that we can plot the theoretical efficient frontier. This we will use as a refererence to compare our simulation results with.

Question 1.

Table 1: Monthly returns for 3 Jan 2000

Ticker	Monthly Returns
AAPL	-0.084
AMGN	-0.076
AMZN	-0.083

Ticker	Monthly Returns
AXP	-0.038
BA	-0.002

Answer: Using the tidyverse and tidyquant packages, stock prices from the DOW Jones constituents can be downloaded. Removing tickers with no continuous trading history leads to the exclusion of removed_symbols, resulting in 27 remaining constituents. Monthly net returns for each constituent on January 3, 2000, are then displayed.

Question 2.

Table 2: Annualized Sharpe Ratios for Dow Jones constituents

Ticker	Sharpe Ratio
AAPL	2.320
AMGN	1.231
AMZN	1.677
AXP	1.057
BA	1.300

Answer: Calculating the sample mean, μ, and the variance-covariance matrix, Σ, the Sharpe-Ratio (SR) is obtained as $SH = \mu/\sqrt{diag(\Sigma)}$, assuming a zero risk-free rate. The stock providing the highest SR is identified as UNH.

Question 3.

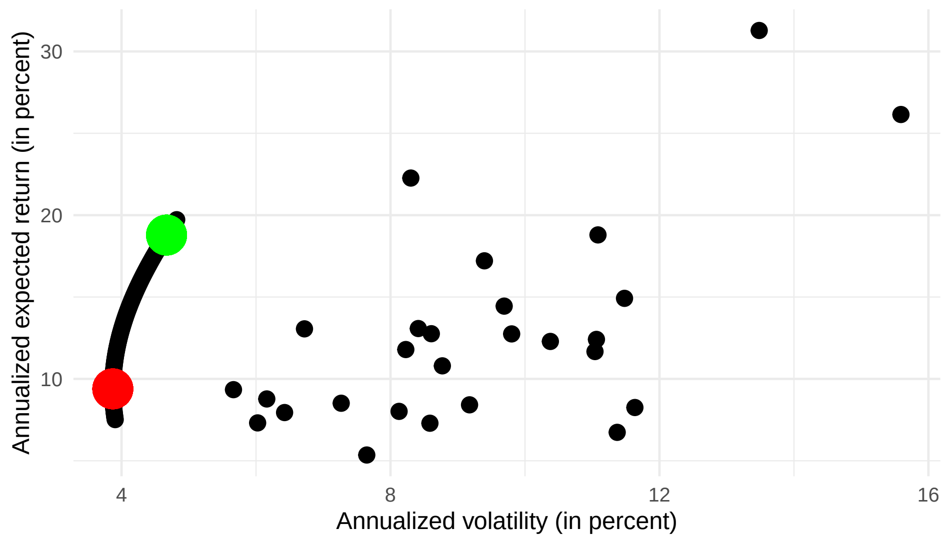
Table 3: Weights for the Dow Jones constituents characterising the efficient frontier

c	AAPL	AMGN	AMZN	AXP
-0.10	0.158	0.06	0.061	-0.085
-0.09	0.157	0.06	0.061	-0.085
-0.08	0.155	0.06	0.060	-0.084
-0.07	0.154	0.06	0.059	-0.084
-0.06	0.153	0.06	0.059	-0.084

Answer The analysis proceeds with the computation of minimum variance portfolio weights, efficient tangency portfolio weights, and the construction of the efficient frontier. The resulting data frame details a sequence of portfolio weights ( ) derived from linear combinations of the minimum variance and efficient tangency portfolios.

Question 3.

Figure 1: Efficient Frontier



Answer Plotting the efficient frontier, the curvature reveals significant diversification rewards, particularly with the minimum variance and efficient tangency portfolios. Notably, these portfolios exhibit returns of mvp_return & eff_return and volatilities of mvp_volatility & eff_volatility, respectively.

Question 4.

Ticker	Efficient tangency portfolio weights
AAPL	0.14512226
AMGN	0.05823478
AMZN	0.05493521
AXP	-0.08168918
BA	0.04655767

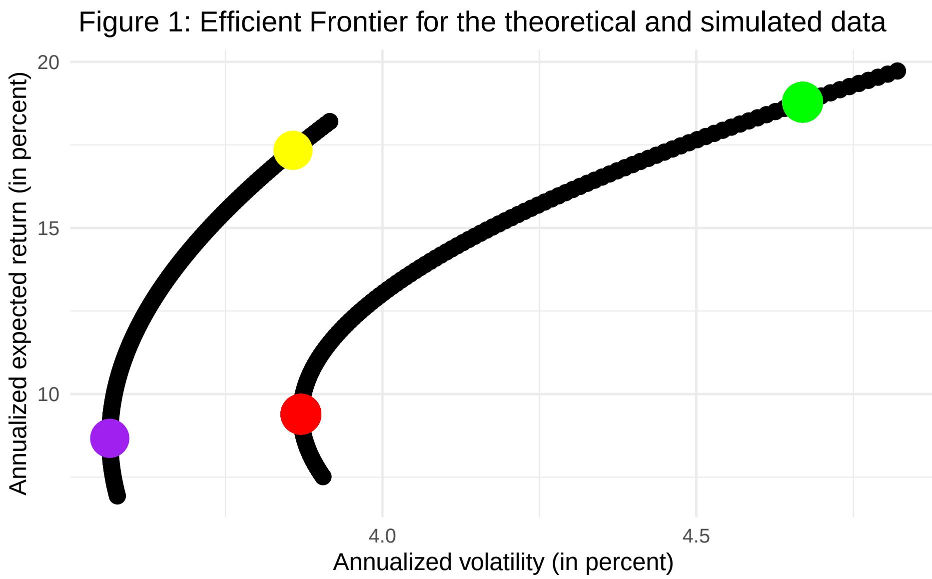
Answer: Efficient tangency portfolio weights, allowing for both long and short positions, are assessed. The portfolio, characterized by a low annualized volatility and a high annualized return of …, respectively, emphasizes the diversification benefits and the risk associated with e.g., the APPLE stock (with the highest weight of …). Issues include the allowance of short-selling and reliance on historical data, since most of the litterature finds evidence of the weak market form hypothesis. The maximum attainable SR, represented by the efficient tangency portfolio, exceeds individual assets' SR. The theoretical benefits of diversification in the market portfolio are thus evident.

Simulation techniques are introduced, involving the function simulate_returns to obtain empirical estimates of minimum variance and efficient tangency portfolios. In line with Brandt 2010, discrepancies between theoretical and empirical estimates arise, with empirical values exhibiting a negative bias and overestimating diversification benefits.

Question 5.

Answer: The usual building bloks of a function sript in R is: i) the name of the function. 2) the inputs the function should use. 3) Lastly, inside the {} sign, what the function should do with these inputs. The function name here is simulate_returns. A very giving name as to what the function is going to do, i.e., simulate returns from a multivariate normal distribution. Next, the inputs are the number of periods to simulate, here set to 200. expected_returns is the return vector of assets, here set to the empirical return vector mu. Finally, covariance_matrix is the variance-covariance matrix, which is set to the outside which we have worked with so far. The function uses MASS::mvrnorm to simulate from a multivariate normal distribution, with n (200) periods, and from the mu and Sigma return- and variance matrices defined outside the function. The output of the function is a matrix with simulated returns in each row, and asset names in each column. **Remember a source here**

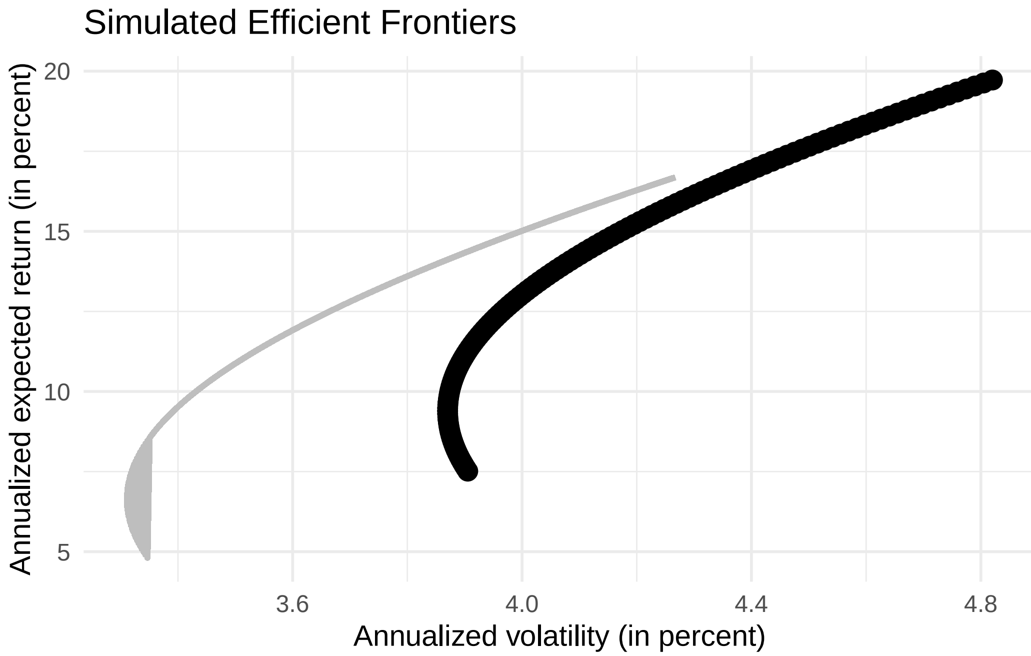
Question 6.



	True Values	Empirical Values
Minimum variance return	9.392	8.667
Minimum variance volatility	3.870	3.566
Efficient tangent return	18.785	17.335
Efficient tangent volatility	4.669	3.858

Answer: In the above figure we see the efficient frontier for the theoretical and empirical values. The red and green dots represent the theoretical efficient tangent portfolio and theoretical minimum variance portfolio, respectively. The yellow and purple dots represent the empirical counterparts, respectively. Empirical efficient frontiers, differing from theoretical estimates, highlight the overestimation of diversification benefits in plug-in estimates. Longer sample periods tend to converge toward true theoretical parameters.

Question 7.



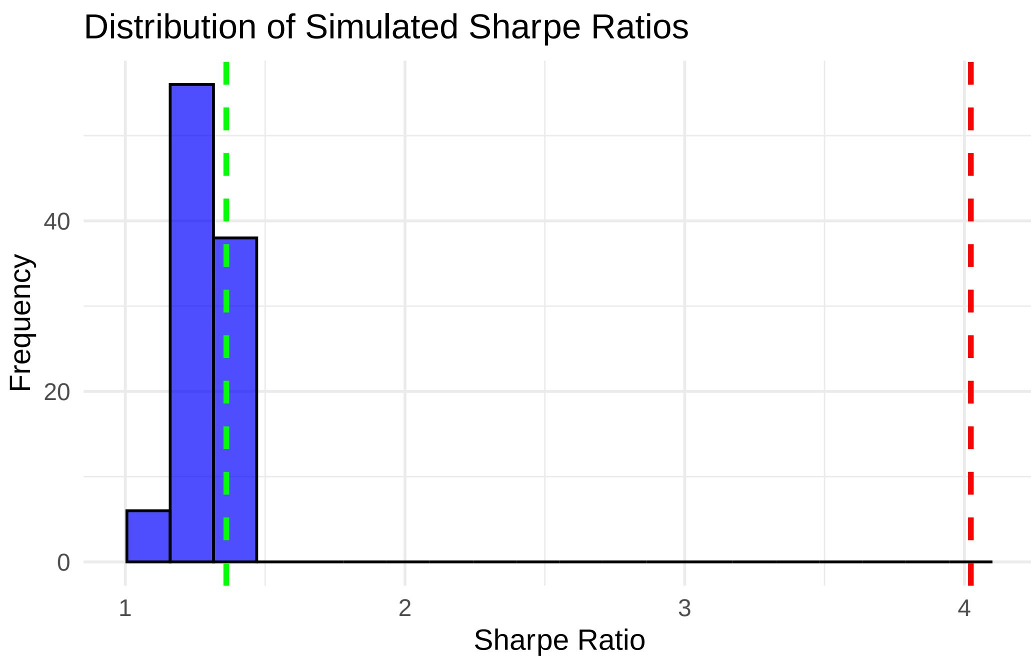
Answer: In the figure above we see that the empirical efficient frontiers seem to differ quite a lot from the theoretical estimates. The main takeaway is that the plug-in estimates seem to overestimate the diversification benefits massively, and thereby provide an inaccurate description of the efficient frontiers and most of the simulated efficient frontiers seem to be centered around the curved part of the efficient frontier around the minimum variance portfolio. Changing the sample periods towards higher periods, i.e., periods = 1000 will result in convergence towards the true theoretical parameters (application of the law of large numbers), which implies that with periods = 10000 for example the plug-in estimated efficient frontiers will all lay on the theoretical efficient frontier.

Question 8.

Ticker	Simulated Sharpe-ratios
AAPL	1.306
AMGN	1.315
AMZN	1.268
AXP	1.274
BA	1.318

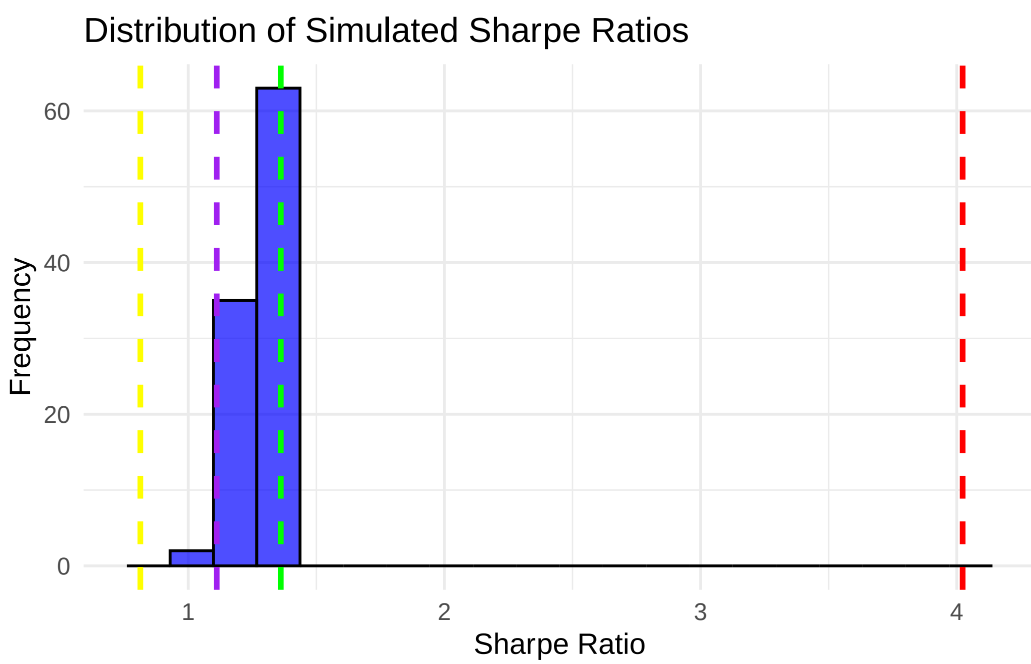
Answer In the figure above we see the simulated sharpe-ratios. Simulated Sharpe ratios appear lower than theoretical values, reflecting limitations in small sample periods. The distribution of SRs centers around 1.3, deviating significantly from theoretical SR.

Question 9.



Answer In the figure above we see the distribution of the SR's. The red dashed line represents the theoretical SR of the efficient tangent portfolio. The green dashed line represents the mean of the theoretical SR´s. The blue boxes represents the distribution of the simulated SR´s. The distribution of SRs centers around 1.3, deviating significantly from theoretical SR. It is centered at the mean of the theoretical SR´s.

Question 10.



Answer: Longer sample periods makes it converge towards the true mean of the sharpe ratio, which is indicated by the green dashed line. We can conclude that assets are NOT normally distributed in small sample periods and we should allow for fatter tails, such that we would get more observations closer to the red dashed line. Two ways to improve upon the plug-in estimates shortfall are i) allowing for no-short-selling, so imposing one extra restiction in the optimization problem, and ii) naive portfolio allocation, such that all the assets have an equal weight. In the figure above we see the SR based on the naive portfolio weights (the yellow dashed line) and the SR based on no-short selling (the purple dashed line).

We find that the two backtesting procedures does not seem to help in solving the plug-in estimates shortfall. In line with (Brandt 2010) we conclude that estimation of smaller moments affects the larger moments and vice-versa. Likewise, we have showed the poor finite sample properties, in that the SR of the plug-in estimates are substantially inferior to the true SR. Lastly, we showed in line with the theoretical argument of Green & Hollifield 1992, that portfolio constraints may actualy hurt the performance of plug-in estimates, which is the case for our analysis. Considering the CAPM's rejection in most literature, the results may stem from both inferior estimation methods and violations of CAPM assumptions. A suggestion is made to revisit the analysis using multifactor models like the APT model.

References