Mandatory Assignment 2

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April 19, 2024

Exercise 1

From the database we read the tables $crsp\_montlhy$ and $factors\_ff3\_monthly$ into two seperate variables. Then we merge these dataframes into one and get:

Table 1: The first 5 rows of data from tidy_finance_*.sqlite database

permno	month	ret_excess	mktcap	mktcap_lag	mkt_excess
10028	1993-03-01	-0.103	6.329	7.032	0.023
10028	1993-04-01	0.386	8.791	6.329	-0.030
10028	1993-05-01	0.198	10.549	8.791	0.029
10028	1993-06-01	-0.136	9.045	10.549	0.003
10028	1993-07-01	0.190	10.784	9.045	-0.003

Where: *permno* is the unique security identifier.

month referes he reference month for the data.

ret_excess is stock's excess return over the risk-free rate for the month.

mktcap consists of the company's market capitalization at month-end.

mktcap_lag is the company's market capitalization from the previous month.

mkt_excess represent the market's excess return over the risk-free rate for the month.

Exercise 2

Since we already have the variable *mktcap_lag* in the dataset we do not have to create it. When making the other variable *mktcap_lag*_12 we simply shift the *mktcap_lag* 12 times. Finally we can create our new variable *mom*_12 which represent the 12-month momentum.

When creating these lag-variables we are left with a lot of NaN rows so we make sure to drop every row which contains these NaN values.

Table 2: A section of the data now with mktcap_lag_12 and mom_12

permno	month	ret_excess	mktcap	mktcap_lag	mkt_excess	mktcap_lag_12	mom_12
10028	1994-03-01	-0.173	13.567	16.350	-0.048	7.032	132.494
10028	1994-04-01	0.126	15.306	13.567	0.007	6.329	114.356
10028	1994-05-01	-0.162	13.577	15.306	0.006	8.791	74.123
10028	1994-06-01	-0.030	13.210	13.577	-0.030	10.549	28.704
10028	1994-07-01	-0.114	11.742	13.210	0.028	9.045	46.049

Since the formula for market capitalization is:

$$MC = SharesOutstanding * StockPrice$$

This means that keeping the Shares outstanding constant, the relative change in price is gonna be the same as the relative change in market capitaliaztion. In the case of a stock split or something else that changes the numbers of shares outstanding the historical data usually gets adjusted aswell.

Exercise 3

We create 10 portfolios breakpoints based on the deciles based of the now sorted variable $Mom^{12}_{i,t}$. We remember to drop all rows with NaN values.

Table 3: Equal-weighted average values

Portfolio nr	Average Momentum	Average mktcap
1	-56.598	337.211
2	-31.888	1060.879
3	-18.345	1795.338
4	-7.836	2463.303
5	1.659	2797.222
6	11.383	3113.262
7	22.668	3187.822
8	38.064	3025.275
9	65.233	2532.454
10	247.424	1353.556

Looking at this table we can see that the portfolios the lowest average momentum and the highest average momentun, consists of stocks which also have the lowest average market capitalization. This could suggest that smaller companies are more volatile compared to larger companies and therefore doesn't have these large drop or raise in the their market capitalization during the 12 month period.

The negative average momentum means that its market capitalization has dropped over the 12 month period. This suggest that the strategy for these portfolios would be to short them.

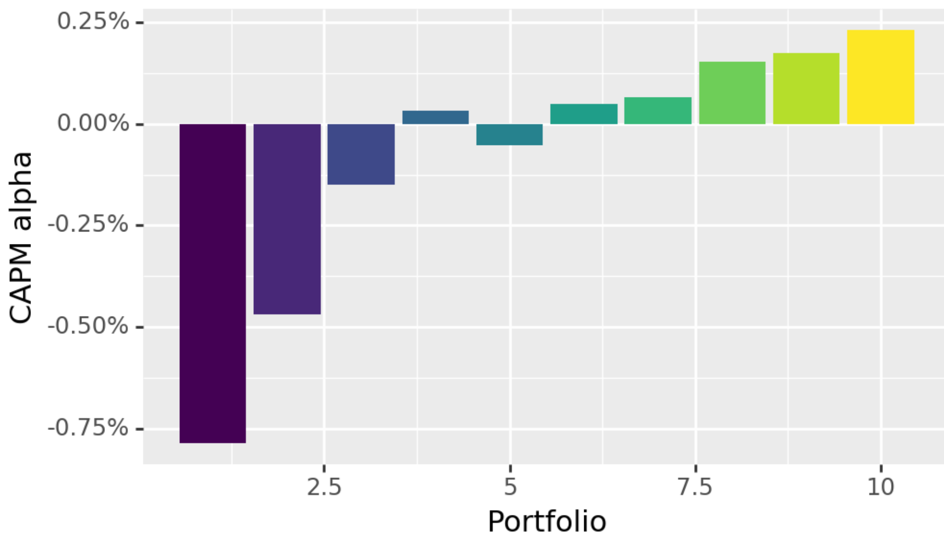


Figure 1: CAPM alphas of momentum-sorted portfolios

Table 4: Momentum-sorted portfolios with CAPM alphas, beta and excess return

Portfolio nr	alpha	beta	excess return
1	-0.0079	1.6060	0.0014
2	-0.0047	1.3198	0.0029
3	-0.0015	1.1293	0.0050
4	0.0003	1.0400	0.0064
5	-0.0005	0.9689	0.0051
6	0.0005	0.9177	0.0058
7	0.0007	0.9349	0.0061
8	0.0015	0.9344	0.0069
9	0.0018	1.0362	0.0077
10	0.0023	1.2271	0.0094

Given table 4 and figure 1, we know that most of the portfolios have a positive alpha, with the exception of the first two deciles, portfolio 1 and 2, which have a negative alpha. This suggests that the higher momentum deciles, portfolio 3 through 10, have provided returns above what would be expected given their market risk as measured by the beta.

The beta values of the portfolios goes from slightly below 1 to above 1 where portfolio 1 showing the highest beta of 1.606. This indicates that the first portfolio, which is the lowest momentum group, is more volatile and has a higher systematic risk than the market. As we move to higher momentum deciles, the beta generally decreases.

Overall all we see that momentun strategy yield the best alpha's when going long on the higher momentum deciles, especially portfolio 10, and shorting the lower deciles portfolio 1 and 2. Taking these portfolios into account we can clearly see that they are not truly market neutral but close. As to whether or not they deliver excess returns, we can clearly see that some of the portfolios does indeed have excess return, but by no means are they 'abnormal'.

Exercise 4

In our analysis, we focus on forecasting future monthly stock returns using a momentum strategy that is generalized to include 60 lags of each stock's own past returns and their squared values. Specifically, the model can be described by the regression equation:

$$r_{i,t+1} = \sum_{k=1}^{60} b_k r_{i,t-k} + \sum_{k=1}^{60} c_k r_{i,t-k}^2$$

where $r_{i,t+1}$ is the future monthly excess return for stock i, b_k and c_k are the coefficients corresponding to the lagged return and the squared lagged return, respectively.

We start of by removing all of the stock that had a market capitalization below the 20% quantile a given month. The aproach simply consist of filtering for the NYSE stocks, and deriving the 20th percentiel of each month, followed by filtering the stock which fulfills the condition of being below the 20% quantile.

We therefor end up with excluding 3231 stocks. Futhermore, we create columns for each the 60 lags: $r_{i,t-2}, ..., r_{i,t-61}$, and similarly for the squared values of each lag: $r_{i,t-2}^2, ..., r_{i,t-61}^2$

To ensure the dependent variable, future monthly returns $r_{i,t+1}$, is centered around zero, we demean it by subtracting the overall mean of all monthly returns from each individual return. This transformation is mathematically represented as:

$$y = r_{i,t+1} - \mu_r$$

where μ_r is the average of all observed monthly returns. This adjustment helps to remove any inherent bias in the data, centering the dependent variable and enhancing the interpretability and stability of the regression analysis.

We standardize predictor variables by demeaning and scaling to unit variance cross-sectionally for each month. Specifically, stock returns $r_{i,t-k}$ and their squares $r_{i,t-k}^2$ are transformed using the formulas:

$$X = \left[\frac{r_{i,t-k} - \bar{\mu}_{r_{t-k}}}{\sigma_{r_{t-k}}}, \frac{r_{i,t-k}^2 - \bar{\mu}_{r_{t-k}^2}}{\sigma_{r_{t-k}^2}^2} \right]$$

This process normalizes the features, ensuring no single predictor dominates due to scale differences, which gives us a more effective regression analysis.

We estimate the coeffecients applying a ridge regression, which is an extension of the traditional OLS method. This approach uses a regularization term to adjust the coefficient estimates, thus enhancing the model's generalizability and stability. The coefficients, are denoted as $\hat{\beta} = [\hat{b}, \hat{c}]$, and are calculated using the formula:

$$\hat{\beta}^{\text{ridge}} = \left(X^T X + \lambda I\right)^{-1} X^T y$$

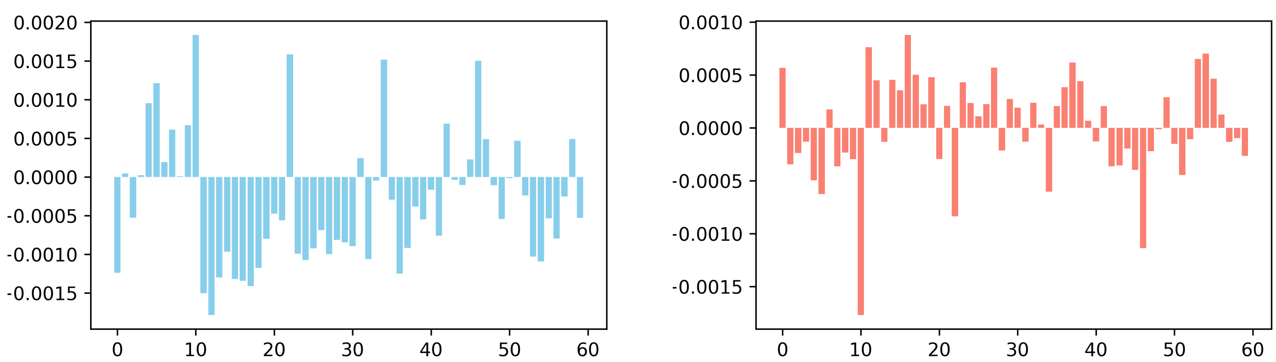
Here, λ acts as the regularization parameter that determines the degree of shrinkage applied to the coefficients. Higher values of λ cause greater shrinkage, which mitigates the risk of overfitting by penalizing the size of the coefficient values.

We determine the optimal λ by assessing the model's performance across different λ values. This ensures that the chosen *lambda* achieves a balance between fitting the historical data and performing well on unseen data. We evaluate the performance of the model by monitoring changes in both the coefficient of determination (R^2) and the mean square error (MSE). The choice of λ is ultimately based on achieving the highest R^2.

Table 5: The scaled R^2 values

Lambda	Scaled R^2
0	13700.395
10	13700.395
100	13700.395
500	13700.391
1000	13700.379
10000	13698.850

By scaling the R^2 values by a factor of 10.000.000, it is clear that the optimal λ for our model is 0. This implies that the regularization term does not enhance the model's performance in our specific case. Given that the model did not improve with increased λ suggests that the model, when fitted using ordinary least squares without regularization, is already well-suited to the data. This indicates that the underlying data is either not prone to overfitting, or the existing features are all relevant and do not introduce significant multicollinearity. Therefore, the simplest model without regularization proves to be the most effective for capturing the dynamics of our data.



(a) Values of b_k for Lambda = 0

(b) Values of c_k for Lambda = 0

Figure 2: Coefficients obtained with lambda=0

Analyzing the coefficients obtained with $\lambda = 0$ provides an insights into the patterns in stock returns. The coefficients b_k, display a mix of positive and negative values. Notably, certain lags show more substantial positive values, which could be indicative of a momentum effect. This effect implies that stocks exhibiting positive returns in these specific past periods are likely to continue performing well. Conversely, the negative coefficients could be suggestive of a short-term reversal, where stocks that performed poorly might experience a bounce-back in the subsequent periods.

The coefficients c_k, associated with the squared returns, show a similar mix of positive and negative values. The presence of these values could point towards a pattern of volatility clustering. Higher absolute values of c_k may imply that stocks with larger fluctuations in past returns, regardless of the direction of those returns, could be expected to show significant movements in the future. This can be interpreted as evidence of a risk premium, where stocks with higher historical volatility are expected to yield higher returns, possibly compensating for the increased level of risk.

Leveraging these insights, a trading strategy can be formulated that incorporates momentum signals from the b_k coefficients and adjusts for risk based on the c_k values. For example, one might construct a portfolio that overweights stocks with the strongest positive b_k values, indicating strong past performance, while also considering the volatility indicated by c_k. This strategy could involve scaling investment sizes based on the magnitude of past returns and their variability, aiming to strike a balance between capitalizing on the momentum effect and managing the inherent risk associated with volatility. The ultimate goal of such a strategy would be to maximize returns while controlling for risk, adapting the positions as new data becomes available and as market conditions evolve.