Mandatory Assignment 2

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Introduction

Exercise 1

The CRSP Monthly dataset contains both observations before 1962 and after 2020. We remove those observations such that the dataset only contains data from 1962-2020. Thereafter, we only keep stocks that have exactly 708 observations of excess return. This ensures that there are no stocks with interrupted observations in our dataset, as there is exactly 708 months between January 1962 and December 2020. Our investment universe now consists of 119 different stocks with an average monthly excess return of 0.77%

Exercise 2 The portfolio choice problem for a transactions-cost adjusted certainty equivalent maximization with risk aversion parameter γ is given by

$\omega^*_{t+1} := \arg\max \left(\hat{\omega}'\mu - \nu_t(\omega, \omega_{t^+}, \beta) - \frac{\gamma}{2}\omega'\hat{\Sigma}\omega\right)$

Where $\omega \in \mathbb{R}^N$, $\iota'\omega = 1$

In the mandatory assignment the proposed transaction costs are specified as

$TC(\omega, \omega_{t^+}) = \lambda(\omega - \omega_{t^+})'\Sigma(\omega - \omega_{t^+})$

To follow the proofs presented in Hautsch & Voigt (2019) we define $\lambda \equiv \frac{\beta}{2}$ where $\beta > 0$ is just a cost parameter like λ.

The optimal portfolio thus takes the form

$\omega^*_{t+1} := \arg\max \left(\hat{\omega}'\mu - \frac{\beta}{2}(\omega - \omega_{t^+})'\Sigma(\omega - \omega_{t^+}) - \frac{\gamma}{2}\omega'\hat{\Sigma}\omega\right) = \arg\max\ \omega'\mu^* - \frac{\gamma}{2}\omega'\Sigma^*\omega$

Where

$\Sigma^* = \left(1 + \frac{\beta}{\gamma}\right)\Sigma$

And

$\mu^* = \mu + \beta\Sigma\omega_{t^+}$

With these new return parameters, we can derive a closed-form solution for the mean-variance efficient portfolio. We compute the mean-variance efficient portfolio by solving for γ:

$\omega^*_{t+1} = \frac{1}{\gamma}\left(\Sigma^{*-1} - \frac{1}{\iota'\Sigma^{*-1}\iota}\Sigma^{*-1}\iota\iota'\Sigma^{*-1}\right)\mu^* + \frac{1}{\iota'\Sigma^{*-1}\iota}\Sigma^{*-1}\iota$

$= \frac{1}{\gamma+\beta}\left(\Sigma^{-1} - \frac{1}{\iota'\Sigma^{-1}\iota}\Sigma^{-1}\iota\iota'\Sigma^{-1}\right)(\mu + \beta\Sigma\omega_{t^+}) + \omega^{mvp}$

$= \omega_{t+1} + \frac{\beta}{\gamma+\beta}(\omega_{t^+} - \omega^{mvp})$

Where ω_{t+1} is the efficient portfolio without transaction costs and risk aversion parameter $\gamma + \beta$.

$\omega^{mvp} = \frac{1}{\iota'\Sigma^{-1}\iota}\Sigma^{-1}\iota$ is the minimum variance allocation.

We see that the optimal weights are a linear combination of the efficient portfolio without transaction costs and the difference between the weights of the minimum variance portfolio and the portfolio before reallocation. The weight $\frac{\beta}{\beta+\gamma}$ only depends on the risk aversion and the cost parameter. Thus, the weight $\frac{\beta}{\beta+\gamma}$ is not affected by Σ. Only ω_{t+1} and ω^{mvp} is affected by Σ. Therefore, the effect of making transaction costs proportional to volatility is ambigous and depends on how β and Σ affect each other.

A simpler case discussed in the lectures is when we model exogenous quadratic transactions costs. Here the effect of higher volatility has a clear effect. Periods with high volatility shifts the optimal portfolio allocation towards the global minimum-variance portfolio. This makes sense as the risk needs to be reduced in high volatility periods.

From a supply/demand point of view however, endogonous transaction costs linked to volatility intuitively makes sense. In a high volatile environment, investors reallocate their portfolio more frequently to maintain optimal portfolio weights. Economic theory suggests that a higher demand must yield a higher price. Therefore, linking transaction costs to volatility makes sense.