Mandatory Assignment 2

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2024-05-10

Introduction

Exercise 1

The CRSP Monthly dataset contains both observations before 1962 and after 2020. We remove those observations such that the dataset only contains data from 1962-2020. Thereafter, we only keep stocks that have exactly 708 observations of excess return. This ensures that there are no stocks with interrupted observations in our dataset, as there is exactly 708 months between January 1962 and December 2020. Our investment universe now consists of 119 different stocks with an average monthly excess return of 0.77%

Exercise 2

Bullet 1 The portfolio choice problem for a transactions-cost adjusted certainty equivalent maximization with risk aversion parameter γ is given by

$$\omega_{t+1}^* = \arg\ \max\ \left(\hat{\omega}'\mu - \nu_t(\omega, \omega_{t^+}, \lambda) - \frac{\gamma}{2}\omega'\hat{\Sigma}\omega\right)$$

Where $\omega \in \mathbb{R}^N$, $\iota'\omega = 1$

In the mandatory assignment the proposed transaction costs are specified as

$$\nu_t(\omega, \omega_{t^+}, \lambda) = TC(\omega, \omega_{t^+}) = \lambda(\omega - \omega_{t^+})'\Sigma(\omega - \omega_{t^+})$$

To follow the proofs presented in Hautsch & Voigt (2019) we define $\lambda \equiv \frac{\beta}{2}$ where $\beta > 0$ is just a cost parameter like λ.

The optimal portfolio thus takes the form

$$\begin{aligned}\omega_{t+1}^* &= \arg\ \max\ \left(\hat{\omega}'\mu - \frac{\beta}{2}(\omega - \omega_{t^+})'\Sigma(\omega - \omega_{t^+}) - \frac{\gamma}{2}\omega'\hat{\Sigma}\omega\right) \\ &= \arg\ \max\ \omega'\mu^* - \frac{\gamma}{2}\omega'\Sigma^*\omega\end{aligned}$$

Where

$$\Sigma^* = \left(1 + \frac{\beta}{\gamma}\right)\Sigma \text{ and } \mu^* = \mu + \beta\Sigma\omega_{t^+}$$

With these new return parameters, we can derive a closed-form solution for the mean-variance efficient portfolio with the specified transaction costs:

$$\begin{aligned}
\omega^*_{t+1} &= \frac{1}{\gamma}\left(\Sigma^{*-1} - \frac{1}{\iota'\Sigma^{*-1}\iota}\Sigma^{*-1}\iota\iota'\Sigma^{*-1}\right)\mu^* + \frac{1}{\iota'\Sigma^{*-1}\iota}\Sigma^{*-1}\iota \\
&= \frac{1}{\gamma+\beta}\left(\Sigma^{-1} - \frac{1}{\iota'\Sigma^{-1}\iota}\Sigma^{-1}\iota\iota'\Sigma^{-1}\right)(\mu + \beta\Sigma\omega_{t^+}) + \omega^{mvp} \\
&= \omega_{t+1} + \frac{\beta}{\gamma+\beta}(\omega_{t^+} - \omega^{mvp})
\end{aligned}$$

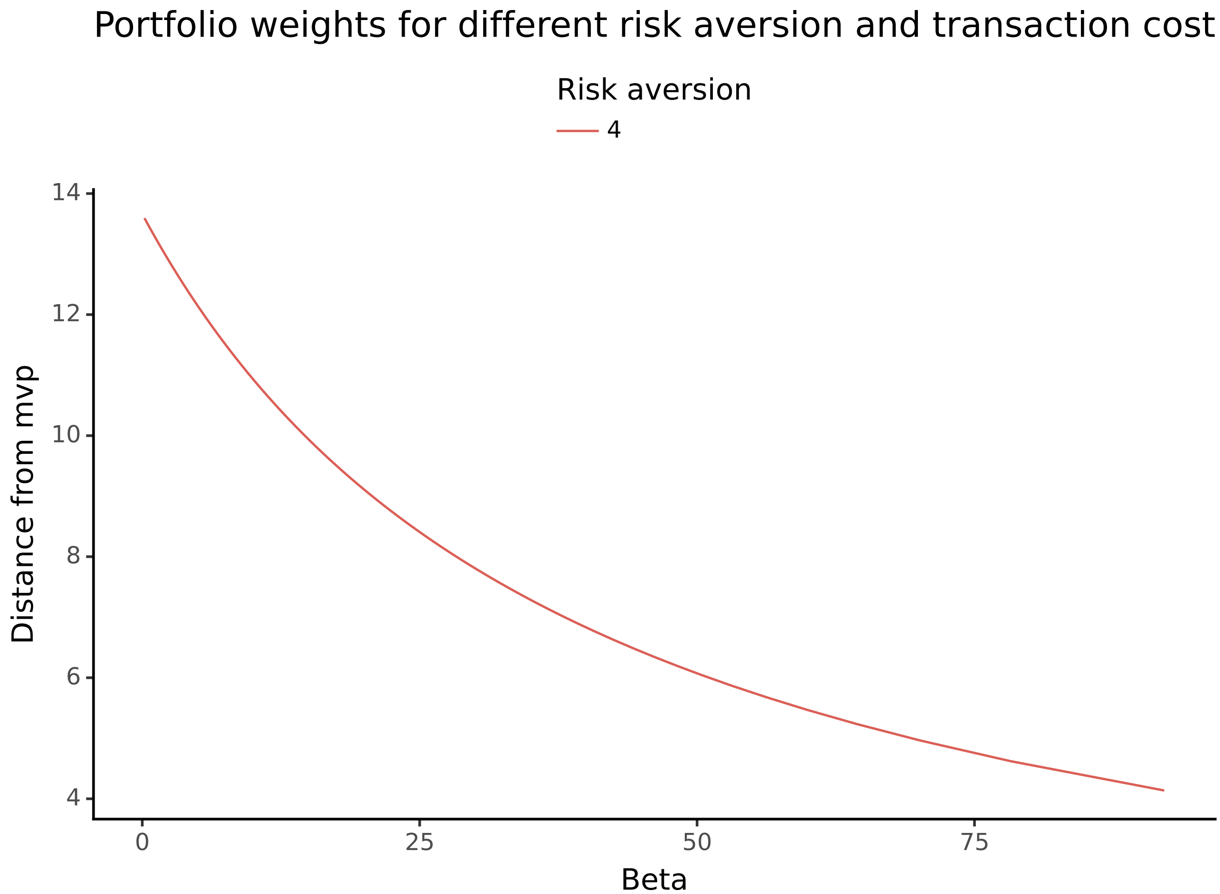
where ω_{t+1} is the efficient portfolio without transaction costs and risk aversion parameter $\gamma + \beta$. $\omega^{mvp} = \frac{1}{\iota'\Sigma^{-1}\iota}\Sigma^{-1}\iota$ is the minimum variance portfolio (mvp).

We see that the optimal weights are a linear combination of the efficient portfolio without transaction costs and the difference between the mvp and the portfolio just before reallocation. The weight $\frac{\beta}{\beta+\gamma}$ only depends on the risk aversion and the cost parameter. Thus, the weight $\frac{\beta}{\beta+\gamma}$ is not affected by Σ. Only ω_{t+1} and ω^{mvp} is affected by Σ. Therefore, the regulatory effect of making transaction costs proportional to volatility is ambiguous and depends on how β and Σ affect each other.

A simpler case discussed in the lectures is when we model exogenous quadratic transactions costs. Here the effect of higher volatility has a clear effect. Periods with high volatility shifts the optimal portfolio allocation towards the global mvp. Thus, assuming quadratic transaction costs not related to volatility, provides a result that intuitively makes sense. However, this type of transaction costs are not very realistic (Hautsch & Voigt 2019)

From a supply/demand point of view, endogenous transaction costs linked to volatility makes sense. In a high volatile environment, investors reallocate their portfolio more frequently to maintain optimal portfolio weights. Economic theory suggests that a higher demand must yield a higher price. Therefore, linking transaction costs to volatility makes sense.

Bullet 2 We now write a function that computes the optimal weight for different values of the transaction cost parameter $\beta \equiv \lambda * 2$. This is done by computing the optimal portfolio weights for rising betas, when we keep the initial allocation (in our case the mvp) constant. The optimal portfolio weights are presented relative to the mvp in the graph below. The "distance from mvp" is measured by the sum of absolute deviations from the mvp to the efficient portfolio.



We see that rising betas shifts the optimal portfolio allocation towards the mvp. The optimal allocation is drawn towards the efficient portfolio without transaction costs but the mvp act as an anchor in the presence of transaction costs. In our case, a rising beta has the same effect, as in the case with exogenous quadratic transactions costs. The rising beta means higher transaction costs, and the optimal portfolio is drawn towards the mvp. **Exercise 3**

The objective of the exercise is to backtest three different portfolios and compare their performance. The portfolios are:

1. Naive portfolio: weights are equal for all companies.
2. Mean-variance portfolio: weights are determined by the mean-variance optimization with a no-short-selling constraint.
3. Hautsch et al. portoflio: weights are theoretically optimal with ex-ante adjustment for transaction costs.

For portfolio 1. the weights are calculated as

$$w_i = 1/N, \ \forall i = 1, 2, ..., N$$

where N is the number of companies.

For portfolio 2. the weights are calculated as

$$w = \arg\min \frac{1}{2} w' \hat{\Sigma} w \text{ s.t.} \sum_{i=0}^{N} w_i = 1 w_i \geq 0, \ \forall i = 1, 2, ..., N$$

where $\hat{\Sigma}$ is some estimated covariance matrix of returns. The estimator for the covariance matrix will be explained later.

For portfolio 3. the weights are calculated as explained in the previous exercise.

Throughout the exercise we will use a transaction cost of $200bp$ and risk-aversion of $\gamma = 4$.

Returning to the estimator of returns vector μ and the covariance matrix Σ. We use a rather simple sample average of past returns as our estimator for the returns while we will use the Ledoit-Wolf shrinkage estimator which is given by:

$$\hat{\Sigma} = \alpha \hat{\Sigma}_{\text{target}} + (1 - \alpha) \hat{\Sigma}_{\text{sample}}$$

where *alpha* is our linear shrinkage parameter, $\hat{\Sigma}_{\text{target}}$ is the target matrix and $\hat{\Sigma}_{\text{sample}}$ is the sample covariance matrix. The target matrix is given by:

$$\hat{\Sigma}_{\text{target}} = I_N \left(\frac{1}{N} \sum_{i=1}^{N} var(r_{it}) \right)$$

where I_N is the identity matrix of size $N \times N$ and $var(r_{it})$ is the variance of the returns of company i. The linear shrinkage parameter is given by: $\alpha = \frac{\hat{\pi}}{\hat{\gamma}}$, where $\hat{\pi}$ is the average pairwise sample covariance and $\hat{\gamma}$ is Frobenuis norm of the matrix $\hat{\Sigma}_{\text{sample}} - \hat{\Sigma}_{\text{target}}$.

In our backtest, we slice the dataset into 600 months of accessible data and 120 months of data which we use as out-of-sample data for testing the performance for each portfolio. We choose to test our portfolios on the last 120 months of data taking computation time into account and at the same time we want to have a reasonable amount of data to test the performance of the portfolios.

We assume that the portfolios are equally weighted and thus equal to the naive portfolio - only playing a role for portfolio 3 which takes the transaction cost of moving from one portfolio to another into account. Coming into 2011 (starting period), the optimal portfolios are created on based on the estimates of $\hat{\Sigma}$ and $\hat{\mu}$ which are estimated on all available data up to the period. After having the optimal portfolios, we use the returns of the current month to calculate the performance of the portfolios. Likewise in the following month, the portfolio weigths are updated using the new estimates of $\hat{\Sigma}$ and $\hat{\mu}$. This procedure is repeated until the end of dataset.

The results of our backtest of the portfolios are presented in the table below. Here, we see from the mean return that the mean-variance portfolio that does not take transaction cost into account performs very poorly with a negative mean return. This can be attributed to the large turnover in the portfolio. However, it is the portfolio with the lowest standard deviation which is aligned with what you might expect. The naive portfolio and third portfolio have very similar properites though the turnover in the naive portfolio is relatively small.

Table 1

Strategy	Mean	Standard Deviation	Sharpe Ratio	Turnover
Naive	11.87%	14.63%	0.811	0.04%
MV	-1.64%	11.51%	nan	160.41%
MV (TC)	11.86%	14.62%	0.811	0.06%

Discussion on backtest: Our backtest strategy only uses the data that is available at the time of the portfolio creation. This means that we do not use any future data to estimate the optimal portfolios, and therefor no look-ahead bias is present in our backtest. This makes our backtest realistic and reliable. However, the dataset which we use to test is obviously not true out-of-sample data and the results will be biased based on the chosen period of testing. For instance, should the drawdown of the market around the Great-Financial Crisis be in the testing period or not, and thus could certain portfolios be favored over others based on the testing period. Alternatively, we could generate a dataset from random drawings based on our estimated $\hat{\Sigma}$ and $\hat{\mu}$ and use this as out-of-sample data. This type of backtest would be more robust to the choice of testing period. However, it would be based on estimator of $\hat{\Sigma}$ and $\hat{\mu}$ which are not the true values and thus not resembling true data.