Advanced Macroeconomics: Business Cycles

Lecture 7: Consumption under Uncertainty

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Agenda

- The consumption Euler equation and its drawbacks
- Consumption under uncertainty: Precautionary saving
- Rule-of-thumb households
- The connection to endogenous borrowing constraints (collateralized borrowing)

Introduction

- In the RBC and NK models studied so far (and many other macroeconomic models), a key building block is the Euler equation for consumption:

$$U'\left(C_t\right) = \beta\left(1 + R_t\right)\mathrm{E}_t\left[U'\left(C_{t+1}\right)\right] \tag{1}$$

- We obtain this equation by combining the representative household's first-order conditions for consumption and saving.
- As such, this equation tells us how the household seeks to smooth its consumption over time.
  - It also tells us how consumption/saving choices are affected by the interest rate, the discount factor, and the intertemporal elasticity of substitution (a feature of $U\left(\cdot\right)$).
- The consumption Euler equation is associated with the permanent income hypothesis / life cycle model of Friedman and Modigliani, dating back to the 1950's, and is familiar from undergraduate macro.

Introduction

- What's wrong with the Euler equation?
- Well...
  - Does not feature credit/liquidity constraints (though these can easily be added, as we shall see).
  - Abstracts from individual income/consumption risk and (when log-linearized) from uncertainty.
  - Aggregate consumption data appears to react more "smoothly" to income and interest rate shocks than the Euler equation implies (motivation for habits in consumption).
  - At the individual level, marginal propensities to consume (MPCs) out of transitory income shocks are much higher than implied by the permanent income hypothesis (PIH).
  - Further empirical studies point to various problems - for example, implied interest rates from estimated Euler equations do not track actual interest rates well.

Introduction

- For these reasons, a massive literature has suggested improvements or alternatives (see last slides).
- Today we will review some of these theories.
- In this course, you have seen / will see some of them in action in other contexts:
  - Collateral constraints will be useful to understand the impact of financial factors on the business cycle.
  - Rule-of-thumb behavior may help our baseline models account for the effects of government spending shocks.
- Important to state, however, that the PIH still offers a number of important insights; probably applies to a number of households (or certain aspects/life phases of many households); and is therefore still relevant in many settings.

Consumption under uncertainty

- In the baseline RBC and NK models considered so far, we have implicitly assumed the existence of *complete financial markets*.
- This means that a set of so-called Arrow-Debreu securities exist, allowing each household to purchase insurance against every possible future state of the world.
- By trading such securities, households obtain *perfect risk-sharing*:
    - No *idiosyncratic* (i.e., household-specific) income risk or uncertainty.
    - Only *aggregate* uncertainty relevant (e.g., a good or bad technology shock).
    - Means that "if I wake up poor, we all wake up poor".
    - Empirical studies show that aggregate risk is much smaller than idiosyncratic risk (effectively, GDP varies by less than many people's individual income).

Consumption under uncertainty

- A large literature studies what happens under incomplete markets and idiosyncratic income risk (e.g., Aiyagari, 1994).
- Let's see how the consumption/saving choices of a household unfold when there is uncertainty about income (and, possibly, the interest rate).
- We will study various types of utility functions in order to gain various insights.
- Bottom line: uncertainty matters!

Consumption under uncertainty

- Consider a household facing the problem:

$$\begin{aligned} Max\ U &= \mathrm{E}_0 \left[\sum_{t=0}^{\infty} \beta^t u(c_t) \mid I_0 \right] \\ s.t.\ a_{t+1} &= R_{t+1} \left(a_t + \widetilde{y}_t - c_t \right), \end{aligned}$$

  where a_t denotes assets (or savings, or the negative of debt), I_0 is the information set at period 0, and $\widetilde{y}_t$ indicates that labor income y_t is stochastic.
- In particular, we assume that labor income is $i.i.d.$ with mean zero - income is entirely unpredictable.
- We can derive the implied Euler equation (*Exercise: check this!*):

$$u'(c_t) = \beta R_{t+1} \mathrm{E}_t u'(c_{t+1}). \tag{2}$$

- The usual Euler equation (but the E_t now covers also individual income risk).

The random walk hypothesis

- To be specific, assume the household has quadratic utility in consumption:

$$u\left(c_t\right) = \alpha c_t - \frac{\gamma c_t^2}{2},\ \alpha > 0,\ \gamma > 0. \tag{3}$$

- In this case, we have *linear* marginal utility:

$$u'\left(c_t\right) = \alpha - \gamma c_t.$$

- Further, consider for simplicity the special case when R_t is constant and equal to $1/\beta$, i.e., $\beta R = 1$.
- We then obtain from (2):

$$\begin{aligned} u'\left(c_t\right) &= \mathrm{E}_t u'\left(c_{t+1}\right) \Leftrightarrow & (4) \\ \left(\alpha - \gamma c_t\right) &= \mathrm{E}_t\left(\alpha - \gamma c_{t+1}\right) \Leftrightarrow & \\ c_t &= \mathrm{E}_t c_{t+1}. & (5) \end{aligned}$$

The random walk hypothesis

- *In expectation*, the household wants to perfectly smooth consumption.
- A straightforward implication is that:

$$\mathrm{E}_t \left[c_{t+1} - c_t \right] = 0 \tag{6}$$

- The household's expected consumption growth in optimum is zero.
    - In other words, consumption is a *random walk*! Changes in consumption cannot be predicted.

The random walk hypothesis

- Why? We can show that the change in the household's consumption level between two periods is a function of *changes in expected future income (i.e., revised expectations)*:

$$c_{t+1} - c_t = (R - 1) \sum_{s=t+1}^{\infty} R^{-(s-t)} \left[\mathrm{E}_{t+1} \widetilde{y}_s - \mathrm{E}_t \widetilde{y}_s \right].$$

  (this can be done by writing up the level of consumption in periods t and $t + 1$, and subtracting one from the other - see next slide).
- In other words, the household only changes its consumption when news about future income arrive between periods t and $t + 1$.
- Obviously, by definition this news was unknown at time t, and thus unpredictable.

Certainty equivalence

- Let's solve for the level of consumption today.
- By solving forward the budget constraint ($a_{t+1} = R_{t+1}\left(a_t + \widetilde{y}_t - c_t\right)$), letting $t \to \infty$, and ruling out explosive paths for asset holdings (No Ponzi condition), we obtain the *intertemporal* budget:

$$\sum_{t=0}^{\infty} R^{-t} c_t = a_0 + \sum_{t=0}^{\infty} R^{-t} \widetilde{y}_t .$$

- We can take expectations and use the fact that consumption is a random walk, implying that $\mathrm{E}_0 c_t = c_0$, to rewrite this as:

$$\begin{aligned} c_0 \frac{1}{1-\frac{1}{R}} &= a_0 + \mathrm{E}_0 \sum_{t=0}^{\infty} R^{-t} \widetilde{y}_t \Leftrightarrow \\ c_0 &= (1-\beta)\left[\underbrace{a_0 + \mathrm{E}_0 \sum_{t=0}^{\infty} R^{-t} \widetilde{y}_t}_{\text{expected permanent income}}\right], \end{aligned} \tag{7}$$

where we have used the formula for the sum of an infinite series ($\sum_{t=0}^{\infty} a^t = \frac{1}{1-a}$, letting $a = \frac{1}{R}$ in this case), and that $\beta = 1/R$.

Certainty equivalence

- The household thus behaves in accordance with the permanent income hypothesis, consuming in each period a constant fraction of *expected* lifetime income.
- Importantly, the household behaves *as if* the expected future income was certain (even though it is not!)
- In other words, the household exhibits *certainty equivalence* - the presence of uncertainty does not affect consumption.

Certainty equivalence

- This result can be traced back to our assumption of quadratic utility, and thus, linear marginal utility.
- Specifically, this assumption allowed us to go from (4) to (5):

$$u'(c_t) = \mathsf{E}_t u'(c_{t+1}) \Leftrightarrow c_t = \mathsf{E}_t c_{t+1}.$$

- In words, with quadratic utility, the expected marginal utility of consumption equals the marginal utility of expected consumption.
- As we will see later, the result breaks down when marginal utility is not linear.

The random walk hypothesis

- First, let's see what the data has to say about the random walk hypothesis (6), which implies that:

$$c_{t+1} - c_t = \varepsilon_{t+1}, \ \varepsilon_{t+1} \sim i.i.d$$

- This is a testable implication: If *something* known at period t predicts the *change* in consumption between periods t and $t+1$, the result breaks down!
- In a widely cited paper, Hall (1978) showed that lagged income indeed **does not** predict consumption changes - the theory seems to hold.
- Subsequent studies have found that:
    - The interest rate should be allowed to affect consumption growth (would show up in (6) if we had not assumed $\beta R = 1$).
    - Consumption growth **does** respond to *expected* income growth (excess sensitivity), and responds too little to *unexpected* income growth (excess smoothness).

The random walk hypothesis: Rule-of-thumb households

hand to mouth

- One famous extension of Hall's (1978) work is the paper by Campbell and Mankiw (1989).
- They assume that a fraction λ of total consumption is done by "hand-to-mouth" consumers, who simply consume their disposable income each period.
- This modifies (6) to:

$$c_{t+1} - c_t = \lambda \left(y_{t+1} - y_t \right) + \left(1 - \lambda \right) \varepsilon_{t+1}. \tag{8}$$

- However, $\left(y_{t+1} - y_t \right)$ and ε_{t+1} are potentially correlated, so (8) cannot be estimated directly.
- Can use IV estimation if we can find instruments that are correlated with income growth but uncorrelated with consumption growth.

The random walk hypothesis: Rule-of-thumb households

- Campbell and Mankiw (1989) use lagged income growth and other lagged variables, which are orthogonal to consumption growth according to the RW hypothesis.
- Using this approach, they find that income changes are strongly significant predictors of consumption growth.
- Their estimates of λ are around $0.4 - 0.5$, suggesting that almost half of consumers are rule-of-thumb households (while the rest obey the PIH).
- As we will see in a few weeks, inserting rule-of-thumb households into the NK model allows it to better account for the effects of fiscal policy.
    - More recently, Kaplan *et al*. (2014) have documented that rule-of-thumb households are not necessarily poor, but just illiquid ("the wealthy hand-to-mouth").

Consumption under uncertainty: Precautionary saving

- Let's now revisit the certainty equivalence result and see how it changes with more realistic utility functions than quadratic utility.
- Suppose the household solves the same problem as before, but now has CRRA utility:

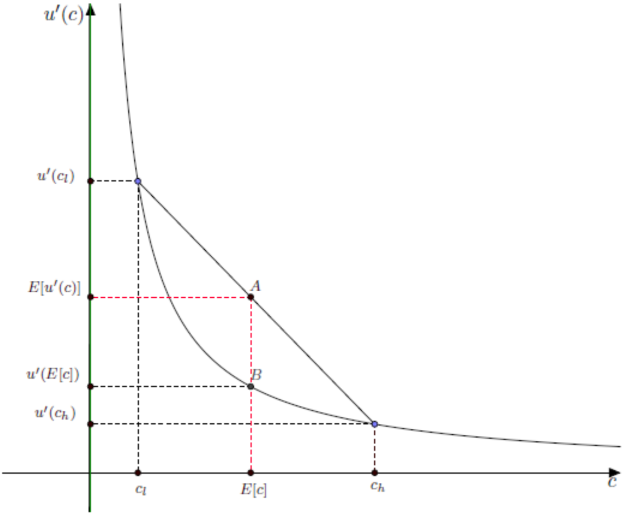
$$u(c_t) = \frac{c_t^{1-\theta}}{1-\theta},\ \theta > 0,\ \theta \neq 1. \tag{9}$$

- Marginal utility is now not linear, but convex:

$$u'(c_t) = c_t^{-\theta}.$$

- As usual, the second derivative of $u'(\cdot)$–i.e., $u'''(\cdot)$– is a measure of the convexity of $u'(\cdot)$.
- To see this, plot $u'(c_t)$ as a function of c_t:

Consumption under uncertainty: Precautionary saving



Consumption under uncertainty: Precautionary saving

- Suppose consumption fluctuates around a mean of c: in each period, it may be c_l or c_h with probability $\frac{1}{2}$.
- Point B denotes the *marginal utility of expected consumption*; $u'(\mathrm{E}[c])$.
- Point A denotes the *expected marginal utility of consumption*; $\mathrm{E}[u'(c)]$; since it is the average of $u'(c_l)$ and $u'(c_h)$.
- Importantly, with CRRA utility, these two points are not identical – in contrast to the case of quadratic utility considered earlier. Specifically (by Jensen's inequality):

$$\mathrm{E}\left[u'(c)\right] > u'(\mathrm{E}[c]). \tag{10}$$

Consumption under uncertainty: Precautionary saving

- Why is this important? Recall the Euler equation when we assumed $\beta R = 1$, i.e., (4):
$$u'(c_t) = \mathrm{E}_t u'(c_{t+1}).$$
- Under quadratic utility, we saw that this implied $c_t = \mathrm{E}_t c_{t+1}$. But if this was to hold, then by (10) we would have:
$$u'(c_t) = u'(\mathrm{E}_t c_{t+1}) < \mathrm{E}\left[u'(c_{t+1})\right].$$
- If marginal utilities of consumption are unequal, it means that saving an additional amount today and consuming it tomorrow, where marginal utility is higher, would improve the household's utility!

Consumption under uncertainty: Precautionary saving

- In other words, $c_t = \mathrm{E}_t c_{t+1}$ cannot be optimal, and it is instead optimal to save more today when future income is uncertain.
- The additional saving is called *precautionary* saving: The household attempts to self-insure against low future income.
- The amount of precautionary saving is tied to the convexity of $u'(\cdot)$, which, as we saw above, is determined by $u'''(\cdot)$.
- In fact, this determines the so-called *coefficient of relative prudence*, defined as $-\frac{u'''(c)}{u''(c)}c$.
    - With CRRA utility as assumed above, this becomes (*check it!*): $\frac{\theta(1+\theta)c^{-\theta-2}}{\theta c^{-(1+\theta)}}c = 1+\theta > 0$.
    - With quadratic utility, it is 0 - there is no precautionary saving. In this case, $u'(\cdot)$ is just a straight line, so points A and B coincide.

Consumption under uncertainty: Precautionary saving

- Another way to see that precautionary savings arise is to take a second-order Taylor expansion of (4) around c_t: (*Exercise: do it!*)

$$\begin{aligned} u'(c_t) &= \mathrm{E}_t u'(c_{t+1}) \\ &\approx \mathrm{E}_t u'(c_t) + u''(c_t)\,\mathrm{E}_t(c_{t+1}-c_t) + \frac{1}{2}u'''(c_t)\,\mathrm{E}_t(c_{t+1}-c_t)^2 \end{aligned}$$

$$u'(c_t) = u'(c_t) + u''(c_t)\,\mathrm{E}_t(c_{t+1}-c_t) + \frac{1}{2}u'''(c_t)\,\mathrm{E}_t(c_{t+1}-c_t)^2 \Leftrightarrow$$

$$\mathrm{E}_t\left(\frac{c_{t+1}-c_t}{c_t}\right) = \underbrace{-\frac{u'''(c_t)\,c_t}{u''(c_t)}}_{\textit{prudence}}\frac{1}{2}\mathrm{E}_t\left(\frac{c_{t+1}-c_t}{c_t}\right)^2. \qquad (11)$$

- This shows that consumption *growth* is an increasing function of the coefficient of relative prudence.
  - Higher consumption *growth* means less consumption today, and thus, higher (precautionary) saving.

Precautionary saving: a word of caution

- Observe that precautionary saving arises only when marginal utility of consumption is non-linear and convex.
- But keep in mind: When we solve a DSGE model (or have Dynare do it), we (log)-linearize the equilibrium conditions.
  - If the Euler equation contains "$c_t^{-\theta}$", we log-linearize it and obtain "$-\theta\widehat{c}_t$".
- This means that linearized models do not feature precautionary saving!
  - It also means that estimating log-linearized Euler equations, as many authors have done (cf. above), suffers from omitted variable bias (Carroll, 2001: "Death to the Log-Linearized Consumption Euler Equation").
- If we believe that precautionary saving is empirically relevant and want our DSGE model to reflect this, we need to:
  - Take a higher-order Taylor approximation (second order will do, as above),
  - Solve the model with a "global" numerical solution method.

Precautionary saving: how to learn more

- Precautionary savings instead play an important role in most models with heterogeneous agents ("HANK" models).
- Typically, an integral part of these models is that financial markets are incomplete; which introduces idiosyncratic risk, thus changing the problem of the household.
- HANK models are the topic of another elective course at this department (*Advanced Macro: Heterogeneous-agents models*), and will therefore not be covered in this course.

Liquidity constraints and the buffer-stock model

- Let's now consider a combination of the "buffer-stock" model of consumption of Carroll (1997) with liquidity constraints as in Deaton (1991).
    - This model takes precautionary savings seriously.
- We will assume that the household is impatient, meaning that $\beta < \frac{1}{R}$.
- As before, the household starts out with some level of assets. Its impatience would induce it to run down asset holdings to increase consumption.
- However, due to precautionary motives, the household realizes that, all else equal, this increases the probability of hitting the liquidity constraint in the future.
    - In isolation, this induces the household to save.

Liquidity constraints and the buffer-stock model

- The problem is largely the same as above:

$$\begin{aligned} Max\ U &= \mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t) \\ s.t.\ a_{t+1} &= R(a_t + \widetilde{y}_t - c_t). \end{aligned}$$

- However, the liquidity constraint now restricts assets to stay non-negative (no access to credit):

$$a_t \geq 0.$$

- Useful to rewrite the problem in terms of "cash holdings" or "liquidity", defined as $x_t = a_t + \widetilde{y}_t$. The constraints are then:

$$x_{t+1} = R(x_t - c_t) + \widetilde{y}_{t+1}, \tag{12}$$

$$c_t \leq x_t. \tag{13}$$

- The latter follows from the fact that if the household is liquidity constrained, consumption cannot exceed cash holdings.

Liquidity constraints and the buffer-stock model

- We can find the Euler equation by combining the first-order conditions for c_t and x_{t+1}, which read as follows; letting λ_t and μ_t denote the Lagrange multipliers on (12) and (13) (*Exercise: do this, and derive (14)!)*:

$$c_t: \; u'(c_t) - \lambda_t R - \mu_t = 0,$$

$$x_{t+1}: \; -\lambda_t + \beta R \mathrm{E}_t \lambda_{t+1} + \beta \mathrm{E}_t \mu_{t+1} = 0.$$

- Combine these to obtain:

$$u'(c_t) = \beta R \mathrm{E}_t u'(c_{t+1}) + \mu_t, \tag{14}$$

  where $\mu_t > 0$ if and only if the liquidity constraint binds.
- However, even when the constraint does *not* bind, the household engages in precautionary savings, as the constraint may become binding in the future!

Liquidity constraints and the buffer-stock model

- It is generally not possible to solve this kind of model analytically, but various numerical methods exist that do the trick.
- Once solved, the model yields a number of insights (see also graphs on next slides):
  - For very low levels of x_t, the borrowing constraint binds, and consumption changes *almost* 1-for-1 with income.
  - For intermediate levels of x_t, the constraint sometimes binds (i.e., a_t sometimes reaches 0), but the household generally seeks to maintain a positive level of assets - its "buffer-stock" savings!
  - At high levels of x_t, the constraint is unlikely to bind, and the household behaves more like certainty-equivalence consumers.
- As discussed at length by Carroll (1997), this enables the buffer-stock model to account for a number of empirical findings that seem at odds with the PIH.

Fig. 1 from Deaton (1991)

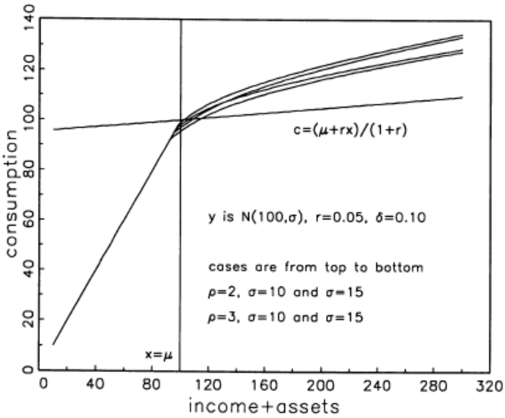


FIGURE 1.—Consumption functions for alternative utility functions and income dispersions.

Fig. 2 from Deaton (1991)

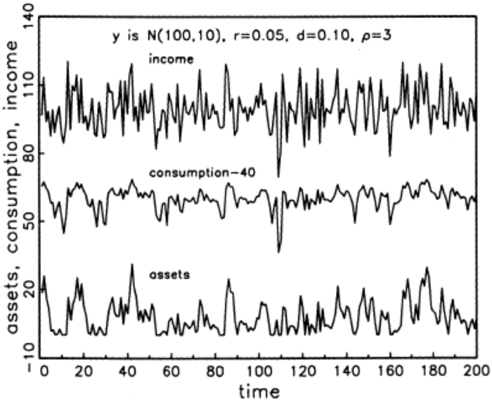


FIGURE 2.—Simulations of income, consumption, and assets, with white noise income.

Fig. 3 from Carroll (1997)

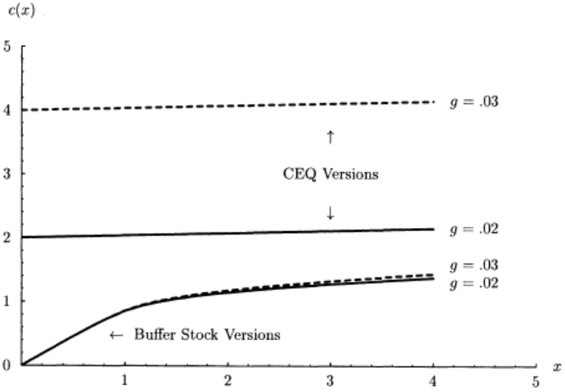


FIGURE III
Marginal Propensity to Consume out of Human Wealth

The marginal propensity to consume (MPC)

- As discussed by Carroll (1997), the buffer-stock model implies a much higher MPC than PIH models.
  - In the latter, the MPC out of transitory current income shocks is around $0.01 - 0.02$ within-quarter.
- Instead, models with a buffer-stock foundation can generate much higher MPC's, depending on the calibration.
  - This enables these models to match empirical studies, which tend to find MPC's around $0.15 - 0.25$ within-quarter (e.g., Parker *et al.* (2013) for the US; or Fagereng *et al.* (2021) for Norway).
  - Importantly, the MPC out of *future* (unrealized) income is instead smaller due to the uncertainty and the risk of hitting future borrowing constraints (previous slide).
- This is a key characteristic of HANK models.
  - Notably, Kaplan *et al.* (2018) show that the transmission of monetary policy is quite different in a HANK model with high MPC's, as compared to the standard NK model.

Connection to endogenous borrowing constraints

- The liquidity constraint considered above tied the maximum borrowing of the household to some fixed (exogenous) amount.
- In reality, many households obtain loans by pledging as collateral their most important financial asset: their house.
  - If a household has home equity, it can borrow against it to finance non-durable consumption.
- This implies that the credit limit faced by a household is not fixed, but time-varying and endogenous.
  - Specifically, if the house price increases, so does the credit availability of home owners!
  - This is a central point of the paper by Iacoviello (2005), which we will study next week.

Connection to endogenous borrowing constraints

- In the model above, the borrowing (or liquidity) constraint was simply:

$$a_t \geq 0.$$

- With collateralized borrowing, as you will see next week, the household instead faces a borrowing constraint of the form:

$$B_t \leq \theta \frac{\mathrm{E}_t Q_{t+1} H_t}{1 + R_t}, \tag{15}$$

  where θ may be interpreted as the *loan-to-value ratio* faced by households, who borrow against the expected discounted value of their current house.
- With a collateral constraint, we still arrive at an Euler equation like (14) - but now μ_t may move for different reasons than before!

Connection to endogenous borrowing constraints

- Unlike before, the household's borrowing capacity is endogenous and time-varying.
- In particular, an increase in expected future house prices drives up the borrowing limit (cf. 15), allowing the household to obtain more credit.
    - In other words, the multiplier μ_t declines, reflecting a relaxation of the borrowing constraint.
- This is a potentially powerful financial amplification mechanism of macroeconomic fluctuations, as we will discuss next week.

Conclusion

- The Euler equation (especially in linearized form) has several shortcomings when confronted with the data.
- Under individual income risk, precautionary saving plays an important role - but does not feature in linearized DSGE models.
- Credit or liquidity constraints are crucial in order to make models of consumption fit the data (both micro and macro).
- For this reason, some version of credit-constrained households (or rule-of-thumb households) are now routinely featured in business cycle models.
    - Typically alongside standard PIH households.

Reading list

- Lecture notes by Gourinchas.
- Carroll (1997): The buffer-stock model of consumption.

Additional Reading

- Campbell and Deaton (1991): Aggregate consumption is "too smooth" to fit the PIH.
- Campbell and Mankiw (1989): Rule-of-thumb households important.
- Deaton (1991): Liquidity constraints matter.
- Fuhrer (2000): Consumption habits perform well in macroeconomic models.
- Johnson, Parker and Souleles (2006) and Jappelli and Pistaferri (2014) (among many others): At the household level, MPCs out of transitory income are substantially larger than what the PIH implies.